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Models of resource allocation optimization when solving the control problems in organizational systems

V Menshikh, A Samorokovskiy, O Avsentev

Voronezh Institute of the Ministry of Internal Affairs of Russia, Voronezh, Russia.

E-mail: samorokovskii_an@mail.ru

Abstract. The mathematical model of optimizing the allocation of resources to reduce the time for management decisions and algorithms to solve the general problem of resource allocation. The optimization problem of choice of resources in organizational systems in order to reduce the total execution time of a job is solved. This problem is a complex three-level combinatorial problem, for the solving of which it is necessary to implement the solution to several specific problems: to estimate the duration of performing each action, depending on the number of performers within the group that performs this action; to estimate the total execution time of all actions depending on the quantitative composition of groups of performers; to find such a distribution of the existing resource of performers in groups to minimize the total execution time of all actions. In addition, algorithms to solve the general problem of resource allocation are proposed.

1. Introduction
One of the control optimization methods in organizational systems is optimization of operation of the executive subsystem [1]. This subsystem, when accomplishing the control tasks, performs some set of actions in order to form control actions on the managed system. These actions are performed in a certain order; execution of separate consecutive actions may depend on previous actions. Separate actions can be performed in parallel.

The execution time of each action depends on the amount of the resource allocated (in general, nonlinearly), under which in the organizational systems, as a rule, decision-makers act. Taking into account this circumstance, we will further assume that the resource is of a discrete nature [2].

In order to reduce the performance time of all actions, various resource allocation options may be used. With this purpose, it is necessary to determine the optimal resource allocation choice before the beginning of performance of the set of actions. As a criterion for optimization is the a priori estimated time of implementation of the whole set of actions for the formation of the control action [3–6].

The purpose of the work is development of a mathematical model for resource allocation optimization in order to reduce the time for taking managerial decisions and algorithms for solving the general resource allocation problem.

2. Problem description and formalization of the resource allocation option selection optimization problem

Let us consider the resource allocation option optimization problem for reduction of the total time for performance of a certain task.

It will be considered that the task implies execution of some set of partially ordered acts by groups of performers. And each action is assigned to a certain group of performers. The action, in turn, is a set of partially ordered operations (subactions), each can be performed by any performer from the respective group. Hence, the action performance time depends on the number of performers.

By resource allocation we shall mean the increase of the number of performers in the groups. This may lead to a reduction of the time of action performance by the groups, to which performers are added, and hence the total performance time of the entire set of actions. The mentioned problem is a complex three-level combinatorial problem, whose evolving required solving of the following specific problems:

1) assessment of the duration of performance of each action depending on the number of performers in the group that performs this action;

2) assessment of the total performance time of all actions depending on the quantitative composition of the group of performers;

3) finding such allocation of available resource of performers by the groups that minimizes the total performance time of all actions [7–10].

Let us formalize this problem. Let us write $\Delta = \{D_1, D_2, \ldots, D_n\}$ — a set of actions (n — number of actions). On set Δ, we will determine partial order relation $E \in \Delta \times \Delta$ such that (D_i, D_j), if action D_i must precede action D_j. It will be considered that for each arbitrary action $D \in \Delta$ a group of performers is set $G(D) \in \Theta = \{G_1, \ldots, G_m\}$ (m — number of groups of performers, $m \leq n$) and action performance time $T(D)$.

Each action $D_i \in \Delta$ supposes performance of set of operations $D_i = \{d_i^1, \ldots, d_i^{ki}\}$ (k_i — number of operations when performing actions D_i). Each operation d_i^j, in turn, is characterized by its performance time t_i^j.

Assume that on set D_i is also set partial order relation $E_i \in D_i \times D_i$ such that $(d_i^j, d_i^k) \in E_i$, if operation d_i^j must precede operation d_i^k.

For each resource allocation option, each group G_j is characterized by available number of performers $N(G_j)$.

3. Assessment of duration of performance of each action.

Initially, let us consider the problems of assessment of performance duration of the entire aggregate of actions for the set numbers of performers in each group $N_1, N_2, \ldots, N_m$.

At the first stage it is necessary to assess duration T_i of performance of each action D_i. Obviously, T_i is determined by the values of duration of operations t_i^j, partial order relation E_i and number of performers N_i in the group that performs this action. Considering that t_i^j and E_i will not subsequently change, and the number of performers during resource allocation may vary, let us consider $T_i = f(N_i)$.

Let us address the problem of assessment of values $T_i = f(N_i)$. Let us represent the process of performance of operations of action D_i as a sequence of steps, such that each step is a performance of one operation by one performer. Let us write the performers of the group $G \in \Theta$, performing action D_i. Write

$$X_{js}^k = \begin{cases} 1, \text{ if at the } k\text{-th step, performer } g_s \text{ performs operation } d_i^j; \\ 0, \text{ if otherwise.} \end{cases}$$

Let us address the description of functional dependencies between the operations and the performers. Write A_i^j — the moment of beginning of performing operation d_i^j, then $A_i^j + t_i^j$ — the moment of its ending. The action ends after the last operations performance step. Hence,

$$T_i = \max(A_i^j(X) + t_i^j)X_{js}^k, \text{ where } i = 1, ..., n. \tag{1}$$

Let us set partial order relation E_i by matrix $B_i = (b_{st}^i), s, t = 1, ..., k_j$, where

$$b_{st}^i = \begin{cases} 1, & \text{if action } d_i^s \text{ is preceded by action } d_i^t; \\ 0, & \text{if otherwise.} \end{cases}$$

Let us describe the constraints for the sequence of performance of steps:

$$b_{st}^i(A_i^S(X) + t_i^S) \le A_i^t(X) \tag{2}$$

— a condition that operation d_i^S must end before the beginning of operation d_i^t;

$$A_i^S((X) + t_i^S)X_{sj}^{k-1}X_{tj}^k \le A_i^t X_{sj}^{k-1} X_{tj}^k \tag{3}$$

— a condition that if on step $k - 1$ is performed operation d_i^S, and on the k-th step — operation d_i^t, then d_i^S must end before the beginning of d_i^t;

$$X_{sj}^k X_{tj}^k = 0, s \ne t \tag{4}$$

— a condition that at each step only one operation is performed.

$$\sum_{j=1}^{ki} \sum_{k=1}^{ki} X_{sj}^k = 1 \tag{5}$$

— a condition that each operation is performed at one step only.

The general problem of assessment of performance time for action D_i looks like a problem of nonlinear Boolean programming of finding X^*, minimizing (1) under constraints (2)–(5).

4. Assessment of the total time of performance of all actions

Value T is determined by the values of durations T_i of certain actions, defined at the previous stage, as well as by partial order relation E and distribution of the groups of performers by actions [15, 16, 17, 18].

Let us again assume that performance of the actions is a sequence of steps such that each step is performance of one action by a respective group of performers. Write

$$Y_i^{km} = \begin{cases} 1, & \text{if at } k \text{ step group of performers } G_m \text{ fulfills action } D_i, \\ 0, & \text{if otherwise.} \end{cases}$$

$$P_i^m = \begin{cases} 1, & \text{if action } D_i \text{ is fulfilled by group of performers } G_m, \\ 0, & \text{if otherwise.} \end{cases}$$

$A(Y)$ — moment of the beginning of performing action D_i.

The entire aggregate of actions ends when the last action ends, i.e.

$$T = \max(A_i(Y) + T_i), \text{where } i = 1, ..., n. \tag{6}$$

Let us set partial order relation E by matrix $B = (b_{st}), s, t, ..., k_i$, where

$$b_{st} = \begin{cases} 1, & \text{if action } D_s \text{ precedes action } D_t, \\ 0, & \text{if otherwise.} \end{cases}$$

By analogy with the previous stage, let us determine the sequence of actions

$$b_{st}(A_s(Y) + T_s) \leq A_t(Y) \tag{7}$$

$$(A(Y) + T_s)Y_s^{(k-1)m}Y_t^{km} \leq A_t(Y), \text{where } Y_s^{(k-1)m}Y_t^k = 0, s \neq t, \tag{8}$$

$$\sum_{k=1}^{p}\sum_{s=1}^{p} Y_t^{km} = 1. \tag{9}$$

Moreover, unambiguity of setting the group of performers for each action is determined by condition

$$\sum_{k=1}^{p} Y_t^{km} = 1. \tag{10}$$

5. Allocation of the resource of performers

In order to minimize the total performance time of all actions.

Write $Z = \{Z_1, ..., Z_m\}$ — an option of allocation of the resource of performers by groups Z_j, — resource allocated to group G_j.

Then, the performance time of action D_i, which is performed by group G_j is defined as $T_i = T_i(X, Z)$, where T_i — a function describing the solution of problem (1)–(6).

The total performance time for all actions is defined as $T = T(Y, T_i(X, Z))$, where T — a function describing the result of solving problem (1)–(6) at the second stage [19,20,21].

For values $Z = \{Z_1, ..., Z_m\}$, such that $Z_i \in \tilde{Z}$, where $\tilde{Z}$ — a set of integers, the following constraints shall be met

$$\sum_{i=1}^{m} Z_i \leq \hat{Z}, \tag{11}$$

$$Z_i \geq 0. \tag{12}$$

Thus, the general resource allocation problem looks like

Find

$$(X^*, Y^*, Z^*) = \underset{X,Y,Z}{Argmin}\, T(Y, T_i(X, Z))$$

under constraints (2)–(5), (7)–(12).

6. Algorithms of solving the general resource allocation problem

In order to construct an algorithm for finding the optimal option of resource allocation, it is necessary to determine groups G_i, which perform their actions D_i over maximum time $T(D)$. It is conditioned by the fact that the use of the resource with maximum action performance duration leads to a more considerable reduction of the action performance time. $t_i^j(M_i)$ — duration of operation when adding M number of performers. It is necessary to find the maximum number of performers $M_i^{\max}$, which is appropriate to be added to groups G_i, for maximum reduction of the performance time of all actions $T(D)$.

If the number of resource units is small, then the problem can be solved by complete enumeration. In other cases, only approximate solution can be found. Within this work, we shall present a general description of two algorithms without their detailing [1].

1) At the first stage, using an algorithm falling within the class of "greedy algorithms", we will find the approximate solution for using the allocated resource, which allows us to maximally use available resource $Z^{\max}$.

Assume that all groups of performers are ordered in descending order of the total duration of actions being performed.

Initially we allocate such a minimal part of the resource that ensures maximal reduction of the total time of performance of all actions.

Similarly, we allocate the resource for the second and subsequent groups of performers until it is exhausted. Graphically, it looks like as follows (Figure 1).
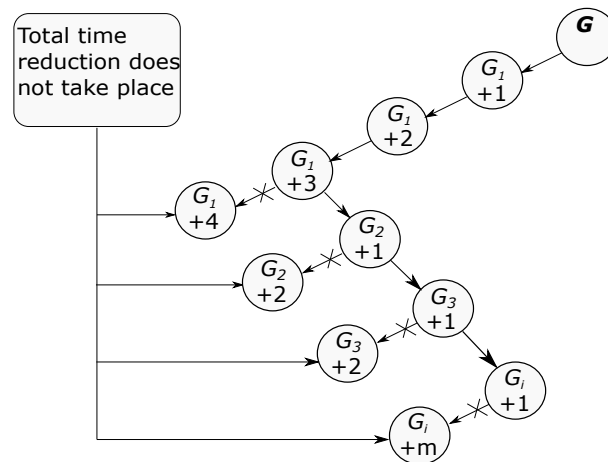

Figure 1. Resource allocation option based on the "greedy" algorithm

At subsequent stages, the obtained result is improved based on the selection of alternative options of resource allocation by groups.

First all rational, i.e. resulting in the total time reduction, options of allocation of the last performers out of the resource are considered. If in this case the total time is improved, then we will memorize the new option.

Then we consider the options of allocation of the two last performers etc. The process continues until all options are considered or the problem solving time limit is exhausted.

2) Another problem solving algorithm can be based on the ideas of "gradient descent" algorithm. Each time, one resource unit is allocated and an option that ensures maximum time reduction is selected (Figure 2). The algorithm continues until the entire resource is exhausted.
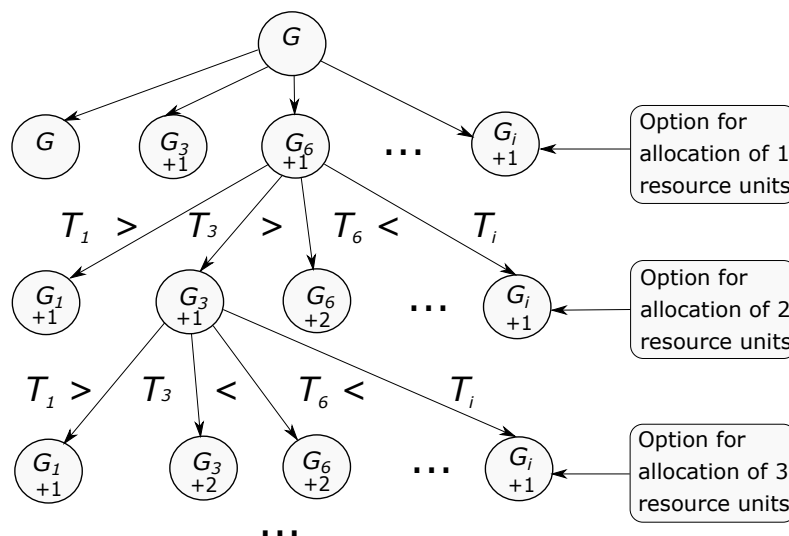

Figure 2. Resource allocation algorithm by the "gradient descent" method

A numerical example of a solution of a problem. Let us suppose that groups of performers g1, g2,..., g5 to form control actions must perform partially ordered actions $d_1, d_2, ...d_8$ (Figure 3).
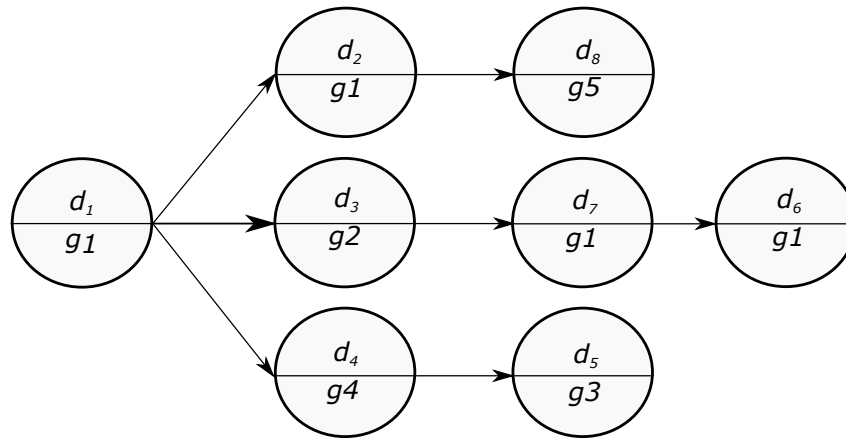

Figure 3. Graph of actions sequence execution.

On the basis of the graph of actions sequence the schedule of actions depending on the execution time of each action is constructed (Figure 4).
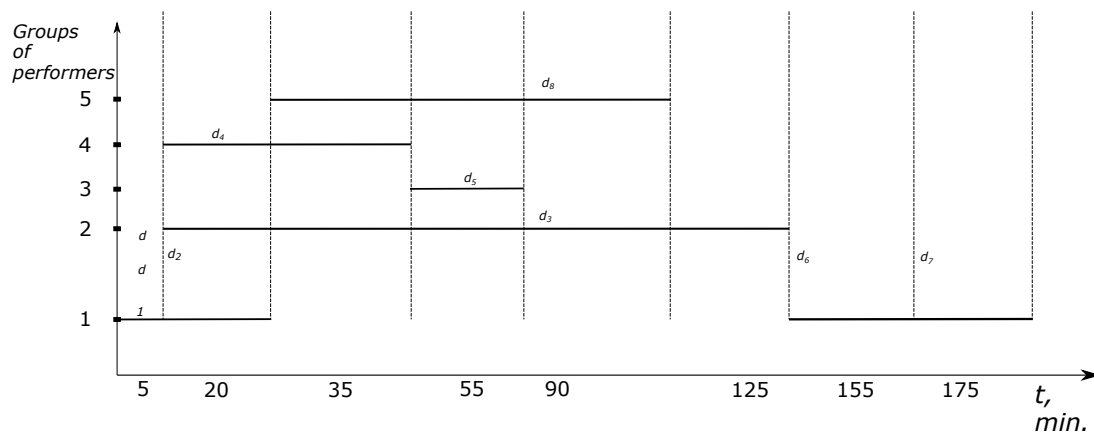

Figure 4. Schedule of actions depending on time.

The time for the implementation of individual actions can be reduced by attracting additional resources. Suppose that two more performers can be drawn into groups. The dependence of the reduction in the execution time of actions is shown in Table 1.

It can be seen from the table that the maximum reduction in the action time (40 minutes) occurs when two people are added to the group g2.

7. Conclusion

The developed model and algorithms can be used for optimizing the selection of the option of resource allocation in subsystems of the control element in order to reduce the total time of development of controlling actions in organizational systems of control of various purpose.

Table 1. Dependence of task execution time on resource usage.

Actions		Performers							
		group g1				group g2	group g3	group g4	group g5
		d1	d2	d6	d7	d3	d4	d5	d8
		Time, min.							
Attraction of a reserve	0	5	15	30	20	120	30	20	70
	1	5	13	25	17	100	30	20	70
	2	5	13	20	15	80	30	20	70
Time gain by action		0	2	5	3	20	0	0	0
Absolute time gain		15				40	0		

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