ChatGPT

Foundational Texts on Convergence and Brownian Motion

Standard references rigorously develop weak convergence of processes and Brownian limits. In particular, **Billingsley's** classic *Convergence of Probability Measures* (2nd ed., 1999) systematically treats tightness (Prohorov's theorem) and weak convergence in $D[0,1]$ [1]. **Ethier & Kurtz** (1986; reprint 2009) gives a modern treatment of convergence for Markov processes: Chapter 3 covers convergence of probability measures [2], and Chapter 7 develops invariance principles (diffusion approximations) [3]. **Kallenberg's** *Foundations of Modern Probability* (2nd ed. 2002) is another comprehensive source (see also references in listing Kallenberg 2002). These texts (and others such as Revuz–Yor 1999, Rogers–Williams 1994, Karatzas–Shreve 1991) present convergence criteria, Skorokhod topology, martingale methods, etc. As noted by Berkes *et al.* (2014), the classical works of Doob (1949), Donsker (1952), and Prohorov (1956) **built the theory of weak convergence** on metric spaces [4], laying the groundwork treated in Billingsley (1999) [1].

- **Donsker's Theorem (Functional CLT):** All the above sources cover the *invariance principle*. For example, Pitman and Yor explain that a centered random walk $(S_{\lfloor nt\rfloor}/\sqrt n)_{t\ge0}$ converges in law to Brownian motion in $C[0,\infty)$ [5]. In other words, suitably interpolated partial sums of i.i.d. variables converge to a Wiener process (this is Donsker's theorem) [5]. Ethier & Kurtz also develop these limits in Chapter 7 ("Invariance Principles") [3], and Billingsley's text gives a classical proof (via tightness and finite-dimensional convergence). (Advanced refinements – e.g. strong approximations by Komlós–Major–Tusnády – are discussed in the literature but beyond the scope here.)

- **Tightness and Compactness Criteria:** Key tools are *Prohorov's theorem* and *Billingsley's criterion* for tightness. Billingsley's book states that a family of probability measures on a complete separable metric space is relatively compact iff it is tight [1]. The "Billingsley criterion" (Theorem 12.3 in Billingsley 1968/1999) gives practical conditions to verify tightness of stochastic processes. For instance, recent work on fractional processes explicitly invokes Billingsley's criterion to reduce tightness verification to bounding increments (see Theorem 12.3 usage in). In short, Billingsley (1999) provides the detailed criteria and proofs for these compactness conditions, and Ethier & Kurtz also discuss them in the context of Skorokhod space [2].

- **Poisson and Lévy Processes:** Foundational treatments of jump processes can be found in standard monographs. J. F. C. **Kingman's** *Poisson Processes* (1993) is a concise graduate text on the theory of Poisson processes. For Lévy processes and infinite divisibility, see **Jean Bertoin** (Cambridge, 1996) or **Ken-iti Sato** (Cambridge, 1999), which develop the full Lévy–Khintchine theory. **Applebaum** (2009) *Lévy Processes and Stochastic Calculus* treats both theory and stochastic integration. These works rigorously cover convergence results (e.g. convergence of compound Poisson to Brownian motion) and properties of jump processes. (For convergence of martingales and semimartingale approximations, see also **Jacod–Shiryaev** *Limit Theorems for Stochastic Processes*.)

- **Fractional Brownian Motion:** Extensions to *long-memory* Gaussian processes are treated in specialized sources. For example, **Mishura**'s book *Stochastic Calculus for Fractional Brownian*

Motion and Related Processes (LNM 1929, 2008) develops theory and limit theorems for fBM [6]. **Biagini et al.** (2008) and **Decreusefond & Üstünel** are other texts on fractional BM. The mixed fBM model is studied by **Cheridito (2001)** [7]. These references support a thesis on extensions of Donsker's principle to fractional Brownian limits and related convergence results.

Key References: Among high-quality sources are Billingsley (1999) [1] (definitive on weak convergence), Ethier & Kurtz (1986) [2] [3] (convergence for processes), and specialized monographs such as Bertoin (1996), Sato (1999), Mishura (2008) [6], etc. Pitman & Yor's *A Guide to Brownian Motion and Related Processes* (2011) surveys many such texts. These works are rigorous, graduate-level treatments of the requested topics: convergence in law/probability, invariance principles, Poisson/Lévy foundations, tightness criteria, and fBM.

Sources: Textbooks and survey papers as cited above [4] [5] [1] [2] [3] [6] [7], among others.

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[1] [5] [6] [7] A guide to Brownian motion and related stochastic processes
https://www.stat.berkeley.edu/~aldous/205B/pitman_yor_guide_bm.pdf

[2] [3] Markov Processes: Characterization and Convergence - Stewart N. Ethier, Thomas G. Kurtz - Google Books
https://books.google.com/books/about/Markov_Processes.html?id=zvE9RFouKoMC

[4] tugraz.elsevierpure.com
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