Complexidade de Algoritmos, 2020/2
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Use a técnica da perturbação para resolver o somatório abaixo, sendo $h = \lfloor \log 2n \rfloor$:

$$\sum_{i=0}^{h} (h - i) \cdot 2^i$$

Resposta:

$$\sum_{i=0}^{h} (2^i . h - 2^i . i) =$$

$$= \sum_{i=0}^{h} 2^i . h - \sum_{i=0}^{h} 2^i . i$$

$$= h . \underbrace{\sum_{i=0}^{h} 2^i}_{1)} - \underbrace{\sum_{i=0}^{h} 2^i . i}_{2)}$$

1) $\sum_{i=0}^{h} 2^i = 2^0 + 2^1 + \dots + 2^h \boxed{+ 2^{h+1}}$

$$\sum_{i=0}^{h} 2^i + 2^{h+1} = 2^0 + \sum_{i=0}^{h} 2^{i+1}$$

$$\sum_{i=0}^{h} 2^i + 2^{h+1} = 2^0 + \sum_{i=0}^{h} 2 . 2^i$$

$$\sum_{i=0}^{h} 2^i + 2^{h+1} = 2^0 + 2 \sum_{i=0}^{h} 2^i$$

$$2^{h+1} = 2 \sum_{i=0}^{h} 2^i - \sum_{i=0}^{h} 2^i$$

$$2^{h+1} - 1 = \sum_{i=0}^{h} 2^i$$

2) $\sum_{i=0}^{h} 2^i . i = 2^0 . 0 + 2^1 . 1 + \dots + 2^h . h + \boxed{2^{(h+1)} . (h+1)}$

$$\sum_{i=0}^{h} 2^i . i + 2^{h+1} . (h+1) = 0 + \sum_{i=0}^{h} 2^{i+1} . (i+1)$$

$$\sum_{i=0}^{h} 2^i . i + 2^{h+1} . (h+1) = \sum_{i=0}^{h} 2^{i+1} . i + 2^{i+1} . 1$$

$$\sum_{i=0}^{h} 2^i . i + 2^{h+1} . (h+1) = \sum_{i=0}^{h} 2^{i+1} . i + \sum_{i=0}^{h} 2^{i+1}$$

$$\cancel{\sum_{i=0}^{h} 2^i . i} + 2^{h+1} (h+1) = 2 \sum_{i=0}^{h} 2^i . i + 2 \sum_{i=0}^{h} 2^i$$

$$2^{h+1} . (h+1) = \sum_{i=0}^{h} 2^i . i + 2 . \boxed{(2^{h+1} - 1)} \rightarrow 1)$$

$$2^{h+1} . (h+1) - 2 . (2^{h+1} - 1) = \sum_{i=0}^{h} 2^i . i$$

$$\sum_{i=0}^{h} (h-1).2^i = 1) - 2)$$

$$= h(2^{h+1}-1) - \left((h+1)2^{h+1} - 2(2^{h+1}-1)\right)$$

$$= h(2 \cdot 2^h - 1) - \left((h+1) \cdot (2 \cdot 2^h) - 2(2 \cdot 2^h - 1)\right)$$

$$= h \cdot 2 \cdot 2^h - h - (h \cdot 2 \cdot 2^h + 2 \cdot 2^h - 4 \cdot 2^h + 2)$$

$$= \cancel{2h \cdot 2^h} - h - \cancel{2h \cdot 2^h} \cancel{-2 \cdot 2^h} + \overset{2}{\cancel{4}} \cdot 2^h - 2$$

$$= 2 \cdot 2^h - h - 2$$

$$= 2 \cdot 2^{(\log_2 n)} - \log_2 n - 2$$

$$= 2n - \log_2 n - 2$$ $\therefore$ Complexidade $O(n)$