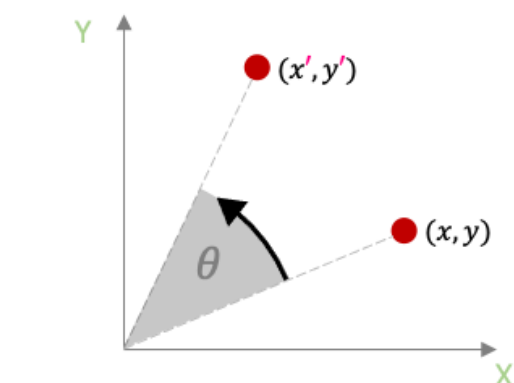


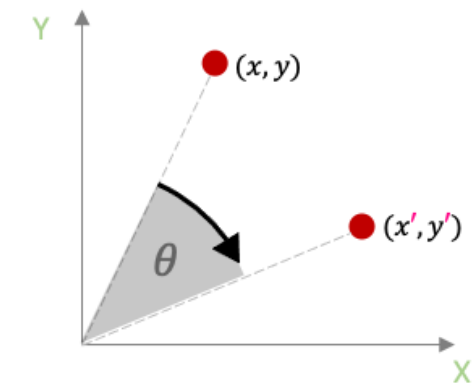
We just saw how we added a minus sign in front of the *sine* of the angle in the Y rotation matrix.

I want to go into a little more detail on why that is the case, and also use this moment to briefly talk about the direction of 3D rotations and how that is connected to coordinate-system handedness.

In 2D, we intuitively talk about clockwise and counterclockwise rotations:



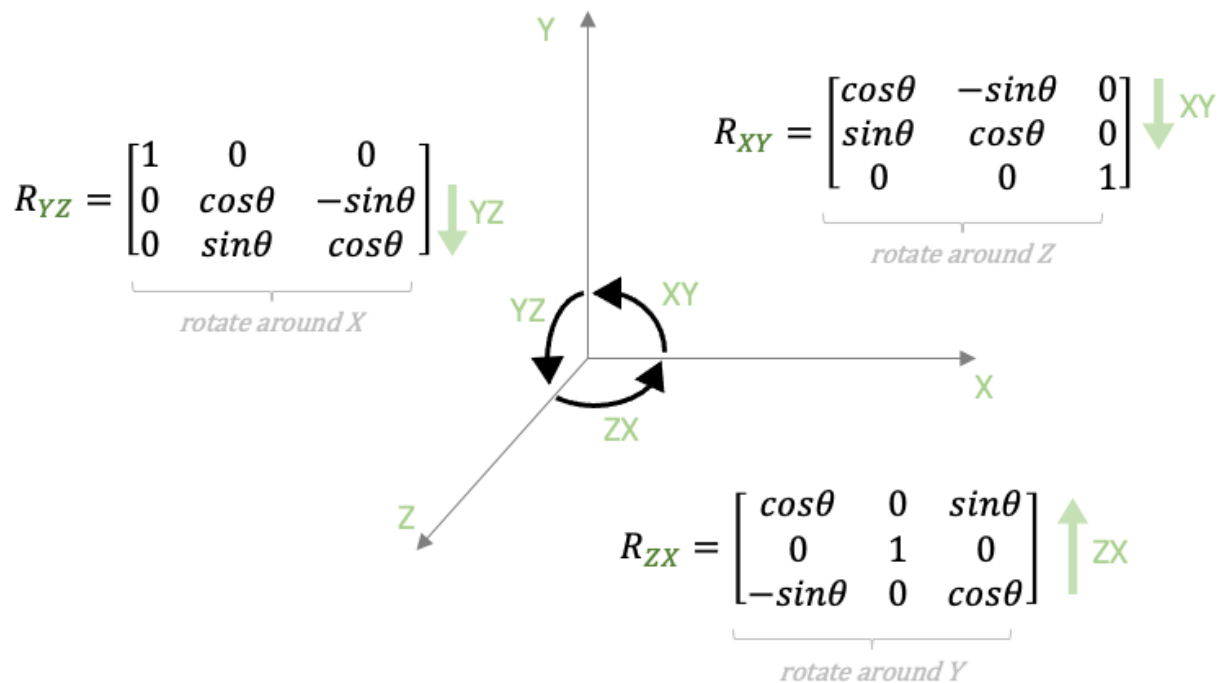
$$\begin{aligned} x' &= x \cos \theta - y \sin \theta \\ y' &= x \sin \theta + y \cos \theta \end{aligned}$$



$$\begin{aligned} x' &= x \cos \theta + y \sin \theta \\ y' &= -x \sin \theta + y \cos \theta \end{aligned}$$

Clockwise and counterclockwise only really exist in 2D, but since humans need to create some convention for the direction of a rotation in 3-dimensions (and higher-dimensions), we create this idea of a "3D clock." At the end of the day, it's just an arbitrary human decision on how we want to perceive our rotations, and what happens to our object as we *increase* or *decrease* the value of the angles.

One well-established convention is to use the left-hand or right-hand cross-product direction rule to determine the rotation direction of our system.



The image above shows how we try to maintain a counterclockwise rotation by flipping the sign of the rotation matrix entries.

This brings me to the main point of this discussion. Do you see how in the above image the Z-axis is pointing "outside" the screen? This means that these rotation matrices will perform this counterclockwise rotation for a **right-handed** coordinate system! If we really want to follow the proper cross-product direction in a **left-handed** system, we should change the signs of our matrix entries to account for that.

For most left-handed coordinate systems (like ours), the rotation matrices **should** be:

$$R_{\mathbf{x}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{\mathbf{y}} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{\mathbf{z}} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Flipping the *sine* of the angles make sure we follow the proper convention of cross-product direction.

That being said, you'll see that I will **not** change my rotation matrices to the left-hand form in my code. Some programmers might not like this decision, but I personally still like to see my angles grow the way they are currently.