Forecasting functional time series

Rob J Hyndman

MONASH University

Mortality rates

Fertility rates

Some notation

Let $y_t(x)$ be the observed (smoothed) data in period t at age x, $t = 1, \dots, n$.

$$y_t(x) = \mu(x) + \sum_{k=1}^{K} \beta_{t,k}\, \phi_k(x) + e_t(x)$$

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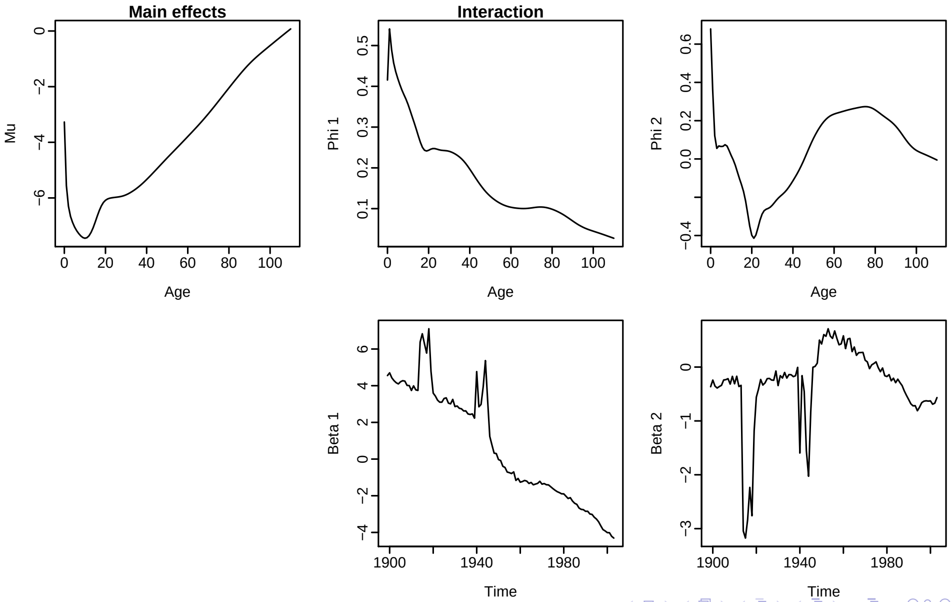
- $\phi_k(x)$ are eigenfunctions
- $e_t(x) \overset{\text{iid}}{\sim} \text{N}(0, v(x))$ is random error

Functional PC



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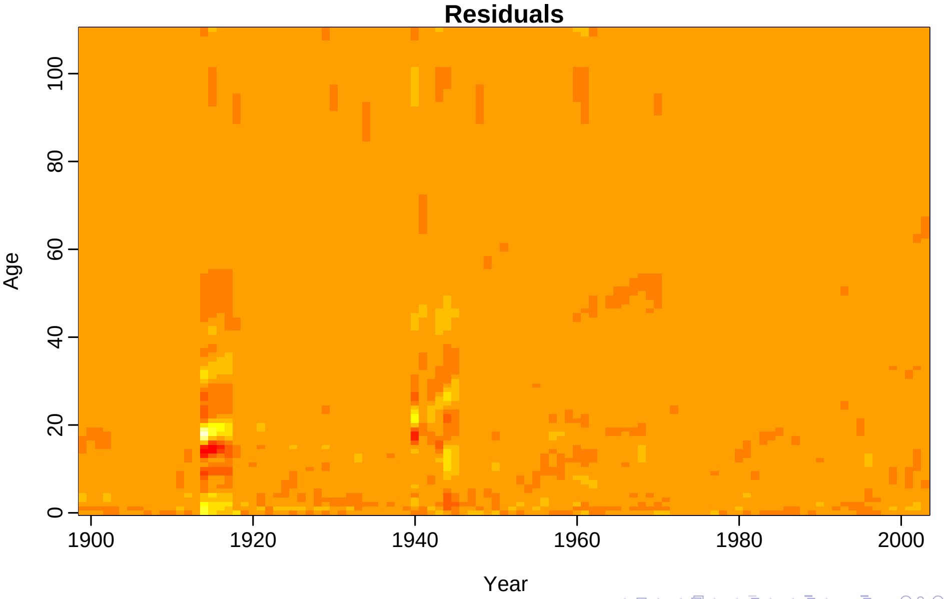
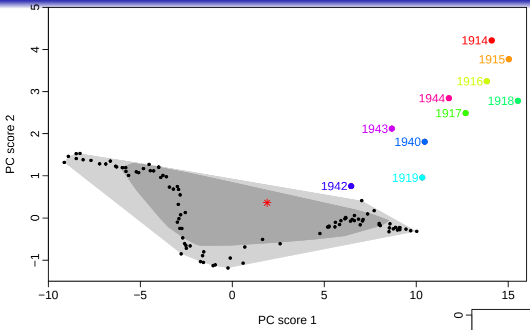


Outliers

$$y_t(x) = \mu(x) + \sum_{k=1}^{K} \beta_{t,k}\,\phi_k(x) + e_t(x)$$

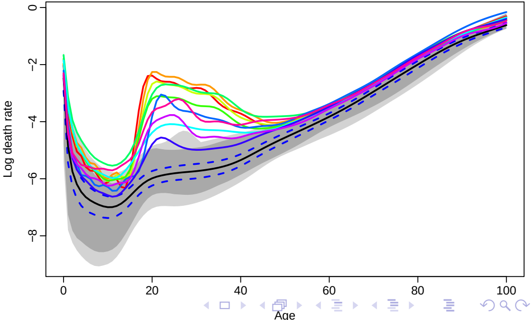
- Outliers in $\{y_t(x_i)\}$ will show up as outliers in $\{\beta_{t,1}, \beta_{t,2}, \dots\}$.

Bagplots

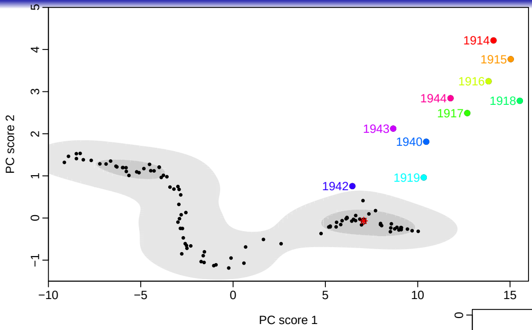


Red asterisk: Median
Dark grey: 50% region
Light grey: outer region

Bagplot for the French male mortality data.



Functional HDR boxplot



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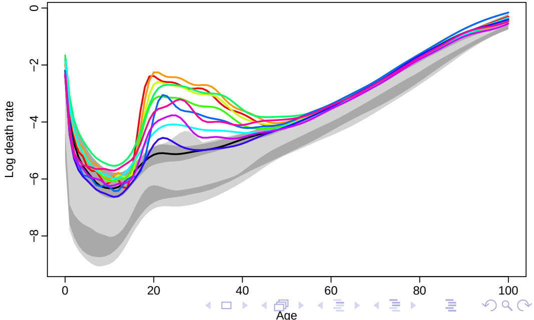

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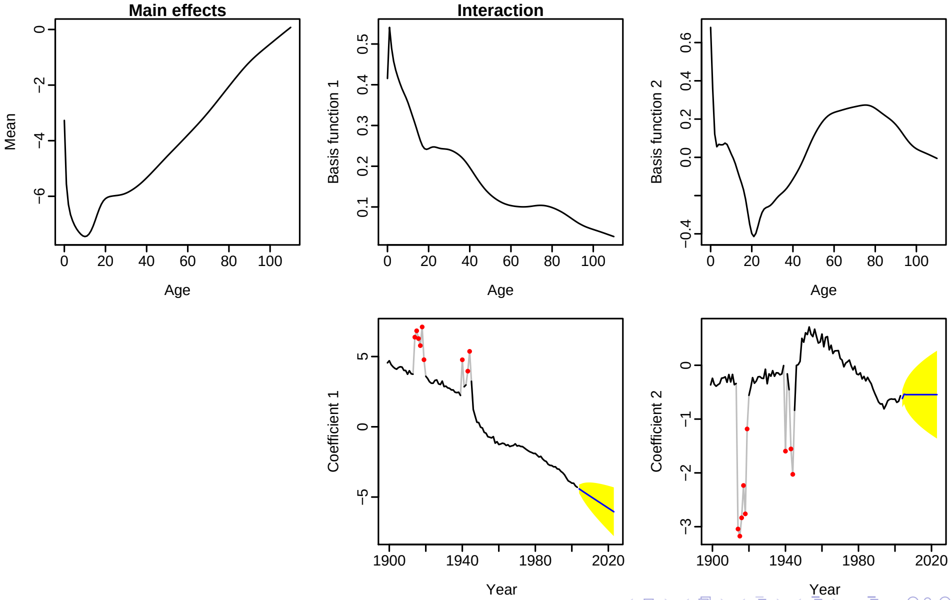
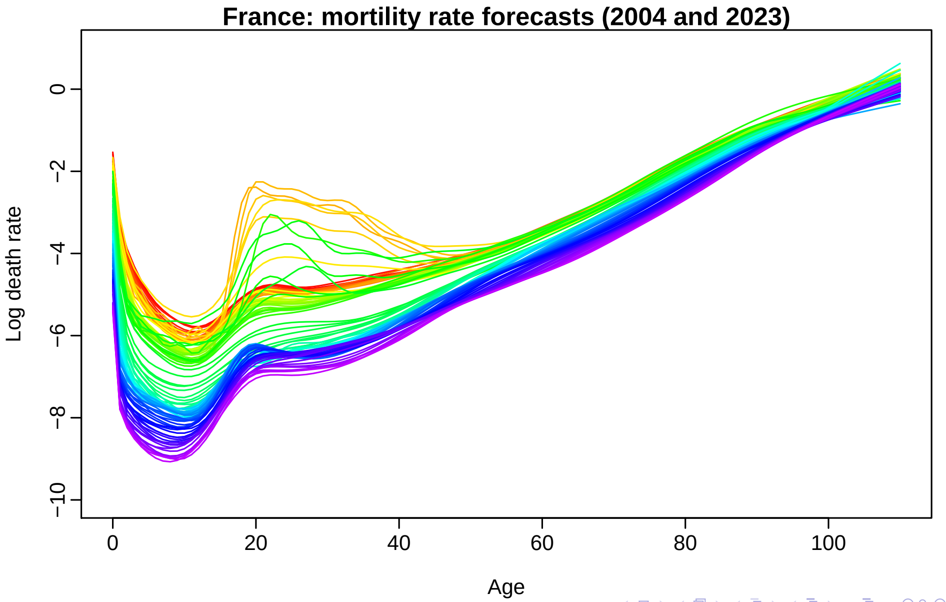
- Outliers are treated as missing values.
- Univariate ARIMA models are used for forecasting.

Forecasting the PC scores



Forecasts of $y_t(x)$



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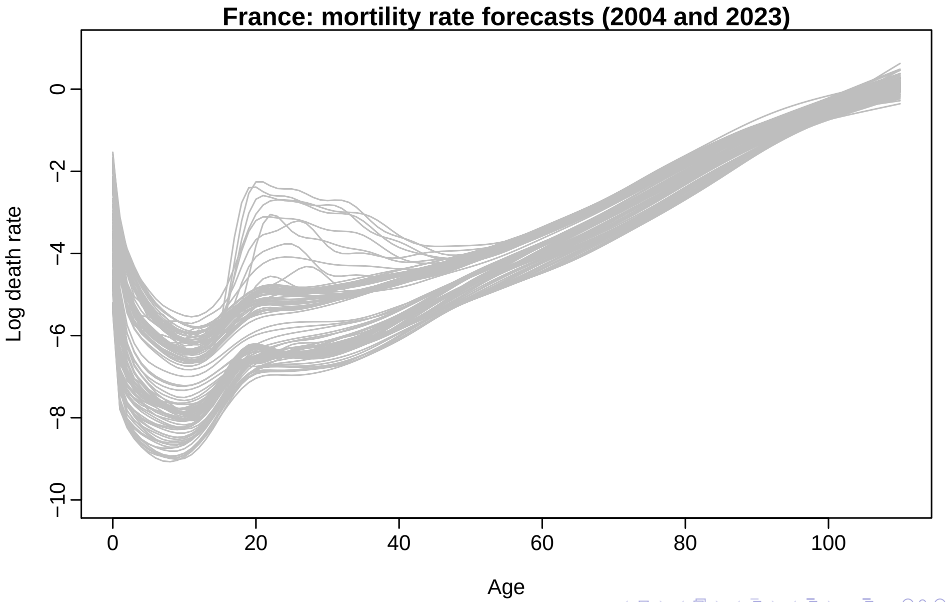

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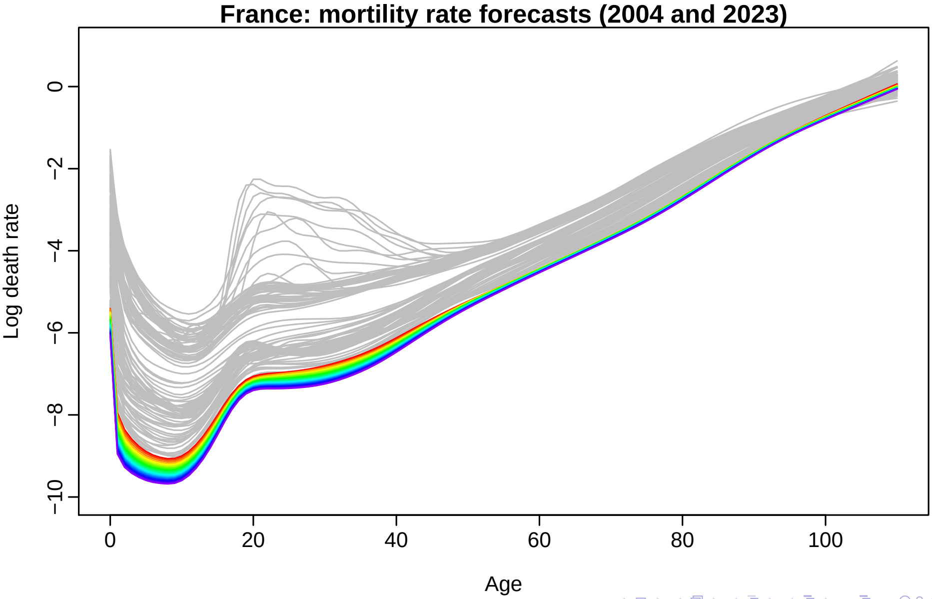

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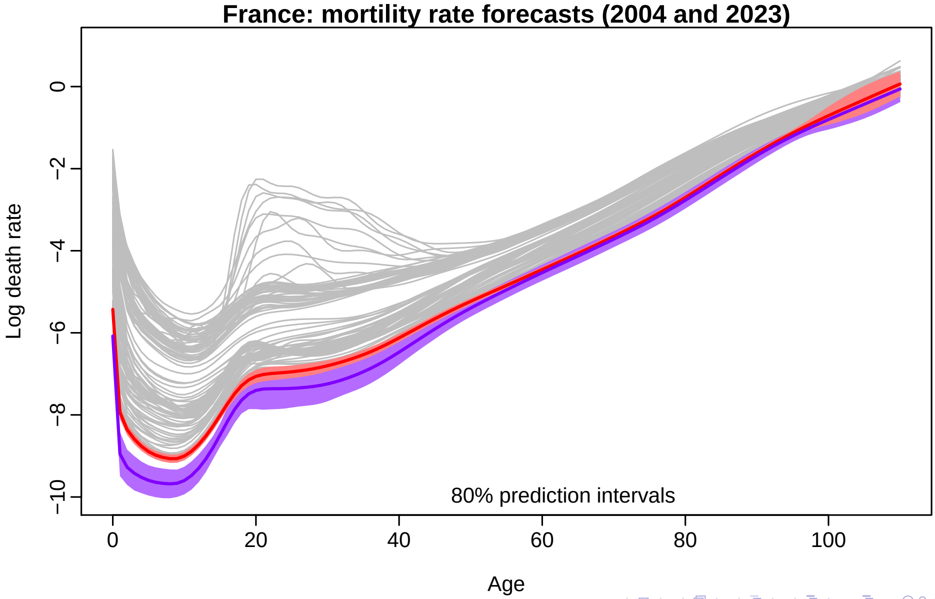
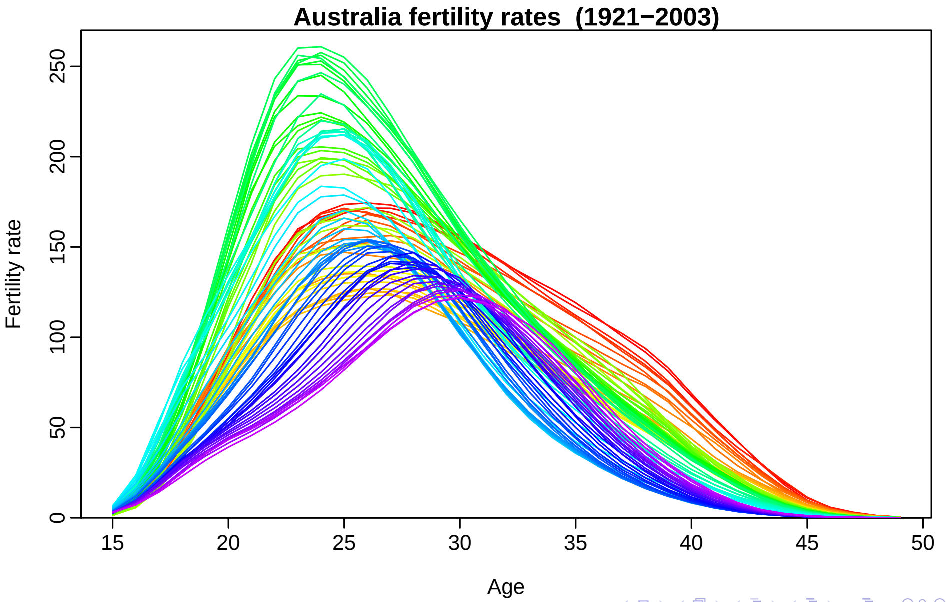


Fertility application



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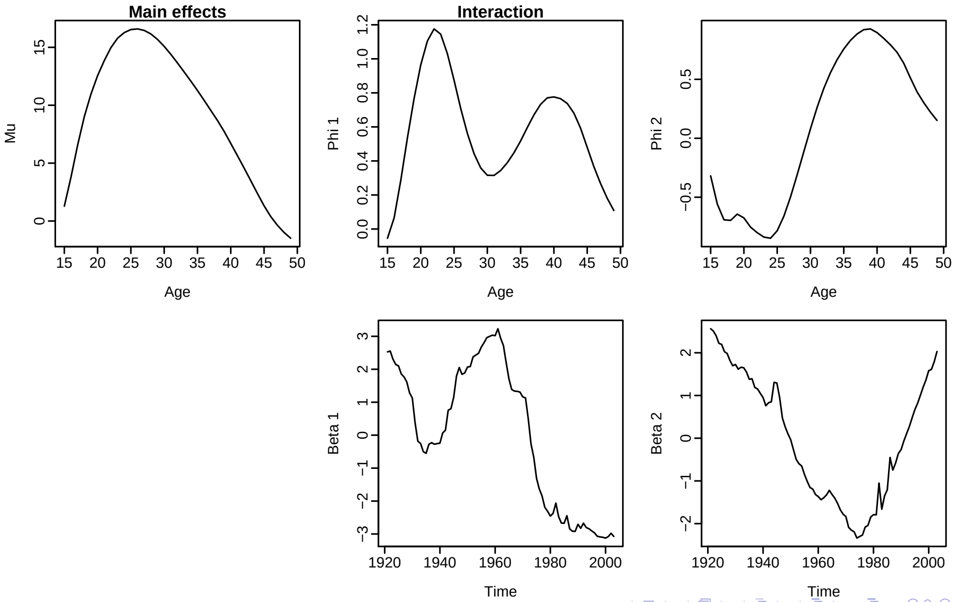

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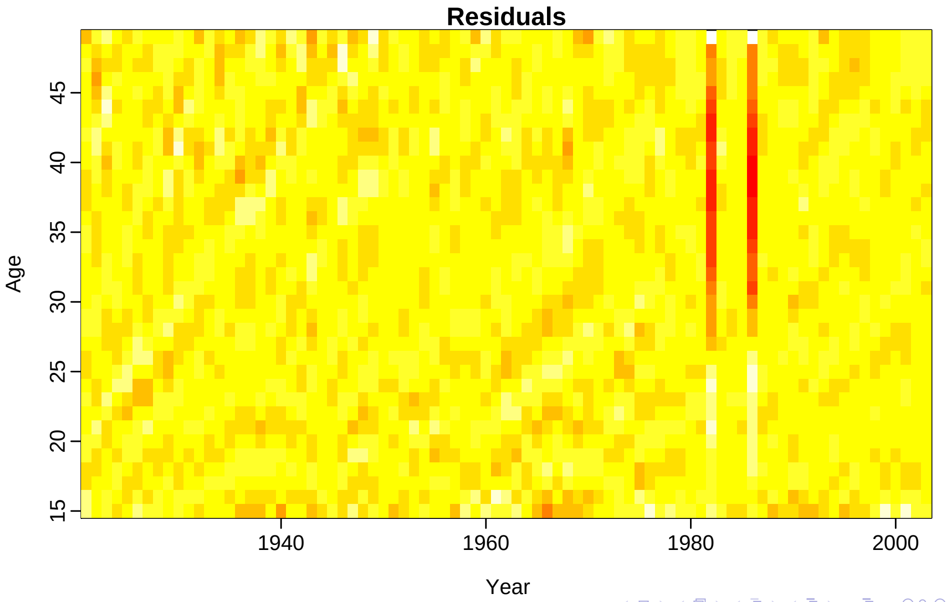

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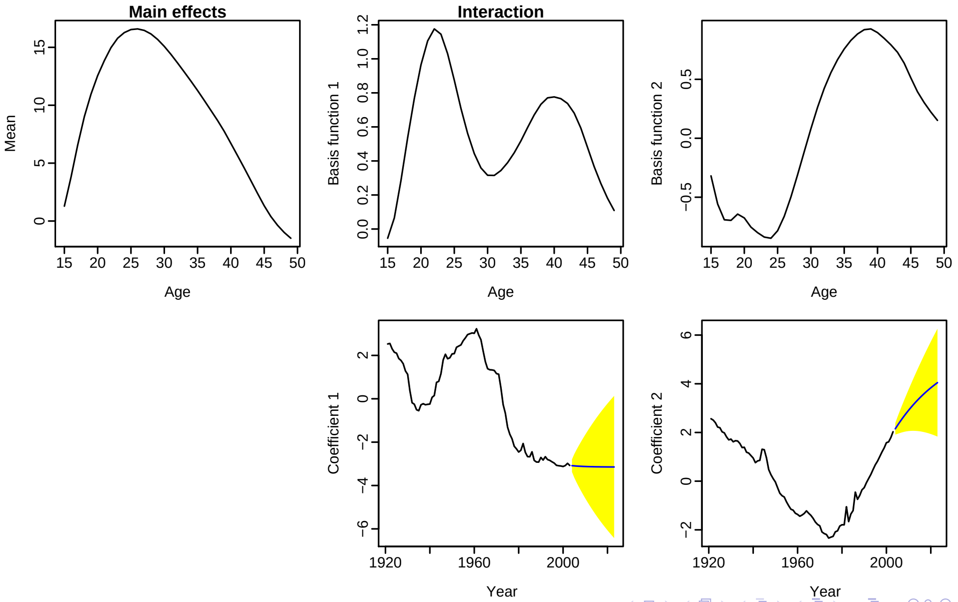

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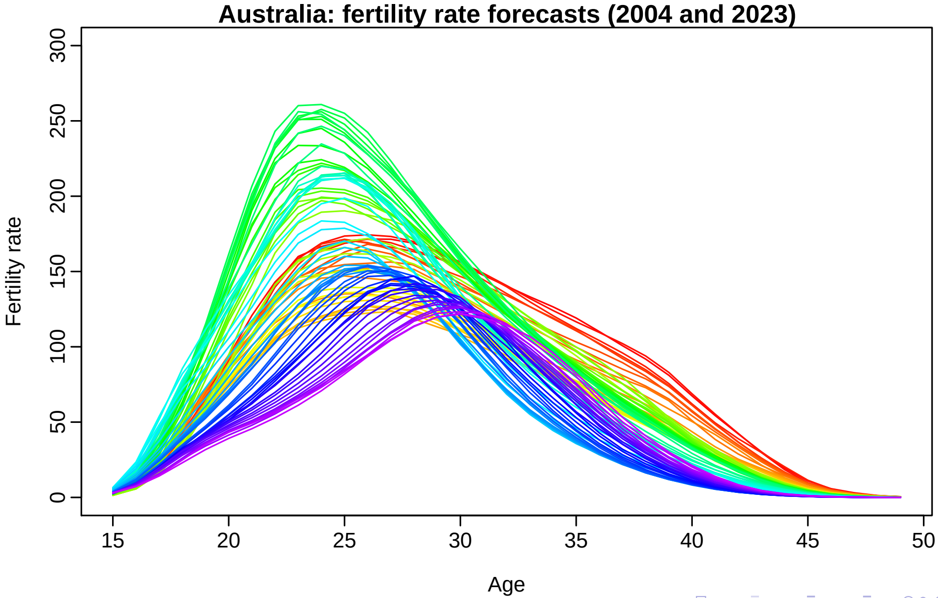

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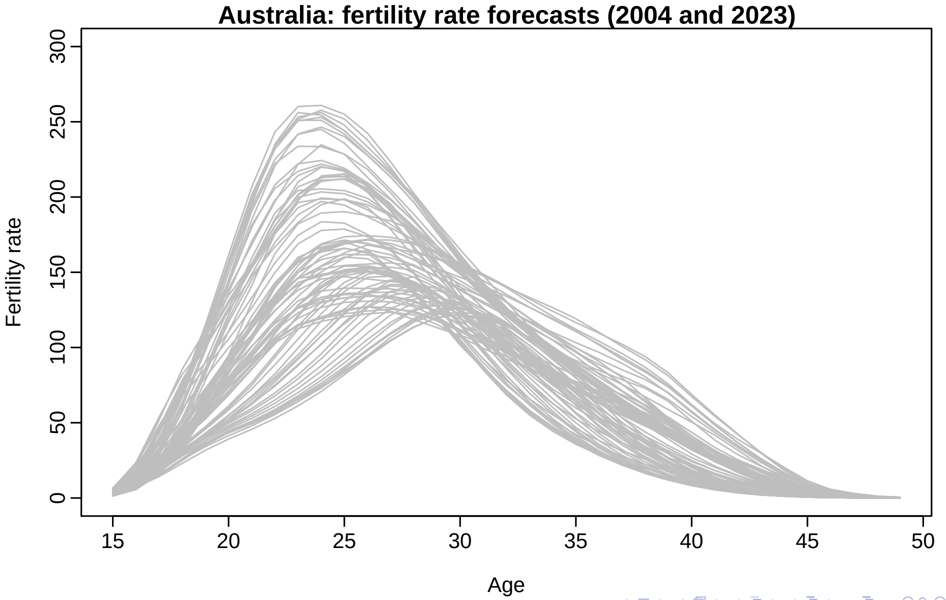

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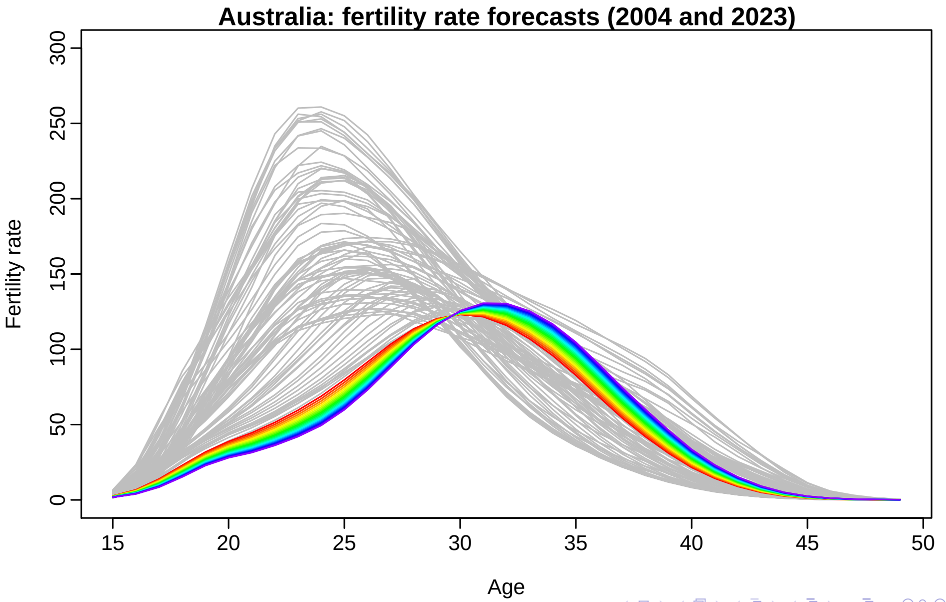

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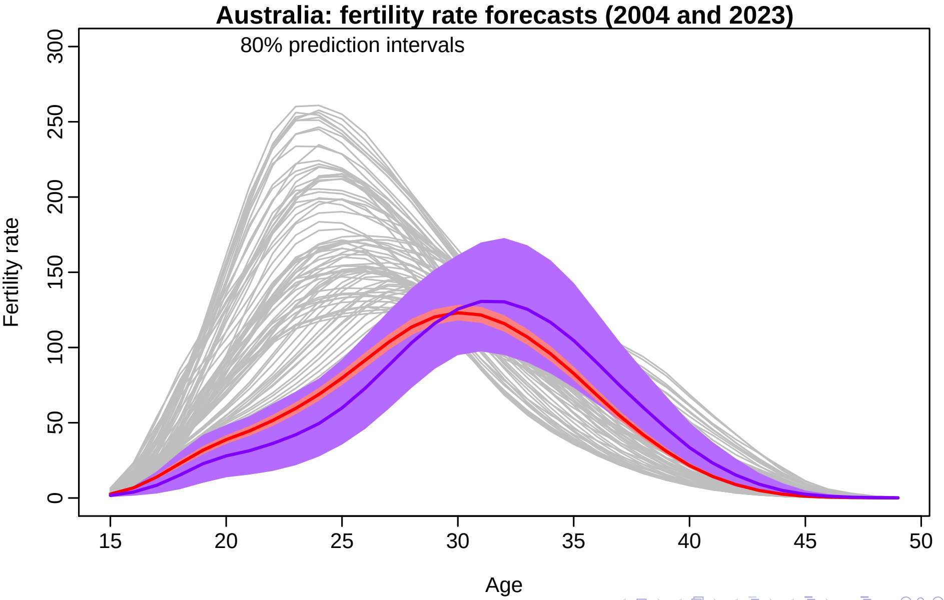

Research team

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- choosing a basis better suited to prediction *(rather than explaining historical variation)*

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More information

Papers and R code available at

www.robhyndman.info