MONASH University

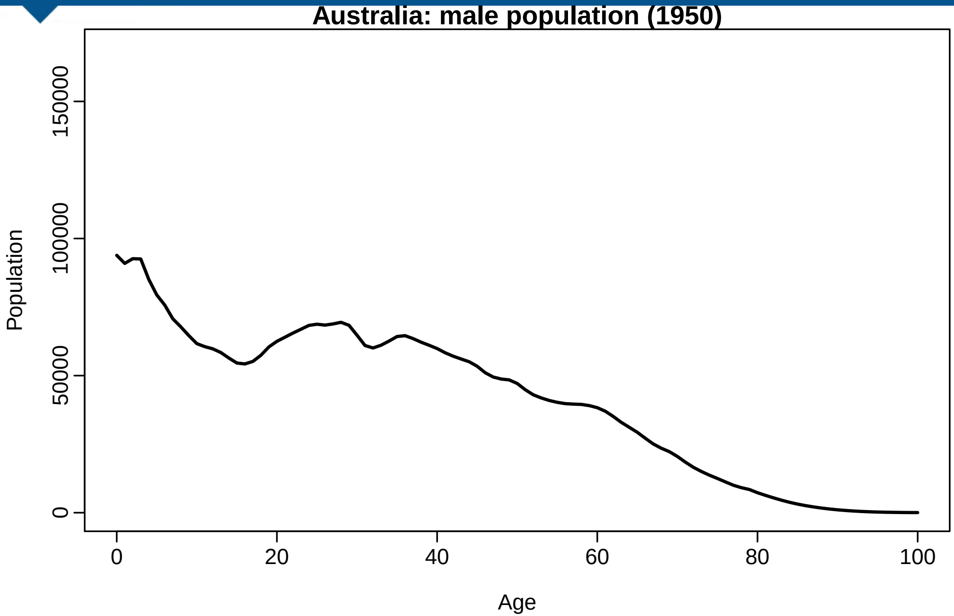
Rob J Hyndman

Functional time series

with applications in demography

8. Stochastic population forecasting

Annual age-specific population



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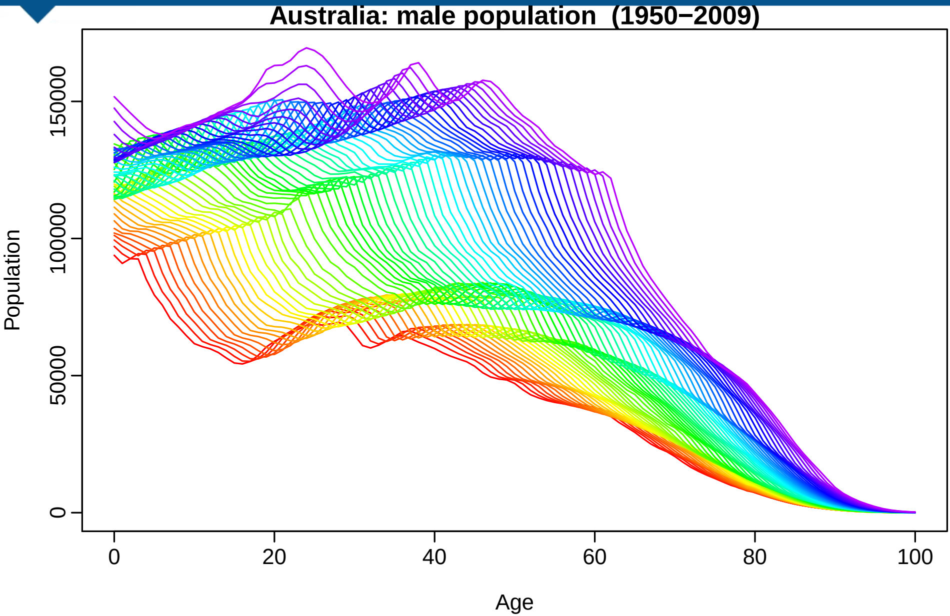

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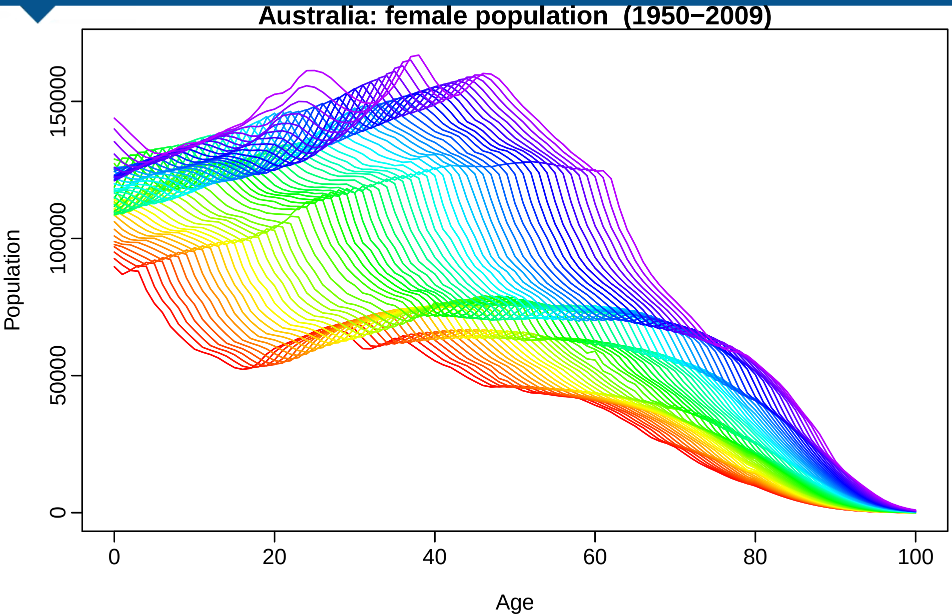

Stochastic population forecasts

Key ideas

- Population is a function of **mortality**, **fertility** and **net migration**.
- Build an age-sex-specific **stochastic model** for each of mortality, fertility & net migration.
- Use the models to **simulate future sample paths** of all components.
- Compute future births, deaths, net migrants and populations from simulated rates.
- Combine the results to get **age-specific stochastic population forecasts**.

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Demographic growth-balance equation

Demographic growth-balance equation

$$P_{t+1}(x+1) = P_t(x) - D_t(x, x+1) + G_t(x, x+1)$$
$$P_{t+1}(0) = B_t \quad - D_t(B, 0) \quad + G_t(B, 0)$$

$x = 0, 1, 2, \ldots$.

$P_t(x) =$ population of age x at 1 January, year t

$B_t =$ births in calendar year t

$D_t(x, x+1) =$ deaths in calendar year t of persons aged x at the beginning of year t

$D_t(B, 0) =$ infant deaths in calendar year t

$G_t(x, x+1) =$ net migrants in calendar year t of persons aged x at the beginning of year t

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The available data

The following data are available:

$P_t(x) =$ **population** of age x at 1 January, year t

$E_t(x) =$ **population** of age x at 30 June, year t

$B_t(x) =$ **births** in year t to females of age x

$D_t(x) =$ **deaths** in year t of persons of age x

From these, we can estimate:

- $m_t(x) = D_t(x)/E_t(x) =$ central death rate in calendar year t;
- $f_t(x) = B_t(x)/E_t^F(x) =$ fertility rate for females of age x in calendar year t.
- $D_t(x, x+1)$ and $D_t(B, 0)$.

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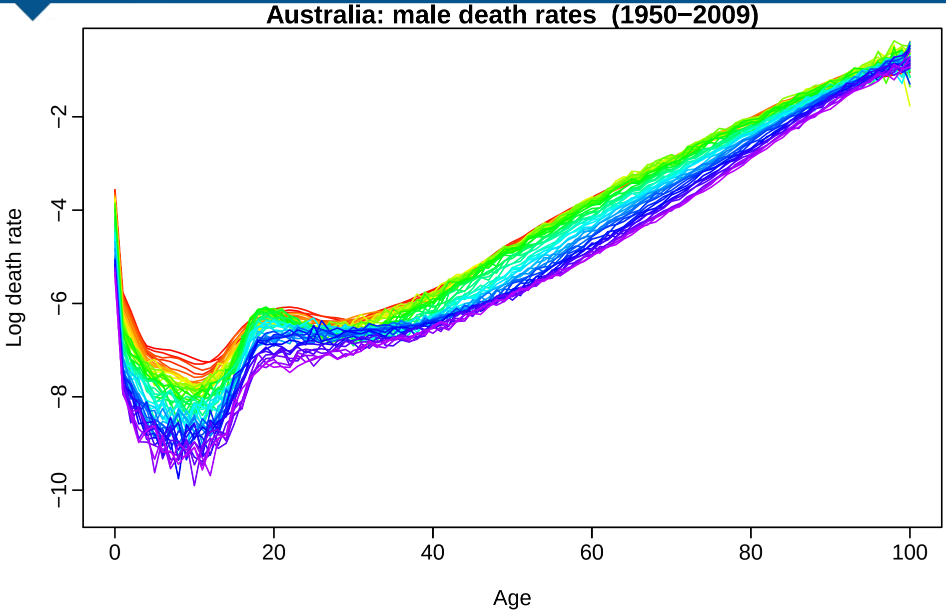
Mortality rates



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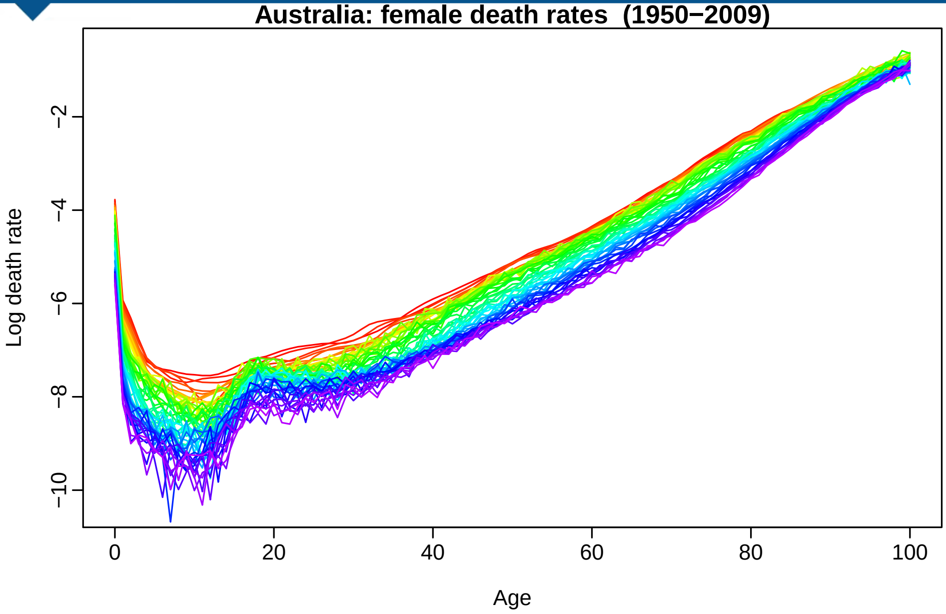
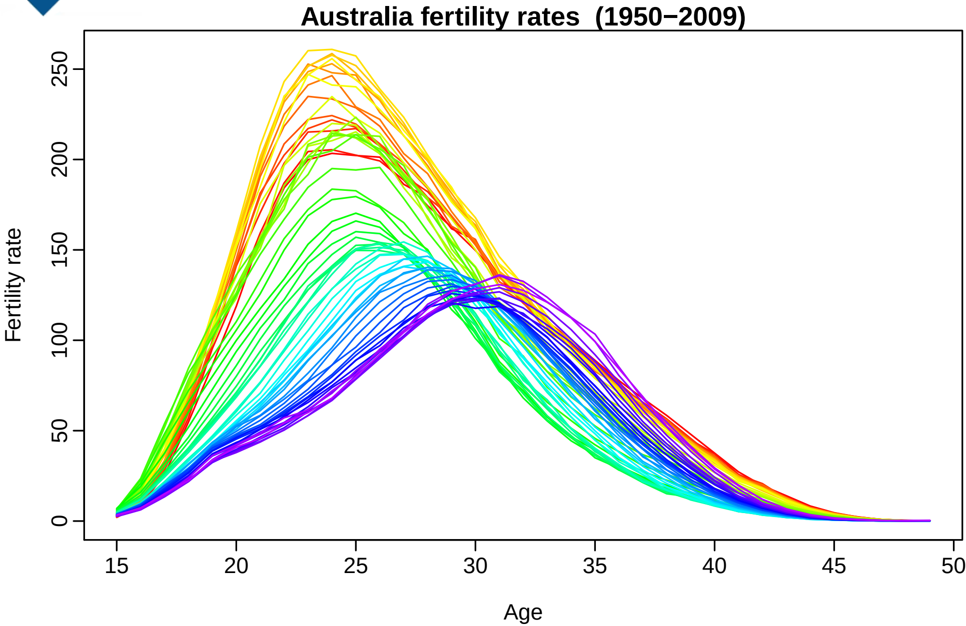


Fertility rates



Net migration

We need to *estimate* **migration** data based on difference in population numbers after adjusting for births and deaths.

Note: "net migration" numbers also include **errors** associated with all estimates. i.e., a "residual".

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$$G_t(x, x+1) = P_{t+1}(x+1) - P_t(x) + D_t(x, x+1)$$
$$G_t(B, 0) = P_{t+1}(0) \qquad - B_t \qquad + D_t(B, 0)$$

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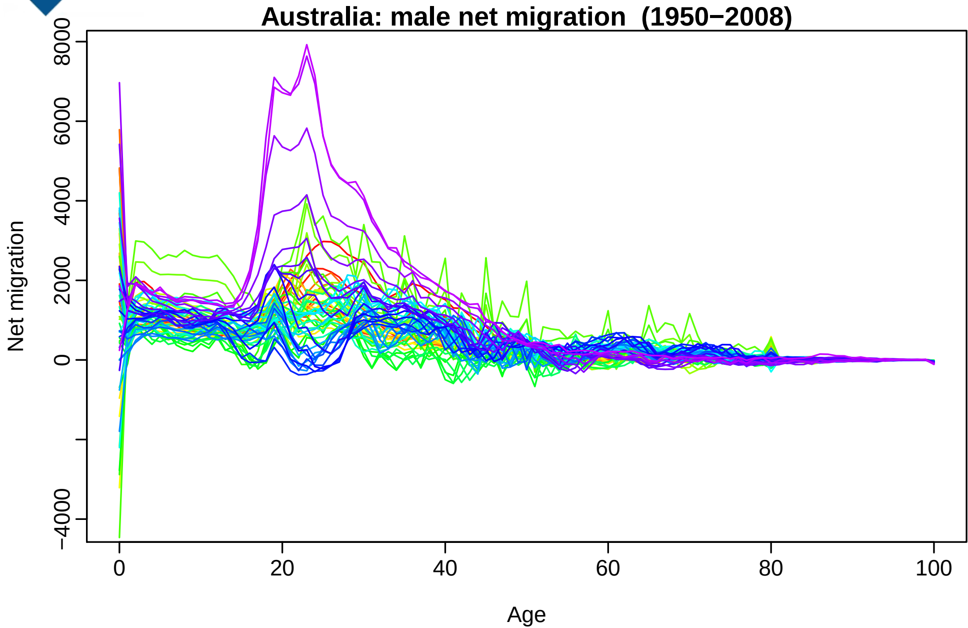
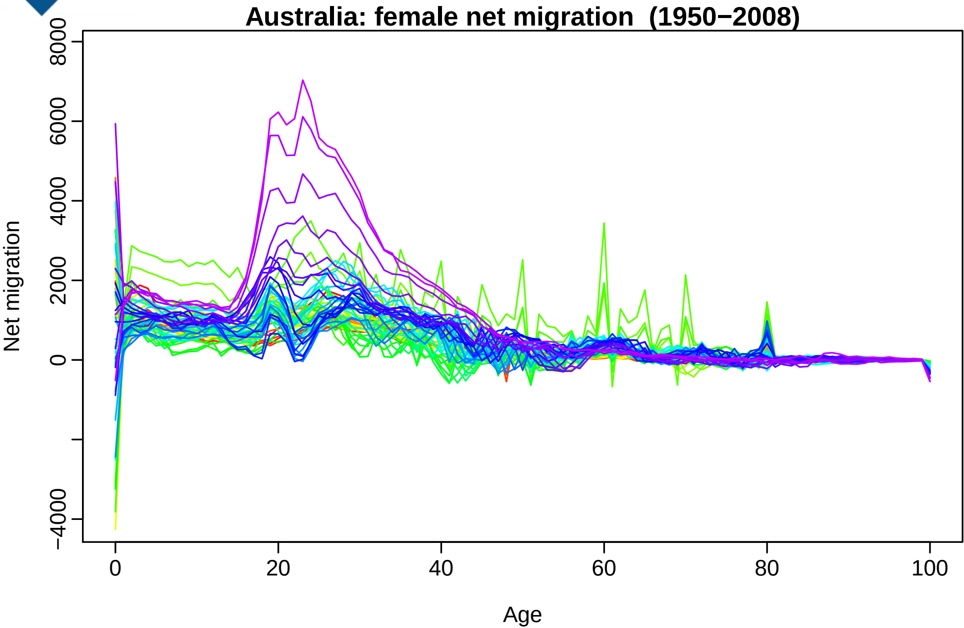
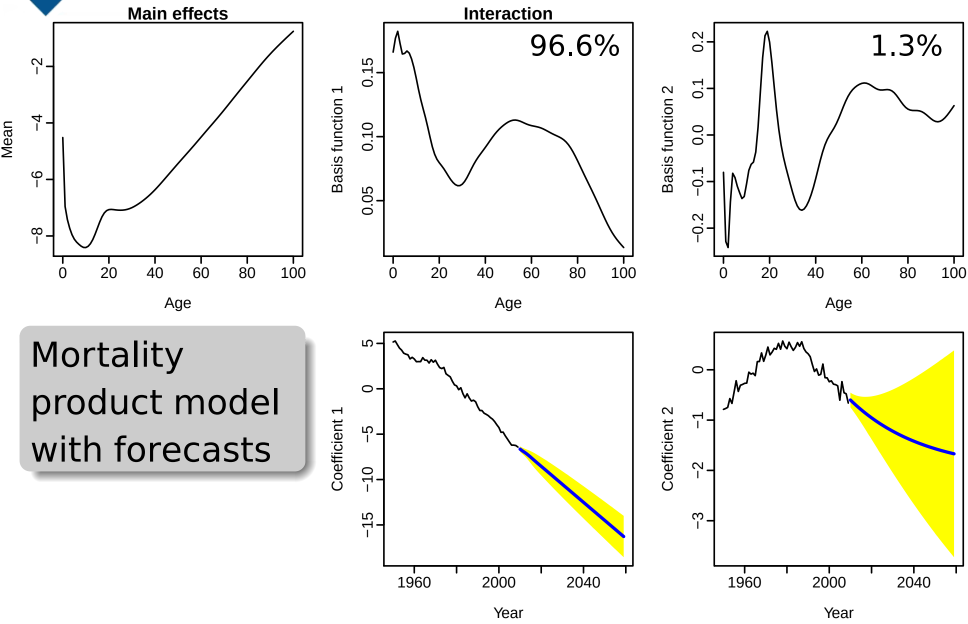


Net migration

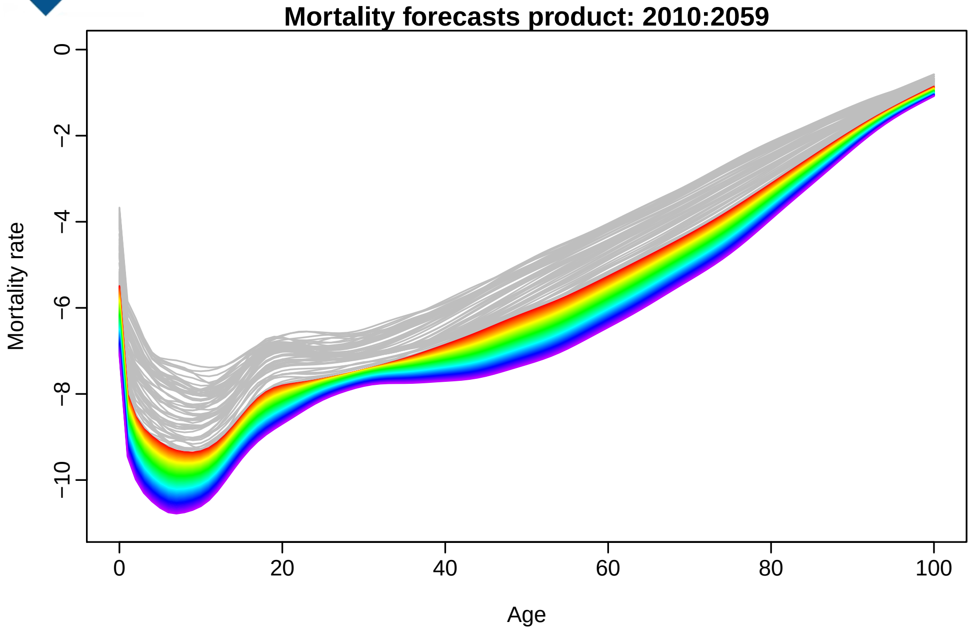


Functional time series model



Mortality product model with forecasts

Mortality forecasts



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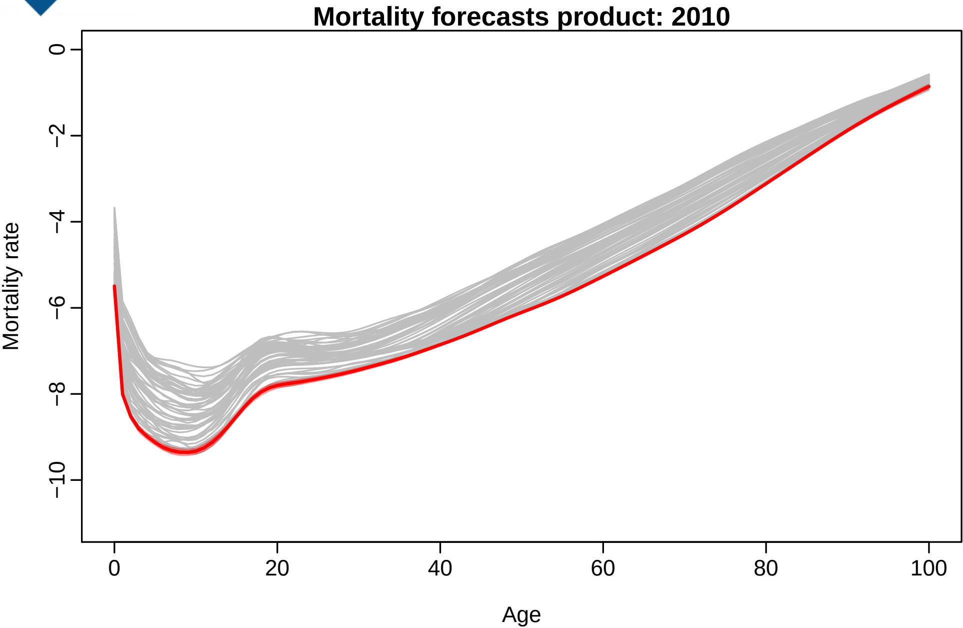

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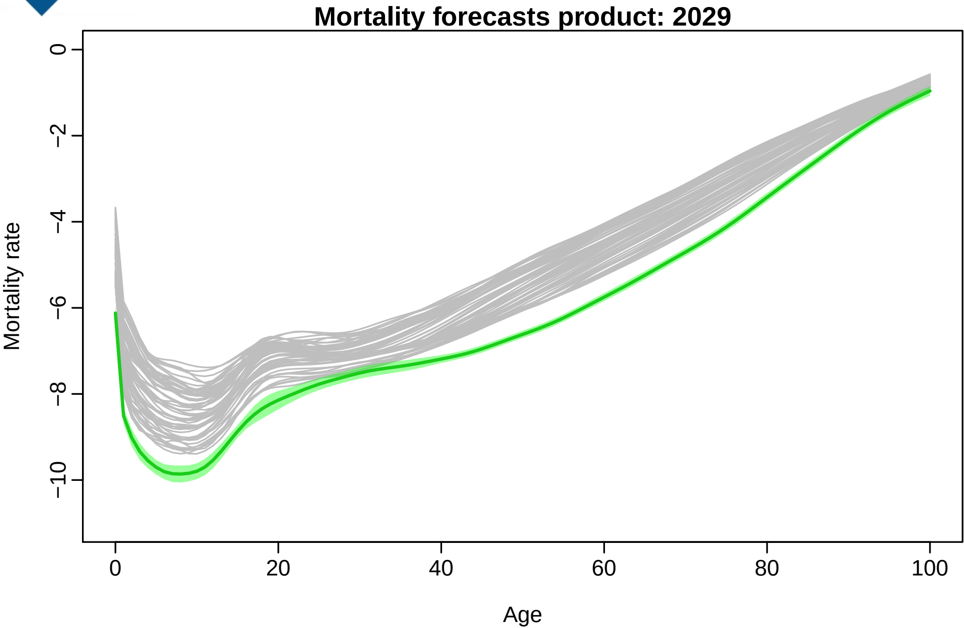

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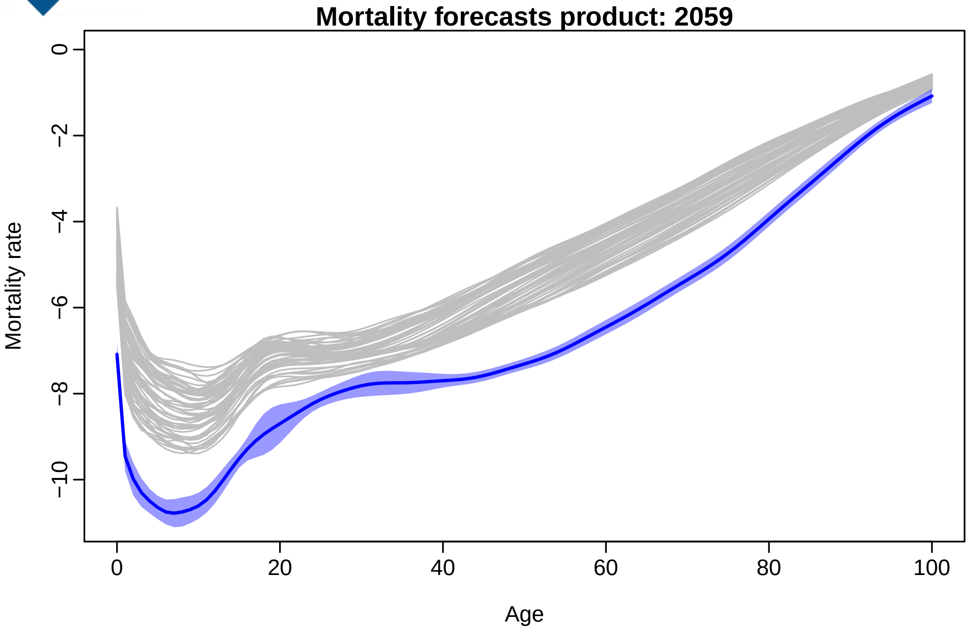

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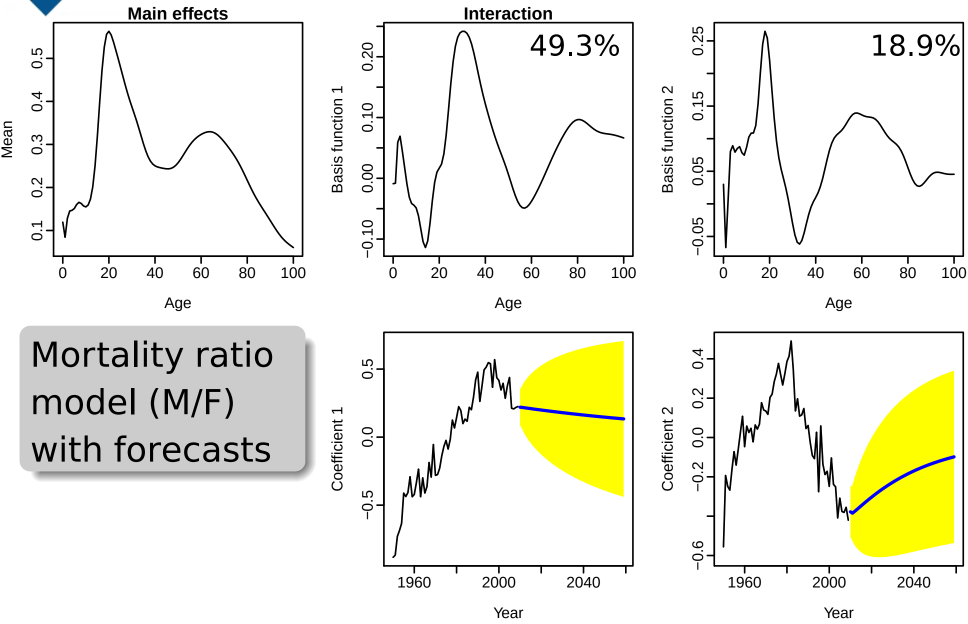
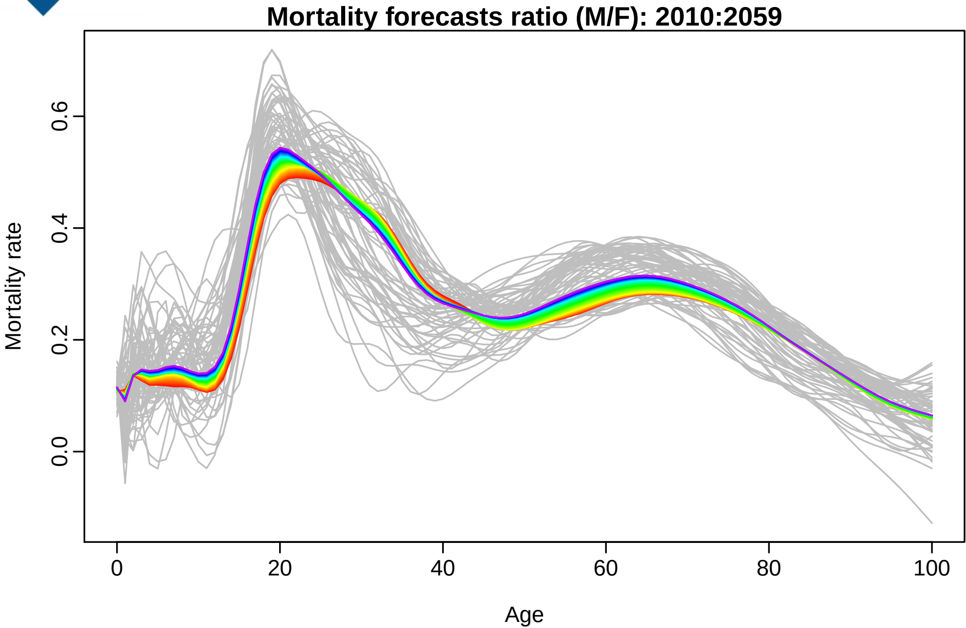


Mortality ratio model (M/F) with forecasts

Mortality forecasts



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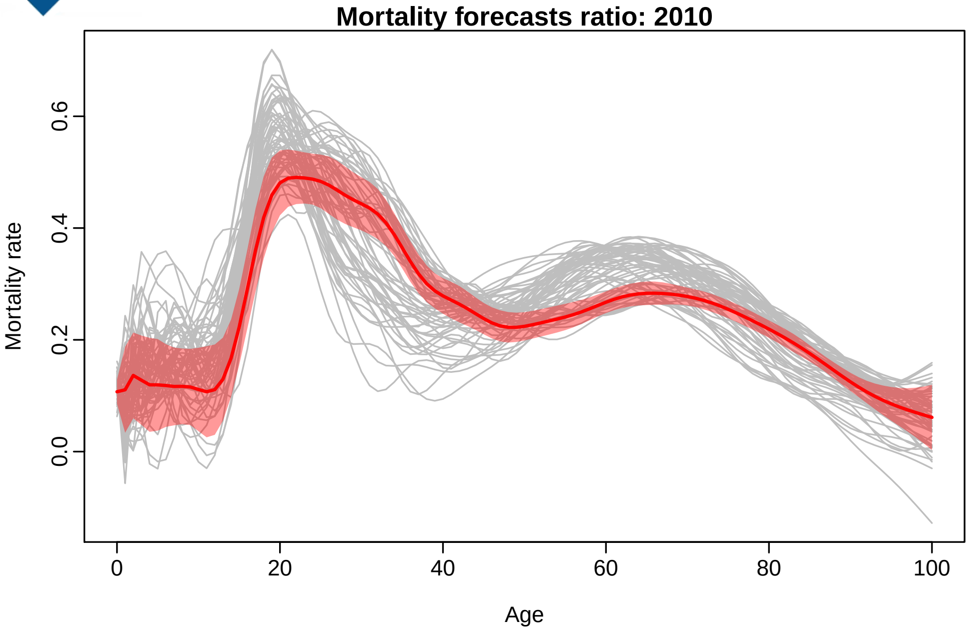

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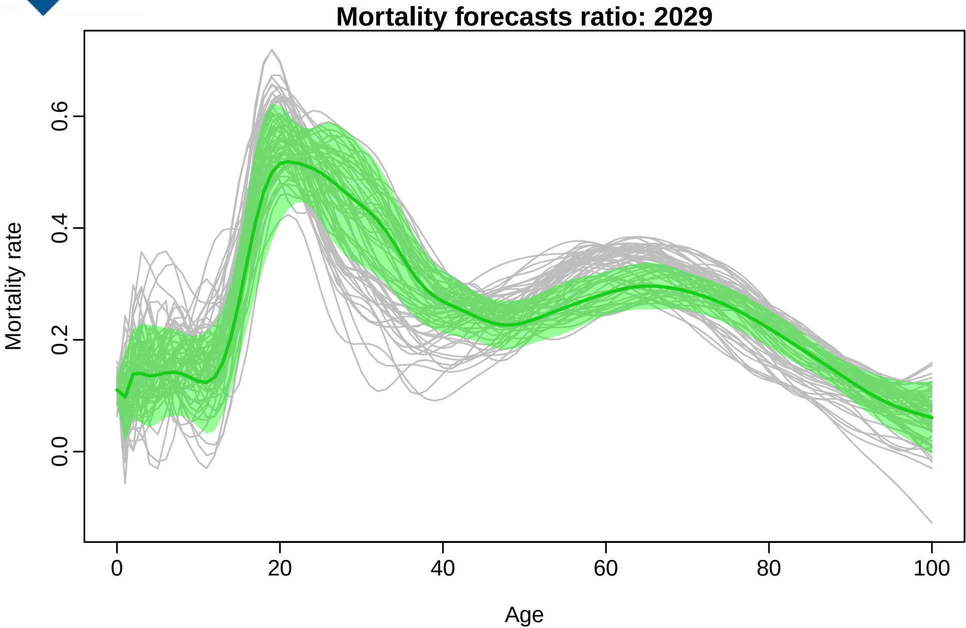

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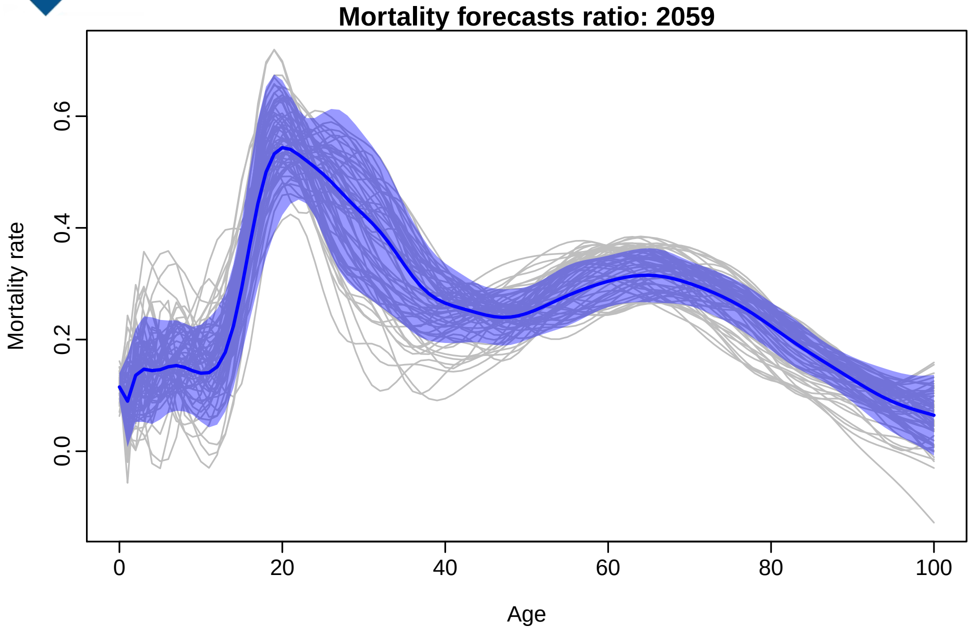

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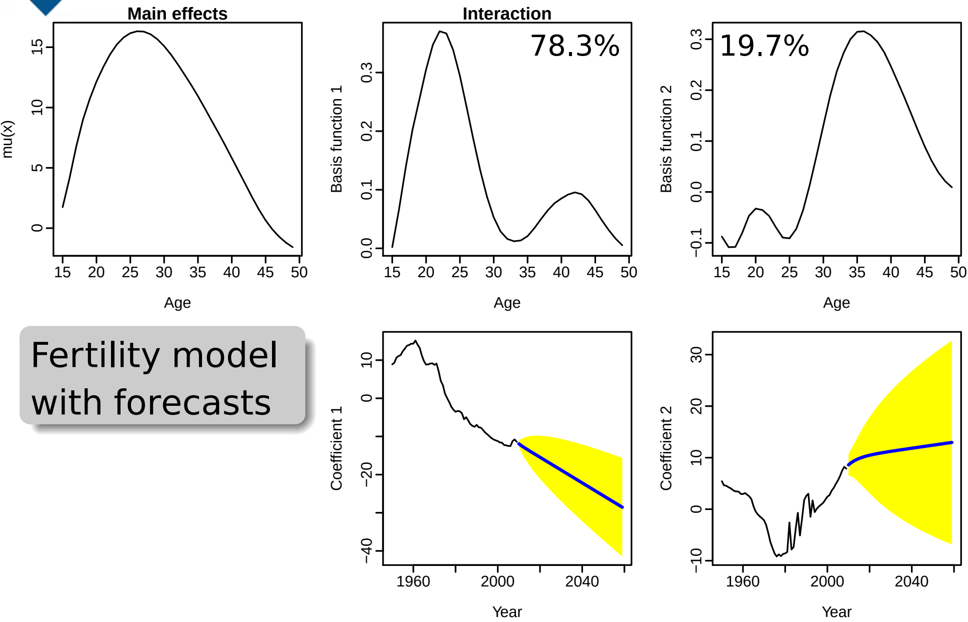

Fertility model with forecasts

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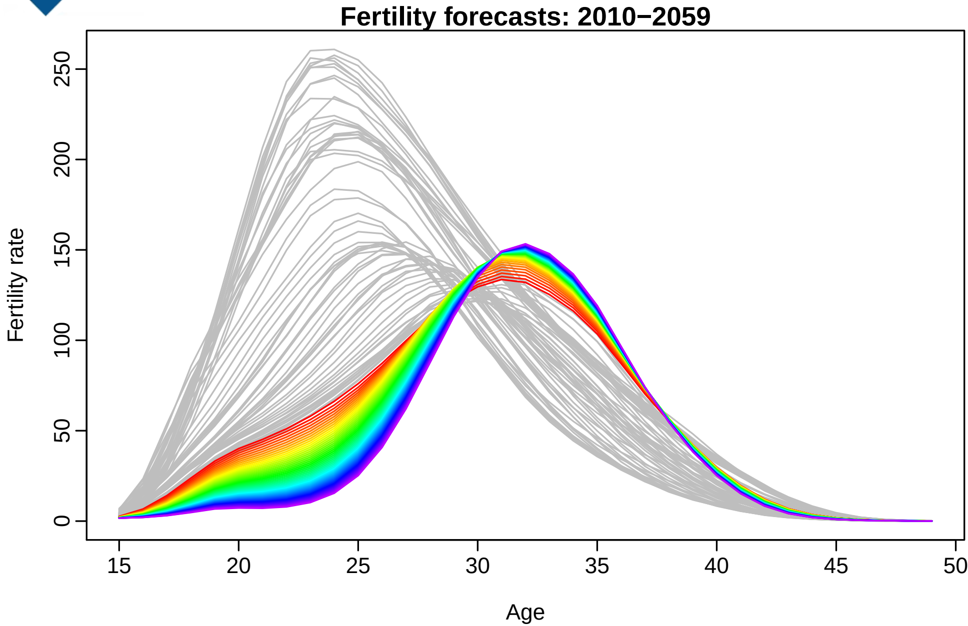

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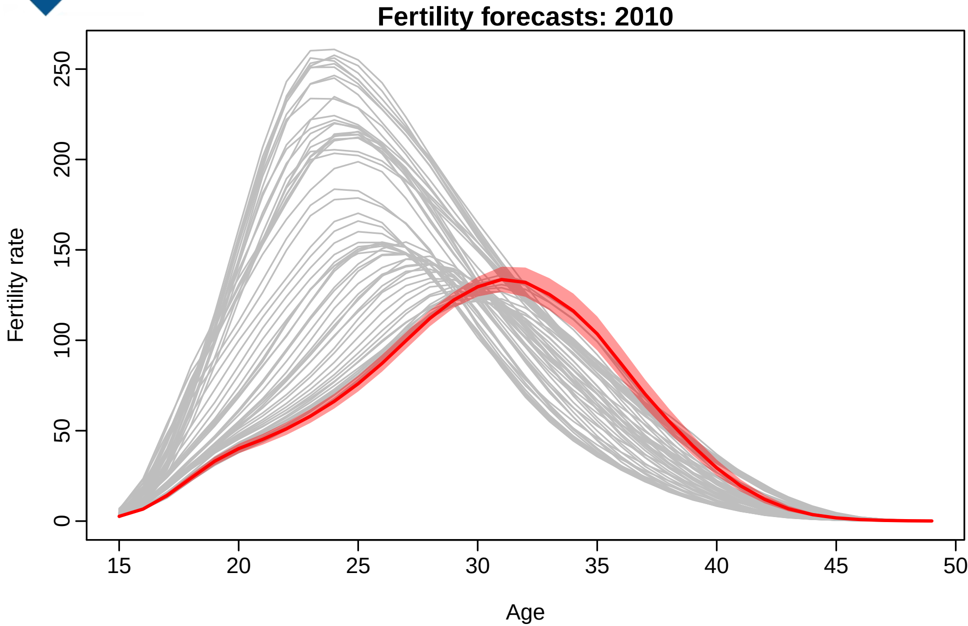

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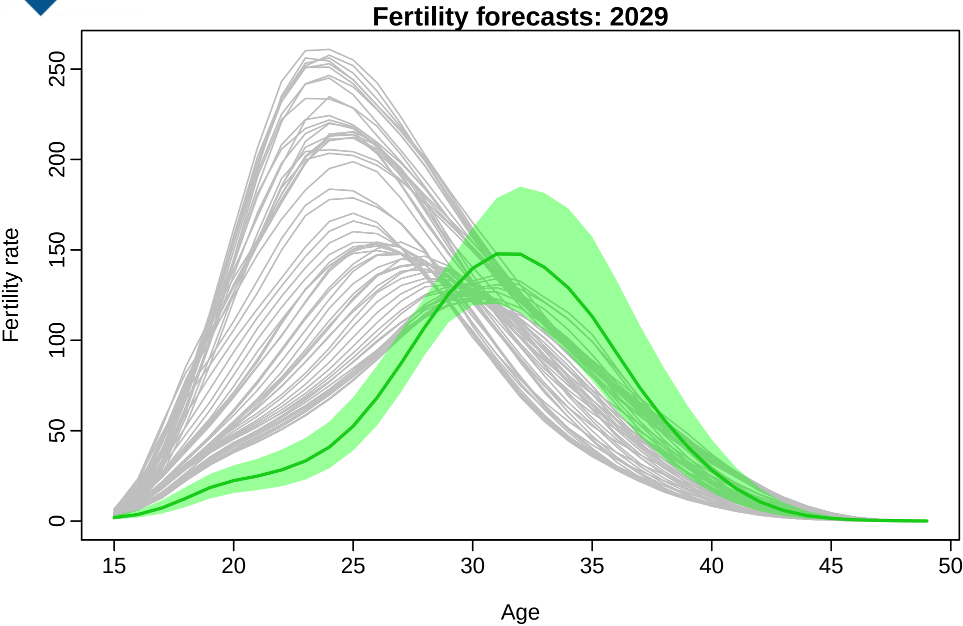
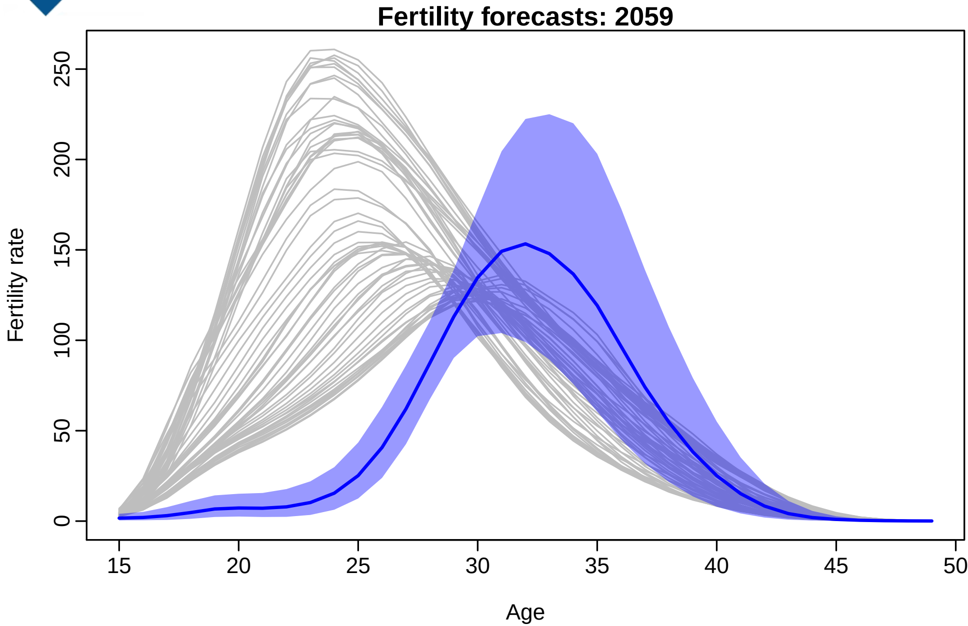


Fertility forecasts



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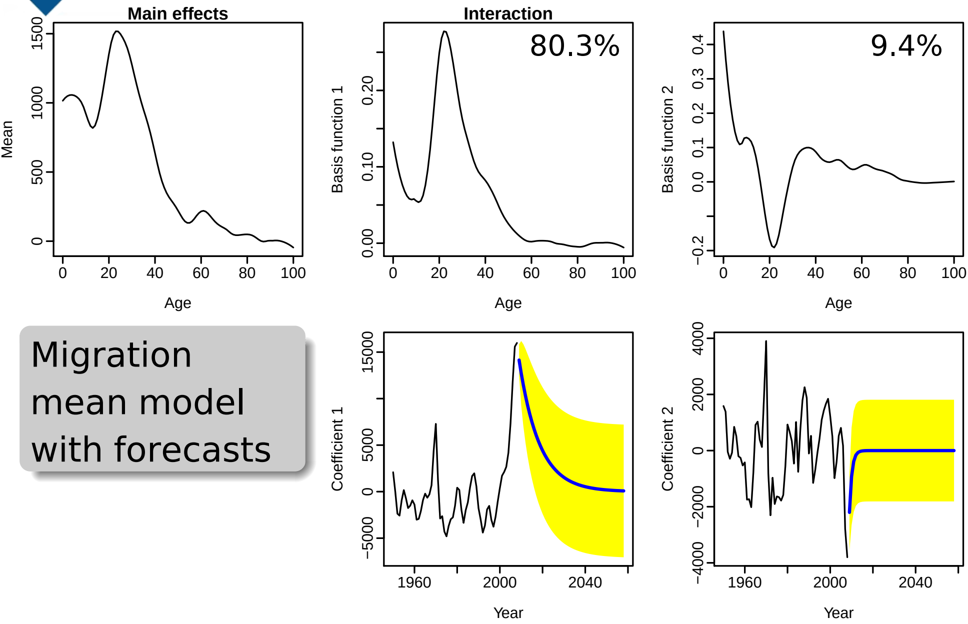
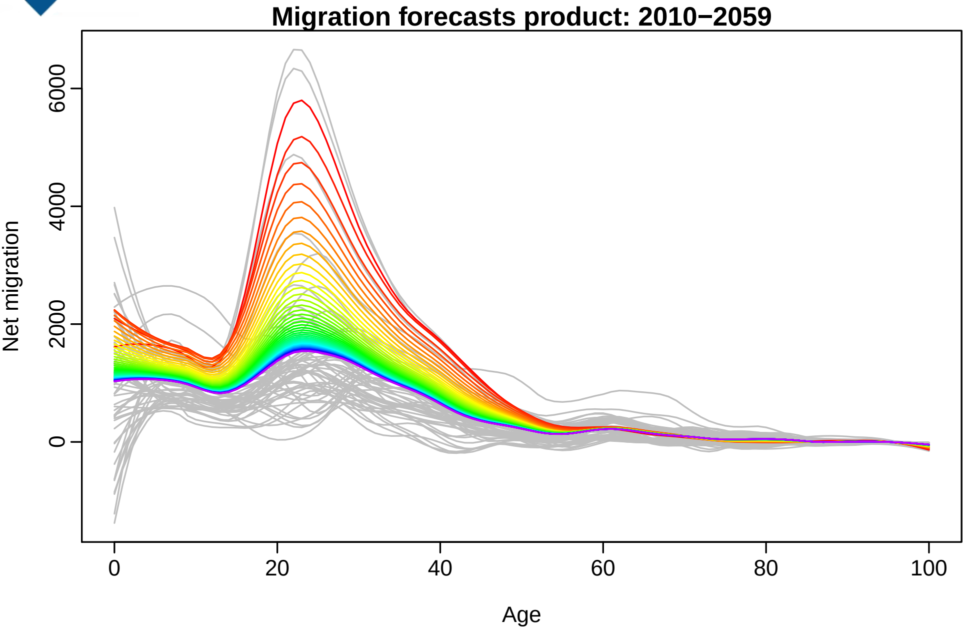


Migration mean model with forecasts

Migration forecasts



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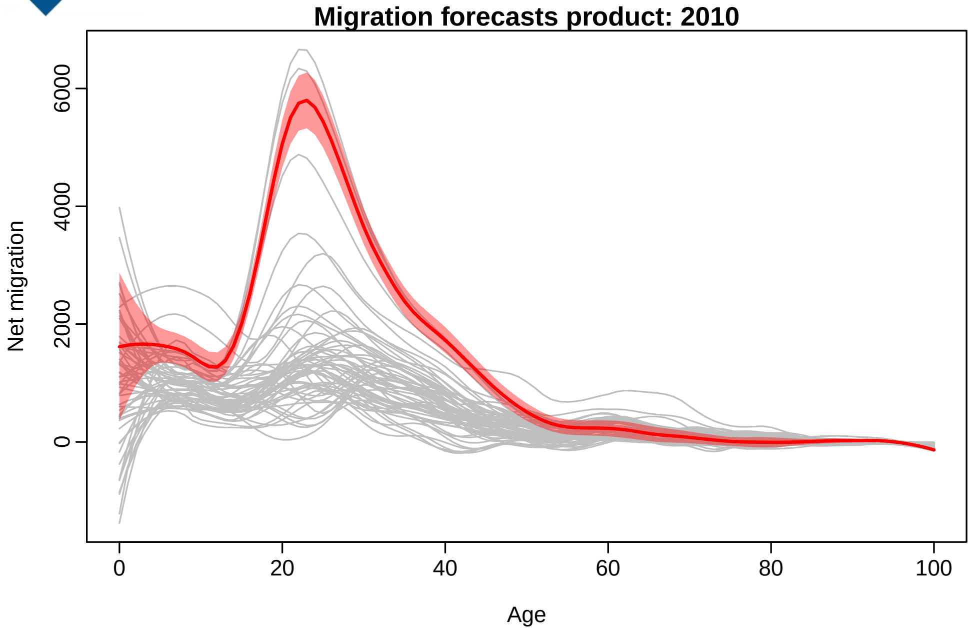

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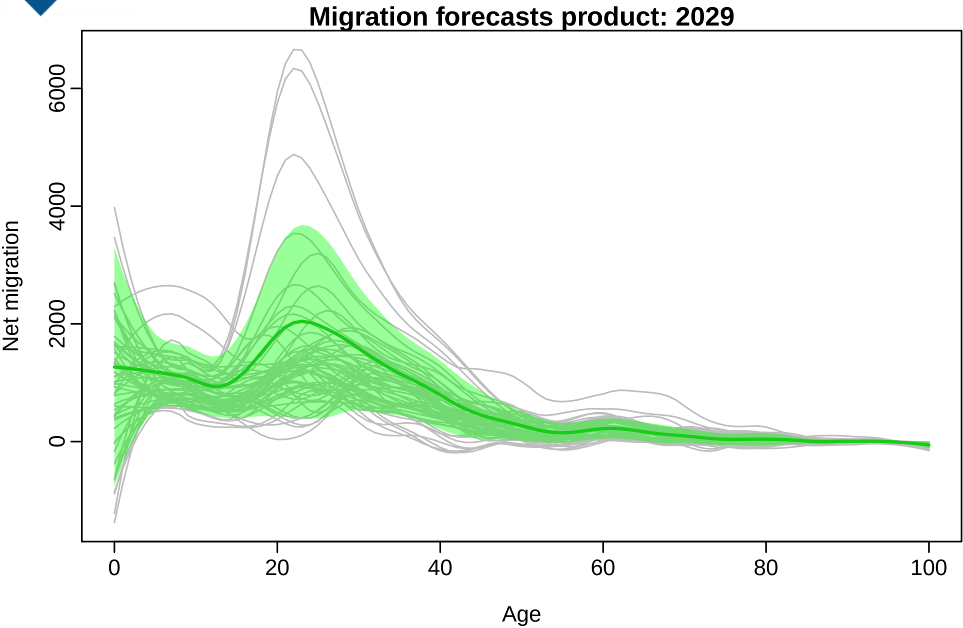

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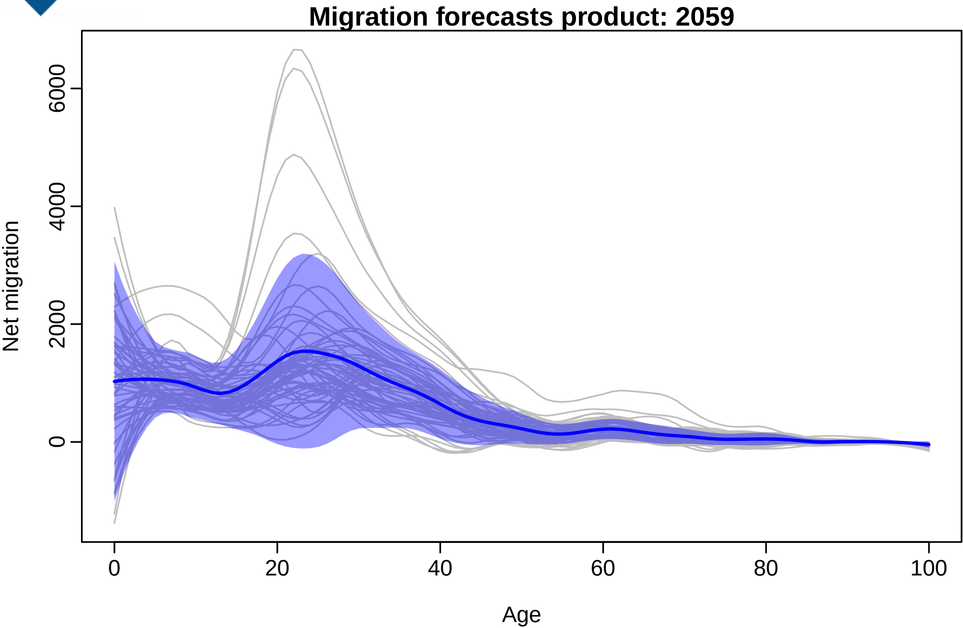

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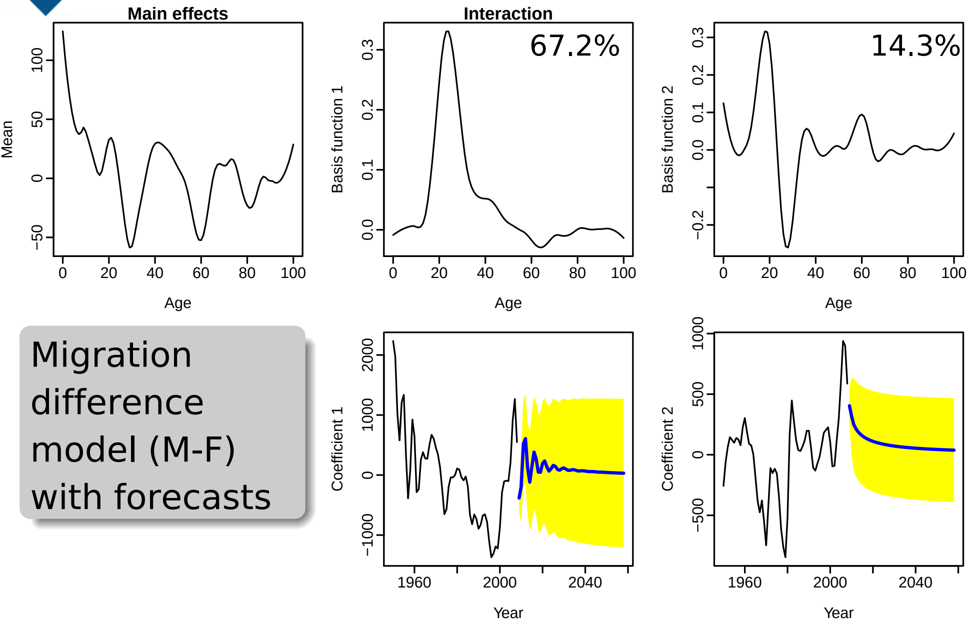

Migration difference model (M-F) with forecasts

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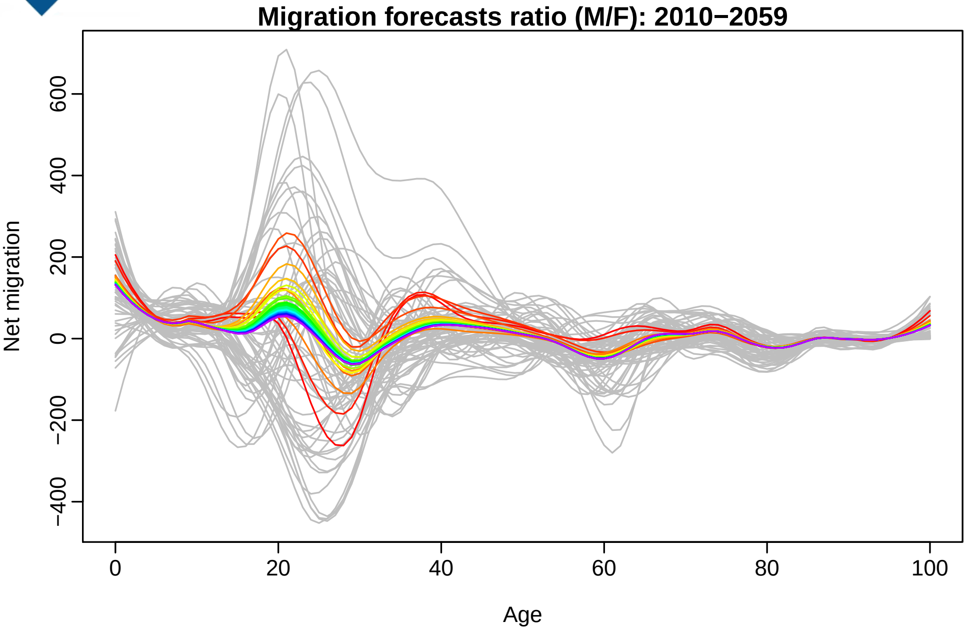

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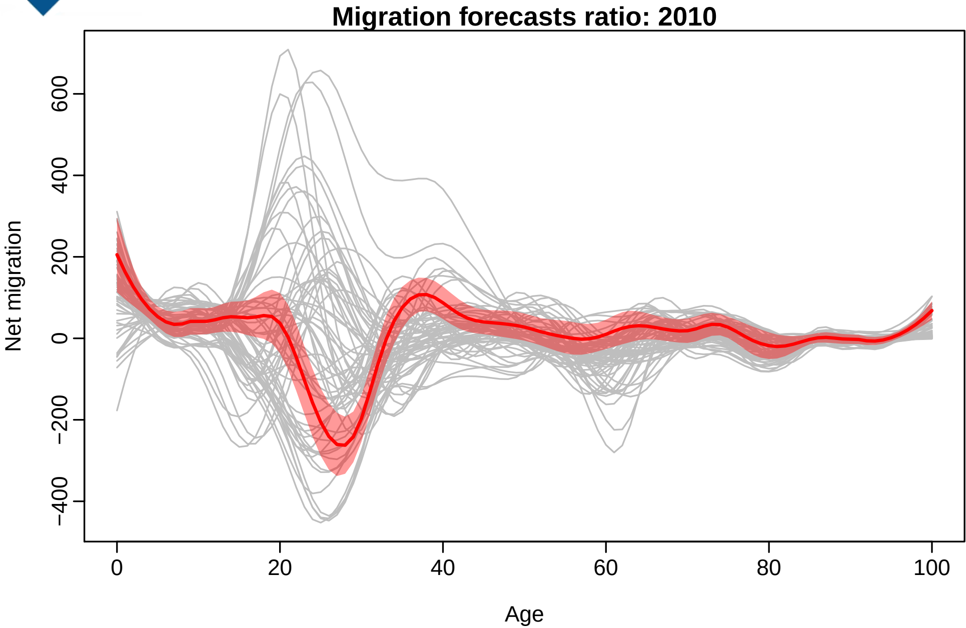

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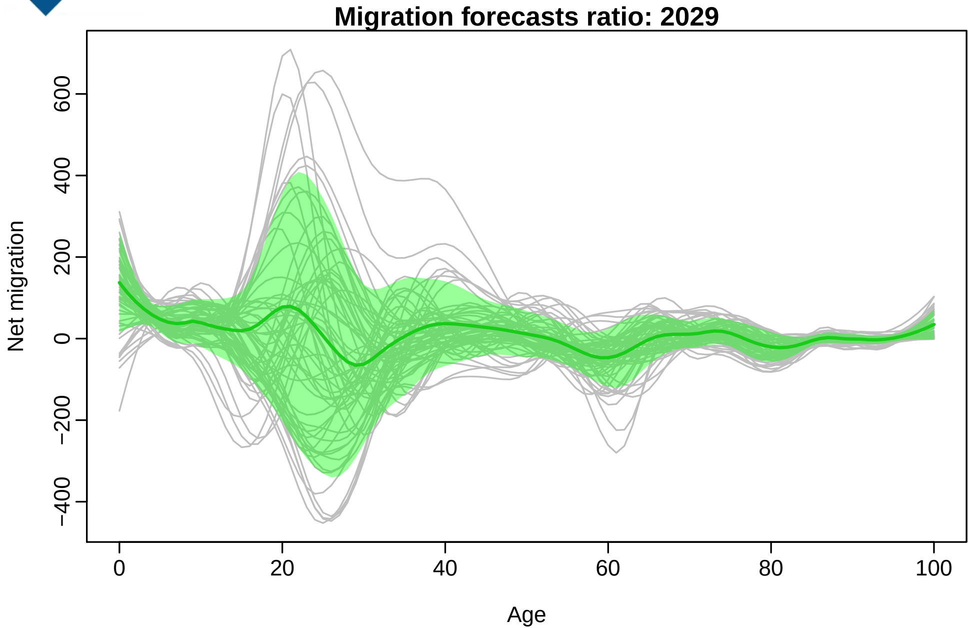

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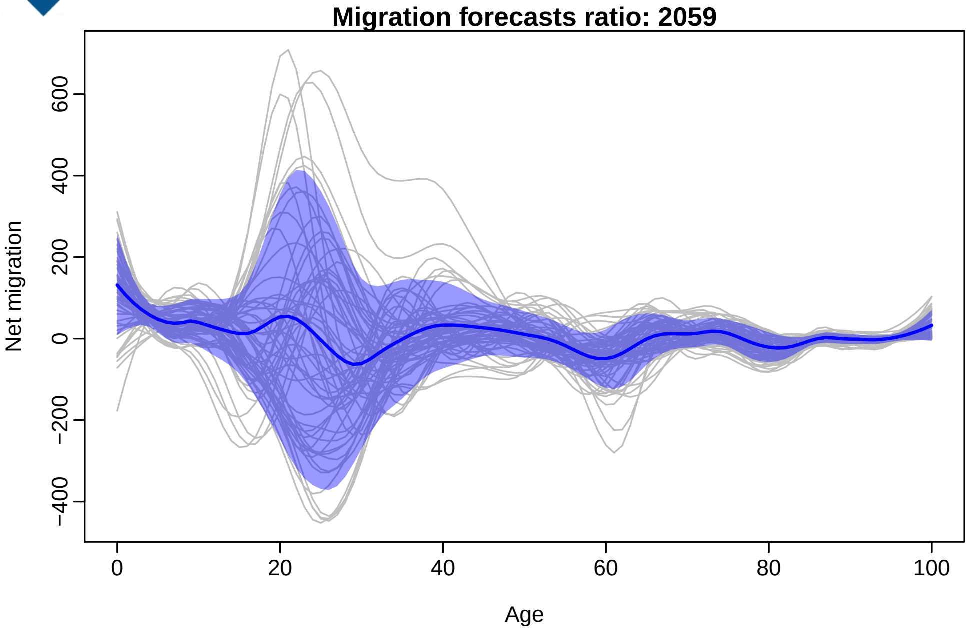

Stochastic population forecasts

Component models

- Data: age/sex-specific mortality rates, fertility rates and net migration.
- Models: Functional time series models for mortality (M/F), fertility and net migration (M/F) assuming independence between components and coherence between sexes.
- Generate random sample paths of each component conditional on observed data.
- Use simulated rates to generate $B_t(x)$, $D_t^F(x, x+1)$, $D_t^M(x, x+1)$ for $t = n+1, \dots, n+h$, assuming deaths and births are Poisson.

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Simulation

Demographic growth-balance equation used to get population sample paths.

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$x = 0, 1, 2, \ldots.$

- 10000 sample paths of population $P_t(x)$, deaths $D_t(x)$ and births $B_t(x)$ generated for $t = 2010, \ldots, 2059$ and $x = 0, 1, 2, \ldots,$.
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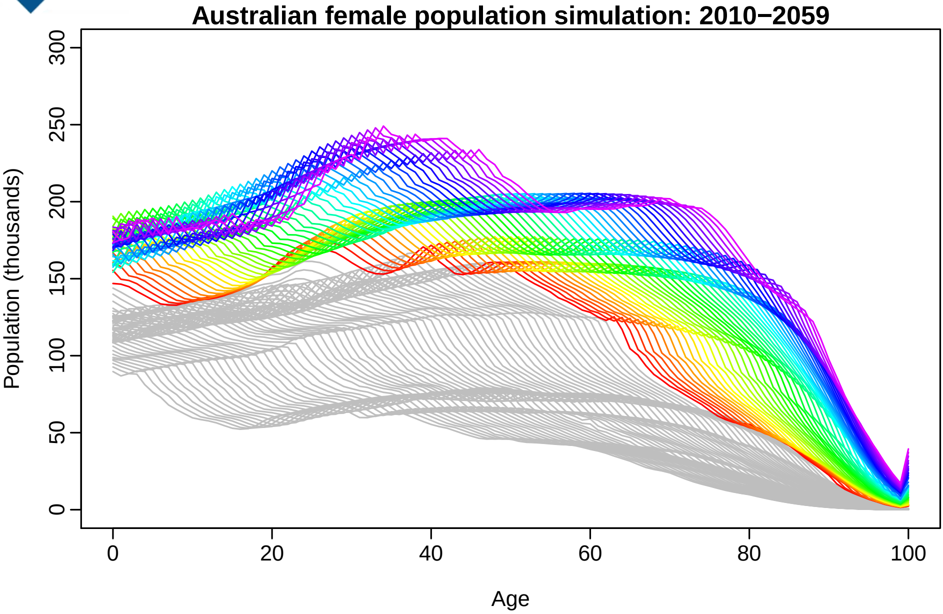
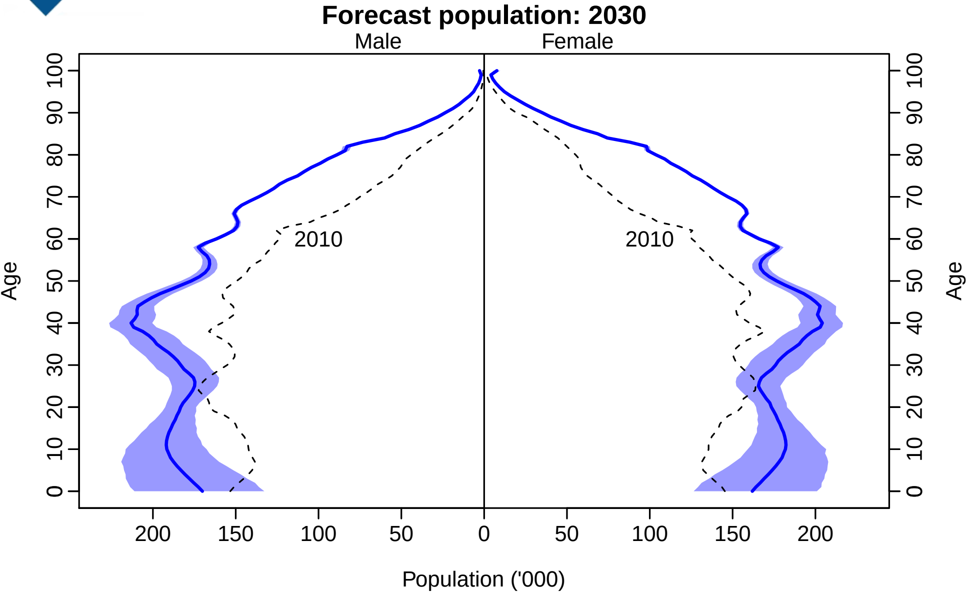
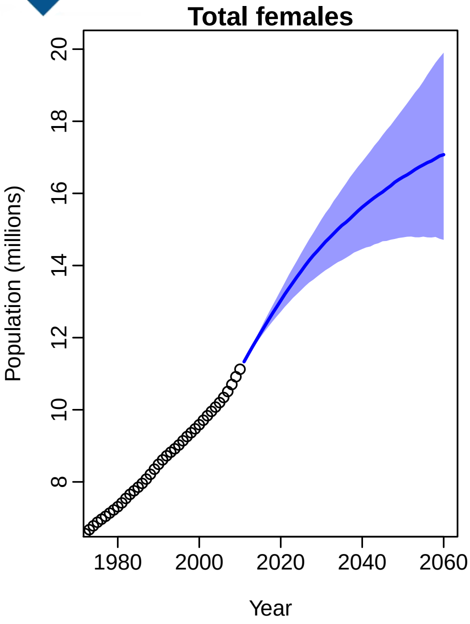
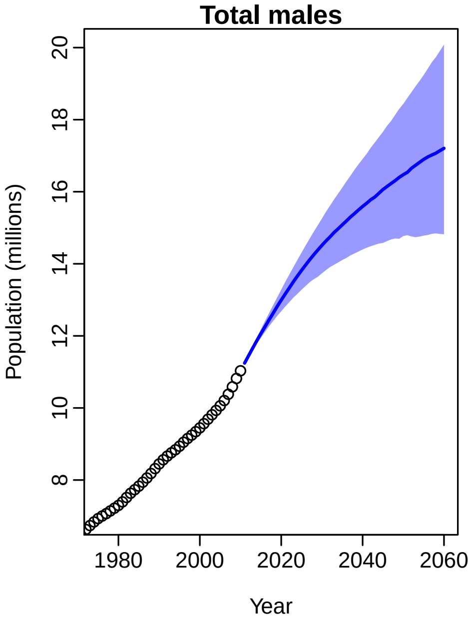

Australian female population simulation: 2010−2059
Population (thousands)
0
50
100
150
200
250
300
Age
0
20
40
60
80
100

Simulation


Forecast population: 2030
Male
Female
2010
2010
Age
Age
Population ('000)
0
10
20
30
40
50
60
70
80
90
100
200
150
100
50
0
50
100
150
200

Simulation


Total females
Population (millions)
8
10
12
14
16
18
20
1980
2000
2020
2040
2060
Year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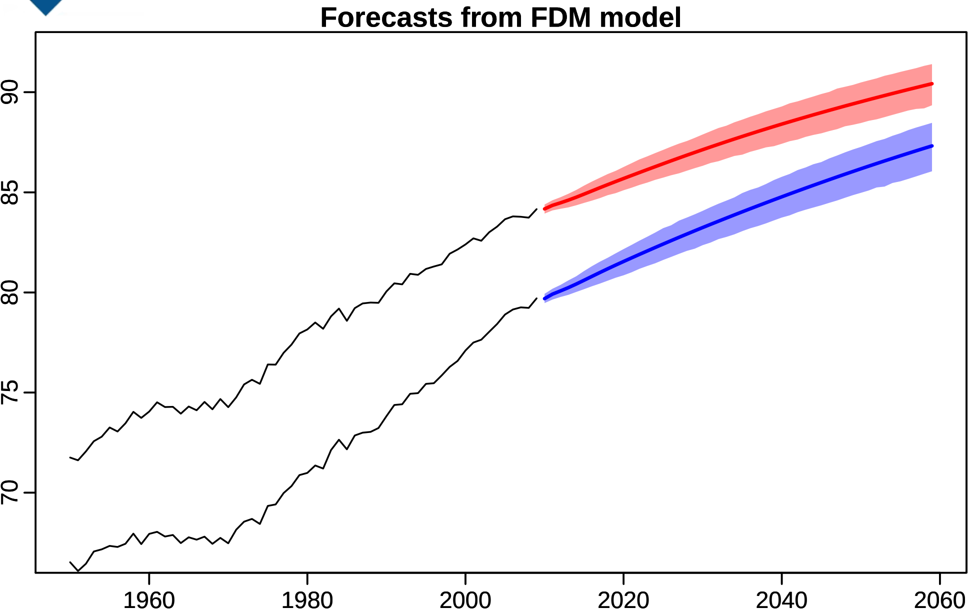
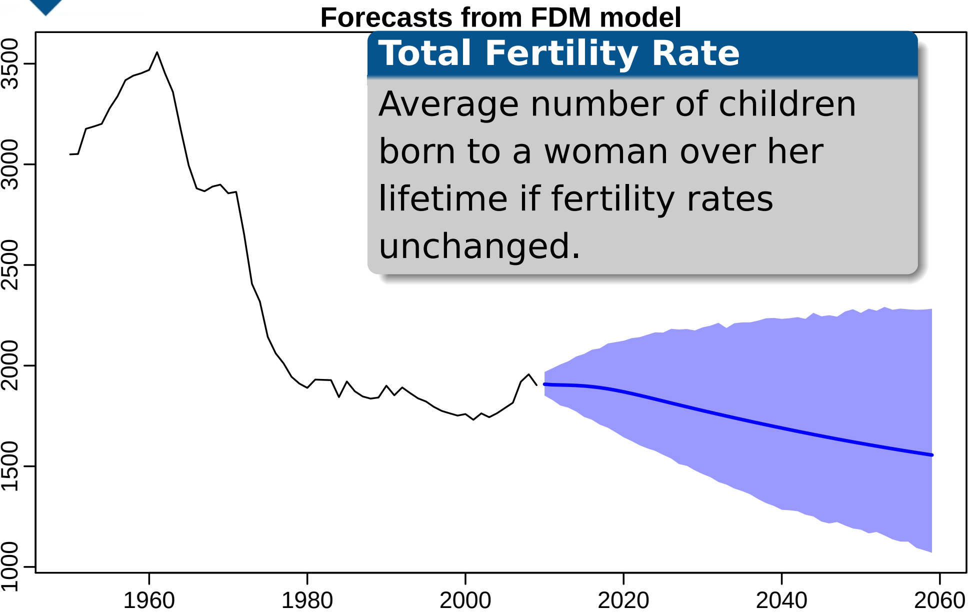
Forecasts of life expectancy at age 0



Forecasts of TFR

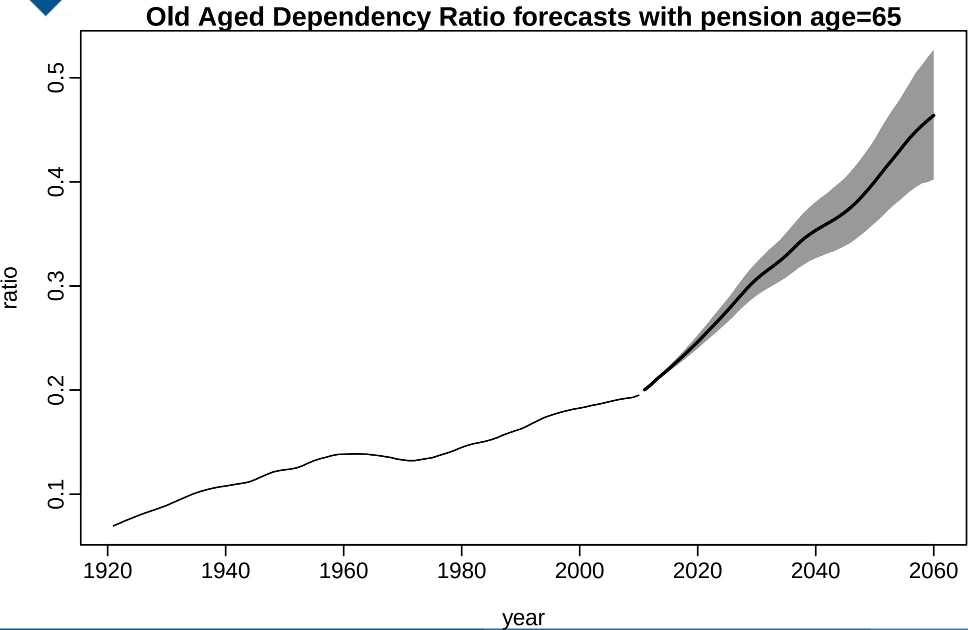


Total Fertility Rate

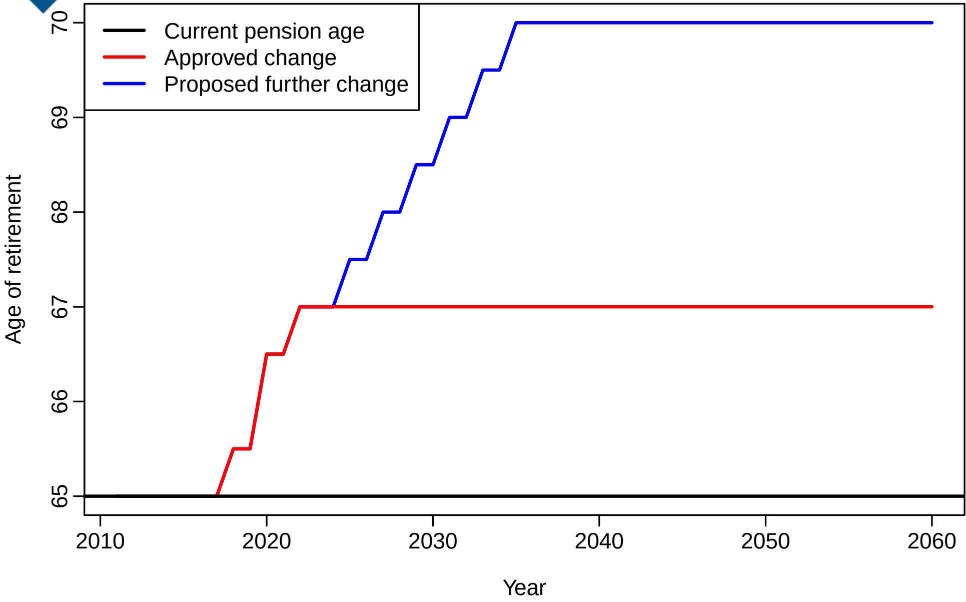
Average number of children born to a woman over her lifetime if fertility rates unchanged.

Old-age dependency ratio

Old-age dependency ratio

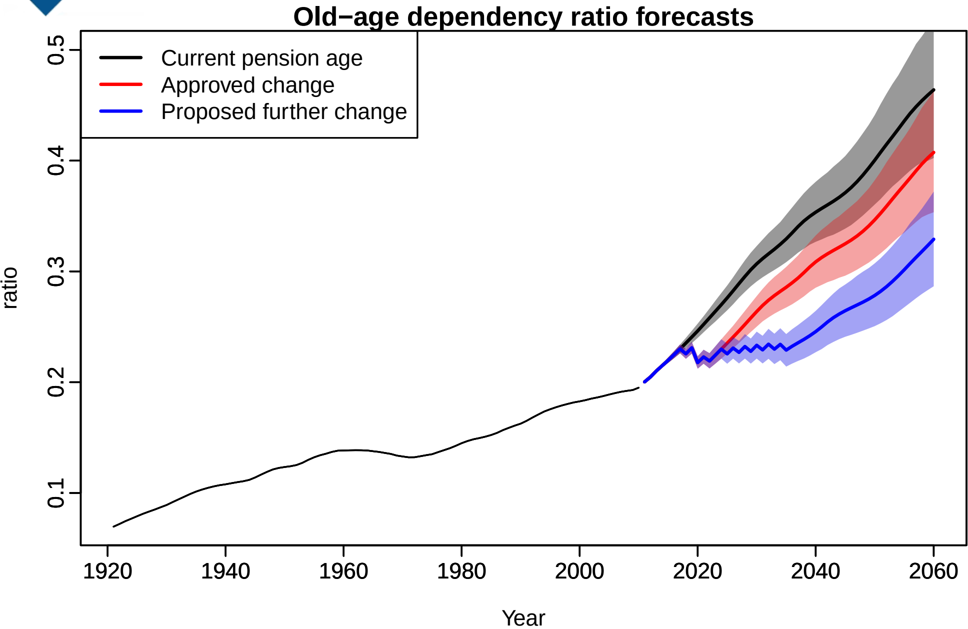


Pension age

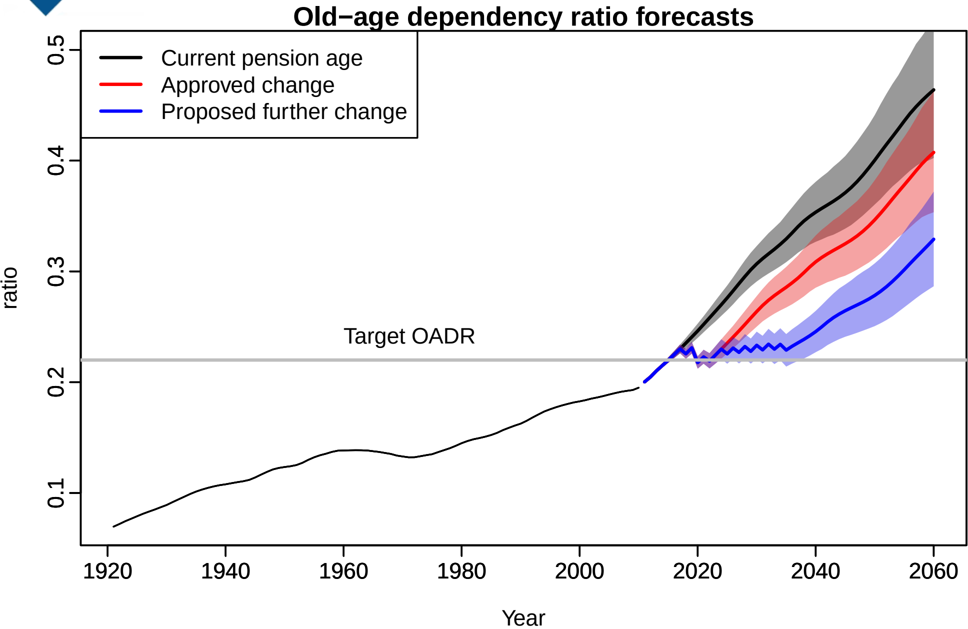


Current pension age
Approved change
Proposed further change
Age of retirement
65
66
67
68
69
70
2010
2020
2030
2040
2050
2060
Year

Pension age

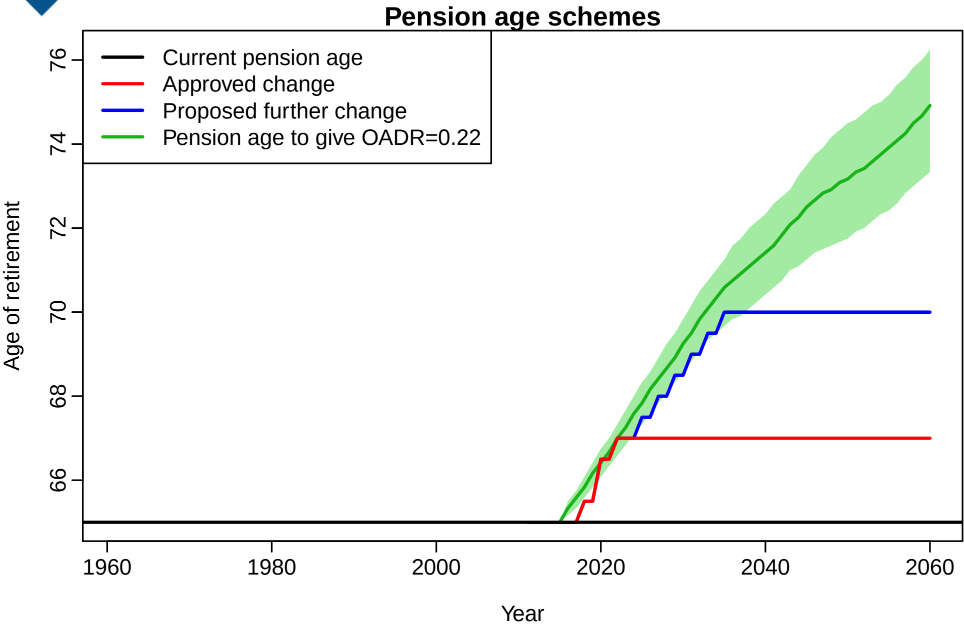

Old−age dependency ratio forecasts
Current pension age
Approved change
Proposed further change
ratio
0.1
0.2
0.3
0.4
0.5
1920
1940
1960
1980
2000
2020
2040
2060
Year

Pension age

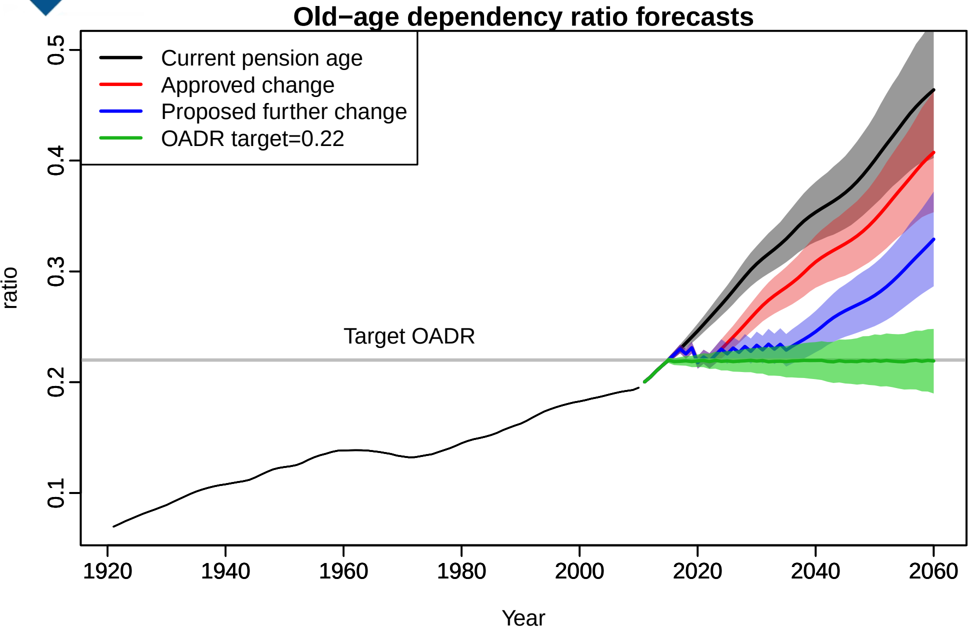


Old−age dependency ratio forecasts
Current pension age
Approved change
Proposed further change
Target OADR
ratio
0.1
0.2
0.3
0.4
0.5
1920
1940
1960
1980
2000
2020
2040
2060
Year

Pension age


Pension age schemes
Current pension age
Approved change
Proposed further change
Pension age to give OADR=0.22
Age of retirement
66
68
70
72
74
76
1960
1980
2000
2020
2040
2060
Year

Pension age


Old−age dependency ratio forecasts
Current pension age
Approved change
Proposed further change
OADR target=0.22
Target OADR
ratio
0.1
0.2
0.3
0.4
0.5
1920
1940
1960
1980
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2020
2040
2060
Year

Advantages of stochastic simulation approach

- Functional data analysis provides a way of forecasting age-specific mortality, fertility and net migration.
- Stochastic age-specific component simulation provides a way of forecasting many demographic quantities with prediction intervals.
- No need to select combinations of assumed rates.
- True prediction intervals with specified coverage for population and all derived variables (TFR, life expectancy, old-age dependencies, etc.)
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Model extensions

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- allow fertility to be modelled using parity data
- allow interaction between fertility and migration?
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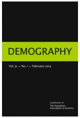
Selected references

Hyndman, Booth (2008). “Stochastic population forecasts using functional data models for mortality, fertility and migration”. *International Journal of Forecasting* **24**(3), 323–342



Hyndman, Booth, Yasmeen (2013). “Coherent mortality forecasting: the product-ratio method with functional time series models”. *Demography* **50**(1), 261–283.

Hyndman (2014). *demography: Forecasting mortality, fertility, migration and population data*. `cran.r-project.org/package=demography`