ASSIGNMENT 1

Due date: May 29, 2024 16:00 (Waterloo time)

WARNING: To receive credit for this assignment, you must check "I agree" to the academic integrity declaration. Please keep all the rules in mind as you complete your work.

Coverage: Modules 1, and 2

Please read the instructions on the reference pages on assignments carefully to ensure that you submit correctly.

Please check the pinned FAQ in the discussion forum for corrections and clarifications. Before answering the questions, please see the section "Other required reading" on the page Course information > Weekly work > Course calendar.

1 Written Component

W1. *[10 marks]* For each of the following expressions, write the expression in order notation. Simplify each expression as much as possible.
Solution:

(a) $100n + n \log n + 10000$

(b) $5m^2 + 3n^2 + 7$

(c) $(4m)(2 \log n)$

(d) $\frac{n(n-1)}{2}$

(e) $3n \times i$, where $i < n$

W2. *[14 marks]*

(a) What does the value returned represent? For this question, M is a Multiset.

```
count(M, item):
    count = 0
    while Is_In(M, item):
        Delete(M, item)
        count = count + 1
    n = 0
    while n < count:
        Add(M, item)
        n = n + 1
    return count
```

Let n represent the number of items in M for parts b and c. Justify your answers.

(b) Express the worst-case running time of *count* in terms of the running times of operations used. Refer to the runtimes of `Is_In` as $I(n)$, `Delete` as $D(n)$, and `Add` as $A(n)$.

(c) For each of the following, give the worst-case running time of `count` in terms of n, where each method has the following run-time.

i. `Is_In` is $\Theta(1)$, `Delete` is $\Theta(1)$, and `Add` is $\Theta(1)$.

ii. `Is_In` is $\Theta(n)$, `Delete` is $\Theta(n)$, and `Add` is $\Theta(n)$.

iii. `Is_In` is $\Theta(\log n)$, `Delete` is $\Theta(n)$, and `Add` is $\Theta(n)$.

iv. `Is_In` is $\Theta(\log n)$, `Delete` is $\Theta(\log n)$, and `Add` is $\Theta(\log n)$.

v. `Is_In` is $\Theta(\log n)$, `Delete` is $\Theta(1)$, and `Add` is $\Theta(1)$.

W3. *[5 marks]* Given the interface for the ADT Multiset from class, write pseudocode for the function `remove(M, item, n)` that removes up to n instances of `item` from M. If there are fewer than n instances of `item`, remove all instances of `item`.

W4. *[8 marks]* Each subquestion gives information about two different implementations of the same operations. In each case, either specify which implementation is preferable and briefly explain why, or explain why there is not enough information available to determine which is a better choice.

(a) A has a worst-case runtime in $O(n \log n)$ and B has a best-case runtime in $\Omega(n)$.

(b) C has a worst-case runtime in $O(\log n)$ and D has a worst-case runtime in $O(n^2)$.

(c) E has a worst-case runtime in $O(n^2)$ and F has a best-case runtime in $\Omega(n^2)$.

(d) G has a worst-case runtime in $O(n^2)$ and H has a best-case runtime in $\Omega(n^3)$.

W5. *[9 marks]*

Analyze the following pseudocode and give a tight (Θ) bound on the running time as a function of n. Show your work. A formal proof is not required, but you should justify your answer (in all cases, n is assumed to be a positive integer).

(a)
```
x = 0
for i = 1 to n + 12 do
   x = x * 4
   for j = 389 to 20100
      for k = 2i to 3i
         x = x * 77
```

(b)
```
x = 0
for i = 1 to ceiling(log(n))
   for j = 1 to i
      for k = 1 to 10
         x = x + 1
```

(c)
```
s = 0;
for i = 1 to n*n do
   for j = 1 to i*i do
      s = s + 1;
```