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MCDM based on Reciprocal Judgment Matrix: a Comparative Study of E-VIKOR and E-TOPSIS Algorithmic Methods with Interval Numbers

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Abstract: Interval number is a useful tool to handle the uncertainty brought by human factors in multi-criteria decision-making (MCDM) process. When faced with MCDM problems in real-world, the experts may be uncomfortable giving precise upper and lower bounds of the interval ratios on multi-criteria. Based on the reciprocal judgment matrices given by the experts through pairwise comparisons, the satisfaction degree of the multiple alternatives on single criterion is defined and the interval ratios was elicited by a linear programming model. The TOPSIS and VIKOR methods are extended with interval number and algorithmic E-VIKOR and E-TOPSIS methods are proposed. Finally, a numerical example of ranking indirect-fire weapon system alternative is given and a comparative experimental study is carried out based on the experts reciprocal judgment matrices generated by Monte Carlo simulation. The result illustrates the feasibilities and distinctive features of the two algorithmic methods.

Keywords: Multi-criteria decision-making (MCDM), reciprocal judgment matrix, E-VIOKR, E-TOPSIS, interval numbers

1 Introduction

Multi-Criteria Decision-Making (MCDM), also called as Multi-Criteria analysis, is often applied in the decision-making with multiple objectives in the field of Operations Research [1,2,3]. Especially, these objectives are conflicting with each other under the preference structure supplied by the decision-maker. The fundamentals of MCDM can be described as follows [1]: (1) construction of evaluation criteria related with decision-making goals for the alternatives; (2) generation of alternatives for achieving the decision-making goals; (3) computation of the alternatives value by the value functions for multiple criteria; (4) application of a normalized Multi-Criteria Analysis (MAC) methods; (5) searching the optimal alternative as the final decision-making result; (6) if the final alternative is unacceptable, multi-criteria optimization process is necessary to be carried out.

There are many literatures correlate with the MCDM problems and the techniques for these problems in various applications [1,2,3,4,5,6,7,8,9]. These methods were divided into four categories by Guitoni and Martel given as follows [4,5]: (1) elementary approaches, including Lexicographic method, weighted sum, Disjunctive method, Conjunctive methods and Maxi-min method; (2) the single synthesizing criterion methods, including multi-attribute value theory (MAVT),TOPSIS, simple multi-attribute rating technique (SMART),UTA (utility theory additive), MAUT (multi-attribute utility theory), EVAMIX,AHP (analytical hierarchy process), Fuzzy maxi-min and Fuzzy weighted sum; (3) the outranking synthesizing methods, including PROMETHEE, ELECTRE, ORESTE ,MELCHIOR and REGIME; and (4) the mixed methods, including Fuzzy conjunctive method , Fuzzy disjunctive method, QUALIFLEX and Martel and Zaras method.

As a commonly used classical MCDM method with cardinal information, TOPSIS accounts for a ratio scale on the multiple criteria given by the experts as AHP matrix [6]. In TOPSIS, the importance weights of multiple criteria and the judgment ratios of alternatives under multiple criteria are given by crisp values, and both weights and ratios are normalized into indices without dimensions for the consequential aggregation [1]. The main principle of TOPSIS is that the optimal alternatives

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should has the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS), and from the distances of which the preference order ranking of the alternatives was derived for the final decision-making [2]. In the recent research, TOPSIS method is widely used in various fields, including material selection [7], energy project [4], and supply chain management [8], and achieved a lot. However, an obvious drawback of TOPSIS is that it only focuses on the distances of the criteria value from the PIS and NIS without the relative importance of these distances. As one feasible and applicable method to implement within MCDM, the VIKOR approach was introduced [2] for multi-criteria optimization problem of complex alternatives and received a broad acceptance. Based on conflicting and different dimensions criteria, VIKOR method compares the closeness of all the alternatives with ideal alternative and performs a compromise ranking with mutual concessions.

Because of the different normalization methods and different aggregation functions used by the TOPSIS and VIKOR methods, a detailed and in-depth comparative analysis of the original TOPSIS and VIKOR was carried out by Opricovic and Tzeng [1]. Besides, TOPSIS and VIKOR are examined as two different MCDM methods by Reza Raei for some observation data samples from Tehran Stock Exchange to search for an appropriate alternative [9]. Besides, the E-VIKOR method with TOPSIS method was developed to help the decision-maker decide the optimal projects developmental strategy [10]. For TOPSIS and VIKOR method, the importance weights of multiple criteria and the judgment ratios of alternatives under multiple criteria are difficult to be given by crisp values for the experts when little information for judgment is available. Handling the uncertainty by interval number is receiving considerable attention by the recent researchers. The Extended VIKOR method for decision making problem with interval numbers by M. K. Sayadi [11] is compared with the extended TOPSIS method proposed by Jahanshahloo [3] in obtaining the compromise solution.

However, in fact, Camerer and Weber [12] suggest that an expert may be uncomfortable giving such precise upper and lower bounds of the interval ratios on multi-criteria. Yager and Kreinovich proposed [13] a formulation to obtain the upper and lower bounds of the interval ratios in a statistical method. Moreover, Guo [14] studied the Linear programming model for estimating and combining interval ratios based on pairwise subjective comparisons of the possibilities of the events.

The contributions of this paper are summarized as follows: based on the reciprocal judgment matrices given by the experts with regard to the satisfaction degree of the multiple alternatives on single criterion, the interval ratios are elicited by a linear programming model. The TOPSIS and VIKOR methods are extend for MCDM problems with interval number and algorithmic E-VIKOR and E-TOPSIS methods are examined by the experts reciprocal judgment matrices generated by Monte Carlo simulation.

The rest of the paper is organized as follows. Section 2 presents the elicitation of interval probability from reciprocal judgment matrix by pairwise comparisons. The extended VIKOR with interval numbers for MCDM, E-VIKOR and its algorithmic process are given in Section 3. The extended TOPSIS with interval numbers for MCDM, E-TOPSIS and its algorithmic process are given in Section 4. An illustrative numerical example examines the two algorithmic methods and clarifies the main experimental results. Conclusions and future work are drawn in Section 6.

2 Elicitation of interval probability from reciprocal judgment matrix by pairwise comparisons

In this section, Alternative Satisfaction Level (ASL) is proposed to indicate the comparative satisfaction level of the alternative with multi-criteria, which has two components: an alternative and a certain criterion, its expression is given as a function with two parameters.

Definition 1. Alternative Satisfaction Level (ASL), denoted as $ASL(S_t,C_u)$, is an index indicating the judgment ratio of an alternative, S_t, on a criterion, C_u, when the alternative is checked and measured in the choosing process by the experts.

Because of the unavoidable uncertainty of the prediction with limited information, $ASL(S_t,C_u)$ is an estimated value, based on the experiential knowledge of experts. It can be denoted as an interval number as follows:

$$ASL(S_t,C_u) = [L_i^{C_u-}, L_t^{C_u+}] \tag{1}$$

where the upper and lower bounds of $ASL(S_t,C_u)$ are $L_t^{C_u+}$ and $L_t^{C_u+}$, restricted by the following inequalities:

$$0 \le L_t^{C_u-} \le L_t^{C_u+} \le 1 \tag{2}$$

It is clear that $ASL(S_t,C_u) \in [0,1]$. If $L_t^{C_u-} = L_t^{C_u+}$, $ASL(S_t,C_u)$ degenerates into a real number. And the center and width of the interval probability, $ASL(S_t,C_u)$ are respectively defined as follows [15]:

$$m(ASL(S_t,C_u)) = \frac{1}{2}(L_t^{C_u-} + L_t^{C_u+}) \tag{3}$$

$$w(ASL(S_t,C_u)) = L_t^{C_u+} - L_t^{C_u-} \tag{4}$$

Considering that there are $k(k \le m)$ alternatives in a set of candidate alternatives $S^* = \{S_t, t = 1,2,\ldots,m\}$, having the corresponding functions to satisfy a certain criterion, C_u. For all the k alternatives, there are interval probability sets $ASL(S,C_u)^*$ containing k elements

$$ASL(S,C_u)^* = \{ASL(S_t,C_u) = [L_t^{C_u-}, L_t^{C_u+}], \\ t = 1,2,K,m, u = 1,2,K,n\} \tag{5}$$

which represent all the possible $ASL(S_t,C_u)$ of the candidate alternatives with the requirement for capability C_u. Therefore, Definition 2 is given as follows [14,16,17]:

Definition 2. For $\forall L_t^{C_u} \in [L_t^{C_u-}, L_t^{C_u+}]$, there is an equation, $\sum_{t=1}^{k} L_t^{C_u} = 1$.

Theorem 1. The interval set $ASL(S,C_u)^*$ satisfies Definition 5 if and only if, the following conditions hold [14,18]:

$$L_t^{C_u+} + L_1^{C_u-} + \ldots + L_{t-1}^{C_u-} + L_{t+1}^{C_u-} + \ldots + L_k^{C_u-} \\ \leq 1, \forall t = 1,2,\ldots,k. \tag{6}$$
$$L_t^{C_u-} + L_1^{C_u+} + \ldots + L_{t-1}^{C_u+} + L_{t+1}^{C_u+} + \ldots + L_k^{C_u+} \\ \geq 1, \forall t = 1,2,\ldots,k.$$

Proof: The sufficient condition: If Definition 2 holds, that is,

$$\sum_{t=1}^{k} L_t^{C_u} = 1 \Leftrightarrow 1 \leq \sum_{t=1}^{k} L_t^{C_u} \leq 1 \tag{7}$$

Then we have

$$\forall L_t^{C_u} + L_1^{C_u-} + \ldots + L_{t-1}^{C_u-} + L_{t+1}^{C_u-} + \ldots + L_k^{C_u-} \\ \leq L_t^{C_u} + L_1^{C_u} + \ldots + L_{t-1}^{C_u} + L_{t+1}^{C_u} + \ldots + L_k^{C_u} = 1, \tag{8}$$
$$\forall L_t^{C_u} + L_1^{C_u+} + \ldots + L_{t-1}^{C_u+} + L_{t+1}^{C_u+} + \ldots + L_k^{C_u+} \\ \geq L_t^{C_u} + L_1^{C_u} + \ldots + L_{t-1}^{C_u} + L_{t+1}^{C_u} + \ldots + L_k^{C_u} = 1,$$

Since

$$L_t^{C_u} \in [L_t^{C_u-}, L_t^{C_u+}] \tag{9}$$

It is easy to check and see that

$$L_t^{C_u+} + L_1^{C_u-} + \ldots + L_{t-1}^{C_u-} + L_{t+1}^{C_u-} + \ldots + L_k^{C_u-} \\ \leq 1, \forall t = 1,2,\ldots,k, \tag{10}$$
$$L_t^{C_u-} + L_1^{C_u+} + \ldots + L_{t-1}^{C_u+} + L_{t+1}^{C_u+} + \ldots + L_k^{C_u+} \\ \geq 1. \forall t = 1,2,\ldots,k.$$

This proves that Definition 2 is a sufficient condition of Theorem 1.

The necessary condition: If Theorem 1 holds,

According to Eq. (1)

$$L_t^{C_u-} \leq L_t^{C_u} \leq L_t^{C_u+}$$

Then we have [14]

$$\forall t : L_t^{C_u} + L_1^{C_u-} + \ldots + L_{t-1}^{C_u-} + L_{t+1}^{C_u-} + \ldots + L_k^{C_u-} \\ \leq L_t^{C_u+} + L_1^{C_u-} + \ldots + L_{t-1}^{C_u-} + L_{t+1}^{C_u-} + \ldots + L_k^{C_u-} \leq 1 \tag{11}$$
$$\forall t : L_t^{C_u} + L_1^{C_u+} + \ldots + L_{t-1}^{C_u+} + L_{t+1}^{C_u+} + \ldots + L_k^{C_u+} \\ \geq L_t^{C_u-} + L_1^{C_u+} + \ldots + L_{t-1}^{C_u+} + L_{t+1}^{C_u+} + \ldots + L_k^{C_u+} \geq 1$$

hold.

Thus,

$$L_t^{C_u} + L_1^{C_u-} + \ldots + L_{t-1}^{C_u-} + L_{t+1}^{C_u-} + \ldots + L_k^{C_u-} \leq 1 \\ \leq L_t^{C_u} + L_1^{C_u+} + \ldots + L_{t-1}^{C_u+} + L_{t+1}^{C_u+} + \ldots + L_k^{C_u+} \tag{12}$$

Clearly, there exists $L_h^{C_u-} \leq L_h^{C_u} \leq L_h^{C_u+}$, $h \in \{1,2,\ldots,k\}$, $h \neq t$, which satisfies the above condition. So,

$$\sum_{t=1}^{k} L_t^{C_u} + (k-1)(L_1^{C_u-} + L_2^{C_u-} \ldots + L_k^{C_u-}) \leq k \\ \leq \sum_{t=1}^{k} L_t^{C_u} + (k-1)(L_1^{C_u+} + L_2^{C_u+} + \ldots + L_k^{C_u+}) \tag{13}$$

Which we can translate into the following:

$$\sum_{t=1}^{k} L_t^{C_u}(k-1) = k - \sum_{t=1}^{k} L_t^{C_u} \Leftrightarrow \sum_{t=1}^{k} L_t^{C_u} = 1 \tag{14}$$

This proves that Definition 2 is a sufficient condition of Theorem 1.

Definition 3. The First-Ignorance of $ASL(S,C_u)^*$ denoted as $I^1(ASL(S,C_u)^*)$, is defined by the sum of the width of the intervals as follows [14,18]:

$$I^1(ASL(S,C_u)^*) = \frac{1}{k}\sum_{t=1}^{k} w(ASL(S_t,C_u)) \\ = \frac{1}{k}\sum_{t=1}^{k}(L_t^{C_u+} - L_t^{C_u-}) \tag{15}$$

Similar definitions and theorems have been used in the literature [16,17] as the constraints and operations of the interval probability.

When there is little information available for the experts to predict $SSL(S_t,C_u)$, a precise estimation of $L_t^{C_u-}$ and $L_t^{C_u+}$ is difficult to achieve. In fact, an expert is more comfortable stating a personal preference towards a set of alternatives by means of pairwise comparison and determining which one has more possibility to satisfy a certain criterion. Wang [19] introduces a goal programming model to obtain interval weights from imprecise preference in MCDA. Guo [14] elicits the interval-valued probabilities, based on a linear and quadratic programming model from subjective pairwise comparisons for the likelihood among several events. The First ignorance model based on linear programming approach is employed in this study to minimize the imprecision of pairwise comparisons.

Considering the pairwise comparison process for each pair of candidate alternatives in a finite set $S^* = \{S_t, t = 1,2,\ldots,m\}$, the possible judgment score from experts on alternative S_t and S_h $(t,h \in N^+, t,h \le m)$ is denoted as a_{th}, which is an integer number [1,9].

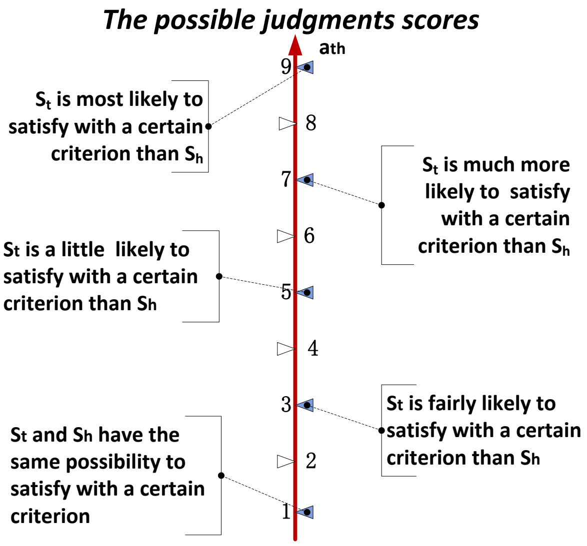

Fig. 1: The possible judgment score axes and the explanations of the scores

As shown in Fig. 1, $a_{th} = 1$ represents that S_t and S_h have the same possibility to satisfy a certain criterion, $a_{th} = 3$ indicates that S_t is more likely to satisfy a certain criterion than S_h, $a_{th} = 5$ means that S_t is a little more likely to satisfy a certain criterion than S_h, $a_{th} = 7$ denotes that S_t is much more likely to satisfy a certain criterion than S_h and when $a_{th} = 9$, S_t is most likely to satisfy a certain criterion. The other numbers 2, 4, 6 and 8 are used analogically. Additionally, an assumption must be noted to explain the relationship between a_{th} and a_{ht} [14,18].

Assumption 1. The comparison results a_{th} and a_{ht} hold the condition $a_{th} \times a_{ht} = 1$.

This means that when an expert makes a comparison, it is not possible that in the first comparison, S_t will be more likely to satisfy a certain criterion than S_h, but in the second, S_h and S_t have the same possibility to satisfy a certain criterion.

Next, we have a $k \times k$ comparison matrix $A(C_u)$ with regard to k alternatives, which can satisfy a certain criterion, C_u, requirement as follows [14,18]:

$$A(c_j)_{k\times k} = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1k} \\ 1/a_{12} & 1 & \cdots & a_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ 1/a_{1k} & 1/a_{2k} & \cdots & 1 \end{bmatrix} \tag{16}$$

Assumption 2. Even if there are multiple experts, there is only one $k \times k$ comparison matrix $A(C_u)$ for a certain criterion, C_u, which represents the integrated preference information after a conference discussion.

The interval ratio $ASL(S_t,C_u)/ASL(S_h,C_u)$ can be calculated by interval arithmetic as follows [14,20]:

$$ASL(S_t,C_u)/ASL(S_h,C_u) = [L_1^{C_u-}/L_h^{C_u+}, L_t^{C_u+}/L_h^{C_u-}] \tag{17}$$

Assumption 3. [14,20,21] the given pairwise comparison a_{th} should belong to the estimated interval ratio $ASL(S_t,C_u)/ASL(S_h,C_u)$, that is

$$a_{th} \in [L_t^{C_u-}/L_h^{C_u+}, L_t^{C_u+}/L_h^{C_u-}] \tag{18}$$

$$\Leftrightarrow L_t^{C_u} - /L_h^{C_u+} \le a_{th} \le L_t^{C_u+}/L_h^{C_u-} \tag{19}$$

$$\Leftrightarrow \begin{cases} L_t^{C_u-} - a_{th}L_h^{C_u+} \le 0, \\ L_t^{C_u+} - a_{th}L_h^{C_u-} \ge 0, \\ L_t^{C_u-} \ge \varepsilon \end{cases} \tag{20}$$

where ε is a very small positive real number.

To determine the interval set $ASL(S,C_u)^*$ with the smallest First ignorance (Definition 3), obtaining the interval probabilities from the expert opinion can be derived by the following optimization model [14,18]:

$$\begin{aligned} &\min_{L_t^{C_u+}, L_t^{C_u-}} I^1(ASL(S,C_u)^*) = \frac{1}{k}\sum_{t=1}^{k}(L_t^{C_u+} - L_t^{C_u-}) \\ &s.t. L_t^{C_u+} + L_1^{C_u-} + \ldots + L_{i-1}^{C_u-} + L_{i+1}^{C_u-} + \ldots + L_k^{C_u-} \le 1 \\ &\forall t = 1,2,\ldots,k, \\ &L_t^{C_u-} + L_1^{C_u+} + \ldots + L_{t-1}^{C_u+} + L_{t+1}^{C_u+} + \ldots + L_k^{C_u+} \ge 1, \\ &\forall t = 1,2,\ldots,k \\ &L_t^{C_u-} - a_{th}L_h^{C_u+} \le 0 \quad \forall (t = 1,2,\ldots,k, h > t), \\ &L_t^{C_u+} - a_{th}L_h^{C_u-} \ge 0 \quad \forall (t = 1,2,\ldots,k, h > t), \\ &L_t^{C_u-} \ge \varepsilon \quad \forall t = 1,2,\ldots,k, \\ &L_t^{C_u+} - L_t^{C_u-} \ge 0 \quad \forall t = 1,2,\ldots,k. \end{aligned} \tag{21}$$

3 The algorithmic E-VIKOR method with Interval numbers for MCDM

The interval numbers are often considered as a useful tool when determining the precise values of the criteria is of difficulty or impossibility. Therefore, the fundamental of the extended VIKOR with interval numbers for solving the MCDM problems has studied by some researchers [2, 10, 11]. We extend the recent methods to E-VIKOR with the possibility degree of interval numbers. At first, we assume that a ratios matrix on multiple criteria with interval numbers is formulated as:

$$\begin{array}{lcccc} & c_1 & c_2 & \cdots & c_n \\ S_1 & [f_{11}^L, f_{11}^U] & [f_{12}^L, f_{12}^U] & \cdots & [f_{1n}^L, f_{1n}^U] \\ S_2 & [f_{21}^L, f_{21}^U] & [f_{22}^L, f_{22}^U] & \cdots & [f_{2n}^L, f_{2n}^U] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ S_m & [f_{m1}^L, f_{m1}^U] & [f_{m2}^L, f_{m2}^U] & \cdots & [f_{mn}^L, f_{mn}^U] \end{array} \qquad (22)$$
$$W = [w_1, w_2, L, w_n]$$

where $S_1, S_2, \ldots, S_m$ are candidate alternatives for decision makers to choose, $C_1, C_2, \ldots, C_n$ are multiple criteria with which all the alternatives performance can be measured, f_{ij} is the ratio of an alternative S_i with respect to criterion C_j and its upper and lower bounds are f_{ij}^U and f_{ij}^L. w_j is the weight of criterion C_j.

The algorithmic E-VIKOR method with interval numbers is comprised of the following steps [2, 10]:

Step 1: Identification of the PIS and NIS.

$$S^* = \{f_1^*, \ldots, f_n^*\} = \{(\max_i f_{ij}^U | j \in I) \text{ or } (\min_i f_{ij}^L | j \in J)\} \qquad (23a)$$

$$S^- = \{f_1^-, \ldots, f_n^-\} = \{(\min_i f_{ij}^L | j \in I) \text{ or } (\max_i f_{ij}^U | j \in J)\} \qquad (23b)$$

where the criteria are divided into two types, benefit criteria and cost criteria, which are indicated by index I and J, respectively. It is clearly that S^* is the PIS and S^- represents NIS.

Step 2: Calculation of intervals $[R_i^L, R_i^U]$ and $[x_i^L, x_i^U]$, $i = 1, 2, \ldots, m$.

$$R_i^L = \max\left\{ w_j \left(\frac{f_j^* - f_{ij}^U}{f_j^* - f_j^-} \right) \middle| j \in I, w_j \left(\frac{f_{ij}^L - f_j^*}{f_j^- - f_j^*} \right) \middle| j \in J \right\}$$
$$i = 1, \ldots, m \qquad (24a)$$

$$R_i^U = \max\left\{ w_j \left(\frac{f_j^* - f_{ij}^L}{f_j^* - f_j^-} \right) \middle| j \in I, w_j \left(\frac{f_{ij}^U - f_j^*}{f_j^- - f_j^*} \right) \middle| j \in J \right\}$$
$$i = 1, \ldots, m \qquad (24b)$$

$$x_i^L = \sum_{j \in I} w_j \left(\frac{f_j^* - f_{ij}^U}{f_j^* - f_j^-} \right) + \sum_{j \in J} w_j \left(\frac{f_{ij}^L - f_j^*}{f_j^- - f_j^*} \right)$$
$$i = 1, \ldots, m \qquad (25a)$$

$$x_i^U = \sum_{j \in I} w_j \left(\frac{f_j^*}{f_{ij}^L} \right) + \sum_{j \in J} w_j \left(\frac{f_{ij}^U - f_j^*}{f_j^- f_j^*} \right)$$
$$i = 1, \ldots, m \qquad (25b)$$

Step 3: Calculation of interval $Q_i = [Q_i^L, Q_i^U]$ in the following formulation:

$$Q_i^L = v \left(\frac{x_i^L - x^*}{x^- - x^*} \right) + (1 - \lambda) \left(\frac{R_i^L - R^*}{R^- - R^*} \right) \qquad (26a)$$

$$Q_i^U = v \left(\frac{x_i^U - x^*}{x^- - x^*} \right) + (1 - \lambda) \left(\frac{R_i^U - R^*}{R^- - R^*} \right) \qquad (26b)$$

where

$$x^* = \min_i x^L, x^- = \max_i x_i^U$$
$$R^* = \min_i R_i^L, R^- = \max_i R_i^U$$

In general, it is supposed that $\lambda = 0.5$ and it represents the strategy weight of "the majority of criteria".

Step 4: Selection of the best alternative that has minimum Q_i, a new method for comparison of interval numbers as follows [10, 11]:

Let $Q_i = [Q_i^L, Q_i^U]$ and $Q_t = [Q_t^L, Q_t^U]$ be two interval numbers that the decision-makers have to choose minimum one between them.

When $Q_i^L = Q_i^U$ and $Q_t^L = Q_t^U$, that is, both interval numbers Q_i and Q_t are exact real numbers, then we can have [22]:

$$p(Q_i \geq Q_t) = \begin{cases} 1 & \text{if } Q_i > Q_t \\ 1/2 & \text{if } Q_i = Q_t \\ 0 & \text{if } Q_i < Q_t \end{cases} \qquad (27)$$

When $Q_i^U = Q_i^L = Q_i$ and $Q_t^U \neq Q_t^L$, we can have

$$p(Q_i \geq Q_t) = \begin{cases} 1 & \text{if } Q_i > Q_t^U \\ \frac{Q_i - Q_t^L}{Q_t^U - Q_t^L} & \text{if } Q_t^L \leq Q_i \leq Q_t^U \\ 0 & \text{if } Q_i < Q_t^L \end{cases} \qquad (28)$$

When $Q_i^U \neq Q_i^L$ and $Q_t^U = Q_t^L = Q_t$, $p(Q_i \geq Q_t)$ is formulated as

$$p(Q_i \geq Q_t) = \begin{cases} 1 & \text{if } Q_i^L > Q_t \\ \frac{Q_i^U - Q_t}{Q_i^U - Q_i^L} & \text{if } Q_i^L \leq Q_t \leq Q_i^U \\ 0 & \text{if } Q_i^U < Q_t \end{cases} \qquad (29)$$

Generally, the most common case is that $Q_i^U \neq Q_i^L$ and $Q_t^U \neq Q_t^L$, then, the two interval numbers, Q_i and Q_t, are shown in Fig. 2 [22]. It is easily to see that a shadowed rectangle with two parts divided by straight line $y = x$, marked as s' and s'', respectively, in different colors. It is formed by four peaks (Q_i^L, Q_t^L), (Q_i^L, Q_t^U), (Q_i^U, Q_t^L) and (Q_i^U, Q_t^U). s, $s^{'}$ and $s^{''}$ represent the corresponding area in the Fig. 2, respectively. The area of the whole rectangle is s. The area that $y > x$ belongs to the area $s^{''}$ and the rest area is $s^{'}$.

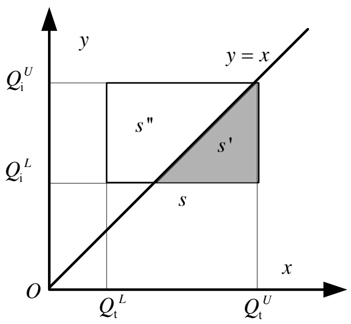

Fig. 2: The relationship between two general interval numbers

Definition 4. Let $Q_i = [Q_i^L, Q_i^U]$ and $Q_t = [Q_t^L, Q_t^U]$ and $Q_i^U \neq Q_i^L$ and $Q_t^U \neq Q_t^L$, then the definition of the possibility degree of $Q_i \geq Q_t$ can be given as follows:

$$p(Q_i \geq Q_t) = \frac{s^{'}}{s^{''}} \tag{30}$$

where $s = (Q_i^U - Q_i^L)(Q_t^U - Q_t^L)$.

Consequentially, the possibility degree of $Q_t > Q_i$ is derived by the formulation [11,22]:

$$p(Q_t \geq Q_i) = \frac{s^{''}}{s^{'}} \tag{31}$$

Step 5: According to the degree of possibility of all the Q_i, we assume that the acceptable degree is above 0.5 and a ranking of all the alternatives.

4 The algorithmic E-TOPSIS method with Interval numbers for MCDM

The algorithmic E-TOPSIS method with Interval numbers for MCDM is proposed in this section [1,3,7,8,9].

Step 1: Identification and construction of evaluation criteria for all the alternatives according to the decision-making Goals.

Step 2: Generation of alternatives for achieving the decision-making goals;

Step 3: Computation of the alternatives ratios in interval number by the value functions on multiple criteria;

Step 4: Identification of the weights of multiple criteria.

Step 5: Construction of the interval judgment matrix and the interval normalized judgment matrix.

Based on the ratios matrix on multiple criteria with interval numbers given by Eq. (14), the normalized values $\bar{\Delta}_{ij}^L$ and $\bar{\Delta}_{ij}^U$ can be calculated as follows [3]:

$$\bar{\Delta}_{ij}^L = f_{ij}^L \Big/ \sqrt{\sum_{j=1}^{m} (f_{ij})^2 + (f_{ij}^U)^2}, \quad j = 1,\ldots,m, i = 1,\ldots,n \tag{32a}$$

$$\bar{\Delta}_{ij}^U = f_{ij}^U \Big/ \sqrt{\sum_{j=1}^{m} (f_{ij}^L)^2 + (f_{ij}^U)^2}, \quad j = 1,\ldots,m, i = 1,\ldots,n \tag{32b}$$

It clearly that the normalized interval number $[\bar{\Delta}_{ij}^L, \bar{\Delta}_{ij}^U]$ is originated from interval number $[f_{ij}^L, f_{ij}^U]$. Beyond all doubt, the normalized interval number $[\bar{\Delta}_{ij}^L, \bar{\Delta}_{ij}^U]$ is belonging to range $[0,1]$.

Step 6: Construction of the interval weighted normalized judgment matrix.

Let us take the different weight of each criterion into consideration. The elements of a weighted normalized interval judgment matrix can be given as follows [7,8]:

$$\bar{v}_{ij}^L = w_j \bar{\Delta}_{ij}^L, j = 1,\ldots,n, i = 1,\ldots,m \tag{33a}$$

$$\bar{v}_{ij}^U = w_j \bar{\Delta}_{ij}^U, j = 1,\ldots,n, i = 1,\ldots,m \tag{33b}$$

where w_j is the weight of the criterion j and $\sum_j^n w_j = 1$.

Step 7: Identification of negative ideal solution and positive ideal solution.

As a consequence, the negative ideal solution and positive ideal solution can be identified as [9]

$$\bar{S}^+ = \{\bar{v}_1^+,\ldots,\bar{v}_m^+\} = \{(\max_i \bar{v}_{ij}^U | j \in I), (\min_i \bar{v}_{ij}^L | j \in J)\}, \tag{34a}$$

$$\bar{S}^- = \{\bar{v}_1^-,\ldots,\bar{v}_m^-\} = \{(\min_i \bar{v}_{ij}^L | j \in I), (\max_i \bar{v}_{ij}^U | j \in J)\}, \tag{34b}$$

where the criteria are divided into two types, benefit criteria and cost criteria, which are indicated by index I and J, respectively.

Step 8: Calculation of the distance of each alternative from negative ideal solution and positive ideal solution, respectively.

Based on the n-dimensional Euclidean distance, the distance of each alternative from the negative ideal solution is formulated as [1,3]:

$$\bar{d}_i^- = \left\{ \sum_{j\in I} (\bar{v}_{ij}^U - \bar{v}_j^-)^2 + \sum_{j \in J} (\bar{v}_{ij}^L - \bar{v}_j^-)^2 \right\}^{\frac{1}{2}}, i = 1,\ldots,m. \tag{35a}$$

Table 1: Reciprocal judgment matrix on C_1

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
S_1	1	2.1	2.1	2.1	3.15	3.15	3.15	9.45	9.45	9.45
S_2	0.525	1	1.05	1.05	1.575	1.575	1.575	4.725	4.725	4.725
S_3	0.525	1.05	1	1.05	1.575	1.575	1.575	4.725	4.725	4.725
S_4	0.525	1.05	1.05	1	1.575	1.575	1.575	4.725	4.725	4.725
S_5	0.35	0.7	0.7	0.7	1	1.05	1.05	3.15	3.15	3.15
S_6	0.35	0.7	0.7	0.7	1.05	1	1.05	3.15	3.15	3.15
S_7	0.35	0.7	0.7	0.7	1.05	1.05	1	3.15	3.15	3.15
S_8	0.1167	0.2333	0.2333	0.2333	0.35	0.35	0.35	1	1.05	1.05
S_9	0.1167	0.2333	0.2333	0.2333	0.35	0.35	0.35	1.05	1	1.05
S_{10}	0.1167	0.2333	0.2333	0.2333	0.35	0.35	0.35	1.05	1.05	1

Table 2: Reciprocal judgment matrix on C_2

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
S_1	1	1.1	1.1	1.1	3.3	3.3	3.3	9.9	9.9	9.9
S_2	1.1	1	1.1	1.1	3.3	3.3	3.3	9.9	9.9	9.9
S_3	1.1	1.1	1	1.1	3.3	3.3	3.3	9.9	9.9	9.9
S_4	1.1	1.1	1.1	1	3.3	3.3	3.3	9.9	9.9	9.9
S_5	0.3667	0.3667	0.3667	0.3667	1	1.1	1.1	3.3	3.3	3.3
S_6	0.3667	0.3667	0.3667	0.3667	1.1	1	1.1	3.3	3.3	3.3
S_7	0.3667	0.3667	0.3667	0.3667	1.1	1.1	1	3.3	3.3	3.3
S_8	0.1222	0.1222	0.1222	0.1222	0.3667	0.3667	0.3667	1	1.1	1.1
S_9	0.1222	0.1222	0.1222	0.1222	0.3667	0.3667	0.3667	1.1	1	1.1
S_{10}	0.1222	0.1222	0.1222	0.1222	0.3667	0.3667	0.3667	1.1	1.1	1

Table 3: Reciprocal judgment matrix on C_3

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
S_1	1	1.05	1.05	1.05	4.2	4.2	4.2	8.4	8.4	8.4
S_2	1.05	1	1.05	1.05	4.2	4.2	4.2	8.4	8.4	8.4
S_3	1.05	1.05	1	1.05	4.2	4.2	4.2	8.4	8.4	8.4
S_4	1.05	1.05	1.05	1	4.2	4.2	4.2	8.4	8.4	8.4
S_5	0.2625	0.2625	0.2625	0.2625	1	1.05	1.05	2.1	2.1	2.1
S_6	0.2625	0.2625	0.2625	0.2625	1.05	1	1.05	2.1	2.1	2.1
S_7	0.2625	0.2625	0.2625	0.2625	1.05	1.05	1	2.1	2.1	2.1
S_8	0.1313	0.1313	0.1313	0.1313	0.525	0.525	0.525	1	1.05	1.05
S_9	0.1313	0.1313	0.1313	0.1313	0.525	0.525	0.525	1.05	1	1.05
S_{10}	0.1313	0.1313	0.1313	0.1313	0.525	0.525	0.525	1.05	1.05	1

Meanwhile, the distance of each alternative from the positive ideal solution is formulated as:

$$\bar{d}_i^+ = \left\{ \sum_{j\in I} (\bar{v}_{ij}^L - \bar{v}_j^+)^2 + \sum_{j\in J} (\bar{v}_{ij}^U - \bar{v}_j^+)^2 \right\}^{\frac{1}{2}}, i = 1,\dots,m. \tag{35b}$$

Step 9: Calculation of the closeness coefficient of each alternative to positive ideal solution.

Once the value of $\bar{d}_j^+$ and $\bar{d}_j^-$ is obtained for each alternative, a closeness coefficient can be calculated from them to help the decision-makers rank all the alternatives.

The closeness coefficient of the alternative S_i with respect to $\bar{S}^+$ is defined as [3]

$$\bar{R}_i = \bar{d}_i^- / (\bar{d}_i^+ + \bar{d}_i^-), i = 1,\dots,m \tag{36}$$

Step 10: Ranking all the alternatives in a preference order according to the value of closeness coefficient.

It can be seen that with the value of $\bar{R}_j$ approaching to 1, the alternative S_i is becoming to be closer to $\bar{S}^+$ and farther from $\bar{S}^-$. Therefore, we can use the closeness coefficient $\bar{R}_j$ to rank all the alternatives and determine which one is the optimal alternative for the decision-making goals.

5 Numerical Experiment and Results

5.1 Experiment description

Based on the scenario carried out by Jussi [23], the illustrative example, demonstrates the application of the

Table 4: Reciprocal judgment matrix on C_4

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
S_1	1	3.3	3.3	3.3	2.2	2.2	2.2	9.9	9.9	9.9
S_2	0.3667	1	1.1	1.1	0.7333	0.7333	0.7333	3.3	3.3	3.3
S_3	0.3667	1.1	1	1.1	0.7333	0.7333	0.7333	3.3	3.3	3.3
S_4	0.3667	1.1	1.1	1	0.7333	0.7333	0.7333	3.3	3.3	3.3
S_5	0.55	1.65	1.65	1.65	1	1.1	1.1	4.95	4.95	4.95
S_6	0.55	1.65	1.65	1.65	1.1	1	1.1	4.95	4.95	4.95
S_7	0.55	1.65	1.65	1.65	1.1	1.1	1	4.95	4.95	4.95
S_8	0.1222	0.3667	0.3667	0.3667	0.2444	0.2444	0.2444	1	1.1	1.1
S_9	0.1222	0.3667	0.3667	0.3667	0.2444	0.2444	0.2444	1.1	1	1.1
S_{10}	0.1222	0.3667	0.3667	0.3667	0.2444	0.2444	0.2444	1.1	1.1	1

Table 5: The Interval normalized decision matrix

	C1L	C1R	C2L	C2R	C3L	C3R	C4L	C4R
S_1	0.1372	0.1372	0.1041	0.1041	0.1029	0.1029	0.1402	0.1402
S_2	0.0653	0.0653	0.0947	0.0947	0.098	0.098	0.0425	0.0467
S_3	0.0622	0.0653	0.0861	0.0947	0.0934	0.098	0.0425	0.0467
S_4	0.0622	0.0653	0.0861	0.0947	0.0934	0.098	0.0425	0.0467
S_5	0.0415	0.0435	0.0287	0.0316	0.0233	0.0245	0.0637	0.0637
S_6	0.0415	0.0435	0.0287	0.0316	0.0233	0.0245	0.0579	0.0637
S_7	0.0415	0.0435	0.0287	0.0316	0.0233	0.0245	0.0579	0.0637
S_8	0.0138	0.0145	0.0096	0.0105	0.0117	0.0123	0.0129	0.0142
S_9	0.0138	0.0145	0.0096	0.0105	0.0117	0.0123	0.0129	0.0142
S_{10}	0.0138	0.0145	0.0096	0.0105	0.0117	0.0123	0.0129	0.0142

proposed E-VIKOR and E-TOPSIS Algorithmic Methods with Interval Numbers for weapon system alternative selection. The goal of the case study is to analyze and assess which indirect fire systems would be the optimal system alternative by supporting the future mechanized infantry forces to fulfill a series of capabilities.

More specifically, the candidate weapon system alternatives indexed by $i = 1,2,...,10$ are a variety of unmanned vehicles, intelligence and reconnaissance alternatives, information alternatives, command and control alternatives and indirect (or direct) fire alternatives, denoted by a set, $S^* = \{S_1,S_2,S_3,S_4,S_5,S_6,S_7,S_8,S_9,S_{10},\}$. With regard to the capability requirement [24], the weapon systems portfolio should meet with four capability requirements in future operations. The four criteria are given as the required four capabilities for the weapon system alternative decision-making goals, indexed by $u = 1,2,3,4$ are Reconnaissance and Intelligence capability (C_1), Orientation capability (C_2), Command and Decision capability (C_3) and Action capability (C_4).

The experts are requested to given four reciprocal judgment matrices by pairwise comparison of all the alternative with regard to four capability, respectively. Monte Carlo method is employed to generate acceptable consistency reciprocal judgment matrix ($CR < 0.1$) to simulate the judgments from the experts. (See Table 1, Table 2, Table 3 and Table 4).The weights of four criteria are given as the same value, $w_1 = w_2 = w_3 = w_4 = 0.25$

5.2 Results analysis and discussion

The interval normalized decision matrix is shown in Table 5, however, some ratios are precise point values, For example, the upper and lower bounds of the ratio of alternative S_1 with respect to criterion $C_1 \sim C_4$ are equal to each other. PIS and NIS computed by E-VIKOR is given in Table 6 and Table 7 shows the x_i and R_i interval numbers. Q_i interval numbers is shown in Table 8 and Table 9 shows The degree of possibility of all the Q_i by comparison. The final result of the alternatives ranking by E-VIKOR is listed in Table 10. S_4 and S_3 both rank the top and the worst three alternatives are S_8, S_9 and S_{10}.

The Interval weighted normalized judgment matrix by E-TOPSIS is shown in Table 11. Distance of each alternative from the positive idea solution and negative idea solution by E-TOPSIS are given in Table. 12 and Table 13. Closeness coefficient and ranking by E-TOPSIS are shown in Table 14. The final ranking order of the alternatives determined by E-TOPSIS is clear and distinctive. S_2 ranks the top and in the bottom lays S_5. S_4 and S_3 rank near to the top and the worst three alternatives are S_7, S_6 and S_5.

Table 6: PIS and NIS computed by E-VIKOR

	C1L C1R	C2L C2R	C3L C3R	C4L C4R
fi∗	0.1372	0.1041	0.1029	0.0142
fi−	0.0138	0.0096	0.0117	0.1402

Table 7: x_i and R_i interval numbers

	x_i^L	x_i^U	R_i^L	R_i^U
S_1	0.25	0.25	0.25	0.25
S_2	0.2403	0.2487	0.1456	0.1456
S_3	0.2403	0.2905	0.1456	0.1519
S_4	0.2403	0.2905	0.1456	0.1519
S_5	0.6948	0.7098	0.2148	0.218
S_6	0.6833	0.7098	0.2148	0.218
S_7	0.6833	0.7098	0.2148	0.218
S_8	0.7419	0.75	0.2486	0.25
S_9	0.7419	0.75	0.2486	0.25
S_{10}	0.7419	0.75	0.2486	0.25

Table 8: Q_i interval numbers

Alternative	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
Q_i^L	0.5096	0	0	0	0.7774	0.7661	0.7661	0.9854	0.9854	0.9854
Q_i^U	0.5096	0.0083	0.0795	0.0795	10	0.8074	0.8074	1	1	1

Table 9: The degree of possibility of all the Q_i by comparison

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
S_1	0.5	0	0	0	1	1	1	1	1	1
S_2	1	0.5	0.052	0.052	1	1	1	1	1	1
S_3	1	0.948	0.5	0.5	1	1	1	1	1	1
S_4	1	0.948	0.5	0.5	1	1	1	1	1	1
S_5	0	0	0	0	0.5	0.6365	0.6365	1	1	1
S_6	0	0	0	0	0.3635	0.5	0.5	1	1	1
S_7	0	0	0	0	0.3635	0.5	0.5	1	1	1
S_8	0	0	0	0	0	0	0	0.5	0.5	0.5
S_9	0	0	0	0	0	0	0	0.5	0.5	0.5
S_{10}	0	0	0	0	0	0	0	0.5	0.5	0.5

Table 10: Ranking of all the alternatives by E-VIKOR

Alternatives	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
Rank	4	3	1	1	5	6	6	7	7	7

Table 11: The Interval weighted normalized judgment matrix by E-TOPSIS

	C1L	C1R	C2L	C2R	C3L	C3R	C4L	C4R
S_1	0.0343	0.0343	0.026	0.026	0.0257	0.0257	0.0351	0.0351
S_2	0.0163	0.0163	0.0237	0.0237	0.0245	0.0245	0.0106	0.0117
S_3	0.0156	0.0163	0.0215	0.0237	0.0233	0.0245	0.0106	0.0117
S_4	0.0156	0.0163	0.0215	0.0237	0.0233	0.0245	0.0106	0.0117
S_5	0.0104	0.0109	0.0072	0.0079	0.0058	0.0061	0.0159	0.0159
S_6	0.0104	0.0109	0.0072	0.0079	0.0058	0.0061	0.0145	0.0159
S_7	0.0104	0.0109	0.0072	0.0079	0.0058	0.0061	0.0145	0.0159
S_8	0.0035	0.0036	0.0024	0.0026	0.0029	0.0031	0.0032	0.0035
S_9	0.0035	0.0036	0.0024	0.0026	0.0029	0.0031	0.0032	0.0035
S_{10}	0.0035	0.0036	0.0024	0.0026	0.0029	0.0031	0.0032	0.0035

Table 12: Distance of each alternative from the positive idea solution

DU1	DU2	DU3	DU4	DU5	DU6	DU7	DU8	DU9	DU10
0.0446	0.0279	0.0287	0.0287	0.0538	0.0535	0.0535	0.0635	0.0635	0.0635

Table 13: Distance of each alternative from the negative idea solution

DU1	DU2	DU3	DU4	DU5	DU6	DU7	DU8	DU9	DU10
0.0637	0.0575	0.0562	0.0562	0.0301	0.031	0.031	0.0448	0.0448	0.0448

Table 14: Closeness coefficient and ranking by E-TOPSIS

Alternatives	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
Rank	4	1	2	3	10	9	8	5	7	6

6 Conclusion

In this paper, we studied the extended VIKOR and TOPSIS algorithmic methods with interval number for MCDM problem based on reciprocal judgment matrix and carried out a comparative experiment.

When faced with MCDM problems in real-world, the experts may be uncomfortable giving crisp ratios or precise upper and lower bounds of the interval ratios on multi-criteria. Based on the reciprocal judgment matrices given by the experts with regard to the satisfaction degree of the multiple alternatives on single criterion, the interval ratios was elicited by a linear programming model. The TOPSIS and VIKOR methods are extend for MCDM problems with interval number and algorithmic E-VIKOR and E-TOPSIS methods are examined by the experts reciprocal judgment matrices generated by Monte Carlo simulation. A numerical example of ranking indirect-fire weapon system alternative is given and illustrates the feasibilities and distinctive features of the VIKOR and TOPSIS algorithmic methods with interval number.

Improvements can be made for future studies in the following ways: The programming model of elicitation of interval probability from reciprocal judgment matrix by pairwise comparisons can be improved. Furthermore, the TOPSIS and VIKOR can be extended with other classical methods (Fuzzy triangle number and ANP methods. etc.) for MCDM problems.

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