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Location in the Hotelling duopoly model with demand uncertainty

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Abstract

Demand uncertainty is introduced into a Hotelling environment with fixed prices by allowing random shocks to the desirability of the firm's product. Given these random shocks, the choice of location affects the average level of demand as well as the riskiness of demand: reducing the distance to the other firm raises expected demand and payoff but also lowers the degree of differentiation between the firms, thus raising demand uncertainty. Risk averse firms will locate away from the center and, depending on degree of risk aversion, markup, and size of the market, may locate on either side of the quartile points.

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1. Introduction

The principle of minimum differentiation introduced by Hotelling (1929) represents a starting point in the theory of optimal location. When two firms selling a homogeneous product with constant marginal cost of production are situated along a linear market, the firms will locate as close to each other as

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possible if demand is inelastic. If, furthermore, consumers are distributed uniformly along the market line then the firms will locate in the center of the market. The reason is the desire of the firm to gain as much hinterland as possible; if one of the firms locates away from the center it will get a smaller share of the fixed market.

The robustness of the principle of minimum differentiation has been questioned in various extensions. The principle no longer applies in the case of elastic demand by Smithies (1941), the case of a larger number of firms and location on a plane instead of along a line by Eaton and Lipsey (1975), and the case of costly relocation by Hay (1976). Interestingly, the principle continues to hold in the nonhomogeneous products case of De Palma et al. (1985), and Ben-Akiva et al. (1989). [1]

Extension of the basic Hotelling model to the case of uncertainty has been largely neglected. One reason may be that uncertainty appears to be immaterial for the principle of minimum differentiation to hold, at least in the context of the original formulation. Suppose that the distribution of consumers along the market line is unknown rather than uniform, then demand is uncertain but the incentive for firms to agglomerate is no different from the situation without uncertainty. Similarly when transportation costs or the constant production costs are random or when consumers face a random transportation cost, the principle of minimum differentiation is not affected – a firm can risklessly increase expected payoff by locating closer to the rival.

A few uncertainty models remain close to the Hotelling formulation. [2] In Devletoglou (1965) firms locate apart because consumers display bounded rationality so that within a particular area they choose sellers randomly. De Palma et al. (1985) and Anderson et al. (1992, Ch. 9) assume uncertainty concerning the preferences of individual consumers for brand differences. Due to the law of large numbers demand uncertainty disappears at the firm level and firms agglomerate in spite of the brand differences.

[1] Endogenous price determination in Hotelling's model has led to corrections invalidating the minimum differentiation result even in the original framework. D'Aspremont et al. (1979) showed the original Hotelling model to be invalid; the profit function is neither continuous nor quasi-concave so that no pure strategy Bertrand–Nash equilibrium exists. Later Osborne and Pitchik (1987) calculated a mixed strategy subgame perfect equilibrium solution and Anderson (1988) was able to derive a solution in pure strategies given sufficiently convex transportation costs. The principle of minimum differentiation ceases to apply in these models because price setting becomes more competitive when firms locate closer together.

[2] A large body of literature has focused on uncertainty models that differ substantially from the original Hotelling framework, considering instead consumers who settle *at a point* rather than along a line. Here firms choose optimal location between input source(s) and the market place (see for instance the literature survey in Park and Mathur, 1990). In this environment the principle of minimum differentiation could still hold as pointed out by Stuart (1979): firms may agglomerate because consumers mimimize their search as well as transportation costs.

Our model introduces demand uncertainty as deriving from some unobservable aspect of the product, that is revealed to the firm once it commits to the location decision. Consumers uniformly consider this characteristic in addition to the price and the transportation cost so that demand becomes uncertain to the firm a priori. If the firm is effectively or in fact risk averse then it must consider expected payoff as well as risk in establishing location, so that the optimal location is no longer obviously in the center of the market. Section 2 presents the model and the basics of the firms' location decisions under demand uncertainty. Section 3 presents the equilibrium locations, which are further examined in Section 4.

2. Risk and return in the location decision

Consider the Hotelling environment of consumers with perfectly inelastic demand uniformly distributed along a bounded line $[0, \ell]$ and two competing firms. The location z_i, (for both firms: $i = 1,2$) is measured relative to the leftmost market bound, 0. Assume without loss of generality that each firm stays on its own market half, including the midpoint, with firm 1 on the left half. The distance between the firms $x \equiv z_2 - z_1$ then is nonnegative. To introduce demand uncertainty and focus directly on the effect of demand uncertainty two deviations from the Hotelling model are adopted. First, for simplicity, the product prices are assumed fixed and equal. Second, stochastic terms ε_i with zero means are introduced that represent random aspects to the quality of each firm's product. [3] These random aspects are observed after each firm has committed to the location decision. [4]

The indirect utility of a consumer at location z depends negatively on the product price and on the sum of the normalized transportation cost $|z_i - z|$ and the (negative) quality shock ε_i. Aggregating over the uniformly distributed consumers provides a demand curve that is contingent on the realization of $\varepsilon \equiv \varepsilon_1 - \varepsilon_2$.

[3] Although we will in the text interpret the demand uncertainty as caused by shocks to the product's quality, several other reasonable interpretations are available. For concreteness consider the classic example of ice cream vendors on a bounded linear beach. In our basic interpretation of quality shocks, demand may be lower than expected when the ice cream turns out to be partly melted or when the vendor appears to have a bad cold. Alternatively, shocks to the desirability of the location may occur: demand may be lower than expected when there are flies or stray dogs around the cart or maybe a mean-looking group of bikers. It is also possible that the vendors carry slightly differentiated brands and are unsure about how much the consumers will like their product.

[4] In an alternative scenario in which consumers cannot observe the random quality aspects directly, but learn after visiting a firm, the basic results of our model would continue to hold with only minor modifications.

Normalizing total market demand to ℓ, the quantity demanded of firm 1 is specified by three cases:

$$q_1 = \begin{cases} \ell & \text{for} \quad \varepsilon < -x \\ \frac{1}{2}(z_1 + z_2 - \varepsilon) & \text{for} \quad \varepsilon < |x| \, . \\ 0 & \text{for} \quad \varepsilon > x \end{cases} \tag{1}$$

The quantity demanded of firm 2 is easily obtained from the relation $q_1 + q_2 = \ell$. Fig. 1 displays quantity demanded as a function of ε. Firm 1 will get all of the market demand when its advantage over firm 2 in the ex post desirability of the product (low ε) exceeds the maximum possible difference in travel distance between the two firms. At $\varepsilon = -x$ a discontinuous drop in demand occurs since all consumers to the right of firm 2 are indifferent between the firms. As ε increases, the quantity demanded from firm 1 drops steadily; at $\varepsilon = 0$ the consumers located between the firms are split evenly. When $\varepsilon > x$, even the consumers to the left of z_1 prefer firm 2.

It is instructive to view how a change in location affects quantity demanded in Fig. 1. An increase in z_1 (moving the quantity demanded to the dotted line) raises the demand in the central region by increasing firm 1's hinterland; this implies an increase in the *expected payoff* to firm 1, and provides the basic intuition for the Hotelling result. An additional effect arises in our framework, however: the increase in z_1 also moves the firms closer together causing less differentiation between the firms by distance so that smaller shocks can now cause extreme demand outcomes. In Fig. 1 this is indicated as a narrowing of the central region. While this narrowing does not to a first order affect firm 1's expected demand, it does affect the firm's *risk*.

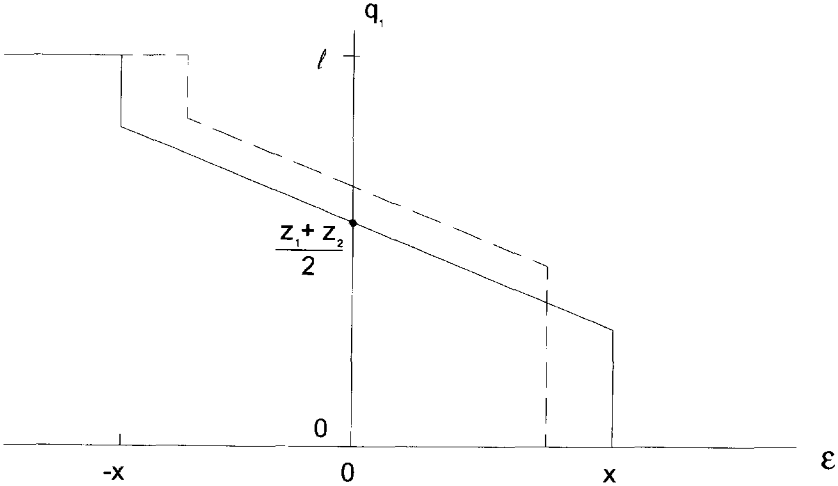

Fig. 1. Demand uncertainty.

3. Location under demand uncertainty

In the following we assume risk averse firms (or firms with convex production costs that are effectively risk averse) to address explicitly the effect of demand risk on the location decision. Each firm maximizes the expected utility of profit, with p defined as the markup of price over a constant marginal cost and concave $u(pq_i)$ as the utility over profit. [5] From Eq. (1):

$$\mathrm{E}[u(pq_1] = \int_{-\infty}^{-x} u(p\ell)\mathrm{d}F(\varepsilon) + \int_{-x}^{x} u[p\tfrac{1}{2}(z_1 + z_2 - \varepsilon)]\mathrm{d}F(\varepsilon) + \int_{x}^{\infty} U\mathrm{d}F(\varepsilon) \tag{2}$$

where $F(\varepsilon)$ is the distribution of ε with density $f(\varepsilon)$, continuously differentiable on its support. Firm 1 chooses location z_1 to maximize expected utility, under the Nash assumption taking the location of firm 2 as given. The use of Leibnitz's rule yields the following first-order condition: [6]

$$[u(p\ell) - u(pz_2)]f(-x) + [u(0) - u(pz_1)]f(x) + \tfrac{1}{2}p\int_{-x}^{x} u'[p\tfrac{1}{2}(z_1 + z_2 - \varepsilon)]\mathrm{d}F(\varepsilon) = 0. \tag{3}$$

Fig. 1 illustrates the meaning of the different terms in Eq. (3). The first two terms relate to the narrowing of the central part of the distribution at $\varepsilon = -x$ and $\varepsilon = x$; the last term represents the increase in quantity demanded over the central part of the distribution. These terms refer to the location-induced effects of increased risk and mean return, respectively.

Eq. (3) may be simplified by integrating by parts the integral expression:

$$\tfrac{1}{2}p\int_{-x}^{x} u'[p\tfrac{1}{2}(z_1 + z_2 - \varepsilon)]\mathrm{d}F(\varepsilon) = \int_{-x}^{x} u[p\tfrac{1}{2}(z_1 + z_2 - \varepsilon)]\mathrm{d}f(\varepsilon) - u(pz_1)f(x) + u(pz_2)f(-x). \tag{4}$$

Substitute Eq. (4) into Eq. (3) to obtain

$$u(p\ell)f(-x) + u(0)f(x) - 2u(pz_1)f(x) + \int_{-x}^{x} u[p\tfrac{1}{2}(z_1 + z_2 - \varepsilon)]\mathrm{d}f(\varepsilon). \tag{5}$$

[5] A slightly different notation $u(p, q_i)$ allows for an interpretation of risk neutrality, but with a convex cost function and p interpreted as the price, in which case $u(\cdot)$ is q_i.

[6] Leibnitz's rule applies when differentiating integral expressions with variable limits. Sufficient (but by no means necessary) for the second-order condition to hold is that ε has a uniform density.

Eq. (5) may be further simplified by restricting $f(\varepsilon)$. For instance, assuming a symmetric and unimodal density it follows directly that $f(-x) = f(x)$ and also that the integral expression must be positive (since $u(\cdot)$ decreases in ε and is therefore highest when $\mathrm{d}f(\varepsilon)$ is positive). A much stronger simplification emerges in the next section where we consider a uniform distribution for the random shock.

4. Specific examination of the equilibrium location

Assume a uniform distribution for ε over the whole range, say from $-b$ to b where for convenience $b > \ell$. This implies $f'(\varepsilon) = 0$ over the range from $-x$ to x so that the integral expression in Eq. (5) vanishes. The first order condition then becomes:

$$u(pz_1^*) = \tfrac{1}{2}[u(p\ell) + u(0)]. \tag{6}$$

Fig. 2 displays the equilibrium location implied by Eq. (6). Clearly $0 < z_1 \leq \ell/2$. Only under risk neutrality do we obtain the basic result that $z_1 = \ell/2$. This is the result obtained in De Palma et al. (1985) in the absence of risk aversion (as well as aggregate demand uncertainty).

One would expect an increase in risk aversion to bring the equilibrium location closer to the boundary as the firm worries more about securing at least part of its

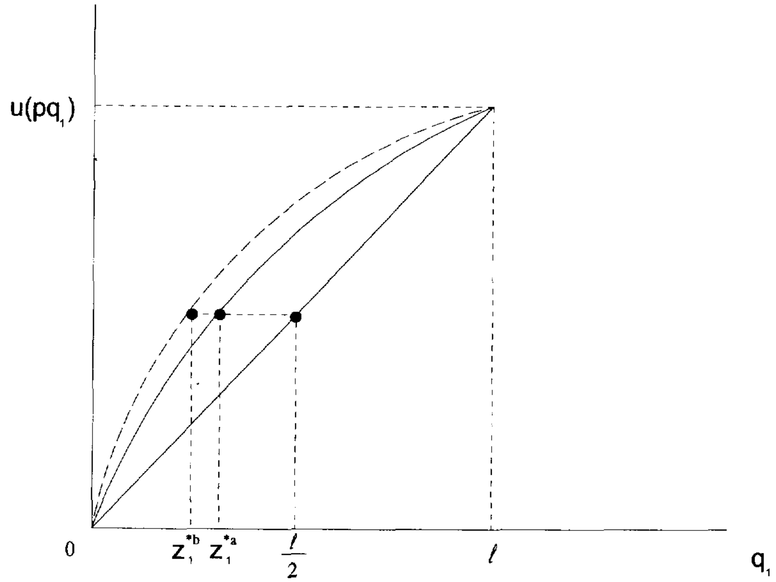

Fig. 2. Risk aversion and location.

demand. Formally, an Arrow–Pratt increase in risk aversion is represented by a concave transformation of the utility function yielding $G[u(\cdot)]$ with $G(\cdot)$ strictly concave and monotonic. Since von Neumann–Morgenstern utility functions are unique up to linear transformations we have two degrees of freedom to set, *without loss of generality* if the utility function is bounded, $G[u(z)] = u(z)$ and $G[u(0)] = u(0)$. The leftmost curve in Fig. 2 is the result of such an increase in risk aversion: equilibrium location moves from z_1^{*a} to z_1^{*b}. It should also be clear that depending on the degree of risk aversion the equilibrium location could be to the left or to the right of the quartile point, i.e., $z_1^* = \ell/4$, which, assuming symmetric location of the other firm, is optimal in terms of minimizing aggregate transportation costs as first discussed by Smithies (1941).

It is obvious from Eq. (6) that $\mathrm{d}z_1^*/\mathrm{d}z_2 = 0$. This is not true in general as follows from Eq. (5). For the uniform distribution, however, two opposite effects exactly cancel: a decrease in z_2 decreases firm 1's expected market share, inciting firm 1 to move closer since it now has less to lose; it also decreases the distance between the firms, inciting firm 1 to move back to reduce risk. In the asymmetric case where firm 2 is, say, more risk averse than firm 1 this does not affect firm 1's location decision, but *does* raise firm 1's expected utility as firm 2 staying closer to its boundary raises firm 1's mean demand and also reduces risk.

An increase in market size ℓ from Eq. (6), affects optimal location as follows:

$$0 < \mathrm{d}z_1^*/\mathrm{d}\ell = \tfrac{1}{2}\left[u'(p\ell)/u'(pz_1)\right] < 1/2. \tag{7}$$

The inequalities follow since marginal utility is positive but decreasing. An increase in ℓ raises z_1^* but by less than half as much since firm 1 has more to lose and spends part of the increased windfall by increasing the distance between the firms and protecting the hinterland.

What happens to the hinterland as a fraction of market size, z_1/ℓ, when the market size increases? From Eq. (7):

$$\frac{\mathrm{d}(z_1^*/\ell)}{\mathrm{d}\ell} = \frac{-2u'(pz_1^*)z_1^* + u'(p\ell)\ell}{2u'(pz_1^*)\ell^2}. \tag{8}$$

The sign depends on whether $u'(pz_1)z_1$ increases or decreases in z_1. Which in turn depends on the rate of relative risk aversion, $R(z_1)$, in the utility function. A *sufficient* condition for a negative effect is $R(z_1) > 1$ since then $u'(pz_1)z_1$ decreases in z_1. Intuitively, as ℓ rises for a strongly risk averse firm the firm has more to lose and prefers to raise its *relative* distance to the other firm. The same result arises if firm 2's reaction to the change in ℓ is considered as well. Concerning the relative distance, x/ℓ, in the symmetric Nash equilibrium, firm 2 moves opposite to firm 1 so that the *size* of the change in x/ℓ is twice as large, while the *direction* of the change is opposite to z_1/ℓ and thus determined by the relative risk aversion of the firms.

An exogenous increase in the markup, an increase in price or decrease in marginal cost, from Eq. (6) has the following effect on the optimal location:

$$\frac{\mathrm{d} z_1^*}{\mathrm{d} p} = \frac{-2u'(pz_1^*)z_1^* + u'(p\ell)\ell}{2u'(pz_1^*)p}. \tag{9}$$

The effect of markup on location tends to be negative. Similarly as for Eq. (8) a sufficient condition for a negative effect is $R(z_1) > 1$. Higher markup means the firm has more potential profits to protect and intends to do so by retreating closer to the boundary.

As the comparative statics discussion reveals, the introduction of demand uncertainty has added a new dimension to the Hotelling environment. Assuming risk aversion, the avoidance of risk now implies a tradeoff of protecting the firm's own hinterland versus encroaching on the opponent's hinterland. This basic risk–return tradeoff is not likely to be affected by a cumbersome relaxation of some of the simplifying assumptions of our analysis. In particular we believe that allowing flexible and unequal prices would add the familiar additional reason for dispersed location but would yield few additional insights. Note, however, that existence of an equilibrium is no longer assured in this case and may require 'enough' demand uncertainty as discussed for instance in Anderson et al. (1992, Ch. 9).

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