PHYS 4330 Theoretical Mechanics

Homework # 10

Submission deadline: 16 April 2024 at 11:59 pm Eastern Time

Submission Instructions: Homework is submitted on Gradescope to Homework 10.

IMPORTANT NOTE: *Only 2* of these problems will be graded for credit (at 10 points each)!! We will not disclose which problems are being graded before the deadline. This means that you will need to submit completed answers to all questions or risk getting a 0 if we grade a problem you didn't do.

1. Three oscillators of equal mass, *m,* are coupled such that the potential energy of the system is given by:

$$U = \frac{1}{2}\left[k_1(x_1^2 + x_3^2) + k_2 x_2^2 + k_3(x_1 x_2 + x_2 x_3)\right]$$

with $k_3 = \sqrt{2k_1 k_2}$.

(a) Calculate the eigenfrequencies of the system.
(b) What is the physical interpretation of the zero-frequency mode?

2. Two equal masses, *m,* are constrained to move on a frictionless plane, one on the positive *x* axis and one on the positive *y* axis. They are attached to two identical springs, with force constant *k,* whose other ends are attached to the origin. In addition, the two masses are connected to each other by a third spring with force constant k_{12}. The springs are chosen so that the system is in equilibrium with all three springs relaxed (length equal to rest length). Consider a small displacement of the masses from the equilibrium position.
a) What are the eigenfrequencies?
b) Find the eigenvectors of the system.
c) Describe (using words) the 2 normal modes of the system.

3. Consider a system of carts and springs like that in Figure 1 **EXCEPT** that there are ***three*** equal-mass carts and ***four*** identical springs. Solve for the three eigenfrequencies, the three eigenvectors, and describe the motion (using words) of the corresponding normal modes.

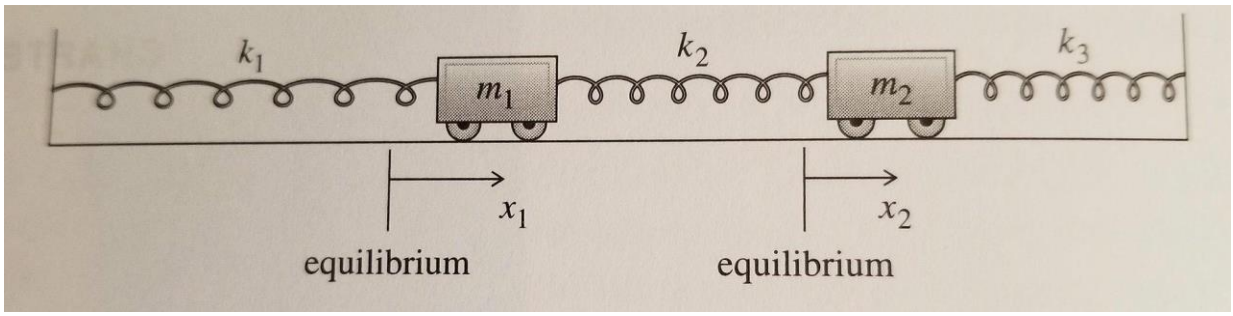

Figure 1