PHYS 4330 Theoretical Mechanics

Homework # 8

Submission deadline: 19 March 2024 at 11:59 pm Eastern Time

Submission Instructions: Homework is submitted on Gradescope to Homework 8.

IMPORTANT NOTE: *Only 2* of these problems will be graded for credit (at 10 points each)!! We will not disclose which problems are being graded before the deadline. This means that you will need to submit completed answers to all questions or risk getting a 0 if we grade a problem you didn't do.

1. Calculate the differential scattering cross section $\frac{d\sigma(\theta)}{d\Omega}$ and the total cross section σ_t for the elastic scattering of a particle from an impenetrable sphere, the potential is given by $U(r) = 0$ for r > a and $U(r) = \infty$ for r ≤ a, where a is the radius of the sphere.

The total scattering cross section is $\sigma_t = \int \frac{d\sigma(\theta)}{d\Omega} d\Omega$

2. The effective gravitational acceleration measured at the surface of the Earth is the superposition of gravity and centrifugal acceleration:

$$\vec{g}_{eff} = \left(\vec{g} - \vec{\omega} \times (\vec{\omega} \times \vec{r})\right)$$

[do not just assume that $\vec{g}$ = 9.81, calculate it out at each position from weight and Newton's law of gravitation.]

For your calculations, use the given constants at the end of the problem statement.

Calculate the effective gravitational accelerations at
(a) the North Pole, and
(b) the Equator.

(c) If a long jumper can jump 9m at the North Pole, how far can the athlete jump at the equator? Ignore wind resistance, temperature and a runway made of ice. Assume that the long jumper lifts off under an angle of 45°.

(do not include the Coriolis effect in your calculations)

[R_p = 6356.7523 km, R_e = 6378.1370 km, M = 5.972 x 10^{24} kg, ω = 7.2921159 x 10^{-5} rad/s, G = 6.67408 x 10^{-11} $m^3/(kg\ s^2)$]

[Problem 3 on next page]

3. a) Define a coordinate system on the Earth at a colatitude of θ (The angle measured down from the North pole to the point on the Earth). The coordinate system origin at this point on Earth is oriented such that z is pointed radially outward along the radius of Earth, y is pointed to the North, and x is pointed to the East. Draw a picture of this coordinate system.

b) Take the velocity of the object to be given in the form $\dot{r} = (\dot{x}, \dot{y}, \dot{z})$. The Earth rotates at some constant angular velocity ω pointed "up" along the axis of rotation.
[This means that ω is not directed exactly along any of the coordinate defined in (a)]

Assume that the object *only* moves under the influence of gravity *and* the Coriolis Force. Using equation (10.34) from your textbook with **S** = **0**, **SHOW** that the general expression for the x, y, and z accelerations for an object just above the surface of the Earth are of the form:

$$\ddot{x} = 2\omega(\dot{y}cos\theta - \dot{z}sin\theta)$$
$$\ddot{y} = -2\omega\dot{x}cos\theta$$
$$\ddot{z} = -g + 2\omega\dot{x}sin\theta$$

c) We will now make a *zeroth order* approximation where ω = 0 (as if there isn't any rotation of the Earth). Assume that the object is thrown with some initial velocity $\vec{v}_0 = (v_{0x}, v_{0y}, v_{0z})$. Using the initial condition and the *zeroth order* approximation for the accelerations from part (b), find the velocities $\dot{x}, \dot{y}, and\ \dot{z}$. Plug these velocity equations into the general accelerations from part (b).

d) Now we explore the specific case of a ball thrown vertically upward (+z) with an initial speed v_0 from a point on the ground at colatitude of θ *[Assume that the ball is thrown from the origin of your coordinate system (0,0,0)]*. Change the general expression from part (c) by setting the initial velocity in x and y to 0. Now integrate the accelerations found at the end of part (c), with the initial conditions, to find the deflection from the Coriolis Force in the x and y direction when the ball hits the ground. *[Hint: use the z expression to find the time when the ball will return to the ground and plug it into the x and y expressions.]*