PHYS 4330 Theoretical Mechanics

Homework # 6

Submission deadline: 27 February 2024 at 11:59 pm Eastern Time

Submission Instructions: Homework is submitted on Gradescope to Homework 6.

IMPORTANT NOTE: *Only 2* of these problems will be graded for credit (at 10 points each)!! We will not disclose which problems are being graded before the deadline. This means that you will need to submit completed answers to all questions or risk getting a 0 if we grade a problem you didn't do.

1. Consider a particle of mass *m* moving under the influence of a conservative force *F* whose potential energy is *U*. Find the Hamiltonian *H* of the particle and show that the canonical equations (Hamilton's equations of motion) reduce to Newton's equations of motion (Newton's 2nd law). Need to show how to get *H* starting from Lagrangian and treat the problem using 3-dimensional Cartesian coordinates.

2. A particle of mass *m* moves in one dimension under the influence of a force

$$F = -kx^{-3}e^{\alpha * t}$$

where α and *k* are positive constants.
(a) Calculate the Hamiltonian function *H* of the particle (starting from L).
(b) Derive Hamilton's equations of motion for the particle. Combine them into a single second order differential equation to confirm you have the correct solution.
(c) Is the Hamiltonian *H* conserved? Explain your reasoning.
(d) Is the total mechanical energy *E* conserved? Explain your reasoning.
[you do not need to solve the differential equations in part b]

3. Consider the Atwood machine pictured below (near the surface of Earth so there is gravity pointing down the image). Suppose that the pulley is a uniform disk of mass *M* and radius *R*. The pulley rotates without slipping. Using *x* as your generalized coordinate:
(a) Find the Lagrangian using a single generalized coordinate (*x*).
(b) Write down the generalized momentum *p* of the entire system.
(c) Find the Hamiltonian *H*
(d) Find Hamilton's equations and use them to find the acceleration $\ddot{x}$.
[you do not need to solve the resulting equation of motion]