PHYS 4330 Theoretical Mechanics

Homework # 5

Submission deadline: 20 February 2024 at 11:59 pm Eastern Time

Submission Instructions: Homework is submitted on LMS to the Assignment HW 5.

IMPORTANT NOTE: *Only 2* of these problems will be graded for credit (at 10 points each)!! We will not disclose which problems are being graded before the deadline. This means that you will need to submit completed answers to all questions or risk getting a 0 if we grade a problem you didn't do.

1. A sphere of radius ρ is constrained to roll without slipping on the lower half of the inner surface of a hollow cylinder of inside radius R (see Figure 1). Gravity points downward in the figure.

(a) Draw the diagram and label the variables for the angular position and rotation of the sphere.
Determine:
(b) the Lagrangian L,
(c) the equation of constraint,
(d) Lagrange's equations of motion,
(e) the frequency of small oscillations for the angular position of the sphere.

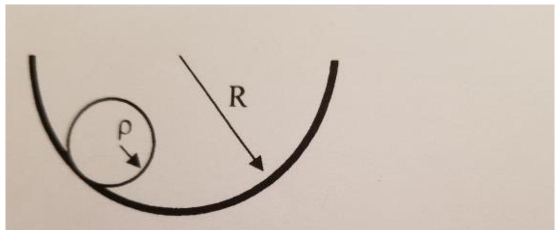


Figure 1

The moment of inertia of a sphere is $I = (2/5)(m\rho^2)$.

[For part (e), the only variable that should be in the expression for the acceleration of the angular position should be the angular position of the sphere itself.]

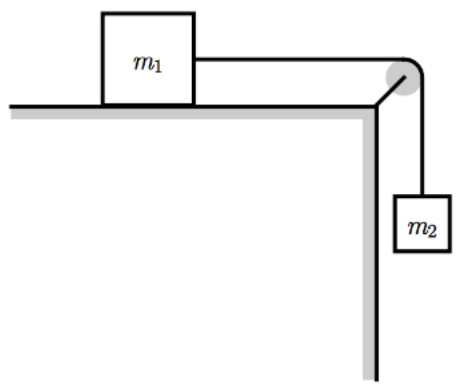
2. Consider a modified Atwood machine as show in the Figure below. Mass m_1 sits on a frictionless table. The masses are released from rest. Gravity points down the page.

(a) Sketch the modified Atwood machine and clearly label you coordinates.

(b) Write the equation of constraint for your system.

(c) Write down the Modified Lagrangian for the system.

(d) Find the acceleration of the blocks.



(e) Solve for λ in terms of m_1 , m_2 , and g. What physical property is represented by λ?

[Problem 3 on next page]

3. Consider a double Atwood machine constructed as follows:

A mass $4m$ is suspended from a string that passes over a massless pulley on a frictionless bearing. The other end of the string supports a second similar pulley, over which passes a string supporting a mass of $3m$ at one end and m at the other. The system is initially held at rest and gravity points downward.

(a) Draw a sketch of the double Atwood machine and label your coordinates.
(b) Use the equations of constraint to reduce the number of coordinates to 2 (generalized coordinates).
(c) Determine the Lagrangian, L for the system.
(d) Using Euler-Lagrange equation, find the acceleration of the $4m$ mass after the system is released from rest.