PHYS 4330 Theoretical Mechanics

Homework # 7

Submission deadline: 12 March 2024 at 11:59 pm Eastern Time

Submission Instructions: Homework is submitted on Gradescope to Homework 7.

IMPORTANT NOTE: *Only 2* of these problems will be graded for credit (at 10 points each)!! We will not disclose which problems are being graded before the deadline. This means that you will need to submit completed answers to all questions or risk getting a 0 if we grade a problem you didn't do.

1. Two particles, whose reduced mass is μ, interact via a potential energy energy $U = -\frac{k}{r}$, where r is the distance between them and $k > 0$.

(a) Make a sketch showing $U(r)$, the centrifugal potential energy $U_{cf}(r)$, and the effective potential energy $U_{eff}(r)$. Treat the angular momentum l as a known, fixed constant. [*Hint:* Think back to the derivation of θ(r)]

(b) Find the "equilibrium" separation r_0, the distance at which the two particles can circle each other with constant r.

(c) By making a Taylor expansion of $U_{eff}(r)$ about the equilibrium point r_0 and neglecting all terms in $(r - r_0)^3$ and higher, find the frequency of small oscillations about the circular orbit if the particles are perturbed a little from the separation r_0.

2. A particle is moving in an attractive central-force field described by $F(r) = -k/r^3$.

a) Find the second order differential equation that describes the orbit for this particle.

[*Hint: Think back to lecture and how we can find the orbit r(θ). It will also be useful to make the substitution that* $u = 1/r$]

b) Solve the resulting differential equation for the case where $l^2 = \mu k$. Report your answer as *r(θ).*

c) Sketch the orbit for the particle moving in this central-force field.

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3. A mass m is free to slide on a frictionless table and is connected by a string, of length l which passes through a hole in the table, to a mass M which hangs below (see figure below). Assume that M moves in a vertical line only, and assume that the string always remains taut.

a) Find the Lagrange equations of motion for the variables r and θ shown in the figure. *(you do not need to find solutions to these differential equations)*

b) Under what condition does m undergo circular motion? Solve for the circular motion radius, r_0, in this case.

c) What is the frequency of small oscillations (in the variable r) about this circular motion? Ignore all terms that are greater than first order in your expansion.