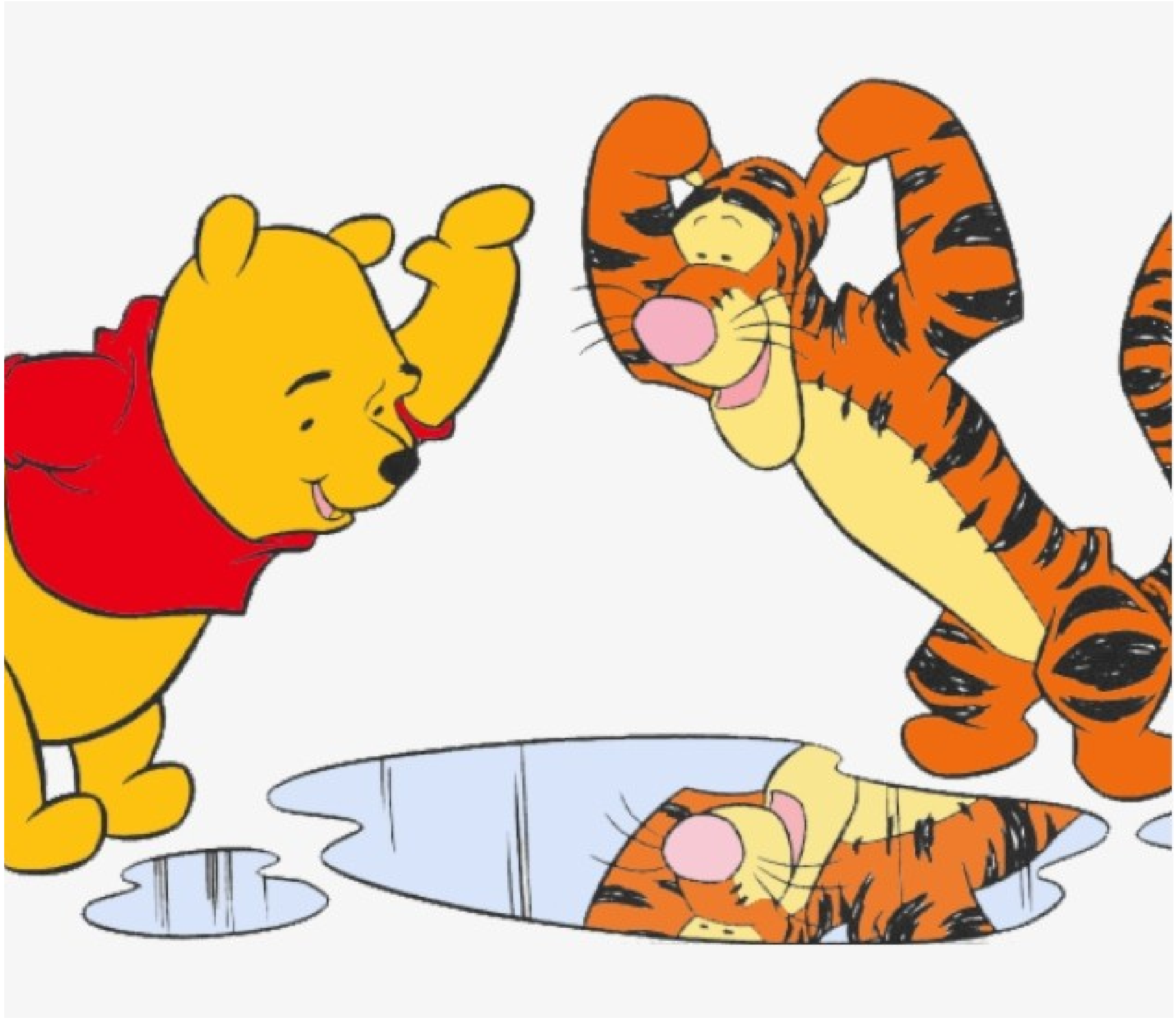


Class 18 (03/25/24)

Electromagnetic Waves in Matter

- Reflection & Refraction of Light in the Context of Maxwell’s Equations -

Key Results:

Electromagnetic Waves Traveling in Vacuum

$$\tilde{\vec{E}}(z,t) = \tilde{\vec{E}}_0 e^{i(kz-\omega t)}, \quad \tilde{\vec{B}}(z,t) = \tilde{\vec{B}}_0 e^{i(kz-\omega t)}$$

are solutions of the wave equations for $\vec{E}$ and $\vec{B}$: $\quad \vec{\nabla}^2\vec{E} = \mu_0\varepsilon_0 \dfrac{\partial^2\vec{E}}{\partial t^2}$ and $\vec{\nabla}^2\vec{B} = \mu_0\varepsilon_0\dfrac{\partial^2\vec{B}}{\partial t^2}$.

Electromagnetic fields propagate at the speed of light $c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}} \approx 3\times 10^8$ m/s.

Electromagnetic waves are transverse waves, i.e. $\vec{E}$ and $\vec{B}$ are $\perp$ to $\vec{k}$ (direction of propagation) because $\vec{\nabla}\cdot\vec{E} = 0$ and $\vec{\nabla}\cdot\vec{B} = 0$ in vacuum.

The electric field $\vec{E}$ is perpendicular to $\vec{B}$ because of $\vec{\nabla}\times\vec{E} = -\dot{\vec{B}}$.

Electromagnetic Waves Traveling in Matter

Today: Matter $\Rightarrow$ linear dielectric and / or magnetic materials which do not absorb the electro-magnetic wave; (recall linear means $\varepsilon = \varepsilon_0 (1 + \chi_e)$ and $\mu = \mu_0 (1 + \chi_m)$ are inde-pendent of external fields $\vec{E}$ and $\vec{H}$)

Maxwell's equation for matter:

$$\vec{\nabla} \cdot \vec{E} = 0, \quad \vec{\nabla} \cdot \vec{B} = 0, \quad \vec{\nabla} \times \vec{E} = -\dot{\vec{B}}, \quad \vec{\nabla} \times \vec{B} = \mu \varepsilon \dot{\vec{E}}$$

The only difference to Maxwell's equations for vacuum is $\mu \varepsilon$ instead of $\mu_0 \varepsilon_0$.

$\vec{\nabla}^2 \vec{E} = \mu \varepsilon \ddot{\vec{E}}$ and $\vec{\nabla}^2 \vec{B} = \mu \varepsilon \ddot{\vec{B}}$.

Main result:

In matter, electromagnetic waves propagate at a speed slower than the speed of light.

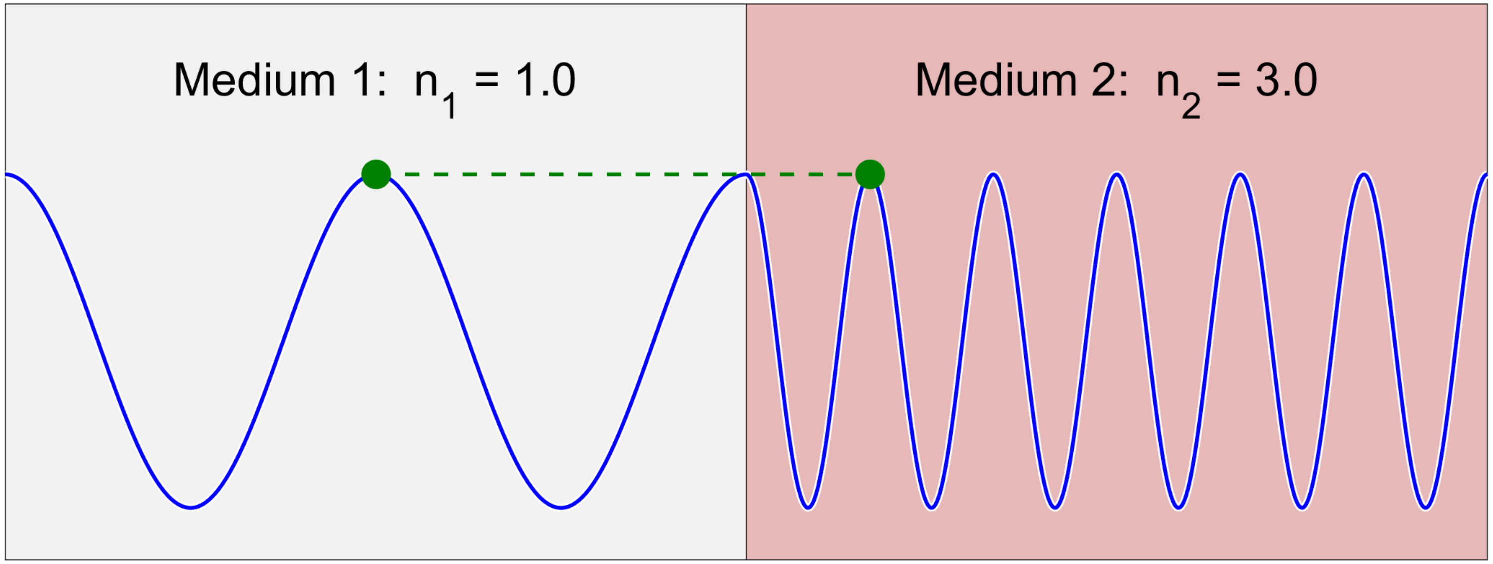
wave propagation speed $v = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{c}{n}$

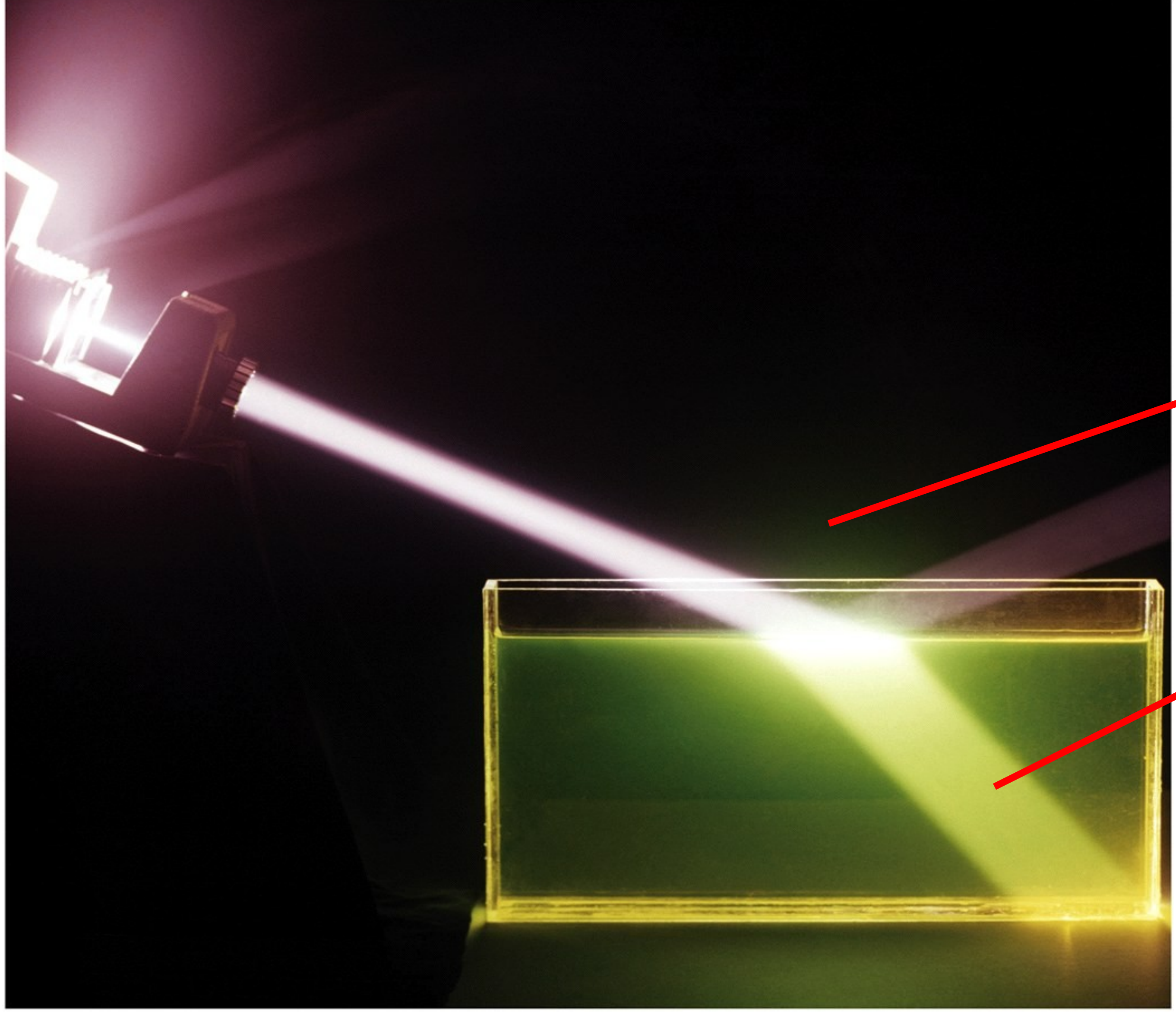
with $n > 1$ because $n = \sqrt{\frac{\varepsilon \mu}{\varepsilon_0 \mu_0}}$ (index of refraction).

When a wave passes from vacuum into matter, the wave frequency remains the same, the wave velocity and wavelength change.

vacuum $c = \lambda \cdot F$, matter $v = \frac{c}{n} = \frac{\lambda \cdot F}{n} = \lambda' \cdot F$

with $\lambda' = \frac{\lambda}{n}$.





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<u>Law of Reflection:</u>

Angle of Incidence = Angle of Reflection

θ_1 incident = θ_1 reflection

<u>Law of Refraction (Snell's Law):</u>

$n_1 \sin \theta_1 = n_2 \sin \theta_2$

Both law's are explained by the requirement that all three waves, the incident wave, reflected wave and transmitted wave have to be in phase at the boundary between the two different media.

Evolution of Snell’s Law: Huygen’s Principle & Fermat’s Principle to Maxwell’s Equations

Willebrord Snell
(1580-1626)

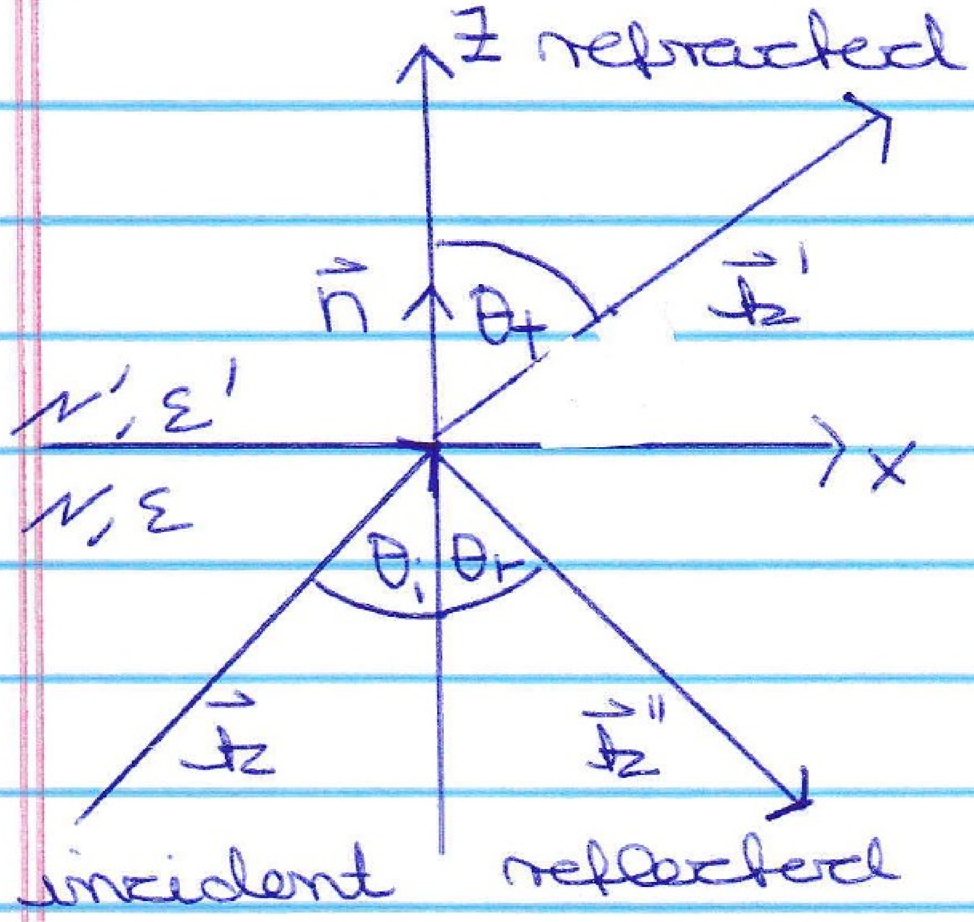
Willebrord Snellius was a Dutch astronomer and mathematician, known in the English-speaking world as Snell. In the west, especially the English speaking countries, his name is attached to the law of refraction of light.

Christiaan Huygens
(1629 – 1695)

Christiaan Huygens was a Dutch physicist, mathematician, astronomer and inventor, who is widely regarded as one of the greatest scientists of all time and a major figure in the scientific revolution. Wikipedia

Fermat's principle (1650) states that “light travels between two points along the path that requires the least time, as compared to other nearby paths.”

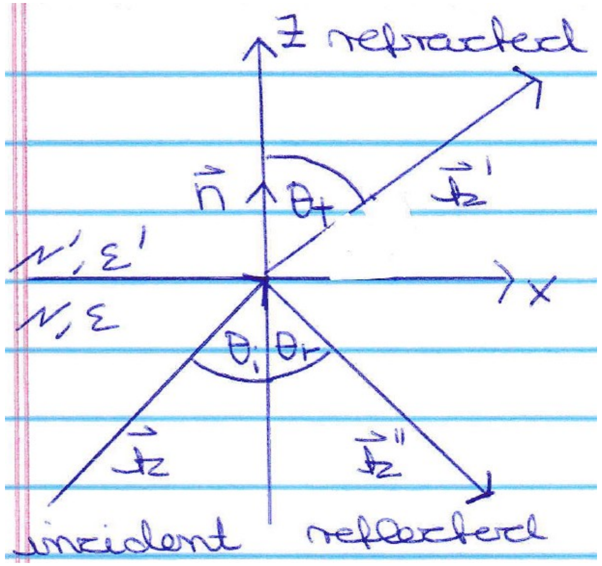
James Clerk Maxwell
1831 – 1879

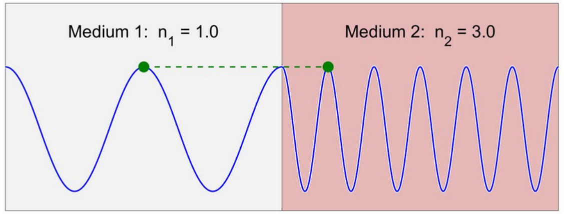


$$\vec{k}' = k' \sin\theta_t \hat{x} + k' \cos\theta_t \hat{z}$$

$$\vec{k} = k \sin\theta_i \hat{x} + k \cos\theta_i \hat{z}$$

$$\vec{k}'' = k \sin\theta_r \hat{x} - k \cos\theta_r \hat{z}$$





incident, reflected & refracted waves have to be in phase at $z=0$

wave $\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$, phase angle $\vec{k}\cdot\vec{r} - \omega t$

$\vec{k}\cdot\vec{r} = \vec{k}'\cdot\vec{r} = \vec{k}''\cdot\vec{r}$ for $z = 0$;

for $\vec{k}$ in x, z plane: $k \sin\theta_i = k' \sin\theta_t = k \sin\theta_r$

$\vec{k}\cdot\vec{r} = k_x \cdot x + k_y \cdot y + k_z \cdot z$; $k_y = 0$ because $\vec{k}$ in x, z plane,

$k_z \cdot z = 0$ because $z=0$ at boundary, therefore $k_x \cdot x = k'_x \cdot x = k''_x x$

$k = k''$

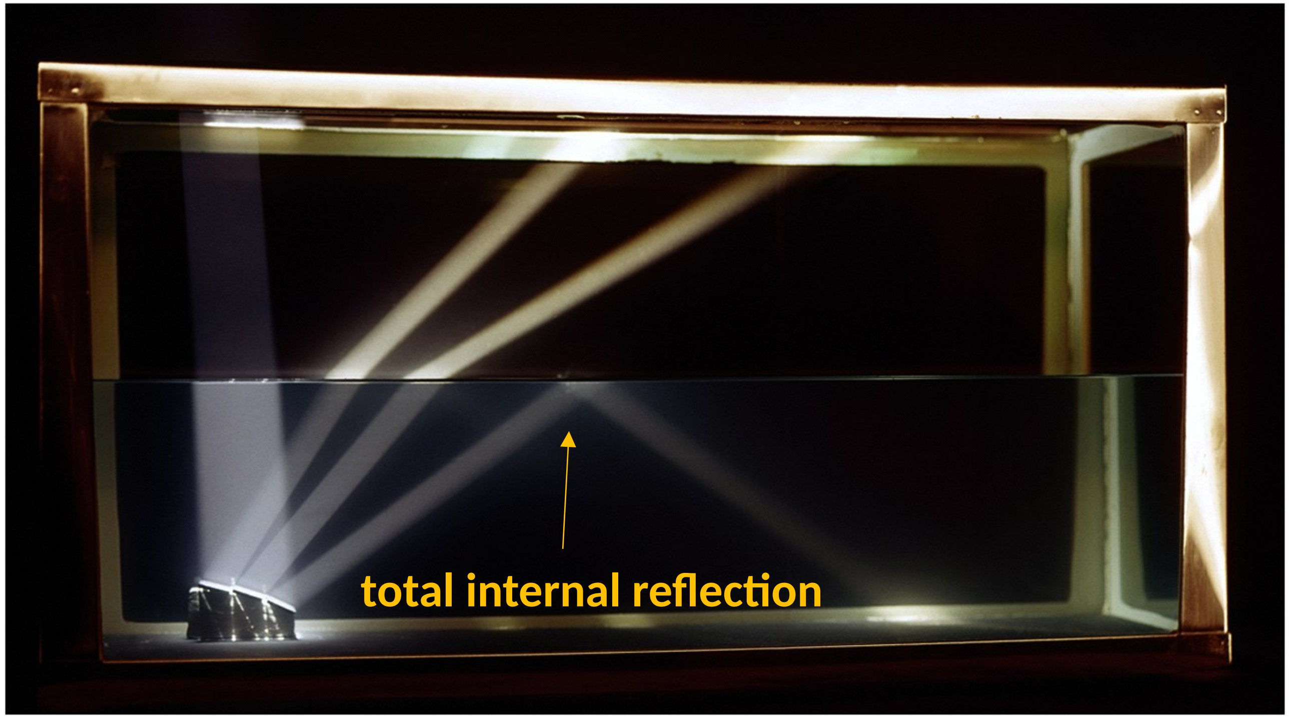
$k \sin\theta_i = k' \sin\theta_t = k \sin\theta_r$

↳ $k \sin\theta_i = k \sin\theta_r \to \theta_i = \theta_r$ (angle of incidence = angle of refrac-tion)

↳ $k \sin\theta_i = k' \sin\theta_t$ (Snell's law)

$\sin\theta_i / \sin\theta_t = k'/k = \sqrt{\dfrac{\mu'\varepsilon'}{\mu\varepsilon}} = \dfrac{n'}{n} \to n \sin\theta_i = n' \sin\theta_t$

Total Internal Reflection

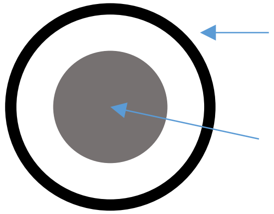


Ken Kay/Fundamental Photographs halliday_9e_fig_33_23b

Electromagnetic wave travels from a material 1 (n_1) to a material 2 (n_2) with $n_1 > n_2$.
The critical angle for total internal reflection is $\theta_c = \sin^{-1}(n_2/n_1)$.
Derived from Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

Very important application: Optical fibers which consist of the core (n_1) and a cladding layer (n_2). Total internal reflection keeps the light in the fiber!

<u>Optical fiber (cross section)</u>



cladding layer

core

Polarization: Same word for two different things in electromagnetic theory

Meaning 1:

(Macroscopic) electric polarization **P** of a dielectric material.

$\mathbf{P}=\langle \mathbf{p}_i \rangle/V$ is (average) electric dipole moment per volume.

P is related to dielectric displacement **D and** electric field **E** as: $\mathbf{D}= \varepsilon_0 \mathbf{E}+\mathbf{P}$, $\mathbf{P}= \varepsilon_0 \chi_e \mathbf{E}$.

Meaning 2:

Polarization of the electro-magnetic (em) wave **E**(**r**,t).

Polarization of the em wave describes the direction of the electric field vector **E**(**r**,t).

Linear, circular, or elliptic polarization (see also class 17).

Brewster Angle

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Convertible transmission-reflection time-domain terahertz spectrometer

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(Received 19 February 2001; accepted for publication 11 May 2001)

The creation of reliable instrumentation for performing complex reflectance and transmittance measurements of dielectrics, metals, and superconductors in the frequency range from 60 GHz to 1.5 THz is reported. The system allows continuous variation of the THz radiation incidence angle from 25° to 80° without major realignment of the optics and provides the signal-to-noise ratio of 1000:1. © *2001 American Institute of Physics.* [DOI: 10.1063/1.1384433]

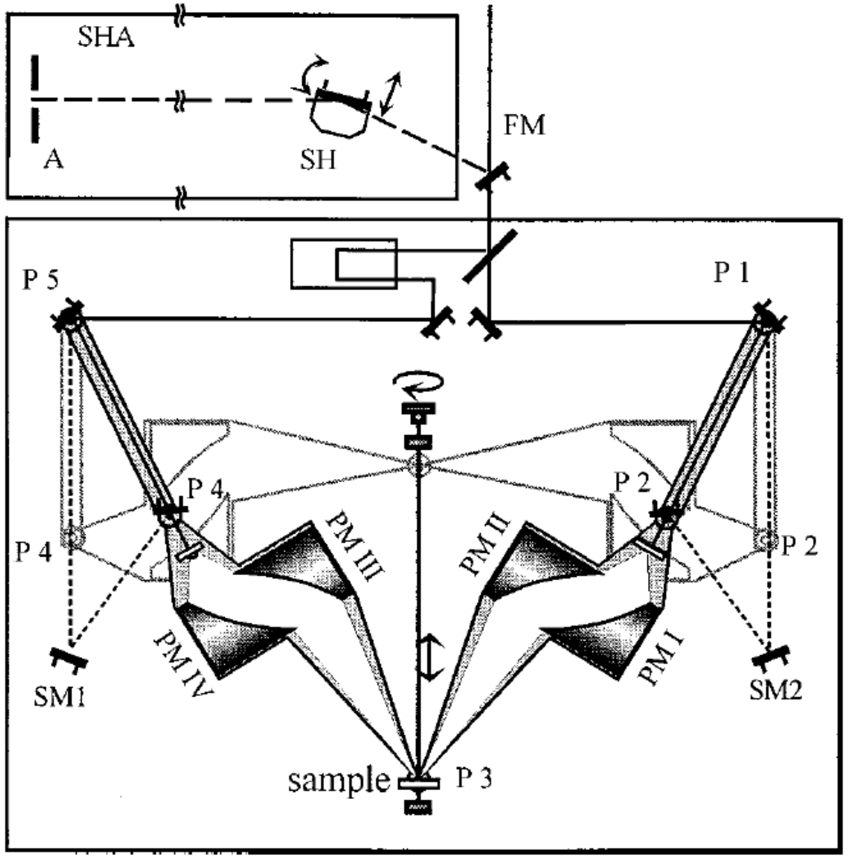
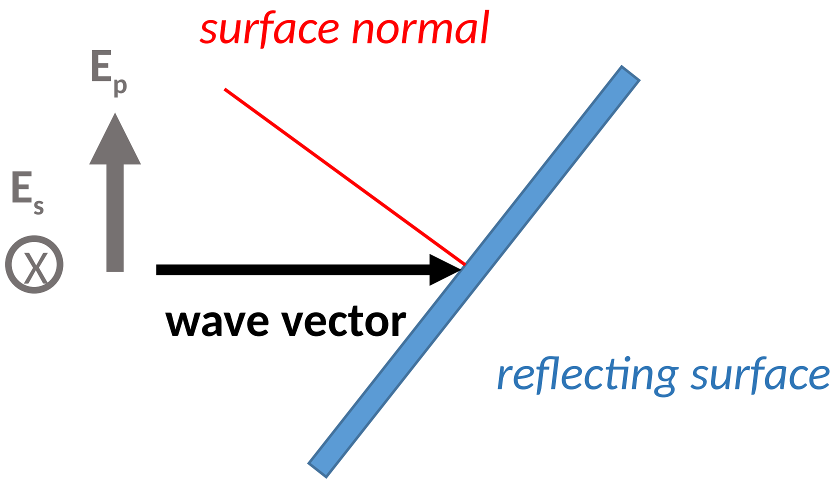
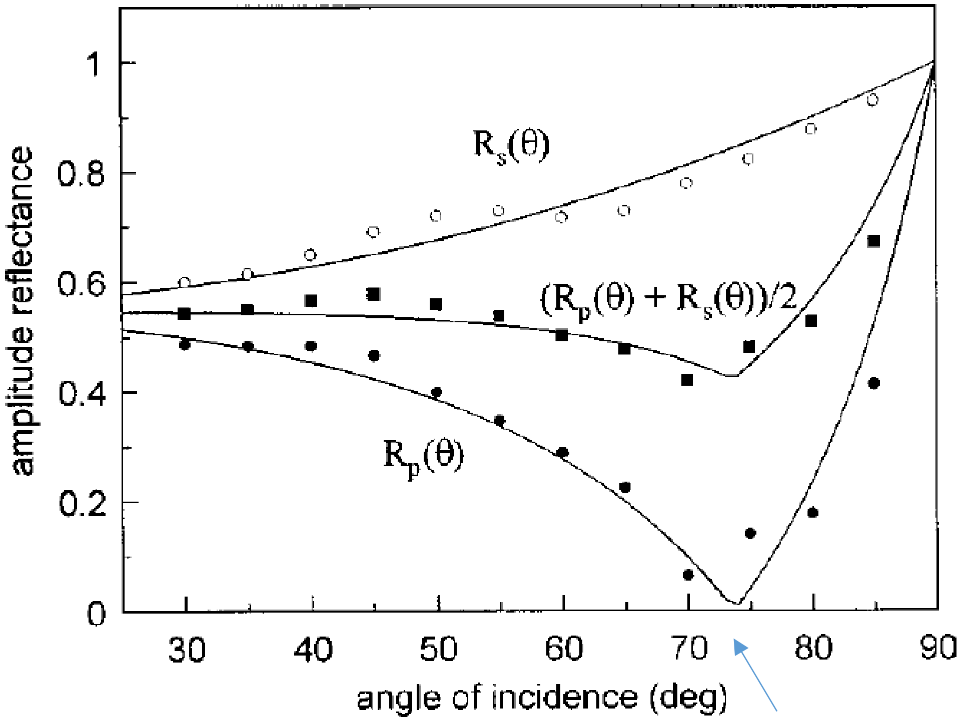


FIG. 2. The reflection configuration: (FM) flipper mirror, (SHA) sample holder adjuster, (SH) sample holder, and (A) aperture.

Measurement of the Brewster Angle of Silicon for Electromagnetic Waves in the Terahertz Frequency Band ($f=10^{12}$ Hz)

The <u>plane of incidence</u> of the electromagnetic wave is described by the wave vector **k** and the surface normal of the reflecting surface.





Brewster angle ≈ $\tan \theta_B = n_2/n_1$

FIG. 6. Amplitude reflectivities of high-resistivity silicon for *s* and *p* polarizations at 0.4 THz. Solid lines are the ideal Fresnel reflectivities.

$n_{air}=1$, n_{Si} (@1 THz)=3.4, Brewster angle θ_B=74°

s: polarization (electric field vector) of the em wave is perpendicular (≈ <u>s</u>enkrecht) to the plane of incidence at the silicon surface.

p: polarization (electric field vector) of the em wave is parallel (≈ <u>p</u>arallel) to the plane of incidence at the silicon surface.