Square Well Worksheet

A general space solution for a particle in a region where V(x)=0 is $\psi(x)$ = A sin kx + B cos kx.

1. We will deduce the form of the solutions for a potential that is symmetric about x=0, that is, the potential is zero for -L/2<x<L/2 and infinite for |x|>L/2

a) What are the boundary conditions at $x = \pm\frac{L}{2}$ on $\psi(x)$ for a particle in the box?

$$\psi(x) = 0 \text{ @ } x \pm \frac{L}{2}$$

b) Rewrite the general solution above for each of the boundary conditions. (You will have two equations.)

$$\psi(x) = A\sin(kx) + B\cos(kx)$$

where $x = \pm\frac{L}{2}$ = 0

$$\psi\left(\frac{L}{2}\right) = A\sin\left(k\left(\frac{L}{2}\right)\right) + B\cos\left(k\left(\frac{L}{2}\right)\right) = 0 \tag{1}$$

$$\psi\left(-\frac{L}{2}\right) = A\sin\left(k\left(-\frac{L}{2}\right)\right) + B\cos\left(k\left(-\frac{L}{2}\right)\right) = 0$$

You can rewrite $\sin(-x)$ as $-\sin(x)$ and $\cos(-x)$ as $\cos(x)$

$$-A\sin\left(k\left(\frac{L}{2}\right)\right) + B\left(\cos\left(k\left(\frac{L}{2}\right)\right)\right) = 0 \tag{2}$$

Adding and taking the difference of these two above functions 1 & 2

$$1) + 2) = B\cos\left(k\left(\frac{L}{2}\right)\right) = 0$$

$$1) - 2) = 0 = 2A\sin\left(k\left(\frac{L}{2}\right)\right)$$

c) What can you conclude about the allowed values of k for sin(kx) solutions and cos(kx) solutions?

For $\sin(kx)$

$$k = \frac{n\pi}{L} \text{ where } n \text{ is an even integer}$$

For $\cos(kx)$

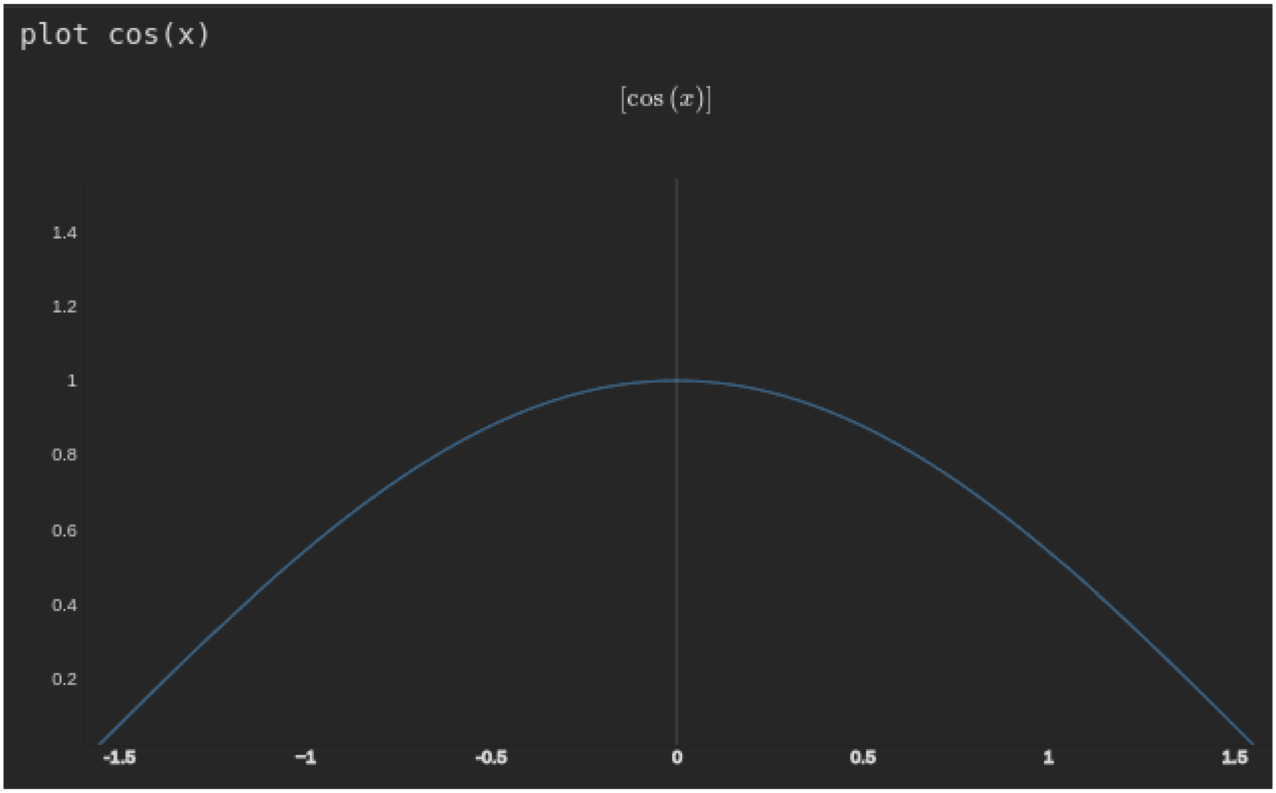
$$k = \frac{n\pi}{L} \text{ where } n \text{ is an odd integer}$$

d) What is the functional form of the ground state (longest wavelength) wavefunction for this well?

For ground state $n = 1$

$$k = \frac{\pi}{L}$$

$$\psi_1 = B\cos\left(\frac{\pi x}{L}\right)$$



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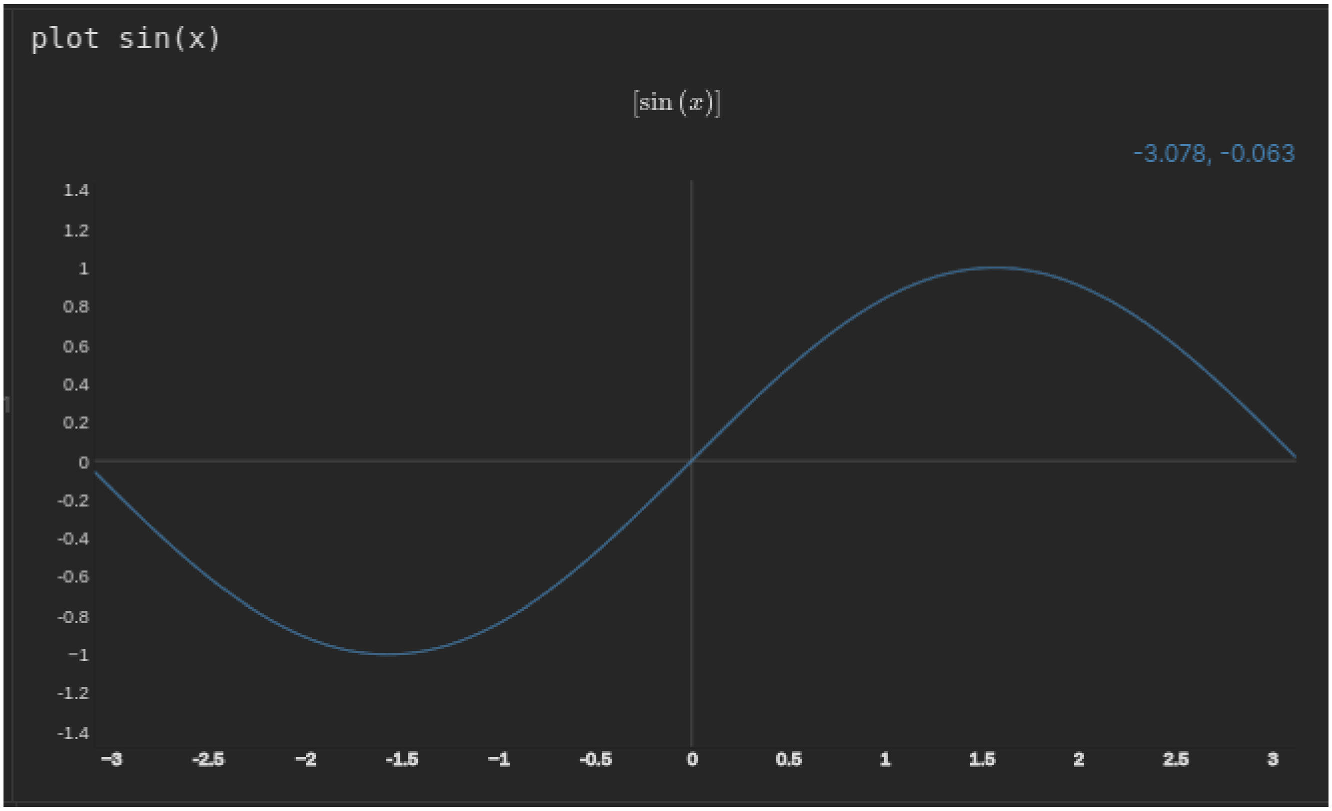
e) What is the wavefunction for the first excited state wavefunction for this well?

$$k = \frac{2\pi}{L}$$

$$\psi_2 = A \sin\left(\frac{2\pi x}{L}\right)$$



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