Compton Scattering Theory

Section I: Inverse-Square Law from Radiation Detection

Expected

- Include a curve-fit showing that the number of detected decays follows the inverse-square law as a function of distance.

Hints

- Fit the data from Section I (of the Compton Lab Instructions Manual) to the function $\frac{1}{r^a}$ where a is the free parameter. Don't forget to include a scaling constant to the curve-fit. Include appropriate error bars.

Gamma-radiation is a form of ionizing radiation. γ-rays are energetic photons, similar to visible light but at much higher frequency, and hence energy. They interact with matter by a variety of mechanisms, such as photoelectric effect, Compton scattering, and electron-positron pair production. The probability of the interaction depends both on its energy and the γ-ray interaction cross section of the material. γ-rays can be emitted by Gamma decay processes, which are a decay of an excited nucleus into a lower energy state. In this process, parent and daughter atoms are the same chemical element, and the emitted γ-ray and recoiling nucleus each have a well defined energy after the decay. γ-rays and X-rays overlap the range of the energy, are distinguished only by their source. When photons are generated by a nuclear reaction, they are called γ-rays. γ-rays extend to higher frequencies than X-rays. γ-ray emission follows the inverse square law.

The interaction of charged particles and photons with matter is electromagnetic in nature, and results in the loss of energy or absorption of photons. Alpha particles lose energy predominantly by colliding with electrons in atoms and electrons lose energy in both the collision with electrons in atoms and by emitting radiation when their trajectory is affected by the field of nucleus. Photons lose energy by colliding with electrons in atoms either by photoelectric (low energy photons) or the Compton effect (medium range photons<2MeV). High-energy photons lose energy by creating electron-positron pairs.

The distribution of energy of γ-ray emission for a certain source is shown in the energy spectra. The energy spectrum of Enriched Uranium (Figure 1) shows the number of decays (or counts) as a function of energy of its γ-radiation. The photopeak of a source describes the energy distribution of emitted γ-rays of a particular nuclear transition.

Section II: Detection and Attenuation

In order to study Compton scattering, you must be able to measure the energy of photons. This is achieved with a NaI crystal. When ionizing radiation such as γ-radiation propagates into a scintillator NaI crystal, electrons in the crystal are excited and the radiation loses the energy. Rapid interatomic transfer of excited electron energy eventually leads to emission of a burst of luminescence. This burst of luminescence is called scintillation. The integral of the luminescent response is a measure of the energy of the ionizing radiation. The dimensions of the photodetector used in this lab is $2'' \times 2''$.

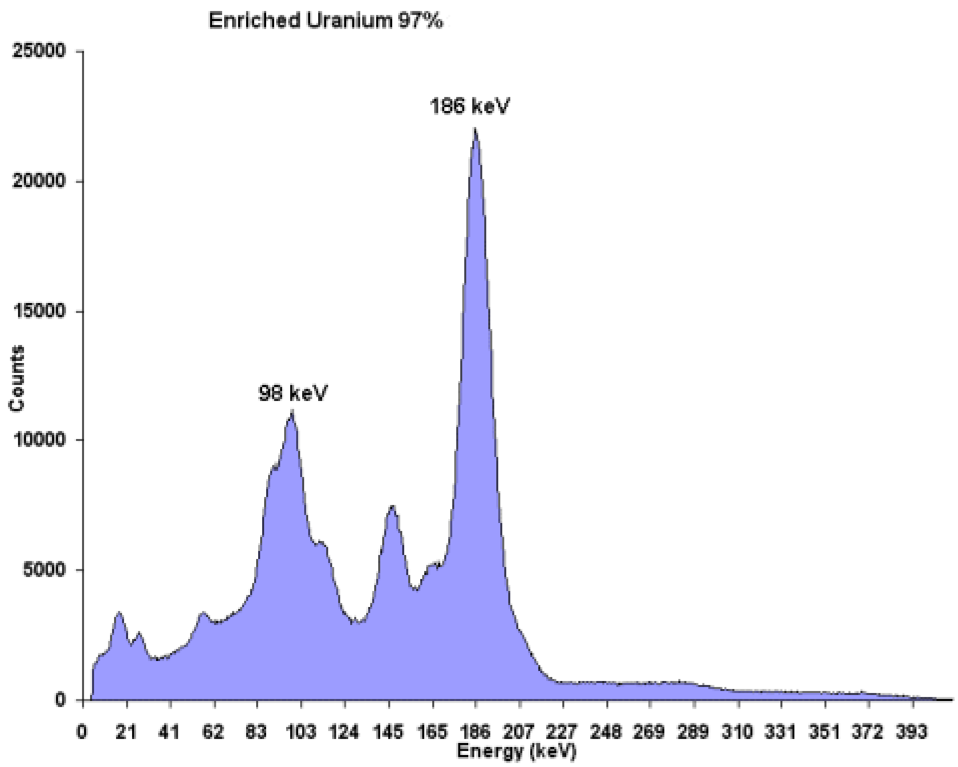

Figure 1: The γ-ray spectrum of Enriched Uranium with the γ-energy photopeak at 186 keV

This scintillation light is detected by a photomultiplier tube (PMT), which converts single photons to a measurable electrical signal through the photoelectric effect. When a photon hits a metallic cathode of a PMT, an electron is ejected from the surface of the cathode. The ejected electron is directed to the electron multiplier caused by the electric field between the focusing electrode and cathode. The electron multiplier consists of an array of electrodes which has a sequentially increasing electrical potential (dynodes). When the electron hits the surface of the first dynode, several secondary electrons will be produced. These secondary electrons are further accelerated by the electric field between the first and the second dynodes. By repeating this process, sufficiently large amount of electrons, 10^5-10^7 per incident electron, will be produced.

When the PMT is combined with a scintillation counter, the amplitude of the current signal is directly proportional to the energy of the incident photons.

Finally, the electrical signal is digitized, and proportionally converted to counts. Thus, the number of counts is proportional to the energy of the incident photon. This concept is called spectroscopy, and an energy distribution measured in this matter is called a spectrum. By fitting a Gaussian to the γ-ray emission energy photopeak in a spectrum, we can accurately determine the energy of the ionizing radiation, which you will do for the Compton scattering portion.

Attenuation

Expected

- Determine the attenuation coefficient of the aluminum sheets of Section II by fitting a curve to the transmission probability as a function of thickness.

The transmission probability for a gamma-ray of a certain energy through a certain medium is

$$T = e^{-\mu x} \tag{1}$$

where x is the thickness of the material and μ is its attenuation coefficient. Attenuation occurs when an incident gamma-ray loses energy after interacting with an electron in the attenuation material. The transmission probability, T, is $\frac{I}{I_0}$ where I is the total counts of incident photons at the gamma energy at a certain thickness, and I_0 is total counts without any attenuation. In general, the attenuation coefficient is unique for different radiation energies.

Section III: Compton Scattering

Expected

- Fit a gaussian to the shifting photopeaks. The energy is the x-value where the peak occurs. Fit a curve to the shifting photopeak energy (E') as a function of scattering angle θ from the data collected in Section III to find the rest mass of an electron.
- Include a plot of the data collected and of the curve fit.

Compton scattering is the inelastic scattering of a high-energy photons (γ-ray) usually by an electron. In 1920, A.H. Compton observed the scattering of monochromatic high-energy electromagnetic waves from a carbon target and found that the scattered wave had a longer wavelength than the incident wave, and hence a lower energy. It was also observed that the change of the energies between the incident and scattered waves increased with scattering angle.

The result was inconsistent with classical wave theory where the frequency is a property of the incoming electromagnetic wave and cannot be altered by the change of the direction due to scattering. Compton modeled the data by assuming a particle nature of the photon and applied conservation of energy and momentum to the collision between the photon and the electron. At the time of the observation, the particle nature of light suggested by the photoelectric effect was still being debated; the Compton experiment gave clear and independent evidence of the particle-like behavior of light. Compton was awarded the Nobel Prize in 1927 for the discovery of the Compton Effect.

The Compton Effect is based on conservation of momentum. Knowing, relativistically, the momentum of a particle is $\frac{hv}{c}$ and its energy is $E = hv$, after a collision, where the new velocity of the particle is v' and the new momentum is p, we can use equation (2) to find the change in energy of the particle.

$$hv + mc^2 - hv' = \sqrt{pc^2 + (mc^2)^2}, \tag{2}$$

where m is the mass of the scattered electron. The change in wavelength of a γ-ray after being scattered through applying the conservation of momentum is

$$\Delta\lambda = \frac{hc}{E'} - \frac{hc}{E} = \frac{h}{mc}(1 - cos\theta) \tag{3}$$

In equation (3), E is the energy of incident photons, E' is the energy of photons scattered in direction of θ, m is the electron mass, c is the speed of light, and θ is the scattering angle. When photons scatter, E' will shift horizontally as a function of θ. The derivation for equation (3) comes from the Napolitano text. Rewriting this, we get equation (4), which is the new energy as a function of scattering angle. E is the initial energy without scattering. Hint: E is the energy used for calibration.

$$\frac{1}{E'} = \frac{1}{mc^2}(1 - cos\theta) + \frac{1}{E} \tag{4}$$

Rewriting (4), we get a clearer picture of how to use a curve fit to find m, using equation (5).

$$E' = \frac{E}{1+(E/mc^2)(1-cos\theta)} \tag{5}$$

Section IV: Differential Cross-Section of Aluminum Rod and Classical Radius of an Electron

Note: This procedure is also described in the supplementary material found in the LMS site: filename: compton_scattering_supplementary material.pdf

Expected

- Use a curve-fit to determine the classical radius of an electron.
- Include a plot of $\frac{d\sigma}{d\Omega}$ as a function of scattering angle θ using the measurements made in Section III of the Instruction Manual.

Hint

- Combine the equations (6) and (7) and solve for Σ_γ' as a function of scattering angle θ, and fit the curve to Σ_γ' to solve for *r*.
- You should already have gaussian fits for the shifting photopeaks from the previous section, so now all you need to do is integrate those using the quadrature function from the Scipy library. This will give you the total # of counts (= total # of decays) within the photopeak.

The differential cross-section for Compton scattering, first proposed by Klein and Nishina (eq. 6), is $\frac{d\sigma}{d\Omega}$ where $d\sigma$ is the cross-sectional area component in the plane of the initial electrons, and $d\Omega$ is the solid angle of the scattered photons scattered at an angle θ.

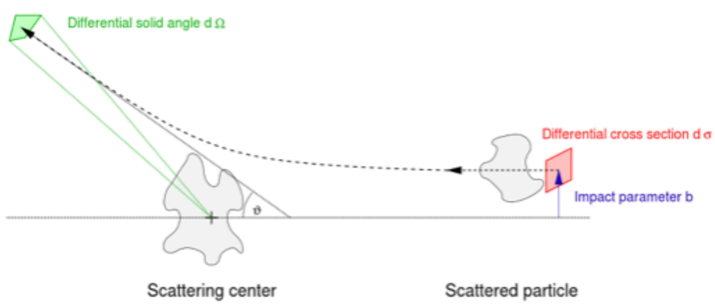

Figure 2: The differential cross-section $\frac{d\sigma}{d\Omega}$ of a particle scattered off a scattering center.

In our setup, the scattering center shown in Figure 2 is the scattering volume of the aluminum or unknown rod, and the scattered particle is the one emitted from the Cesium-137 source.

$$\frac{d\sigma}{d\Omega} = \frac{r^2}{2}\left[\frac{1+\cos^2\theta}{[1+\alpha(1-\cos\theta)]^2}\right]\left[1 + \frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right] \tag{6}$$

In the Klein-Nishina differential cross-section, $\alpha = \frac{E_\gamma}{mc^2}$ where E_γ is the unscattered γ-ray energy of the source, which for Cs-137 is 661.7 keV, and m is the rest energy of an electron, determined from the Compton Scattering portion of the lab. The purpose of this section of the lab is to determine r, the classical radius of an electron.

Experimentally, the differential cross-section, taking into account the material of the scattering rod, equation (6) becomes equation (7).

$$\frac{d\sigma}{d\Omega} = \frac{\Sigma_\gamma'}{n_\theta I \Delta\Omega t \epsilon} \tag{7}$$

Σ_γ' is the integral of the shifted photopeak (= total # of decays) as a function of θ, t is the length of time of the scan (120 s), $\epsilon$is the efficiency of the detector, shown by the graph in figure 3. To determine the changing ϵ, use the linear curve-fit for the particular case of a 2” x 2” scintillator as $\epsilon = \; - 0.50E \; + \; 0.58$ where E is in MeV (this means that for Cs-137, E= 0.6617MeV). This fit is accurate to the hundredth. Remember in finding Σ_γ', sum the number of decays of the shifting photopeaks in the Compton Scattering portion above.

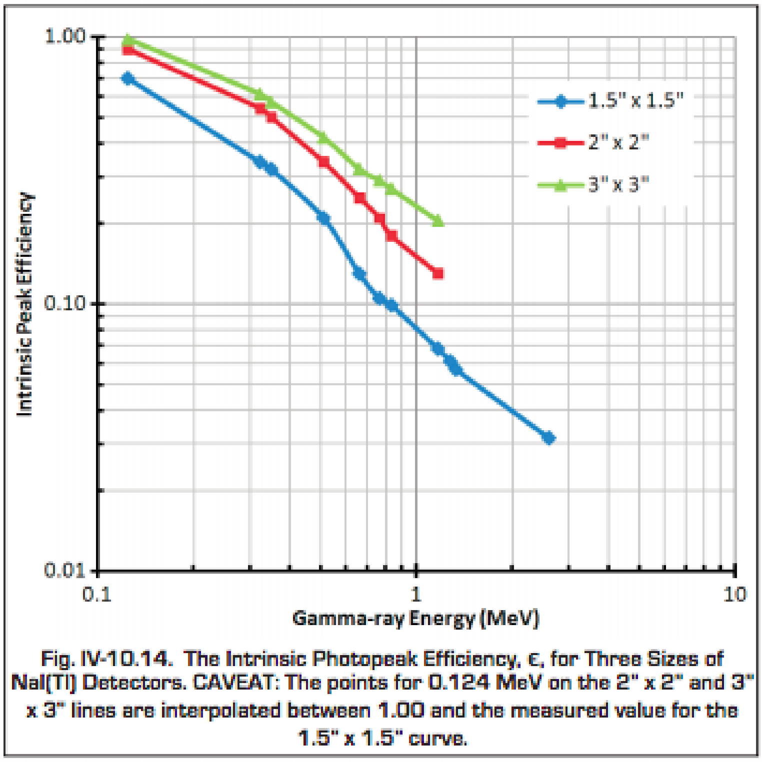

Figure 3: The efficiency of the photodetector depending on the energy of the radiation.

n_θ is dependent on the electrons scattered from the aluminum rod,

$$n_\theta = \rho V N_A \sum_i w_i \frac{Z_i}{M_i}, \tag{8}$$

where ρ is the density of the scatterer in g/cm^3, V is the scattering volume of the rod, and N_A is Avogadro's number. Z_i is the atomic number of the ith particle of the scattering material and M_i is the molar mass of the material of the rod. The weight fraction of the scattering rod $\sum_i w_i$, by definition for a pure material, sums to 1. The scattering volume is different from the entire volume of the rod. The radiation from the Cs-137 source is assumed to be a rectangular beam with a width of the scattering rod and height of the lead hole the source sits in. The volume of the scattered material is therefore a shortened cylinder with a height of approximately 2 cm.

I in equation (7) is the number of incident γ-rays/cm^2 per second at the scattering rod. There are essentially two approaches in computing the differential cross-section using the data from the aluminum rod based on how I (flux of γ-rays) is estimated.

Approach 1:

Using the intrinsic activity of the Cs-137 source, one can setup the following equation for the photon flux at the scattering rod:

$$I = \frac{A_0 f}{4\pi R_1^{\ 2}}, \tag{9}$$

where A_0 is the activity of the source, 10mCi, f is the fraction of gamma-ray emissions, 0.851 for Cs-137, and R_1 is the distance from the source to the center of the scattering rod (=31cm). Remember to convert the units of Ci (Curies) to # of decays.

Approach 2:

Use the measurement of the Cs-137 photopeak (at 661.7keV) with the detector at 0° without the aluminum rod in place (this was done at the beginning of Section III of the Instruction Manual). Assume the inverse square law holds and use that to compute the incident flux at the scattering rod. The photon flux at the detector will be equal to the total # of decays (= # of counts) from the integral of the photopeak which is then divided by the area of the detector and the scan time (=120s). Use the inverse square law to scale the photon flux measured at the detector in order to get the estimate at the scattering rod.

Use both approaches in estimating the value of "*I*", the incident photon flux at the scattering rod, and compare the computed values of *r*, the classical electron radius from the curve fit.

You will also need the following information to complete the computation:

The solid angle of the scintillation detector is

$$\Delta\Omega = \frac{\pi (D/2)^2}{R_2^{\ 2}}, \tag{10}$$

where *D* is the diameter of the scintillator (5.08 cm) and R_2 is the distance from the center of the scattering rod to the front of the detector.

In the final part of the experiment, you will try to deduce the composition of the unknown material. Note that the differential cross-section is identical, and you have measured the total decays ($\Sigma_\gamma{}'$) as a function of scattering angle. Therefore the only unknown is the # of electron scatterers in the material given by equation (8). Look at the unknown material and take a guess as to what you think it is composed of. Then look through the literature and use the documented composition of the manufactured material to compute the measured value of n_θ. You may need to tweak the weight fraction of each element to get the best fit.