Quantum Physics 1
Class 24

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Zeeman Effect

Last Time:

Hydrogen Atom:

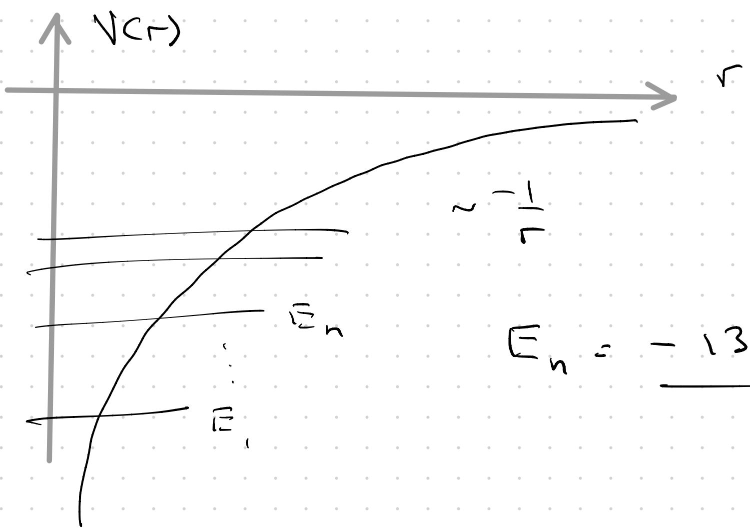
$$V(r) \sim -\frac{1}{r}$$

$$\Psi_{n\ell m}(r,\theta,\phi) = R_{n,\ell}(r)\, Y_{\ell,m}(\theta,\phi)$$

$$R_{n,\ell} = r^{\ell} F_n(r)\, e^{-\sqrt{-2m_e E_n/\hbar^2}\, r}$$

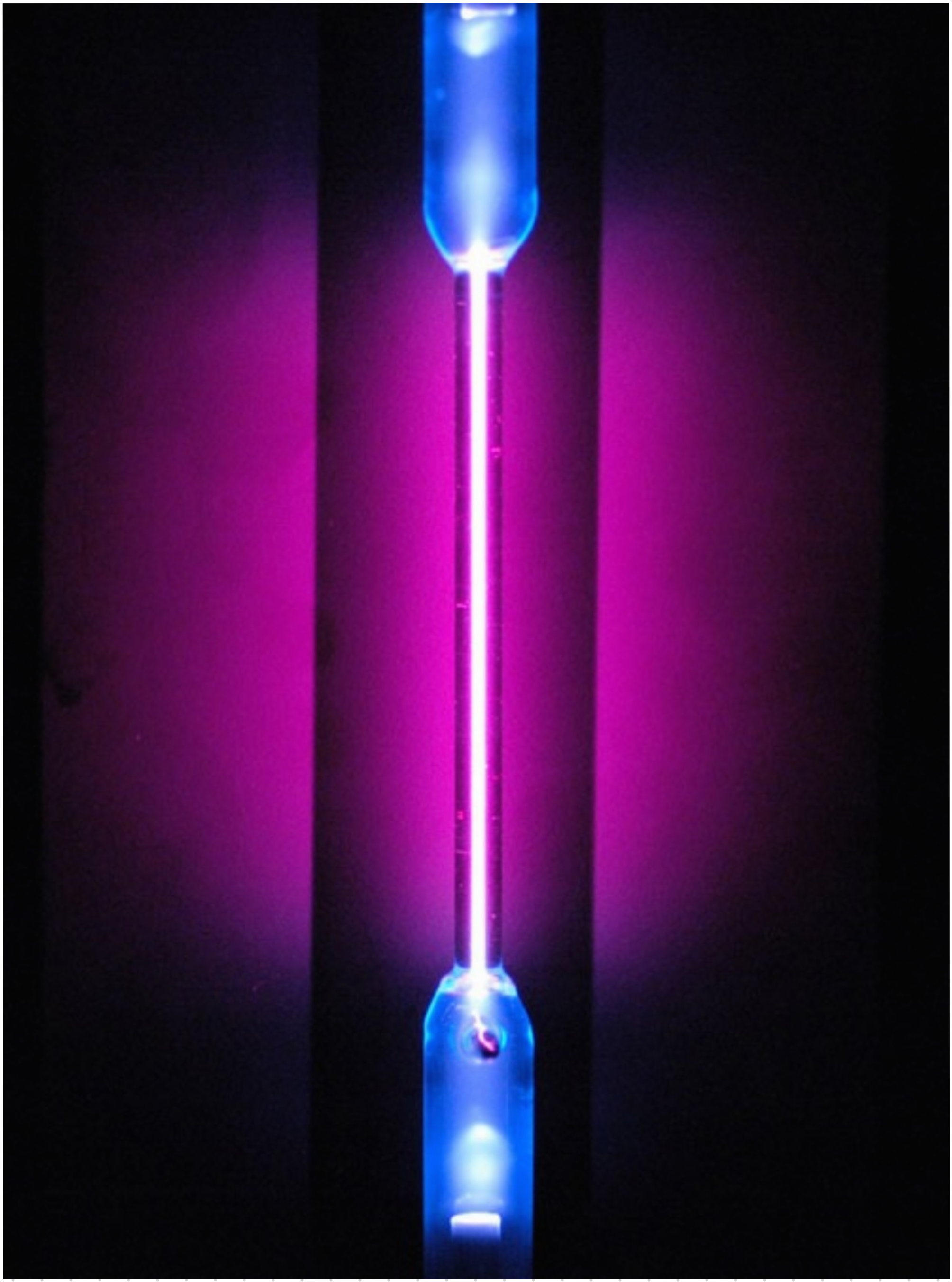
$$\hat{L}^2 Y_{\ell,m}(\theta,\phi) = \ell(\ell+1)\hbar^2 Y_{\ell,m}(\theta,\phi)$$

$$\hat{L}_z Y_{\ell,m}(\theta,\phi) = m\hbar\, Y_{\ell,m}(\theta,\phi)$$



$$E_n = \frac{-13.6\,eV}{n^2}$$

Hydrogen Lamp



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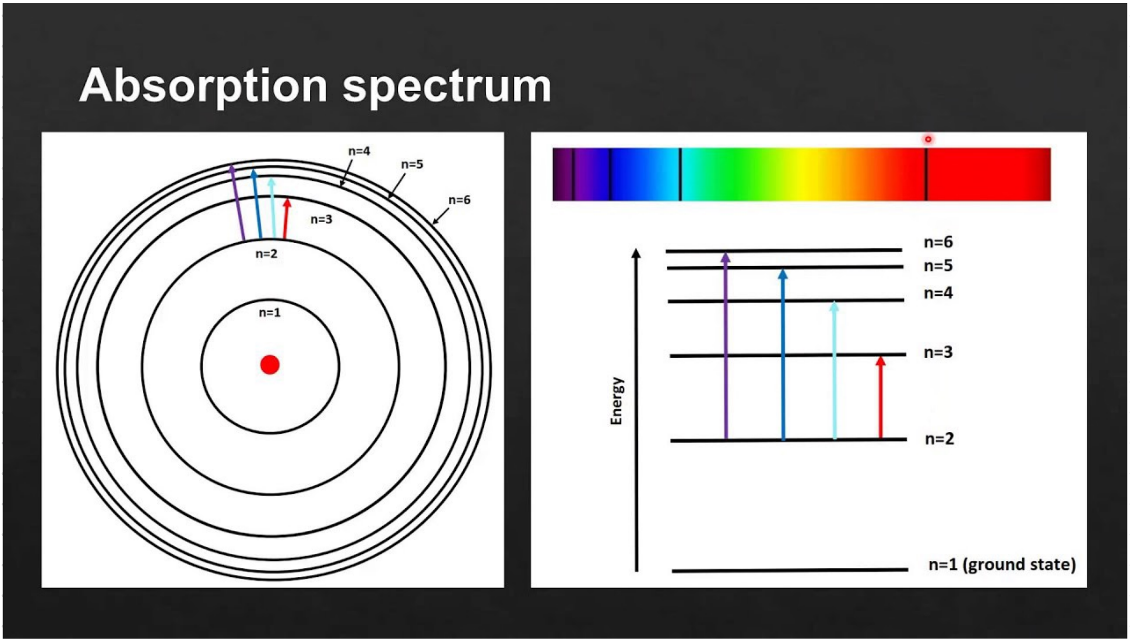

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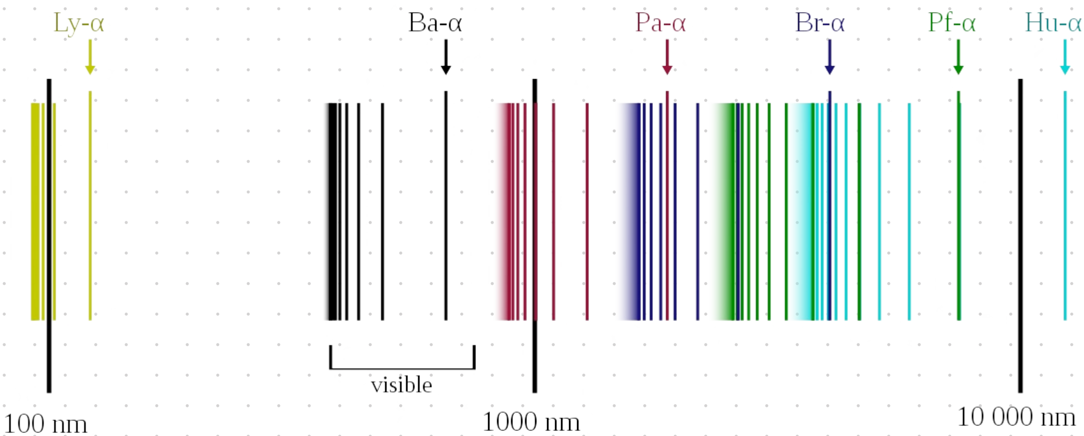
Emission Spectrum



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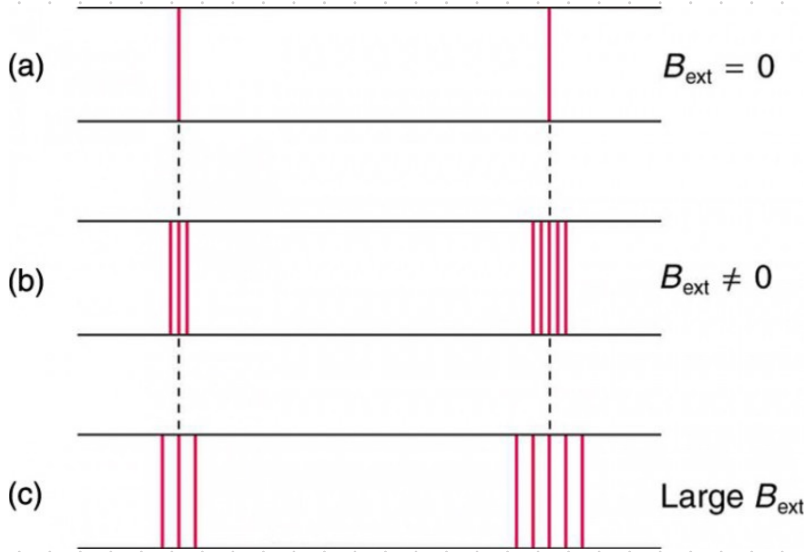
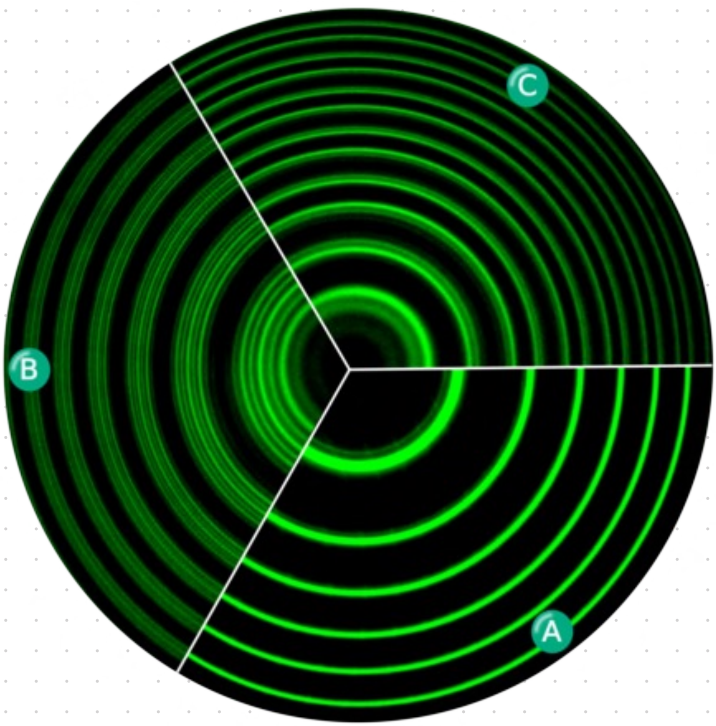
Emission lines in a magnetic field.





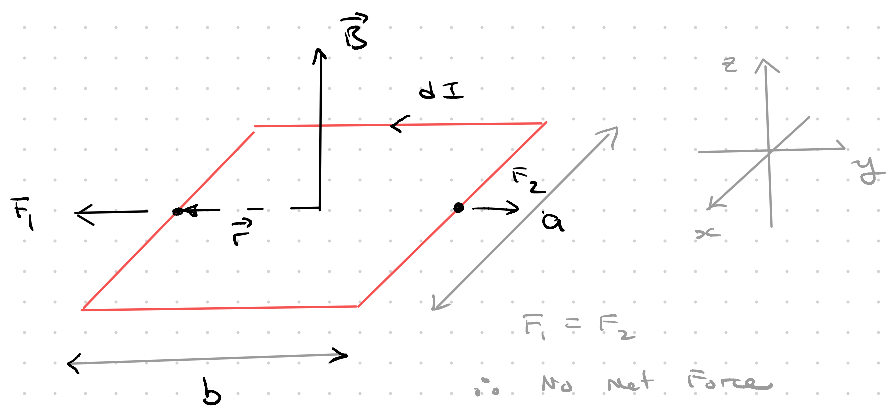
The spectral lines of mercury vapor lamp at wavelength 546.1 nm, showing anomalous Zeeman effect. (A) Without magnetic field. (B) With magnetic field, spectral lines split as transverse Zeeman effect. (C) With magnetic field, split as longitudinal Zeeman effect.: The spectral lines were obtained using a Fabry–Pérot interferometer.

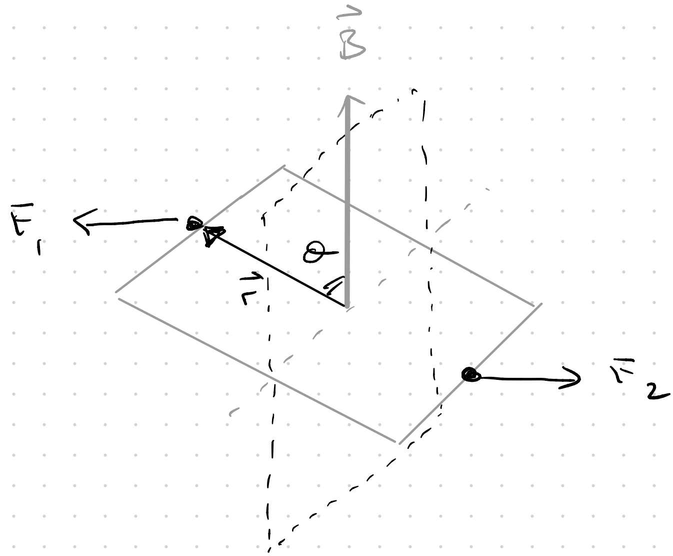
What about magnetic fields on hydrogen atom?
(e^- in the hydrogen atom?)

- Consider classical H-atom (see textbook)

- e^- revolving around the nucleus.
- " e^- current" produces a magnetic field.
  ** Qualitatively wrong but gives a rough idea and can be used to estimate real world behavior
- What is magnetic field generated by moving e^-? $\vec{\mu}$?

Consider a current carrying loop:





$F_1 = F_2$

But $\tau \neq 0$

Force on loop?

$$\vec{F} = \cancel{q\vec{E}} + q\vec{v} \times \vec{B}$$

$$dF_1 = dq \frac{d\vec{l}}{dt} \times \vec{B} = I\, d\vec{l} \times \vec{B}$$

$$F_1 = I a B$$

Torque: $|\vec{\tau}| = 2\,|\vec{r} \times \vec{F}|$

$$= 2\,\frac{b}{2}\, I B \cdot \sin\theta \cdot a$$

$$= \underbrace{a\,b\,I}_{\text{magnetic dipole, } \vec{\mu}}\, B \sin\theta$$

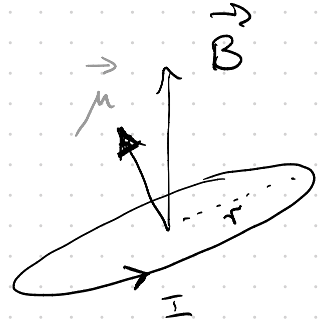
$$= \mu B \sin\theta$$

Potential Energy? (Work done to orient $\vec{\mu}$; ie move the loop)

$$U = \int_{\frac{\pi}{2}}^{\theta} \tau \, d\theta = \int_{\pi/2}^{\theta} \mu B \sin\theta \, d\theta$$

$$= -\mu B \cos\theta$$

$$\boxed{U = -\vec{\mu} \cdot \vec{B}}$$



Now $|\vec{\mu}|$ = Area × current

$$= \pi r^2 \cdot I$$

$$= \pi r^2 \frac{q}{T} \; ; \; T = \frac{1}{f}$$

$$= \pi r^2 q \left(\frac{2\pi r}{v}\right)^{-1} \; ; \; T = \frac{2\pi r}{v}$$

$$= \pi r^2 \left(\frac{vq}{2\pi r}\right)$$

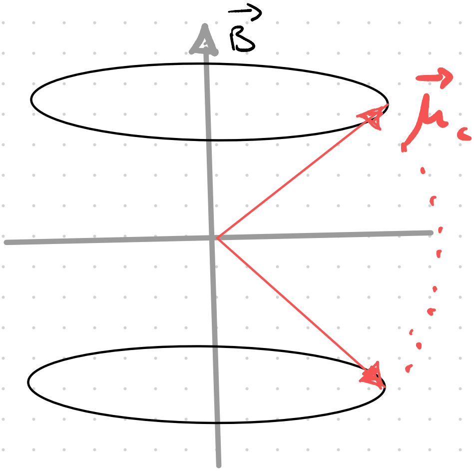
$$|\vec{\mu}| = \frac{qrv}{2} = \frac{q}{2} \frac{rp}{m_0} = \frac{q}{2m} L$$

$\therefore$ Potential Energy:

$$\boxed{-\vec{\mu} \cdot \vec{B} = \frac{-q}{2m_0} \vec{L} \cdot \vec{B}}$$

NB: Classically μ_c is continuous and can take on only values between

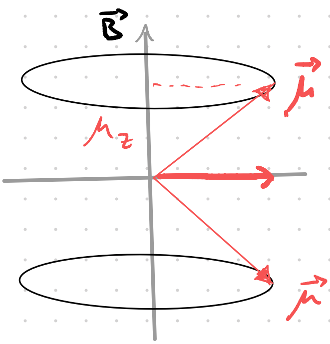
$$-(\mu_c)_{max} \leq \mu_c \leq (\mu_c)_{max}$$



Quentum μ : Only discrete values!

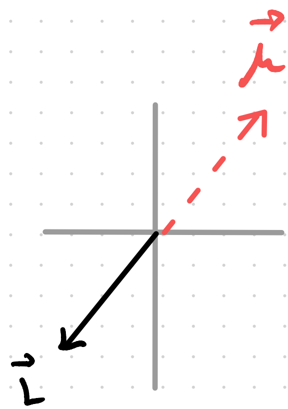
Because L & L_z are

discrete : $\sqrt{l(l+1)}\,\hbar$, $m\hbar$



For $q = -e$:

$$\boxed{\mu = \frac{q}{2M_0}\hat{L} = \frac{-e}{2M_0}\hat{L}}$$



In-class 24.1, 24.2
24.3

How can we describe the hydrogen atom in $\vec{B}$?

$$\hat{H} = \underbrace{\frac{-\hbar^2}{2m_0}\nabla^2 + V(r)}_{H_0} - \frac{q}{2m_0}\vec{L}\cdot\vec{B} \quad ; \quad q = -e$$

$$V(r) \sim -\frac{1}{r}$$

$$\hat{H} = \left(H_0 - \frac{q}{2m_0}\vec{L}\cdot\vec{B}\right)$$

What are the eigenfunctions?

In-class 24.4

Zeeman effect:

Zeeman energy — energy splitting

* consider degeneracy of 2 only