JOHNSON NOISE

Reference: Text: *Melissinos and Napolitano (M&N)* Chapter 3.6.

Objective: To determine Boltzmann constant by amplifying signals from thermally dependent electron movements in a resistor.
In this experiment, the voltage variance across several resistors that generate Johnson Noise will be measured as a function of resistance at room temperature. From this curve, one will be able to calculate the Boltzmann constant. (*Note: You are encouraged to read the reference material on this experiment as important background material for this experiment*)

Section I: Experimental Approach

For the analysis, you will need to measure the gain of the amplifier and band pass filter as shown in Figure 1 as a function of frequency. Several resistors are then connected sequentially to the input of the frequency constrained amplifier and the voltage variance measured using an oscilloscope (Figure 2). You will then record the variance in voltage across a resistor as a function of input resistance.

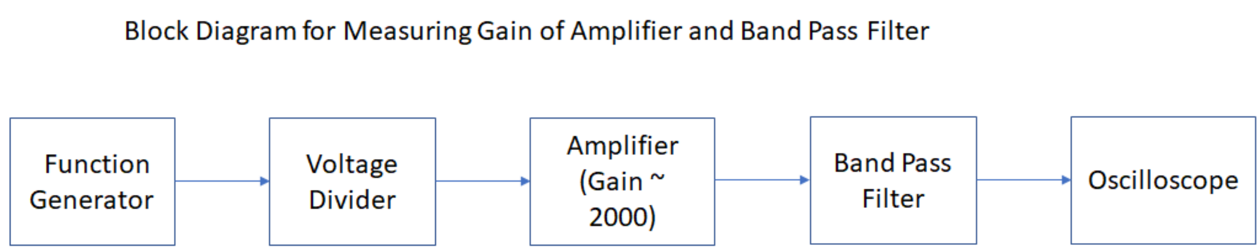

Figure 1: Method of measuring gain of the amplifier & band pass filter

Figure 1 shows the overall arrangement for quantifying the gain of the amplifier and band pass filter as a function of frequency. The respective elements in the block diagram (Figure 1) perform the following function:

- **Function Generator**: Produces a sine wave with varying frequency and fixed amplitude is used for initiating the measurement
- **Voltage Divider**: Converts the input signal from the Function Generator and reduces to small values (~ 1-3 mV (amplitude) - note that you will be amplifying the signal by a factor of 2000 and need the amplitude to be always lower than the battery supply voltage (~ 9V)), typically unattainable by the Function Generator. The output from the voltage divider string drives the amplifier stage.
- **Amplifier**: This is constructed by combining two op-amps to get a gain of ~ 2000, with the first op-amp having the lower gain of ~10 and the second op-amp at a gain of ~ 200. These op-amps

are wired as **non-inverting** amplifiers. The final output signal is the input for the Band Pass Filter.

- **Band Pass Filter**: This limits the frequency range over which the variance in voltage is measured. The range is constrained to be between 5kHz-20kHz, by using a combination of a Low Pass and a High Pass filter, constructed from resistors and capacitors, connected in tandem.
- **Oscilloscope**: The voltage output from the Band Pass Filter is measured as a function of frequency with this instrument. The amplitude (peak or peak-to-peak) of the signal is documented with varying frequency (using the Function Generator). This final "Gain" curve is used in the analysis for computing the Boltzmann Constant.

After measuring the Gain curve, you will use the combination of amplifier and band pass filter to measure the Johnson Noise of an assortment of resistors by measuring the variance in voltage. The setup for performing this measurement is shown in Figure 2:

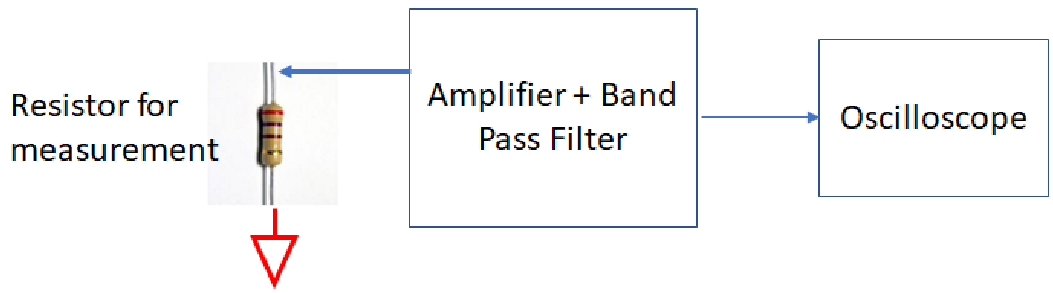

Figure 2: Setup for Johnson Noise Measurements

Section II: Construction of Individual Components

Section II.1 Voltage Divider

A voltage divider will be used when we calibrate the gain of the system (see Figure 1). It is very difficult to generate low voltage signals from a Function Generator (~ mV). For this reason, we must pass a high enough voltage through the voltage divider, but then attenuate it (at R2) before it gets amplified.

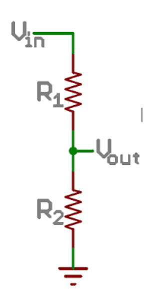

Figure 3. Voltage divider (from Electronics Lab)

Section II.2 Amplifier

The amplifier consists of two operational amplifiers (op-amps) wired in tandem to get the final high gain of ~ 2000. The pinout diagram for the op-amps used in this lab is shown below. Use two 9V rechargeable batteries to supply the positive (pin 7) and negative power (pin 4) inputs.

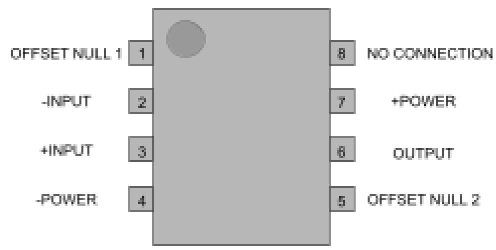

Figure 3. Pinout diagram

Op-amps can be used to multiply input voltage by a constant. The input and feedback resistors determine the gain coefficient for each stage of amplification shown in figure 4. The final amplification is achieved by using both op-amps in series. Derive, using the Golden Rules from the Electronic Lab, the gain for each stage of the non-inverting amplifier. Note that the same valued resistor to ground (R1) is used for both stages of amplification.

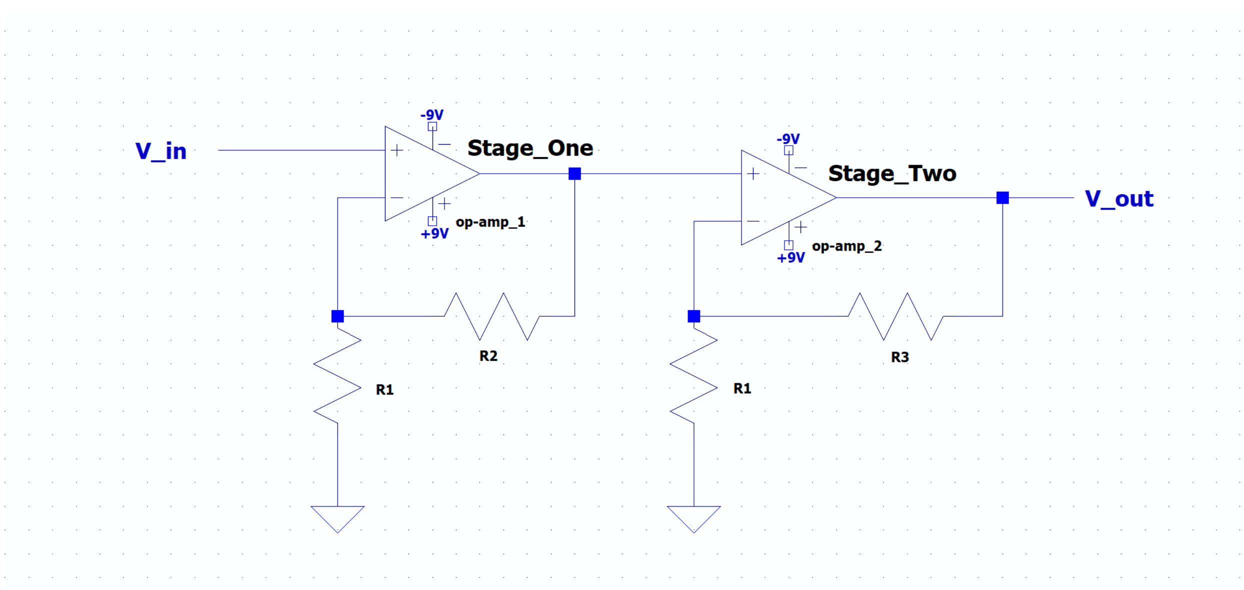

Figure 4. Dual Stage Amplifier

Choose values of R1 and R2 to generate a gain of about 10 for Stage One of the amplifier. Similarly, choose values of R1 and R3 to generate a gain of about 200 for Stage Two of the amplifier. Check the gain of each stage independently by using a Function Generator with a 10kHz sine wave operating at a suitable amplitude to perform the measurement <u>without</u> saturating the op-amp (recall that the DC powered levels are at +/- 9V). (*Note: In the example shown in the text (M&N), the first stage has a gain of 11 and the second stage a gain of 221.* ***Resistor values for each of the two stages are also documented in the text***.)

Section II. 3 Band-Pass Filter

The large factor of amplification is used to amplify the thermal noise of a resistor, but in doing so, all of the other unwanted noise also gets amplified. In order to more clearly see the Johnson noise, it is important to cut out high and low frequencies. You will do this by making a band pass filter. Johnson noise typically has frequency components in between 5 kHz and 20 kHz. Build a bandpass filter with these as the lower and upper limits.

High-pass filter:

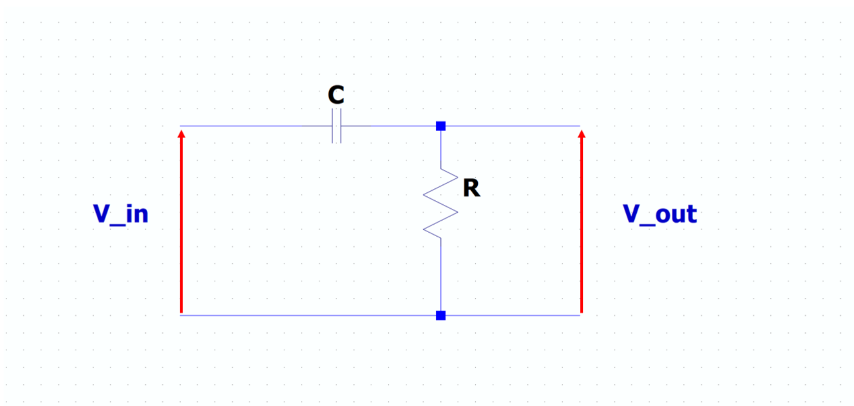

Figure 5: High Pass Filter

High-pass filters attenuate signals of low frequencies. The cutoff frequency is dependent on the resistor and capacitor values: $f_c = \frac{1}{2\pi\tau} = \frac{1}{2\pi RC}$. This works because low frequency signals cannot transfer charge quickly enough to make it through the capacitor. Any built up charge due to low frequency signals is discharged through the resistor and sent to ground. Use a 0.1 μF capacitor and construct a high pass filter with a 5 kHz cut-off frequency.

Low-pass filter:

Low-pass filters work very much in the same way as high pass filters.

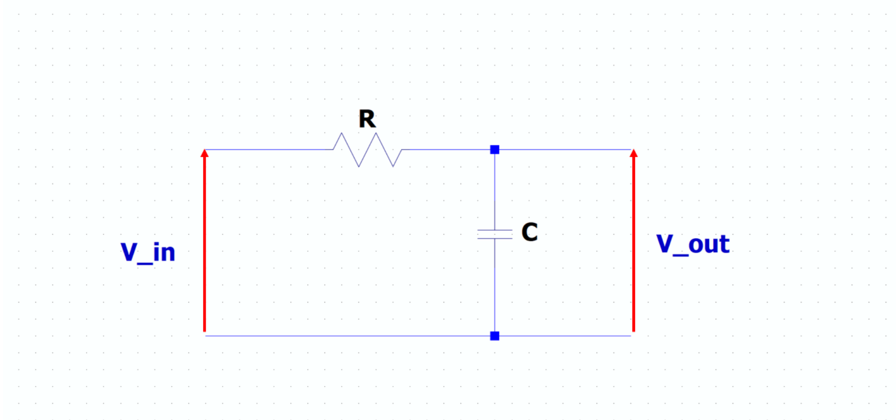

Figure 6: Low Pass Filter

The capacitor connected to ground means all high frequency signals are sent to ground, allowing only the low frequencies to pass. Again, the cutoff frequency is $f_c = \frac{1}{2\pi\tau} = \frac{1}{2\pi RC}$. Use a 0.1 μF capacitor and construct a high pass filter with a 20 kHz cut-off frequency.

Section III INSTRUCTIONS

Section III.1 Calibration of Circuit Gain

- Wire the voltage divider (Figure 2) with a divider ratio of ~100
- Determine the resistor and capacitor values to limit the frequency range to that of the Johnson noise (5 kHz - 20 kHz) and build these components in series to create the band-pass filter.
- Build two non- inverting amplifiers in series such that the final amplification is ~2000 (Section II.2).
- Send the signal from a function generator through these circuit elements in series (Figure 1) and display both the original and final signal on the oscilloscope.
- Record the gain of your circuit for frequencies between 1kHz and 100kHz. (*Ques: How does one define gain in this measurement?*)

Section III.2 Johnson Noise Measurement

- Remove the function generator and the voltage divider from the setup and put the resistor you want to use for the Johnson Noise measurements between the non-inverting input of the first stage amplifier and ground.
- Set the time scale on the oscilloscope to 0.25ms/div.

- Save the oscilloscope output for at least six resistors between approximately 10Ω and 10kΩ
- Record the room temperature
- The final circuit arrangement is shown in the Appendix

Section III.3 Johnson Noise Measurement using TEACHSPIN "Noise Fundamentals" Equipment.

This is an opportunity to work with the TEACHSPIN "Noise Fundamentals" equipment where special care has been taken in fabricating and setting up the circuits, which are similar to what you built, to generate the signals for measuring the Boltzmann constant. Now, you will understand the functionality of each piece of equipment which leads up to the final measurement of the variance in voltage as a function of input resistance. The relevant parts of the manual will be on display next to the equipment. The hardware is already setup - so all you need to do is to collect data for the various resistor values. See the separate document on TEACHSPIN Approach to Measuring Johnson Noise.

To include in the Report:

Section III.1:

- A table and a plot of the gain of your circuit for frequencies between 1kHz and 100kHz. Write down your definition of gain for this circuit

Section III.2:

- Record the room temperature
- Include a digital image of the final circuit arrangement and label the various blocks of the circuit
- Using **matplotlib** and the data from the first measurement (Section III.1 from the Johnson Noise Lab Instructions manual), determine the numerical value of this integral and its error
- For each input resistance, determine $\langle V^2 \rangle$
- Plot the variance as a function of R and determine the slope and its error assuming a linear behavior with a non-zero intercept
- Using equation 3 from the Theory document ($< V^2 > = 4kTR \int_0^\infty g^2(v)dv$) , find the value for the Boltzmann constant and the associated uncertainty

Section III.3:

Plot the computed Johnson Noise ($<V_J^2(t)>$) as a function of R_{IN} and use the slope to compute the Boltzmann Constant using equation (4) from the Johnson Noise - TEACHSPIN Approach document. The frequency bandwidth is obtained by subtracting the High Pass Filter corner frequency from the Low Pass Filter corner frequency.

APPENDIX

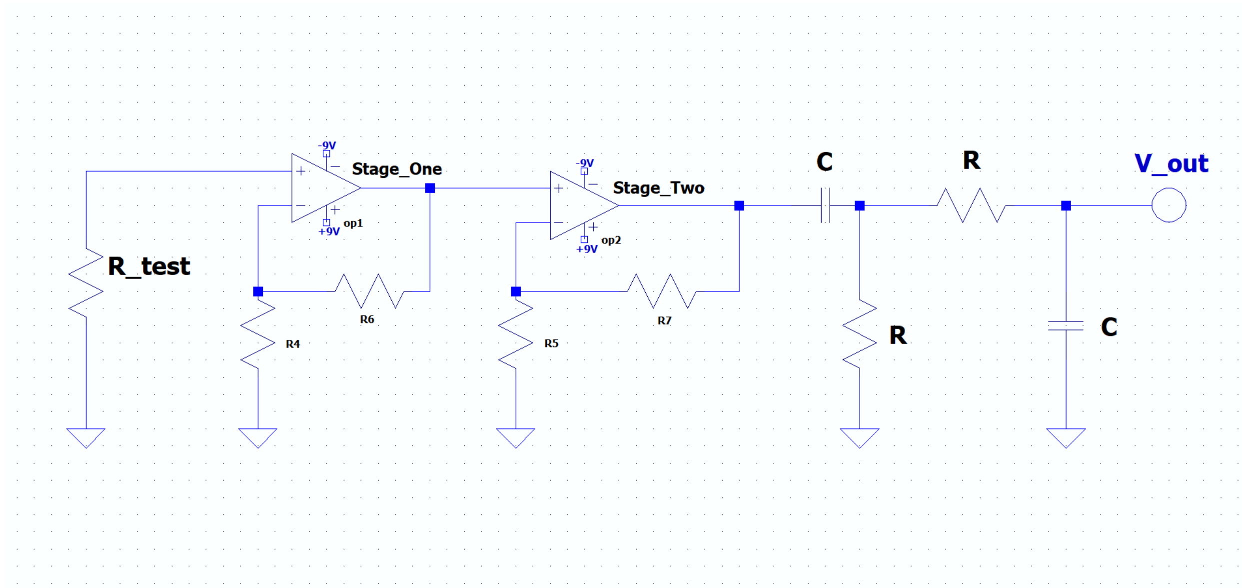

Figure 7: Final Circuit for performing Johnson Noise measurements

The figure above summarizes the final arrangement for doing the measurements. *R_test* , the particular resistor under test, is the resistor that will generate a certain level of Johnson Noise depending on the resistance and ambient temperature. *V_out* is the amplified bandpass limited signal that will be captured on the scope for analysis.