Inclass 21.1. Show that  $\hat{L}_z \equiv x\hat{p}_y - y\hat{p}_x = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$ .

$$x = rsin\theta cos\phi$$
$$y = rsin\theta sin\phi$$
$$z = rcos\theta$$

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = \begin{pmatrix} \sin\theta \cos\phi & \frac{\cos\theta \cos\phi}{r} & \frac{-\sin\phi}{r\sin\theta} \\ \sin\theta \sin\phi & \frac{\cos\theta \sin\phi}{r} & \frac{\cos\phi}{r\sin\theta} \\ \cos\theta & \frac{-\sin\theta}{r} & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial r} \\ \frac{\partial}{\partial \theta} \\ \frac{\partial}{\partial \phi} \end{pmatrix}$$

Inclass 21.2. Show that  $\hat{L}^2$  and  $\hat{L}_z$  commute, that is:  $[\hat{L}^2, \hat{L}_z] = 0$ .

Inclass 21.3. (a) A bullet of mass 0.005 kg with a speed of 1000 m/s flying by 1 meter from me. Estimate the angular momentum quantum number l for this system. Hint: approximate

 $|L| = \sqrt{l(l+1)}\hbar \approx l\hbar$ . (b) What is the difference in angular momentum between adjacent states.

$$r_{\perp} = 1 m$$

$$p = mv$$

Inclass 21.4. Show that  $\left[\hat{L}_x,\hat{L}_y\right]=i\hbar\hat{L}_z$   $\hat{L}_x=y\hat{p}_z-z\hat{p}_y;~~\hat{L}_y=z\hat{p}_x-x\hat{p}_z;~~\hat{L}_z=x\hat{p}_y-y\hat{p}_x$