N. IV.4 #1, 2
N. IV.5 #2

1. Work out dim. of $\{m\}$ for $SU(3)$

- This is the irreducible rep. furnished by the totally symmetric tensor carrying "m" indicies.

$\{3\}$ has 10 dimensions

1
2 3
4 5 6
7 8 9 10

Pg 230)

$\{4\}$

φ^{3333}
φ^{3331} φ^{3332}
φ^{3311} φ^{3312} φ^{3322}
φ^{3111} φ^{3112} φ^{3122} φ^{3222}
φ^{1111} φ^{1112} φ^{1122} φ^{1222} φ^{2222}

$\{5\}$

1
2 3
4 5 6
7 8 9 10
11 12 13 14 15
16 17 18 19 20 21

$\{3\} \to 10$ $\{2\} \to 6$

$\{4\} \to 15$ $\{1\} \to 3$

$\{5\} \to 21$

$\to$ for "M" indicies the dimension is

$$\boxed{1 + \sum_{i=2}^{M+1} (i)}$$

for $M = 1$

$$1 + (2) = 3 \checkmark$$

for $M = 2$

$$1 + (2 + 3) = 6 \checkmark$$

for $M = 3$

$$1 + (2 + 3 + 4) = 10 \checkmark \quad \ldots$$

IV.4 #2 Structure Constants for SU(2) & SU(3)

structure constant for SU(2) should be same as SO(3), b/c the algebras are isomorphic

$$\boxed{\varepsilon^{abc}}$$

SU(3) Structure constant using Gell-mann

$[T^a, T^b] = i f^{abc} T^c$ for $T^a = \frac{1}{2}\lambda^a$

$\lambda_3\ \lambda_8$ diagonal (α) Structure constants

$\lambda_1, \lambda_2, (\lambda_3)$ SU2 & top left (β) f^{123} #1

$\lambda_4\ \lambda_5$ opposite diagonal (γ) $f^{\beta\gamma} = f^{3\gamma} = f^{3\delta}$ #2

$\lambda_6\ \lambda_7$ b. right (δ) f^{38} #3

f^{67} & f^{45} #4

f^{123} $\quad [T^1, T^2] = i f^{123} T^3$

$$\begin{pmatrix} 0 & -i & \\ i & 0 & \\ & & \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & \\ 1 & 0 & \\ & & \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & \\ 1 & 0 & \\ & & \end{pmatrix}\begin{pmatrix} i & 0 & \\ 0 & -i & \\ & & \end{pmatrix} - \begin{pmatrix} 0 & -i & \\ i & 0 & \\ & & \end{pmatrix}\begin{pmatrix} -i & 0 & \\ 0 & i & \\ & & \end{pmatrix} = \begin{pmatrix} 2i & 0 & 0 \\ 0 & -2i & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= 2i(1)\lambda_3 = i(1)T_3$$

$f^{123} = 1$

~~$[\lambda_1, \lambda_3] = 0$~~

$$\begin{vmatrix} & & 1 \\ & 0 & \\ 1 & & \end{vmatrix} \qquad \begin{vmatrix} 0 & 1 & \\ 1 & 0 & \\ & & \end{vmatrix}$$

#2 $[\lambda_1, \lambda_4] = \begin{vmatrix} 0 & 1 & \\ 1 & 0 & \\ & & \end{vmatrix}\begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} - \begin{vmatrix} & & 1 \\ & 0 & \\ 1 & & \end{vmatrix}\begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$

f^{14x}

$f^{147} = -\frac{1}{2}$

$$= \begin{vmatrix} & 0 & 1 \\ & -1 & 0 \\ & & \end{vmatrix} = -\frac{1}{2} i T^7$$

f^{13X}

$$\begin{pmatrix}0&1\\1&0\end{pmatrix} \cdot \begin{pmatrix}0&0&0\\0&0&-i\\0&0&0\end{pmatrix} - \begin{pmatrix} & & -i\\ & & \\ i & & \end{pmatrix} \begin{pmatrix}0&0&0\\0&0&0\\0&i&0\end{pmatrix} = \begin{pmatrix}0&0&0\\0&0&-i\\0&-i&0\end{pmatrix} = -\frac{1}{2} i T^6$$

$f^{156} = -\frac{1}{2}$ ✓

$-\frac{1}{2} = f^{147}, f^{156}, -f^{2+X}, f^{25X}, f^{34}, f^{36}$

f^{45}

$$\begin{pmatrix} & & 1\\ & & \\ 1 & & \end{pmatrix} \begin{pmatrix}i&0&0\\0&0&0\\0&0&-i\end{pmatrix} - \begin{pmatrix} & & -i\\ & & \\ i & & \end{pmatrix} \begin{pmatrix}-i&0&0\\0&0&0\\0&0&i\end{pmatrix}$$

$$= 2i \begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix} = i\left(\lambda_3 + \sqrt{3}\lambda_8\right)$$

$$f^{45[38]} = \sqrt{3}/2 = f^{67[38]}$$

#3 T^{ij} rotates like a vector
$6/2 = 3$ components ($i\,j\,k$ / xyz).
It has to transform like a tensor and a 3 element tensor transforming like a tensor is a 3-d vector.

The 3-d vector furnishes the vector representation of $SO(3)$, the rotation group