Problem Set 2

ECON4570 : Problem Set 2

Due by 11:59pm on 10/01/2024

1. Assume that you are in charge of the central monetary authority in a mythical country. You are given the following historical data on the quantity of money (X) and national income (Y)(both in million of dollars). Also assume **Assumptions 1, 2, 3 hold and the variance is (conditional) homoskedasticity**:

Year	Quantity of Money (X)	National Income (Y)
1989	2.0	5.0
1990	2.5	5.5
1991	3.2	6.0
1992	3.6	7.0
1993	3.3	7.2
1994	4.0	7.7
1995	4.2	8.4
1996	4.6	9.0
1997	4.8	9.7
1998	5.0	10.0

(a) Estimate the regression of national income Y on the quantity of money X and provide estimates of their standard errors. (Do this by HAND with a calculator and show your work).

$$Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i$$

$$\hat{\beta}_2 = \frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n \cdot \bar{X}^2} =$$

$\sum X_i Y_i - n\bar{X}\bar{Y} = 13.09$

$147.18 - 138.384 = 8.796$

$\hat{\beta}_2 = 13.09 / 8.796 = \underline{1.7155}$

$\sum X_i Y_i$

$\Rightarrow 10 + 13.75 + 19.2 + 25.2 + 23.76 + 30.8 + 35.28 + 41.4 + 46.56 + 50 = 295.95$

$\bar{X} = 3.72$

$\bar{Y} = 7.55$

$n = 10$

$\Rightarrow 280.86$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \cdot \bar{X} \rightarrow 7.55 - 1.7155 \cdot 3.72$$

$$\Leftrightarrow 1.168144$$

$$\hat{\beta}_1 = 1.168144$$

$$\boxed{\hat{Y}_i = 1.168144 + X_i \cdot 1.7153}$$

$$se(\hat{\beta}_2) = \sqrt{\frac{s^2}{\sum X_i^2 - n\bar{X}^2}} \qquad s^2 = \frac{\sum_{i=1}^{n} \hat{\varepsilon}_i^2}{n-2} \qquad \hat{\varepsilon}_i = Y_i - \hat{Y}_i$$

$$se(\hat{\beta}_1) = \sqrt{\frac{s^2}{n} \cdot \frac{\sum X_i^2}{\sum X_i^2 - n\bar{X}^2}}$$

X	ε
2	0.401256
2.5	0.043606
3.2	-0.657104
3.6	-0.343224
3.3	0.371366
4.0	-0.329344
4.2	0.027596
4.6	-0.0758524
4.8	0.298416
5.0	0.255356

$$\sum \hat{\varepsilon}_i^2 = 1.11732181$$

$$s^2 = \frac{\sum \hat{\varepsilon}_i^2}{n-2} = \frac{1.11732181}{8}$$

$$\Leftrightarrow 0.139665$$

$$se(\hat{\beta}_2) = \sqrt{\frac{0.139665}{\sum X_i^2 - n\bar{X}^2}}$$

$$\Leftrightarrow 8.796$$

$$\sqrt{\frac{0.139665}{8.796}} = 0.1260$$

$$se(\hat{\beta}_1) = \sqrt{\frac{0.139665}{n} \cdot \frac{\sum X_i^2}{\sum X_i^2 - n\bar{X}^2}}$$

$$\Leftrightarrow \sqrt{0.0139665 \cdot \frac{147.18}{8.796}}$$

$$\Leftrightarrow 0.4834215$$

(b) How do you interpret the intercept and the slope of the regression line?

The intercept is the average national income w/out any money in circulation. This may be illogical because it lays outside the sample size. The slope means that, on average, if the money supply increases by 1, then national income increases by 1.168.

(c) Test the significance of X at 5% significance level by assuming classical assumptions (stated in class) are satisfied.

$H_0: \beta_2 = 0 \quad H_1: \beta_2 \neq 0$

Under H_0

$$t = \frac{\hat{\beta}_1 - \beta_1}{se(\hat{\beta}_1)} = \frac{\hat{\beta}}{se(\hat{\beta}_1)}$$

$$t^{act} = \frac{1.7133 - 0}{0.1260} \approx 13.62$$

$$t_{9,\frac{\alpha}{2}} \approx t_{8,0.025} \approx 2.306$$

$|t_{act}| > t_{8,0.025}$

* We reject the null hypothesis @ 5% significance level, the quantity of money is significant in national income.

(d) If you had sole control over the money supply and wished to achieve a level of national income of 12.0 in 1999, at what level would you set the money supply? Explain.

$$\boxed{\hat{Y}_i = 1.168144 + X_i \cdot 1.7153}$$

Using this

$\Leftrightarrow 12 = 1.168144 + X \cdot 1.7155$

$\therefore \underline{X \approx 6.31411}$ million on average.

However, 12 million is outside data set, $\therefore$ may not hold.

(e) Compute the coefficient of determination R^2. Explain the meaning of the estimated R^2.

$$R^2 = 1 - \frac{SSR}{TSS} = 1 - \frac{\sum \varepsilon_i^2}{\sum (Y_i - \bar{Y})^2} = 1 - \frac{(n-2)s^2}{\sum X_i^2 - n\bar{Y}^2}$$

$$\Leftrightarrow 1 - \frac{8 \cdot 0.139665}{597.03 - 570.025} = 0.95863$$

$R^2 = 0.95863$, which means 95.863% of change in national income is due to change in money supply

(f) Construct 95% confidence interval for the parameter of X. Interpret this confidence interval.

Confidence interval for $\hat{\beta}_2$

$$\left[\hat{\beta}_2 - t_{8,\frac{\alpha}{2}} \cdot se(\hat{\beta}_2),\ \hat{\beta}_2 + t_{8,\frac{\alpha}{2}} \cdot se(\hat{\beta}_2)\right]$$

$t_{8,0.025} = 2.306$

$t \cdot se(\hat{\beta}_2) = 2.306 \cdot 0.1260 = 0.290556$

1.7155 ± 0.290556

$[1.424944,\ 2.006056]$

This interval represents the β_2 parameter w/ a 95% confidence.