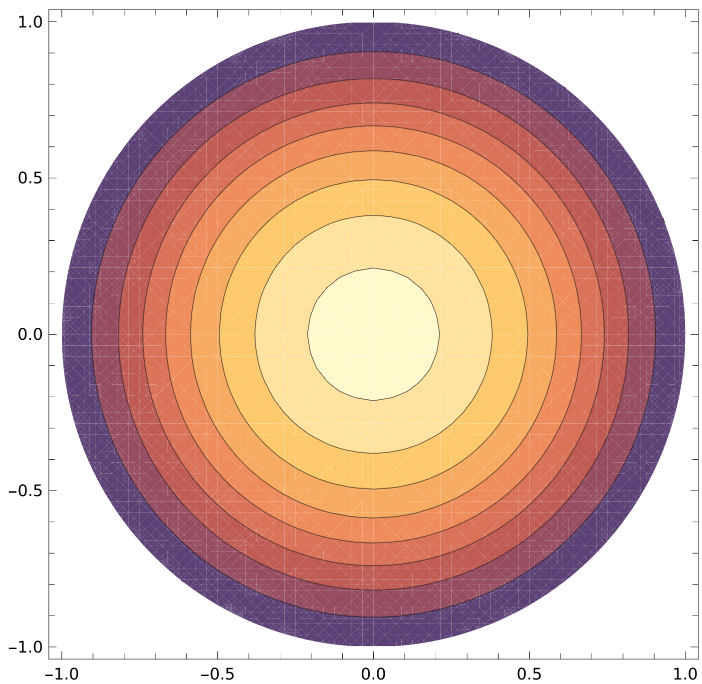
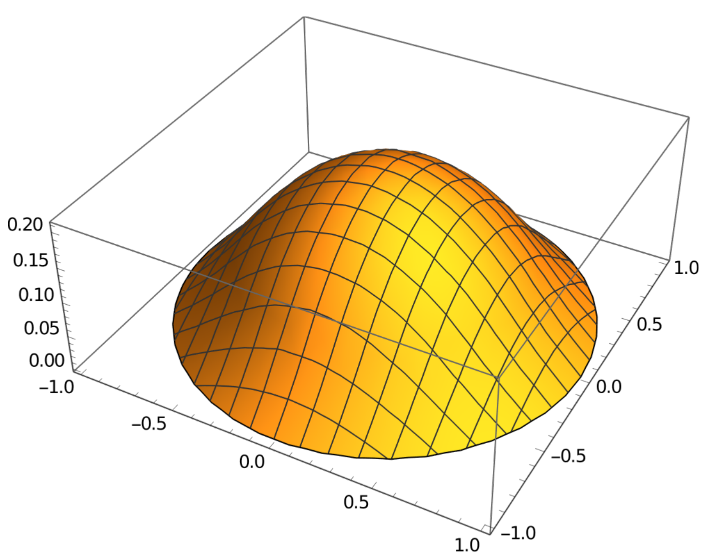
In[∘]:=
```
deqn = Laplacian[V[x, y], {x, y}] == Piecewise[{{-1, x^2 + y^2 < 1/2}}, 0];
region = Disk[];
boundary = DirichletCondition[V[x, y] == 0, x^2 + y^2 == 1];
solution = NDSolveValue[{deqn, boundary}, V, {x, y} ∈ region];
ContourPlot[solution[x, y], {x, y} ∈ region, PlotRange → All]
Plot3D[solution[x, y], {x, y} ∈ region, PlotRange → All]
```

Out[∘]=



Out[∘]=



In[•]:=
```
matrix = {{1, 0, 4, 1}, {7, 4, 1, 0}, {0, 0, 2, 5}, {9, 1, 0, 3}};
matrix // MatrixForm
{val, vec} = Eigensystem[matrix]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 4 & 1 \\ 7 & 4 & 1 & 0 \\ 0 & 0 & 2 & 5 \\ 9 & 1 & 0 & 3 \end{pmatrix}$$

Out[•]=
```
{{8.66..., -0.910... + 4.39... i, -0.910... - 4.39... i, 3.16...},
 {{0.523..., 0.948..., 0.751..., 1},
  {-0.515... + 0.473... i, 0.722... + 0.132... i, -0.524... - 0.791... i, 1},
  {-0.515... - 0.473... i, 0.722... - 0.132... i, -0.524... + 0.791... i, 1},
  {8.41..., -75.5..., 4.30..., 1}}}
```

In[•]:=
```
N[(matrix . vec[[1]])] == N[(val[[1]] * vec[[1]])]
```

Out[•]= True

In[•]:=
```
bvec = {4, 1, 0, 2};
LinearSolve[matrix, bvec]
```

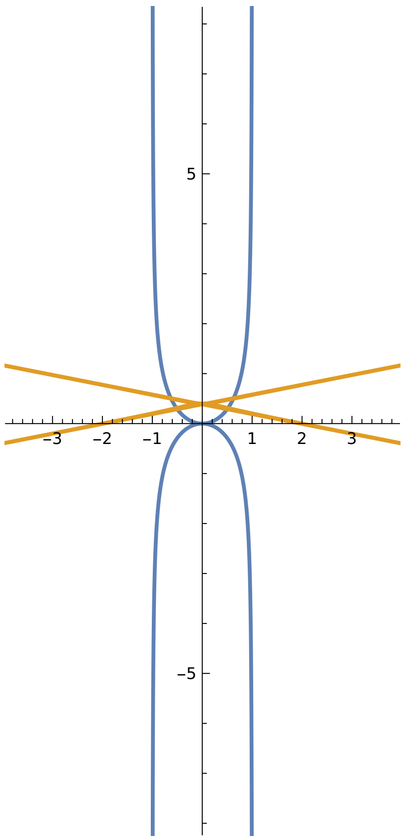
Out[•]=

$$\left\{\frac{242}{551}, -\frac{422}{551}, \frac{545}{551}, -\frac{218}{551}\right\}$$

In[∘]:=
```
r[θ_] = Abs[Tan[θ]];
r1[θ_] = 2 / (Cos[θ]^2 + 5 Sin[θ]);

PolarPlot[{r[θ], r1[θ]}, {θ, 0, 2 Pi}]
```

Out[∘]=



In[∘]:=
```
solution1 = FindRoot[r[θ] == r1[θ], {θ, 0.5}];
solution2  = FindRoot[r[θ] == r1[θ], {θ, 3}];
sol1 = θ /. solution1
sol2 = θ /. solution2
rval1 = r[θ] /. solution1;
rval2 = r[θ] /. solution2;
polarIntersects = {{sol1, rval1}, {sol2, rval2}}
```

Out[∘]= 0.542759

Out[∘]= 2.59883

Out[∘]= {{0.542759, 0.603186}, {2.59883, 0.603186}}

In[∘]:=
```
cartesianIntersects = CoordinateTransform["Polar" → "Cartesian", polarIntersects]
```

Out[∘]= {{0.446979, 0.30789}, {2.14022, 1.47424}}

In[6]:=
```
f[r_, θ_, ϕ_] = Sin[θ] E^(-r^2)/(1+r);
Integrate[f[r, θ, ϕ] r^2 Sin[θ], {r, 0, Infinity}, {θ, 0, Pi}, {ϕ, 0, 2 Pi}]
```

Out[7]=
$$\frac{\pi^2 \left(e - e\sqrt{\pi} + \pi\,\mathrm{Erfi}[1] - \mathrm{ExpIntegralEi}[1]\right)}{2\,e}$$

In[8]:=
```
N[%]
```

Out[8]= 2.16052

In[17]:=
```
field = Grad[f[r, θ, ϕ], {r, θ, φ}, "Spherical"]
```

Out[17]=
$$\left\{-\frac{e^{-r^2}\,\mathrm{Sin}[\theta]}{(1+r)^2} - \frac{2\,e^{-r^2}\,r\,\mathrm{Sin}[\theta]}{1+r},\ \frac{e^{-r^2}\,\mathrm{Cos}[\theta]}{r\,(1+r)},\ 0\right\}$$

Out[1]=
$$\left\{-\frac{e^{-r^2}\,\mathrm{Sin}[\theta]}{(1+r)^2} - \frac{2\,e^{-r^2}\,r\,\mathrm{Sin}[\theta]}{1+r},\ \frac{e^{-r^2}\,\mathrm{Cos}[\theta]}{1+r},\ 0\right\}$$

In[41]:=
```
cfunc = TransformedField["Spherical" → "Cartesian", f[r, θ, ϕ], {r, θ, φ} → {x, y, z}]
cfield  = Grad[cfunc, {x, y, z}]
VectorPlot3D[{cfield}, {x, 0, 1}, {y, 0, 1}, {z, 0, 1}]
```

Out[41]=

$$\frac{e^{-x^2-y^2-z^2}\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}\left(1+\sqrt{x^2+y^2+z^2}\right)}$$

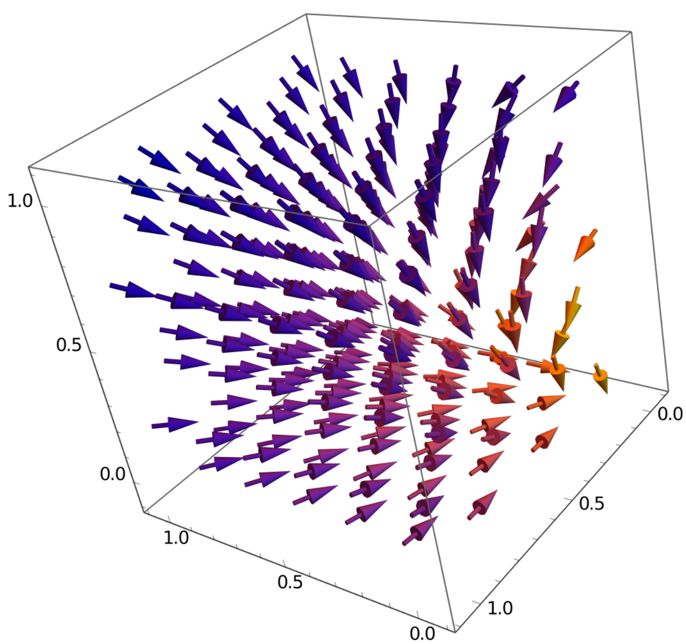
Out[42]=

$$\left\{-\frac{e^{-x^2-y^2-z^2}\,x\sqrt{x^2+y^2}}{(x^2+y^2+z^2)\left(1+\sqrt{x^2+y^2+z^2}\right)^2}-\frac{e^{-x^2-y^2-z^2}\,x\sqrt{x^2+y^2}}{(x^2+y^2+z^2)^{3/2}\left(1+\sqrt{x^2+y^2+z^2}\right)}+\frac{e^{-x^2-y^2-z^2}\,x}{\sqrt{x^2+y^2}\sqrt{x^2+y^2+z^2}\left(1+\sqrt{x^2+y^2+z^2}\right)}-\frac{2\,e^{-x^2-y^2-z^2}\,x\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}\left(1+\sqrt{x^2+y^2+z^2}\right)},\right.$$

$$-\frac{e^{-x^2-y^2-z^2}\,y\sqrt{x^2+y^2}}{(x^2+y^2+z^2)\left(1+\sqrt{x^2+y^2+z^2}\right)^2}-\frac{e^{-x^2-y^2-z^2}\,y\sqrt{x^2+y^2}}{(x^2+y^2+z^2)^{3/2}\left(1+\sqrt{x^2+y^2+z^2}\right)}+\frac{e^{-x^2-y^2-z^2}\,y}{\sqrt{x^2+y^2}\sqrt{x^2+y^2+z^2}\left(1+\sqrt{x^2+y^2+z^2}\right)}-\frac{2\,e^{-x^2-y^2-z^2}\,y\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}\left(1+\sqrt{x^2+y^2+z^2}\right)},$$

$$\left.-\frac{e^{-x^2-y^2-z^2}\sqrt{x^2+y^2}\,z}{(x^2+y^2+z^2)\left(1+\sqrt{x^2+y^2+z^2}\right)^2}-\frac{e^{-x^2-y^2-z^2}\sqrt{x^2+y^2}\,z}{(x^2+y^2+z^2)^{3/2}\left(1+\sqrt{x^2+y^2+z^2}\right)}-\frac{2\,e^{-x^2-y^2-z^2}\sqrt{x^2+y^2}\,z}{\sqrt{x^2+y^2+z^2}\left(1+\sqrt{x^2+y^2+z^2}\right)}\right\}$$

Out[43]=



In[196]:=

```
v[x_] = Piecewise[{{0, -4 < x < -1}, {10^100, x > 4 && x < -4}, {2, -1 ≤ x < 1}, {0, 1 ≤ x < 4}}]
boundary = DirichletCondition[ψ[x] == 0, x == 4 && x == -4];
{eval, evec} = NDEigensystem[{-(1/4) Laplacian[ψ[x], {x}] + v[x] * ψ[x]} , ψ[x], {x, -4, 4}, 6]
evec = evec + eval;
Show[Plot[{evec, eval}, {x, -10, 10}, PlotRange → Full],
 Plot[v[x], {x, -10, 10}, Filling → Bottom]]
```

Out[196]=

```
 0                                                                                  -4 < x < -1
 10 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000   x > 4 && x < -4
 2                                                                                  -1 ≤ x < 1
 0                                                                                  True
```

Out[198]=

```
{{0.0518675, 0.0576191, 0.463369, 0.514127, 1.25977, 1.39244},
 {InterpolatingFunction[ Domain: {{-4., 4.}} Output: scalar ][x],
  InterpolatingFunction[ Domain: {{-4., 4.}} Output: scalar ][x],
  InterpolatingFunction[ Domain: {{-4., 4.}} Output: scalar ][x],
  InterpolatingFunction[ Domain: {{-4., 4.}} Output: scalar ][x],
  InterpolatingFunction[ Domain: {{-4., 4.}} Output: scalar ][x],
  InterpolatingFunction[ Domain: {{-4., 4.}} Output: scalar ][x]}}
```

Out[200]=