Quantum HW 32–35 Paul Len

32.a Prove j_x for $\psi(x) = Ce^{-K_i x} + De^{K_i x} = 0$

$$j_x = \frac{\hbar}{2mi}\left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x}\right)$$

1

$$\frac{\hbar}{2mi}\cancel{\left[(Ce^{-\kappa x}+De^{\kappa x})(-\kappa Ce^{-\kappa x}+\kappa De^{\kappa x}) - (Ce^{-\kappa x}+De^{\kappa x})(-\kappa Ce^{-\kappa x}+\kappa De^{\kappa x})\right]}$$

$$j_x = \frac{\hbar}{2mi}(0) = 0$$

b.

$$\frac{\partial \psi^* \psi}{\partial t} = -\frac{\partial j_x}{\partial x}$$

$$\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$$

1

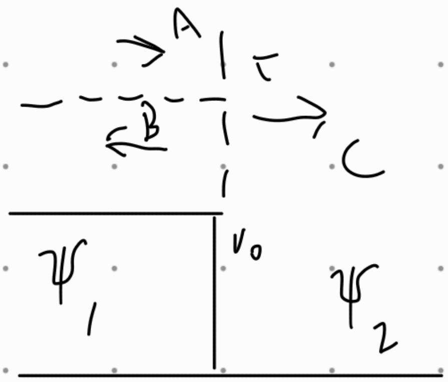
$$\Psi^* \Psi = \psi^* \psi \cancel{e^{-iEt/\hbar}} \cancel{e^{iEt/\hbar}}$$

Therefore $\frac{\partial \Psi^* \Psi}{dt} = 0$, $\rightarrow \frac{\partial j_x}{\partial x} = 0$

& j_x is constant.

Since we proved that $j_x = 0$ for $x > 0$ it logically follows that j_x is also 0 for $x < 0$.

Question 33.



for $-x$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{2m(E-V_0)}{\hbar^2}\psi$$

for $+x$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{2mE}{\hbar^2}\psi$$

$$\psi_1 = \underbrace{Ae^{ikx}}_{\text{Incident}} + \underbrace{Be^{-ikx}}_{\text{Reflected}} \quad \text{for } x<0$$

$$k = \frac{\sqrt{2m(E-V_0)}}{\hbar} = \sqrt{k_1'^2 - \frac{2mV_0}{\hbar}}$$

$$\psi_2 = \underbrace{Ce^{ik_1'x}}_{\text{transmitted}} + \cancel{De^{-ik_1'x}} \quad \text{for } x>0$$

no wave from left

$$k_1' = \frac{\sqrt{2mE}}{\hbar}$$

$$\psi_1(0) = \psi_2(0) \rightarrow A + B = C$$

Solve this system of eqs

&

$$\frac{\partial\psi_1(0)}{\partial x} = \frac{\partial\psi_2(0)}{\partial x} \rightarrow \frac{\cancel{ik}(A-B)}{\cancel{ik}} = \frac{\cancel{i}k_1' C}{\cancel{ik}}$$

$$A - B = \frac{k_1'}{k}\cancel{C}(A+B)$$

$$A - B = \frac{k_1'}{k}A + \frac{k_1'}{k}B$$

$$A - \frac{k_1'}{k}A = B + \frac{k_1'}{k}B$$

$$A\left(1 - \frac{k_1'}{k}\right) = B\left(1 + \frac{k_1'}{k}\right)$$

$$\frac{B}{A} = \left(\frac{k - k_1'}{k + k_1'}\right) = \sqrt{R}$$

$$R = \left|\frac{\sqrt{\cancel{\frac{2m}{\hbar^2}}(E-V_0)} - \sqrt{\cancel{\frac{2m}{\hbar^2}}E}}{\sqrt{\cancel{\frac{2m}{\hbar^2}}(E-V_0)} + \sqrt{\cancel{\frac{2m}{\hbar^2}}E}}\right|$$

$$R = \left| \frac{\sqrt{E-V_0} - \sqrt{E}}{\sqrt{E-V_0} + \sqrt{E}} \right|^2$$

$$T = 1 - R = 1 - \left| \frac{\sqrt{E-V_0} - \sqrt{E}}{\sqrt{E-V_0} + \sqrt{E}} \right|^2$$

$$\boxed{T = \frac{4\sqrt{E-V_0}\sqrt{E}}{\left(\sqrt{E-V_0} + \sqrt{E}\right)^2}}$$

4.26

Solve for $\frac{C}{A}$

for F

$A + B = F + G \quad x = 0$

$Fe^{k_1 a} + Ge^{-k_1 a} = Ce^{ika} \quad @\, x = a$

$ik(A - B) = k_1(F - G) \quad @\, x = 0$

$k_1(Fe^{k_1 a} - Ge^{-k_1 a}) = ikCe^{ika} \quad @\, x = a$

$$Fe^{k_1 a} - \cancel{Ge^{-k_1 a}} = \frac{ik}{k_1} Ce^{ika}$$
$$Fe^{k_1 a} + \cancel{Ge^{-k_1 a}} = Ce^{ika} \quad +$$

$$2Fe^{k_1 a} = Ce^{ika} + \frac{ik}{k_1} Ce^{ika}$$

$$2Fe^{k_1 a} = Ce^{ika}\left(1 + \frac{ik}{k_1}\right)$$

$$F = \frac{Ce^{ika}\left(1 + \frac{ik}{k_1}\right)}{2e^{k_1 a}} = \frac{C}{2} e^{(ik - k_1)a}\left(1 + \frac{ik}{k_1}\right)$$

for G

$$Fe^{K_1 a} + Ge^{-K_1 a} = Ce^{ika}$$

$$-Fe^{K_1 a} - Ge^{-K_1 a} = \frac{ik}{K_1} Ce^{ika}$$

$$2Ge^{-K_1 a} = Ce^{ika} - \frac{ik}{K_1} Ce^{ika}$$

$$2Ge^{-K_1 a} = Ce^{ika}\left(1 - \frac{ik}{K_1}\right)$$

$$G = \frac{Ce^{ika}\left(1 - \frac{ik}{K}\right)}{2e^{-K_1 a}}$$

$$G = \frac{C}{2}e^{a(ik+K)}\left(1 - \frac{ik}{K}\right)$$

Adding 3 & 4

$$1 + \frac{B}{A} = \frac{C}{A}\left(\cosh(K_1 a) - \frac{ik}{K_1}\sinh(K_1 a)\right)$$

$$1 - \cancel{B}/A = \frac{C}{A}e^{ika}\left[\cosh(K_1 a) + \frac{ik}{K_1}\sinh(K_1 a)\right]$$

Putting F & G into eq 1

$$A + B = \frac{C}{2}e^{a(ik+K_1)}\left(1 + \frac{ik}{K_1}\right) + \frac{C}{2}e^{a(ik-K)}\left(1 - \frac{ik}{K}\right)$$

$$= Ce^{ika}\left(\left[\frac{e^{-K_1 a} + e^{K_1 a}}{2}\right] - \frac{ik}{K_1}\left[\frac{e^{K_1 a} - e^{-K_1 a}}{2}\right]\right)$$

$$= Ce^{ika}\left(\cosh K_1 a - \frac{ik}{K_1}\sinh K_1 a\right)$$

$$A+B/A = \frac{C}{A}e^{ika}\left(\cosh(K_1 a) - \frac{ik}{K_1}\sinh(K_1 a)\right)$$

$$1 + \frac{B}{A} = \frac{C}{A}\left(\cosh(K_1 a) - \frac{ik}{K_1}\sinh(K_1 a)\right)$$

$$ik(A - B) = K_1\left(\frac{C}{2}e^{(ik-K_1)a}\left(1 + \frac{ik}{K_1}\right) + \frac{C}{2}e^{a(ik+K)}\left(1 - \frac{ik}{K}\right)\right)$$

$$A - B = Ce^{ika}\left(\frac{K_1}{ik}\left[\frac{e^{K_1 a} - e^{-K_1 a}}{2}\right] + \left[\frac{e^{K_1 a} + e^{-K_1 a}}{2}\right]\right)$$

$$Ce^{ika}\left[\cosh(Ka) + \frac{ik}{K_1}\sinh(K_1 a)\right]$$

$$1 - B/A = \frac{C}{A}e^{ika}\left[\cosh(K_1 a) + \frac{ik}{K_1}\sinh(K_1 a)\right]$$

$$2 = \frac{C}{A}e^{ika}\left[2\cosh(K_1 A) + i\left(\frac{K_1}{k} - \frac{k}{K_1}\right)\sinh(K_1 a)\right]$$

$$\frac{C}{A} = \frac{2e^{-ika}}{\left[2\cosh(K_1 A) + i\left(\frac{K_1}{k} - \frac{k}{K_1}\right)\sinh(K_1 a)\right]}$$

$$A = \frac{Ce^{ika}}{2ika}\left[\left(K^2 - K_1^2\right)\frac{e^{K_1 a} - e^{-K_1 a}}{2} + 2i\frac{e^{K_1 a} + e^{-K_1 a}}{2}\right]$$

$$\frac{A}{C} = \frac{e^{ika}}{2ikx}\left[\left(K^2 - K_1^2\right)\sinh(K_1 a) + 2i\cosh(K_1 a)\right]$$

✓

$$T = \frac{|C|^2}{|A|^2} = \left[\left(\frac{C}{A}\right)\left(\frac{C}{A}\right)^*\right] = \frac{4}{\left[2\cosh(K_1 A) + i\left(\frac{K_1}{k} - \frac{k}{K_1}\right)\sinh(K_1 a)\right]} \cdot \frac{1}{\left[2\cosh(K_1 A) - i\left(\frac{K_1}{k} - \frac{k}{K_1}\right)\sinh(K_1 a)\right]}$$

$$\cosh^2(K_1 a) = 1 + \sinh^2(K_1 a)$$

$$T = \frac{4}{4\cosh^2(K_1 a) + \left\{\frac{K_1^2 - k^2}{K_1 k}\right\}^2 \sinh^2(K_1 a)}$$

$$T = \frac{4}{4 + \frac{4(K_1 k)^2 + (K_1^2 - k^2)^2}{(K_1 k)^2}\sinh^2(K_1 a)} \rightarrow \frac{4}{4 + \frac{(K_1^2 + k^2)^2}{(K_1 k)^2}\sinh^2(K_1 a)}$$

$$T = \frac{1}{1 + \frac{(K^2 + k^2)^2}{4(Kk)^2}\sinh^2(K_1 a)}$$

✓

35.

a. Qualitatively, the particle w/ higher mass is probably less likely to penetrate, due to the fact it has the same energy.

b. $K_1 = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$

$$T = 16\frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)e^{-2K_1 L}$$

$$T = 16\,\frac{3}{10}\left(1 - \frac{3}{10}\right)e^{-2\frac{\sqrt{2m(10-3)}}{\hbar}10^{-14}}$$

$$T = 3.36\,e^{-\frac{2}{\hbar}10^{-14}\sqrt{2m(1.12E-12)}}$$

$T_{proton} = 0.0000305369$

$T_{Deutron} = 0.000000244207$