# CHAPTER 11

## THE NUCLEUS

In disentangling the problems of the atom, one of the major steps has been the recognition that it is useful to speak of individual orbits of the electrons in the atoms. This, to be sure is only an approximation, in fact a crude approximation, but still it provides a quite invaluable starting point for the study of complex atoms containing large numbers of electrons.

When physicists became reasonably certain that the nucleus was constructed of protons and neutrons, questions were raised concerning the orbital behavior of these particles. Could nuclear structure be interpreted on the general pattern of atomic structure by attributing to the various neutrons and to the various protons within the nucleus something like individual orbits and individual states?...

Consider one nucleon in the nucleus traveling along its orbit among the other nucleons. If the collision mean free path is $\lambda$ this nucleon would collide with the other neutrons and protons in the nucleus and its orbit would be lost after it had gone the distance of its free path... . Now it is a very difficult problem to decide the length of the mean free path, but if one takes somewhat literally the strength of the interactions between the neutron and other components of the nucleus, one is led to a value that seems discouragingly short... .

In spite of this argument, evidence has been accumulating for the last few years... to the effect that orbits do exist. The best known feature of this evidence has been the discovery of the so-called magic numbers. They are the numbers 2, 8, 20, 50, 82, 126. When a nucleus contains a number of either neutrons or protons equal to one of the magic numbers, it is particularly stable... .

It would appear that for some reason the mean free path must be longer than is given by a somewhat crude estimate of its length. One possible reason for this may be the Pauli principle, according to which collisions between two particles may be forbidden when, after the collision, one of the particles would go to an occupied state.

**Enrico Fermi**

We might well refer to the period from 1910 to 1930 as the "golden age" of atomic physics (see Table 11-1). During this period, the basics of atomic physics were understood for the first time with the development of quantum mechanics. Atoms were described by the energies and angular momenta of electrons. The fundamental structure of matter was established at an energy scale from a few eV to 50 keV (x ray energies).

The period from about 1930 to 1950 may well be referred to as the "golden age" of nuclear physics. During this period, the study of the nucleus was the frontier of fundamental science. The first modern accelerator technology was developed and the energy frontier was pushed to 100 MeV. The basic constituents of the nucleus were established to be neutrons and protons. Since the size of the nucleus was known to be a few femtometers, physicists were led to conclude that a new "strong" force was responsible for the interaction between protons and neutrons. The strength of this force relative to the strength of the electromagnetic force is responsible for the small size of the nucleus compared to the atomic size. Together with the discovery of the nuclear constituents came new fundamental questions. One major difficulty was that the force that gives the nucleus its small size also causes a large interaction probability between protons and neutrons, yet protons and neutrons were observed to have stable configurations ("nuclear orbits") inside nuclei. Another major challenge was to understand why certain nuclei would spontaneously decay, whereas others were stable.

## 11-1 DISCOVERY OF THE NEUTRON

By 1932, physicists had measured both the atomic mass number $A$ and the nuclear charge $Ze$ of the elements. Since the nucleus was found to contain nearly all the mass of the atom and to have the electric charge of $Z$ protons, it was natural to expect that the nuclear mass was due to $Z$ protons ($A = Z$). For all nuclei (except a single proton), however, it was found that $A > Z$. The discovery of the nucleus had left physicists with an interesting problem: *Why are A and Z different?* Since protons were known to be very massive compared to electrons, one plausible scenario was that the nucleus is a bound state of $A$ protons and $A - Z$ electrons, which would give the nucleus the observed mass and charge. This interesting possibility was qualitatively supported by the observation that electrons were emitted in certain types of nuclear decays, called *beta* decays. Recall that beta decays were discovered a decade before the nucleus was discovered!

*If the nucleus decays into an electron plus other particles, is it not natural to assume that the electron is present inside the nucleus?* An important clue comes from the energies of the emitted electrons, which are typically an MeV. The kinetic energy of the beta decay electrons imposed a very serious problem for a nucleus made of

**TABLE 11-1**
**FRONTIERS IN PHYSICS.**

| Period | Energy | Matter | Composition |
|---|---|---|---|
| 1910–30 | keV | Atoms | Electrons + nucleus |
| 1930–50 | MeV | Nucleus | Protons + neutrons |
| 1950–80 | GeV | Hadrons | Quarks |
| 1980–present | TeV | Quarks + leptons | ??? |

electrons and protons because an electron confined in such a small space as the nuclear size has a small wavelength and large kinetic energy. If we approximate the nucleus as a box with a size of a few fm, then the maximum wavelength of an electron confined inside the nucleus is about 5 fm. The momentum of such an electron is

$$p = \frac{h}{\lambda}, \tag{11.1}$$

or

$$pc = \frac{hc}{\lambda} = \frac{1240\,\text{MeV}\cdot\text{fm}}{5\,\text{fm}} \approx 250\,\text{MeV}. \tag{11.2}$$

Such an electron is relativistic; the electron kinetic energy is

$$E_k \approx pc \approx 250\ \text{MeV}. \tag{11.3}$$

This energy is far greater than the observed electron energies from nuclear decays. Another problem with electrons bound inside the nucleus is that the observed force between an electron and a nucleus is not strong enough to bind the electron. In other words, protons and neutrons experience the strong force but electrons do not. The electromagnetic potential energy between an electron and a gold nucleus at a distance of 5 fm is

$$\begin{aligned} V &= \frac{Zke^2}{r} = \frac{(79)(1.44\ \text{MeV}\cdot\text{fm})}{(5\ \text{fm})} \\ &\approx 23\ \text{MeV}. \end{aligned} \tag{11.4}$$

This energy is not sufficient to bind the electron in a volume of nuclear size because, as we have seen, the kinetic energy of an electron confined to such a small space is an order of magnitude greater:

$$250\ \text{MeV} \gg 23\ \text{MeV}. \tag{11.5}$$

Of course, the electromagnetic force does bind electrons and nuclei into atoms, but the strength of the force results in an atomic size of about 0.1 nm, or about $10^5$ times the nuclear size.

The solution to the nuclear puzzle of why $A$ and $Z$ are not identical was provided by James Chadwick in 1932 with the discovery of the neutron. (The year 1932 was a "banner" year for physics, as Carl Anderson also discovered the positron and John Cockroft and Ernest Walton made the first artificial disintegration of the nucleus.)

A new type of radiation particle ($Y$) was discovered by bombarding a beryllium foil with alpha particles. The reaction was

$$\alpha + {}^{9}\text{Be} \rightarrow (\text{new nucleus}) + Y. \tag{11.6}$$

The superscript specifies the atomic mass number of beryllium ($A = 9$). The atomic number of beryllium is $Z = 4$. The alpha particle (the helium nucleus) has $A = 4$ and $Z = 2$. This reaction had been studied by several physicists, and it was known that the $Y$ particle was electrically neutral and that it could penetrate a few centimeters of lead. Irène Curie and Frédéric Joliot scattered $Y$ particles from a paraffin target. Paraffin contains hydrogen atoms that are bound by only a few electronvolts; paraffin is a source of protons that are essentially free. Protons were observed to be freed from the paraffin in the reaction,

$$Y + p \rightarrow Y + p. \tag{11.7}$$

The kinetic energy of the protons was measured to be 5.7 MeV. If the $Y$ particle was a photon, then a large photon energy (about 50 MeV) is needed to produce protons with an energy of 5.7 MeV.

## EXAMPLE 11-1

If the $Y$ particle is the photon, calculate the photon kinetic energy required to produce 5.7-MeV protons.

## SOLUTION:

The process is Compton scattering—not the usual scattering of an energetic photon from an *electron*, but rather the scattering of an energetic photon from a *proton*. The photon interacts with all charged particles! We can use the Compton formula (4.170) with $m$ replaced by the proton mass to get an exact answer. We may simplify the analysis, however, by observing that the scattered proton is nonrelativistic since its kinetic energy is much smaller than its mass energy (5.7 MeV << 940 MeV). Therefore, a significant momentum is transferred to the proton but not too much kinetic energy, similar to a photon scattering from a "brick wall." The maximum momentum is transferred from the photon to the proton when the scattering is backwards. If the initial photon momentum is $p_x$, the change in momentum of the photon is

$$\Delta p \approx 2 p_x.$$

The kinetic energy of the recoiling proton is

$$E_k = \frac{\Delta p^2}{2M} = \frac{2 p_x^{\,2}}{M},$$

where $M$ is the proton mass. Solving for $p_x$, we have

$$p_x = \sqrt{\frac{ME_k}{2}}.$$

The initial photon energy ($E$) is

$$E = p_x c = \sqrt{\frac{Mc^2 E_k}{2}}$$
$$= \sqrt{\frac{(940 \text{ MeV})(5.7 \text{ MeV})}{2}}$$
$$\approx 50 \text{ MeV}.$$ ■

The exact energy of the $Y$ particles was not known; however, Chadwick did not believe that the $Y$ particles could be produced with a kinetic energy as large as 50 MeV, because the typical energy scale of nuclear decays had been established to be a few MeV.

Part of the great contribution of Chadwick was to show that the particle $Y$ was *not* the photon. To establish the identity of the mysterious $Y$ particle, Chadwick scattered the $Y$ particles from several different targets: hydrogen, helium, nitrogen, oxygen, and argon. The experimental arrangement is indicated in Figure 11-1. Chadwick measured the kinetic energy of the recoiling nucleus and then compared his result to that expected from Compton scattering (see Example 11-1) for each target. The results were found to be clearly inconsistent with Compton scattering. Furthermore, Chadwick observed that the scattering *rate* for $Y$ particles was greater than that of Compton scattering by several orders of magnitude.

The data of Chadwick showed that the $Y$ particle is more efficient than a massless photon at transferring *energy* to the target particle. The scattering data could be explained if the $Y$ particle had a mass that is approximately equal to the proton mass and an interaction with the proton that was significantly stronger than the photon–proton interaction. For example, if the $Y$ particle has the same mass as the proton, then a $Y$ particle kinetic energy of 5.7 MeV is sufficient to scatter protons with a kinetic energy of 5.7 MeV. Chadwick had demonstrated that the $Y$ particle had a mass approximately equal to the proton mass. The $Y$ particle is called the *neutron* ($n$), from the Italian *neutrone* meaning "the large neutral one."

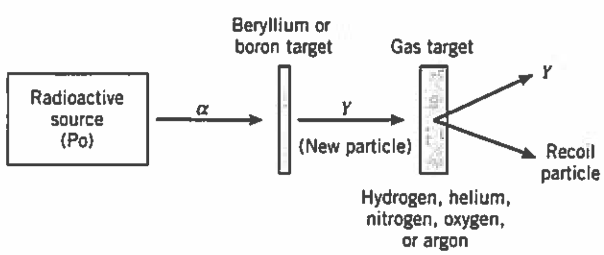



**FIGURE 11-1 The experiment of Chadwick.**
A new particle ($Y$) is produced when alpha particles from the radioactive decay of polonium hit a beryllium or boron target. The $Y$ particle is observed to transfer its energy to various gas particles, indicating it is massive. The mass of the $Y$ particle is determined by conservation of energy in the boron target from the reaction $\alpha + {}^{11}\text{B} \to {}^{14}\text{N} + Y$. The $Y$ particle is called the neutron.

By the preceding analysis, Chadwick knew that the neutron mass ($m_n$) was approximately equal to the proton mass ($m_p$). Chadwick made an accurate determination of the neutron mass by analysis of the reaction,

$$\alpha + {}^{11}\text{B} \to {}^{14}\text{N} + n, \tag{11.8}$$

and applying conservation of energy and momentum. Chadwick chose a boron target because the masses of boron ($m_B$) and nitrogen ($m_N$) were well known at the time of the experiment. By conservation of energy,

$$\frac{1}{2}m_\alpha v_\alpha^2 + m_\alpha c^2 + m_B c^2 = \frac{1}{2}m_N v_N^2 + m_N c^2 + \frac{1}{2}m_n v_n^2 + m_n c^2, \tag{11.9}$$

where $v_\alpha$ is the speed of the incoming alpha and $v_N$ and $v_n$ are the speeds of the outgoing nitrogen and neutron. (All the particles are nonrelativistic.) Conservation of momentum gives

$$m_\alpha \mathbf{v}_\alpha = m_n \mathbf{v}_n + m_N \mathbf{v}_N. \tag{11.10}$$

Since the neutron mass is much smaller than the nitrogen mass, $m_n \ll m_N$, the neutron gets nearly all the kinetic energy of the final state by conservation of momentum. The nitrogen nucleus gets just enough kinetic energy to conserve momentum. Therefore, energy conservation (11.9) gives

$$\frac{1}{2}m_n v_n^2 + m_n c^2 \approx \frac{1}{2}m_\alpha v_\alpha^2 + m_\alpha c^2 + m_B c^2 - m_N c^2. \tag{11.11}$$

Solving for the mass energy of the neutron,

$$m_n c^2 \approx \frac{\frac{1}{2}m_\alpha v_\alpha^2 + m_\alpha c^2 + m_B c^2 - m_N c^2}{1 + \frac{v_n^2}{2c^2}}. \tag{11.12}$$

All quantities on the right-hand side were known by Chadwick except the neutron speed. Chadwick measured the neutron speed by allowing the neutron to collide with a proton and then measuring the resulting proton speed, applying momentum conservation with $m_n \approx m_p$. Chadwick's result for the neutron mass energy was 938 MeV with an experimental error of 1.8 MeV. The experimental error includes Chadwick's errors due to his knowledge of the alpha, boron, and nitrogen masses.

More refined measurements show that the mass energy of the neutron is

$$m_n c^2 = 939.57 \text{ MeV}. \qquad (11.13)$$

The mass of the neutron is slightly greater than the mass of the proton. The difference in the neutron and proton mass energies is

$$m_n c^2 - m_p c^2 = 1.29 \text{ MeV}. \qquad (11.14)$$

## 11-2 BASIC PROPERTIES OF THE NUCLEUS

The nucleus is a bound state of protons and neutrons. Protons and neutrons are collectively called *nucleons*. Before embarking on the description of the nucleus, we make a few remarks about the nature of the strong force between nucleons.

### The Force Between Nucleons

The neutron and the proton have nearly identical masses. In other fundamental respects, the neutron is similar to the proton except that the neutron has zero electric charge. For example, the proton and neutron have identical intrinsic angular momentum quantum numbers:

$$s = \frac{1}{2}. \qquad (11.15)$$

The neutron and proton are observed to have identical strong interactions. The strong force between any two nucleons is observed to be *attractive*. This strong force does not reflect the most fundamental aspect of the strong interaction because the nucleons are composite objects (see Chapter 6). The strong attraction between a neutron and a proton is analogous to the electromagnetic attraction of two neutral hydrogen atoms to form the $H_2$ molecule. In the molecule, the fundamental electromagnetic attraction is between the charges, electrons and protons. The molecule is formed as sort of a residual effect; when the atoms are close together, they feel the composite charges of each other, which results in a net attraction. The approximate range of the residual electromagnetic force between atoms is just the size of the atom. In a nucleus, the fundamental strong interaction is between quarks. The quarks have the property (strong charge which is also called color) that causes the force, but the neutron and proton have no *net* color. The neutron and proton are *color neutrals* just like the hydrogen atom is electrically neutral. When a neutron and a proton are close together, they feel the composite quarks, resulting in a net attraction. The *residual* strong interaction between two nucleons has a short range because the nucleons have no net color. The approximate range of the residual strong force between nucleons is just the size of the nucleon (about 1 fm). Nucleons inside a nucleus feel the residual strong force only from their immediate neighbors. The fundamental aspects of the strong interaction between quarks is discussed in Chapter 18.

### The Deuteron

The deuteron is a bound state of one neutron and one proton. When we combine two particles with $s = 1/2$, there are two possibilities for the resulting intrinsic angular momentum quantum number: zero or one. The strong interaction between a neutron and a proton depends on the orientation of the intrinsic angular momentum vectors, that is, the strong interaction is *spin dependent*. The strong force is spin dependent because it is mediated by gluon exchange and the gluons have intrinsic angular momentum ($s = 1$). If the intrinsic angular momentum vectors of a proton and neutron are not parallel, then the force is not strong enough to hold the neutron and the proton together. The state with antiparallel spins is not stable. A bound state of two neutrons, which by the Pauli exclusion principle would have to have opposite intrinsic angular momentum vectors, does not exist in nature. (The bound state of two protons also does not exist in nature; besides the Pauli exclusion principle, the electromagnetic repulsive force also prevents the binding of two protons.) For a neutron and proton with spins parallel, the force is strong enough to hold the neutron and proton together in a bound state. The bound state has

$$s = 1, \qquad (11.16)$$

and is called the deuteron. The binding energy of the deuteron is 2.22 MeV. The proton ($m_p$), neutron ($m_n$), and deuterium ($m_d$) masses are related by

$$m_d c^2 + 2.22 \text{ MeV} = m_p c^2 + m_n c^2. \quad (11.17)$$

The atom made up of a bound state of a deuteron and an electron is called deuterium. Deuterium exists on earth with a natural abundance of about $1.5 \times 10^{-4}$ times that of hydrogen. The deuterons that are found on earth (inside deuterium atoms) were made in nuclear reactions in the early universe.

## Nuclear Stability

The nuclear states are summarized in Appendix J. Some nuclei are stable against all decays, whereas others decay spontaneously. The decays of nuclei are discussed in more detail in the next section. There are a total of 254 stable nuclei found in nature and many more unstable ones. Figure 11-2 shows a plot of the number of protons versus the number of neutrons for the stable nuclei. The unstable nuclei tend to have values of $A$ and $Z$ very close to that of a stable nucleus. From this plot, we see that for small values of $Z$ ($Z < 20$) the numbers of neutrons and protons tend to be equal. At larger values of $Z$, however, there are more neutrons than protons in the nucleus.

The stability of a nucleus increases with increasing $A$ because there are more nucleons that are all attracting each other. There are two additional factors, however, that *reduce* the stability of the nucleus: (1) the Pauli exclusion principle and (2) the Coulomb repulsion of the protons. We may think of the nucleus as neutrons and protons in a box. The nucleons will occupy quantized energy levels similar to the electrons in an atom. The neutrons and protons are observed to obey the Pauli exclusion principle, that is, no two protons and no two neutrons can have an identical set of quantum numbers. (The Pauli exclusion principle applies to all particles with $s = 1/2$.) For example, the $^{12}C$ nucleus has six protons and six neutrons. This configuration has a lower energy and is therefore more stable than seven protons and five neutrons ($^{12}N$) or five protons and seven neutrons ($^{12}B$). The Pauli exclusion principle favors a nucleus in which the number of neutrons and protons are equal. If this was the only factor, the stable nuclei would lie along the 45 degree line of Figure 11-2. This is the case for small $Z$; however, at large $Z$ there is a significant deviation from equal numbers of neutrons and protons. A nucleus with $A - Z > Z$ is more stable than a nucleus with $A = Z$ because in the later case a greater fraction of the nucleons repell each other by the electromagnetic force. At small values of $Z$ the proton repulsion is unimportant compared to the Pauli exclusion principle but at large $Z$ it becomes important. The heaviest stable nucleus is $^{209}Bi$, which has 83 protons and 126 neutrons.

## Nuclear Binding Energy

The sum of the masses of the individual components of a nucleus, the protons and neutrons, is greater than the mass of the nucleus. This excess energy is called the nuclear binding energy. The amount of binding energy is a function of the atomic mass number. The binding energy per nucleon versus $A$ is shown in Figure 11-3. The binding energy per nucleon rises very rapidly with increasing $A$ and reaches a broad maximum of about 8.5 MeV at about $A = 55$. For larger values of $A$, the binding energy per nucleon drops gradually to about 7 MeV for very high values of $A$. Note that the alpha particle ($A = 4$) has an exceptionally large binding energy (about 7 MeV per nucleon) compared to other light nuclei.

### EXAMPLE 11-2

The mass of the alpha particle ($m_\alpha$) is 3727.41 MeV/c$^2$. Calculate the binding energy and binding energy per nucleon of the alpha particle.

### SOLUTION:

The alpha particle is a bound state of two neutrons and two protons. The binding energy of the alpha particle is

$$E_b = 2m_p c^2 + 2m_n c^2 - m_\alpha c^2,$$

or

$$\begin{aligned} E_b &= (2)(938.27 \text{ MeV}) + (2)(939.57 \text{ MeV}) \\ &\quad - 3727.41 \text{ MeV} \\ &\approx 28.3 \text{ MeV}. \end{aligned}$$

The binding energy per nucleon of the alpha particle is

$$\frac{E_b}{A} \approx \frac{28.3 \text{ MeV}}{4} = 7.1 \text{ MeV}. \quad \blacksquare$$

### EXAMPLE 11-3

The mass of the $^{238}U$ nucleus ($m_U$) is 221697.7 MeV/c$^2$. Calculate the binding energy and binding energy per nucleon in $^{238}U$.

### SOLUTION:

The $^{238}U$ nucleus has 92 protons and 146 neutrons. The total binding energy is

$$E_b = 92m_p c^2 + 146m_n c^2 - m_U c^2,$$

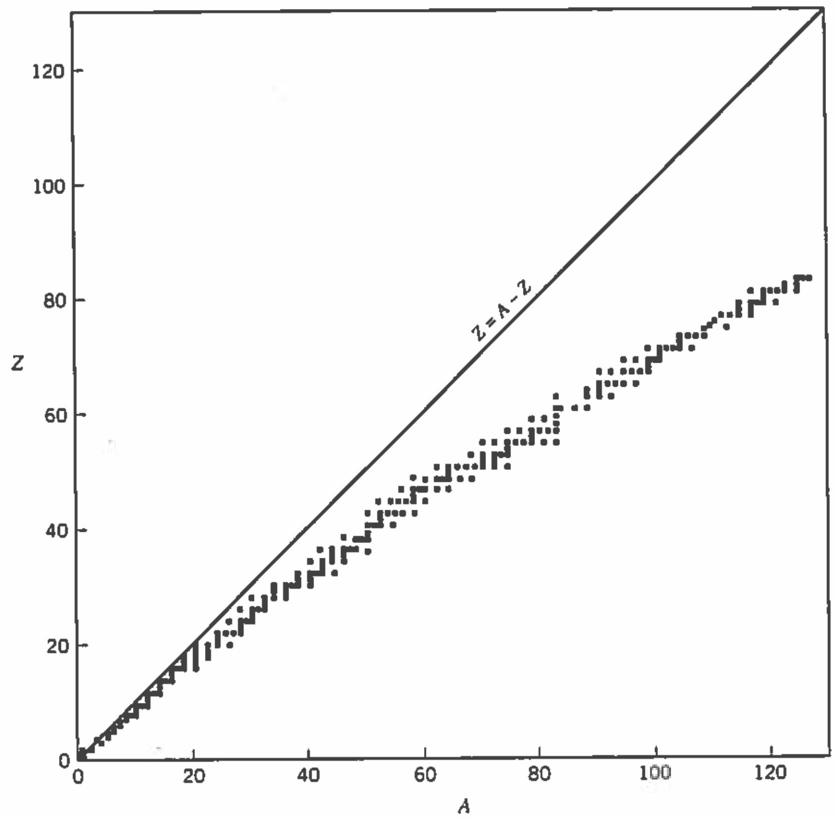



**FIGURE 11-2 Number of protons (*Z*) versus number of neutrons (*A* − *Z*) for stable nuclei.**

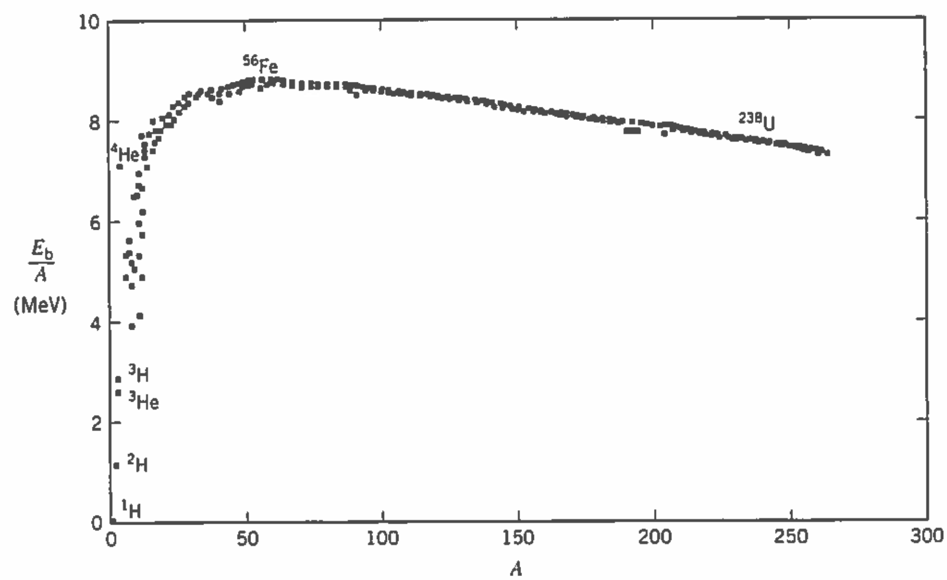



**FIGURE 11-3. Binding energy per nucleon versus atomic mass number (*A*).**

or

$$E_b = (92)(938.3\text{ MeV}) + (146)(939.6\text{ MeV}) - 22{,}1697.7\text{ MeV} \approx 1810\text{ MeV}.$$

The binding energy per nucleon is

$$\frac{E_b}{A} \approx \frac{1810\text{ MeV}}{238} \approx 7.6\text{ MeV}. \qquad \blacksquare$$

## 11-3 NUCLEAR MODELS

### Liquid Drop Model

Scattering experiments show that nuclei have constant densities (see Figure 6-14). This has led to a model of the nucleus as a very dense incompressible spherical *liquid drop*. In the liquid drop model, the nuclear binding energy as a function of atomic mass number $A$ (see Figure 11-3) is semiempirically fit to the form

$$E_b = C_1 A - C_2 A^{2/3}, \qquad (11.18)$$

where $C_1$ and $C_2$ are positive constants. The first term in the binding energy (11.18) is proportional to the number of nucleons ($A$) because each nucleon feels the presence of all the other nucleons. The second term accounts for the fact that the nucleons on the surface of the liquid drop have fewer neighbors. (The surface area of the drop is proportional to $A^{2/3}$.) We find that this expression (11.18) gives a fairly good representation of the observed nuclear binding energies (Figure 11-3) with $C_1$=16 MeV and $C_2$=18 MeV.

The calculation of the binding energy (11.18) may be refined by taking into account the Coulomb energy of proton repulsion and the effect of the Pauli exclusion principle. The reduction of the binding energy due to the Coulomb repulsion ($\Delta E_b^{\text{Coulomb}}$) is proportional to $Z^2$ and inversely proportional to the nuclear radius ($A^{1/3}$),

$$\Delta E_b^{\text{Coulomb}} \approx -\frac{C_3 Z^2}{A^{1/3}}. \qquad (11.19)$$

The constant $C_3$ is found from the data on binding energies to be about 0.711 MeV. The Pauli exclusion principle favors $A = 2Z$, and the effect of the exclusion principle is less important at large values of $A$. Accordingly, the reduction of the binding energy ($\Delta E_b^{\text{Pauli}}$) may be written in the form

$$\Delta E_b^{\text{Pauli}} \approx -\frac{C_4 (A-2Z)^2}{A}. \qquad (11.20)$$

The constant $C_4$ is found from the data on binding energies to be about 23.7 MeV. Finally, a fifth correction, also due to the Pauli exclusion principle, may be added to the binding energy, which accounts for the fact that nuclei with an even numbers of both neutrons and protons (*even–even* nuclei) are observed to be especially stable, while those with odd numbers of neutrons and protons (*odd–odd* nuclei) tend to be unstable. The last term is not needed if $Z$ is even and $A - Z$ is odd or vice versa (*even–odd* nuclei).

The resulting empirical formula for the nuclear binding energy is for even–odd nuclei,

$$E_b^{\text{even-odd}} = (15.75\text{ MeV})A - (17.8\text{ MeV})A^{2/3} - \frac{(0.711\text{ MeV})Z^2}{A^{1/3}} - \frac{(23.7\text{ MeV})(A-2Z)^2}{A}; \qquad (11.21)$$

for even–even nuclei,

$$E_b^{\text{even-even}} = E_b^{\text{even-odd}} + \frac{11.18\text{ MeV}}{\sqrt{A}}; \qquad (11.22)$$

and for odd–odd nuclei,

$$E_b^{\text{odd-odd}} = E_b^{\text{even-odd}} - \frac{11.18\text{ MeV}}{\sqrt{A}}. \qquad (11.23)$$

This parameterization of the binding energy is called the *Weizsaecker formula*.

### EXAMPLE 11-4

Calculate the binding energy of $^{56}$Fe and compare the result with the liquid drop model.

### SOLUTION:

The binding energy is

$$E_b = Zm_p c^2 + (A-Z)m_n c^2 - Mc^2,$$

where $M$ is the nuclear mass, $A$ = 56, and $Z$ = 26. Consulting Appendix J to find the nuclear mass, we have

$$E_b = (26)(938.27\text{ MeV}) + (30)(939.57\text{ MeV}) - (52090.2\text{ MeV}) = 492\text{ MeV}.$$

From leading terms of the Weizsaecker formula we have

$$E_b \approx (15.75\text{ MeV})(56) - (17.8\text{ MeV})(56)^{2/3} = 621.5\text{ MeV}.$$

The Coulomb correction is

$$\Delta E_b^{\text{Coulomb}} \approx -\frac{(0.711\text{ MeV})Z^2}{A^{1/3}} = -\frac{(0.711\text{ MeV})(26)^2}{(56)^{1/3}} = -125.6\text{ MeV}.$$

The Pauli exclusion principle correction is

$$\Delta E_b^{\text{Pauli}} \approx -\frac{(23.7\text{ MeV})(A-2Z)^2}{A} = -\frac{(23.7\text{ MeV})(4)^2}{56} = -6.8\text{ MeV}.$$

The even–even correction is

$$\Delta E_b^{\text{even-even}} \approx \frac{11.18\text{ MeV}}{\sqrt{A}} = \frac{11.18\text{ MeV}}{\sqrt{56}} = 1.5\text{ MeV}.$$

The calculated binding energy is

$$E_b = 621.5\text{ MeV} - 125.6\text{ MeV} - 6.8\text{ MeV} + 1.5\text{ MeV} = 491\text{ MeV},$$

which deviates from the actual value (492 MeV) by about 0.2%. ■

## Shell Structure

There are more stable nuclei with even numbers of neutrons and protons than with odd numbers. Furthermore, when the number of neutrons is

$$2, 8, 20, 28, 50, 82, \text{or } 126, \tag{11.24}$$

the nucleus is observed to be particularly stable. These numbers are referred to as *magic numbers*. The magic numbers also apply to protons, except that there is no element number 126; when the number of protons in a nucleus is equal to a magic number, then the nucleus is also very stable. Nuclei in which the number of neutrons and the number of protons are *both* equal to magic numbers ($^{4}_{2}\text{He}$, $^{16}_{8}\text{O}$, $^{40}_{20}\text{Ca}$, $^{48}_{20}\text{Ca}$, $^{208}_{82}\text{Pb}$) tend to be particularly stable.

The existence of the magic numbers is a signature for a *shell structure* of the nucleus analogous to the atomic shell structure. Recall that atoms that have the most stable electron configurations are those that have a number of electrons (2, 10, 18, 36, 54, or 86) corresponding to certain filled subshells. In the nuclear shell model, each nucleon has an independent wave function analogous to our description of electrons in atoms. We now examine the origin of the magic numbers. The total angular momentum (**J**) of a single neutron or proton is given by the sum of its spin (**S**) and orbital (**L**) parts:

$$\mathbf{J} = \mathbf{L} + \mathbf{S}. \tag{11.25}$$

The spin quantum number $s$ is equal to 1/2. The orbital quantum number $\ell$ is equal to a nonnegative integer (0, 1, 2, ...), and as in atomic physics these states are called $s, p, d$, and so on. The possible values of the total angular momentum quantum number ($j$) are given by the addition rule for angular momentum. For $s$ states,

$$j = \frac{1}{2}. \tag{11.26}$$

For $p$ states,

$$j = \frac{1}{2} \quad \text{or} \quad \frac{3}{2}. \tag{11.27}$$

For $d$ states,

$$j = \frac{3}{2} \quad \text{or} \quad \frac{5}{2}, \tag{11.28}$$

and so on.

In nuclear physics, the principle quantum number $n$ is defined so that the energy levels (in increasing energy) for $s$ states are called $1s, 2s, 3s, \ldots$; the energy levels for $p$ states are $1p, 2p, 3p$, and so on. In atomic systems there is no $1p$ state. We have defined the quantum numbers in atomic physics so that the states with nearly the same energy (e.g., $2s$ and $2p$) have the same value of the quantum number $n$. There is no corresponding symmetry for the nucleus. For this reason, the states with the lowest energy for each value of the quantum number $\ell$ are labeled starting with $n = 1$.

In atoms we found that states with lower values of orbital angular momentum have lower energy. The same is true for the nucleus. In addition, there is a strong spin dependence to the force between nucleons. The spin dependence results in nuclear fine-structure splitting that is quite large for large values of $\ell$. For a given value of $\ell$, the energy is lower when L and S are parallel compared to the case where L and S are antiparallel. Thus, the larger of the two values of the quantum number $j$ has the lower energy. (Recall that the deuteron has proton and neutron spins parallel.) The energy levels for the shell model are indicated in Figure 11-4.

The liquid drop model assumes that the nucleons are strongly coupled to each other in the nucleus. In contrast, the shell model assumes that each nucleon moves as an independent particle. Both models have their virtues and deficiencies. More sophisticated nuclear models have been made by combining the features of the liquid drop and shell models.

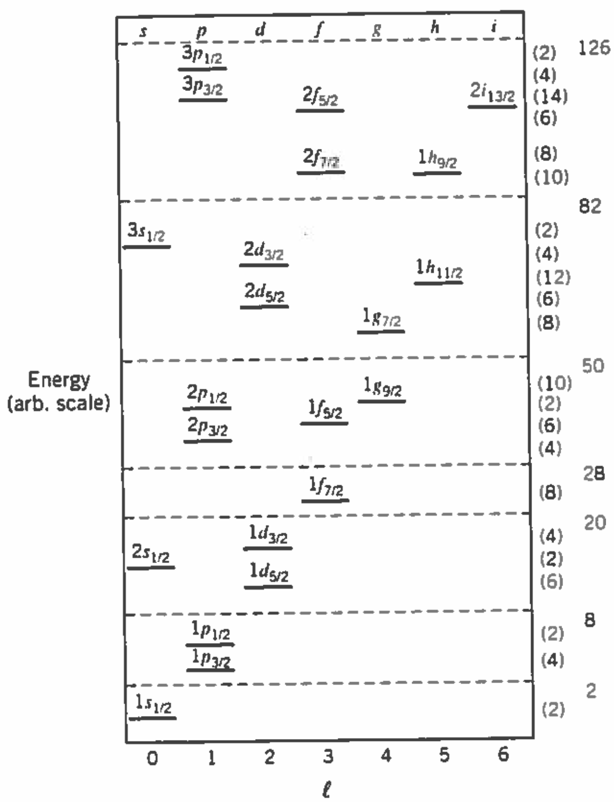



**FIGURE 11-4 Schematic diagram of the nuclear shell model.**
The numbers in parenthesis indicate the number of nucleons per subshell. The numbers on the far left indicate the total number of nucleons in filled subshells.

## 11-4 RADIOACTIVE DECAYS

### Mean Lifetime and Half-Life

The first nuclear disintegrations, those of a uranium compound, were discovered by Becquerel in 1896. Radioactive decay is a random process. Any given particle has a certain probability per unit time of spontaneous decay. The decay probability is independent of the previous life of the particle. If $N(t)$ is the number of particles in a sample as a function of time, then the decay rate $(-dN/dt)$ is proportional to $N$,

$$-\frac{dN}{dt} = \lambda N. \tag{11.29}$$

The constant of proportionality ($\lambda$) has dimensions of inverse time. If we begin with $N_0$ particles, then the number of particles as a function of time is

$$N(t) = N_0 e^{-\lambda t}. \tag{11.30}$$

The number of particles decreases exponentially.

When we speak of a single particle, we usually refer to its mean lifetime. The mean lifetime ($\tau$) of a particle is

$$\tau = \frac{1}{\lambda}, \tag{11.31}$$

as was shown in Example 2-1. For a large sample of particles, $1/e$ of them (about 37.8%) will have *not decayed* after a time $\tau$. In nuclear physics, lifetimes are usually specified by the half-life ($t_{1/2}$) the time after which one-half of the sample has decayed. The half-life is shorter than the mean-life, $\tau$ and $t_{1/2}$ are related by

$$e^{-t_{1/2}/\tau} = \frac{1}{2}, \tag{11.32}$$

or

$$t_{1/2} = \tau \ln 2. \tag{11.33}$$

The exponential decay law for $^{222}$Rn is shown in Figure 11-5.

### Alpha Decay

Alpha particles were first classified as the decay products of natural radioactivity that could not penetrate matter. The $\alpha$ particle, or helium nucleus ($^4$He), has an exceptionally large binding energy per nucleon:

$$E_b = 28 \text{ MeV}, \tag{11.34}$$

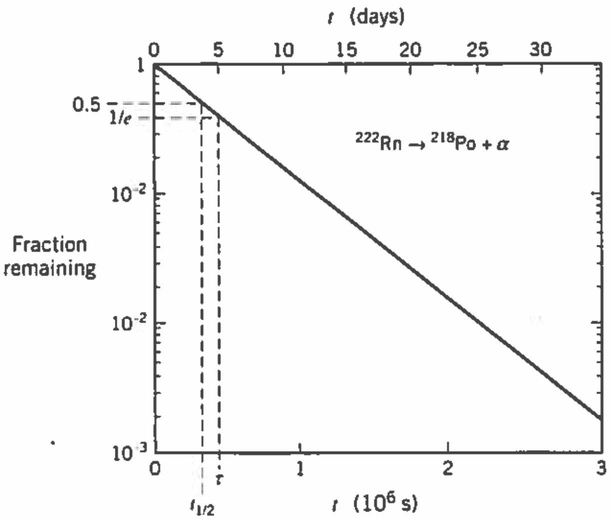



**FIGURE 11-5 The exponential decay law.**
Radon-222, part of the uranium series, decays by alpha emission with a half-life of 3.82 days. For an initial sample of $^{222}$Rn at $t$ = 0, we plot the fraction that has not decayed as a function of time.

or 7 MeV per nucleon. The large binding energy accounts for the very high stability of the alpha particle. We have considered $\alpha$ decay already as an example of barrier penetration. Quantum mechanical tunneling explains the small variation in $\alpha$ energies and the large range of lifetimes (see Figure 7-17). There is a finite probability per unit time that the four nucleons will find themselves in the form of an $\alpha$ particle outside of the nucleus.

The number of neutrons and protons are both conserved in $\alpha$ decay; they are simply rearranged. In an $\alpha$ decay, the $Z$ of the decaying or *parent* nucleus is lowered by 2 and the value of $A$ is lowered by 4. The energy released in $\alpha$ decay ($Q$) appears as kinetic energy of the $\alpha$ particle and the resulting *daughter* nucleus. If $M_P$ is the mass of the parent nucleus, $M_D$ is the mass of the daughter nucleus, and $m_\alpha$ is the mass of the alpha particle, then the energy released in the decay is

$$Q = (M_P - M_D - m_\alpha)c^2. \tag{11.35}$$

Not all nuclei can decay by alpha emission because of conservation of energy. Alpha decay occurs if and only if $Q > 0$. Since the decay has only two particles in the final state, the $\alpha$ particle and the daughter nucleus have equal and opposite momentum ($p$). By energy conservation,

$$Q = \frac{p^2}{2M_D} + \frac{p^2}{2m_\alpha}, \tag{11.36}$$

or

$$Q = \frac{p^2}{2m_\alpha}\left(1 + \frac{m_\alpha}{M_D}\right)$$
$$= \frac{p^2}{2m_\alpha}\left(1 + \frac{4}{A-4}\right). \tag{11.37}$$

Solving for the kinetic energy of the alpha particle, we have

$$\frac{p^2}{2m_\alpha} = \frac{Q(A-4)}{A}. \tag{11.38}$$

## EXAMPLE 11-5

Calculate the kinetic energy of an alpha decay from $^{238}$U.

## SOLUTION:

The decay product of $^{238}$U is $^{234}$Th. The decay process is

$$^{238}\mathrm{U} \rightarrow {}^{234}\mathrm{Th} + \alpha.$$

The energy released in the decay is

$$Q = M_P c^2 - M_D c^2 - m_\alpha c^2.$$

Numerically, we have

$$Q = 221{,}697.68\ \mathrm{MeV} - 217{,}965.99\ \mathrm{MeV}$$
$$-3727.41\ \mathrm{MeV}$$
$$\approx 4.3\ \mathrm{MeV}.$$

The kinetic energy (11.38) of the alpha particle is

$$E = \frac{Q(A-4)}{A} \approx \frac{(4.3\ \mathrm{MeV})(234)}{238} \approx 4.2\ \mathrm{MeV}.$$ ■

Figure 11-6 shows a photograph taken by C. F. Powell and G. Occhialini of the decay sequence

$$^{228}\mathrm{Th} \rightarrow {}^{224}\mathrm{Ra} + \alpha, \quad (t_{1/2} = 1.9\,\mathrm{y}); \tag{11.39}$$

$$^{224}\mathrm{Ra} \rightarrow {}^{220}\mathrm{Rn} + \alpha, \quad (t_{1/2} = 3.6\,\mathrm{d}); \tag{11.40}$$

$$^{220}\mathrm{Rn} \rightarrow {}^{216}\mathrm{Po} + \alpha, \quad (t_{1/2} = 54.5\,\mathrm{s}); \tag{11.41}$$

and

$$^{216}\mathrm{Po} \rightarrow {}^{212}\mathrm{Pb} + \alpha, \quad (t_{1/2} = 0.16\,\mathrm{s}). \tag{11.42}$$

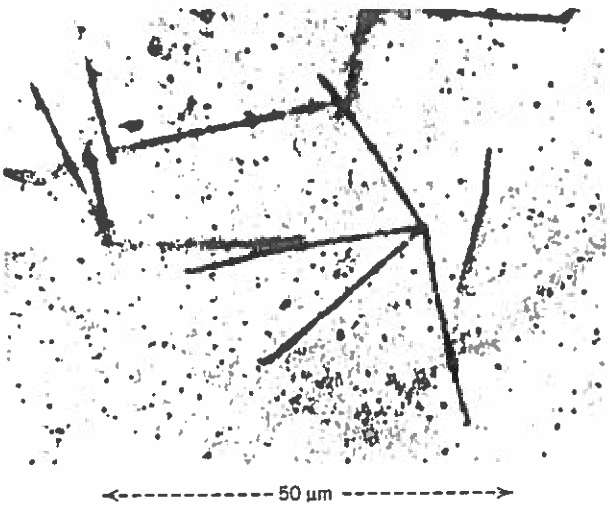



**FIGURE 11-6 Alpha particles from radioactive decays observed in emulsion.**
The four alpha particles are produced in the sequential spontaneous decays of $^{228}$Th, $^{224}$Ra, $^{220}$Rn and $^{216}$Po. From C. F. Powell and G. Occhialini, *Nuclear Physics in Photographs*, Oxford University Press (1947). Reprinted by permission of Oxford University Press.

The picture was made by mixing a sample of $^{228}$Th into a photographic emulsion. The parent nucleus ($^{228}$Th) has a long lifetime so that enough of it must be introduced into the emulsion so that some will decay in a few days. The decay products all have much shorter lifetimes and thus, are "guaranteed" to decay in a relatively short time period. All four alpha particles are observed as heavily ionizing tracks originating from the same point because the nuclei do not move.

Alpha decays decrease the atomic mass number $A$ by four. Therefore, the nuclei that are products of a chain of alpha decays will have atomic mass numbers that differ by four times an integer. The decay of $^{232}$Th produces nuclei with $A = 228, 224, 220,\ldots$ ; $^{238}$U produces nuclei with $A = 234, 230, 226\ldots$; $^{237}$Th produces nuclei with $A = 233, 229, 225,\ldots$ ; and $^{235}$U produces nuclei with $A = 231, 227, 223\ldots$ There are four and only four parent nuclei that give unique chains or *series* of radioactive decay. The radioactive series are listed in Table 11-2. The decays of Figure 11-6 are part of the thorium series.

## Beta Decay

The beta particle is the electron. Nuclear decays in which an electron is emitted are called *beta-minus* ($\beta^-$) decays. In

**TABLE 11-2**
**THE FOUR RADIOACTIVE SERIES.**

| Series | Parent | Lifetime | First Decay | End Product |
|---|---|---|---|---|
| Thorium | $^{232}$Th | $1.40 \times 10^{10}$ y | $^{232}\text{Th} \rightarrow {}^{228}\text{Ra} + \alpha$ | $^{208}$Pb |
| Neptunium | $^{237}$Np | $2.14 \times 10^{6}$ y | $^{237}\text{Np} \rightarrow {}^{233}\text{Pa} + \alpha$ | $^{209}$Bi |
| Uranium | $^{238}$U | $4.17 \times 10^{9}$ y | $^{238}\text{U} \rightarrow {}^{234}\text{Th} + \alpha$ | $^{206}$Pb |
| Actinium | $^{235}$U | $7.04 \times 10^{8}$ y | $^{235}\text{U} \rightarrow {}^{231}\text{Th} + \alpha$ | $^{207}$Pb |

$\beta^-$decay, a neutron is converted into a proton. The simplest example of beta-minus decay is the decay of a free neutron. The neutron mass is larger than the proton mass by about 1.3 MeV. Free neutrons are observed to decay with a mean lifetime of about 1000 seconds. The electron momentum distribution from beta decay is shown in Figure 11-7. In the rest frame of the neutron, the proton and electron do not have opposite momentum and they are not monoenergetic as would be expected for a simple two-body decay ($n \rightarrow p + e$). This was the first indication of the existence of the *neutrino*. The neutrino has zero electric charge, zero mass ($< 7$ eV/$c^2$), and an intrinsic angular momentum quantum number $s = 1/2$. The neutrino is the

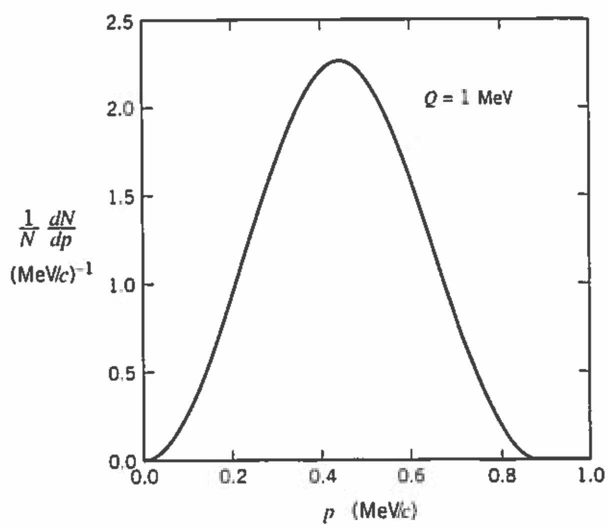



**FIGURE 11-7 Electron momentum distribution from beta decay.**
The shape of the spectrum is characteristic of a final state with three particles. The spectrum is plotted for a reaction $Q$ value of 1 MeV, corresponding to a maximum electron momentum of about 0.85 MeV/$c$.

weak interaction partner of the electron. The particle that is emitted in neutron decay is called the *electron antineutrino* ($\bar{\nu}_e$); the process of beta decay produces one particle and one antiparticle. The neutron beta decay process is

$$n \rightarrow p + e^- + \bar{\nu}_e. \qquad (11.43)$$

Beta decay is an example of the weak interaction.

A neutron inside the alpha particle cannot decay by conservation of energy. The decay process would be $\alpha \rightarrow {}^4\text{Li} + e^- + \bar{\nu}_e$. Such a process is forbidden by energy conservation because the alpha mass is smaller than the $^4$Li mass. (The alpha mass is smaller than the $^4$Li mass because the binding energy of the alpha is so large.)

In many nuclei, the binding energies are such that beta decay is possible resulting in an unstable nucleus. For example, a neutron can decay inside the nitrogen nucleus,

$${}^{16}\text{N} \rightarrow {}^{16}\text{O} + e^- + \bar{\nu}_e. \qquad (11.44)$$

The mass of the $^{16}$N nucleus is 14,906.1 MeV/$c^2$, the mass of the $^{16}$O nucleus is 14,895.2 MeV/$c^2$, and the electron mass is 0.5 MeV/$c^2$. The mass of $^{16}$N is greater than the sum of the $^{16}$O mass plus the electron mass. The decay occurs with a half-life of about 7.13 seconds. In $\beta^-$ decay the value of $Z$ increases by one and $A$ is unchanged.

A second type of beta decay occurs in which a proton is converted to a neutron (the inverse of $\beta^-$decay). This type of beta decay is called *beta-plus* decay ($\beta^+$) because a positively charged particle called the *positron* is emitted. The positron ($e^+$) is the *antiparticle* of the electron. The mass of the positron is identical to the mass of the electron. The $\beta^+$ decay process is

$$p \rightarrow n + e^+ + \nu_e. \qquad (11.45)$$

*This process cannot occur for a free proton* because of energy conservation ($m_p < m_n$). The $\beta^+$ decay (11.45) can occur inside some nuclei. The $\beta^+$ decay transforms the nucleus by increasing $Z$ by one while leaving $A$ unchanged. The condition for $\beta^+$ decay to occur is that the binding energy of the transformed nucleus must exceed the binding energy of the original nucleus by at least $m_nc^2 - m_pc^2 + mc^2$ (where $m$ is the electron mass). Some examples of $\beta^+$ decay are

$${}^{11}\text{C} \rightarrow {}^{11}\text{Be} + e^+ + \nu_e, \qquad (11.46)$$

$${}^{23}\text{Mg} \rightarrow {}^{23}\text{Na} + e^+ + \nu_e, \qquad (11.47)$$

and

$${}^{41}\text{Sc} \rightarrow {}^{41}\text{Ca} + e^+ + \nu_e. \qquad (11.48)$$

## The Discovery of the Neutrino

The need for the neutrino was recognized by Pauli in 1931, from theoretical analysis of the beta decay process. The interaction of neutrinos with matter is so weak that it took 25 years to detect their interactions! Neutrino detection is possible only if an extremely large flux is available. The first direct observation of the neutrino interacting with matter was made by Frederick Reines, Clyde Cowan, Jr., and collaborators in 1956. Their neutrino source was the nuclear reactor at the Savannah River Plant in South Carolina. A nuclear reactor is a source of electron antineutrinos from $\beta^-$ decays.

In 1958, Reines and Cowan repeated their neutrino experiment with greatly improved sensitivity. Figure 11-8 shows the experimental arrangement of Reines and Cowan. The process they observed was

$$\bar{\nu}_e + p \rightarrow n + e^+. \qquad (11.49)$$

The target protons were provided by a 1400 liter liquid detector loaded with a cadmium compound. The region around the neutrino target was instrumented with liquid *scintillator*. The scintillator emits light in response to an energetic gamma ray interaction. The scintillation light is detected by photomultipliers.

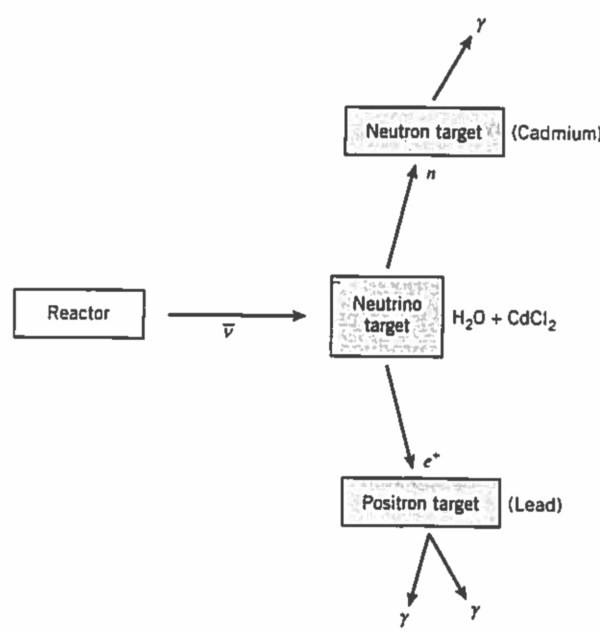



**Figure 11-8 The experiment of Reines and Cowan.**

The presence of the positron from the antineutrino interaction (11.49) was detected by its annihilation with an electron

$$e^+ + e^- \rightarrow \gamma + \gamma. \qquad (11.50)$$

The presence of the neutron was detected by its absorption in cadmium

$$n + {}^{108}\text{Cd} \rightarrow {}^{109}\text{Cd}^* \rightarrow {}^{109}\text{Cd} + \gamma. \qquad (11.51)$$

The experimental signature of the $\overline{\nu}_e$ interaction is the scintillator light from the three gamma rays. (The $\gamma$ from the neutron interaction is produced about $10^{-5}$ s after the $e^+e^-$ annihilation.) The experimenters proved that the scintillation signals were caused by electron antineutrinos from the reactor by making measurements with the reactor both on and off.

## EXAMPLE 11-6

The $\overline{\nu}_e$ flux passing through a 1.4 ton detector is about $10^{15}$ m$^{-2}$s$^{-1}$ and the measured neutrino interaction is $10^{-2}$s$^{-1}$. Estimate the neutrino cross section per target proton.

## SOLUTION:

The total cross section for interaction in the detector is the interaction rate divided by the incident flux:

$$\sigma_T = \frac{R_i}{\Phi_i}.$$

The cross section per proton is obtained by dividing by the number of protons in the target,

$$\sigma = \frac{\sigma_T}{N_p} = \frac{R_i}{\Phi_i N_p}.$$

The number of protons in the target is

$$N_p = \frac{MN_A}{(2)(10^{-3}\text{ kg})},$$

where $M$ is the mass of the target and we have assumed that 1/2 of the target mass is contained in protons. Therefore,

$$\begin{aligned}\sigma &\approx \frac{R_i(2)(10^{-3}\text{ kg})}{\Phi_i MN_A} \\ &\approx \frac{(10^{-2}\text{ s}^{-1})(2)(10^{-3}\text{ kg})}{(10^{15}\text{ m}^{-2}\text{s}^{-1})(1.4\times10^3\text{ kg})(6\times10^{23})} \\ &\approx 10^{-47}\text{ m}^2.\end{aligned}$$

The interaction of the neutrino with matter is indeed "weak." ■

## * *Challenging*

## Fermi's Golden Rule

The dynamics of beta decay was first worked out by Fermi. The number of beta decays per unit time is given by the *Golden Rule*, which states that the transition rate ($W$) is proportional to the density of states ($\rho$) times the square of a matrix element ($|\mathcal{M}|^2$),

$$W = \frac{2\pi}{\hbar}\rho|\mathcal{M}|^2. \qquad (11.52)$$

The matrix element contains the physics of the specific interaction. For the purpose of calculating a transition rate for any given process, $|\mathcal{M}|^2$ is just a number evaluated from the integral of the wave function times its complex conjugate with the interaction potential sandwiched in between. Often, $|\mathcal{M}|^2$ is too complicated to calculate and must be determined from experiment. The most important part of $|\mathcal{M}|^2$ is that it contains a factor of the coupling strength of the force squared ($\alpha_w^2$ for the weak force).

> Fermi's Golden Rule: The transition rate is proportional to the density of final states times a matrix element squared:
>
> $$W = \frac{2\pi}{\hbar}\rho|\mathcal{M}|^2.$$

For one particle, the density of states (2.90) is

$$\rho = \frac{dn_s}{dE} = \frac{dn_s}{dp}\frac{dp}{dE}, \qquad (11.53)$$

and

$$\frac{dn_s}{dp} = \frac{4\pi p^2}{h^3}. \qquad (11.54)$$

(The density of states is discussed further in Chapter 12.) For beta decay we must include both the electron and the neutrino,

$$\frac{d^2 n_s}{dp_e dp_\nu} = \frac{16\pi^2 p_e^2 p_\nu^2}{h^6}. \qquad (11.55)$$

Since we may neglect the kinetic energy of the nucleus, conservation of energy gives

$$E = cp_\nu + E_e, \tag{11.56}$$

where $E$ is the total energy available in the decay, $cp_\nu$ is the neutrino energy, and $E_e$ is the electron energy;

$$dp_\nu = \frac{dE}{c}. \tag{11.57}$$

The density of states factor is

$$\rho = \frac{d^2 n_s}{dE dp_e} = \frac{16\pi^2}{c^3 h^6} p_{e}^2 (E - E_e)^2. \tag{11.58}$$

The transition rate is

$$W = \frac{2\pi}{\hbar}\rho|\mathcal{M}|^2$$

$$= \frac{2\pi}{\hbar}\frac{16\pi^2}{c^3 h^6} p_e^2 (E - E_e)^2 |\mathcal{M}|^2. \tag{11.59}$$

The square root of the transition rate divided by the electron momentum is a linear function of the electron energy:

$$\frac{\sqrt{W}}{p_e} \propto (E - E_e). \tag{11.60}$$

A plot of $\sqrt{W}/p$ versus $E_e$ is called a *Kurie plot* after Franz Kurie, J. R. Richardson, and H. C. Paxton, who first plotted the beta decay data in this manner in 1936. A Kurie plot for promethium-147 is shown in Figure 11-9. Kurie plots have been made for many different beta decays. The Kurie plots prove that the decay rate is proportional to the density of states (11.58). These data show that the Golden Rule works for beta decay.

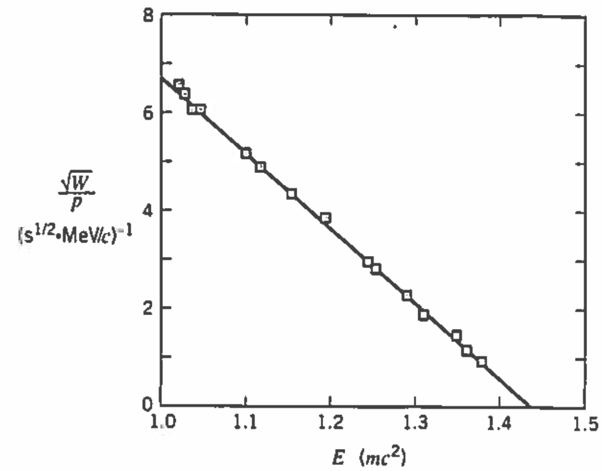



**FIGURE 11-9 Kurie plot of $^{147}$Pm.**
The energy spectrum of the electrons from beta decay is measured. In a plot of the square root of the decay rate divided by the electron momentum versus electron energy, the data lie on a straight line as predicted by Fermi's Golden Rule. After L. M. Langer et al., *Phys. Rev.* 77, 798 (1950).

## Parity Violation

Parity is a transformation that changes the algebraic sign of the coordinate system. Under the parity transformation

$$x' = -x, \tag{11.61}$$

$$y' = -y, \tag{11.62}$$

and

$$z' = -z. \tag{11.63}$$

The parity transformation changes a right-handed coordinate system into a left-handed coordinate system, as indicated in Figure 11-10. If we apply the parity transformation twice to any vector, we return to the original vector.

The electromagnetic and strong interactions are invariant under the parity transformation. In the early 1950s, it was widely assumed that the weak interaction was also invariant under parity. This turned out not to be true! In 1956, Tsung Dao Lee and Chen Ning Yang predicted the nonconservation of parity in the weak interaction. In 1957, a classic experiment was performed by C. S. Wu and her collaborators that demonstrated parity violation in the weak interaction. In the Wu experiment, $^{60}$Co nuclei were aligned in a magnetic field (Figure 11-11). The cobalt undergoes beta decay

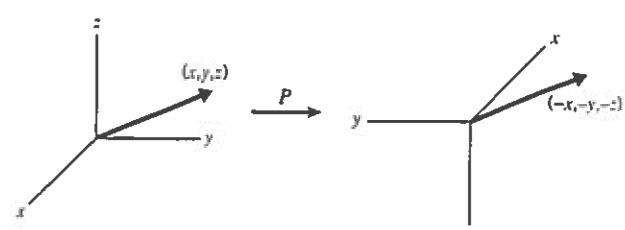



**FIGURE 11-10 The parity transformation.**
Under the parity transformation, a right-handed system is changed into a left-handed system.

$$^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni} + e^- + \bar{\nu}_e, \tag{11.64}$$

with a half-life of 10.5 minutes. The directions of the emitted electrons were measured. If parity were to be conserved in this decay, equal numbers of electrons would be emitted in directions parallel and antiparallel to the magnetic field. Since parity is not conserved in the weak interaction, more electrons are emitted in the direction opposite the magnetic field and, therefore, opposite to the direction of the nuclear spin.

A handy tool for analysis of weak decays is the helicity ($H$) of a particle defined as the component of intrinsic angular momentum (**S**) in the direction of the velocity vector (**v**):

$$H = \frac{\mathbf{S} \cdot \mathbf{v}}{Sv}. \tag{11.65}$$

The helicity is frame dependent. Consider an electron with its spin parallel to the velocity vector ($H = +1$) in Frame $A$ as shown in Figure 11-12. In Frame $B$, which is moving with a velocity **V** in the same direction as **v**

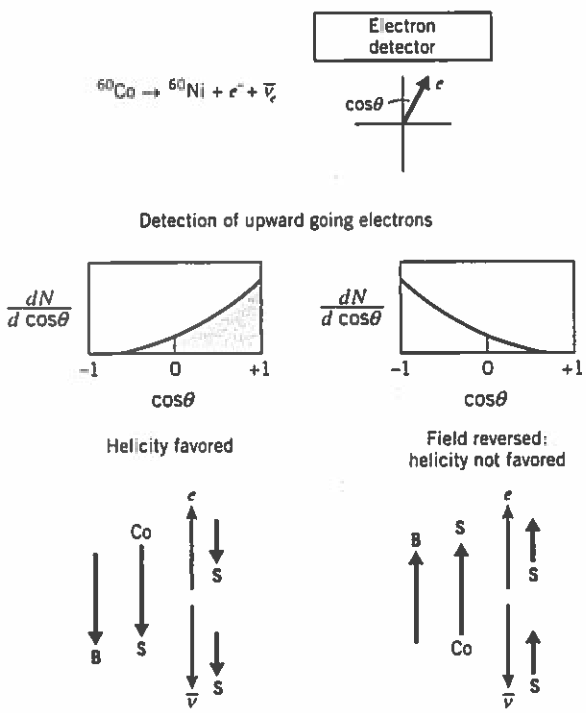



**FIGURE 11-11 The discovery of parity violation in the weak interaction.**
The $^{60}$Co nuclei decay by the weak interaction into $^{60}$Ni nuclei, an electron, and an antineutrino. The experiment was first performed by C. S. Wu et al.

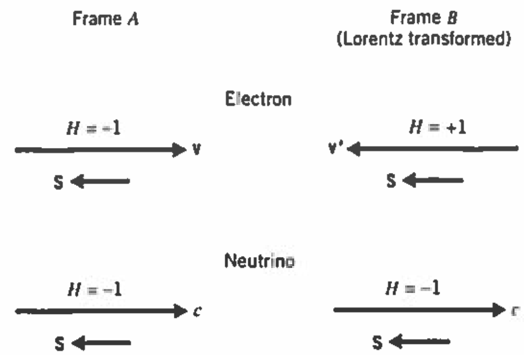



**FIGURE 11-12 Helicity.**

($V > v$), the spin is antiparallel to the velocity vector ($H = -1$). Now consider the case of a neutrino. Since the neutrino is massless, its speed is always equal to the speed of light, $c$. When we transform with any velocity **V** ($V < c$) in the direction of the neutrino, the speed of the neutrino is still $c$. There is no frame in which the neutrino is moving in the opposite direction. We cannot stop a neutrino with a Lorentz transformation!

Now comes an amazing piece of physics. The neutrino is defined to be the particle emitted together with the positron in $\beta^+$ decays. All neutrinos observed in nature are found to have negative helicity. The intrinsic angular momentum of the neutrino always points in a direction opposite the velocity. Such a particle is called a *left-handed* particle. Nobody has ever detected the presence of a *right-handed* neutrino, where the spin points in the same direction as the velocity. The antineutrino is defined to be the particle emitted together with the electron in $\beta^-$ decays. All antineutrinos observed in nature are found to have positive helicity; the antineutrinos are right-handed. *

## Electron Capture

Nuclei that undergo $\beta^+$ decay can also decay by *electron capture*. The electron capture process is

$$p + e^- \rightarrow n + \nu_e, \tag{11.66}$$

occurring inside the nucleus. The emitted neutrino is monoenergetic. Electron capture may occur when $\beta^+$ decay is forbidden by energy conservation because a smaller difference in binding energy is needed for electron capture than for $\beta^+$ decay by twice the electron mass energy (about 1 MeV). Usually, the inner ($K$ shell) electrons are captured, and the process is referred to as $K$ *capture*. An example of electron capture is

$$^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu_e, \qquad (11.67)$$

which occurs with a half-life of 53 days.

When an electron is captured, it leaves a vacancy or *hole* in the atom. The hole will be filled by an electron making a transition from a higher energy level, usually with the emission of an x ray. Occasionally, it is observed that the transition causes an electron to be ejected from the atom. Such an ejected electron is called an *Auger* electron. Auger electrons are emitted with a well-defined energy typically in the keV energy region.

## Gamma Decay

Often a nucleus is left in an excited state after a decay. The nuclear excited state is analogous to the atomic excited state. The excited nucleus (N*) may decay to the ground state by emission of a photon without changing the value of $Z$ or $A$:

$${}^{Z}_{A}N^{*} \rightarrow {}^{Z}_{A}Ni + \gamma. \qquad (11.68)$$

This process is the nuclear analogy of photon emission in atomic systems. The typical energy scale of the photons from nuclear decay is an MeV. Gamma decay is an electromagnetic process.

Table 11-3 lists some commonly used radioactive sources.

**TABLE 11-3**
**PROPERTIES OF SOME COMMONLY USED RADIOACTIVE SOURCES.**

| Source | half-life (y) | Type | Radiation | Energy (keV) |
|---|---|---|---|---|
| $^{22}Na$ | 2.60 | $\beta^{+}$, EC | Positron | 546 (max) |
| | | | Gamma | 511 |
| | | | Gamma | 1275 |
| $^{55}Fe$ | 2.73 | EC | Gamma | 5.89 |
| | | | Gamma | 6.49 |
| | | | Gamma | 14.4 |
| $^{60}Co$ | 5.27 | $\beta^{-}$ | Electron | 318 (max) |
| | | | Gamma | 1173 |
| | | | Gamma | 1332 |
| $^{90}Sr$ | 28.5 | $\beta^{-}$ | Electron | 546 (max) |
| | | $\beta^{-}$ | Electron | 2284 (max, from $^{90}Y$) |
| $^{207}Bi$ | 32.2 | EC | Gamma | 70 |
| | | | Gamma | 1064 |
| | | | Gamma | 1770 |
| | | | Electron | 976 (Auger) |
| | | | Electron | 482 (Auger) |
| | | | Electron | 1048 (Auger) |
| $^{241}Am$ | 432 | $\alpha$ | Alpha | 5486 |
| | | | Alpha | 5443 |
| | | | Gamma | 60 |
| | | | Gamma | 18 |
| | | | Gamma | 14 |

Handbook of Chemistry and Physics.

## The Radioactive Series

The four series of radioactive decays (thorium, actinium, neptunium, and uranium) are shown in Figure 11-13. All of the series contain beta decays as well as alpha decays. The reason for this is that alpha decays change both $A$ and $Z$ by 2 units. The lighter nuclei are more stable when they have a smaller neutron fraction than the heavier nuclei (by the Pauli exclusion principle). The $\beta^{-}$ decays change neutrons into protons as needed to provide the nuclear stability.

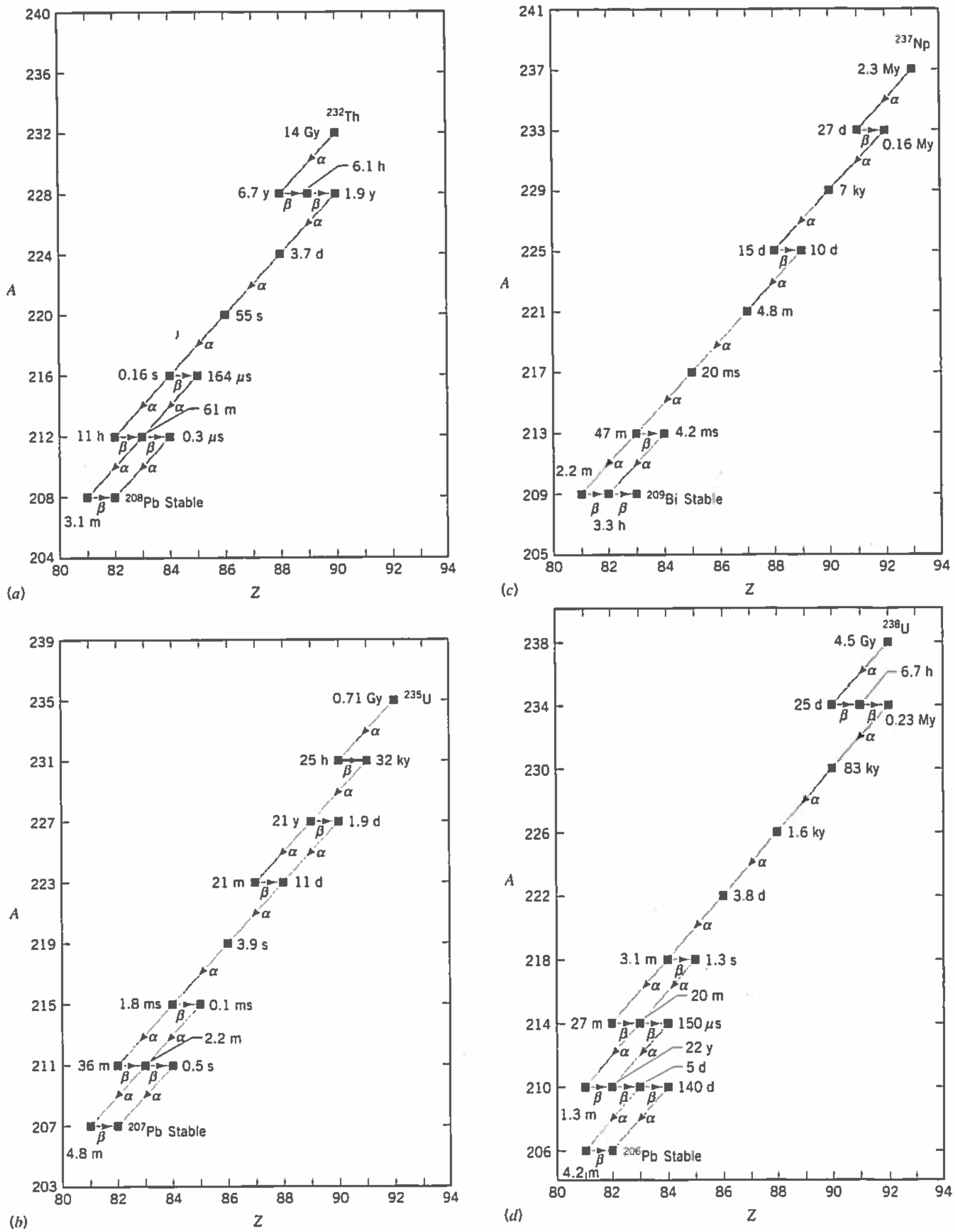
All of the heavy elements were created by collapsing stars in approximately equal numbers. Thorium, uranium, and actinium are all present in the earth; however, there is no neptunium. All of the neptunium that was present in the earth has decayed; the half-life of $^{237}Np$ is

$$t_{1/2} = 2.14 \times 10^{6} \text{ y}, \qquad (11.69)$$

and the age of the earth is much larger than this, about $4.6 \times 10^{9}$ years.

The three radioactive series (thorium, uranium, and actinium) account for most of the activity found in nature. In addition, a small amount of radioactive material, such as $^{14}C$, is continuously produced by cosmic rays. There are a few additional radioactive elements that are not pro-

**FIGURE 11-13 The four radioactive series.**
(*a*) The thorium series begins with $^{232}Th$ which has a half-life of about fourteen billion years. By a sequence of alpha and beta decays, the stable element $^{208}Pb$ is produced. (*b*) The actinium series begins with $^{235}U$ which has a half-life of about seven-hundred million years, and ends with $^{207}Pb$. (*c*)The neptunium series begins with $^{237}Np$ which has a half-life of about two million years, and ends with $^{209}Bi$. (*d*) The uranium series begins with $^{238}U$ which has a half-life of about five billion years, and ends with $^{206}Pb$.



232Th
14 Gy
6.1 h
6.7 y
1.9 y
3.7 d
55 s
0.16 s
164 μs
61 m
11 h
0.3 μs
208Pb Stable
3.1 m
A
Z
(a)
237Np
2.3 My
27 d
0.16 My
7 ky
15 d
10 d
4.8 m
20 ms
47 m
4.2 ms
2.2 m
209Bi Stable
3.3 h
A
Z
(c)
0.71 Gy
235U
25 h
32 ky
21 y
1.9 d
21 m
11 d
3.9 s
1.8 ms
0.1 ms
2.2 m
36 m
0.5 s
207Pb Stable
4.8 m
A
Z
(b)
238U
4.5 Gy
6.7 h
25 d
0.23 My
83 ky
1.6 ky
3.8 d
3.1 m
1.3 s
20 m
27 m
150 μs
22 y
5 d
140 d
1.3 m
206Pb Stable
4.2 m
A
Z
(d)

duced by cosmic rays and are present in the earth. These elements are all lighter than lead, the endpoint of the three naturally occurring radioactive series. These elements must also have lifetimes longer than about a billion years or they would have all decayed. An example is $^{40}$K, which has a half-life of about $1.3 \times 10^9$ years.

## Radioactive Dating

### Carbon Dating

Our atmosphere is continuously bombarded by energetic cosmic ray protons. These protons produce neutrons by the reaction

$$p + \text{nucleus} \rightarrow n + X. \tag{11.70}$$

The flux of neutrons that is generated is about $2 \times 10^4$ m$^{-2}$s$^{-1}$. The neutrons continuously produce $^{14}$C in the earth's atmosphere through the process

$$n + {}^{14}\text{N} \rightarrow p + {}^{14}\text{C}. \tag{11.71}$$

The $^{14}$C isotope is not stable and decays with a half-life of 5730 years by $\beta^-$ decay,

$${}^{14}\text{C} \rightarrow {}^{14}\text{N} + e^- + \bar{\nu}_e. \tag{11.72}$$

The chemistry of $^{14}$C is identical to that of $^{12}$C. Radioactive $^{14}$C forms carbon dioxide just like $^{12}$C does. Since the lifetime of the radioactive $^{14}$C is relatively long, it is distributed throughout the earth's atmosphere and enters all living objects: plants, animals, and people. All living things have been activated with $^{14}$C by the cosmic radiation. The $^{14}$C also ends up in the ocean in inorganic forms of dissolved carbon dioxide, bicarbonate, and carbonate. The fraction of carbon that is in the form of $^{14}$C is measured to be

$$f = 1.3 \times 10^{-12}. \tag{11.73}$$

All living things are radioactive! The activity of a radioactive sample is the number of decays per second.

## EXAMPLE 11-7

Calculate the activity in a living sample that contains 1 kilogram of carbon.

## SOLUTION:

The activity ($R$) of the sample is

$$R = \frac{N}{\tau} = \frac{N \ln 2}{t_{1/2}},$$

where $N$ is the number of $^{14}$C atoms in the sample, $\tau$ is the $^{14}$C mean lifetime, and $t_{1/2}$ is the $^{14}$C half-life. The number of $^{14}$C atoms in the sample is $f$ times the total number of carbon atoms in the sample:

$$N = \frac{f N_A (1 \text{ kg})}{(12)(10^{-3} \text{ kg})}.$$

The activity is

$$R = \frac{(1.3 \times 10^{-12})(6.02 \times 10^{23})(1 \text{ kg})(\ln 2)}{(12)(10^{-3} \text{ kg})(5730 \text{ y})(3.15 \times 10^7 \text{ s/y})} = 250 \text{ s}^{-1}.$$

The activity of a living sample due to radioactive $^{14}$C decays is 250 decays per second per kilogram of carbon. ■

In 1947, Willard F. Libby developed the method of carbon dating. Libby determined that all living objects have a constant fraction of radioactive carbon, but that when they die the $^{14}$C is not replenished and decays exponentially with its 5730-year half-life. By determining the $^{14}$C activity of a once-living object, one can determine how long it has been dead. This technique assumes that the $^{14}$C fraction (11.73) has remained constant (i.e., the cosmic ray flux has not changed). Figure 11-14 shows the

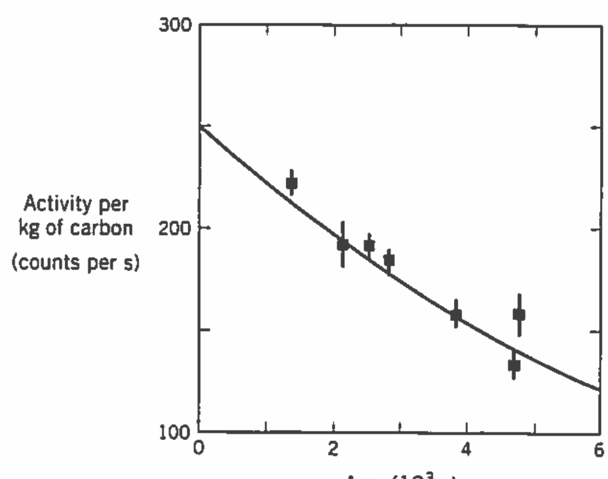



**FIGURE 11-14 Activity of $^{14}$C (disintegrations per second per kilogram of carbon) for several samples of known age.**
The data are from W. Libby, *Radiocarbon Dating* Univ. of Chicago Press (1952).

measured $^{14}C$ activity for several samples of known age. Carbon dating by measuring $^{14}C$ activity is limited to objects less than 25,000 years old (a few half-lifes), because the $^{14}C$ activity is too low in older objects.

A modern version of carbon dating is made possible by use of a mass spectrometer. In a mass spectrometer, one can measure the ratio of $^{14}C$ to $^{12}C$ atoms before the $^{14}C$ atoms decay. This allows a much more sensitive measurement of objects older than 20,000 years, and allows extension of the carbon dating technique to ages of 50,000 years. Another advantage of the mass spectrometer technique is that objects may be dated by using samples that are one thousand times smaller than that needed for measuring $^{14}C$ activity.

## EXAMPLE 11-8

The activity in a piece of bone is measured to be 100 decays per second per kilogram. What is the age of the bone?

## SOLUTION:

Let $T$ be the age of the bone. The activity is

$$R = R_0 e^{-T/\tau},$$

where $R_0$ is the initial decay rate in the bone (250 $s^{-1}$). The age of the bone is

$$T = -\tau \ln\left(\frac{R}{N_0}\right) = \tau \ln\left(\frac{R_0}{R}\right) = \frac{t_{1/2}}{\ln 2} \ln\left(\frac{R_0}{R}\right)$$

$$= \frac{5730\,\text{y}}{\ln 2} \ln\left(\frac{250}{100}\right) \approx 7600\,\text{y}.$$

■

### Age of the Earth

As already mentioned, the material of the earth once contained all heavy elements in roughly equal proportions. As the earth has aged, the radioactive elements have decayed. The absence of the neptunium series (half-life of $2 \times 10^6$ years) in the earth indicates that the earth is much older than a few million years. The existence of the thorium series (half-life of $1.4 \times 10^{10}$ years) in the earth indicates that the earth is less than one-hundred billion years old because the thorium has not yet all decayed. The age of the earth may be determined by *uranium dating*. In uranium dating, one measures the ratio of $^{238}U$ (half-life of $4.17 \times 10^9$ y) to $^{206}Pb$ (stable). Measurements of this ratio on the earth, in meteorites, and on the moon shows that the age of the solar system is about $4.6 \times 10^9$ years.

# 11-5 NUCLEAR REACTIONS

## Nuclear Fusion

An examination of the atomic mass number ($A$) dependence of the binding energy of Figure 11-3 shows that for light elements the binding energy per nucleon is increasing with increasing $A$. Each additional nucleon contributes to the binding energy of the other nucleons. Therefore, if we fuse two light nuclei together, they are more tightly bound than the two original nuclei.

> If two light nuclei are combined into a heavier nucleus (fusion), the sum of the masses of the two lighter nuclei is greater than the mass of the heavy nucleus. Therefore, energy is released when the heavier nucleus is formed.

In the fusion of deuterium and tritium,

$$d + t \rightarrow \alpha + n, \tag{11.74}$$

(equivalently written $^2H + {}^3H \rightarrow {}^4He + n$), an energy of 17.6 MeV is released. This energy appears in the kinetic energy of the alpha particle and the neutron. Table 11-4 lists some fusion reactions and their $Q$ values.

**TABLE 11-4**
**SELECTED FUSION REACTIONS AND THEIR $Q$ VALUES.**

| Reaction | $Q$ (MeV) |
|---|---|
| $p + p \rightarrow d + e^+ + \nu_e$ | 1.4 |
| $He^3 + \alpha \rightarrow {}^7Be + \gamma$ | 1.6 |
| $d + d \rightarrow t + p$ | 3.3 |
| $d + d \rightarrow {}^3He + n$ | 4.0 |
| $p + d \rightarrow {}^3He + \gamma$ | 5.5 |
| $C^{12} + p \rightarrow {}^{13}N + \gamma$ | 7.6 |
| $^3He + {}^3He \rightarrow \alpha + p + p$ | 12.9 |
| $^7Li + p \rightarrow \alpha + \alpha$ | 17.3 |
| $d + t \rightarrow \alpha + n$ | 17.6 |
| $^3He + d \rightarrow \alpha + p$ | 18.3 |

## EXAMPLE 11-9

Calculate the energy released in deuterium-tritium fusion.

## SOLUTION:

The energy released is

$$Q = m_d c^2 + m_t c^2 - m_\alpha c^2 - m_n c^2 ,$$

where the masses correspond to deuterium, tritium, alpha, and neutron. Numerically, we have

$$\begin{aligned} Q &= 1875.63 \ \text{MeV} + 2808.94 \ \text{MeV} - 3727.41 \ \text{MeV} \\ &\quad -939.57 \ \text{MeV} \\ &= 17.6 \ \text{MeV}. \end{aligned}$$

■

### Energy Production in Stars

The process by which energy is produced in stars is nuclear fusion. Sir Arthur Stanley Eddington was the first to realize that the main source of solar energy was the combining of four nucleons into an alpha particle. In 1938, Hans Bethe determined the sequence of reactions that powers most stars, the carbon cycle. The nuclear reactions that make up the carbon cycle are

$$p + {}^{12}\text{C} \to {}^{13}\text{N} + \gamma , \tag{11.75}$$

$${}^{13}\text{N} \to {}^{13}\text{C} + e^+ + \nu_e , \tag{11.76}$$

$$p + {}^{13}\text{C} \to {}^{14}\text{N} + \gamma , \tag{11.77}$$

$$p + {}^{14}\text{N} \to {}^{15}\text{O} + \gamma , \tag{11.78}$$

$${}^{15}\text{O} \to {}^{15}\text{N} + e^+ + \nu_e , \tag{11.79}$$

and

$$p + {}^{15}\text{N} \to {}^{12}\text{C} + \alpha . \tag{11.80}$$

In these reactions, carbon is a catalyst for the burning of four protons to make an alpha particle. The net reaction is $4p \to \alpha + 2e^+ + 2\nu_e + 3\gamma$. The amount of energy released in these reactions is about 25 MeV. The collision cross sections are extremely small, but the collision rate in stars is enormous. The carbon cycle is the dominate source of energy in stars that have internal temperatures greater than about $10^8$ kelvin.

A second set of proton-burning reactions in stars is the proton cycle. The nuclear reactions of the proton cycle are

$$p + p \to d + e^+ + \nu_e , \tag{11.81}$$

$$p + d \to {}^{3}\text{He} + \gamma , \tag{11.82}$$

and

$${}^{3}\text{He} + {}^{3}\text{He} \to p + p + \alpha . \tag{11.83}$$

The net reaction is the same as for the carbon cycle. An alternate sequence can include

$${}^{3}\text{He} + \alpha \to {}^{7}\text{Be} + \gamma , \tag{11.84}$$

$${}^{7}\text{Be} + e^- \to {}^{7}Li + \nu_e , \tag{11.85}$$

$${}^{7}\text{Li} + p \to {}^{8}\text{Be} + \gamma , \tag{11.86}$$

$${}^{8}\text{Be} \to \alpha + \alpha , \tag{11.87}$$

and

$${}^{7}\text{Be} + p \to {}^{8}\text{B} + \gamma , \tag{11.88}$$

$${}^{8}\text{B} + e^- \to {}^{8}\text{Be} + \nu_e , \tag{11.89}$$

$${}^{8}\text{Be} \to \alpha + \alpha . \tag{11.90}$$

At temperatures below about $10^8$ kelvin the proton cycle dominates over the carbon cycle. The temperature of the solar core is about $1.5 \times 10^7$ kelvin, and fusion in the sun is dominated by the proton–proton cycle.

## EXAMPLE 11-10

The age of the sun is roughly 10 billion years. The power output of the sun is about $4 \times 10^{26}$ watts. Show that the energy source of the sun cannot be of gravitational origin.

## SOLUTION:

The gravitational binding energy ($E_b$) of the sun depends on how the mass is distributed. For an object of mass $M$ and radius $R$, the order of magnitude of the energy is (Chap. 1, prob. 46)

$$E_b \approx \frac{GM^2}{R} .$$

For the sun, this energy is

$$\begin{aligned} E_b &\approx \frac{\left(7\times10^{-11} \ \text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}\right)\left(2\times10^{30} \ \text{kg}\right)^2}{7\times10^{8} \ \text{m}} \\ &\approx 4\times10^{41} \ \text{J}. \end{aligned}$$

$$\rho = 10^3\ \mathrm{kg/m^3}.$$

The rate of energy loss per distance traveled by the muon inside the person is

$$-\left(\frac{dE}{dx}\right) \approx (0.2\,\mathrm{MeV\cdot m^2/kg})\rho$$
$$\approx (0.2\,\mathrm{MeV\cdot m^2/kg})(10^3\ \mathrm{kg/m^3})$$
$$= 200\,\mathrm{MeV/m}.$$

Estimate the height, $\Delta x$, of the person to be

$$\Delta x \approx 1.70\,\mathrm{m}.$$

The energy deposited in the person by the muon is

$$\Delta E \approx -\frac{dE}{dx}\Delta x \approx (200\,\mathrm{MeV/m})(1.70\,\mathrm{m})$$
$$\approx 340\,\mathrm{MeV}.$$

■

## Radiation Units and Doses

The SI unit of radioactivity or *activity* is the becquerel (Bq), named after the discoverer of nuclear disintegrations:

$$1\,\mathrm{Bq} = 1\,\mathrm{decay/s}. \quad (11.123)$$

The activity of radioactive sources is commonly stated in units of the curie (Ci),

$$1\,\mathrm{Ci} = 3.7\times10^{10}\ \mathrm{Bq}. \quad (11.124)$$

One curie is the approximate activity of one gram of radium. The activity includes the decay not only of radium but also that of the daughter nuclei. The heavily ionizing tracks produced by alpha particles from radium and its daughter decays are illustrated in Figure 11-24. The experimenters put a tiny speck of radium on a photographic plate and developed it a few days later. The activity of 1 kg of natural uranium is about $1.3\times10^{13}$ Bq and the activity of 1 kg natural thorium is about $4.11\times10^{12}$ Bq.

The energy absorbed per mass is called the *radiation absorbed dose*. The SI unit is the gray (Gy):

$$1\,\mathrm{Gy} = 1\,\mathrm{J/kg} = 6.24\times10^{12}\ \mathrm{MeV/kg}. \quad (11.125)$$

Some types of energy deposits do more biological damage then others. For example, a 5-MeV alpha particle does more biological damage than a 5-MeV electron. We take this into account by defining the *radiation dose equivalent* as the radiation absorbed dose multiplied by a damage factor ($Q$), which depends on the type of radiation. The more dangerous the radiation is to your health, the higher the $Q$. Values of $Q$ are given in Table 11-6. The SI unit of the radiation dose equivalent is the sievert (Sv):

$$1\,\mathrm{Sv} = (1\,\mathrm{Gy})\times Q. \quad (11.126)$$

The average annual background radiation received by a person on the earth is in the range of 0.4 to 4 millisieverts (mSv). The maximum recommended occupational dose is 50 mSv per year. A lethal dose is about 3 sieverts.

### EXAMPLE 11-19

A person works for 4 hours in a room that contains an unshielded 10-mCi $^{60}$Co source. The person keeps a certain distance from the source. At what distance is the estimated radiation dose in 4 hours equal to one-hundredth of the maximum permitted annual occupational dose?

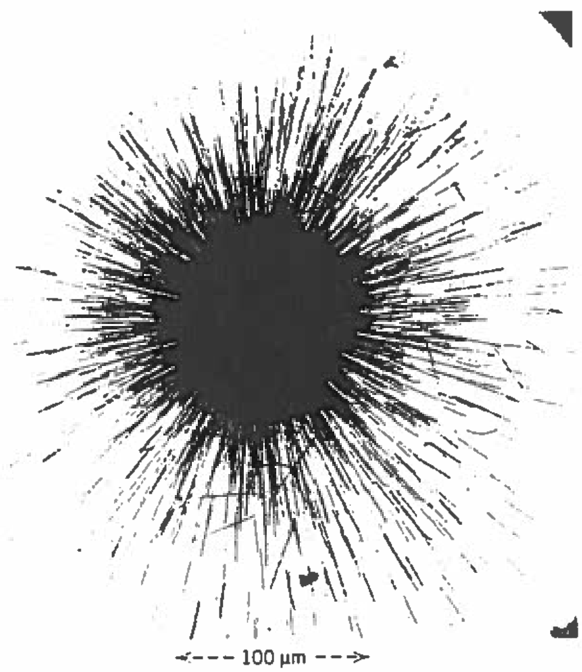



**FIGURE 11-24 Alpha particles from radium observed in emulsion.**
A "speck" of radium, too small to be seen under a microscope, was inserted into emulsion by first depositing the radium on a needle and then shaking the needle over the photographic plate. The emulsion was developed after a few days. From C. F. Powell and G. Occhialini, *Nuclear Physics in Photographs*, Oxford University Press (1947). Reprinted by permission of Oxford University Press.

**TABLE 11-6**
**VALUES OF $Q$ FOR VARIOUS TYPES OF RADIATION.**

| Radiation | $Q$ |
|---|---|
| Gamma | 1 |
| Electron | 1 |
| Protons (10 MeV) | 1 |
| Protons (1 GeV) | 2 |
| Neutrons (thermal) | 3 |
| Neutrons (fast) | 10 |
| Alpha | 20 |

## SOLUTION:

Cobalt-60 is a beta emitter. The decay is

$$^{60}\text{Co} \rightarrow {}^{60}\text{Ni} + e^- + \bar{\nu}_e .$$

($^{60}$Ni is stable.) The typical kinetic energy of the electron is 1 MeV. The angular distribution of the electrons is isotropic. The activity of the source is

$$10\,\text{mCi} = \left(10^{-2}\ \text{Ci}\right)\left(3.7\times10^{10}\ \text{Bq/Ci}\right) = 3.7\times10^{8}\ \text{Bq}.$$

The total energy that appears in electrons in 4 hours is

$$E \approx \left(3.7\times10^{8}\ \text{s}^{-1}\right)(4\,\text{h})\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)(1\,\text{MeV})$$
$$= 5.3\times10^{12}\ \text{MeV}.$$

If all of this energy were absorbed by the person of mass 60 kg, the radiation absorbed dose would be

$$\frac{E}{60\,\text{kg}} = \left(\frac{5.3\times10^{12}\ \text{MeV}}{60\,\text{kg}}\right)\left(\frac{1\,\text{Gy}}{6.24\times10^{12}\ \text{MeV/kg}}\right)$$
$$= 0.014\,\text{Gy}.$$

Since the $Q$ value of beta radiation is unity, the maximum radiation dose is 0.014 Sv. To get one-hundredth of the maximum annual dose, the fraction of the electrons absorbed would be

$$f = \frac{5\times10^{-4}\ \text{Sv}}{0.014\,\text{Sv}} = 3.6\times10^{-2}.$$

At a distance $R$, a person with a cross-sectional area ($A$) absorbs the fraction

$$f = \frac{A}{4\pi R^2},$$

or

$$R = \sqrt{\frac{A}{4\pi f}}.$$

Let us estimate the cross-sectional area of the person to be $A = 0.5\ \text{m}^2$. The distance that the person must keep from the source to limit the dose to 0.5 mSv is

$$R = \sqrt{\frac{0.5\,\text{m}^2}{4\pi\left(3.6\times10^{-2}\right)}} \approx 1\,\text{m}.$$

Notice that the radiation dose varies as the inverse square of the distance to the source. The person is protected by the distance to the source because of the "solid angle" factor. A person 10 m away gets 100 times less radiation (5 μSv). At the other extreme, a person just next to the source for 4 hours will absorb roughly one-half of all the radiation, or about 7 mSv. ∎

Radon gas can be a particularly dangerous health hazard. Radon is produced naturally on earth as part of the uranium series. The fraction of radon in air is about one part in $10^{21}$, on the average. This corresponds to a radiation dose to the lungs of about 0.1 mSv. Radon becomes a health hazard when it accumulates in mines or in the basements of buildings. Part of the radon decay chain is

$$^{222}\text{Ra} \rightarrow {}^{218}\text{Po} + \alpha, \tag{11.127}$$

$$^{218}\text{Po} \rightarrow {}^{214}\text{Pb} + \alpha, \tag{11.128}$$

$$^{214}\text{Pb} \rightarrow {}^{214}\text{Bi} + e^- + \bar{\nu}_e, \tag{11.129}$$

$$^{214}\text{Bi} \rightarrow {}^{214}\text{Po} + e^- + \bar{\nu}_e, \tag{11.130}$$

and

$$^{214}\text{Po} \rightarrow {}^{210}\text{Pb} + \alpha. \tag{11.131}$$

The daughter decay products, which are normally solids, get stuck to dust particles in the air. The half-lifes of $^{218}$Po, $^{214}$Pb, $^{214}$Bi, and $^{214}$Po are all relatively short. Lead-210 has a half-life of 22.3 years. The three alpha particles dominate the radiation damage from radon to your lungs because of their large $Q$ factors. Radon is considered to be a health hazard if its presence in air causes an activity greater than about 150 Bq/m$^3$.

## EXAMPLE 11-20

A person works for an hour in a basement that contains radon gas. The activity of the air in the basement is 300 Bq/m$^3$. Estimate the radiation dose equivalent to the lungs that the person receives during this time.

## SOLUTION:

Each radon decay produces five disintegrations. The $^{222}$Ra decay rate ($R$) is

$$R = \frac{300\,\text{Bq/m}^3}{5} = 60\,\text{Bq/m}^3.$$

The half-life ($\tau_{1/2}$) of $^{222}$Ra is 3.92 days. The number of $^{222}$Ra per volume ($n$) is

$$\begin{aligned} n &= \frac{R\tau_{1/2}}{\ln 2} \\ &= \frac{(60\,\text{Bq/m}^3)(3.92\,\text{d})(24\,\text{h/d})(3600\,\text{s/h})}{0.693} \\ &= 2.9\times 10^7\ \text{m}^{-3}. \end{aligned}$$

The total number density of air molecules is

$$V = \frac{6.02\times 10^{23}}{0.0224\,\text{m}^3} = 2.7\times 10^{25}\ \text{m}^{-3}.$$

The fraction of radon in the air is about one part in $10^{18}$ or 1000 times the normal level. The annual dose to the lungs at this rate would be

$$(10^3)(0.1\,\text{mSv}) = 100\,\text{mSv}.$$

In one hour, the dose is

$$\frac{(100\,\text{mSv})(3600\,\text{s})}{3.16\times 10^7\ \text{s}} \approx 10\,\mu\text{Sv}.$$

A prolonged exposure at this rate is hazardous because the radiation dose is concentrated in the lungs. ■

## EXAMPLE 11-21

A detector for a particle physics experiment is fashioned out of a thin wafer of silicon. The silicon detector is designed to operate for $10^7$ seconds in a flux of energetic charged particles of $10^{10}$ per square meter per second. Make an estimate of the radiation absorbed dose that the silicon will get in the experiment.

## SOLUTION:

Let $A$ and $\Delta x$ be the cross-sectional area and thickness of the silicon wafer and let $\rho$ be the density of silicon. The energy deposited in the wafer in $10^7$ seconds is

$$E = (0.2\,\text{MeV}\cdot\text{m}^2/\text{kg})\rho\Delta x(10^7\ \text{s})(10^{10}\ \text{m}^{-2})A.$$

The mass of the wafer is

$$M = \rho\Delta x A.$$

The radiation absorbed dose is

$$\begin{aligned} \frac{E}{M} &= (0.2\,\text{MeV}\cdot\text{m}^2/\text{kg})(10^7\ \text{s})(10^{10}\ \text{m}^{-2}) \\ &= 2\times 10^{16}\ \text{MeV/kg}, \end{aligned}$$

or

$$\begin{aligned} \frac{E}{M} &= (2\times 10^{16}\ \text{MeV/kg})\left(\frac{1\,\text{Gy}}{6.24\times 10^{12}\ \text{MeV/kg}}\right) \\ &= 3.2\,\text{kGy}. \end{aligned}$$

## CHAPTER 11: PHYSICS SUMMARY

- The nucleus is made up neutrons and protons (nucleons) bound together by the strong interaction. Neutrons and protons have an identical strong interaction behavior. The spin quantum number of both the neutron and proton is

$$s = \frac{1}{2}.$$

- For small values of $Z$, the most stable nuclear configurations are those with approximately equal numbers of neutrons and protons. The reason for this is that the neutrons and protons each obey the Pauli exclusion principle. At large values of $Z$, the stable nuclei have more neutrons than protons. This is due to the electromagnetic repulsion of the protons.

- Nuclei with a magic number (2, 8, 20, 28, 50, 82, or 126) of neutrons or protons are observed to be especially stable. These magic numbers can be accounted for in the shell model of the nucleus, where each nucleon is assumed to move in a stable orbit as an independent particle.
- Fusion is the combining of two elements lighter than iron to make a single nucleus, resulting in an increase in the binding energy per nucleon. Fission is the splitting of an element heavier than iron, resulting in an increase in the binding energy per nucleon.
- The three types of radioactive decays are alpha, beta, and gamma. The alpha decay process is

$$ {}^{A}_{Z}P \rightarrow {}^{A-4}_{Z-2}D + \alpha $$

where $P$ and $D$ are the parent and daughter nuclei. There are two types of beta decays, beta-minus

$$ {}^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + e^{-} + \bar{\nu}_e $$

and beta-plus

$$ {}^{A}_{Z}P \rightarrow {}^{A}_{Z-1}D + e^{+} + \nu_e . $$

Gamma decays are transitions between an excited nuclear state ($P^*$) and a lower energy state ($P$):

$$ {}^{A}_{Z}P^* \rightarrow {}^{A}_{Z}P + \gamma . $$

- The Mössbauer effect occurs when a radioactive nucleus is embedded in a crystal. Nuclear gamma decays can occur without nuclear recoil, resulting in extremely narrow line widths.
- One becquerel is the unit of radioactivity:

$$ 1\,\text{Bq} = 1\,\text{decay/s} . $$

One curie is the approximate activity from one gram of radium:

$$ 1\,\text{Ci} = 3.7 \times 10^{10}\,\text{Bq} . $$

- The radiation absorbed dose is the energy absorbed per mass.

$$ 1\,\text{Gy} = 1\,\text{J/kg} = 6.24 \times 10^{12}\,\text{MeV/kg} . $$

- The radiation dose equivalent is the radiation absorbed dose multiplied by a factor ($Q$) that depends on the type of radiation,

$$ 1\,\text{Sv} = (1\,\text{Gy}) \times Q . $$

Radiation that is more harmful to your health has a higher $Q$ factor. The $Q$ factor ranges from unity for electrons to about 20 for alpha particles.

## REFERENCES AND SUGGESTIONS FOR FURTHER READING

F. Ajzenberg-Selove and E. K. Warburton, "Nuclear Spectroscopy," *Phys. Today* **36**, No. 11, 26 (1983).

P. Armbruster and G. Münzenberg, "Creating Superheavy Elements," *Sci. Am.* **260**, No. 5, 66 (1989).

John N. Bahcall, "The Solar Neutrino Problem," *Sci. Amer.* **262**, No. 5, 54 (1990).

J. M. Blatt and V. Weisskopf, *Theoretical Nuclear Physics*, Wiley (1952).

J. Chadwick, "The Existence of a Neutron," *Proc. of the Royal Soc.* **136A**, 692 (1932).

E. Fermi, "The Nucleus" *Phys. Today* **5**, No. 3, 6 (1952).

H. Frauenfelder and E. M. Henley, *Subatomic Physics*, Prentice-Hall (1991).

R. E. M. Hedges and J. A. J. Gowlett, "Radiocarbon Dating by Accelerator Mass Spectrometry," *Sci. Am.* **254**, No. 1, 100 (1986).

J. Hans D. Jensen, "The History of the Theory of Structure of the Atomic Nucleus," *Sci.* **147**, 1419 (1965).

A. D. Krisch, "Collisions Between Spinning Protons," *Sci. Am.* **257**, No. 2, 42 (1987).

W. F. Libby, *Radioactive Carbon Dating*, Univ. of Chicago Press (1952).

M. G. Mayer, "The Structure of the Nucleus," *Sci. Am.* **184**, No. 3, 22 (1951).

M. Goeppert Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, Wiley (1955).

A. C. Melissinos, *Experiments in Modern Physics*, Academic Press (1966).

M. K. Moe and S. P. Rosen, "Double-Beta Decay," *Sci. Am.* **261**, No. 5, 48 (1989).

E. Segrè, "The Discovery of Nuclear Fission," *Phys. Today* **42**, No. 7, 38 (1990).

E. Segrè, *Nuclei and Particles*, Benjamin (1977).

C. P. Slichter, *Prinicples of Magnetic Resonance*, Springer (1978).

L. Wolfenstein and E. W. Beier, "Neutrino Oscillations and Solar Neutrinos," *Phys. Today* **42**, No. 7, 28 (1990).

## QUESTIONS AND PROBLEMS

*Discovery of the neutron*

1. Recall that in the discovery of the neutron, Chadwick ruled out Compton scattering partly on the basis of the observed scattering rate. Why is the neutron–proton cross section much greater than the photon–proton cross section?
2. In the decay of the neutron,

$$n \to p + e^- + \bar{\nu}_e,$$

what can be said about the relative wavelengths of the three particles?

3. An unknown particle scatters from the nitrogen nucleus and the nucleus is measured to recoil with a maximum kinetic energy of 1.4 MeV. (a) Calculate the energy of the incident particle if the particle is a photon. (b) Calculate the energy of the incident particle if the particle is a neutron.

*Basic properties of the nucleus*

4. Why does the binding energy per nucleon increase with increasing atomic mass number ($A$) for small values of $A$? Why does the binding energy per nucleon decrease with increasing atomic mass number for large values of $A$?
5. Why does carbon-12 have a greater binding energy than nitrogen-12?
6. The binding energy per nucleon of $^{235}$U is 7.59 MeV. (a) Calculate the mass of the uranium nucleus. (b) Calculate the mass of the $^{235}$U atom.
7. Calculate the binding energy per nucleon for all the stable isotopes of manganese, iron, and cobalt. Which nucleus has the largest binding energy per nucleon?
8. Which nucleus do you expect will have a larger binding energy, tritium ($^3$H) or $^3$He? Explain. Calculate the binding energy of each.
9. Which nucleus do you expect to have a larger binding energy, $^{64}$Ni or $^{64}$Zn? Explain. Calculate the binding energy of each.
10. Which nucleus do you expect to have the largest binding energy, $^{12}$Be, $^{12}$B, $^{12}$C, or $^{12}$N? Explain. Calculate the binding energy of each.

*Nuclear models*

11. Calculate the binding energies of $^{55}$Fe, $^{57}$Co, and $^{58}$Ni. Compare the actual binding energies to the Weizsaecker formula of the liquid drop model.

*Radioactive decays*

12. Why are there $\beta^-$ decays in the four radioactive series? Why are there no $\beta^+$ decays?
13. Why do two of the radioactive series start with uranium isotopes?
14. Why is the lifetime of $^{237}$Np so much shorter than the lifetimes of leading nuclei in the other three series?
15. If the activity of a substance drops by a factor of 32 in 5 seconds, what is the radioactive half-life?
16. Can a given nucleus have both $\beta^-$ and a $\beta^+$ decay modes?
17. Why are the photons from nuclear gamma transitions so much larger in energy than photons from atomic transitions?
18. A piece of bone found at the Palace of Knossos contains 100 grams of carbon and has a $^{14}$C activity of 1000 per minute. How long has this chap been dead?
19. (a) What is the initial $^{238}$U decay rate from one gram of $^{238}$U? (b) What is the initial $^{234}$Th decay rate from the same sample?

*Nuclear reactions*

20. (a) A neutron is added to $^{16}$O to make the stable isotope $^{17}$O. Calculate the binding energy of the neutron. (b) Another neutron is added to $^{17}$O to make the stable isotope $^{18}$O. Calculate the binding energy of the neutron.
21. Iron-55 decays by electron capture. (a) What are its end products? Is the resulting nucleus stable? (b) The capture process results in an atomic x ray. Make an estimate of the energy of the x ray. (c) Estimate the radiation dose equivalent (sieverts) that a person receives in 1 hour working 3 meters from a iron-55 source that has an activity of 5 mCi.
22. For each of the following reactions, identify the particle "$X$":

(a) $^{23}\text{Mg} \to e^- + \nu_e + X$, (b) $X \to {}^{186}\text{Os} + \alpha$,
(c) $^{11}\text{C} \to e^+ + \nu_e + X$, (d) $^{12}\text{N} \to {}^{12}\text{C} + \nu_e + X$,
and (e) $p + X \to {}^2\text{H} + e^+ + \nu_e$.

23. Calculate the kinetic energy of an alpha particle from the decay of $^{239}$Pu.
24. The energy ($Q$) released in beta decay is given by

$$Q = M_P c^2 - M_D c^2 - mc^2,$$

where $M_P$ and $M_D$ are the masses of the parent and daughter *nuclei*, and $m$ is the electron or positron mass. If one uses the atomic masses ($M_P'$ and $M_D'$) then show that

$$Q = M_P' c^2 - M_D' c$$

for $\beta^-$ decay, and

$$Q = M_P' c^2 - M_D' c^2 - 2mc^2$$

for $\beta^+$ decay.

25. Why does the proton cycle dominate over the carbon cycle at lower temperatures?

*Nuclear spin*

26. Show that the decay $n \rightarrow pe$ cannot conserve angular momentum.
27. Make a prediction of the spin quantum numbers of the following nuclei (a) $^{12}$C, (b) $^{14}$N, (c) $^{16}$O, and (d) $^{19}$F.

*The Mössbauer effect*

28. Estimate the size of the crystal necessary to cause resonant absorption in $^{57}$Fe.
29. Calculate the source speed that corresponds to change in energy of (a) $10^{-5}$ eV for $^{191}$Ir, and (b) $10^{-8}$ eV for $^{57}$Fe.

*Passage of radiation through matter*

30. Is a 1-curie radioactive source dangerous to handle? Why or why not?
31. A piece of earth one-half meter thick with an area of one square kilometer contains roughly 1 gram of radium, corresponding to an activity of 1 curie. Does this radiation constitute a health hazard?
32. Which is more of a radiation safety problem, radioactive waste with a long half-life or radioactive waste with a short half-life? Why?
33. Can a person absorb a lethal dose of radiation without having a significant change in temperature? Make a rough calculation to support your answer.
34. A person works at a distance of 10 meters from a $^{60}$Co source that is not shielded. Estimate the maximum strength of the source that does not present a radiation hazard to the worker.
35. A person manages to eat 1 milligram of radium. Is this hazardous to the health? Estimate the radiation dose received in one day.
36. Estimate the amount of radium, that if eaten, would double the amount of radiation that a person normally receives from environmental background.
37. A basement room has a size of 250 cubic meters. What mass of $^{222}$Rn in the air will make the room a health-hazard?
38. A physicist wishes to test a transistor for its capability to operate under intense radiation by charged particles. The physicist brings the transistor to an accelerator and places it in an energetic electron beam. About how many electrons does the physicist need to send through the transistor to give it a radiation dose of 10 kilograys?
39. Make an estimate of the radiation dose equivalent received by a person from cosmic rays during one year.
40. The recommended permissible annual oral intake of $^{226}$Ra is about 0.1 µCi. How many $^{226}$Ra atoms is it safe to eat per year?

*Additional problems*

41. The giant "star" shown in Figure 11-24 was made by depositing specks of radium, "in many cases too small to be seen under the microscope," on a photographic plate. (a) Make a rough estimate of the number of radium atoms in a "speck." (b) How many radium nuclei decay from the speck in one day?
42. In the $\beta^+$ decay of $^8$B an energy of 11.15 MeV is released. (a) Calculate the mass of the resulting $^8$C nucleus. (b) What is the binding energy of $^8$C? (c) The $^8$C nucleus is extremely unstable resulting in the following decays:

$$^8\text{C} \rightarrow {^7\text{B}} + p,$$

$$^7\text{B} \rightarrow {^6\text{Be}} + p,$$

and

$$^6\text{Be} \rightarrow {^4\text{He}} + p + p.$$

How much energy is released?