Group Theory HW 7

N.V.1: 1 & 2

N.V.2: 1

N.V.3: 1

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$$I_2 = 0, \quad I_\pi = 1, \quad I_p = \frac{1}{2}, \quad {}^3H: \underbrace{P N}_{D} N + \nu, \quad -\frac{1}{2} \; 0 \to -\frac{1}{2}$$

$$6 = 6$$

$$\overset{2}{He}/\overset{3}{H} \; \pi \quad 4 \quad 2$$

$$\frac{1}{2} \otimes 1 = \frac{3}{2} \oplus \frac{1}{2} \quad \Leftarrow \text{final State}$$

$$|{}^3H + \pi^+\rangle \equiv |1, -\tfrac{1}{2}\rangle$$

$$|{}^3He + \pi^0\rangle \equiv |0, \tfrac{1}{2}\rangle$$

$$\left|j = l + \tfrac{1}{2}, m\right\rangle = \sqrt{\frac{l + m + 1/2}{2l+1}} \left|m - \tfrac{1}{2}, \tfrac{1}{2}\right\rangle + \sqrt{\frac{l - m + \frac{1}{2}}{2l+1}} \left|m + \tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle = \overset{1}{\cancel{\sqrt{\frac{1 + 3/2 + 1/2}{2(1)+1}}}} \left|1, \tfrac{1}{2}\right\rangle + \cancel{\overset{0}{\sqrt{\frac{1 - \frac{3}{2} + \frac{1}{2}}{2(1)+1}}} \left|2, -\tfrac{1}{2}\right\rangle} = \left|1, \tfrac{1}{2}\right\rangle + \varnothing$$

$$\left|\tfrac{3}{2}, \tfrac{1}{2}\right\rangle = \sqrt{\frac{1 + \frac{1}{2} + \frac{1}{2}}{2(1)+1}} \left|0, \tfrac{1}{2}\right\rangle + \sqrt{\frac{1 - \frac{1}{2} + \frac{1}{2}}{2+1}} \left|1, -\tfrac{1}{2}\right\rangle$$

$$= \sqrt{\tfrac{2}{3}} \left|0, \tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}} \left|1, -\tfrac{1}{2}\right\rangle$$

$$\left|j = l - \tfrac{1}{2}, m\right\rangle = -\sqrt{\frac{l - m + \frac{1}{2}}{2l+1}} \left|m - \tfrac{1}{2}, \tfrac{1}{2}\right\rangle + \sqrt{\frac{l + m + \frac{1}{2}}{2l+1}} \left|m + \tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle = -\sqrt{\frac{1 - \frac{1}{2} + \frac{1}{2}}{3}} \left|0, \tfrac{1}{2}\right\rangle + \sqrt{\frac{1 + \frac{1}{2} + \frac{1}{2}}{3}} \left|1, -\tfrac{1}{2}\right\rangle$$

$$\sqrt{\tfrac{1}{3}} \left|0, \tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}} \left|1, -\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle = \left|1,\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{2}{3}}\left|0,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|1,-\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = -\sqrt{\tfrac{1}{3}}\left|0,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\left|1,-\tfrac{1}{2}\right\rangle$$

* conservation of isospin $|0,1\rangle \to \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle$

$$\frac{\left(\sqrt{\tfrac{2}{3}}\right)^2}{\left(-\sqrt{\tfrac{1}{3}}\right)^2} = 2$$

#2 $$\frac{\sigma(\pi^+ + p)}{\sigma(\pi^- + p)} = 3$$

$$\frac{\pi^+ + P \to \pi^+ + P}{\pi^- + P \to \pi^- + P}$$

$\hookrightarrow \pi^0 + n$

— final states

$\left|1,\tfrac{1}{2}\right\rangle$ * only possibility, $A=1$

$\left.\begin{matrix}\left|-1,\tfrac{1}{2}\right\rangle \\ \left|0,-\tfrac{1}{2}\right\rangle\end{matrix}\right\}$ 2 possibilities

$$\left|j=l-\tfrac{1}{2}, m\right\rangle = -\sqrt{\frac{l-m+\frac{1}{2}}{2l+1}}\left|m-\tfrac{1}{2}, \tfrac{1}{2}\right\rangle + \sqrt{\frac{l+m+\frac{1}{2}}{2l+1}}\left|m+\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

$$\left|j=l+\tfrac{1}{2}, m\right\rangle = \sqrt{\frac{l+m+1/2}{2l+1}}\left|m-\tfrac{1}{2}, \tfrac{1}{2}\right\rangle + \sqrt{\frac{l-m+\frac{1}{2}}{2l+1}}\left|m+\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

from Q,

$$\left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle = \left|1, \tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{3}{2}; \tfrac{1}{2}\right\rangle = \sqrt{\tfrac{2}{3}}\left|0; \tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|-1, \tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{1}{2}; \tfrac{1}{2}\right\rangle = -\sqrt{\tfrac{1}{3}}\left|0, \tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\left|-1, \tfrac{1}{2}\right\rangle$$

$$\overset{\pi^-}{} \quad \overset{p}{}$$

$$\left|-1, \tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

$$\pi^0, \; n$$

$$\left|0, -\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{2}{3}}\left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

$$\langle \pi^+ p | \tau | \pi^+ p \rangle = 1^2 \left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle$$

$$\langle \pi^- p | \tau | \pi^- p \rangle = \tfrac{1}{3}\left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle - \tfrac{2}{3}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

$$\langle \pi^- p | \tau | \pi^0 n \rangle = \tfrac{2}{3}\left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle + \tfrac{1}{3}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

$$\frac{1\,\left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle}{\left(\frac{2}{3}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle+\frac{1}{3}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle\right)+\left(\frac{1}{3}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle-\frac{2}{3}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle\right)}$$

$$\cancel{\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle} \quad -\frac{1}{3}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$

ignore this

Expected Value

$$\frac{1}{1/3}=3$$

N.V.2:

#1 SU(3) irrep (4, 0)

$$D(m,n)=\frac{1}{4}(m+1)(m+2)(n+1)(n+2)$$

$$-\frac{1}{4}m(m+1)n(n+1)$$

$$\frac{1}{4}(4+1)(4+2)(1)(2)$$

$$\cancel{-\frac{1}{4}4(4+1)0(1)}$$

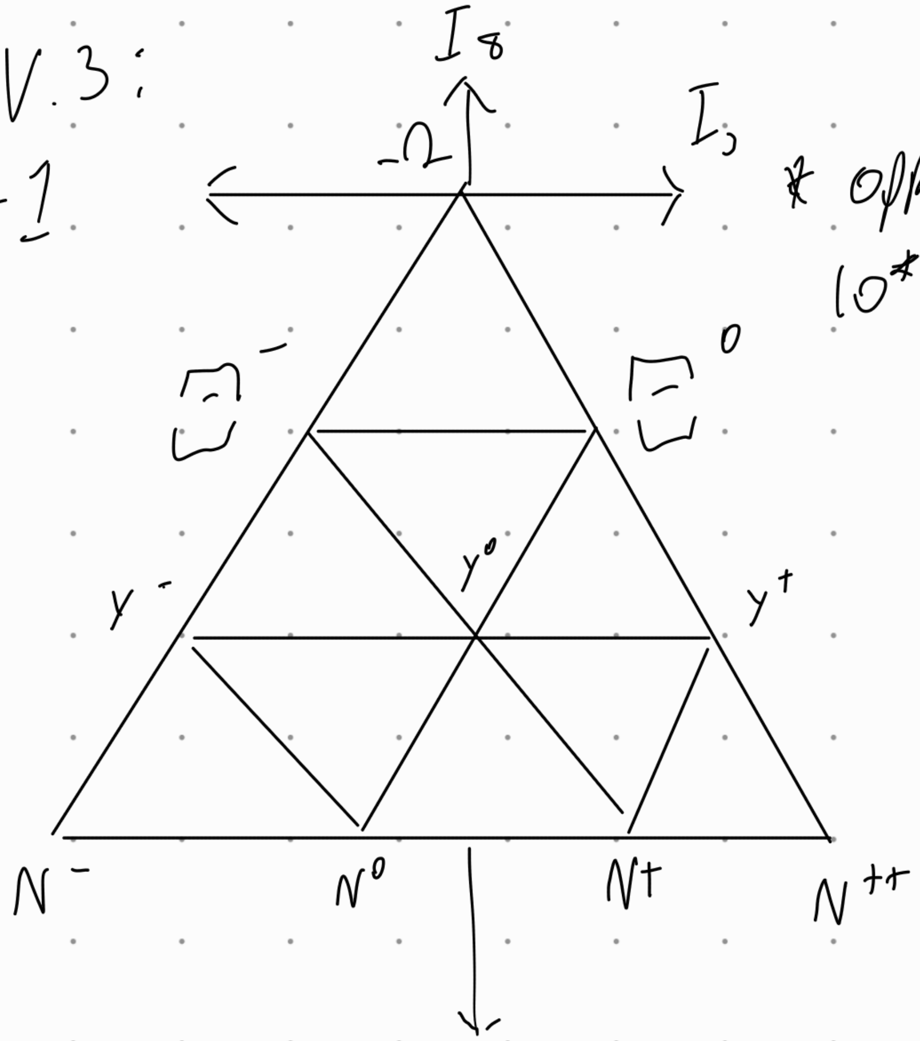
$$\frac{1}{4}(5)(6)(2)$$

$$\frac{1}{2}(30)=15.$$

Solving for all 15 d irreps w/ a numerical solver, we also get <u>(1, 2) as an</u> irreducible representation

N.V.3:

#1



* opposite of 10*