

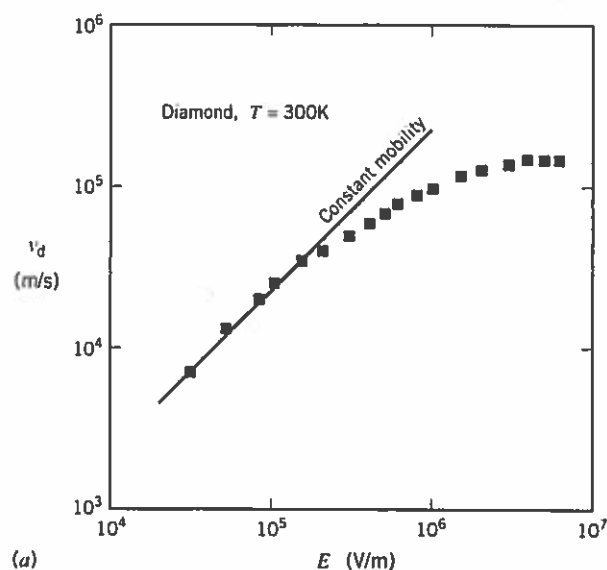


**CONDUCTORS,
INSULATORS, AND
SEMICONDUCTORS**

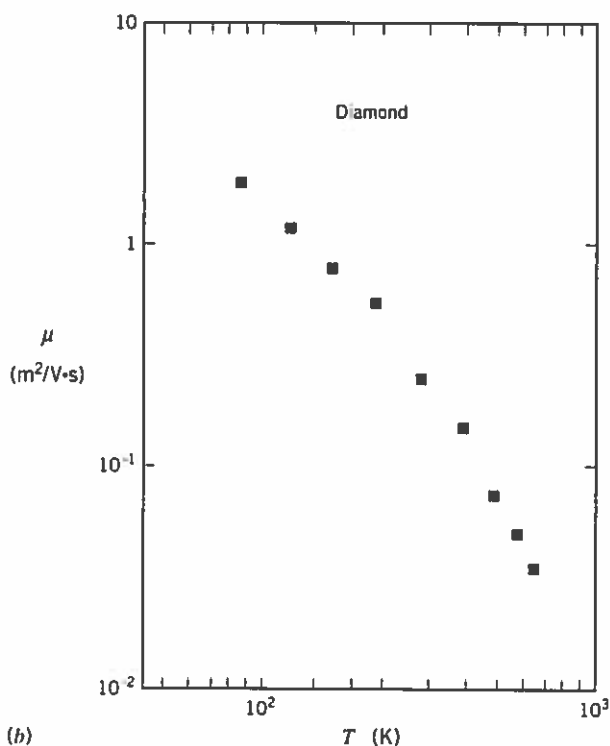
The manufacture of glass, along with the forming of metals, is an art that goes back to prehistoric times. It always seems to me remarkable that our first understanding of the ductility of metals in terms of atomic movements came *after* the discovery of the neutron... The years that passed before anyone tried to get a theoretical understanding of electrons in glass surprises me even more. After all, the striking thing about glass is that it is transparent, and that one does not have to use particularly pure materials to make it so. But, in terms of modern solid-state physics, what does "transparent" mean? It means that, in the energy spectrum of electrons in the material, there is a gap of forbidden energies between the occupied states (the valence band) and the empty states (the conduction band); light quanta corresponding to a visible wavelength do not have the energy needed to make electrons jump across it. This gap is quite a sophisticated concept, entirely dependent on quantum mechanics...

Nevill F. Mott

14-1 ELECTRONIC ENERGY BANDS**14-2 FERMI ENERGY****14-3 HEAT CAPACITY****14-4 OHM'S LAW****14-5 SEMICONDUCTORS****14-6 THE HALL EFFECT**



(a)



(b)

FIGURE 14-13 Electron drift speed in the insulator diamond.

(a) Electron drift speed (v_d) measured as a function of electric field strength (E). (b) The mobility ($\mu = v_d/E$) measured as a function of temperature. The measurements were taken at small values of electric field where the mobility does not depend on E . After F. Nava et al., *Solid State Com.* 33, 475 (1980).

As a final remark we point out that in our derivation of Ohm's law we have also assumed that the number of conduction electrons remains fixed. This is true for a metal but not for an insulator at large E because additional electrons enter the conduction band. Therefore, in an insulator we may write the current density as

$$J = n(E)ev_d(E). \quad (14.70)$$

14-5 SEMICONDUCTORS

Intrinsic Semiconductors

An intrinsic semiconductor is an element in which the energy gap between the outermost filled band (called the *valence band*) and the next highest band is small compared to that of an insulator. The family of intrinsic semiconductors is made up of the first four elements in Group IV of the periodic table (see Figure 14-14). Silicon ($Z = 14$) is the most common intrinsic semiconductor. Its electron configuration is $1s^2 2s^2 2p^6 3s^2 3p^2$. In silicon, the $3s$ and $3p$ bands overlap to make a filled outer band (4 electrons per atom) with an energy gap of about 1.1 eV. The band structure of silicon is shown in Figure 14-15.

p-dopants	Intrinsic semiconductors	n-dopants
5 B $2s^2 2p^1$	6 C $2s^2 2p^2$ $E_g = 5.5 \text{ eV}$	7 N $2s^2 2p^3$
13 Al $3s^2 3p^1$	14 Si $3s^2 3p^2$ $E_g = 1.1 \text{ eV}$	15 P $3s^2 3p^3$
31 Ga $4s^2 4p^1$	32 Ge $4s^2 4p^2$ $E_g = 0.7 \text{ eV}$	33 As $4s^2 4p^3$
49 In $5s^2 5p^1$	50 Sn $5s^2 5p^2$ $E_g = 0.1 \text{ eV}$	51 Sb $5s^2 5p^3$

FIGURE 14-14 The intrinsic semiconductors diamond, silicon, germanium, and tin together with their neighboring elements in the periodic table.

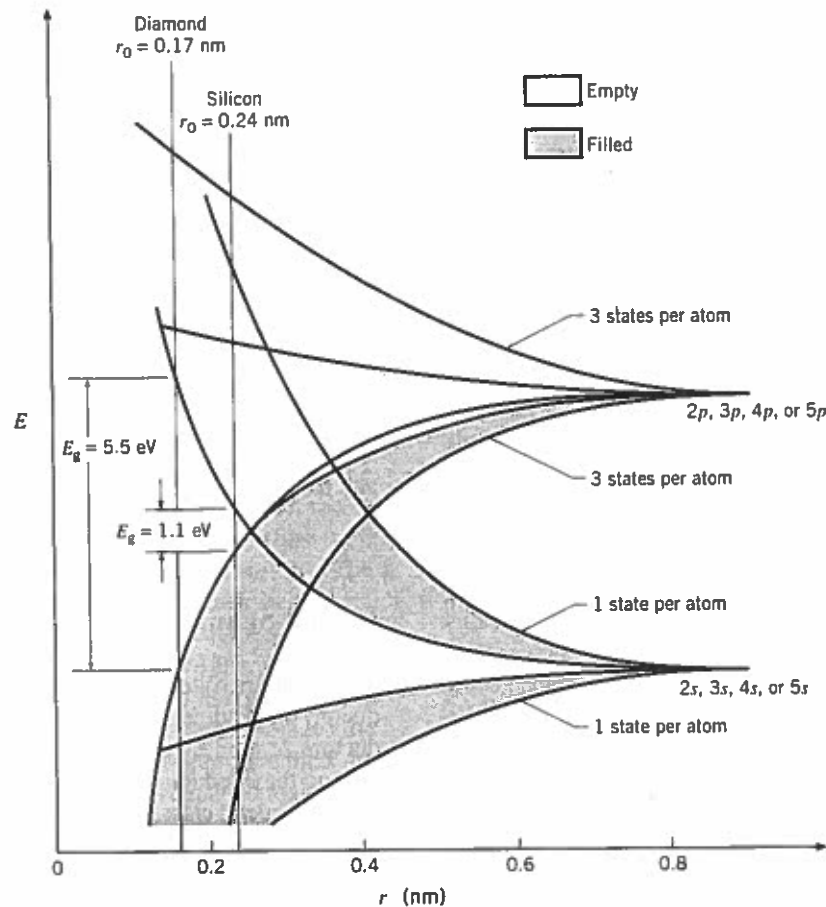


FIGURE 14-15 Band structure of the intrinsic semiconductor.

After R. L. Sproull and W. A. Phillips, *Modern Physics: The Quantum Physics of Atoms, Solids, and Nuclei*, Wiley (1980).

Carbon ($Z = 6$), germanium ($Z = 32$), and tin ($Z = 50$) have the same outer electron configurations as silicon, except the outer shells correspond to $n = 2$, $n = 4$, and $n = 5$. The crystal structure of silicon is a diamond lattice (see Figures 14-2 and 14-3). The band structures of diamond, germanium, and tin are similar to that of silicon except that the spacing between atoms is smaller in diamond and larger in germanium and tin. This leads to an energy gap of 5.5 eV in diamond, 0.7 eV in germanium, and 0.1 eV in tin.

We should point out one subtle feature of the energy gap. The maximum of the valence band need not coincide with the minimum of the conduction band. This is illustrated in Figure 14-16 which shows a plot of electron energy versus wave number (see Figure 14-5). If the maximum of the valence band coincides with the minimum of the conduction band as in Figure 14-16a, an

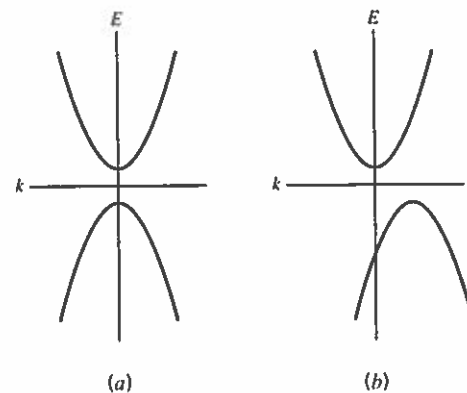


FIGURE 14-16 Semiconductor band gaps.

(a) Energy versus wave number for a semiconductor in which the minimum of the conduction band coincides with the maximum of the valence band. (b) The valence and conduction bands do not coincide.

electron can cross the gap with the absorption of a photon of energy E_g , a *direct transition*. If the valence and conduction bands do not coincide as in Figure 14-16b, an electron can still absorb a photon and cross the gap provided that the appropriate momentum is supplied by a phonon, an *indirect transition*. Since the phonon also has energy of order of magnitude kT_D , the indirect transitions are caused by photons with energies smaller than the energy gap. This effect can be especially important for those semiconductors with small band gaps.

In an intrinsic semiconductor, the band gap is small enough that some electrons are thermally excited to the conduction band, leaving unfilled states, or *holes*, in the valence band. Therefore, the conductivity of a semiconductor increases with temperature.

EXAMPLE 14-9

Make an estimate of the density of electrons in the conduction band in diamond, silicon, germanium, and tin at room temperature.

SOLUTION:

The occupation probability for an electron to be in the conduction band (see Example 14-2) is given by the Maxwell-Boltzmann factor

$$P = e^{-\Delta E/kT},$$

where

$$\frac{\Delta E}{kT} = \frac{E_g}{2kT} \approx \frac{20 E_g}{eV}.$$

The probability that an electron is excited to the conduction band is for diamond ($E_g = 5.5$ eV)

$$P \approx e^{-\Delta E/kT} \approx e^{-(20)(5.5)} \approx e^{-110} \approx 10^{-48}.$$

for silicon ($\Delta E = 1.1$ eV)

$$P \approx e^{-(20)(1.1)} \approx e^{-22} \approx 10^{-10},$$

for germanium ($\Delta E = 0.7$ eV)

$$P \approx e^{-(20)(0.7)} \approx e^{-14} \approx 10^{-6},$$

and for tin

$$P \approx e^{-(20)(0.1)} \approx e^{-2} \approx 10^{-1}.$$

We may estimate that about one electron per atom is near the top of the outer band. The density of these electrons is

$$n \approx \frac{1}{(0.2 \text{ nm})^3} \approx 10^{29} \text{ m}^{-3}.$$

The density of conduction electrons (n_c) is for diamond

$$n_c \approx nP \approx (10^{-48})(10^{29} \text{ m}^{-3}) = 10^{-19} \text{ m}^{-3},$$

for silicon

$$n_c \approx nP \approx (10^{-10})(10^{29} \text{ m}^{-3}) = 10^{19} \text{ m}^{-3},$$

for germanium

$$n_c \approx nP \approx (10^{-6})(10^{29} \text{ m}^{-3}) = 10^{23} \text{ m}^{-3},$$

and for tin

$$n_c \approx nP \approx (10^{-1})(10^{29} \text{ m}^{-3}) = 10^{28} \text{ m}^{-3}. \quad \blacksquare$$

Doped Semiconductors

When impurity atoms are added to an intrinsic semiconductor, the resulting material is called a *doped semiconductor*. A modest doping fraction of 10^{-6} can greatly change the conduction properties of the material. There are two general classes of doped semiconductors as illustrated in Figure 14-17.

In an *n-type* (n for negative) doped semiconductor, the impurity atoms have one more outer electron than the intrinsic semiconductor. An example is germanium doped with arsenic (see Figure 14-14). The arsenic atoms occupy random positions in the crystal lattice. The energy levels of the arsenic atoms are slightly different from germanium. The outer electrons from arsenic reside in energy levels which are slightly below the conduction band. With a small amount of energy from thermal excitations, electrons can be boosted into the conduction band without leaving unfilled states (holes) in the valence band.

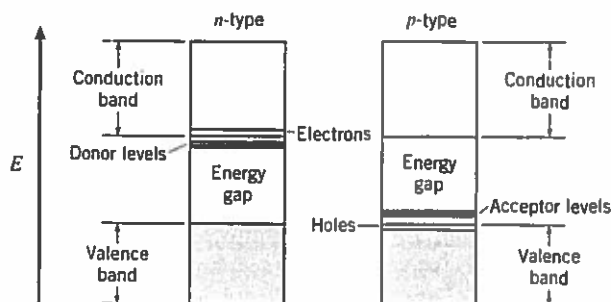


FIGURE 14-17 Energy bands in doped semiconductors.

In a *p*-type (*p* for positive) doped semiconductor, the impurity atoms have one less outer electron than the intrinsic semiconductor. An example is germanium doped with gallium (see Figure 14-14). The gallium atoms occupy random positions in the crystal lattice and leave holes in energy levels which are just above the valence band. Electrons can be thermally excited from the valence band. This creates holes in the valence band without putting electrons in the conduction band.

The *p*-*n* Junction

If we put a *p*-type and an *n*-type material in contact with each other, we create a *p*-*n* junction. An electrical device containing such a junction is called a diode. Some of the electrons near the junction boundary diffuse from the *n*-region into the *p*-region and some of the holes diffuse from the *p*-region into the *n*-region (see Figure 14-18). The movement of these mobile charges leaves behind an excess of fixed positive charge in the *n*-region near the boundary and an excess of fixed negative charge in the *p*-region near the boundary. These fixed charges create a strong electric field (10^6 – 10^8 V/m) that serves to keep the region free of mobile charge. This region is called the *depleted* region. The thickness of the depleted region is a few microns. A potential difference (V_0), called the *contact potential*, is created across the boundary.

The net current across a junction, after the initial movement that creates the contact potential, is zero. This current consists of four components, electrons and holes that can each travel in both directions. Let the current due to electrons moving from the *n*-region to the *p*-region be called I_n . The number of electrons traveling in the opposite

direction, from the *p*-region to the *n*-region, creates a current $-I_n$. Let the current due to holes moving from the *p*-region to the *n*-region be called I_p . The number of holes traveling in the opposite direction, from the *n*-region to the *p*-region, creates a current $-I_p$. Equilibrium is established and no net current is flowing across the junction:

$$I = I_n - I_n + I_p - I_p = 0. \quad (14.71)$$

Figure 14-19a shows an energy level diagram for a *p*-*n* junction. The Fermi energies for the *p*-type and the *n*-type coincide. The *p*-type region contains mobile holes at the top of the valence band that are kept from crossing into the *n*-type region by the contact potential. Similarly, the *n*-type material contains mobile electrons at the bottom of the conduction band that are kept from crossing into the *p*-type region. The contact potential and energy gap are related by $eV_0 = E_g$.

If we connect an external voltage source to the junction with the polarity selected such that an electric field is generated in the same direction as the field caused by the contact potential, this is called *reverse bias* (see Figure 14-19b). In reverse bias, the negative terminal of the voltage source is connected to the *p*-region and the positive terminal to the *n*-region. Since resistance to charge flow in the semiconductor is predominantly at the junction, the external applied voltage will appear at the junction. This destroys the equilibrium condition by making it essentially impossible for charges to cross the barrier in the direction opposing the potential, corresponding to electrons going from the *n*-region to the *p*-region or vice versa for holes. Mobile charges can still diffuse across in the other direction, creating a net current

$$I = -(I_n + I_p). \quad (14.72)$$

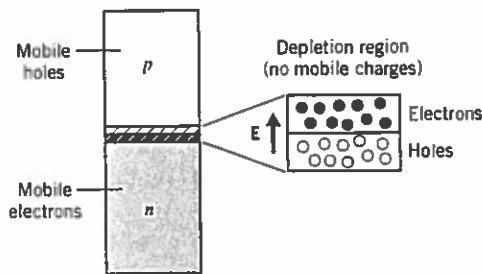


FIGURE 14-18 The *p*-*n* junction.

The depletion region is a few microns thick and contains the fixed charges left over after electrons diffuse from the *n*-region to the *p*-region and holes from the *p*-region to the *n*-region. These charges create an electric field that prevents further diffusion of charge.

This current is independent of the size of the reverse bias voltage and is relatively small because there are very few mobile electrons in the *p*-region or mobile holes in the *n*-region. If the reverse-bias voltage is made large enough, electrons that are not normally mobile because they are bound in the atoms may be freed, causing an avalanche breakdown.

If we connect an external battery such that the applied electric field is opposite that due to the contact potential, this serves to help charges cross the barrier (see Figure 14-19c). This is called *forward bias*. To achieve a forward bias, the positive voltage terminal is connected to the *p*-type semiconductor and the negative to the *n*-type. The potential of the external battery will appear as a lowering

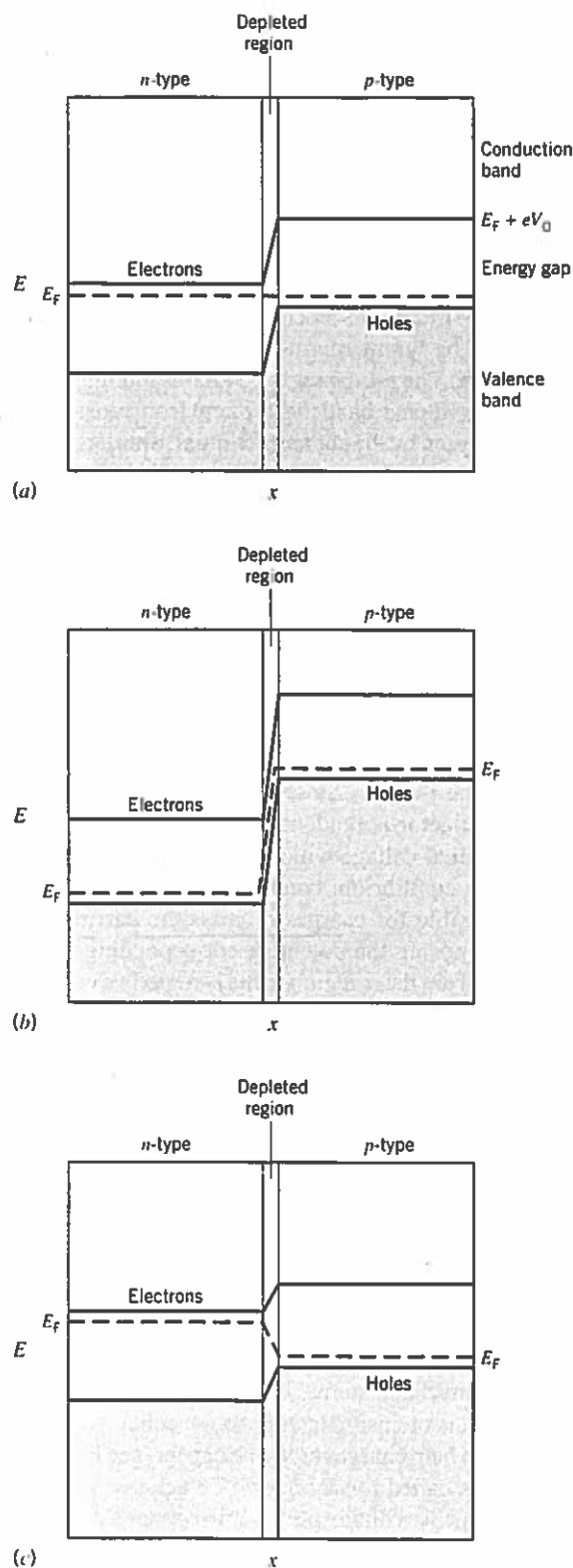


Figure 14-19 Energy levels of electrons and holes in an p - n junction.

(a) No external voltage applied, (b) reverse bias, and (c) forward bias.

of the contact potential of the junction. The effect of reducing the contact potential allows current to flow.

The mobile charge carriers, because they are the most energetic, are described by the Maxwell-Boltzmann distribution. A charge contributes to the current if it has enough energy to cross the barrier. The current due to the mobile electrons crossing from the n -region to the p -region plus that of mobile holes crossing from the p -region to the n -region is given by

$$I_1 = (I_n + I_p) e^{eV/kT}. \quad (14.73)$$

The current due to mobile electrons that have diffused to the p -region and now cross back to the n -region does not depend on the junction voltage because it is energetically favorable for the charges to cross the junction. The same is true for the mobile holes that have diffused to the n -region and cross back to the p -region. This part of the current is just what it was before the external voltage was applied:

$$I_2 = -(I_n + I_p). \quad (14.74)$$

The net current is

$$I = I_1 + I_2 = (I_n + I_p) (e^{eV/kT} - 1). \quad (14.75)$$

The current-voltage characteristics of a diode are shown in Figure 14-20.

Transistors

The *bipolar junction transistor* (BJT) consists of three semiconductor layers of alternating types, either n - p - n or p - n - p . Transistors are capable of power amplification and as such have many uses. The transistor was invented in 1948 by John Bardeen, Walter H. Brattain, and William Shockley (see Figure 14-21).

The physics of the transistor may be understood by analyzing what happens to the electrons (or holes) at each junction. Figure 14-22 shows an energy band diagram for an n - p - n transistor. The three regions are called the emitter, base, and collector. When no external voltage is present (Figure 14-22a), there is an energy barrier between the p -type and n -type regions, as discussed earlier.

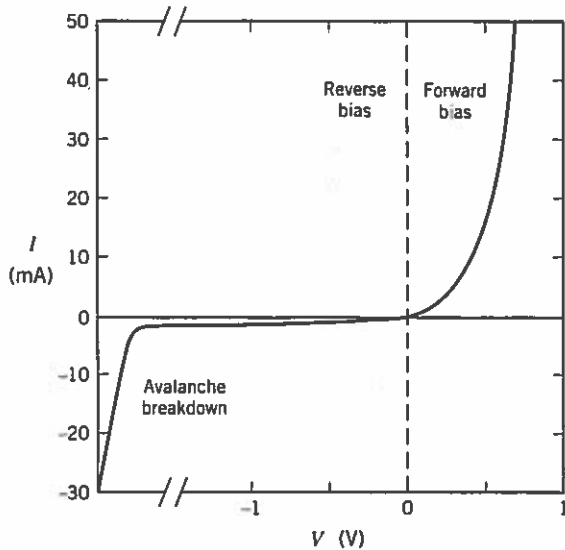


FIGURE 14-20 Current versus voltage in a p - n junction.

This barrier prevents the flow of electrons between the two n -regions. In order to function as an amplifier, the transistor must be biased with external power supplies. These power supplies will provide the energy source for the power amplification. Figure 14-22b shows an energy band diagram for an n - p - n transistor that is connected to external voltage sources. The base-emitter junction is forward biased so that this energy barrier is reduced. The base-collector junction is reverse biased so that this energy barrier is raised. Electrons are ready to travel from the emitter to the collector, but they are hindered from doing so because of the potential barrier in the base. This barrier

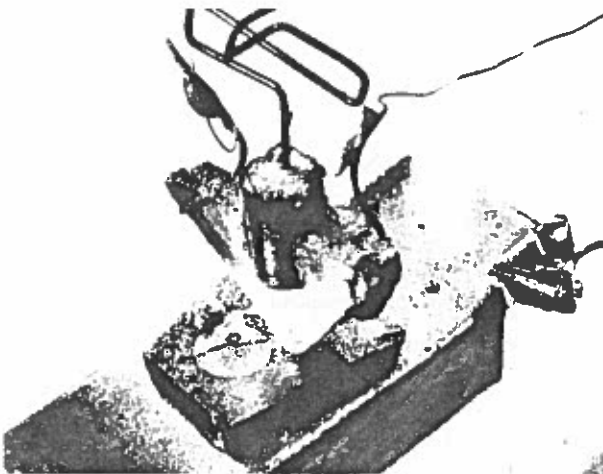


FIGURE 14-21 The first transistor.
Courtesy A. T. & T. Bell Laboratories, reprinted by permission.

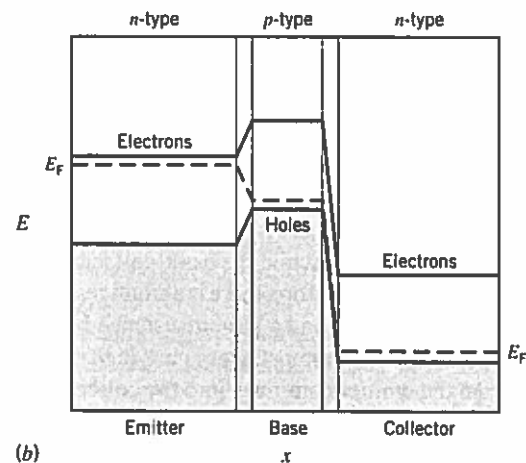
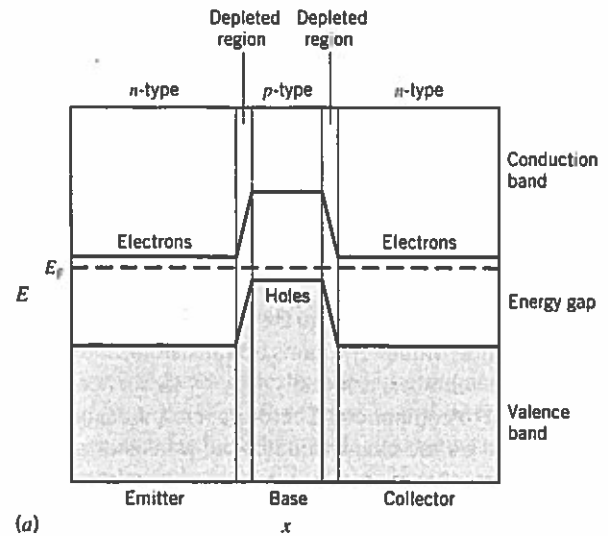


FIGURE 14-22 The n - p - n junction.

(a) No external voltage and (b) emitter-base forward biased and base-collector reverse biased.

is made up of negative charge that has accumulated in the base. If electrons are allowed a path to escape from the base, then the emitter-base barrier is lowered and a small change in height of the barrier allows a large amount of charge to flow across it. A small base-emitter current causes a large emitter-collector current (see Figure 14-23). The n - p - n transistor is designed such that essentially all of the electrons from the emitter that enter the base pass through it into the collector. This is accomplished by making the base very thin and lightly p -doped while the emitter is heavily n -doped. The collector is usually lightly n -doped. The ratio (α) of collector (I_c) to emitter currents (I_e) is of order unity,

$$\alpha = \frac{I_c}{I_e} \approx 1. \quad (14.76)$$

The ratio of collector to base (I_b) currents is called the *gain* (β) of the transistor,

$$\beta = \frac{I_c}{I_b}. \quad (14.77)$$

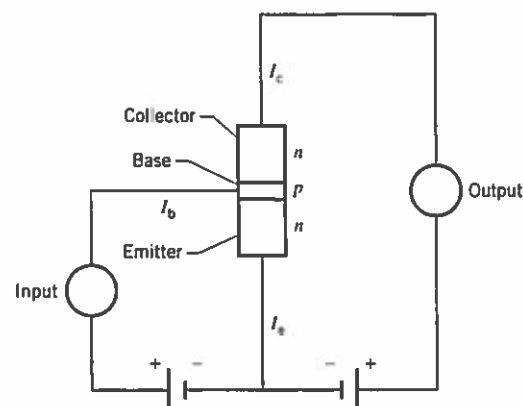
The numerical value of β is in the range 10–1000 with 100 being a typical value. The gains of transistors, even of the same model number, can easily vary by factors of 2 from component to component. Therefore, circuit design must not depend on the exact value of the transistor gain. In addition, every transistor has current and voltage limits beyond which it will “burn out.”

A p - n - p transistor consists of a heavily p -doped emitter and a lightly p -doped collector separated by a lightly n -doped base. The operation of a p - n - p transistor is similar to that of an n - p - n transistor except that the bias voltages are reversed and the current is dominated by the motion of holes.

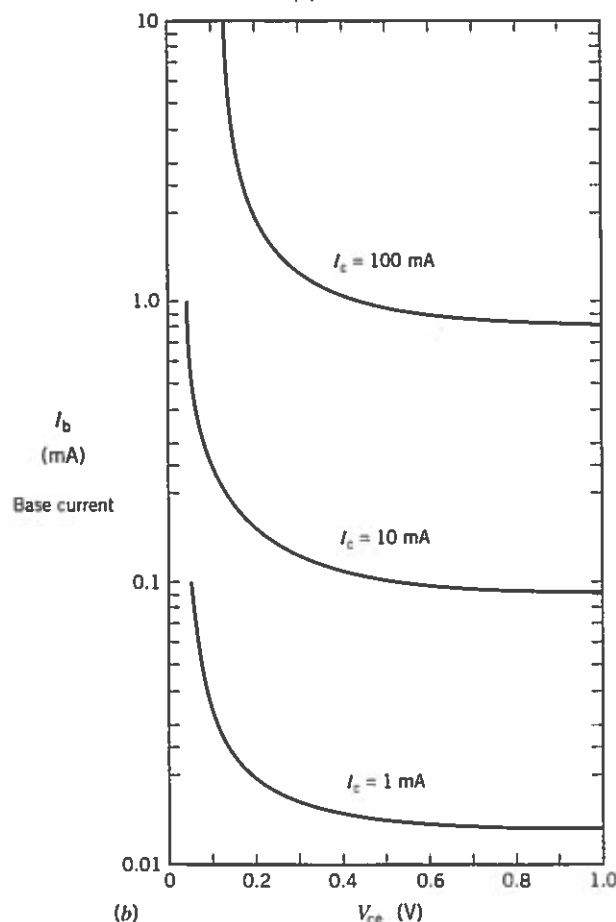
The normal operation of the transistor may be summarized by noting that a small current in the base controls a large current through the collector. The base-emitter junction is forward-biased while the base-collector junction is reverse biased. Therefore, there is a small resistance between emitter and base and a large resistance between collector and base. Since almost every charge that enters the base from the emitter continues into the collector, the current across each junction is the same. Since the collector junction has higher resistance, it has a higher power ($P = I^2R$), and the transistor serves as a power amplifier. The current-voltage characteristics of a common transistor are shown in Figure 14-23b. To operate the transistor as an amplifier, a voltage of about 0.5 V is maintained between the collector and emitter (V_{ce}). The collector current (I_c) is then about 100 times the base current (I_b).

Field-Effect Transistors

The collector current in a bipolar transistor is controlled by the base current. In another type of transistor, the *field-effect transistor* (FET), current is controlled by an electric field. There are two main categories of FETs, the junction field-effect transistor (JFET) and the metal-oxide-semiconductor field-effect transistor (MOSFET). Both the JFET and the MOSFET come in two varieties, p -channel



(a)



(b)

FIGURE 14-23 The transistor as an amplifier. (a) Circuit diagram for an n - p - n transistor in which the base-emitter junction is forward biased and the base-collector junction is reverse biased. (b) Current-voltage characteristics of a typical transistor amplifier. For a given collector-emitter voltage (e.g., $V_{ce} = 0.5$ V), the collector current is approximately 100 times the base current.

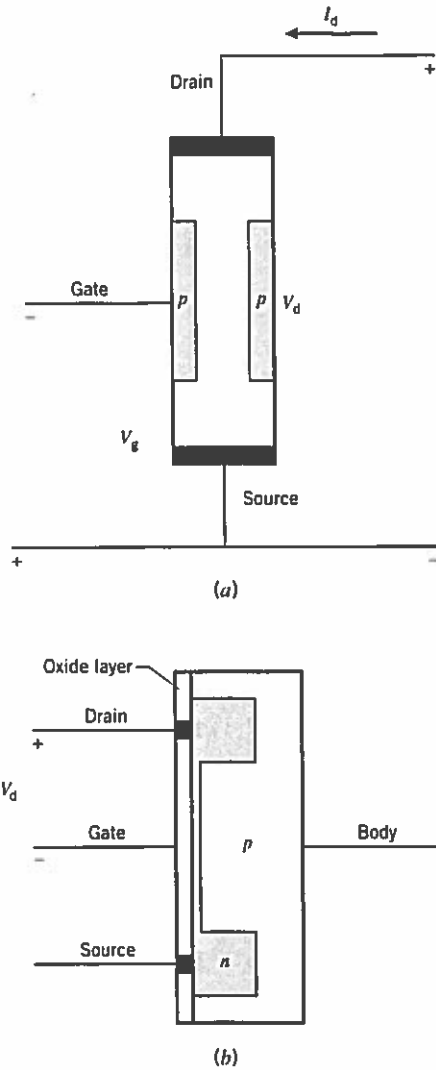


FIGURE 14-24 The field-effect transistor. (a) JFET, and (b) MOSFET.

and n -channel, analogous to p - n - p and n - p - n bipolar transistors. The n -channel JFET is shown in Figure 14-24a. The JFET consists of a semiconductor bar, most commonly silicon, with electrical contacts, called the source and the drain, on each end. The middle of the bar contains a second semiconductor with the opposite doping, called the *gate*. The gate is designed to draw essentially no current. The gate-source is reversed biased, creating an electric field in the semiconductor bar. The term *gate* arises because the amount of voltage bias determines the size of the energy gap that the mobile charges pass through. The three terminals gate, source,

and drain are analogous to the base, emitter, and collector of the bipolar transistor.

The MOSFET differs from the JFET in that the gate consists of a metal electrode that is insulated from the semiconductor by a layer of glass (SiO_2). These devices are sometimes referred to as insulated-gate FETs (IGFETs). The n -channel MOSFET is shown in Figure 14-24b.

14-6 THE HALL EFFECT

Consider a conductive sheet placed in a magnetic field as shown in Figure 14-25. A voltage is applied to the sheet causing a current in the x direction. The current may be due to either mobile electrons (Figure 14-25a) or mobile holes (Figure 25b). For the case of mobile electrons drifting in the negative x direction, there is a magnetic force (F_m),

$$\mathbf{F}_m = -ev_d \times \mathbf{B}, \quad (14.78)$$

on electrons in the negative y direction. This magnetic force causes an accumulation of negative charge on the lower edge of the sheet (Figure 14-25a). This charge generates an electric force (F_e) on the drifting electrons that just cancels the magnetic force. This is called the *Hall effect* after Edwin H. Hall, who first observed it in 1879. If the current is due to mobile holes (Figure 14-25b), then the magnetic force on the holes is in the negative y direction and positive charge accumulates on the lower edge of the sheet. The magnitudes of the forces are

$$F_e = eE = F_m = ev_d B. \quad (14.79)$$

The Hall voltage (V_H) is defined to be the potential difference across the width of the sheet from the accumulated charges:

$$V_H = Ey = v_d By, \quad (14.80)$$

where y is the width of the sheet. The Hall voltage is very small ($V_H \ll 1$ V) even for large magnetic fields because the electron drift speed is small. The polarity of the Hall voltage is determined by the sign of the mobile charges (electrons or holes). Measurement of the Hall voltage provides a determination of the drift velocity. If the current density ($J = nev_d$) is also measured, then measurement of the Hall voltage provides a determination of the density of mobile charges:

$$n = \frac{J}{ev_d} = \frac{JBy}{eV_H}. \quad (14.81)$$

energy. Estimate the number of pulses needed to raise the temperature of the copper target by 10 kelvin. You may neglect the cooling of the target between pulses.

12. What is the typical energy of a phonon in a copper crystal at room temperature? What is the typical energy of a conduction electron?
13. Aluminum has a melting point of 934 kelvin and a density of $2.7 \times 10^3 \text{ kg/m}^3$. Use the data of Figure 14-10 to estimate the Debye temperature of aluminum.

Ohm's law

14. The mean free path of electrons in a very pure sample of copper at 4 kelvin is about 3 millimeters. Calculate the resistivity of the sample.
15. Calculate the mobility of electrons in copper at room temperature. Take the conductivity to be $5.9 \times 10^7 \Omega^{-1} \cdot \text{m}^{-1}$. Compare the mobility of copper with that of diamond at room temperature (see Figure 14-13).
16. Make a rough sketch of the current density (14.70) as a function of electric field in an insulator. Indicate the region where Ohm's law holds and the effect of a saturating drift speed. Assume that at some very large value of electric field that the number of conduction electrons grows exponentially.
17. Use the measured mobility in diamond (Figure 14-13b) to calculate the relaxation time at room temperature.
18. The mobility of silicon at room temperature is measured to be $0.19 \text{ m}^2/\text{V} \cdot \text{s}$. (a) Calculate the relaxation time. (b) Determine the drift speed when an electric field of 10^3 V/m is applied.

Semiconductors

19. The next element after tin in Group IV of the periodic table is lead. Why is lead not an intrinsic semiconductor?
20. Estimate the temperature where the density of conduction electrons in undoped germanium is 10^{15} per cubic meter.
21. Why doesn't Ohm's law hold for conduction in a diode?
22. Why is diamond transparent to visible light whereas a silicon crystal is not?
23. When a germanium crystal is doped with indium, what type of semiconductor results?
24. Calculate the ratio of forward to reverse current in a diode at room temperature for an applied voltage of plus or minus 1 volt.

25. Draw an energy level diagram similar to Figure 14-21 for a $p-n-p$ transistor that is (a) unbiased and (b) biased for operation as an amplifier.
26. The conductivity of a sample of germanium is measured to be $2 \Omega^{-1} \cdot \text{m}^{-1}$ at room temperature. Give an expression for the conductivity as a function of temperature (T) for $T > 300 \text{ K}$. What is the conductivity at $T = 1000 \text{ K}$?

The Hall effect

27. Why do you think it took 101 years from the time the Hall effect was discovered until the quantized Hall effect was discovered?
28. Why was the discovery of the quantized Hall effect made at low temperature?
29. Silver has a density of $1.05 \times 10^4 \text{ kg/m}^3$ and an atomic number of 108. (a) Calculate the density of conduction electrons. (b) A current density of 10^6 A/m^2 flows in a thin sheet of width 10^{-2} m . Determine the strength of magnetic field needed to generate a Hall voltage of $1 \mu\text{V}$.
30. Von Klitzing, Dorda, and Pepper (*Phys. Rev. Lett.* 45 494, 1980) measured the $n = 4$ quantized Hall resistance with high accuracy and obtained a value of $R_H = 6453.17 \pm 0.02 \Omega$. With what accuracy did they determine the electromagnetic coupling constant, α ?
31. Tsui, Störmer, and Gossard (*Phys. Rev. Lett.* 48 1559, 1982) discovered the fractional quantum Hall effect for filling factors of $\nu = 1/3$ and $2/3$ in a device that had a density of charge carriers equal to $1.23 \times 10^{11} \text{ m}^{-2}$. Calculate the values of magnetic field where the FQHE was observed.
32. (a) Prove that the filling factor is dimensionless. (b) A quantum Hall device is designed to have a filling factor of one at $B = 10$ tesla. What is the charge carrier concentration?

Additional problems

33. Calculate the average energy of a conduction electron in gold.
34. What type of semiconductor (p or n) is obtained when silicon is doped with (a) antimony, (b) phosphorous, (c) aluminum, and (d) indium?
35. The longest wavelength that a certain semiconductor can absorb is $1.85 \mu\text{m}$. Calculate the energy gap of the semiconductor.
36. The Fermi temperature (T_F) is defined by

$$E_F \equiv kT_F.$$

Calculate the Fermi temperature for silver ($E_F = 5.48$ eV). What is the physical significance of the Fermi temperature?

- *37. (a) For what approximate value of the electron density does the nonrelativistic expression for the Fermi energy break down? (b) Derive an expression for the Fermi energy of a gas of relativistic electrons, in terms of the electron density. (c) A star of mass 2×10^{30} kg collapses to a radius of 10^4 m. Estimate the Fermi energy of the electrons in the collapsed star.

- *38. *Thermal conductivity and the Wiedemann-Franz law.* (a) Consider the flow of charge q in a metal rod of gradient cross-sectional area A when a voltage V is applied. Show that Ohm's law may be written

$$\frac{dq}{dt} = \sigma A \frac{dV}{dx}.$$

(b) The heat flow in the rod is described with a similar equation. If energy in the form of heat (Q) is supplied at one end of the rod, there will be a temperature gradient (dT/dx) along the length of the rod. The rate at which energy is transferred to the other end of the rod (dQ/dt) may be written

$$\frac{dQ}{dt} = \kappa A \frac{dT}{dx},$$

where the constant of proportionality (κ) is called the *thermal conductivity*. Show that for an ideal gas the thermal conductivity may be written

$$\kappa = \frac{C_v \langle v \rangle d}{3V},$$

where $\langle v \rangle$ is the average speed of a gas molecule, d is the mean free path and V is the volume. (c) Calculate κ for a metal by treating the electrons as a gas and show that

$$\frac{\kappa}{\sigma T} = \frac{\pi^2 k^2}{3e^2} = 2.45 \times 10^{-8} \text{ W} \cdot \Omega / \text{K}^2.$$

39. *Light-emitting diode (LED).* Consider a heavily doped p - n junction that is forward biased so that a large number of electrons and holes diffuse into the junction region. Electrons can combine with the holes. (a)

How much energy is released when an electron falls into a hole? (b) In some semiconductors, such as cadmium selenide, the electron-hole combining process occurs with the emission of a photon. Calculate the wavelength of the photons that are emitted if the band gap in CdSe is 1.8 eV. (c) In other semiconductors, such as silicon and germanium, the energy from the electron-hole combining process does not produce photons. What happens to the energy?

- *40. *The photovoltaic effect.* When light is incident on a p - n junction, photons can create electron-hole pairs. The electrons are able to move a distance d_n and holes are able to move a distance d_p before recombining. (a) Photons incident on the junction create electron-hole pairs at a rate R . Calculate the current density in a forward-biased junction. (b) Calculate the current density in a reverse-biased junction as a function of R . (c) For illumination of an open-circuit diode, a voltage will build up between the terminals. Calculate the voltage as a function of R . (d) What is the maximum possible voltage? (e) Gallium arsenide, which has a band gap of 1.4 eV, is well suited for conversion of solar energy to electrical energy. Why is GaAs better than germanium for solar power conversion?

- *41. *The Richardson-Dushman equation.* Consider the escape of electrons from the surface of a metal. Let the x direction be normal to the surface. (a) Show that the condition for electrons to escape is that the x component of momentum be greater than some critical value (p_c) given by

$$p_c = \sqrt{2m(E_F + \phi)},$$

where ϕ is the work function of the metal. (b) Writing the current density (J) in terms of the number of electrons per volume (n) and making use of the density of states for electrons in a metal, show that

$$J = \frac{2e}{h^3 m} \int_{p_c}^{\infty} dp_x p_x \int_{-\infty}^{\infty} dp_y \int_{-\infty}^{\infty} dp_z \times \frac{1}{e^{(p_x^2 + p_y^2 + p_z^2)/2mkT - E_F/2mkT} + 1}.$$

- (c) Use the fact that $\phi \gg kT$ to show that

$$J \approx \left(\frac{4\pi me}{h^3} \right) (kT)^2 e^{-\phi/kT}.$$