HW: 5

N. IV. 3 #1

Work out example c in detail

$j=2 \quad j'=1$

IV.2.8: $j=1$ $\quad j_+|-1\rangle = \sqrt{2}|0\rangle,\ j_+|0\rangle = \sqrt{2}|1\rangle$

$j_-|1\rangle = \sqrt{2}|0\rangle,\ j_-|0\rangle = \sqrt{2}|-1\rangle$

IV.2.4 $\quad j=2$

$J_+|-2\rangle = 2|-1\rangle,\ J_+|-1\rangle = \sqrt{6}|0\rangle,\ J_+|0\rangle = \sqrt{6}|1\rangle,\ J_+|1\rangle = 2|2\rangle$

$J_-|2\rangle = 2|1\rangle,\ J_-|1\rangle = \sqrt{6}|0\rangle,\ J_-|0\rangle = \sqrt{6}|-1\rangle,\ J_-|-1\rangle = 2|-2\rangle$

$|J=3, M=3\rangle = |m=2, m'=1\rangle$

$J_-|2,1\rangle = 2|1,1\rangle + \sqrt{2}|2,0\rangle$

normalize

$$\frac{1}{\sqrt{2^2+\sqrt{2}^2}} = \frac{1}{\sqrt{6}}$$

$$|3,2\rangle = \frac{1}{\sqrt{6}}\left(2|1,1\rangle + \sqrt{2}|2,0\rangle\right)$$

$$|3,1\rangle = J_-\left[\frac{1}{\sqrt{6}}\left(2|1,1\rangle + \sqrt{2}|2,0\rangle\right)\right]$$

$$= \frac{1}{\sqrt{6}}\Big(2\sqrt{2}\,|1,0\rangle + 2\sqrt{6}\,|0,1\rangle$$

$$+ 2\sqrt{2}\,|1,0\rangle + \sqrt{2}\sqrt{2}\,|2,-1\rangle\Big)$$

$$= \frac{1}{\sqrt{6}}\left(4\sqrt{2}\,|1,0\rangle + 2\sqrt{6}\,|0,1\rangle + 2\,|2,-1\rangle\right)$$

$$|3,1\rangle = \frac{1}{\sqrt{15}}\left(2\sqrt{2}\,|1,0\rangle + \sqrt{6}\,|0,1\rangle + |2,-1\rangle\right)$$

$$(2\sqrt{2})^2 + (\sqrt{6})^2 + (1)^2 = 15$$

$$\Leftrightarrow \frac{1}{\sqrt{15}}$$ norm factor

$$|3,0\rangle = J_-\left[\frac{1}{\sqrt{15}}\left(2\sqrt{2}\,|1,0\rangle + \sqrt{6}\,|0,1\rangle + |2,-1\rangle\right)\right]$$

$$= 2\sqrt{2}\sqrt{6}\,|0,0\rangle + \overset{4}{\cancel{2\sqrt{2}}\sqrt{2}}\,|1,-1\rangle$$

$$+ \overset{6}{\cancel{\sqrt{6}}\sqrt{6}}\,|-1,1\rangle + \sqrt{6}\sqrt{2}\,|0,0\rangle$$

$$+ 2\,|1,-1\rangle + 0\,\cancel{|2,-1\rangle}$$

$$= 2(2)(6)\,|0,0\rangle + 2(2)(2)\,|1,-1\rangle + 6\,|-1,1\rangle$$

$$= 3\sqrt{12}\,|0,0\rangle + 6\,|1,-1\rangle + 6\,|-1,1\rangle$$

$$\frac{1}{\sqrt{(3\sqrt{12})^2 + 6^2 + 6^2}} = \frac{1}{\sqrt{9\cdot 12 + 2(36)}}$$

$$= \frac{1}{\sqrt{180}}$$

$$|3,0\rangle = \frac{1}{\sqrt{5}}\left(\sqrt{3}\,|0,0\rangle + |1,-1\rangle + |-1,1\rangle\right)$$

$$|3,-1\rangle = J_-\left(\sqrt{3}\,|0,0\rangle + |1,-1\rangle + |-1,1\rangle\right)$$

$$\sqrt{3}\sqrt{6}\,|-1,0\rangle + \sqrt{3}\sqrt{2}\,|0,-1\rangle$$
$$+ \sqrt{6}\,|0,-1\rangle + \emptyset$$
$$+ 2\,|-2,1\rangle + \sqrt{2}\,|-1,0\rangle$$

$$(\sqrt{3}\sqrt{6} + \sqrt{2})\,|-1,0\rangle + (\sqrt{3}\sqrt{2} + \sqrt{6})\,|0,-1\rangle + 2\,|-2,1\rangle$$

$$\sqrt{18} + \sqrt{2}$$
$$\sqrt{2}\sqrt{9} + \sqrt{2}$$
$$3\sqrt{2} + \sqrt{2}$$

$$4\sqrt{2}\,|-1,0\rangle + (2\sqrt{6})\,|0,-1\rangle + 2\,|-2,1\rangle$$

$$\frac{1}{\sqrt{(4\sqrt{2})^2+(2\sqrt{6})^2+2^2}} = \frac{1}{\sqrt{60}}$$

$$|3,-1\rangle = \frac{1}{\sqrt{60}}\left[4\sqrt{2}\,|-1,0\rangle + (\,2\sqrt{6}\,)\,|0,-1\rangle + 2\,|-2,1\rangle\right]$$

$$|3,-2\rangle = J_-\left[4\sqrt{2}\,|-1,0\rangle + (\,2\sqrt{6}\,)\,|0,-1\rangle + 2\,|-2,1\rangle\right]$$

$$4\sqrt{2}\,2\,|-2,0\rangle + 4\cancel{\sqrt{2}}^{\,2}\cancel{\sqrt{2}}\,|-1,-1\rangle$$

$$+\,2\cancel{\sqrt{6}}^{\,6}\cancel{\sqrt{6}}\,|-1,-1\rangle + \varnothing$$

$$+\,\varnothing + 2\sqrt{2}\,|-2,0\rangle$$

$$10\sqrt{2}\,|-2,0\rangle + 20\,|-1,-1\rangle$$

normlize

$$\sqrt{(10\sqrt{2})^2+20^2}$$

$$200 + 400 \qquad \frac{1}{\sqrt{600}}$$

$$600$$

$$\frac{1}{\sqrt{6}} \quad \frac{\cancel{1}}{\sqrt{100}}$$

$$|3,-2\rangle = \frac{1}{\sqrt{6}}\left(\sqrt{2}\,|-2,0\rangle + 2\,|-1,-1\rangle\right)$$

$$|3,-3\rangle = j_-\left[\left(\sqrt{2}\,|-2,0\rangle + 2|-1,-1\rangle\right)\right]$$

$$\cancel{0} + \sqrt{2}\sqrt{2}\,|-2,-1\rangle$$
$$+ 2(2)\,|-2,-1\rangle + \cancel{0}$$

$$= 2|-2,-1\rangle + 4|-2,-1\rangle$$
$$= 6|-2,-1\rangle$$

$$|3,-3\rangle = \frac{1}{6}\left(6|-2,-1\rangle\right)$$
$$= |-2,-1\rangle$$

first <u>7</u> combos. <u>8</u> left

for $m+m'=2$

kets $|2,0\rangle$ & $|1,1\rangle$

orthogonal to $\frac{1}{\sqrt{6}}\left(2|1,1\rangle + 2|2,0\rangle\right)$

$$\frac{1}{\sqrt{6}}\left(2|1,1\rangle - 2|2,0\rangle\right) = |2,2\rangle$$

$$|2,1\rangle = J_-\sqrt{2}|1,1\rangle - 2|2,0\rangle$$

$$J_-\left[\frac{1}{\sqrt{6}}\left(\sqrt{2}|1,1\rangle - 2|2,0\rangle\right)\right]$$

$$= \left(\cancel{\sqrt{2}}\cancel{\sqrt{2}}^{\,2}|1,0\rangle + \sqrt{2}\sqrt{6}|0,1\rangle\right.$$

$$\left. - 2(2)|1,0\rangle - 2\sqrt{2}|2,-1\rangle\right)$$

$$= \alpha\left(-\cancel{2}|1,0\rangle + \cancel{2}\sqrt{3}|0,1\rangle - \cancel{2}\sqrt{2}|2,-1\rangle\right)$$

$$\alpha = \frac{1}{\sqrt{(-1)^2 + \sqrt{3}^2 + \sqrt{2}^2}} = \frac{1}{\sqrt{6}}$$

$$|2,1\rangle = \frac{1}{\sqrt{6}}\left(-|1,0\rangle + \sqrt{3}|0,1\rangle - \sqrt{2}|2,-1\rangle\right)$$

$$|2,0\rangle = J_-\left(-|1,0\rangle + \sqrt{3}|0,1\rangle - \sqrt{2}|2,-1\rangle\right)$$

$$\cancel{-\sqrt{6}|0,0\rangle} - \sqrt{2}|1,-1\rangle$$

$$+\sqrt{3}\sqrt{6}|-1,1\rangle \cancel{+\sqrt{3}\sqrt{2}|0,0\rangle}$$

$$-\sqrt{2}(2)|1,-1\rangle + \emptyset$$

$$\left[3\sqrt{2}\,|-1,1\rangle - 3\sqrt{2}\,|1,-1\rangle\right]$$

$$|2,0\rangle\ \frac{1}{\sqrt{2}}\left[|-1,1\rangle - |1,-1\rangle\right]$$

$$|2,-1\rangle = J_-\left[|-1,1\rangle - |1,-1\rangle\right]$$

$$2|-2,1\rangle + \sqrt{2}|-1,0\rangle$$
$$- \sqrt{6}|0,-1\rangle \quad + \emptyset$$

$$2|-2,1\rangle + \sqrt{2}|-1,0\rangle - \sqrt{6}|0,-1\rangle$$

$$\sqrt{2}|-2,1\rangle + |-1,0\rangle - \sqrt{3}|0,-1\rangle$$

$$\alpha = \frac{1}{\sqrt{\sqrt{2}^2, \sqrt{3}^2 + 1^2}} = \frac{1}{\sqrt{5}}$$

$$|2,-1\rangle = \frac{1}{\sqrt{5}}\left[\sqrt{2}|-2,1\rangle + |-1,0\rangle - \sqrt{3}|0,-1\rangle\right]$$

$$|2,-2\rangle = J_-\left[\sqrt{2}|-2,1\rangle + |-1,0\rangle - \sqrt{3}|0,-1\rangle\right]$$

$$\emptyset \quad \sqrt{2}\sqrt{2}|-2,0\rangle$$
$$+ 2|-2,0\rangle + \sqrt{2}|-2,-1\rangle$$
$$- \sqrt{3}\sqrt{6}|-1,-1\rangle$$

$$= \alpha\left[4|-2,0\rangle + \sqrt{2}|-2,-1\rangle - \sqrt{18}|-1,-1\rangle\right]$$

$$\alpha = \frac{1}{\sqrt{4^2+\sqrt{2}^2\,(-\sqrt{18})^2}} = \frac{1}{\sqrt{38}}$$

$$|2,-2\rangle = \frac{1}{\sqrt{38}}\left[4|-2,0\rangle + \sqrt{2}|-2,-1\rangle - \sqrt{18}|-1,-1\rangle\right]$$

5 more states. $7+5=12$ total.

3 more

$m+m' < 1$

K(1)

$|2,-1\rangle, |-1,2\rangle, |1,0\rangle, |0,1\rangle$

$$\frac{1}{\sqrt{13}}\left(2\sqrt{2}\,|1,0\rangle + \sqrt{6}\,|0,1\rangle + |2,-1\rangle\right)$$

$$\frac{1}{\sqrt{6}}\left(-|1,0\rangle + \sqrt{3}\,|0,1\rangle - \sqrt{2}\,|2,-1\rangle\right)$$

existing states

$$\frac{2\sqrt{2}}{\sqrt{15}},\ + \frac{\sqrt{6}}{\sqrt{15}},\ + \frac{1}{\sqrt{13}} \to \frac{8}{15},\ \frac{6}{15},\ \frac{1}{15}$$

$$-\frac{1}{\sqrt{6}},\ + \frac{\sqrt{3}}{\sqrt{6}},\ - \frac{\sqrt{2}}{\sqrt{6}} \to -\frac{1}{6},\ \frac{3}{6},\ -\frac{2}{6}$$

$$\begin{pmatrix} \frac{8}{13} \\ \frac{6}{15} \\ \frac{1}{13} \end{pmatrix} \quad \begin{pmatrix} -\frac{1}{6} \\ \frac{3}{6} \\ -\frac{2}{6} \end{pmatrix} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$a = -\frac{1}{2}$

$b = \frac{1}{2}$

$c = 1$

$$-\frac{1}{6}a + \frac{3}{6}b - \frac{2}{6}c = 0$$

$$\frac{8}{13}a + \frac{6}{15}b + \frac{1}{15}c = 0$$

$$a + b + c = 1$$

orthogonal to above states:

$$|1,1\rangle = \alpha\left(-\frac{1}{2}|1,0\rangle + \frac{1}{2}|0,1\rangle + |2,-1\rangle\right)$$

$$\alpha = \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 1}} = \frac{1}{\sqrt{\frac{3}{2}}} = \sqrt{\frac{2}{3}}$$

$$|1,1\rangle = \sqrt{\frac{2}{3}}\left(-\frac{1}{2}|1,0\rangle + \frac{1}{2}|0,1\rangle + |2,-1\rangle\right)$$

$$|1,0\rangle = J_-\left(-|1,0\rangle + \sqrt{3}|0,1\rangle - \sqrt{2}|2,-1\rangle\right)$$

$$-\frac{1}{2}\sqrt{6}|0,0\rangle - \frac{1}{2}\sqrt{2}|1,-1\rangle$$
$$+\frac{1}{2}\sqrt{6}|-1,1\rangle + \frac{1}{2}\sqrt{2}|0,0\rangle$$
$$+(2)|1,-1\rangle + \emptyset$$

$$\left(-\frac{1}{2}\sqrt{6} + \frac{1}{2}\sqrt{2}\right)|0,0\rangle + \left(2 - \frac{1}{2}\sqrt{2}\right)|1,-1\rangle + \frac{\sqrt{6}}{2}|-1,1\rangle$$

$$|1,0\rangle = \alpha\left[\left(-\frac{1}{2}\sqrt{6} + \frac{1}{2}\sqrt{2}\right)|0,0\rangle + \left(2 - \frac{\sqrt{2}}{2}\right)|-1,1\rangle + \frac{\sqrt{6}}{2}|1,-1\rangle\right]$$

$$\alpha = \frac{1}{\sqrt{\left(-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right)^2 + \left(2-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{6}}{2}\right)^2}}$$

$$\frac{6}{4} - 2\left(\frac{1}{2}\sqrt{6}\right)\left(\frac{1}{2}\sqrt{2}\right) + \frac{1}{4}(2)$$

$$+4 - 2\cancel{(2)}\left(\frac{\sqrt{2}}{\cancel{2}}\right) + \frac{2}{4}$$

$$\frac{3}{2} - \sqrt{3} + \frac{4}{2} + \frac{8}{2} - 2\sqrt{2} + \frac{1}{2} + \frac{3}{2} \leftarrow + \frac{6}{4}$$

$$\frac{14}{2} - \sqrt{3} - 2\sqrt{2}$$

$$|1,0\rangle = \frac{1}{\sqrt{\frac{14}{2} - \sqrt{3} - 2\sqrt{2}}}\left[\left(-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right)|0,0\rangle + \left(2-\frac{\sqrt{2}}{2}\right)|-1,1\rangle + \frac{\sqrt{6}}{2}|1,-1\rangle\right]$$

$$|1,-1\rangle = J_-\left[\left(-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right)|0,0\rangle + \left(2-\frac{\sqrt{2}}{2}\right)|-1,1\rangle + \frac{\sqrt{6}}{2}|1,-1\rangle\right]$$

$$\left(-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right)\sqrt{6}\,|-1,0\rangle + \left(-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right)\sqrt{2}\,|0,-1\rangle +$$

$$\left(2-\frac{\sqrt{2}}{2}\right)2\,|-2,1\rangle + \left(2-\frac{\sqrt{2}}{2}\right)\sqrt{2}\,|-1,0\rangle +$$

$$\frac{\sqrt{6}}{2}\sqrt{6}\,|0,-1\rangle + \emptyset$$

$$\left(-\frac{6}{2}+\frac{1}{2}\sqrt{12}\right)|-1,0\rangle+\left(-\frac{1}{2}\sqrt{12}+1\right)|0,-1\rangle+$$

$$(4-\sqrt{2})|-2,1\rangle+(2\sqrt{2}-1)|-1,0\rangle$$

$$+\ 3|0,-1\rangle$$

$$\left[(2\sqrt{2}-1)+\left(-3+\frac{1}{2}\sqrt{2}\right)\right]|-1,0\rangle+\left(-\frac{1}{2}\sqrt{12}+4\right)|0,-1\rangle+(4-\sqrt{2})|-2,1\rangle$$

$$\alpha\left[\left(\frac{5}{2}\sqrt{2}-4\right)|-1,0\rangle+\left(-\frac{1}{2}\sqrt{12}+4\right)|0,-1\rangle+(4-\sqrt{2})|-2,1\rangle\right]$$

$$\alpha=\frac{1}{\sqrt{\left(\frac{5}{2}\sqrt{2}-4\right)^2+\left(-\frac{1}{2}\sqrt{12}+4\right)^2+(4-\sqrt{2})^2}}$$

$$\frac{25}{4}(2)-\cancel{2}(4)\left(\frac{5}{\cancel{2}}\sqrt{2}\right)+16$$

$$\frac{1}{4}(12)-(2)\left(\frac{1}{2}\sqrt{12}\right)(4)+16$$

$$+16-2(4)(\sqrt{2})+2$$

$$\frac{50}{4}-20\sqrt{2}+48+5-8\sqrt{3}-8\sqrt{2}$$

$$\frac{50}{4}+53-28\sqrt{2}-8\sqrt{3}$$

$$65+\frac{1}{2}-28\sqrt{2}-8\sqrt{3}$$

$$|1,-1\rangle = \frac{1}{\sqrt{65+\frac{1}{2}-28\sqrt{2}-8\sqrt{3}}}\left[\left(\frac{5}{2}\sqrt{2}-4\right)|-1,0\rangle + \left(-\frac{1}{2}\sqrt{12}+4\right)|0,-1\rangle + \left(4-\sqrt{2}\right)|-2,1\rangle\right]$$

That's all 15 state accounted