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In[1]:= rp = {0, 0, d/2}
rm = {0, 0, -d/2}
V[r_] := q/Sqrt[(r-rp).(r-rp)]-q/Sqrt[(r-rm).(r-rm)]
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Out[1]= {0, 0, d/2}
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Out[2]= {0, 0, -d/2}
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In[11]:= -∇{x,y,z} V[{x, y, z}]
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Out[11]=
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$$\left\{ \frac{q x}{\left(x^2 + y^2 + \left(-\frac{d}{2} + z\right)^2\right)^{3/2}} - \frac{q x}{\left(x^2 + y^2 + \left(\frac{d}{2} + z\right)^2\right)^{3/2}}, \right. \\ \left. \frac{q y}{\left(x^2 + y^2 + \left(-\frac{d}{2} + z\right)^2\right)^{3/2}} - \frac{q y}{\left(x^2 + y^2 + \left(\frac{d}{2} + z\right)^2\right)^{3/2}}, \frac{q \left(-\frac{d}{2} + z\right)}{\left(x^2 + y^2 + \left(-\frac{d}{2} + z\right)^2\right)^{3/2}} - \frac{q \left(\frac{d}{2} + z\right)}{\left(x^2 + y^2 + \left(\frac{d}{2} + z\right)^2\right)^{3/2}} \right\}$$

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In[13]:= EfieldCartesian[r_] = {
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$$\frac{q x}{\left(x^2 + y^2 + \left(-\frac{d}{2} + z\right)^2\right)^{3/2}} - \frac{q x}{\left(x^2 + y^2 + \left(\frac{d}{2} + z\right)^2\right)^{3/2}},$$

$$\frac{q y}{\left(x^2 + y^2 + \left(-\frac{d}{2} + z\right)^2\right)^{3/2}} - \frac{q y}{\left(x^2 + y^2 + \left(\frac{d}{2} + z\right)^2\right)^{3/2}}, \frac{q \left(-\frac{d}{2} + z\right)}{\left(x^2 + y^2 + \left(-\frac{d}{2} + z\right)^2\right)^{3/2}} - \frac{q \left(\frac{d}{2} + z\right)}{\left(x^2 + y^2 + \left(\frac{d}{2} + z\right)^2\right)^{3/2}} \right\}$$

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Out[13]=
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$$\left\{ \frac{q x}{\left(x^2 + y^2 + \left(-\frac{d}{2} + z\right)^2\right)^{3/2}} - \frac{q x}{\left(x^2 + y^2 + \left(\frac{d}{2} + z\right)^2\right)^{3/2}}, \right. \\ \left. \frac{q y}{\left(x^2 + y^2 + \left(-\frac{d}{2} + z\right)^2\right)^{3/2}} - \frac{q y}{\left(x^2 + y^2 + \left(\frac{d}{2} + z\right)^2\right)^{3/2}}, \frac{q \left(-\frac{d}{2} + z\right)}{\left(x^2 + y^2 + \left(-\frac{d}{2} + z\right)^2\right)^{3/2}} - \frac{q \left(\frac{d}{2} + z\right)}{\left(x^2 + y^2 + \left(\frac{d}{2} + z\right)^2\right)^{3/2}} \right\}$$

In[14]:= **EfieldSpherical** =

**CoordinateTransform**["Cartesian" → "Spherical", **EfieldCartesian**[[x, y, z]]]

Out[14]=

$$\left\{ \sqrt{\left( \left( \frac{q x}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q x}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}} \right)^2 + \left( \frac{q y}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q y}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}} \right)^2 + \left( \frac{q \left( -\frac{d}{2} + z \right)}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q \left( \frac{d}{2} + z \right)}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}} \right)^2} \right)^{1/2}, \right. \\ \left. \text{ArcTan} \left[ \frac{q \left( -\frac{d}{2} + z \right)}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q \left( \frac{d}{2} + z \right)}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}}, \right. \right. \\ \left. \sqrt{\left( \left( \frac{q x}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q x}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}} \right)^2 + \left( \frac{q y}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q y}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}} \right)^2} \right)^{1/2}}, \right. \\ \left. \text{ArcTan} \left[ \frac{q x}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q x}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}}, \frac{q y}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q y}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}} \right] \right\}$$

In[15]:= **EfieldCylindrical** =

**CoordinateTransform**["Cartesian" → "Cylindrical", **EfieldCartesian**[[x, y, z]]]

Out[15]=

$$\left\{ \sqrt{\left( \left( \frac{q x}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q x}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}} \right)^2 + \left( \frac{q y}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q y}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}} \right)^2} \right)^{1/2}, \right. \\ \left. \text{ArcTan} \left[ \frac{q x}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q x}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}}, \frac{q y}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q y}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}}, \right. \right. \\ \left. \left. \frac{q \left( -\frac{d}{2} + z \right)}{\left( x^2 + y^2 + \left( -\frac{d}{2} + z \right)^2 \right)^{3/2}} - \frac{q \left( \frac{d}{2} + z \right)}{\left( x^2 + y^2 + \left( \frac{d}{2} + z \right)^2 \right)^{3/2}} \right] \right\}$$

In[216]:=

```

Clear[x]
region = RegionUnion[Disk[{0, 0}, 2], Rectangle[{-2, -1}, {2, 1}]]

ufun = NDSolveValue[{ $\nabla_{\{x,y\}}^2 u[x, y] == -1$ ,
  PeriodicBoundaryCondition[u[x, y], x == -2,
    Function[x, x + {4, 0}]]],
  DirichletCondition[
    u[x, y] == 0, -2 < x < 2],
  u, {x, y}  $\in$  region];

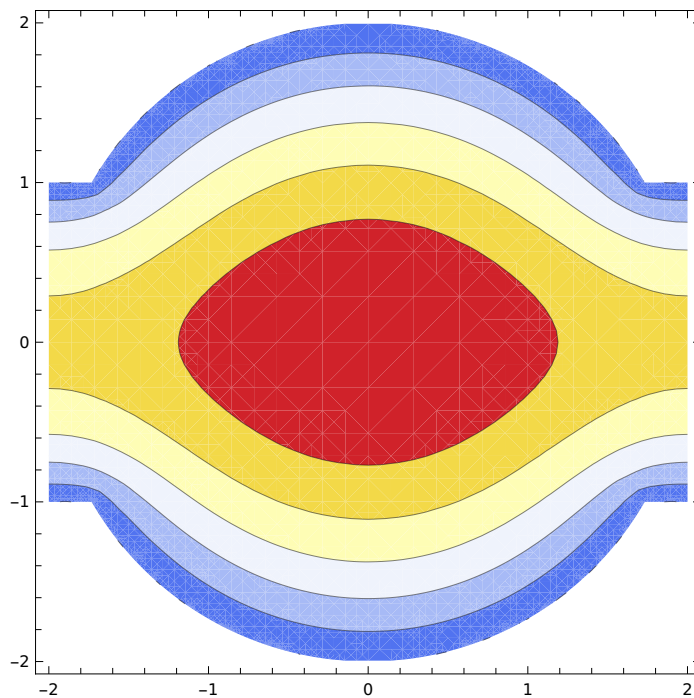
graph = ContourPlot[ufun[x, y], {x, y}  $\in$  region,
  ColorFunction -> "TemperatureMap", AspectRatio -> Automatic]

```

Out[217]=

```
BooleanRegion[#1 || #2 &, {Disk[{0, 0}, 2], Rectangle[{-2, -1}, {2, 1}]}]
```

Out[219]=



```
Rectangle[{-1, -2}, {1, 2}]
```