Analog Computing Lab Report

Objective:

Solve Differential Equations with analog circuits using Op-Amp based integrators, differentiators, adders and inverters.

Equipment

- Breadboard
- Oscilloscope
- Jumper Cables
- Resistor Assortments
- Capacitor Assortments
- Op-Amps (TLE2071CP)
- Digital Multi-meter
- 9v Battery and Harness
- USB Flash Drive
- 2 Function Generators

Section 1

Objective

- Familiarize ourselves with components of the analog computer, including inverters, integrators, differentiators and adders. ### Equipment:
- Same as above

Section 1.1: Inverting Amplifier

Objective:

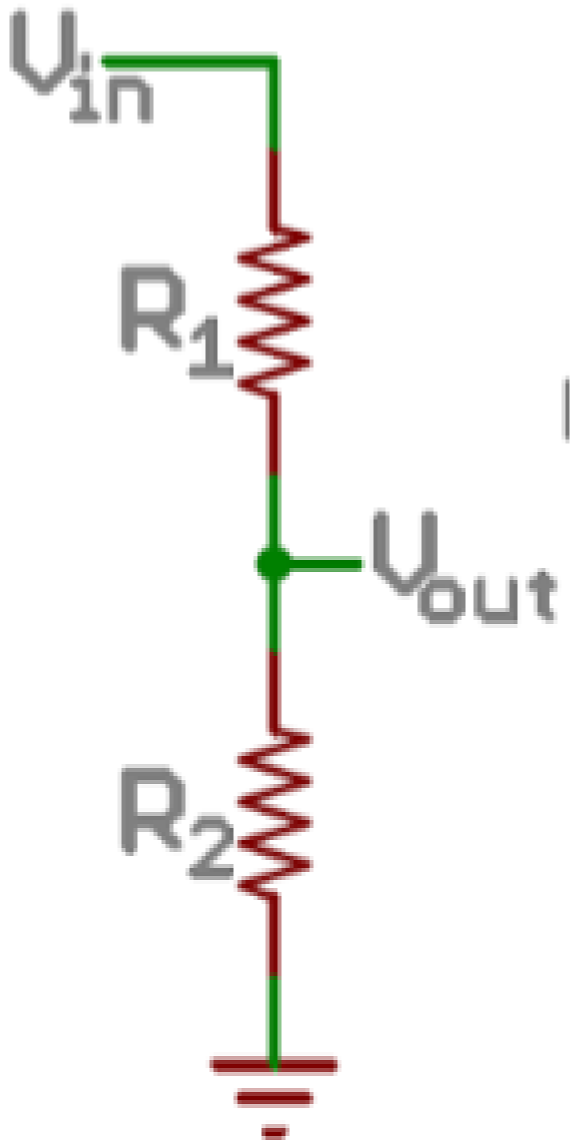
The objective of this portion of the lab is to use an Operational amplifier, or Op-Amp integrated circuit to create an inverting amplifier and test if the theoretically calculated amplification values are in line with the experimentally measured and verified values.

Equipment:

- 4x Resistors
- 1x Op-Amp
- Breadboard
- 9v battery and power harness
- Oscilliscope
- Digital Multimeter
- Jumpers

Procedure:

Begin by selecting 4 appropriate resistors. To experience the effects of the op-amp without overloading the power supply we will need to use a voltage divider that has approximately the inverse of the gain of the inverting Op-Amp. The calculated resistor values will be below in the data subsection, as well as the experimentally measured values. After acquiring these resistors, firstly construct a voltage divider, as shown in figure 1 below.



> Figure 1

After constructing the voltage divider, we can construct the inverting op-amp as per figure 2 below, and connect the V_{out} from the voltage divider to the V_{in} point on the inverting Op-Amp.

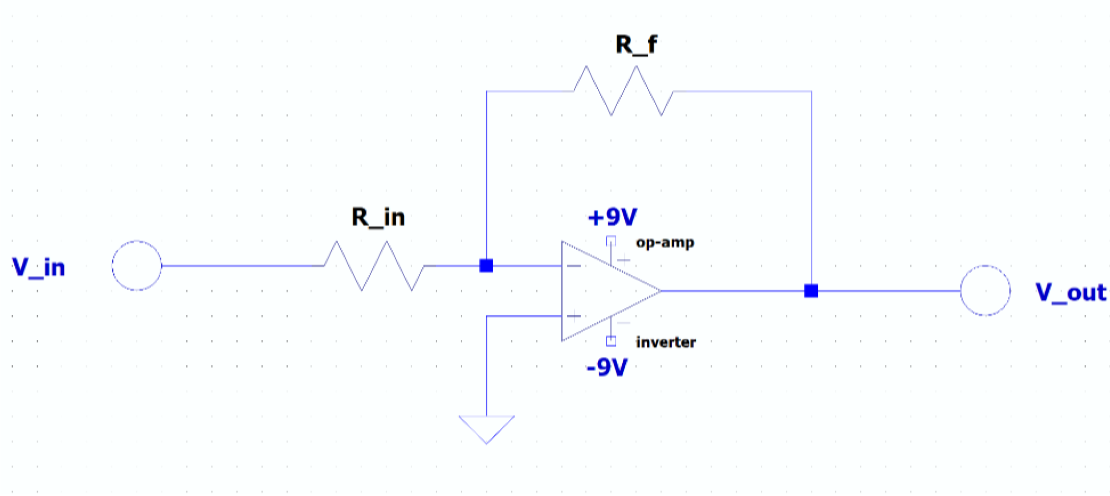


Figure 2

After building the circuit testing the circuit can be done with both a static DC voltage applied through the voltage divider as well as a waveform applied with a function generator. Ideally, 4 tests would be performed with different resistor values, but in our case we were unable to perform these many tests. After these two tests are run and voltage measurements before and after both the voltage divider and op-amp circuit are gathered, we now are able to determine the experimental gain and compare to the theoretical gain.

Data

Resistance values for Voltage divider

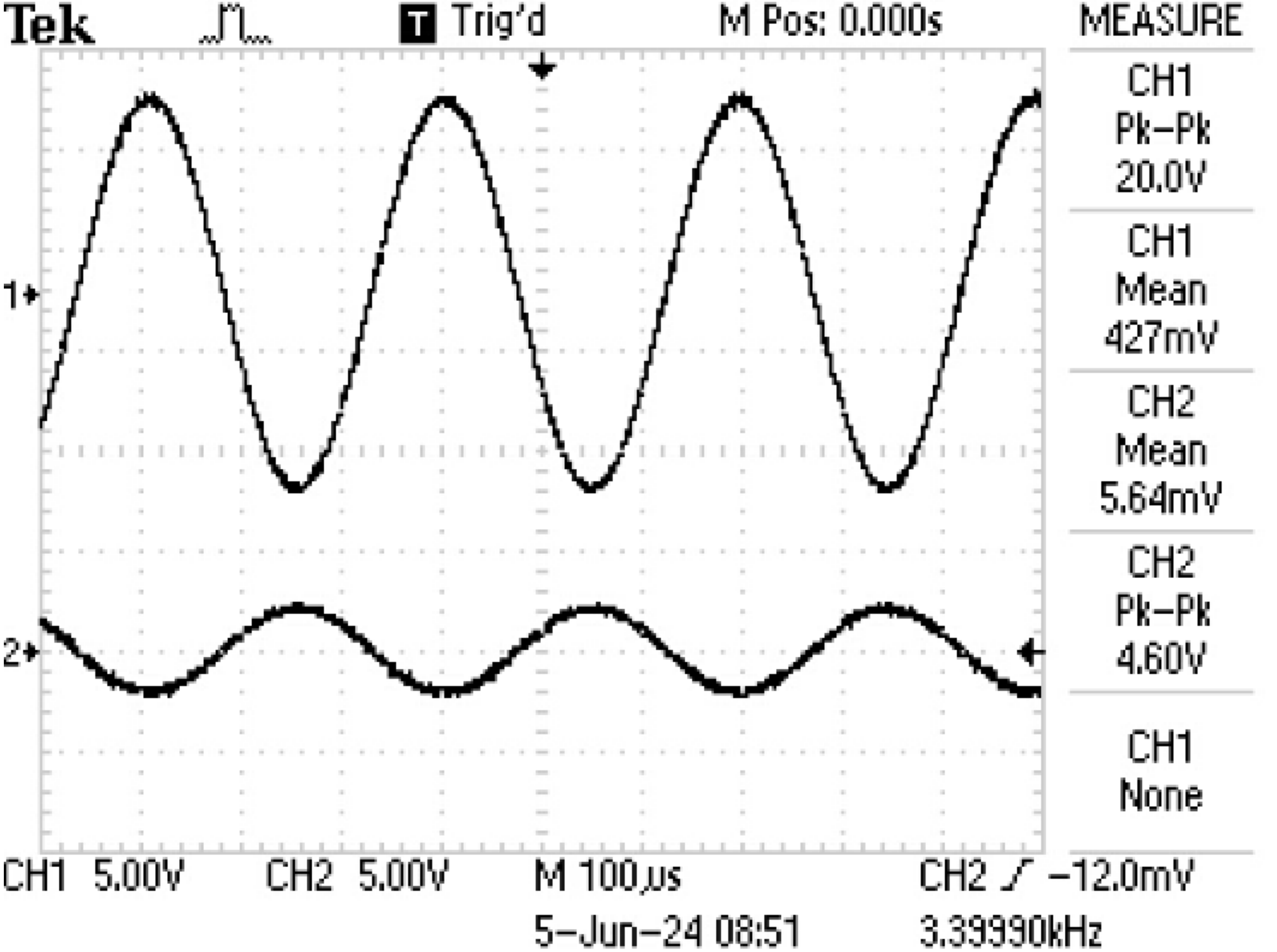
	Resistance (kilo-ohms)
R_1	10 $\pm 0.9\%$
R_2	1.196 $\pm 0.9\%$
R_2 Real	0.6012 $\pm 0.9\%$

Resistance Values for Inverting op-amp

	Resistance (kilo-ohms)
R_{in}	1.209 $\pm 0.9\%$
R_f	4.72 $\pm 0.9\%$

Potential in volts for input and output from circuit

	Potential (Volts)
V_in	9.45 $\pm 0.5\%$
V_out	-2.094 $\pm 0.5\%$



Screenshot of oscilliscope scope showing total combined gain

Analysis

Voltage Divider Calculations:

$$V_{out} = \frac{R_2}{R_1 + R_2}$$

We have a virtual ground point on the op-amp, so we need to consider R_2 to be in parallel with R_{in} on the op-amp circuit when calculating the voltage divider drop.

Voltage divider attenuation calculations:

$$\begin{aligned} & \Delta A = \sqrt{\left(\frac{\partial A}{\partial R_1}\Delta R_1\right)^2 + \left(\frac{\partial A}{\partial R_2}\Delta R_2\right)^2} \\ A = 0.056657117 \qquad & = \sqrt{\left(\left(-(R_2/(R_1+R_2)^2)\right)\cdot \Delta R_1\right)^2 + \left(\left(R_1/(R_1+R_2)^2\right)\cdot \Delta R_2\right)^2} \\ & = 0.0006802704 \\ & = 6.802704 \times 10^{-4} \\ & = 7 \times 10^{-4} \end{aligned}$$

$$A = (5.67 \pm 0.07) \times 10^{-2}$$

Where A is attenuation of the voltage divider. Op-Amp Gain Calculations:

$$\text{Op-Amp gain} = -\frac{R_f}{R_{in}}$$

Op-Amp Resistor Values Gain Calculation

$$\Delta g = \sqrt{\left(\frac{\partial g}{\partial R_i n}\Delta R_i n\right)^2 + \left(\frac{\partial g}{\partial R_f}\Delta R_f\right)^2}$$

$$g = -3.9040529363 \qquad = \sqrt{\left((R_f/R_i n^2)\cdot \Delta R_i n\right)^2 + \left((-(1/R_i n))\cdot \Delta R_f\right)^2}$$

$$= 0.0496904815$$

$$= 4.96904815 \times 10^{-2}$$

$$= 5 \times 10^{-2}$$

$$g = (-3.9 \pm 0.05) \times 10^{0}$$

Combining the attenuation and gain we should have a combined gain of:

$$\Delta t = \sqrt{\left(\frac{\partial t}{\partial g}\Delta g\right)^2 + \left(\frac{\partial t}{\partial a}\Delta a\right)^2}$$

$$t = -0.22113 \qquad = \sqrt{((a)\cdot \Delta g)^2 + ((g)\cdot \Delta a)^2}$$

$$= 0.0039357496$$

$$= 3.9357496 \times 10^{-3}$$

$$= 4 \times 10^{-3}$$

$$t = (-2.21 \pm 0.04) \times 10^{-1}$$

Total gain is -0.221 ± 0.004 Experimentally measured gain:

$$\Delta g_e = \sqrt{\left(\frac{\partial g_e}{\partial V_i n}\Delta V_i n\right)^2 + \left(\frac{\partial g_e}{\partial V_o ut}\Delta V_o ut\right)^2}$$

$$g_e = -0.2215873016[\qquad = \sqrt{\left((-(V_o ut/V_i n^2))\cdot \Delta V_i n\right)^2 + ((1/V_i n)\cdot \Delta V_o ut)^2}$$

$$= 0.0015668588$$

$$= 1.5668588 \times 10^{-3}$$

$$= 2 \times 10^{-3}$$

$$g_e = (-2.22 \pm 0.02) \times 10^{-1}$$

The experimentally measured total gain is -0.222 ± 0.002

The experimentally measured gain and theoretically calculated gain are within 0.001, less then the uncertainty of our measuring equipment. This is evidence that confirms that our circuit performs as expected.

Section 1.2: Integrating Amplifier

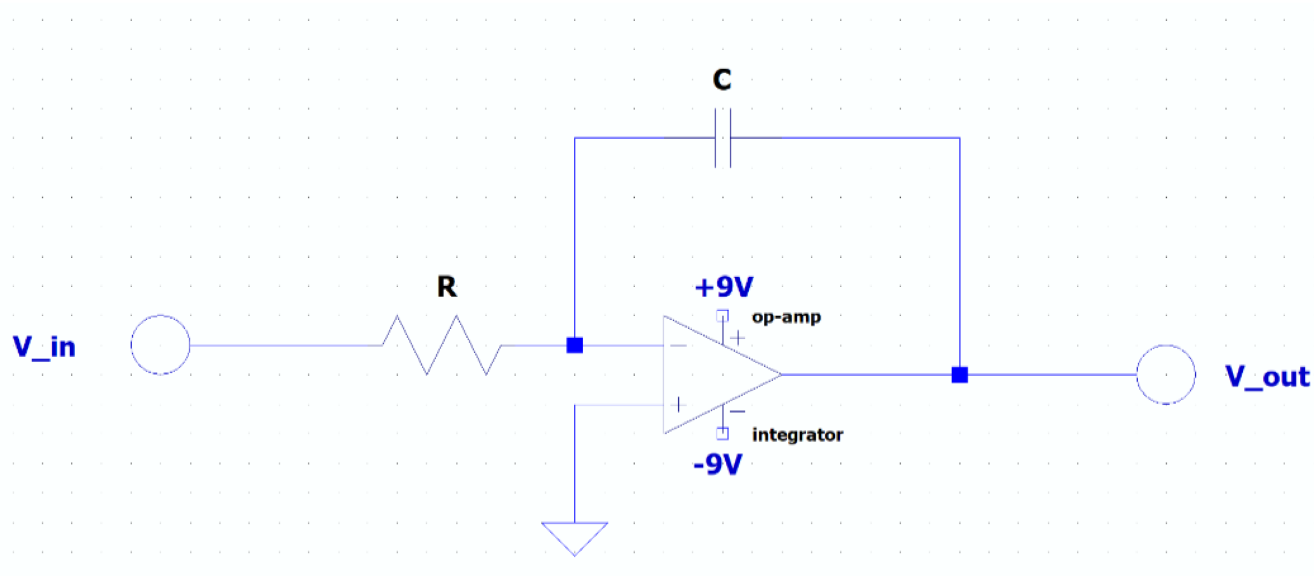
Objective:

The objective of this portion of the lab is to use an Operational amplifier, or Op-Amp integrated circuit to create an integrating amplifier and test if the theorized values and waveforms are in line with the experimentally measured and verified values and waveforms.

Equipment:

- 1x Resistor
- 1x Capacitor
- 1x Op-Amp
- Breadboard
- 9v battery and power harness
- Oscilloscope
- Digital Multimeter
- Jumpers #### Procedure: As before, we first select appropriate values for the capacitor and multimeter. The lab reference material suggests nominal values of 10 $k\Omega$ for the resistor and 0.01 μf for the capacitor, which is what we used. The actual values and error propagation calculations are below in the data section.

After selecting appropriate components, it is now appropriate to construct the circuit on the breadboard according to figure 3 below.



> Figure 3

After the circuit is constructed, we can use the function generator to test the integrating capacities using 2 different waveforms:

- Square wave $\pm$ 1V at 1kHz
- Triangular wave $\pm$ 1V at 1kHz Record data on oscilloscope, using channel one to record raw input from the function generator, and channel 2 to record the waveforms after they've been integrated. Compare this to the theoretically derived values and explain any discrepancies in the results.

To find the theoretically derived values for V_{out} we can use the following formula:

$$V_{out} = V_0(0) - \frac{1}{RC}\int V_{in}dt$$

Where $V_0(0)$ is an initial condition, which we can ignore because our circuit continuously integrates.

Data:

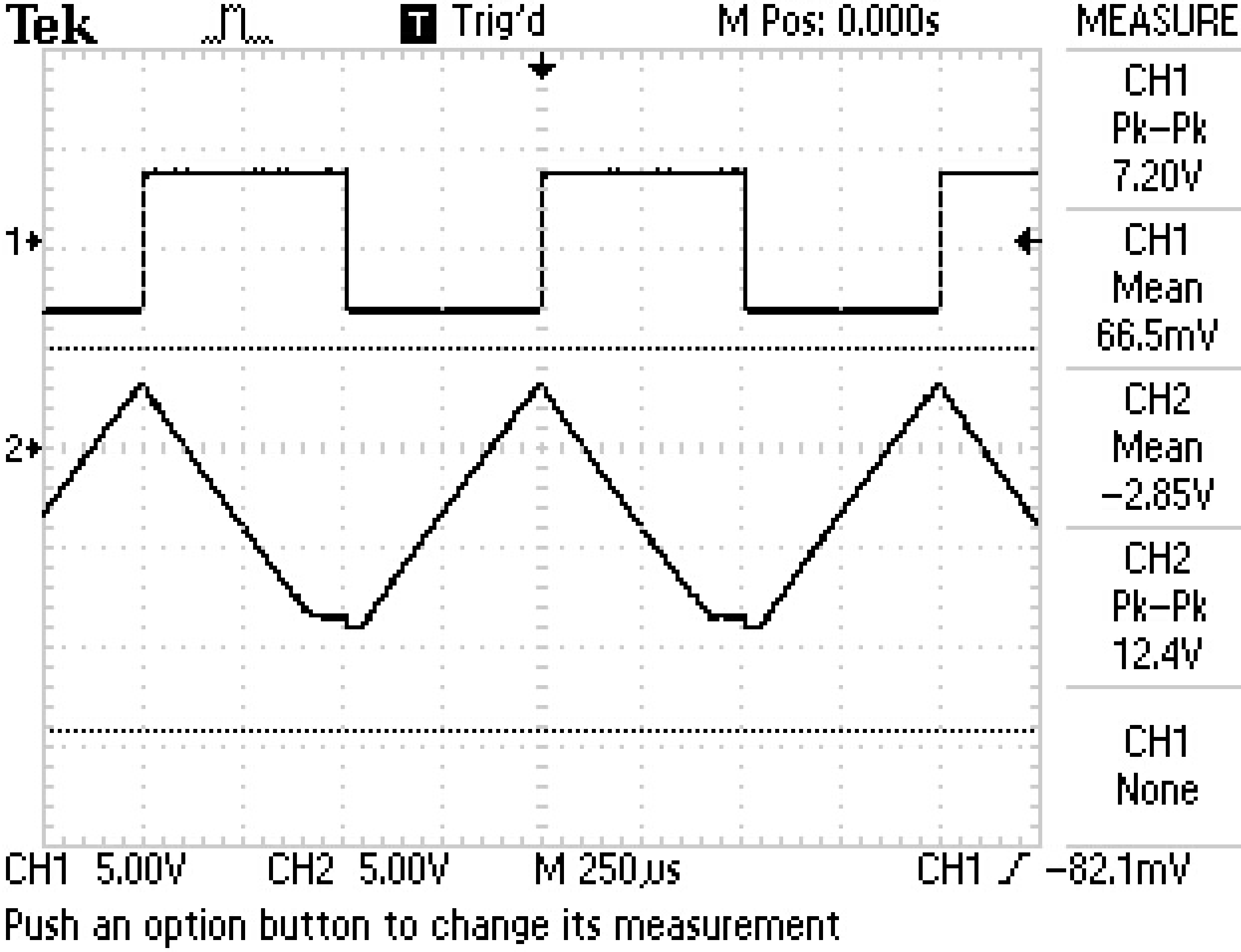
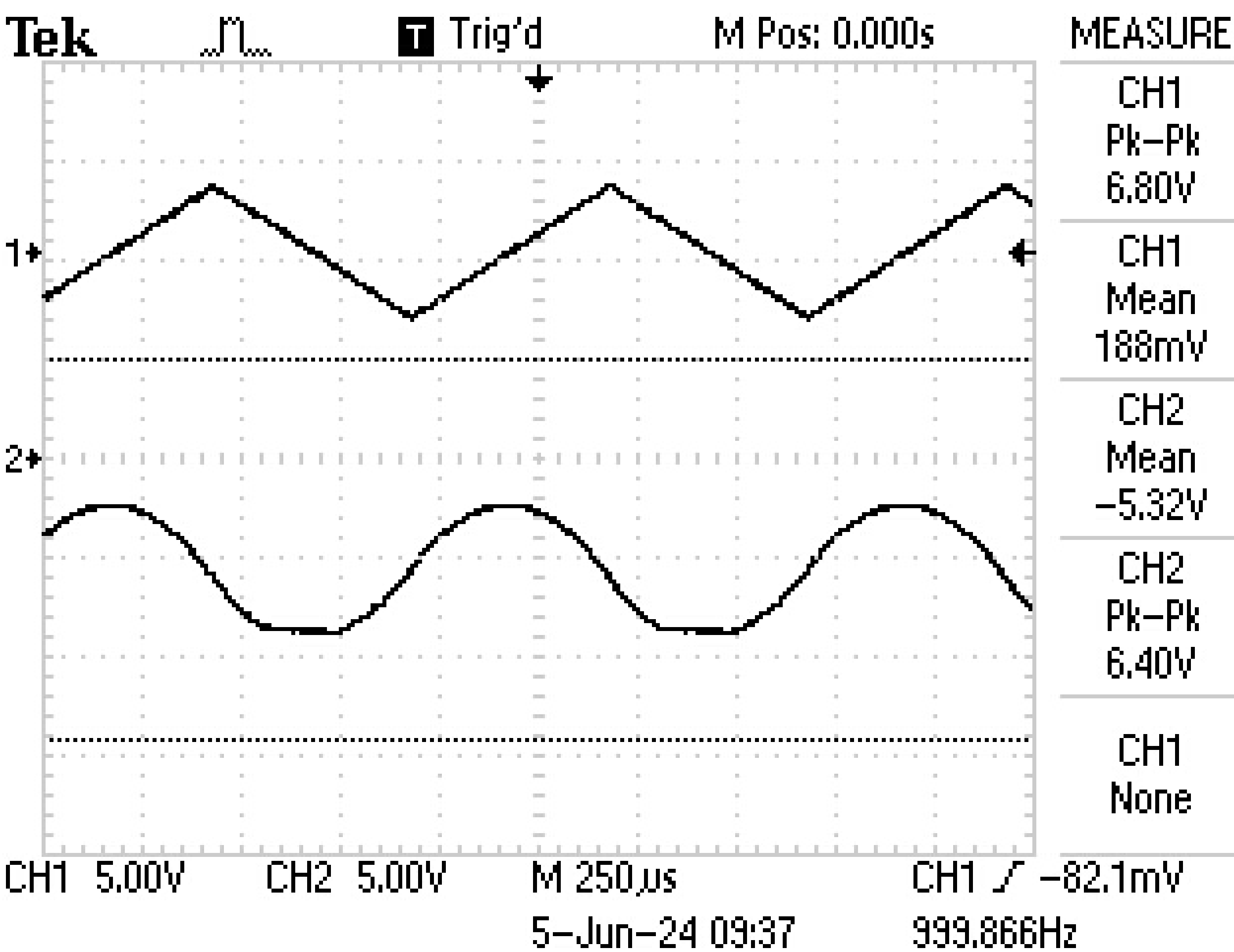


Figure Showing the integration of a square wave



> Figure showing the integration of a triangle wave

	Components
R	10 $k\Omega \pm 0.9\%$
C	0.097 $\mu F \pm 1.9\%$

Values of the circuit components

Analysis

The continuous integration of the waveforms is in line with the theoretical model of $V_{out} = V_0(0) - \frac{1}{RC}\int V_{in}dt$, with some slight variation at the extrema, likely due to the Op-Amp receiving interference or reaching its maximum output.

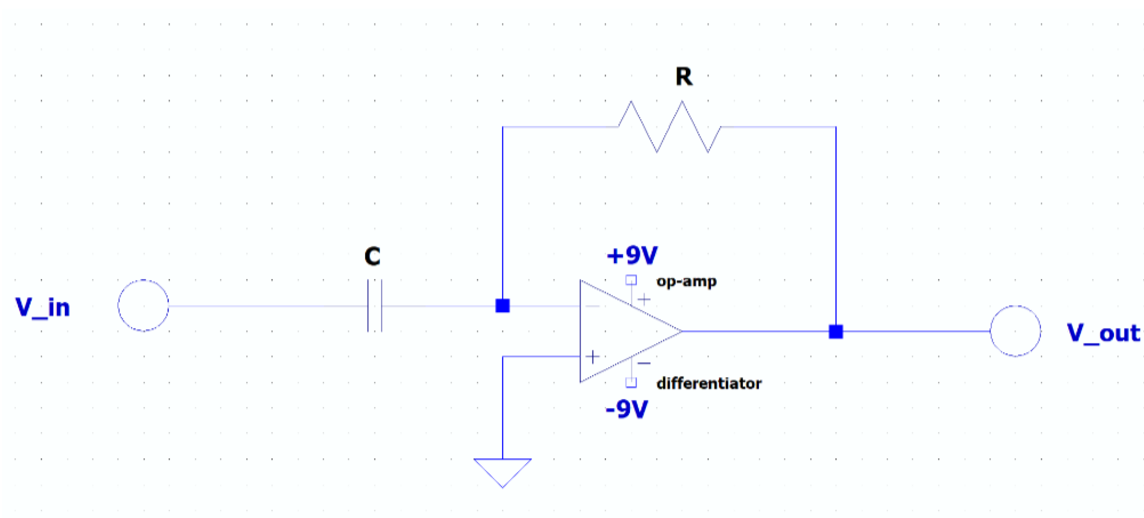
Section 1.3 Differentiating Op-Amp

Objective:

The objective of this portion of the lab is to convert our integrating op-amp circuit into a differentiating op-amp circuit.

Equipment:

- 1x Resistor
- 1x Capacitor
- 1x Op-Amp
- Breadboard
- 9v battery and power harness
- Oscilloscope
- Digital Multimeter
- Jumpers
- differentiating circuit #### Procedure: We will continue to use the same resistors and capacitors from the integrating amplifier for ease of construction, to change the integrating circuit to a differentiating circuit it is a simple matter of swapping the location of the capacitor and resistor. The diagram for this can be seen below in figure 4



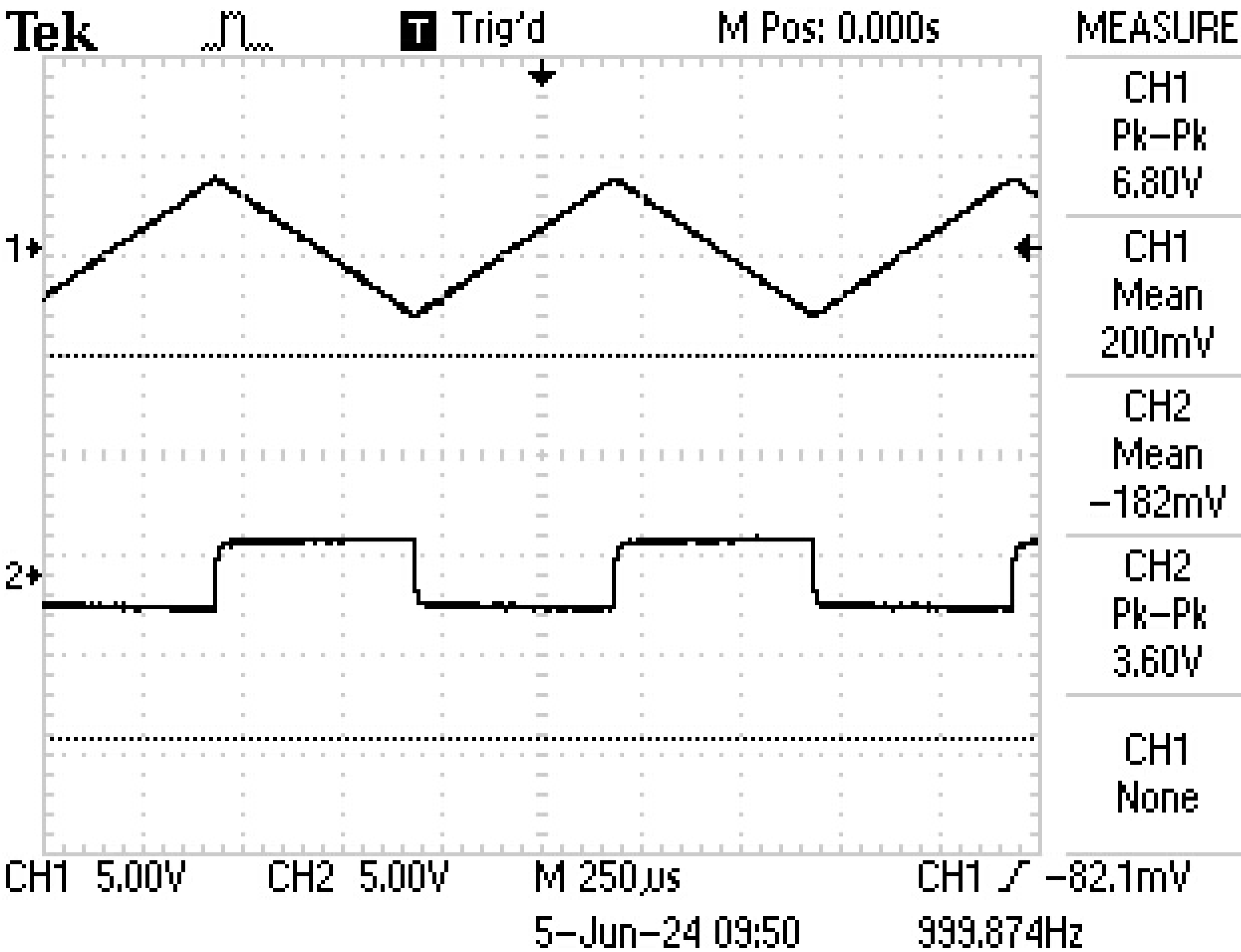
> Figure 4

After this circuit is complete, we will proceed to due the same tests performed upon the integrating amplifier to see if the differentiating amplifier circuit matches its theoretically derived values. To find the theoretically derived values for this portion of the lab we use the following equation:

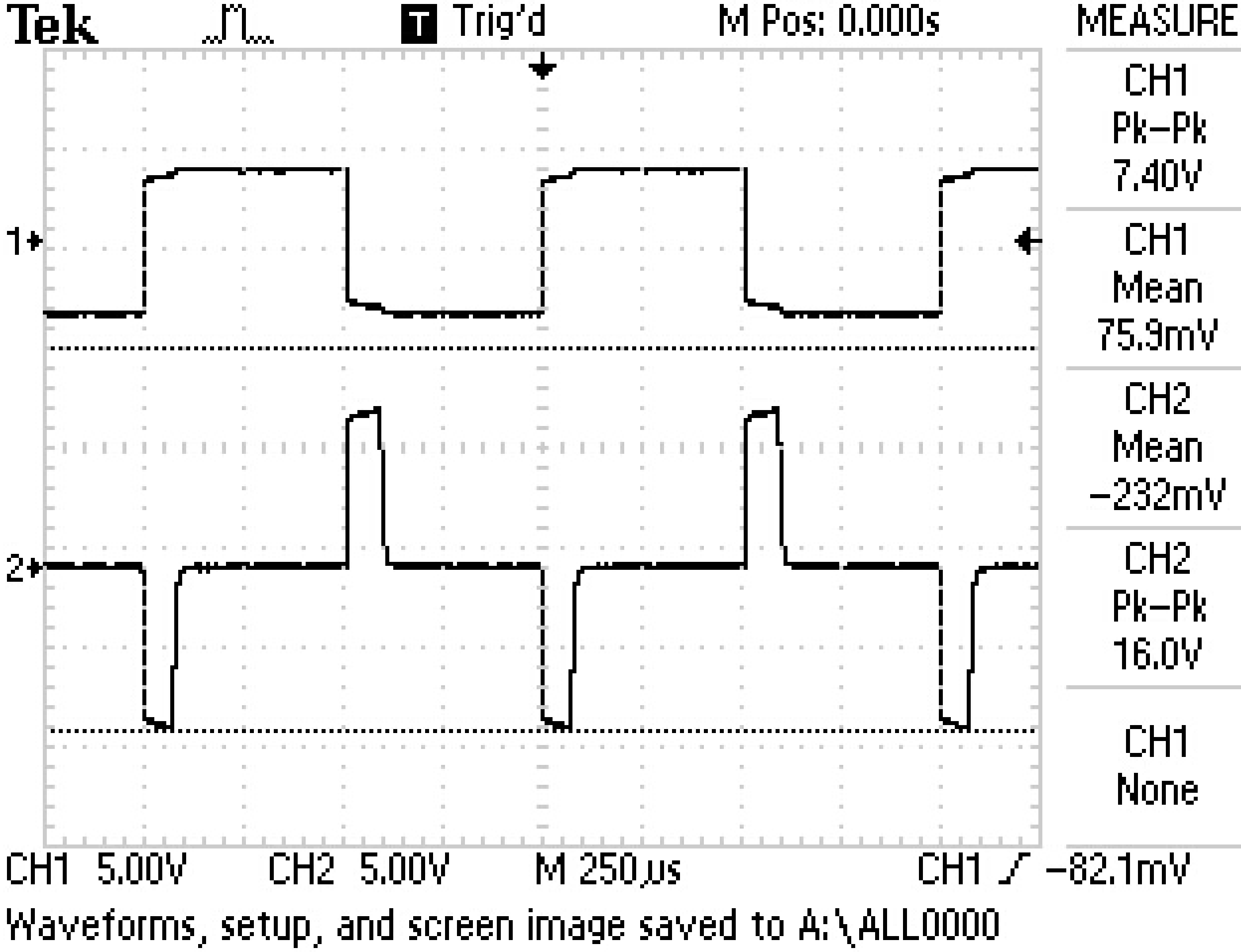
$$V_{out}(t) = V_0(0) - RC\frac{dV_{in}}{dt}$$

Where, as before, $V_0(0)$ is the initial condition term that we can safely ignore for our continuously differentiating case. After computing the theoretical values, we can test the circuit for the three cases as in the integrator section, a square and triangle wave.

Data:



> Figure showing the differentiation of a triangle wave over time. Note the similarities to the integrating section



> Figure showing the differentiation of a square wave.

Components	
R	10 $k\Omega \pm 0.9\%$
C	0.097 $\mu F \pm 1.9\%$

Values of the circuit components

Analysis:

The differentiation of the waveforms follows the given theoretical model, $V_{out}(t) = V_0(0) - RC\frac{dV_{in}}{dt}$. We can also logically think about the derivative of a function like a square wave, and what pattern it would follow, and the circuit produces an inverted derivative. While the derivative of a square wave at the leading and falling edges should be $\pm\infty$, that is obviously not physically possible, and this is represented by the op-amp maxing out its voltage supply at the leading and falling edges.

Section 1.4 Summing Amplifier

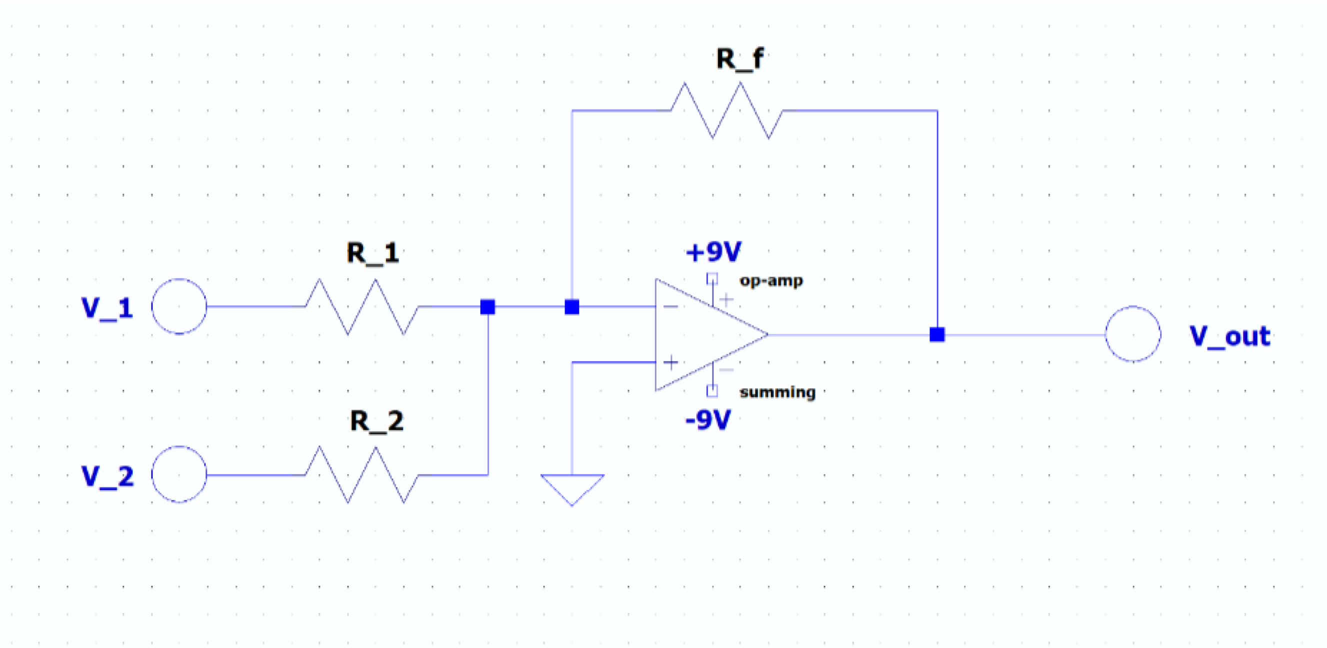
Objective:

The objective of this portion of the lab is to convert our inverting op-amp circuit into a summing op-amp circuit.

Equipment:

- 1x Resistor
- 1x Capacitor

- 1x Op-Amp
- Breadboard
- 9v battery and power harness
- Oscilloscope
- Digital Multimeter
- Jumpers
- inverting circuit #### Procedure: We can convert the inverting circuit into a summing circuit by adding a secondary resistor to the input leg of the op-amp. Choose appropriate resistors, between 1-5kΩ, to find appropriate values of amplification. Keep in mind that any difference in resistors will result in a different weighting for each added component. Follow figure 5 below for more detailed instructions.



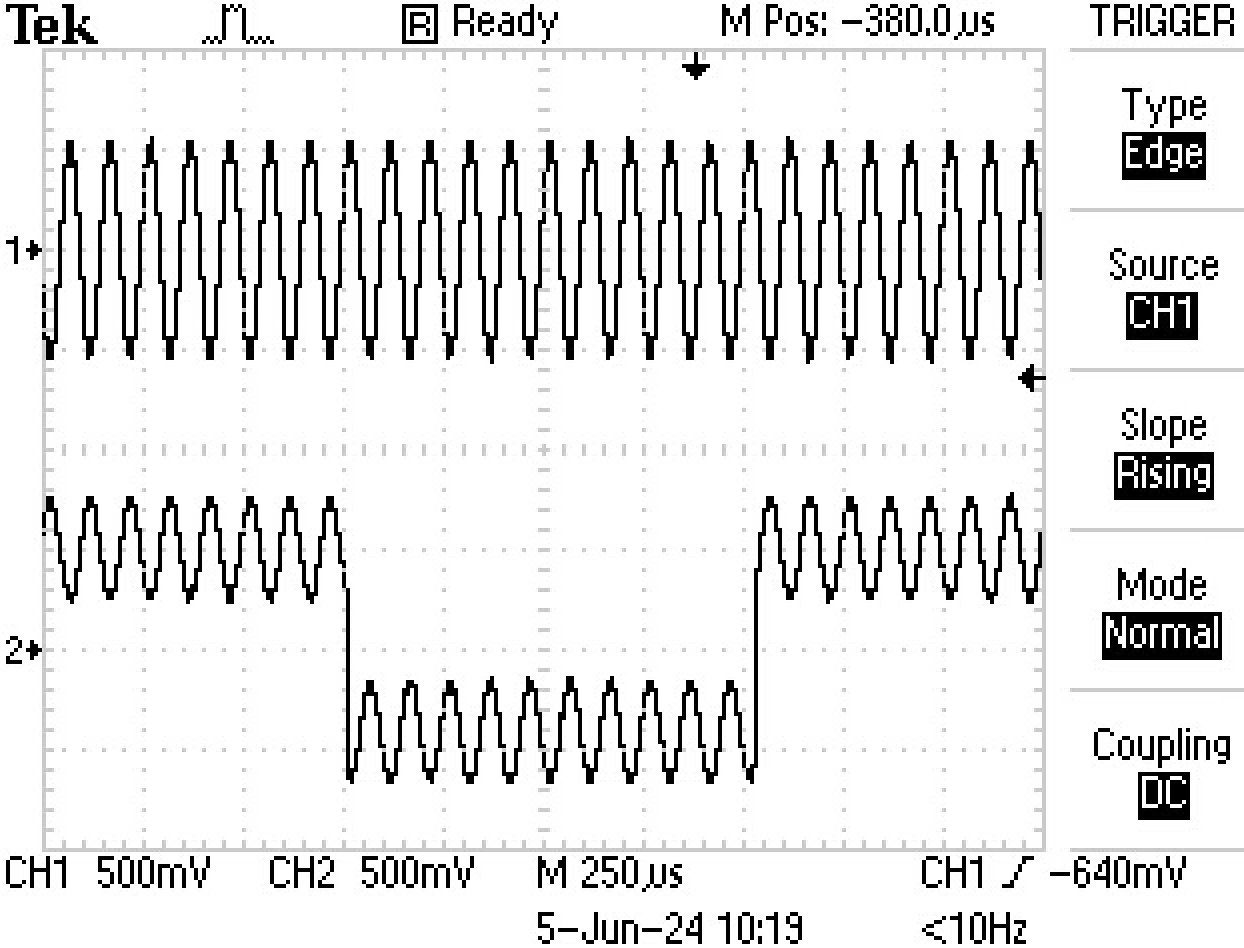
> Figure 5

After constructing the adder, we need to test it. We can test this circuit using 2 waveforms, finally utilizing both function generators. The first function generator will be set to a square wave of about 500 Hz, and the second will be a higher frequency sine wave, at about 10 kHz. We can observe the qualitative effects of this on the oscilloscope to confirm our adder is working correctly.

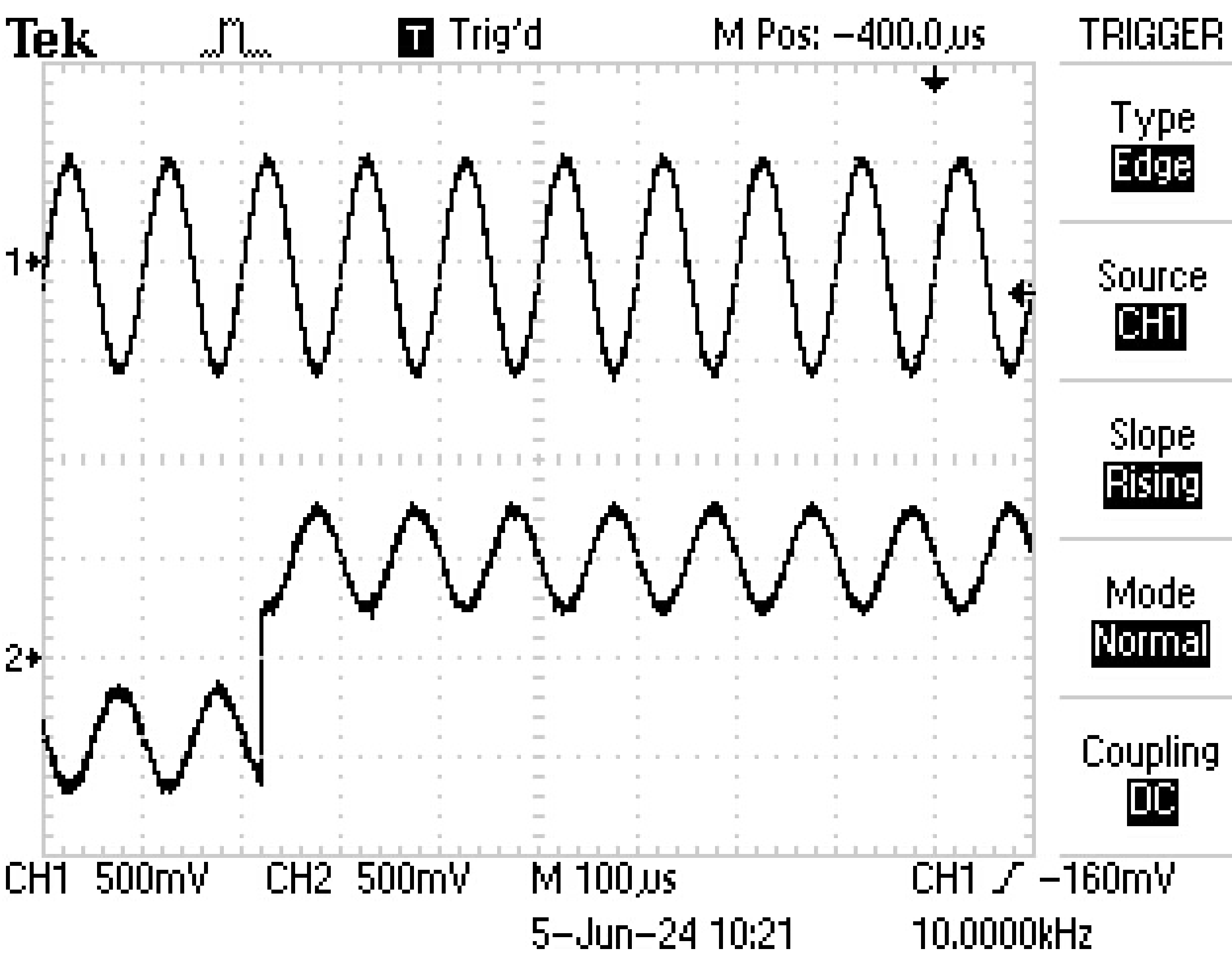
Data

	Components
R_f	3.931 $k\Omega \pm 0.9\%$
R_1	1.782 $k\Omega \pm 0.9\%$
R_2	1.782 $k\Omega \pm 0.9\%$

Values of the components used in the adder circuit.



The figure above shows the result of adding a high frequency sine wave and a lower frequency square wave.



This figure is a more magnefied equivalant of the previous image, showing the rising edge behavior more clearly.

Analysis

The circuit provides a clear and intuitive model for layering waveforms on top of eachother. The square and sine wave example provides a very easy to understand interpretation of the action of the adding circuit, layering the sine waves and the square waves together while preserving both frequencies and waveforms.

Section 2: Solving the simple harmonic oscillator

Objective:

Use the components constructed in section 1 to solve the simple harmonic oscillator's differential equation continuously.

Equipment:

- Breadboard
- Oscilloscope
- Jumper Cables
- Resistor Assortments
- Capacitor Assortments
- Op-Amps (TLE2071CP)
- Digital Multi-meter
- 9v Battery and Harness
- USB Flash Drive
- Function Generator

Procedure:

The first step to building the analog computer and solving these differential equations is to design the aforementioned analog computer. This can be done by analytically thinking about the task at hand, and breaking it down into components that can be built. The task at hand can be accomplished with 2 integrating circuits and an adder. It can be helpful to think of individual signals between components as parts of the differential equation, x, $\dot{x}$ and $\ddot{x}$. The design that we settled on is pictured below (figure 6) as a block form, and in figure 7 as a circuit form.

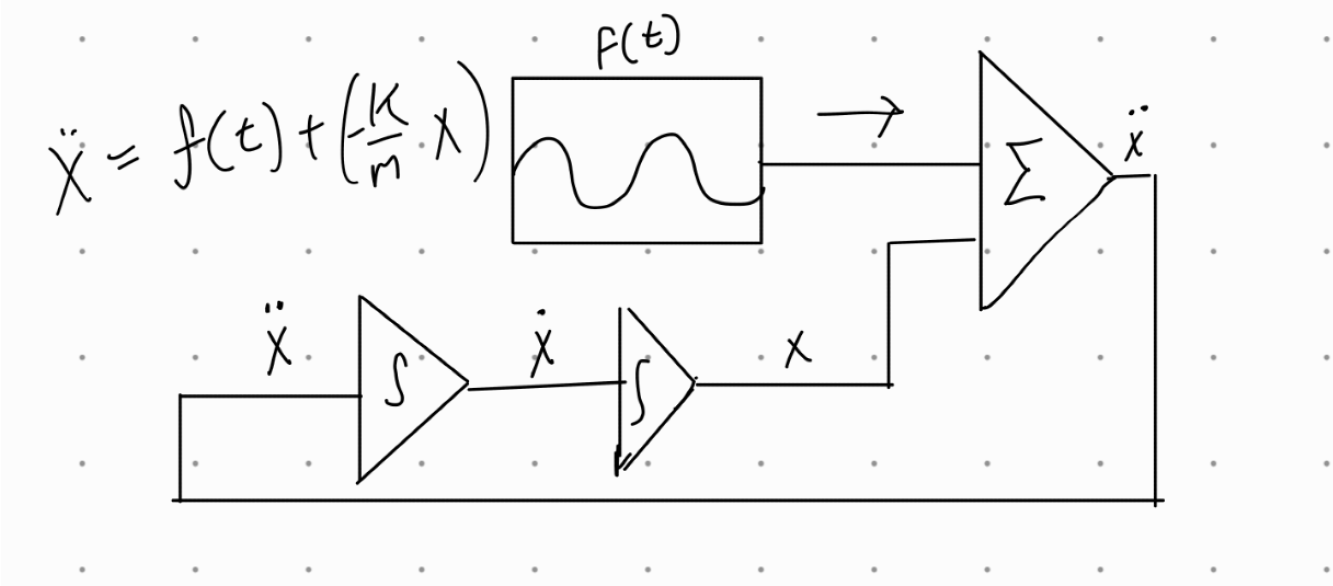

$\ddot{x} = f(t) + \left(-\frac{k}{m}x\right)$
F(t)
Σ
$\ddot{x}$
$\ddot{x}$
$\int$
$\dot{x}$
$\int$
x

Figure 6

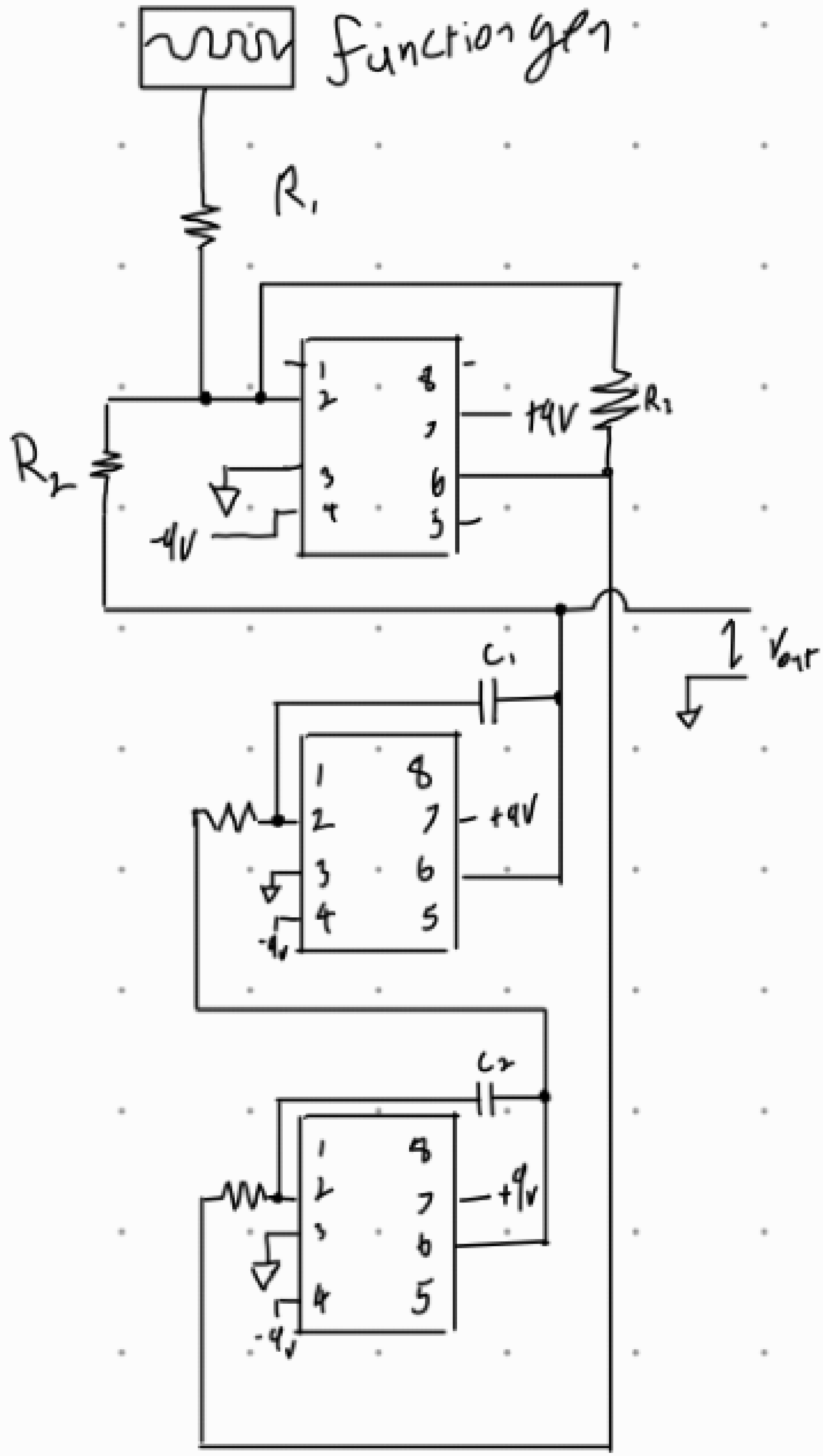


Figure 7

Before we construct this circuit we have to consider what values of resistance and capacitance we should use to ensure the broadest possible range of integration. The literature for this lab suggests values of 0.01 μf and 1 kΩ, which is what we used. It is also worth noting that due to the inverting nature of the op-amps, we will get an inverted answer. This will be proven in the data and analysis section.

After constructing this circuit, we have to provide a driving force. Connecting a function generator to the input leg of the analog adder will yield the required forcing and allow us to observe the harmonic oscillations.

Finally it is time to gather data, beginning by providing a low-frequency square pulse, we can use the oscilloscope to view and capture the resulting oscillatory action. It is also suggested to change values of the resistance on the adder such that continual, non-dampened oscillations can be observed. We made sure to capture data on oscillation amplitude and frequency using the cursor function on the oscilloscope.

Data

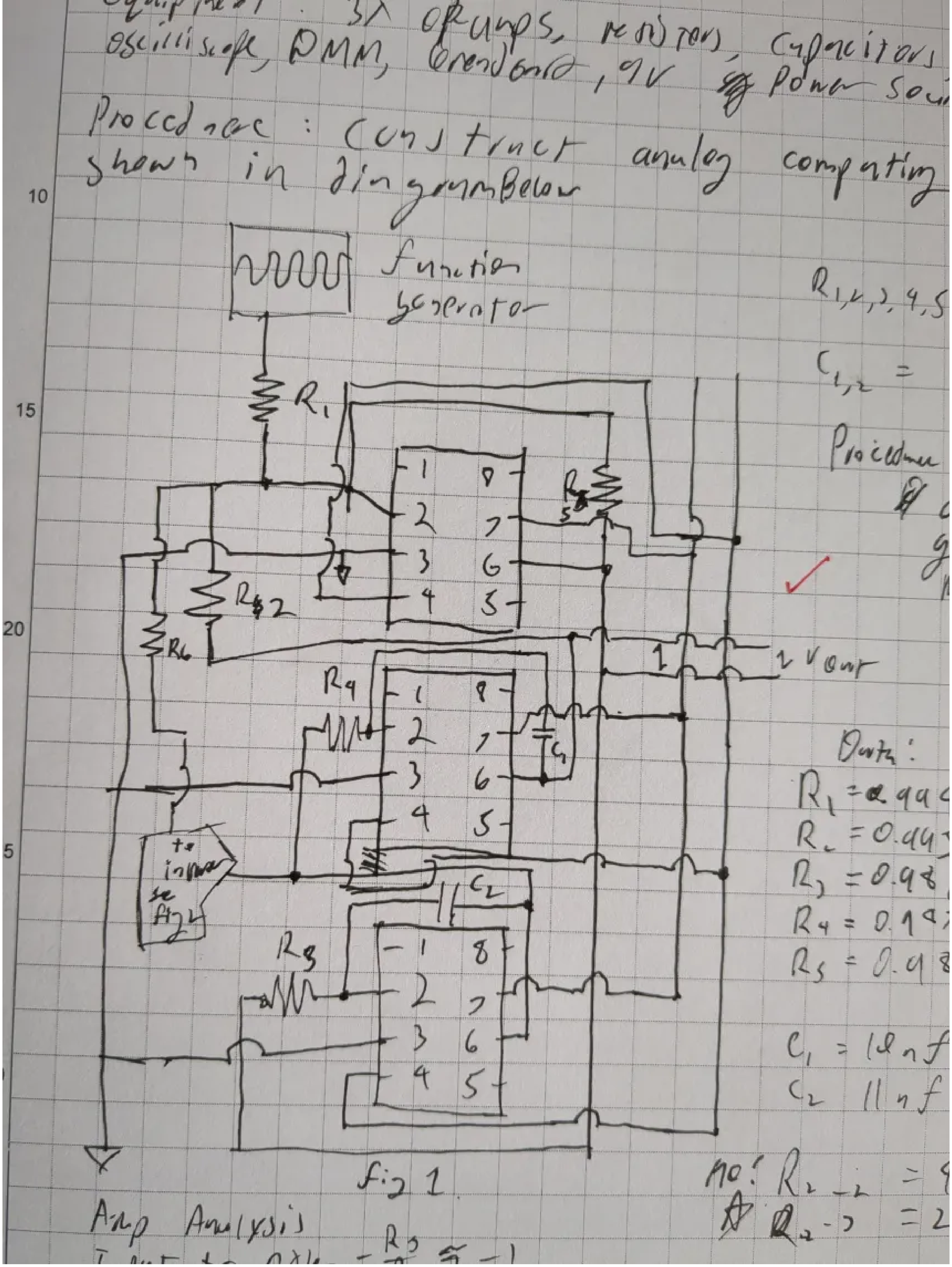


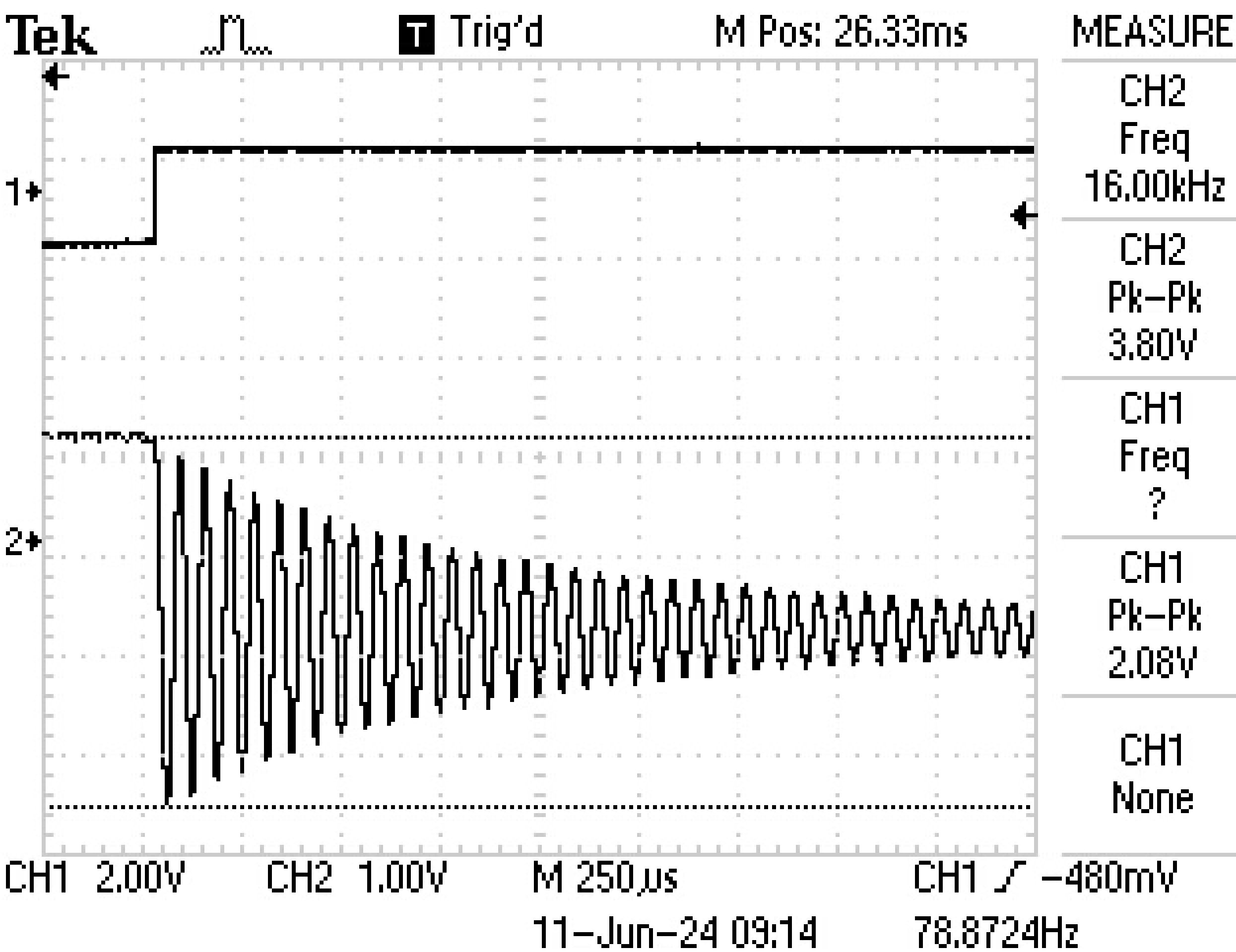
Diagram with numbered components.

Resistors

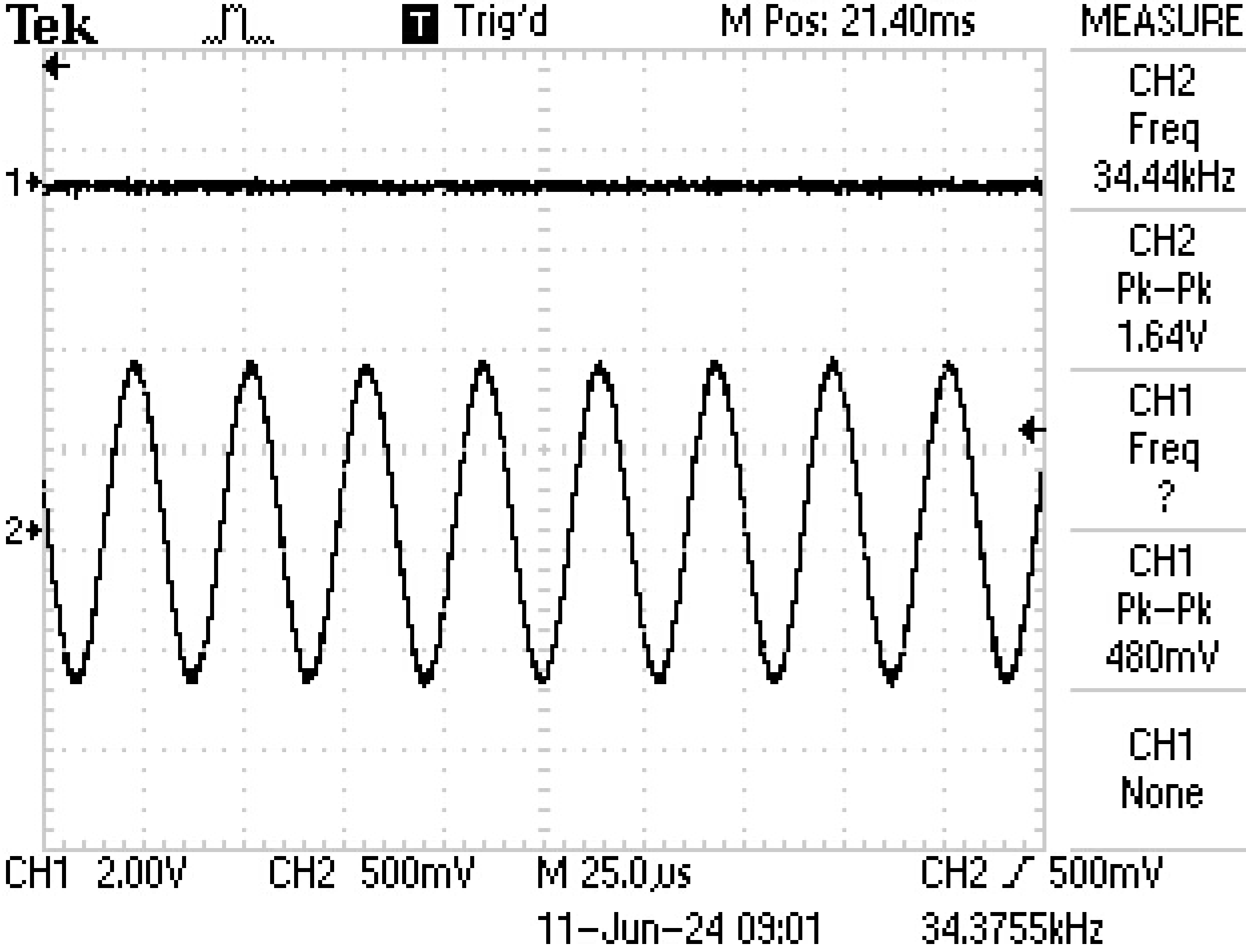
R_1	0.994 $k\Omega$ $\pm 0.9\%$
R_2	0.995

	$k\Omega$ $\pm 0.9\%$
R_3	0.986 $k\Omega$ $\pm 0.9\%$
R_4	0.987 $k\Omega$ $\pm 0.9\%$
R_5	0.986 $k\Omega$ $\pm 0.9\%$

Capacitors	
C_1	10 nF $\pm 1.9\%$
C_2	11 nF $\pm 1.9\%$



The above image shows the oscillations directly after the focing "motion" is applied



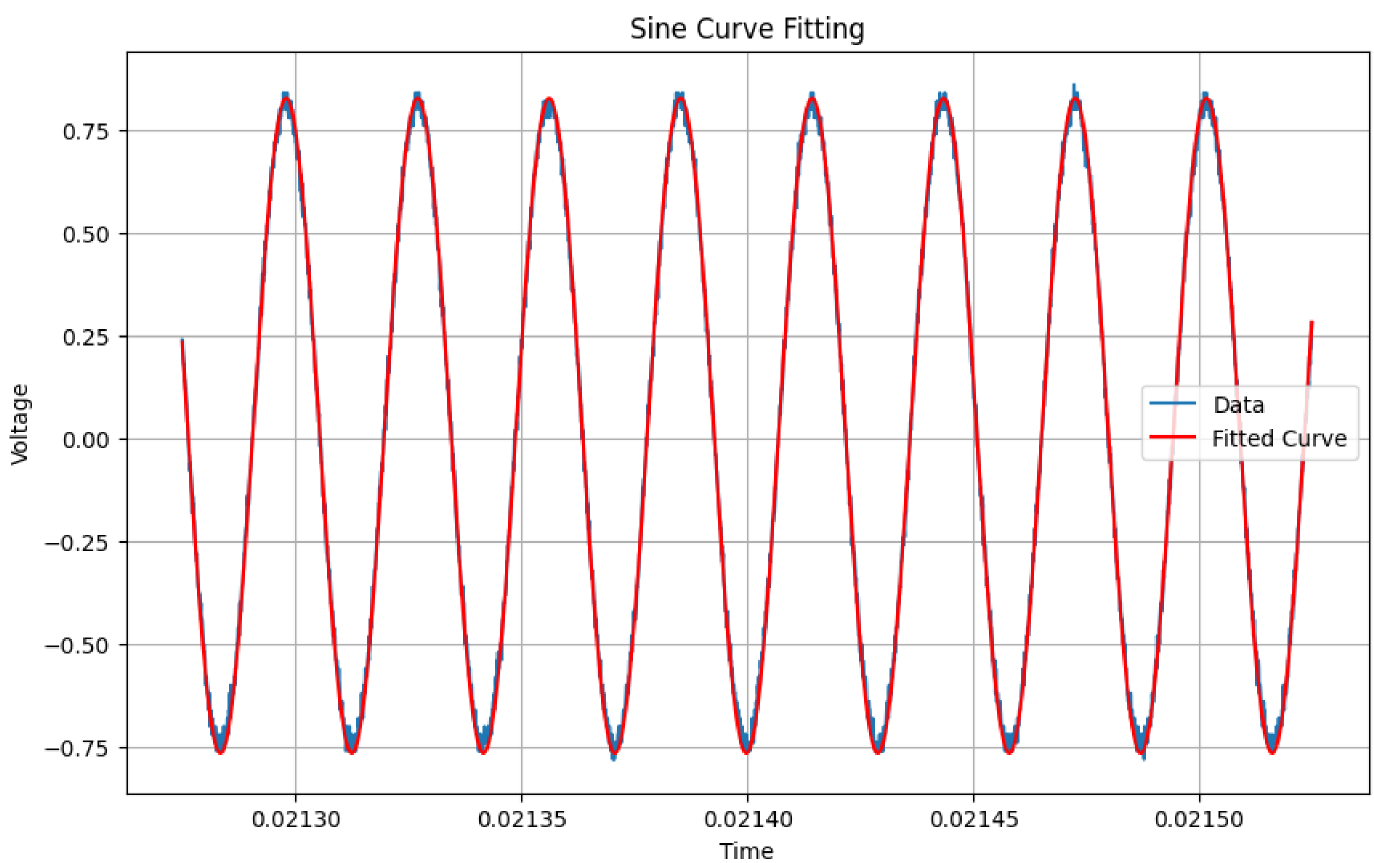
The above image demonstrates the conditions under which the system oscillates by itself by replacing R_2 with a resistor measured at 218.4 ohms $\pm 0.9\%$

Analysis

To find the natural frequency and damping of oscillations, we will fit a sine curve to the data with orthogonal distance regression. We are going to use the stable oscillation condition with R_2

In [51]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit, leastsq
from scipy.odr import Model, RealData, ODR

# Define the decaying sine function
def sine(params, t):
    A, omega, phi, C = params
    return A * np.sin(omega * t + phi) + C

# Read CSV file
def read_csv(filename):
    df = pd.read_csv(filename)
    t = df['t'].values
    V = df['V'].values
    return t, V

# Load data
filename = 'data.csv'
t, V = read_csv(filename)

# Initial guess for parameters
```

```python
A_guess = 15
omega_guess = 215000
phi_guess = 0.3
C_guess = 1
initial_guess = [A_guess, omega_guess, phi_guess, C_guess]

# Define the ODR model
model = Model(sine)
data = RealData(t, V)
odr = ODR(data, model, beta0=initial_guess)

# Run the regression
odr_result = odr.run()

# Extract the fitted parameters
params_fit = odr_result.beta

# Generate fitted curve
V_fit = sine(params_fit, t)

# Plot the data and the fitted curve
plt.figure(figsize=(10, 6))
plt.plot(t, V, label='Data')
plt.plot(t, V_fit, 'r-', label='Fitted Curve')
plt.xlabel('Time')
plt.ylabel('Voltage')
plt.title('Sine Curve Fitting')
plt.legend()
plt.grid(True)
plt.show()

# Print the fitted parameters
print("Fitted Parameters:")
print("A:", params_fit[0])
print("omega:", params_fit[1])
print("phi:", params_fit[2])
print("C:", params_fit[3])
```



```
Fitted Parameters:
A: -0.7963437124475999
omega: 216027.28932029623
phi: -22.07888097138799
C: 0.0307785571238096
```

Following the method described in the appendix of the Analog Computer Theory, we have the following relations:

$$V_{out} = -\frac{1}{RC}\int V_{in} dt$$

and in the context of the analog computer

$$\dot{x} = \alpha \int \ddot{x} dt$$

and

$$x = \beta \int \dot{x} dt$$

Therefore

$$x = \beta \int \alpha \int \ddot{x} dt\, dt$$

We can factor out beta and alpha

$$x = \beta\alpha \iint \ddot{x}\, dt$$

And we know that $\beta\alpha$ is equal to the following:

$$-\frac{1}{R_1 C_1}\left(-\frac{1}{R_2 C_2}\right)$$

Or using our experimentally measured values:

$$\alpha\beta = 9341427493.4278 \qquad \begin{aligned} \Delta ab &= \sqrt{\left(\frac{\partial ab}{\partial R}\Delta R\right)^2 + \left(\frac{\partial ab}{\partial C}\Delta C\right)^2 + \left(\frac{\partial ab}{\partial R_1}\Delta R_1\right)^2 + \left(\frac{\partial ab}{\partial C_1}\Delta C_1\right)^2} \\ &= \sqrt{\begin{aligned} &\left(\left(-(C_1/(R*C*R_1*C_1)^2 * R_1 * C)\right)\cdot \Delta R\right)^2 \\ &+ \left(\left(-(C_1/(R*C*R_1*C_1)^2 * R_1 * R)\right)\cdot \Delta C\right)^2 \\ &+ \left(\left(-(C_1/(R*C*R_1*C_1)^2 * R * C)\right)\cdot \Delta R_1\right)^2 \\ &+ \left(\left(-(R/(R*C*R_1*C_1)^2 * C * R_1)\right)\cdot \Delta C_1\right)^2 \end{aligned}} \\ &= 277740606.63077885 \\ &= 2.7774060663 \times 10^8 \\ &= 3 \times 10^8 \end{aligned}$$

$$ab = (9.3 \pm 0.3) \times 10^9$$

To find the final proportionality constant, the gain of the summing circuit also has to be accounted for. We can use the following to find this gain:

$$-\frac{R_f}{R_{in}}$$

And for our summing amplifier

$$\Delta R = \sqrt{\left(\frac{\partial R}{\partial R_f}\Delta R_f\right)^2 + \left(\frac{\partial R}{\partial R_i}\Delta R_i\right)^2}$$

$$R = -4.5146520147 \qquad \begin{aligned} &= \sqrt{\left((-(1/R_i))\cdot\Delta R_f\right)^2 + \left(\left(R_f/R_i^2\right)\cdot\Delta R_i\right)^2} \\ &= 0.057462139 \\ &= 5.7462139\times 10^{-2} \\ &= 6\times 10^{-2} \end{aligned}$$

$$R = (-4.51 \pm 0.06)$$

Finally, we get our total proportionality constant:

$$\alpha\beta\cdot R =$$

$$\Delta p = \sqrt{\left(\frac{\partial p}{\partial R}\Delta R\right)^2 + \left(\frac{\partial p}{\partial ab}\Delta ab\right)^2}$$

$$p = -41943000000 \qquad \begin{aligned} &= \sqrt{((ab)\cdot\Delta R)^2 + ((R)\cdot\Delta ab)^2} \\ &= 1463548085.9882944 \\ &= 1.463548086\times 10^9 \\ &= 1\times 10^9 \end{aligned}$$

$$p = (-4.2\pm 0.1)\times 10^{10}$$

Applied to the harmonic oscillator differential equation

$$\ddot{x} = -\frac{k}{m}x$$

$$-\frac{k}{m} = (-4.2\pm 0.1)\times 10^{10}$$

and therefore the frequency is related in the following manner:

$$\omega = \sqrt{\frac{k}{m}} = 204939.015319 \pm 31622.7766017$$

And with this, we can see the frequency obtained in the sinusoidal regression in the python script is within the propagated uncertainty values.

Section 3: Damping the harmonic oscillator

Objective:

Using the circuit from section 2 our goal is to introduce a damping term $\frac{\gamma}{m}$ using an inverting op-amp circuit tied into our existing circuit.

Equipment:

Everything from section 2, plus another op-amp chip, resistors and necessary jumper cables.

Procedure:

Begin by designing the appropriate modifications to include the damping term by connecting in to the "$\dot{x}$" signal in the analog computer and adding it to the $\ddot{x}$ term using the existing adder circuit. Using three more resistors, we can adjust the value of damping to something that will be easy to visualize on the oscilloscope. The modifications are outlined in figure 8 in purple.

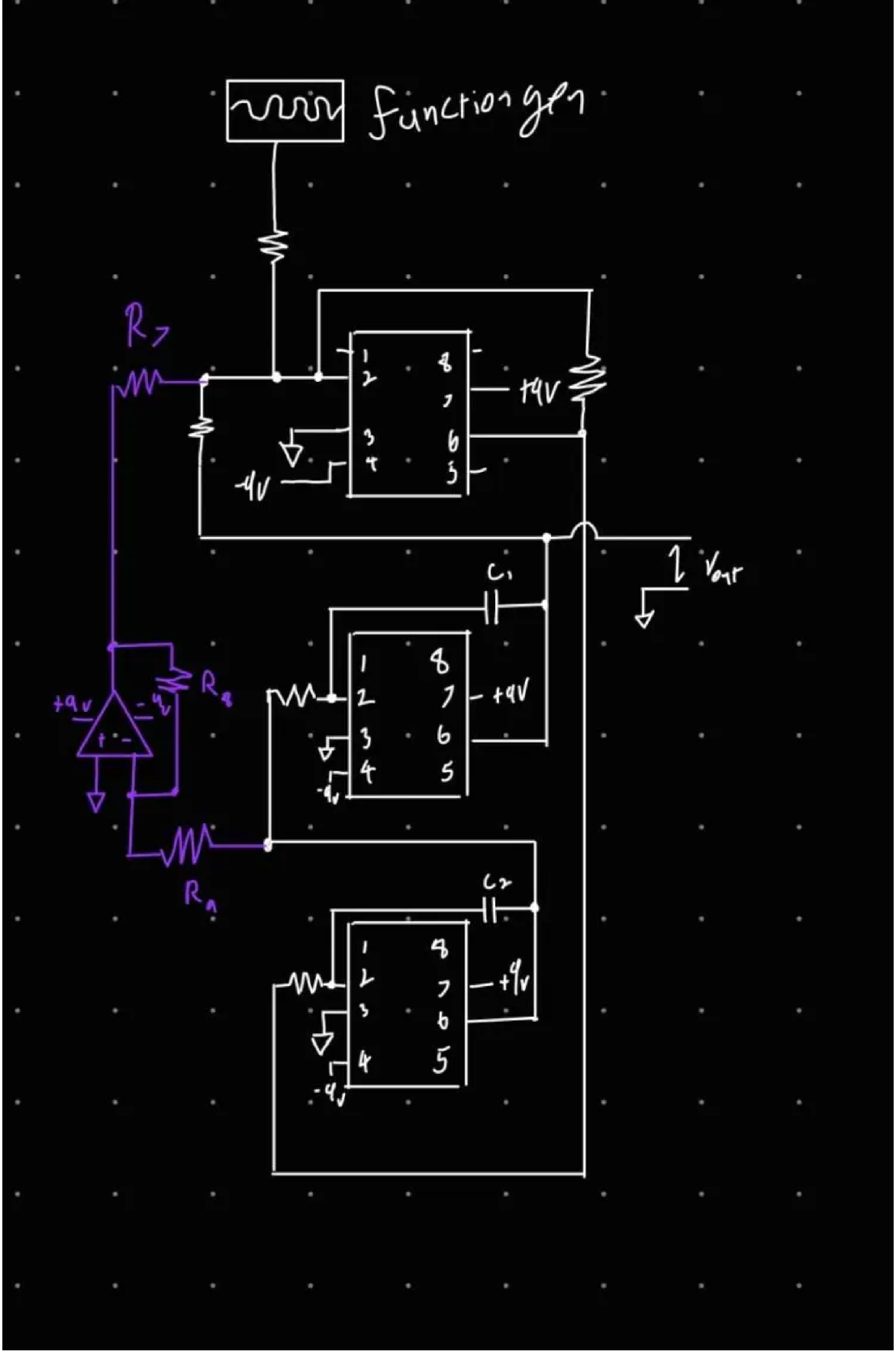


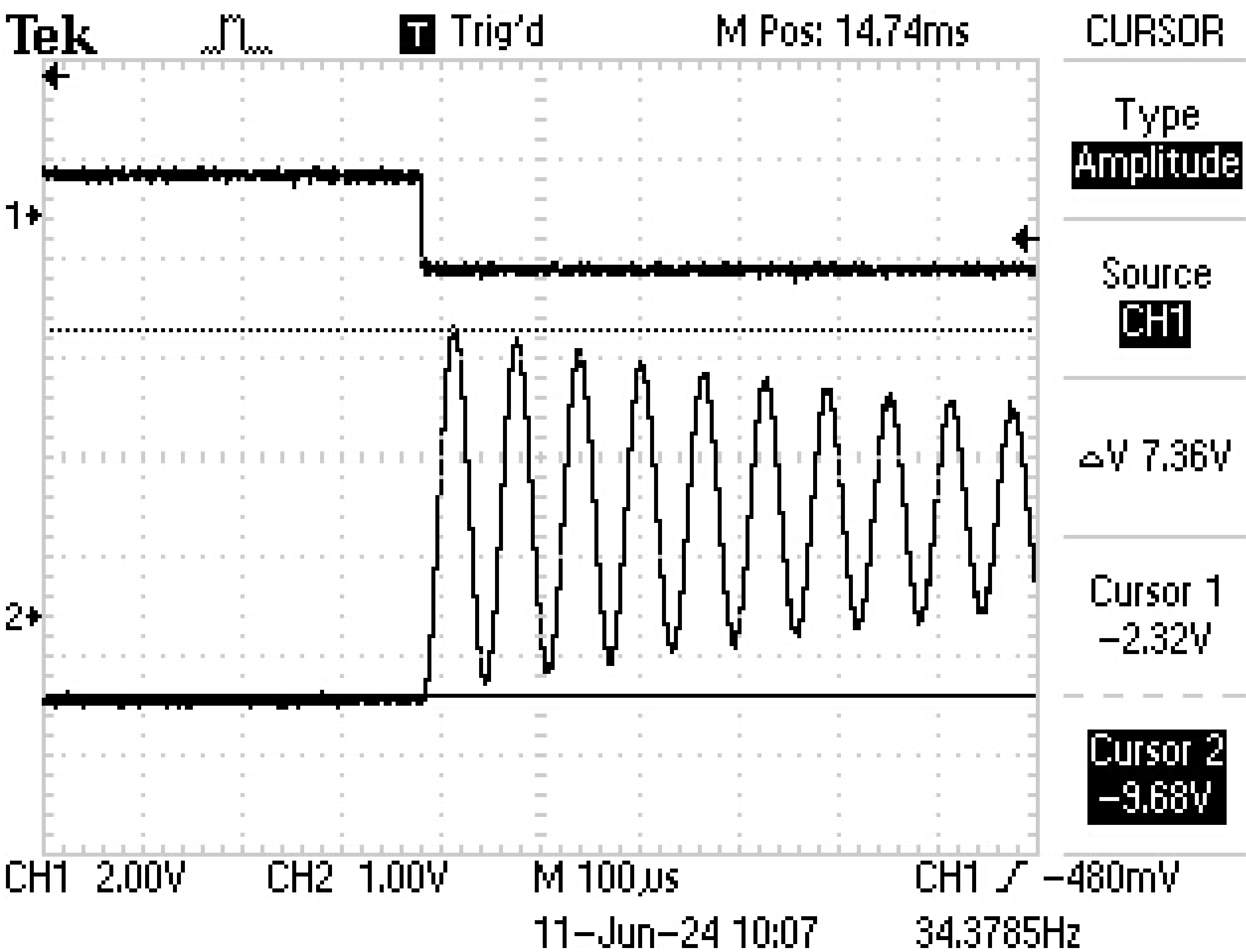
Figure 8

After making the appropriate modifications to the circuit, we can proceed to collect our data. It is important to first take a screenshot of the oscilloscope with the damper portion deactivated to account for any changes in the natural damping of the circuit. Compare this reading to the reading obtained after the damper is connected.

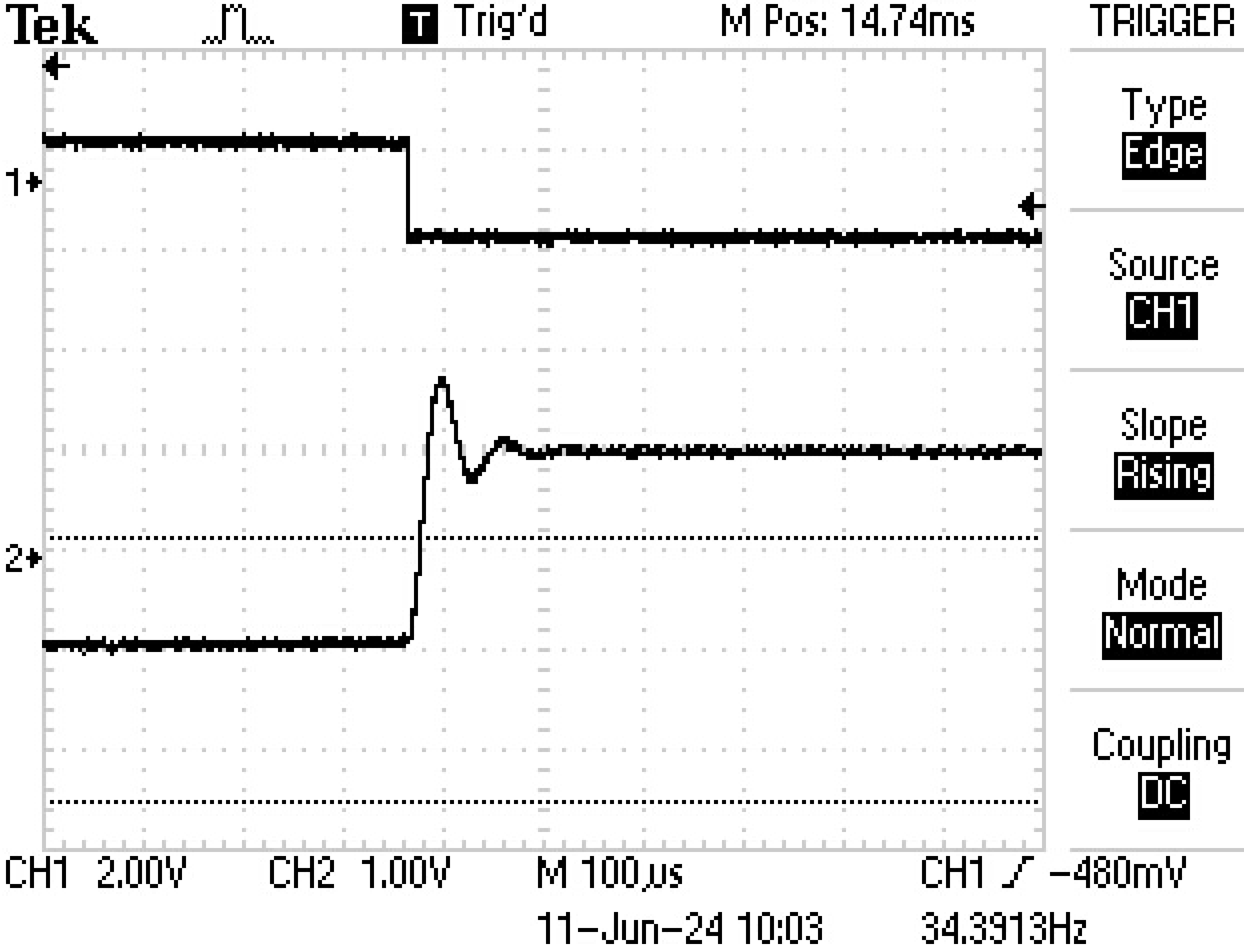
Data

The resistors in table below correspond to the resistors in the purple part of the diagram in figure 8. | | Resistors | |-----|-----------------------------| | R_7 | 0.980 $k\Omega \pm 0.9\%$ | | R_8 | 0.991 $k\Omega \pm 0.9\%$ | | R_9 | 1.789 $k\Omega \pm 0.9\%$ |

With Resistor 9 removed from the circuit and the function generator set to a 34.4hz square wave:



With Resistor 9 reinstalled in the circuit and the function generator set to the same 34.4hz square wave:

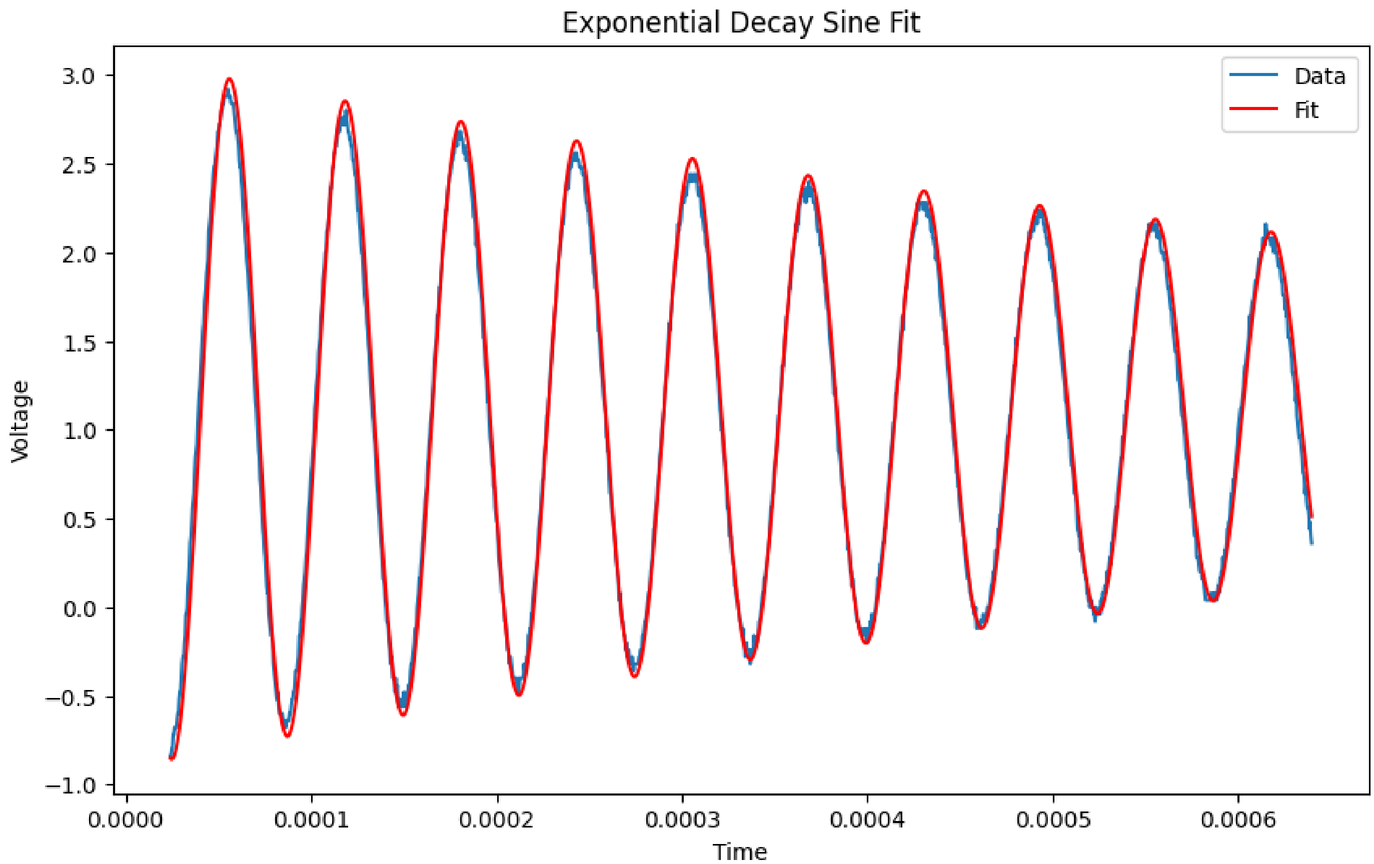


Analysis

In [209…

```
def exp_decay_sine(beta, t):
    A, lambda_, omega, phi,c  = beta
    return A * np.exp(-lambda_ * t) * np.sin(omega * t + phi) + c

def fit_exp_decay_sine_odr(t_data, y_data):
    # Initial guess
    initial_guess = [1, 1000, 100500, 0.5,0]
    model = Model(exp_decay_sine)
    data = RealData(t_data, y_data)
    odr = ODR(data, model, beta0=initial_guess)
    output = odr.run()
    params = output.beta
    y_fit = exp_decay_sine(params, t_data)
    return params, y_fit

data = pd.read_csv('data2.csv')
t_data = data.iloc[:, 0].values
y_data = data.iloc[:, 1].values
#Trimming data so it starst at 0
t_data = t_data - 0.0146
params, y_fit = fit_exp_decay_sine_odr(t_data, y_data)

# Plot the results
plt.figure(figsize=(10, 6))
plt.plot(t_data, y_data,  label='Data')
plt.plot(t_data, y_fit, 'r-', label='Fit')
plt.xlabel('Time')
plt.ylabel('Voltage')
plt.legend()
plt.title('Exponential Decay Sine Fit')
```

```
plt.show()

print(f"Fitted parameters: A={params[0]:.3f}, lambda={params[1]:.3f}, omega={params[2]:.
```




```
Fitted parameters: A=2.004, lambda=1090.942, omega=100664.100, phi=2.210
```

We want to find the theoretical damping coefficient γ. This is going to be equivalent to the coefficient of the inverting amplifier combined with the gain applied to it from the summing amplifier multiplied by alpha from the previous section.

$$\alpha = \frac{1}{R_3C_1}, \text{ Inverter gain} = -\frac{R_8}{R_9}, \text{ Summer gain } = -\frac{R_5}{R_7}$$

$$\frac{\gamma}{m} = -\frac{R_8R_5}{R_3C_1R_7R_9}$$

$$\gamma/2m =$$

-28262.2831133571

$$\begin{aligned}
\Delta\gamma/2m &= \sqrt{\left(\frac{\partial\gamma/2m}{\partial R_9}\Delta R_9\right)^2 + \left(\frac{\partial\gamma/2m}{\partial R_8}\Delta R_8\right)^2 + \left(\frac{\partial\gamma/2m}{\partial R_7}\Delta R_7\right)^2 + \left(\frac{\partial}{}\right. \\ + \left(\frac{\partial\gamma/2m}{\partial R_5}\Delta R_5\right)^2 + \left(\frac{\partial\gamma/2m}{\partial R_3}\Delta R_3\right)^2} \\
&= \sqrt{\begin{aligned}&\left(\left(R_8/(R_3*C_1*R_7*R_9)^2/2*R_5*R_3*C_1*R_7\right)\cdot\Delta R_9\right)^2 \\ &+\left(\left(R_5*-1/2/(R_3*C_1*R_7*R_9)^2*R_3*C_1*R_7*R_9\right)\cdot\Delta R_8\right. \\ &+\left(\left(R_8/(R_3*C_1*R_7*R_9)^2/2*R_5*R_9*R_3*C_1\right)\cdot\Delta R_7\right)^2 \\ &+\left(\left(R_8/(R_3*C_1*R_7*R_9)^2/2*R_5*R_9*R_7*R_3\right)\cdot\Delta C_1\right)^2 \\ &+\left(\left(R_8*-1/2/(R_3*C_1*R_7*R_9)^2*R_3*C_1*R_7*R_9\right)\cdot\Delta R_5\right. \\ &+\left(\left(R_8/(R_3*C_1*R_7*R_9)^2/2*R_5*R_9*R_7*C_1\right)\cdot\Delta R_3\right)^2\end{aligned}} \\
&= 782.20687253 \\
&= 7.8220687253\times 10^2 \\
&= 8\times 10^2
\end{aligned}$$

$$\gamma/2m = (-2.83\pm 0.08)\times 10^4$$

Divided by two to yield the $\frac{\gamma}{2m}$ term we need

This yields our theoretical damping constant introduced from the addition of the inverting amplifier.

The general form for the oscillator with damping:

$$x(t) = A\cdot e^{-\gamma t/2m}\cos(\omega t-\phi)$$

Using exponentially decaying sinusoidal regression, we were able to find the parameters that matched the naturally damped case above.

Fitted parameters: A=2.004, lambda=1090.942, omega=100664.100, phi=2.210

Plugging these in:

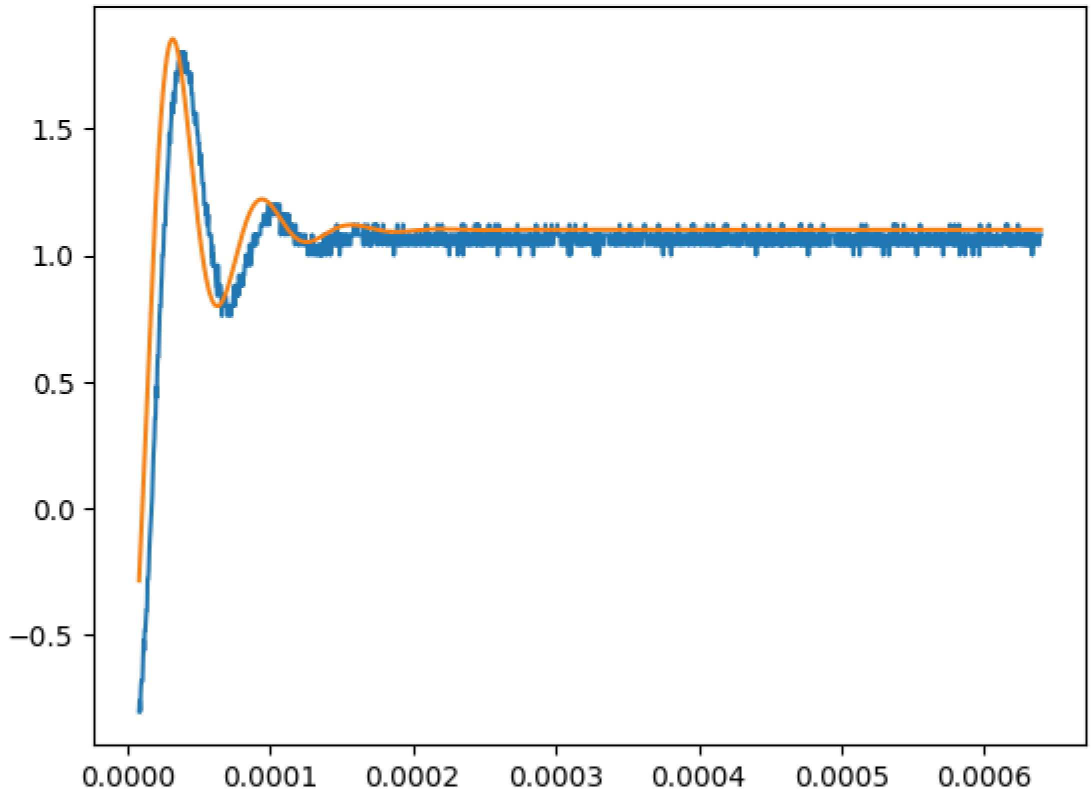
$$x(t) = 2.004\cdot e^{1090.942t}\cos(100664.1t-2.210)$$

Now, with the damping circuit constant of $(-2.83\pm 0.08)\times 10^4$ we can compare and see if this yields the expected result.

$$x'(t) = 2.004\cdot e^{1090.942+(-2.83\pm 0.08)\times 10^4 t}\cos(100664.1t-2.210)$$

In [216…

```
data2 = pd.read_csv("data3.csv")
t_vals2 = data2.iloc[:, 0].values
y_vals2 = data2.iloc[:, 1].values
t_vals2 = t_vals2-0.0146

plt.plot(t_vals2, y_vals2)
def decay_function(t):
    return 2.004 * np.exp((-1090.942 - 28262) * t) * np.cos(100664.1*t-3.5) + 1.1
plt.plot(t_vals2, decay_function(t_vals2))
plt.show()
```



This looks really good! Seems like the theorized values and experimentally verrified values closely align.

Section 4: Nonhomogenous boundary beats

Objective:

Using two function generators we want to observe beating behavior near the natural oscillation frequency of the analog computer.

Equipment:

Everything from section 2 plus a second function generator

Procedure :

Begin by adding a second input to the adder circuit Connect the second function generator to this new input and set it to a sine waveform of approximately the natural harmonic frequency. Adjust the frequency until clear beating motion can be observed, and capture data. It is easiest to use the "Run/Stop" feature of the oscilliscope to clearly capture the beating patterns in the waveforms. A diagram of the experimental setup is in figure 9 below:

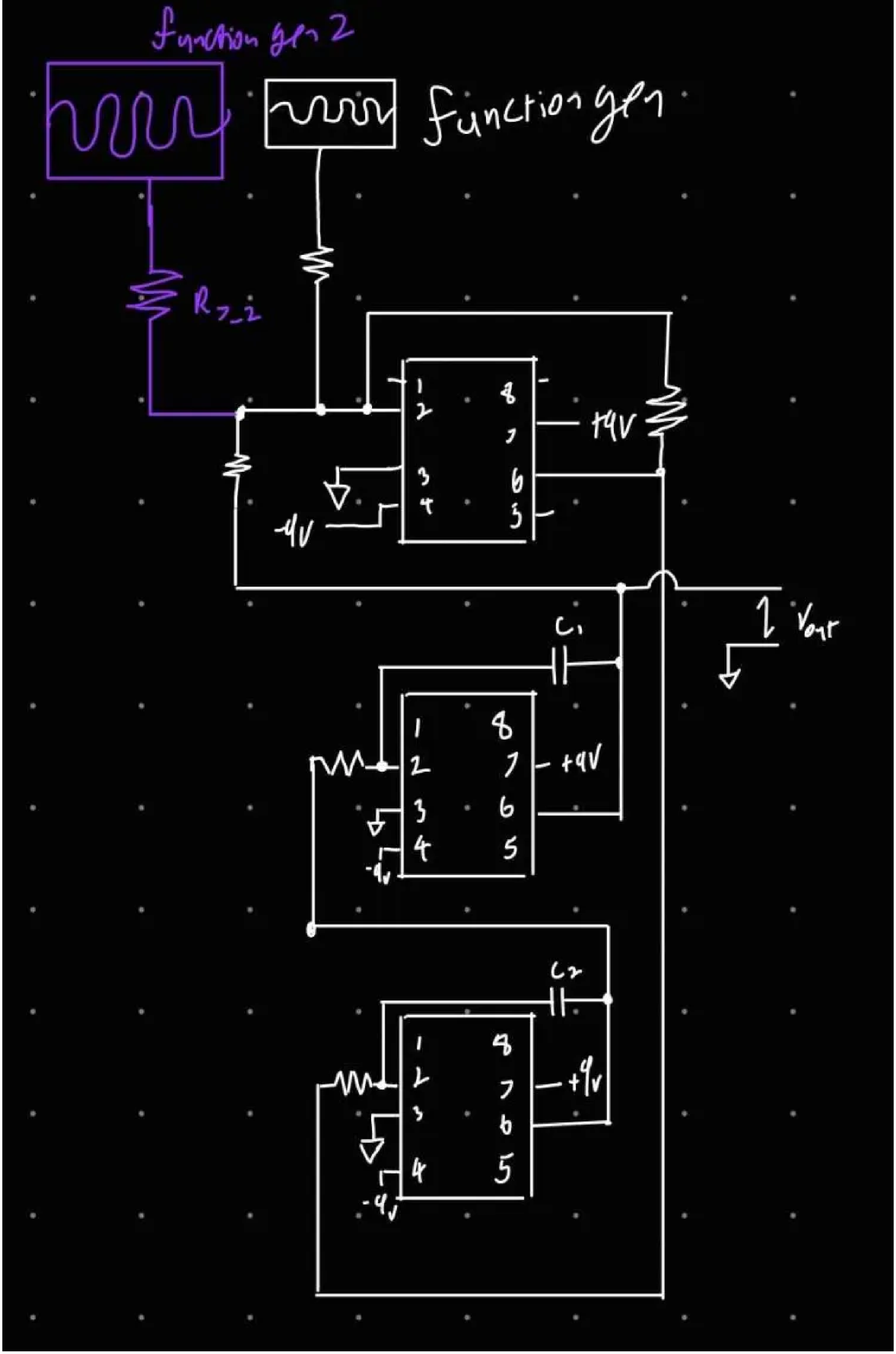
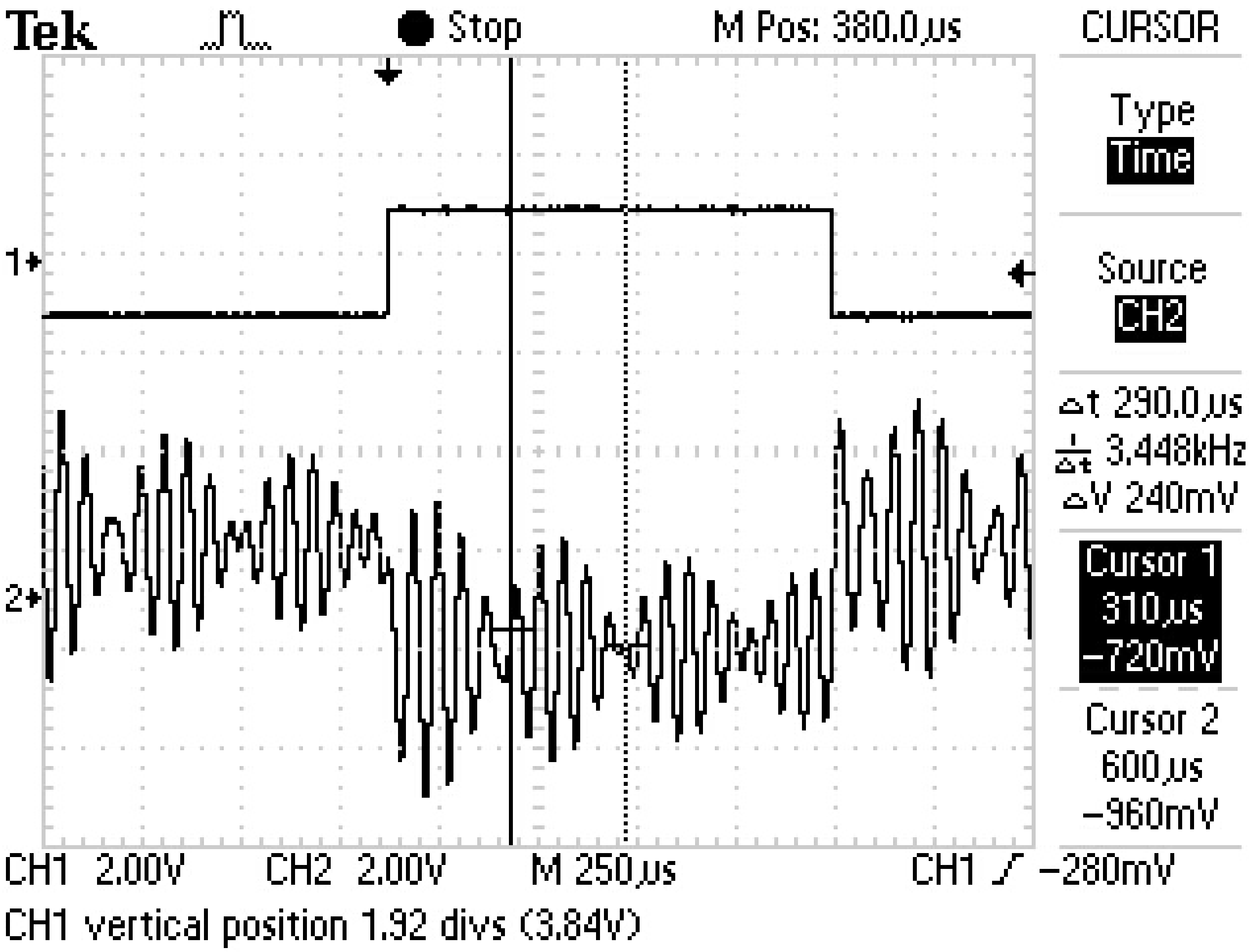

function gen 2
function gen
R_{7_2}
+9V
-9V
C_1
V_{out}
C_2
+9V
-9V
+9V
-9V

Figure 9

Data:

Resistor R_7-2 is measured to be 2.161 $k\Omega \pm 0.9\%$

Function generator #1 is set to a 450hz square wave. Function generator #2 is set to a 17.83kHz sine wave



As seen in the screenshot above, the frequency of the beats is approximately 3.5 kHz.

Analysis:

The general solution to a beat problem is given by the following:

$$V_o(t) = \frac{A}{\omega_0^2 - \omega^2} \cos \omega t + B \sin \omega t$$

The beat frequency should be the difference between the natural frequency of the harmonic oscillator and the frequency of the second function generator.

Calculating the theoretical beat frequency: Natural Harmonic Frequency $\approx$ 15.6 kHz as measured on oscilloscope Function Generator is set to 17.83 kHz

$$17.83 - 15.3 = 2.53\text{kHz}$$

This isn't exactly the expected value, but given the uncertainty in measuring beat frequency due to the unclear edges in our oscilloscope reading, it is a reasonably close value.