Quantum Physics 1
Class 3

Class 3

Wave nature of light

What is light?

- Visible light, x-rays, γ-rays, μ-wave
- Waves!
- Waves interfere!!!

Review:

PDE:

$$\frac{\partial^2 E(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E(x,t)}{\partial t^2} = 0$$

Solve? Use separation of variables,

let $E(x,t) = Q(x)\phi(t)$

$$\therefore \frac{\partial^2 Q(x)}{\partial x^2} = s\,Q(x) \quad \ldots \text{①}$$

$$\frac{\partial^2 \phi(t)}{\partial t^2} = c^2 s\,\phi(t) \quad \ldots \text{②}$$

guess solution: $E(x,t) \sim e^{\pm i(kx-\omega t)}$ where $Q(x) \sim e^{\pm ikx}$, $\phi(t) \sim e^{\pm i\omega t}$

In case of superposition:

If E_1 & E_2 are solutions, then

$E_{tot} = E_1 + E_2$ is also a solution.

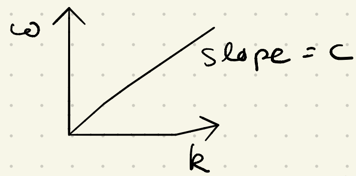
where $E_1 \sim e^{i(k_1x-\omega_1 t)}$ & $E_2 \sim e^{i(k_2x-\omega_2 t)}$

In-class #3.1

Example:

$$E_1 = e^{i(k_1 x - \omega_1 t)}, \quad E_2 = e^{i(k_2 x - \omega_2 t)}$$

where $\omega = ck$

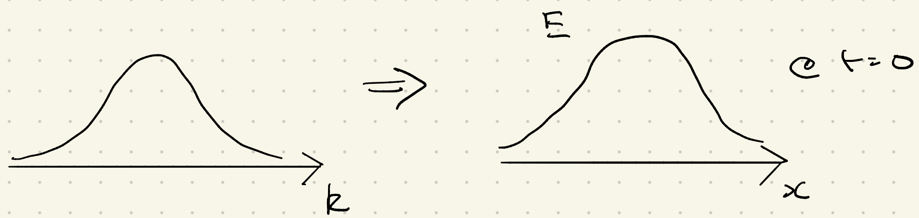


Now, $E_{tot} = E_1 + E_2 = A e^{i(k_1 x - \omega_1 t)} + B e^{i(k_2 x - \omega_2 t)}$

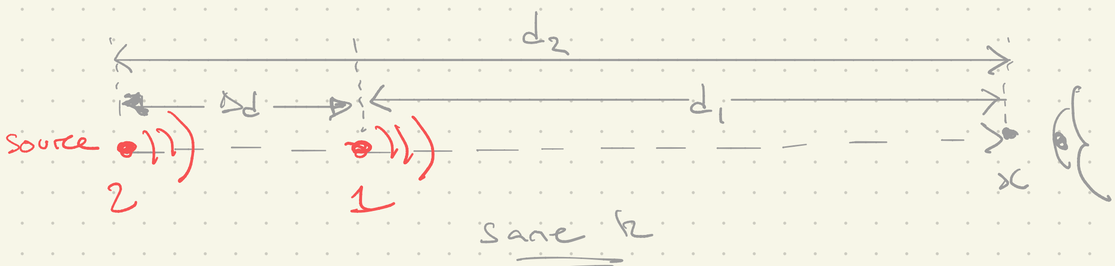
where in general: $E = \int g(k) e^{i(kx - \omega t)} dk$

consider $t = 0$, $E(x,0) = \int g(k) e^{ikx} dx$

w/ distribution of k:



Example #2



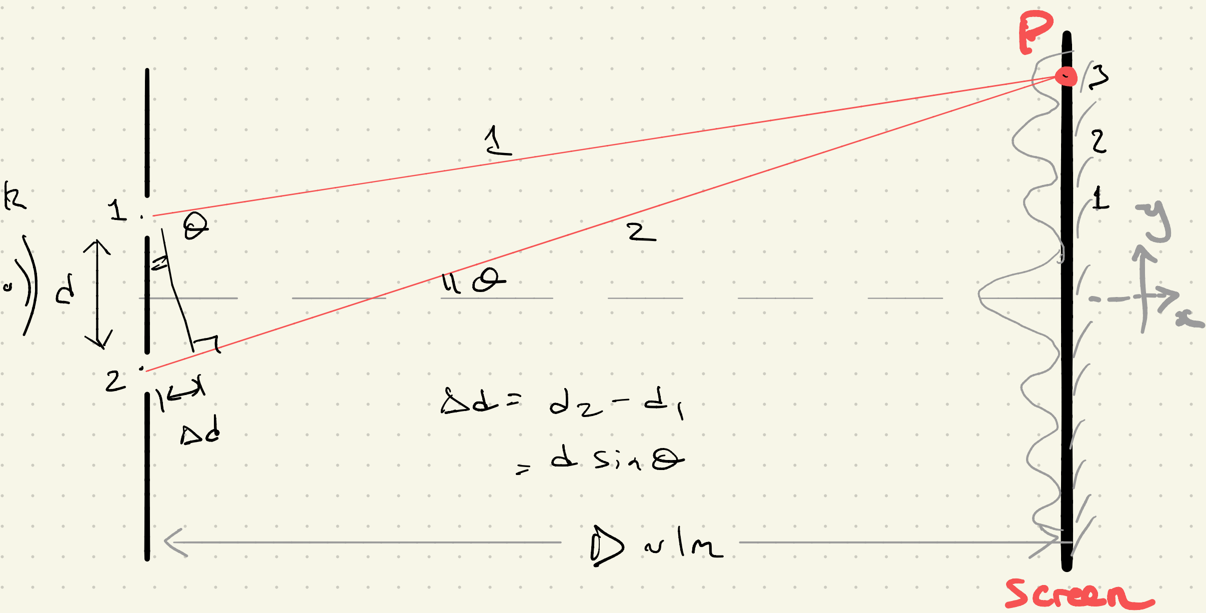
$$E_{tot} = E_1 + E_2 = e^{ikd_1} e^{-i\omega t} + e^{ikd_2} e^{-i\omega t}$$
$$= \left(e^{ikd_1} + e^{ikd_2}\right) e^{-i\omega t}$$
$$= e^{ikd_1}\left(1 + e^{ik(d_2 - d_1)}\right) e^{-i\omega t}$$
$$E_{tot}(x,t) = e^{ikd_1}\left(1 + e^{ik\Delta d}\right) e^{-i\omega t}$$

Note at $t=0$

$$\boxed{E_{tot}(x,0) = e^{ikd_1}\left(1 + e^{ik\Delta d}\right)}$$

Application of above - double slit experiment

Double Slit Experiment



$$E_p \equiv z_p = re^{ikd_1} + re^{ikd_2}$$, E at point P

$$= re^{ikd_1}(1 + e^{ikd\sin\theta})$$; $\Delta d = d\sin\theta$

Intensity distribution : $z_p^* z_p$

claim $I = z_p^* z_p = 4r_0^2 \cos^2\frac{\phi}{2}$; $\phi = kd\sin\theta$

In-class Hints:

Half-angle formula : $\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}}$

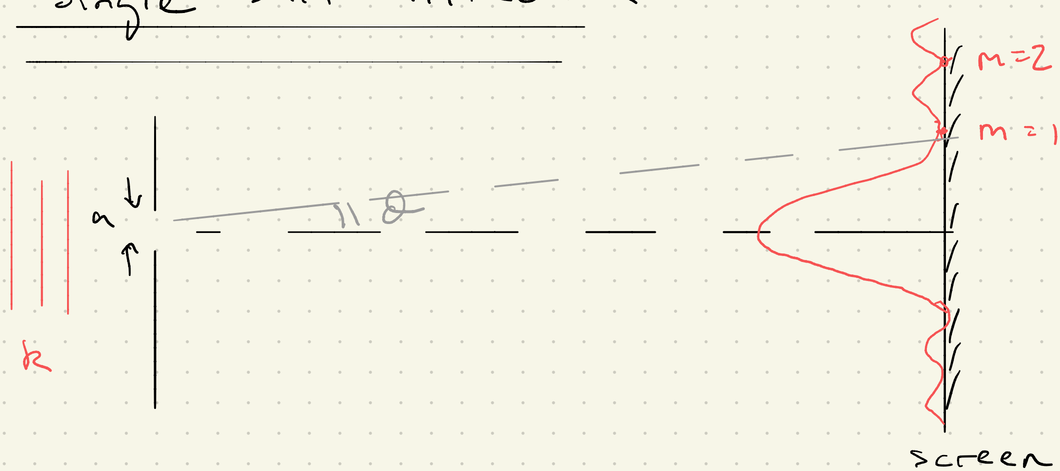
** aside: $(1+e^{ikd\sin\theta})(1+e^{-ikd\sin\theta})$

$$= 2 + e^{ikd\sin\theta} + e^{-ikd\sin\theta}$$

$$= 2 + 2\cos(kd\sin\theta)$$

$$\boxed{\text{In-class \# 3.2}}$$

Single Slit Diffraction



* destructive interference @ $\lambda/2 = d$

$$\therefore \quad \frac{a}{2}\sin\theta = d$$

$$\frac{a}{2}\sin\theta = \lambda/2$$

$$\boxed{a\sin\theta = \lambda}$$

& consider $a/4$, $a/6$, $a/8$, ...

In general, $\boxed{a\sin\theta = m\lambda}$

$m = 1, 2, 3, \ldots$

<u>Note</u>: $I = \left[\dfrac{\sin\beta/2}{\beta/2}\right]^2$; $\beta = \dfrac{2\pi a}{\lambda}\sin\theta$

So for minimum: $\underline{\beta = 2m\pi}$

$m = 1, 2, 3, \ldots$

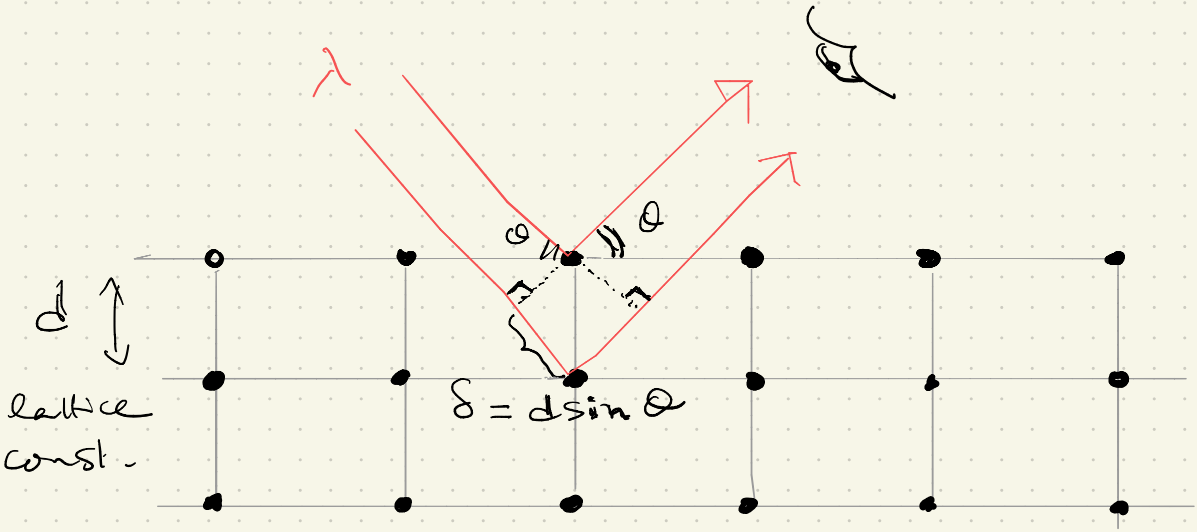
eg) For $m = 1$:

$\sin\theta = \dfrac{\lambda}{a}$ & if $a \gg \lambda \Rightarrow$ NO DIFFRAC.

no diffraction $\rightleftharpoons$ particle-like behavior

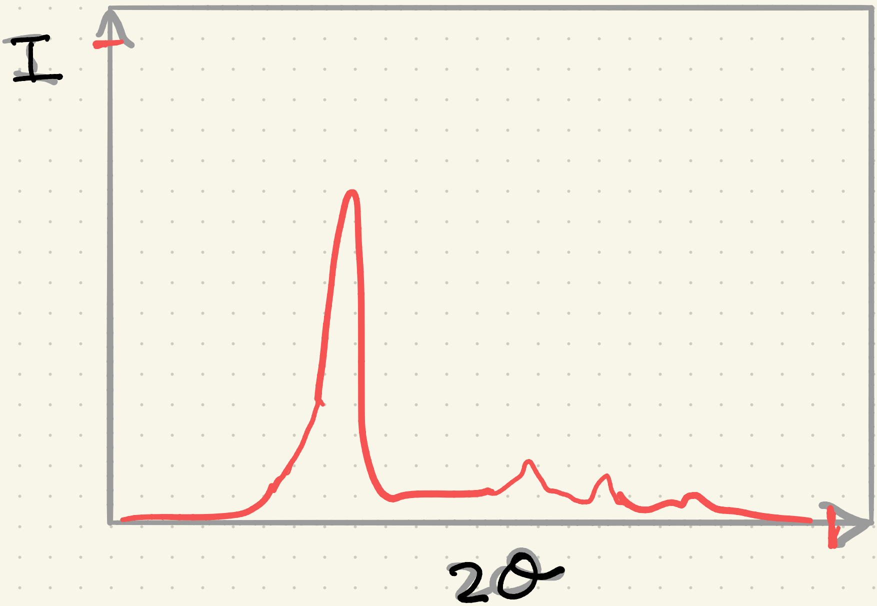
In-class # 3.3

X-ray Diffraction for Crystals.



$$2d\sin\theta = n\lambda \quad \text{(constructive interf.)}$$

$$\sin\theta = \frac{n\lambda}{2d}$$



In-Class #3-4