Class 4 (01/24/24)

Conductors

Static Electric Fields

description of physics

math entity

$$\vec{\nabla} \times \vec{E} = 0 \qquad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot (-\vec{\nabla} V) = -\vec{\nabla}^2 V = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}^2 V = -\frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}^2 V = 0$$

Finding the electric potential V by solving the Poisson's and Laplace's equations is a math problem.

Methods for solving the math problem:

cylindrical coordinates
(Bessel function)

multipole
expansion

Green's
function

separation
of variables

spherical
coordinates
(Legendre Polynomials)

$$\vec{\nabla}^2 V = 0 \Leftarrow \vec{\nabla}^2 V = \frac{-\rho}{\varepsilon_0}$$

image
charges

cartesian coordinates
(Fourier series)

Conductor Materials : Metals

In conductors, e. g. metals, electrons move freely because at the microscopic level the valence band and the conduction band overlap.

Table 1.1
FREE ELECTRON DENSITIES OF SELECTED METALLIC ELEMENTS[a]

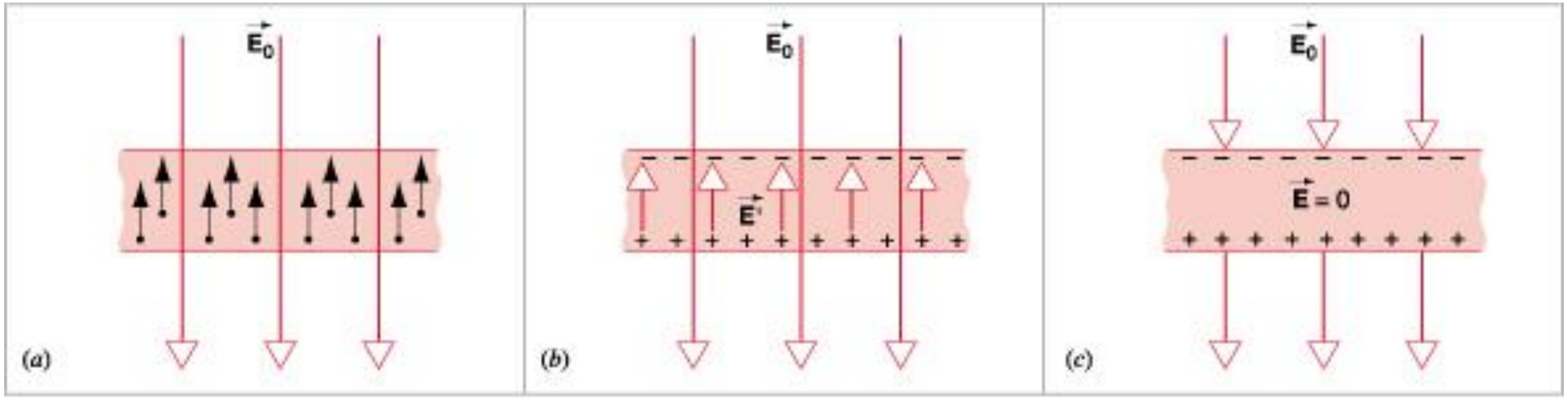
ELEMENT	Z	n ($10^{22}/cm^3$)	r_s(Å)	r_s/a_0
Li (78 K)	1	4.70	1.72	3.25
Na (5 K)	1	2.65	2.08	3.93
K (5 K)	1	1.40	2.57	4.86
Rb (5 K)	1	1.15	2.75	5.20
Cs (5 K)	1	0.91	2.98	5.62
Cu	1	8.47	1.41	2.67
Ag	1	5.86	1.60	3.02
Au	1	5.90	1.59	3.01
Be	2	24.7	0.99	1.87
Mg	2	8.61	1.41	2.66
Ca	2	4.61	1.73	3.27
Sr	2	3.55	1.89	3.57
Ba	2	3.15	1.96	3.71
Nb	1	5.56	1.63	3.07
Fe	2	17.0	1.12	2.12
Mn (α)	2	16.5	1.13	2.14
Zn	2	13.2	1.22	2.30
Cd	2	9.27	1.37	2.59
Hg (78 K)	2	8.65	1.40	2.65
Al	3	18.1	1.10	2.07
Ga	3	15.4	1.16	2.19
In	3	11.5	1.27	2.41
Tl	3	10.5	1.31	2.48
Sn	4	14.8	1.17	2.22
Pb	4	13.2	1.22	2.30
Bi	5	14.1	1.19	2.25
Sb	5	16.5	1.13	2.14

[a] At room temperature (about 300 K) and atmospheric pressure, unless otherwise noted. The radius r_s of the free electron sphere is defined in Eq. (1.2). We have arbitrarily selected one value of Z for those elements that display more than one chemical valence. The Drude model gives no theoretical basis for the choice. Values of n are based on data from R. W. G. Wyckoff, *Crystal Structures*, 2nd ed., Interscience, New York, 1963.

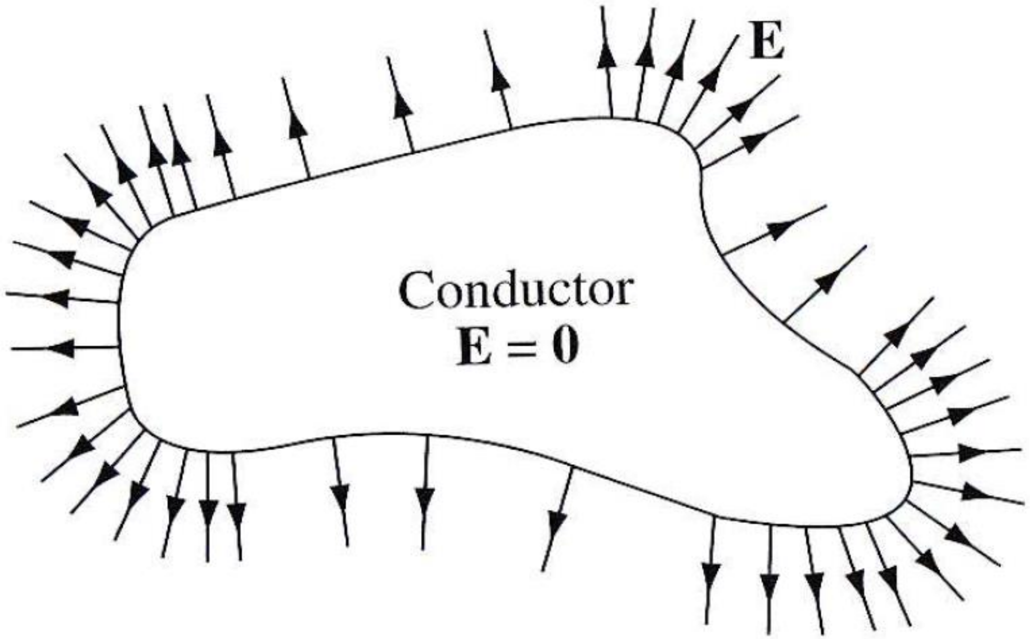
basic facts about conductors

- The static electric field **E** inside the conductor is zero.
- The net electric charge density ρ inside a conductor is zero.
- Any net electrical charges sit on the surface of the conductor.
- The static electric field **E** is always perpendicular at the surface of the conductor. (recall example: shell with surface charge density).
- The component of the static electric field which is parallel to the surface is always 0.
- The magnitude of the static electric field at the surface is: $E = \sigma/\varepsilon_0$
- Surface charge density σ can be calculated as: $\sigma = -\varepsilon_0 \frac{\partial V}{\partial n}$
- The electric potential V has the same magnitude at every point on the surface of the conductor.
- A conducting surface which is grounded is at electric potential V=0.
- Physical ground is an infinite source of negative electric charge.
- Connecting a battery to a conductor sets the electric potential at the conductor surface to the voltage delivered by the battery. The polarity of the electric potential is determined by the polarity of the connecting battery pole.

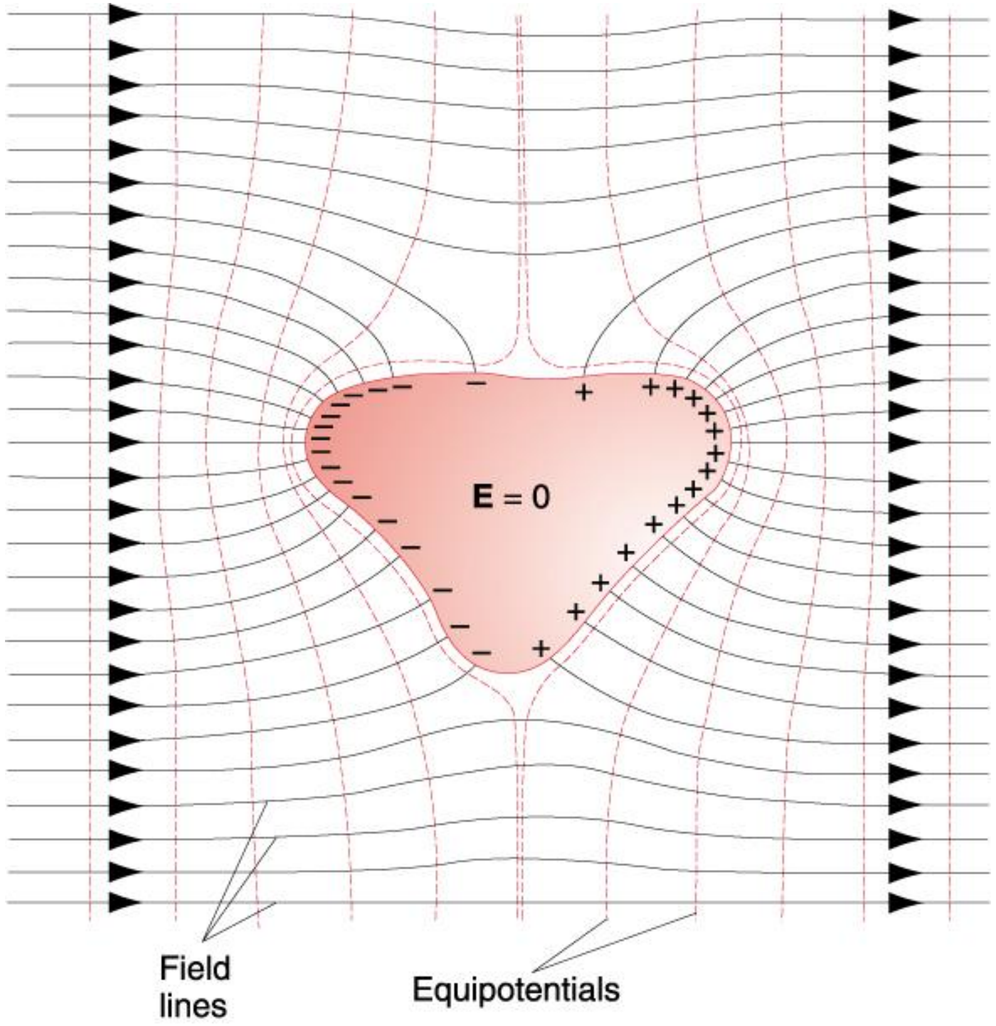
The static electric field **E** inside a conductor is 0 because the net electric charge density inside the conductor is 0.

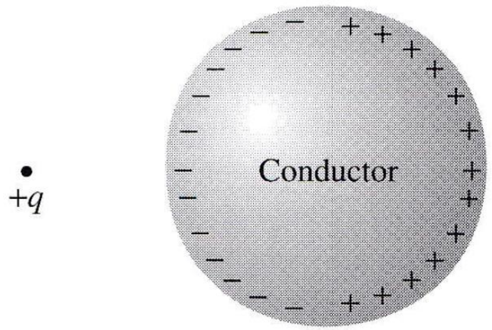


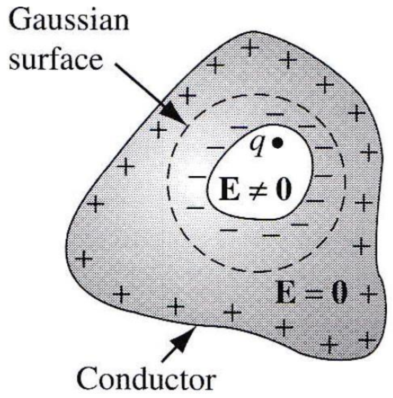
Any net electric charges will move to the conductor surface because they repel as a result of the Coulomb force.
At equilibrium, charges are arranged such that the electrostatic potential energy of the system of charges is minimized.



Examples of induced electric charges







Conductors, physical ground and batteries: