High-temperature series expansion of the $\boxed{\text{Debye function}}$

$$D(x_D) = -\frac{3x_D}{e^{x_D}-1} + \frac{12}{x_D^3}\int_0^{x_D} \frac{x^3\,dx}{e^x-1}$$

$$x_D = \frac{\Theta_D}{T} \quad , \; \Theta_D = \frac{\hbar\omega_D}{k}$$

ω_D : Debye frequency
Θ_D : Debye temperature

high-temperature limit $\quad x_D = \frac{\Theta_D}{T} \ll 1$

first term: $$-\frac{3x_D}{1+x_D+\frac{x_D^2}{2}+\frac{x_D^3}{6}+\dots-1} \simeq -\frac{3x_D}{x_D+\frac{x_D^2}{2}+\frac{x_D^3}{6}+\dots} = -\frac{3}{1+\frac{x_D}{2}+\frac{x_D^2}{6}+\dots}$$

$$\simeq -3\left[1-\left(\frac{x_D}{2}+\frac{x_D^2}{6}\right)+\frac{x_D^2}{4}+o(x_D^3)\right] = -3\left[1-\frac{x_D}{2}+\frac{1}{12}x_D^2\right]$$

$$\simeq -3+\frac{3}{2}x_D-\frac{1}{4}x_D^2$$

the integral: $$\frac{x^3}{e^x-1} = \frac{x^3}{1+x+\frac{x^2}{2}+\frac{x^3}{6}+\dots-1} = \frac{x^2}{1+\frac{x}{2}+\frac{x^2}{6}+\dots} \simeq x^2\left(1-\left(\frac{x}{2}+\frac{x^2}{6}\right)+\frac{x^2}{4}+o(x^3)\right)$$

$$\simeq x^2\left(1-\frac{x}{2}+\frac{1}{12}x^2\right) = x^2-\frac{x^3}{2}+\frac{x^4}{12}$$

$$\int_0^{x_D}\frac{x^3}{e^x-1}dx \simeq \frac{1}{3}x_D^3-\frac{1}{8}x_D^4+\frac{1}{60}x_D^5$$

$$\frac{12}{x_D^3}\int\frac{x^3}{e^x-1}dx \simeq 4-\frac{3}{2}x_D+\frac{1}{5}x_D^2$$

Finally:

$$D(x_D) \simeq \left(-3 + \frac{3}{2} x_D - \frac{1}{4} x_D^2\right) + \left(4 - \frac{3}{2} x_D + \frac{1}{5} x_D^2\right)$$

$$\simeq 1 - \frac{1}{20} x_D^2$$

$$\boxed{D(x_D) \simeq 1 - \frac{x_D^2}{20} + \dots}$$