Class 10 Electric Fields in Matter (02/08/24)

Electrostatics in Vaccum

- Electrostatic Fields **E** in Vaccum: $\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon_0}, \quad \nabla \times \vec{E} = 0. \quad \vec{E} = -\nabla V$

Methods for the calculation of electrostatic fields **E**:

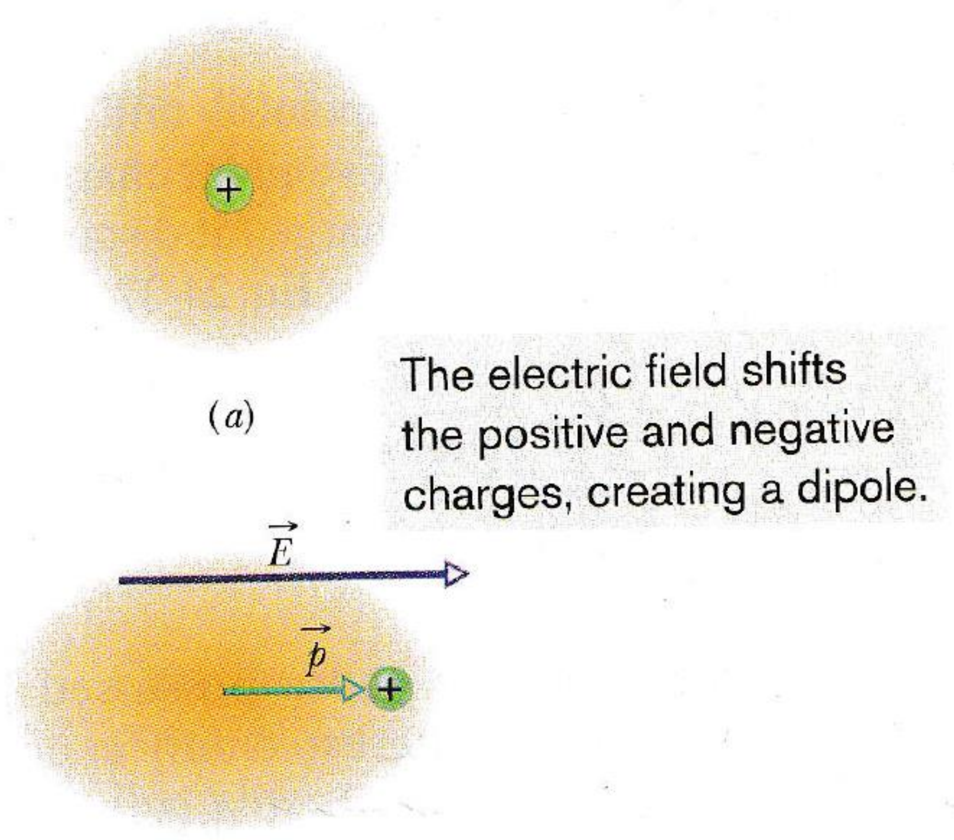
- From known electric charge: Superposition principle for point charges, Gauss law.

- Derived from electric potential V where V is determined by the superposition principle for point charges, or solving Poisson-Laplace Eq. (direct integration of diff.eq., method of images), or solving Laplace Eq. (boundary value problems), or approximation of an electric potential V of an electric charge distribution by a multipole expansion.

Electrostatic Fields in Matter

- Conductors : $\boldsymbol{E}=0$ on the inside, always.
- Dielectric Matter : $\mathbf{E}\neq 0$ on the inside possible.

Electric Polarization of Atoms



Induced dipoles:
Atomic polarizability α : $\vec{p} = \alpha\,\vec{E}$ [α is a scalar]

The Nobel Prize in Physics 2018

Arthur Ashkin

Gerald Mourou

Donna Strickland

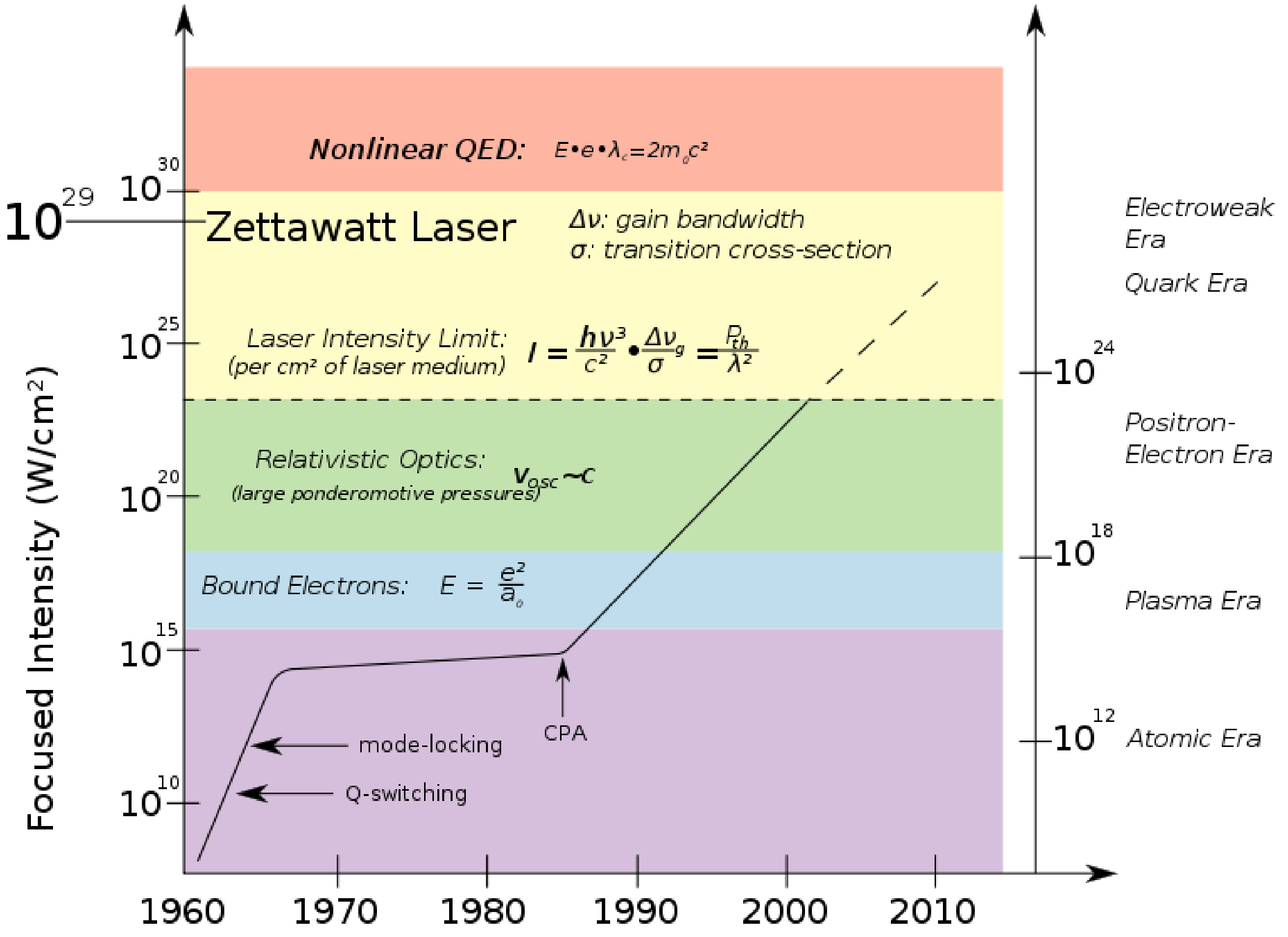
Prize motivation: "for the optical tweezers and their application to biological systems."

Prize motivation: "for their method (chirped laser pulse amplification) of generating high-intensity, ultra-short optical pulses."

The method is useful for producing electric fields sufficiently strong to ionize atoms.

Source: Niklas Elmehed. © Nobel Media

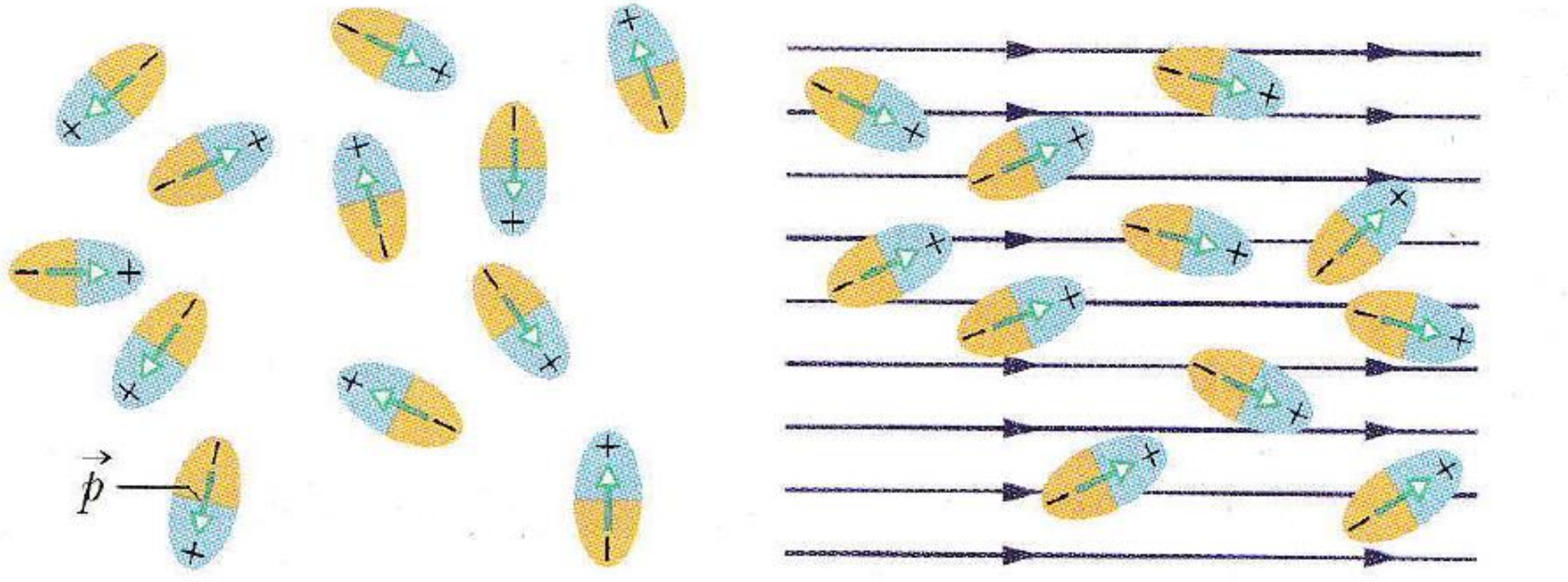
The Development of Intense Laser Light Sources



Source: Wikimedia Commons

Permanent Electric Dipoles in an **E**-Field

E : **F**=-q**E** & **F**=+q**E**



(a) (b)

Polarizability of atoms and molecules

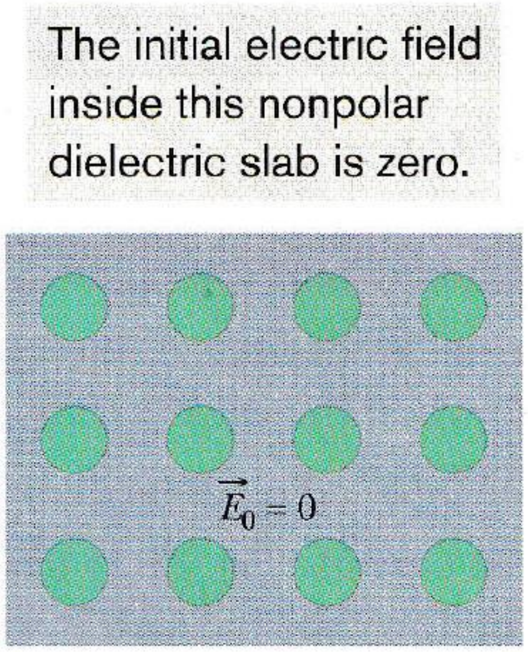
Induced dipoles:

- Atomic polarizability α : $\vec{p} = \alpha\,\vec{E}$ [α is a scalar]

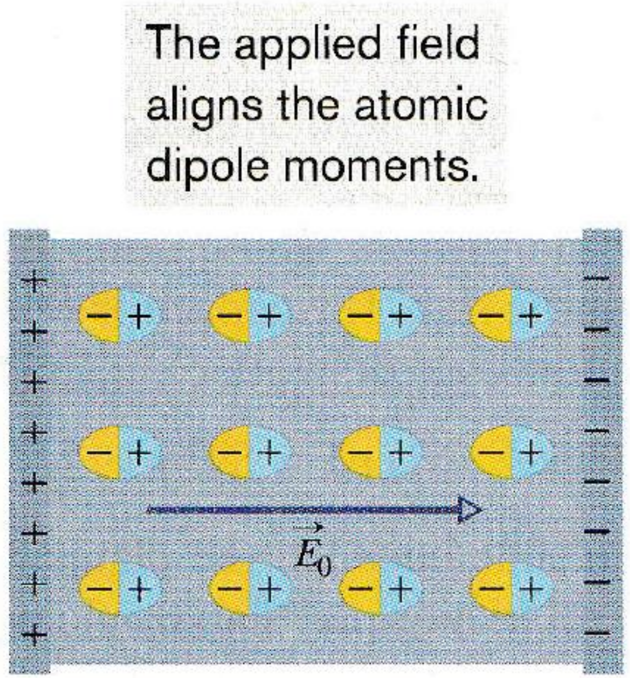
- Molecular polarizability: $\vec{p} = \{\alpha\}\,\vec{E}$

  [{a} is a tensor, **p** and **E** are generally not parallel]

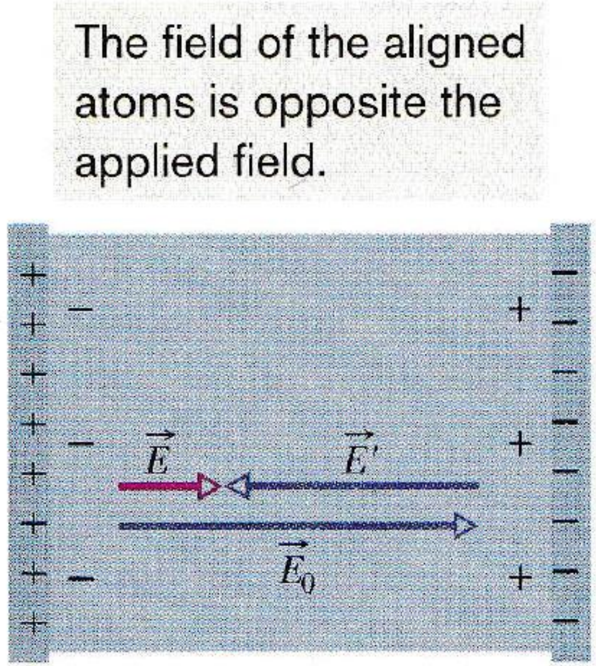
Permanent dipoles: e.g. water molecule



(a)



(b)



(c)

Electric polarization **P** is defined as the average electric dipole moment <**p**$_i$> within the volume of material over which the averaging is performed:
P = <**p**$_i$>/V.

How do we calculate the net electric field E within the dielectric slab for a given polarization **P**?
$\vec{E} = -\vec{\nabla}V$ with V:

Full description of V(r): [in example (c) ρ_{free}=0 and ρ_b=0]

$$V_{total}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left[\int_{V'} \frac{\rho_{free}}{|\vec{r}-\vec{r}'|}\,dV' + \int_{S(A)} \frac{\sigma_b}{|\vec{r}-\vec{r}'|}\,d\vec{A}' + \int_{V'} \frac{\rho_b}{|\vec{r}-\vec{r}'|}\,dV'\right]$$

$\vec{P}\cdot\vec{n} = \sigma_b$ surface charge, $-\vec{\nabla}\cdot\vec{P} = \rho_b$

Macroscopic versus Microscopic Electric Fields in Matter

1. Macroscopic electric fields are the electric fields **E** we measure, e.g. Coulomb-force acting on a test charge **E** = **F** / q, or **E** calculated from measured electric potentials V by voltmeters.
2. The measured macroscopic electric fields in matter are explained theoretically by assuming that the material is filled with (induced or permanent) electric dipoles (or sometimes higher order poles).
3. Macroscopic electric fields in matter are averages over microscopic electric fields present within an atom and molecules. Averaging is performed over volumes large compared to the volume of an individual atom or molecule.

Figuring out (2) and (3) [particularly making sure that averaging procedure is sound] was subject of scientific research and debate (!) for hundreds of years. Today, the questions are soundly answered and we apply the results.

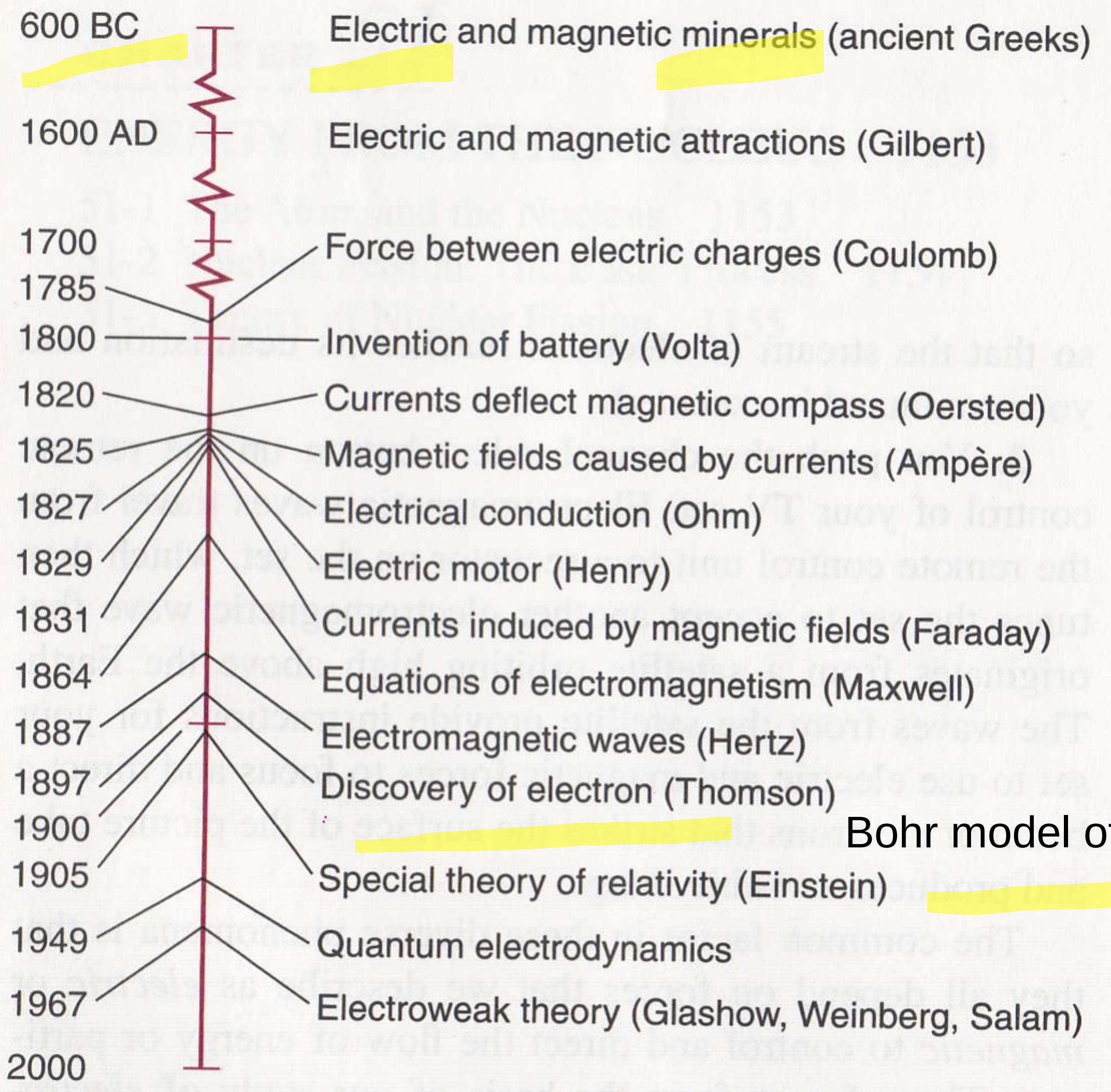


FIGURE 25-1. Time line of major developments in electromagnetism.

Bohr model of the atom: 1913

The Electric Field of an Electrically Polarized Object

- Electric polarization **P** is defined as the average electric dipole moment $\langle \mathbf{p}_i \rangle$ within the volume of material over which the averaging is performed: $\mathbf{P} = \langle \mathbf{p}_i \rangle / V$.
- How can we calculate the electric potential V(**r**) of a charge distribution ρ assuming that it is an electric dipole **p**?
- Well, we use the n=1 term of the multiple expansion of the charge distribution ρ:

$$V_1(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\int_{V'} r'\cos\alpha\,\rho(\vec{r}')\,dV' = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\,\hat{r}\int \vec{r}'\rho(\vec{r}')\,dV' = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

with $r'\cos\alpha = \hat{r}\cdot\vec{r}'$

with $\vec{p} = \int \vec{r}'\rho(\vec{r}')\,dV'$

- Reminder: **r** points to the location of the potential V(**r**), **r**' points to the location of the electric charge density ρ, α is the angle between the two vectors **r**, **r'**

Derivation of the electric potential V(**r**) of an electrically polarized object :

p : electric dipole moment
P : (macroscopic) polarization

P = <**p**$_i$>/V.

$$\vec{p} = \int \vec{P}(\vec{r}\,')\,dV'$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\,\frac{\vec{p}\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^2}$$

(from multipole expansion)

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\vec{P}(\vec{r}\,')\,(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^2}\,dV'$$

$$\vec{\nabla}_{\vec{r}\,'}\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) = \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^2}$$

useful pieces of math

$$V = \frac{1}{4\pi\varepsilon_0}\int_V \vec{P}\cdot\vec{\nabla}_{\vec{r}\,'}\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right)dV'$$

integration by parts: $\vec{\nabla}\cdot(f\vec{A}) = f\vec{\nabla}\cdot\vec{A} + \vec{A}\,\vec{\nabla}f$

$$\vec{A}\,\vec{\nabla}f = \vec{\nabla}\cdot(f\vec{A}) - f\,\vec{\nabla}\cdot\vec{A}$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\int_V \vec{\nabla}_{\vec{r}\,'}\cdot\left(\frac{\vec{P}}{|\vec{r}-\vec{r}\,'|}\right)dV' - \int_V \frac{1}{|\vec{r}-\vec{r}\,'|}\left(\vec{\nabla}_{\vec{r}\,'}\cdot\vec{P}\right)dV'\right]$$

divergence theorem: $\int \vec{\nabla}\cdot\vec{A}\cdot dV = \int \vec{A}\cdot d\vec{A}$

$$V = \frac{1}{4\pi\varepsilon_0}\oint_{A(V)} \frac{1}{|\vec{r}-\vec{r}\,'|}\,\vec{P}\cdot d\vec{A} - \frac{1}{4\pi\varepsilon_0}\int_V \frac{1}{|\vec{r}-\vec{r}\,'|}\left(\vec{\nabla}_{\vec{r}\,'}\cdot\vec{P}\right)dV'$$

$$V = \frac{1}{4\pi\varepsilon_0} \oint_{A(V)} \frac{1}{|\vec{r}-\vec{r}\,'|} \vec{P} \cdot d\vec{A} - \frac{1}{4\pi\varepsilon_0} \int_V \frac{1}{|\vec{r}-\vec{r}\,'|} (\vec{\nabla}_{\vec{r}\,'} \cdot \vec{P})\, dV'$$

$\vec{P} \cdot \vec{n} = \sigma_b$ surface charge, $-\vec{\nabla} \cdot \vec{P} = \rho_b$

analyze units: $\sigma = \frac{Q}{A}$ $[\sigma] = \frac{C}{m^2}$, $\vec{P} = \frac{\vec{p}}{V} = \frac{q \cdot \vec{d}}{V}$ $[\vec{P}] = \frac{C \cdot m}{m^3}$

$$\vec{\nabla} \cdot \vec{P} = \frac{C}{m^2} \cdot \frac{1}{m}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \oint_{S(A)} \frac{\sigma_b}{|\vec{r}-\vec{r}\,'|} d\vec{A}\,' + \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho_b}{|\vec{r}-\vec{r}\,'|} dV'$$

uniform polarization $\vec{P} = \text{const}$, $\vec{\nabla} \cdot \vec{P} = 0$

with $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho_{Free}}{|\vec{r}-\vec{r}'|} dV'$

$$\underset{total}{V(\vec{r})} = \frac{1}{4\pi\varepsilon_0}\left[\int_{V'} \frac{\rho_{free}}{|\vec{r}-\vec{r}'|} dV' + \int_{S(A)} \frac{\sigma_b}{|\vec{r}-\vec{r}'|} d\vec{A}' + \int_{V'} \frac{\rho_b}{|\vec{r}-\vec{r}'|} dV'\right]$$

$\vec{P}$ given, e.g. $\vec{P} = \vec{P}_0$ constant, $\vec{P} = \vec{P}(x,y,z)$

calculate σ_b, ρ followed by calculating $V(\vec{r})$ total

$$\vec{P}\cdot\vec{n} = \sigma_b, \quad -\vec{\nabla}\cdot\vec{P} = \rho$$