Lecture 14 : Ampere's Law

Biot - Savart Law allows computation of the magnetic field from known currents, just like Coulomb's Law allows computation of the electric field from known charge distributions.

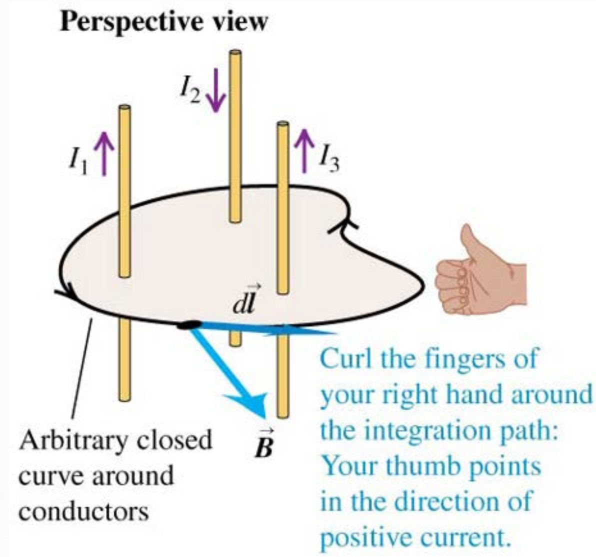
In certain symmetric systems, an easier way to compute the field is Ampere's Law :

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}}$$

where I_{enc} is the net current enclosed (crossing through) the "Amperian" loop.

- Ampere's law is true for all chosen paths, regardless of shape.
- For the case $I_{enc} = 0$, $\oint \vec{B} \cdot d\vec{l} = 0$, but this does not necessarily mean that $\vec{B} = 0$ for the region being analyzed.
- I_{enc} is the net current passing throug the

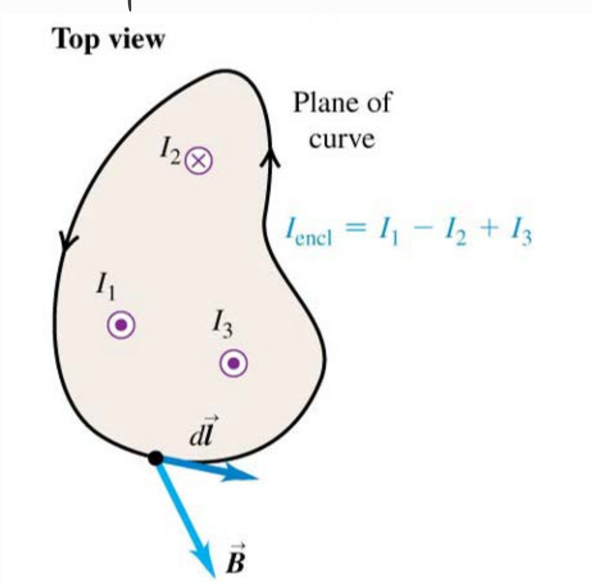
surface area bound by the Amperian path. It is the algebraic sum of the currents passing through the path.



Positive and negative currents are defined using the following right-hand rule:

1. Curl the fingers of your right hand in the direction chosen for the closed Amperian path with your palm to the inside of the path.

2. Your thumb direction is taken as the positive direction for current.



If current is spread throughout a conductor with net current density $\vec{J}_{net}$, the enclosed current in Ampère's law is:

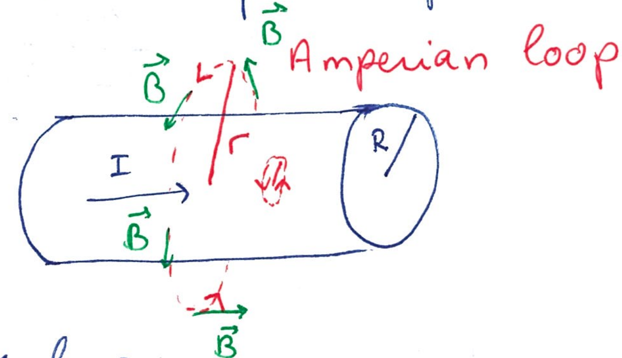
$$I_{enc} = \int \vec{J}_{net} \cdot d\vec{a}$$

where $\int d\vec{a}$ is the area of the surface bounded by the Amperian path.

Using Ampere's law to calculate magnetic fields.

- Choose an Amperian path consistent with the symmetry of the system being considered. (If there is no simple symmetry, you must use Biot-Savart.)
- To be able to extract the magnetic field from the line integral, your Amperian loop needs to follow a closed path on which the magnetic field is constant and/or zero.

Example: A uniform current I flows down a long cylindrical wire with radius R. Find the magnetic field everywhere.



- The magnetic field lines are circular: choose a circular Amperian path bounding the cross-sectional area the current flows through.
- Since $B(r)$ is constant along this path,

$$\oint \vec{B} \cdot d\vec{l} = B \oint dl$$

Outside the wire, we have:

$$B\oint dl = B(2\pi r) = \mu_0 I_{enc} = \mu_0 I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}} \quad \text{for } r > R$$

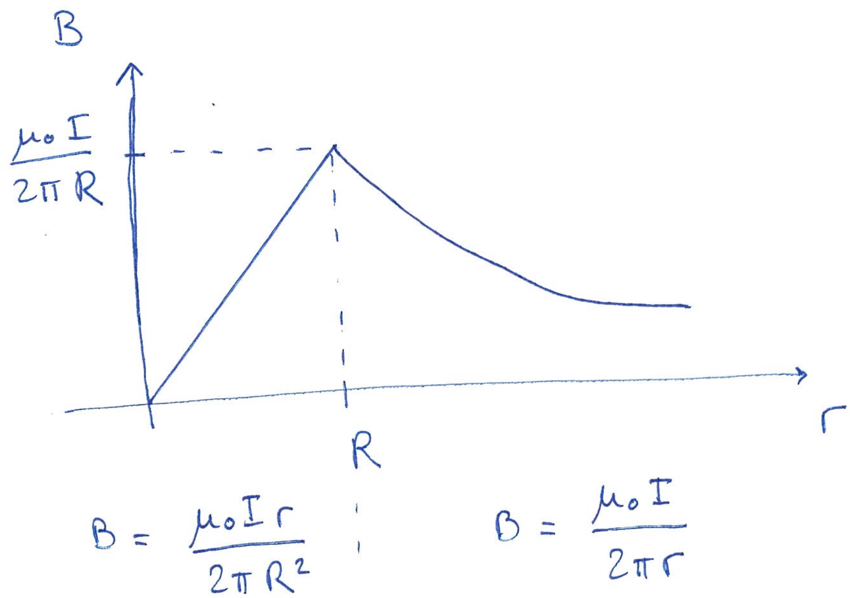
Inside the wire, only a portion of the current flows through the Amperian loop.

The current density is:

$$\vec{J} = \frac{I}{\pi R^2}\hat{k} \quad \text{and} \quad d\vec{a} = da\,\hat{k}$$

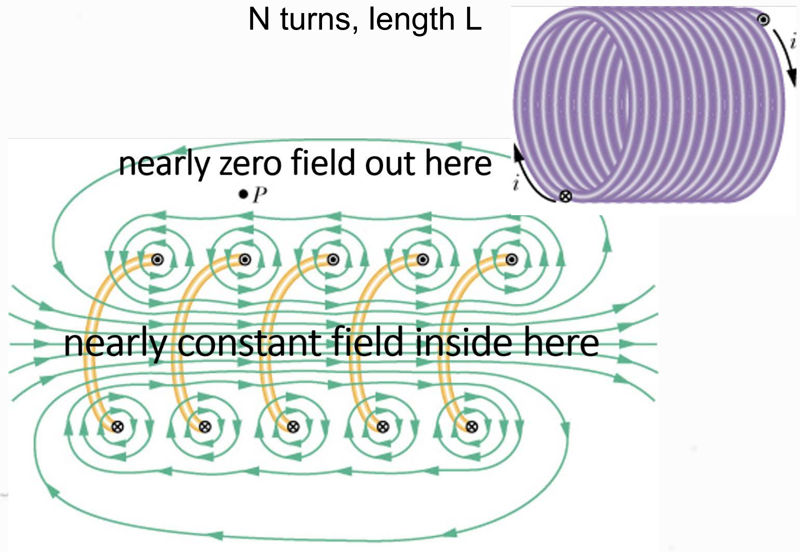
$$\Rightarrow \oint \vec{B}\cdot d\vec{l} = B(2\pi r) = \mu_0 I_{enc} = \mu_0 \int \vec{J}\cdot d\vec{a} = \mu_0 J A$$

$$\Rightarrow \boxed{B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r} \quad \text{for } r < R$$



A solenoid is a helical winding of wire on a cylinder.

An almost uniform field is created within a very long solenoid having tight winding:

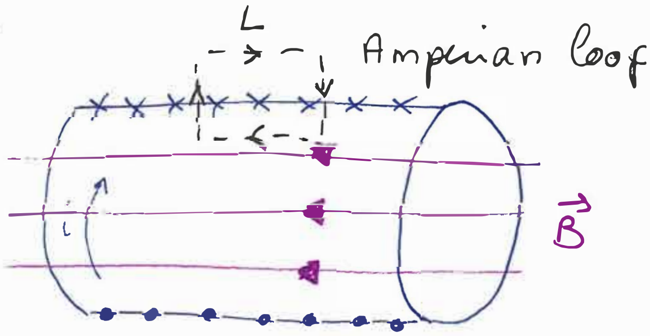


- Field between the wires cancels
- Field outside the coil cancels
- Field inside the coil adds. A very long solenoid acts as a magnetic field "pipe", condensing the nearly uniform magnetic field within it.

Finite sized solenoids have a dipole-field behavior. One end acts as a north pole, and the other end as a south pole.

Example: Using Ampere's Law to calculate the B-field inside a long solenoid

Along the shown amperian loop, we have:



$$\oint \vec{B} \cdot d\vec{l} = BL = \mu_0 N I$$

↖ number of coils enclosed in the loop.

$$\Rightarrow \quad B = \mu_0 \left(\frac{N}{L}\right) I = \underline{\mu_0 n I}$$

Although Ampere's law is always true, it can only be used to find the magnetic field for a few special symmetries:

* straight wire
* sheet of current
* solenoid
* toroid