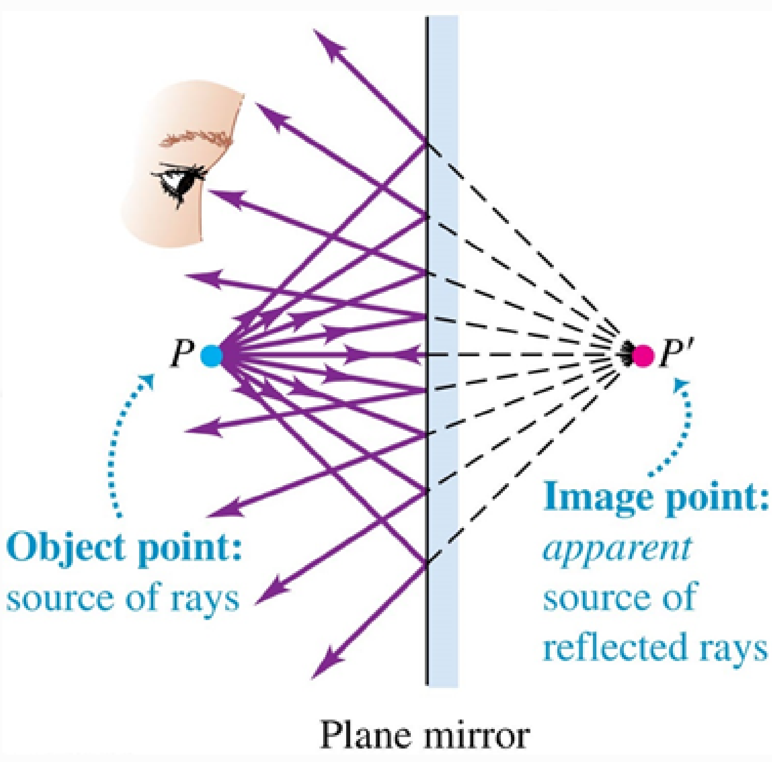
Lecture 24: Geometrical optics

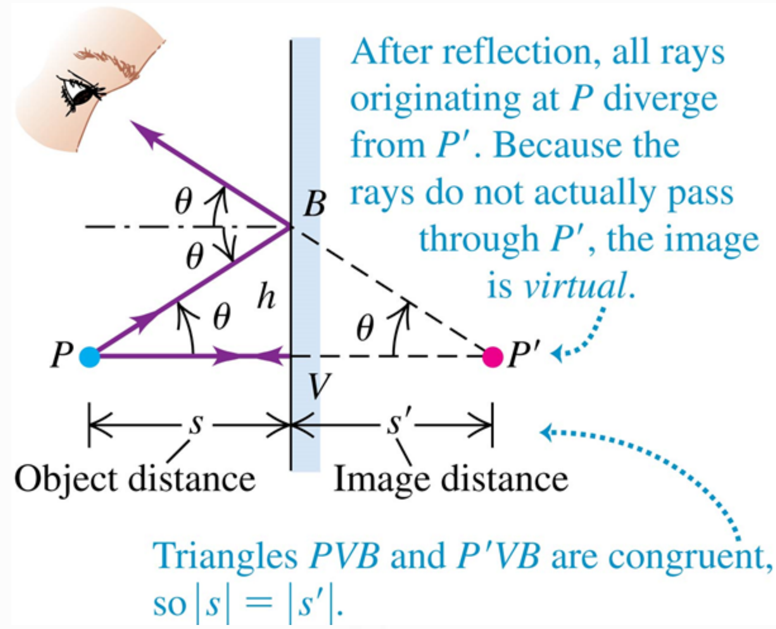
Geometrical optics deals in rays, i.e., trajectories that are orthogonal to the wavefronts and thus parallel to the wavevector (direction of propagation). It is used to study the optical behavior of systems with lengthscales much larger than the wavelength, and emphasizes finding the light path.
In this lecture, we will use rays to understand the principles behind optical devices based on lenses and mirrors.

1- Reflection at a plane surface.



- Light rays from the objec at point P are reflected from a plane mirror
- The angle of reflection = the angle of incidence
- The reflected rays entering the eye look as though they had come from P′ (image point).

- Object distance s is <u>positive</u> because the object is on the same side as the incoming light.
- Image distance s' is <u>negative</u> because the image is NOT on the same side as the outgoing light.



$s = -s'$

<u>Sign conventions</u>

- Object distance (o) - distance of object from mirror surface.
  - * Positive if it is where the light shines from
- Image distance (i) - distance of apparent or projected image from mirror surface.
  - * Positive if it is where the light shines to.
  - * Negative if it is where light doesn't shine.
  - * $i = -o$
- Magnification (m) $= -\frac{i}{o}$ = ratio of image size to object size.

For a plane mirror, the image is virtual, upright, and the same size as the object:

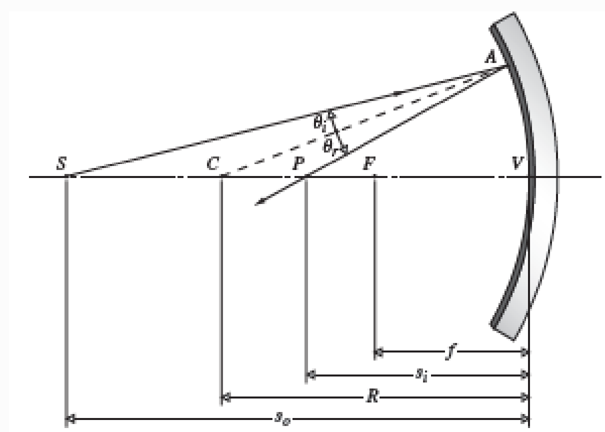
$i = -o$; $m = -\frac{i}{o} > 0$ so upright $m = |1|$ so life size.

Mirrors lead to inversion: they convert a right hand coordinate system to a left handed one. (The mirror image of your right hand is a left hand.)

2 - Reflection at a curved surface.

If a mirror is curved, it can act like a lens.

Where is the image of a point object for a curved mirror?



Ray tracing rules:

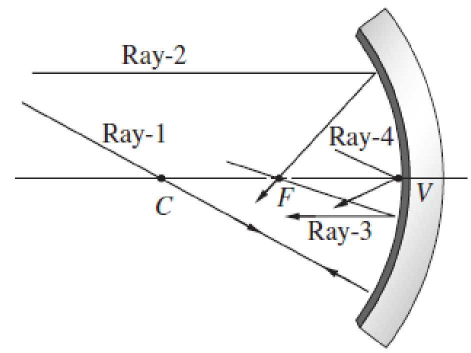
- Any incident ray passing through the center of curvature reflects back out on the path it entered on.

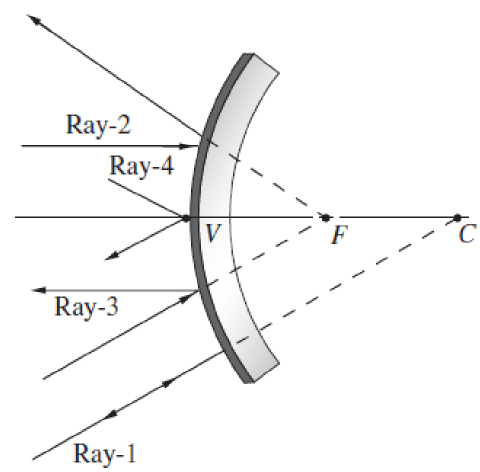
- Any ray passing in along a line through the focal point leaves parallel to the axis after striking the mirror surface.

- Any ray incident parallel to the axis leaves along a line through the focal point.

- center simply obeys $\theta_i = \theta_r$ about the axis.

The distance from the vertex to the focal point is called the focal length.

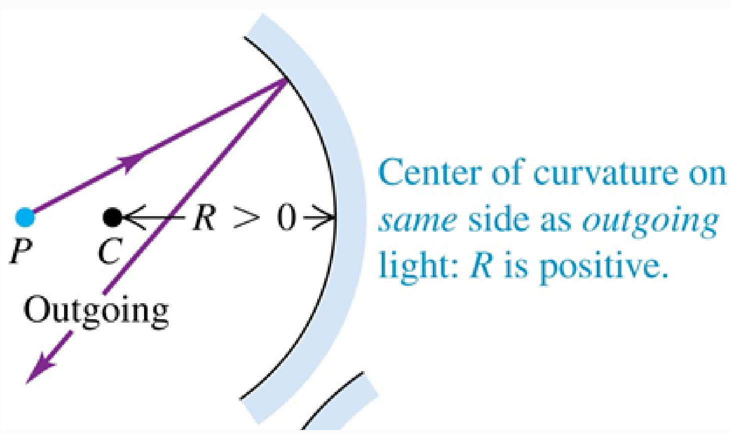


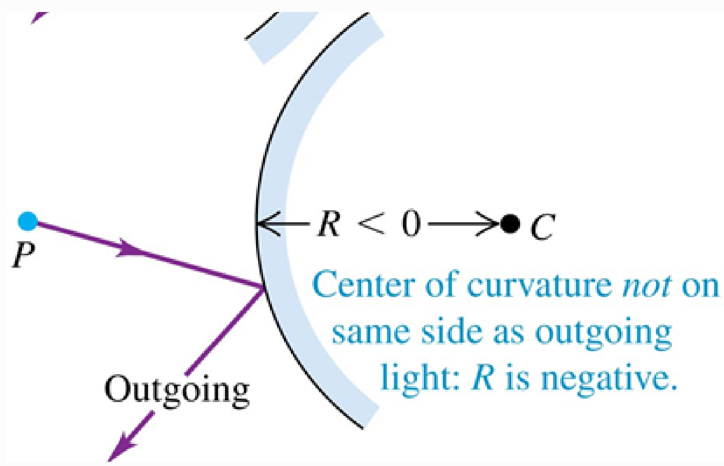


$$\boxed{f = \frac{R}{2}}$$

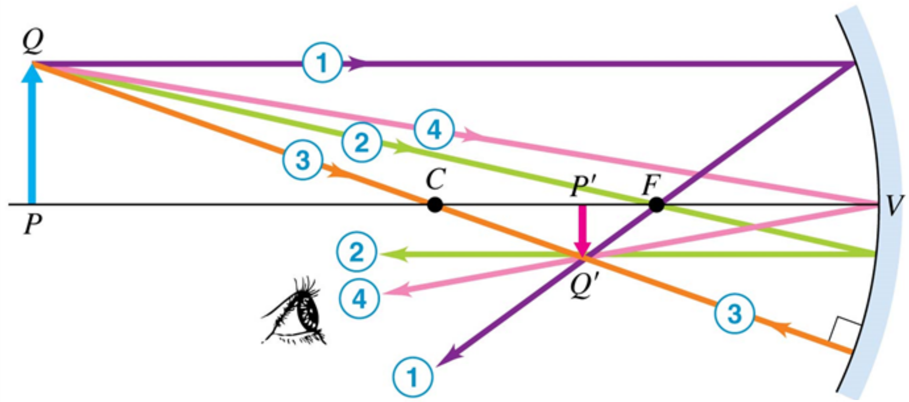
where R is the radius of curvature.

<u>Sign conventions</u>

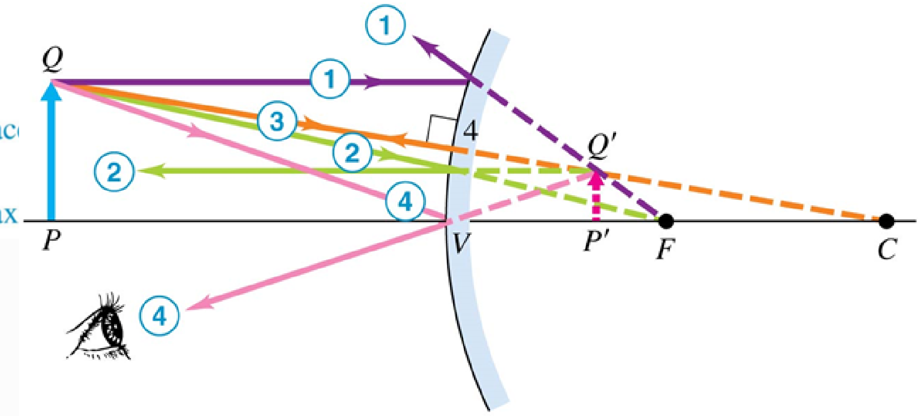




Object-image relationship: $\boxed{\frac{1}{f} = \frac{2}{R} = \frac{1}{o} + \frac{1}{i}}$



1. Ray parallel to axis reflects through focal point.
2. Ray through focal point reflects parallel to axis.
3. Ray through center of curvature intersects the surfac
   and reflects along its original path.
4. Ray to vertex reflects symmetrically around optic ax



1. Reflected parallel ray appears to come from focal point.
2. Ray toward focal point reflects parallel to axis.
3. As with concave mirror: Ray radial to center of curvature intersects the surface normally and reflects along its original path.
4. As with concave mirror: Ray to vertex reflects symmetrically around optic axis.

TABLE 5.4 Sign Convention for Spherical Mirrors

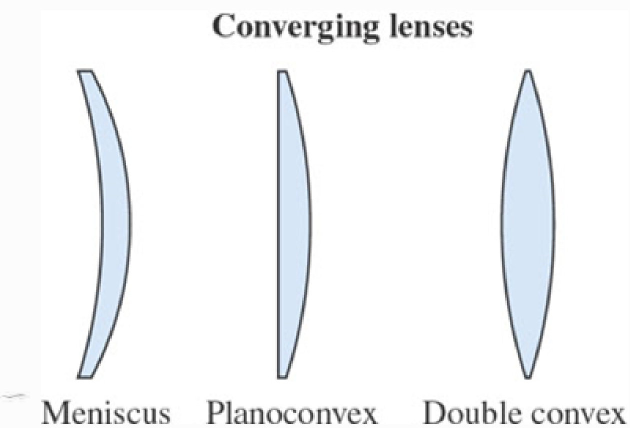
Quantity	Sign +	Sign −
s_o	Left of V, real object	Right of V, virtual object
s_i	Left of V, real image	Right of V, virtual image
f	Concave mirror	Convex mirror
R	C right of V, convex	C left of V, concave
y_o	Above axis, erect object	Below axis, inverted object
y_i	Above axis, erect image	Below axis, inverted image

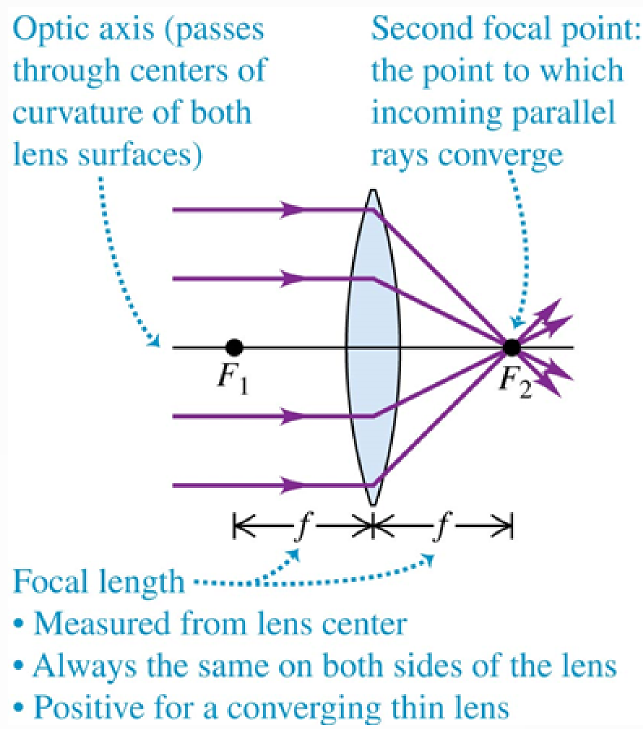
TABLE 5.5 Images of Real Objects Formed by Spherical Mirrors

Concave				
Object	**Image**			
Location	Type	Location	Orientation	Relative Size
$\infty > s_o > 2f$	Real	$f < s_i < 2f$	Inverted	Minified
$s_o = 2f$	Real	$s_i = 2f$	Inverted	Same size
$f < s_o < 2f$	Real	$\infty > s_i > 2f$	Inverted	Magnified
$s_o = f$		$\pm\infty$		
$s_o < f$	Virtual	$\lvert s_i\rvert > s_o$	Erect	Magnified
Convex				
Object	**Image**			
Location	Type	Location	Orientation	Relative Size
Anywhere	Virtual	$\lvert s_i\rvert < \lvert f\rvert$, $s_o > \lvert s_i\rvert$	Erect	Minified

3. Rays passing through thin lenses

A lens is an optical system with two refracting surfaces. The simplest lens has two spherical surfaces close enough together that we can ignore the distance between them; we call this a thin lens.



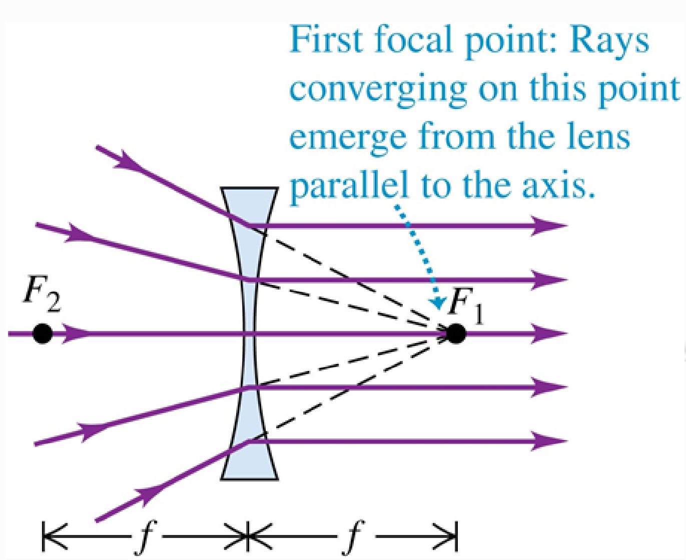


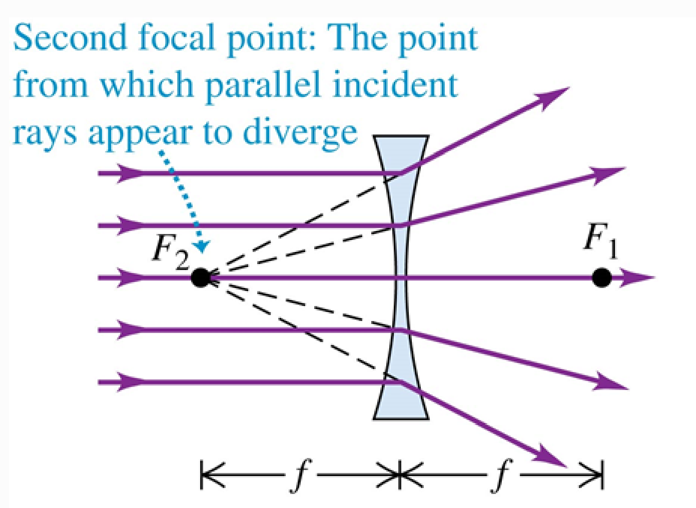
Simple rules for thin lenses:

index of lens material — index of host material

$$\boxed{\frac{1}{f} = \frac{1}{o} + \frac{1}{i} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

focal distance — object distance — image distance — radius of curvature (r_1: first surface; r_2: second surface

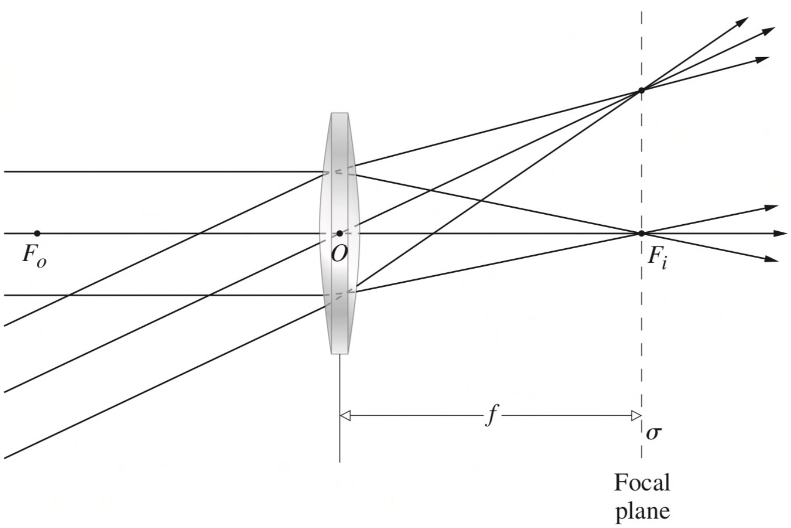




For a diverging thin lens, f is negative.

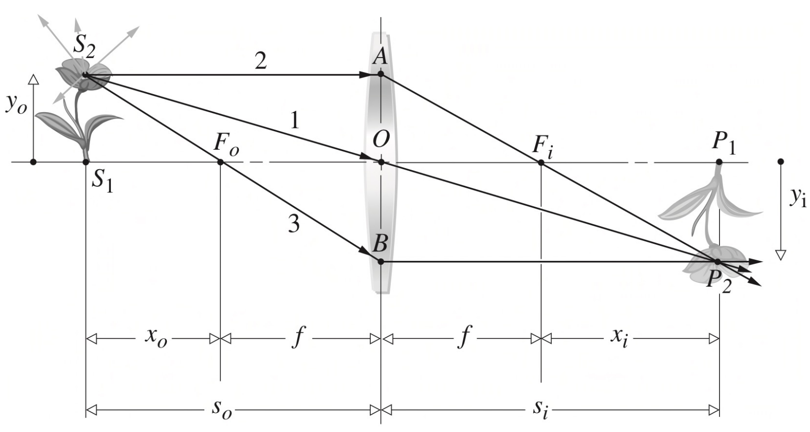
Thin lens ray tracing rules:

- Rays parallel to the optical axis incident from the left side of the lens are deflected through the right focal point.
- Rays passing through the left focal point become parallel to the optical axis. They are collimated.
- Rays passing through the center of the lens remain unchanged.

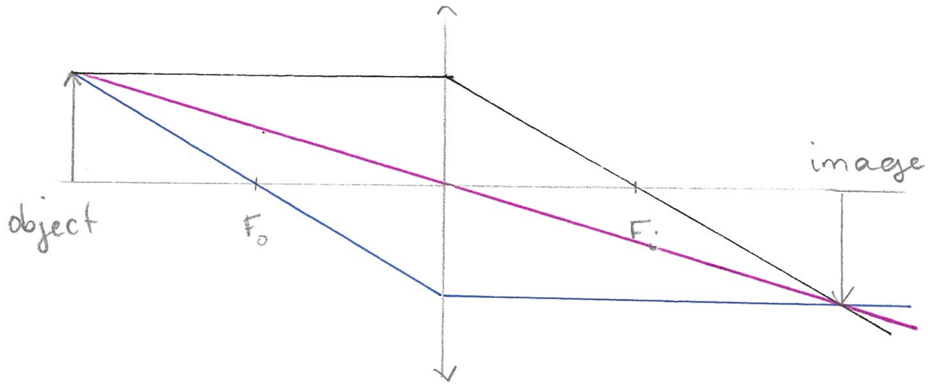


All bundles of parallel rays converge to focal points that lay on one plane: the second or back focal plane. F_o lies on the first or front focal plane.

Each point in the object plane is a point source of spherical waves, and the lens will image them to respective points on the image plane.

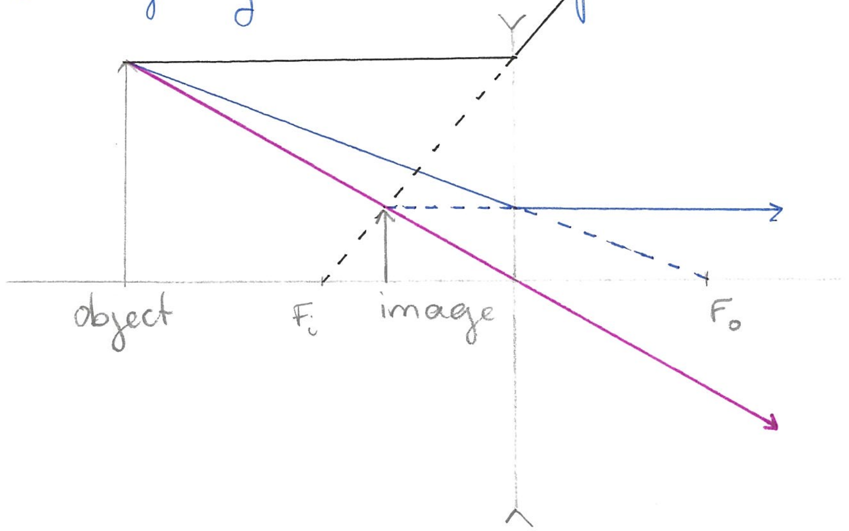


• Converging lens example



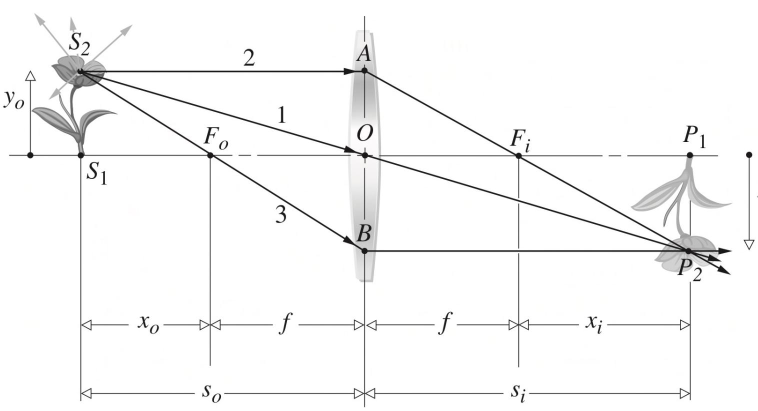
In this configuration, the image is <u>real, inverted, and enlarged</u>.

• Diverging lens example



- Rays parallel to the optica axis appear to come from F_i
- Rays through the center of the lens remain unchang
- Rays directed towards F_o emerge parallel to the optical axis.

Here, image is <u>virtual, upright, and reduced in size</u>.



Triangles S_1S_2O and P_1P_2O are similar, which means that :

$$\boxed{M_T \equiv \frac{y_i}{y_o} = -\frac{s_i}{s_o}}$$

Magnification equation

A positive M_T represents an upright image, while a negative value means the image is inverted.

Example : What is the magnification for an object placed 15 cm away from a thin lens with focal distance $f = 10$ cm

Gaussian lens formula $\Rightarrow$ $s_i = \frac{f s_o}{s_o - f} = \frac{(10)(15)}{15-10} = 30$ cm

Then $M_T = -\frac{s_i}{s_o} = -\frac{30}{15} = \boxed{-2}$

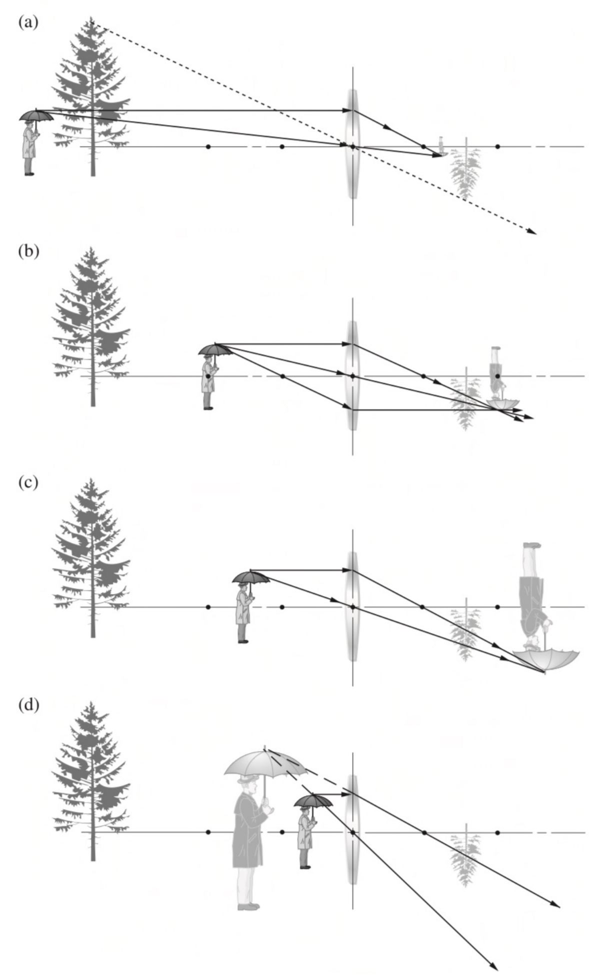


TABLE 5.3 Images of Real Objects Formed by Thin Lenses

Convex				
Object	**Image**			
Location	Type	Location	Orientation	Relative Size
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$s_o = 2f$	Real	$s_i = 2f$	Inverted	Same size
$f < s_o < 2f$	Real	$\infty > s_i > 2f$	Inverted	Magnified
$s_o = f$		$\pm\infty$		
$s_o < f$	Virtual	$\|s_i\| > s_o$	Erect	Magnified

Concave				
Object	**Image**			
Location	Type	Location	Orientation	Relative Size
Anywhere	Virtual	$\|s_i\| < \|f\|$, $s_o > \|s_i\|$	Erect	Minified