Homework Problems 9-   Due 1/30/23

HW Problem 9

2.15. Verify that $\Psi(x,t) = A\cos(kx - \omega t)$ and $\Psi(x,t) = A\sin(kx - \omega t)$ are *not* solutions to the Schrödinger equation for a free particle:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

HW Problem 10

2.16. Show that $c_1\psi_1 + c_2\psi_1$ is a solution to the Schrödinger equation (2.6) provided ψ_1 and ψ_2 are solutions and c_1 and c_2 are arbitrary complex numbers.

HW Problem 11

2.19. For the wave function $\Psi = Ae^{ikx} + Be^{-ikx}$ evaluate the probability current

$$j_x = \frac{\hbar}{2mi}\left(\Psi^*\frac{\partial\Psi}{\partial x} - \Psi\frac{\partial\Psi^*}{\partial x}\right)$$

HW Problem 12

2.31. Normalize the wave function

$$\Psi(x) = \begin{cases} Nx^2(L-x) & 0 < x < L \\ 0 & \text{elsewhere} \end{cases}$$

What is $\langle x\rangle$ for this wave function?

HW Problem 13

2.34. Assume $\psi(x)$ is an arbitrary normalized real function. Calculate $\langle p_x \rangle$ for the wave function $\Psi(x) = e^{ikx}\psi(x)$.

HW Problem 14

Given the Gaussian wavenumber distribution of a wave package

$A(k) = (\frac{2}{\pi})^{\frac{1}{4}}\sqrt{\sigma}e^{-\sigma^2(k-k_o)^2}$ and using

$\Psi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} A(k)e^{ikx}dk$ show that $\Psi(x) \propto e^{ik_0x}e^{-\frac{x^2}{4\sigma^2}}$

Useful formula: $f(y) = ay^2 + by + c = (\sqrt{a}y + \frac{b}{2\sqrt{a}})^2 - \frac{b^2}{4a} + c$

HW Problem 15

Townsend Problem 2.35 Determine <x> and Δx for the wavefunction

$$\psi(x) = \begin{cases} Ae^{\kappa x} & for\ x \leq 0 \\ Ae^{-\kappa x} & for\ x \geq 0 \end{cases}$$

Note that this is the wavefunction for a particle bound to the Dirac Delta function potential. (We will solve this problem later in the semester.)