

# Muon Physics

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The primary research objective of this lab was to produce an accurate measurement for muon lifetimes on earth's surface. Secondary objectives included verifying the lab's equipment for this project through testing frequency response, saturation voltage, FPGA response time and discriminator functionality. Additionally, we used the derived muon lifetimes to derive the Fermi coupling constant as well as the ratio of positive to negative muons.

## Section 1: Frequency Response

The objective of this section was to verify the frequency response of the amplifier over the range of frequencies we will need for this experiment.

### Equipment:

- Muon physics electronics box
- Oscilloscope

### Procedure

- Applied various frequency 100mV peak-to-peak sine wave from a Function Generator to the PMT Input of the electronics box.
- Measured the Amplifier Output and calculated the ratio  $V_{out}/V_{in}$ , representing the amplifier's gain, for each frequency tested.
- Accounted for the attenuation resistors between the amplifier output and the front panel connector by multiplying this ratio by 21 (1050/50), considering a 1% uncertainty in this factor, to determine the actual amplifier gain.

### Data

The `matrepr` extension is already loaded. To reload it, use:  
`%reload_ext matrepr`

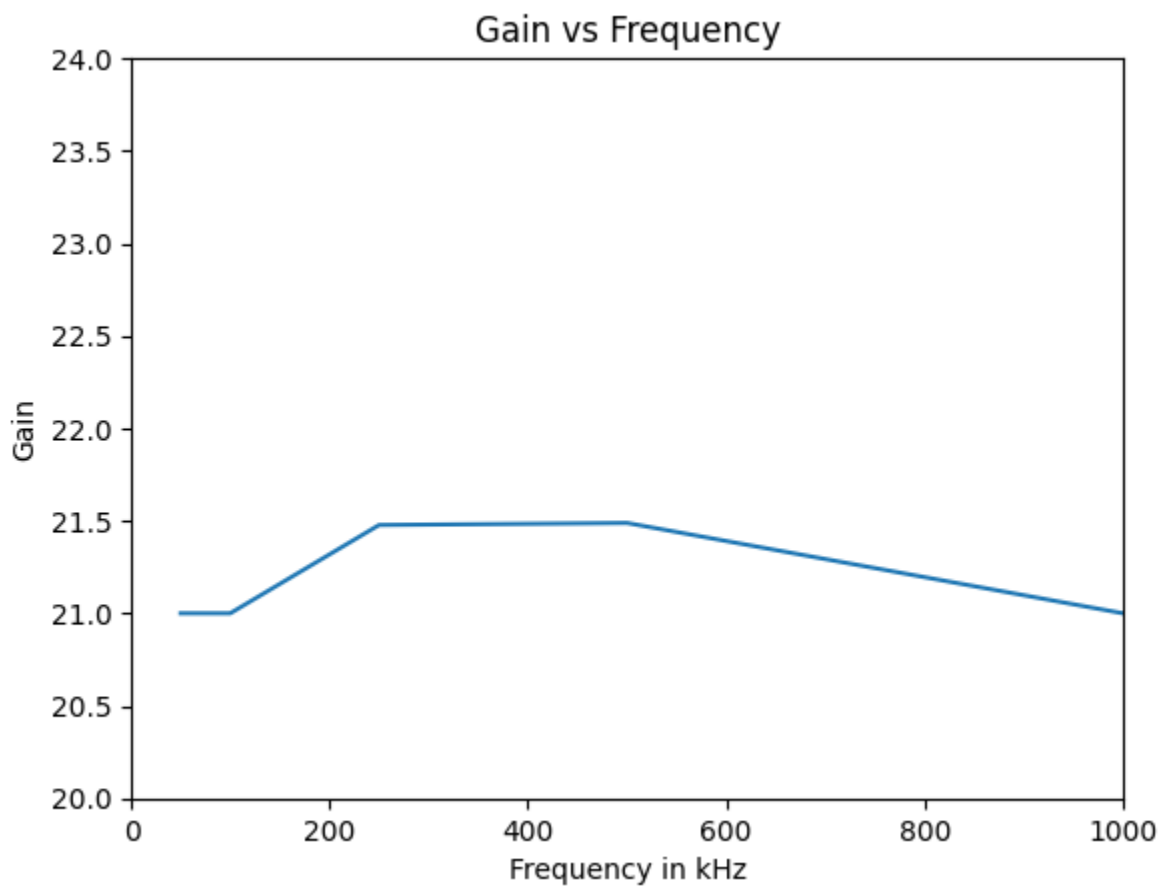


Figure 1: Plot of Gain vs Frequency

### Analysis:

The frequency response for this amplifier circuit is very flat. Throughout the 50kHz-1mHz range, the response remains identical. It is hard to estimate the exact maximum decay rate, but it is likely well in excess of 1 megahertz, or much less than 1 microsecond.

### Conclusions:

The frequency response of the amplifier is very flat. There is a constant and steady gain of 1 throughout the 50kHz to 1mHz regime. This is expected and beneficial to our experiments occurring later on.

## Section 2: Saturation output voltage

The objective of this portion of the lab was to determine the saturation voltage for our amplifier used in this experiment.

### Equipment

- Oscilloscope
- Muon electronics box

### Procedure

- Applied various voltage 100kHz sine waves from a Function Generator to the PMT Input of the electronics box.
- Took measurements of each peak-peak output voltage for each input voltage
- Noted any falloff points or discrepancies

## Data

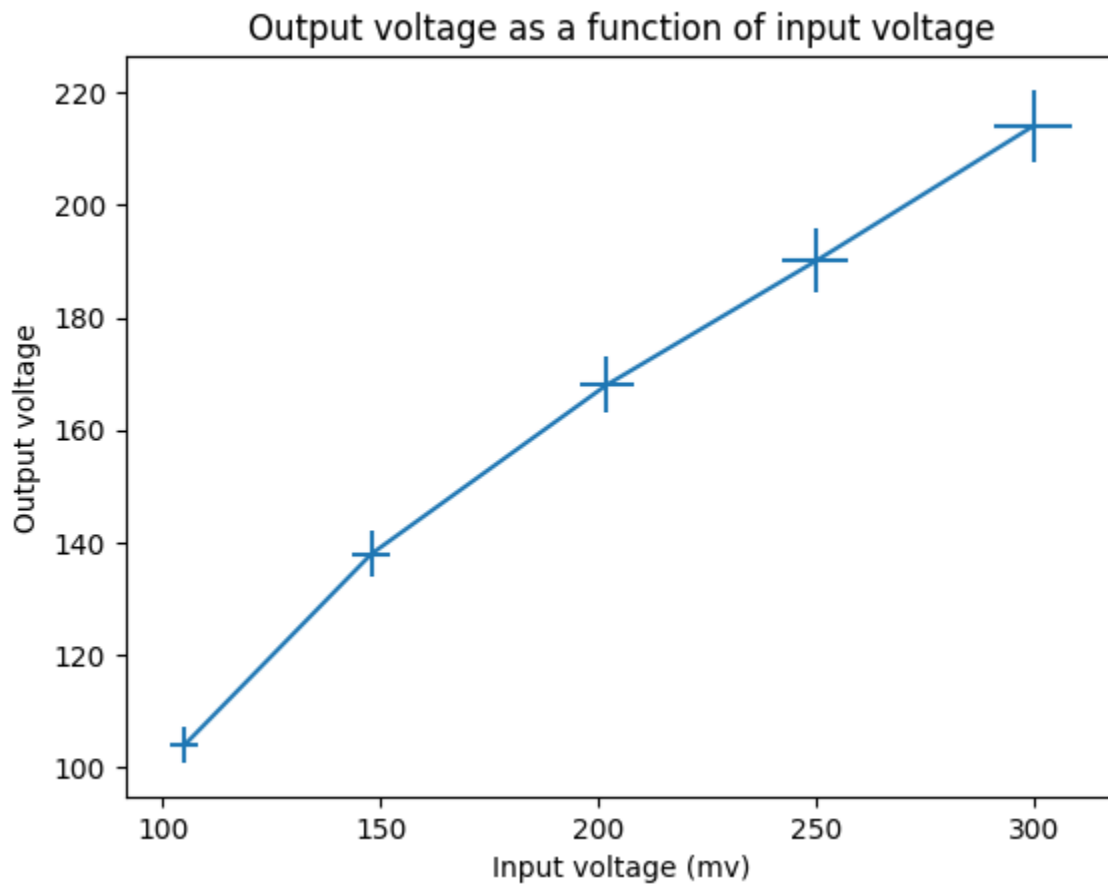


Figure 2: Plot of output voltage as a function of input voltage

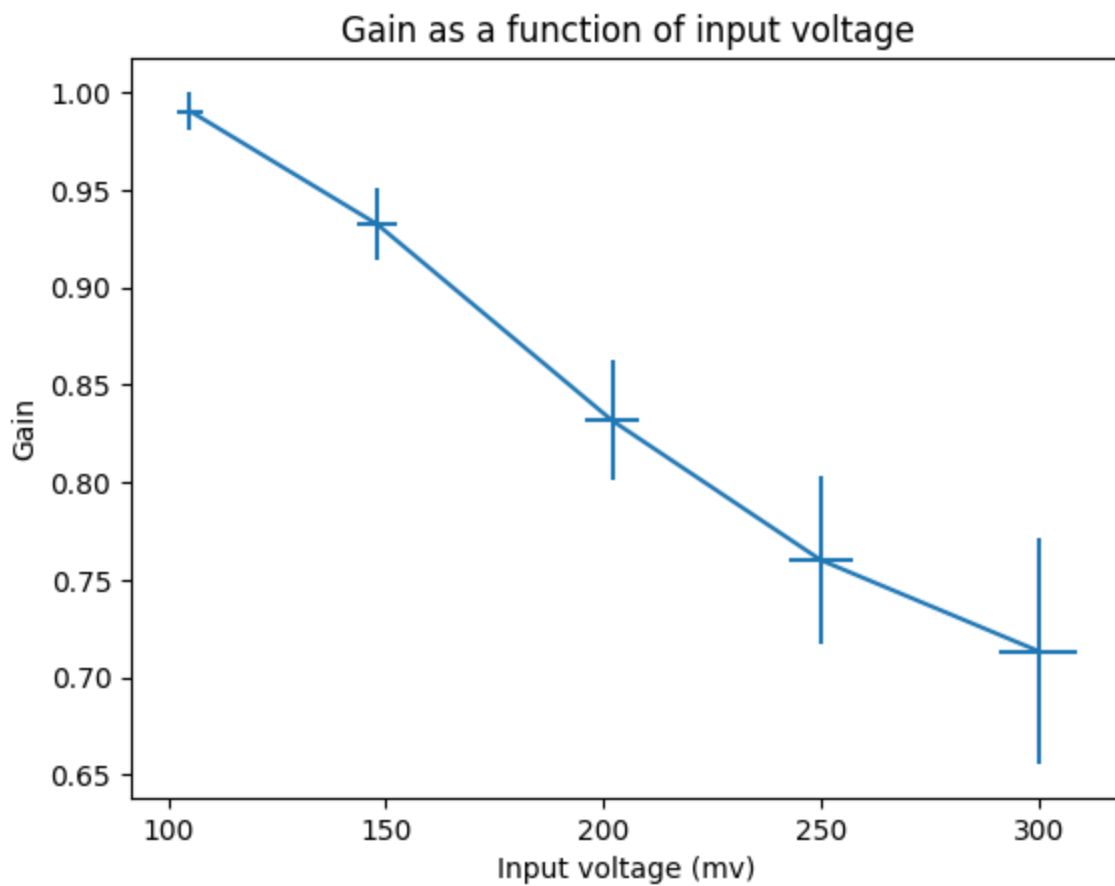


Figure 3: Gain as a function of input voltage

### Analysis:

Voltage saturation began to occur above 100mv. The effect was slight though, and the amplifier continued to perform moderately well past its saturation point. The FPGA did not appear to be affected, and the waveforms appeared to qualitatively be in sync.

## Conclusions

The amplifier performed as expected, with voltage gain dropoff occurring above 100mv. The gain calculated resulted in larger errors towards the higher end of the voltage spectrum due to error propagation between input voltage and output voltage.

## Section 3: Discriminator

The objective of this section was to examine the behavior of the discriminator function of the Muon Physics electronics setup. This was accomplished by adjusting the threshold value and observing the changes in the time difference.

## Equipment

- Oscilloscope
- Muon electronics box
- Digital Multimeter

## Procedure

- Applied a 100kHz 100mV sine signal into the PMT input of the electronics box.
- Adjusted the discriminator threshold to various levels and observed the behavior recorded on oscilloscope.
- Used oscilloscope to measure time interval and voltage max using cursor search feature.
- Repeated for every discriminator value

## Data

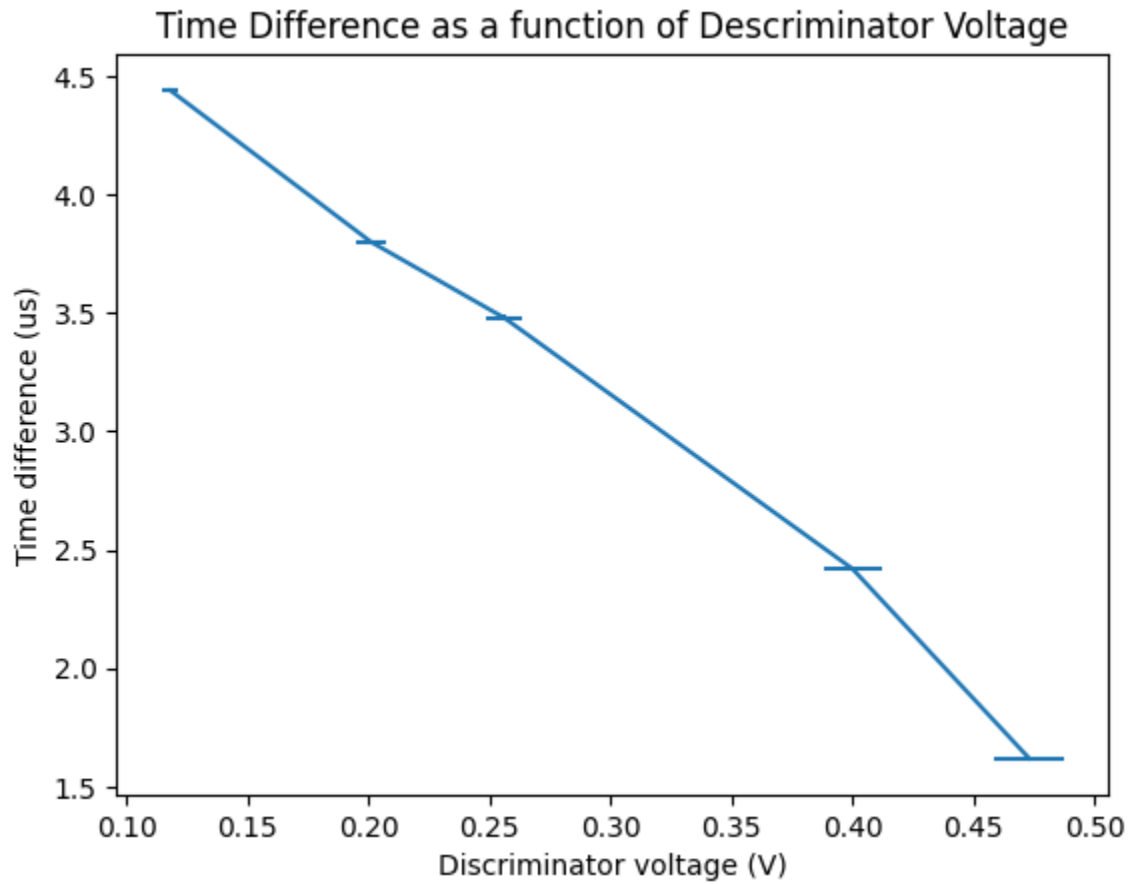


Figure 4: Time difference as a function of Discriminator Voltage

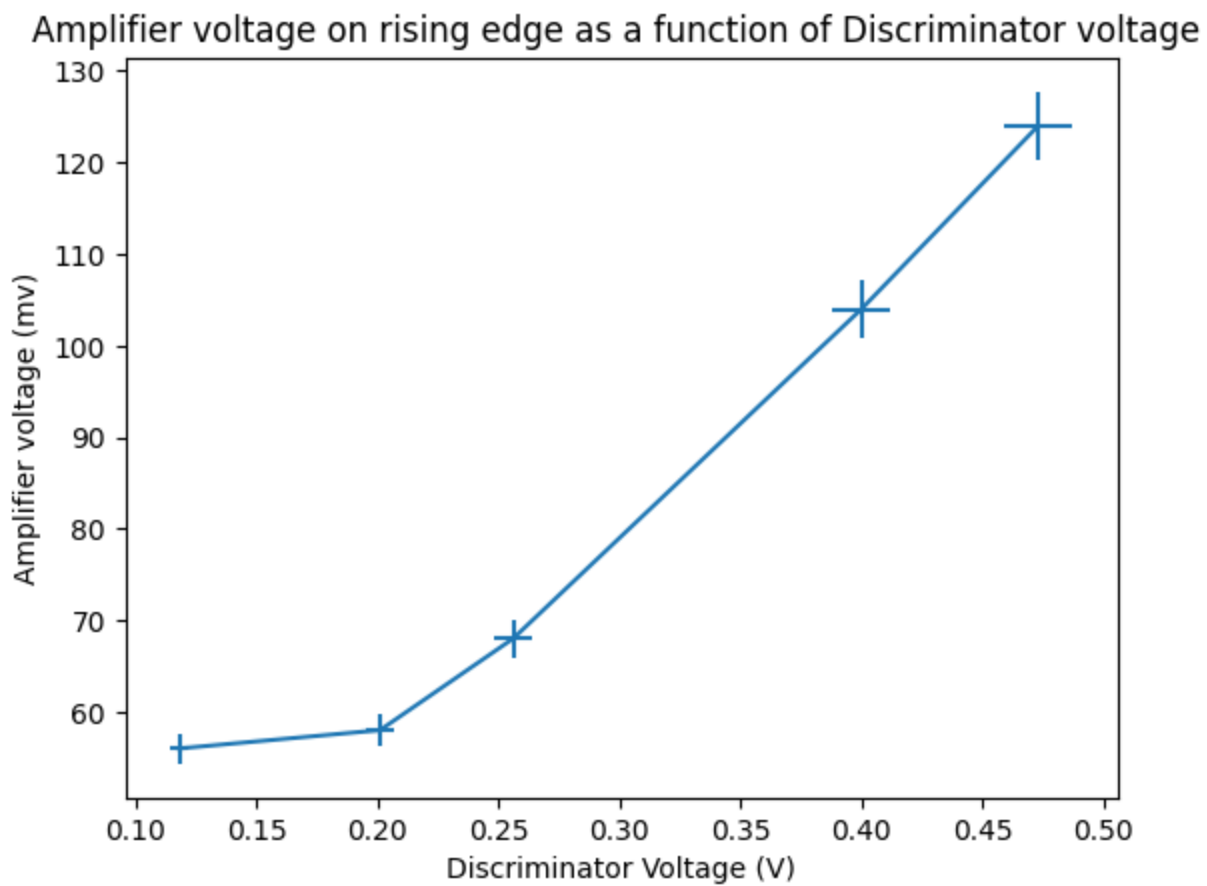


Figure 5 Amplifier voltage on rising edge as a function of discriminator voltage

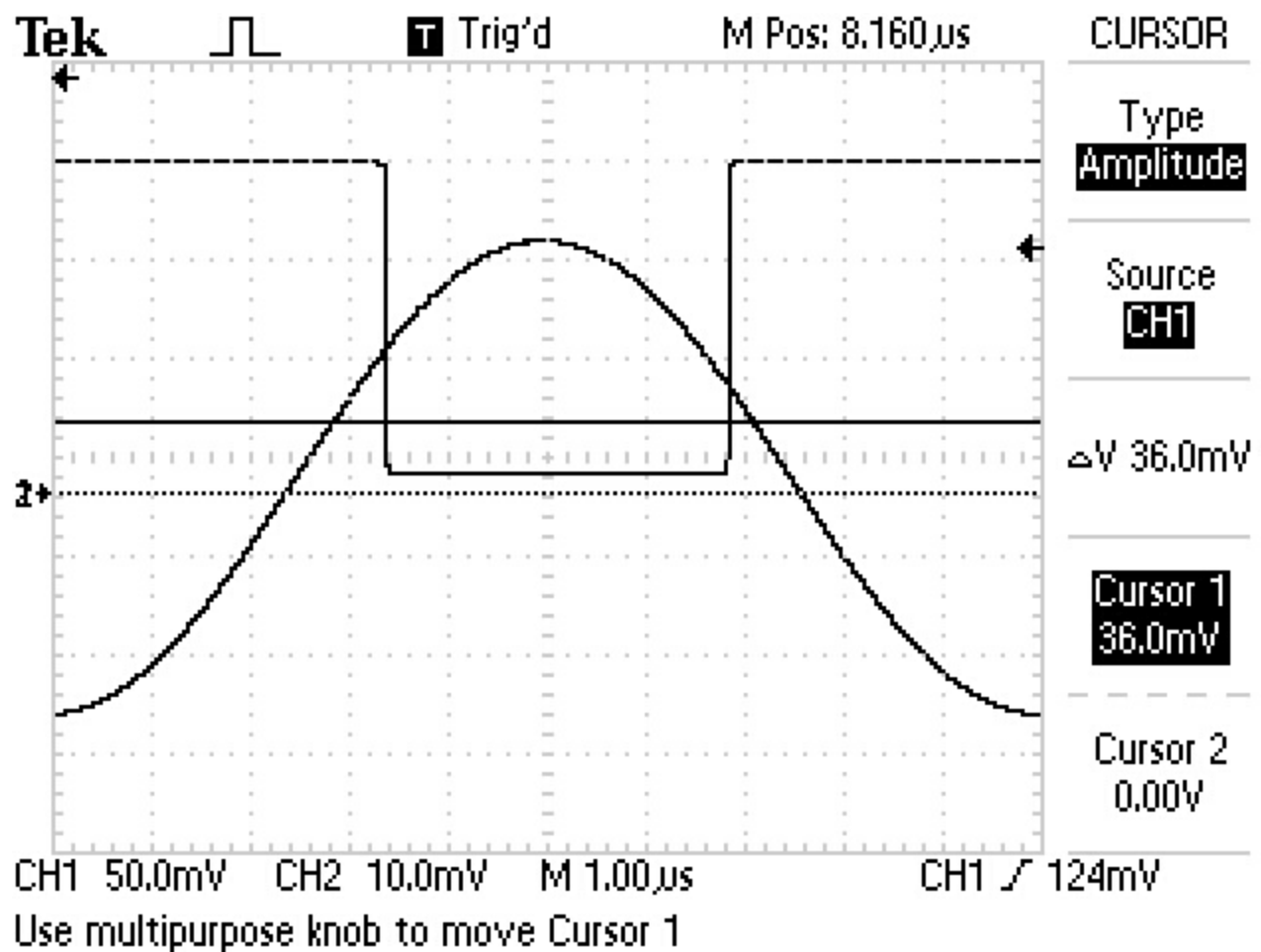


Figure 6: Single discriminator and amplifier signal waveform

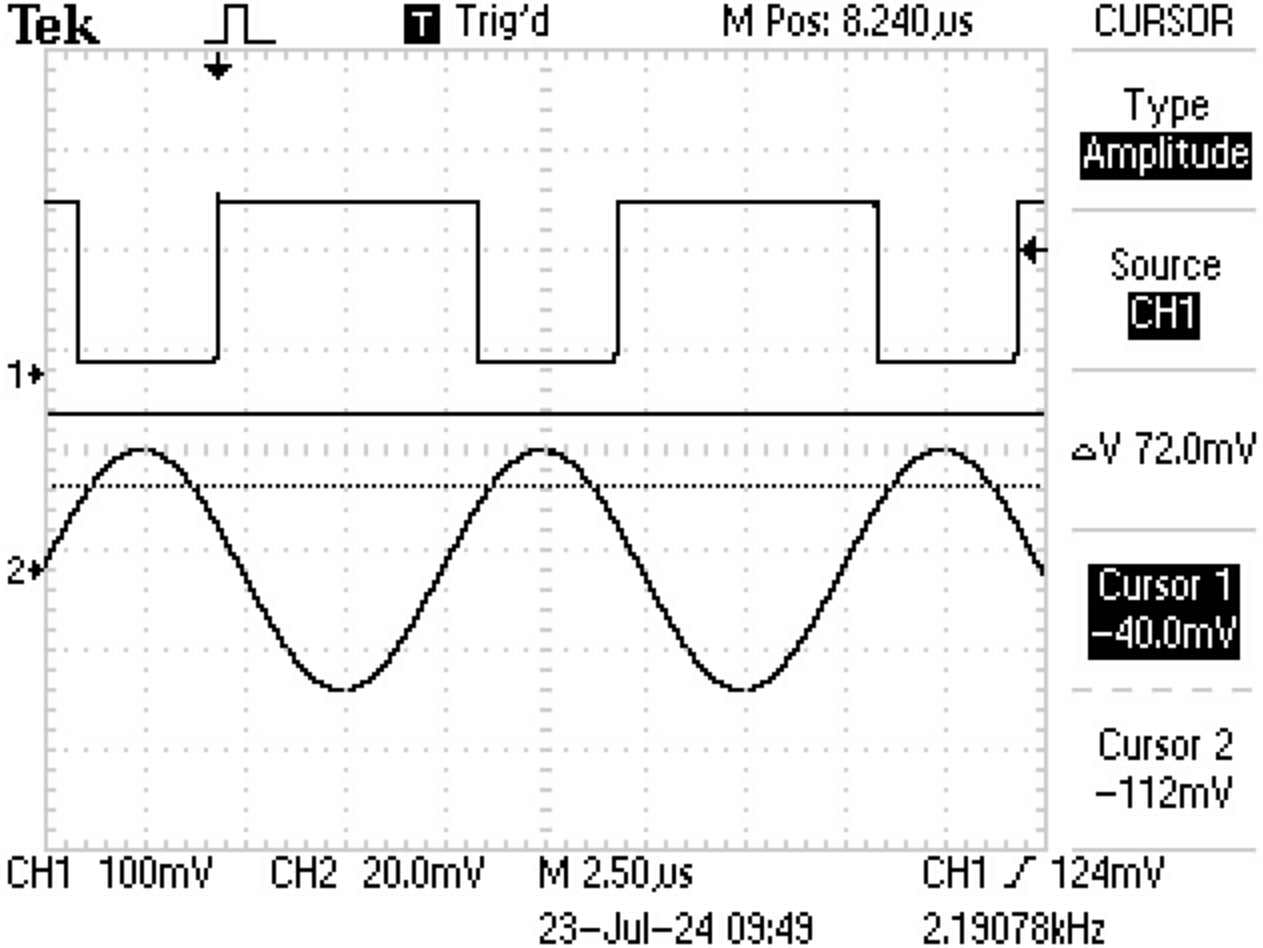


Figure 7: Several discriminator and amplifier signal waveforms

## Analysis:

This section detailed the usage of the discriminator, and its sensitivity to voltage fluctuation. By setting the threshold at any level desired, it allows an arbitrary waveform to be converted into a series of pulses.

## Conclusion:

The discriminator functioned as intended. The waveforms qualitatively looked good and had sharp rising and falling edges. The time difference and amplifier voltage were roughly linear, and error in those may be due to the inprecision of the oscilloscope cursors.

## Section 4: Timing properties of FPGA

The objective of this section was to verify the properties of the FPGA communication interface with the PC used to record muon events later on. Importantly, we determined the maximum and minimum event times observable by the computer.

## Equipment

- Oscilloscope
- Muon electronics box
- Scintillator (and LED pulser)
- Desktop with Muon software

- Digital Multimeter

## Procedure

- We switched from using the function generator to using the pulse input of the scintillator tube.
- Measured the time between successive pulses using the cursors on the oscilloscope.
- Next, we repeated this same measurement using the computer interface.
- These measurements were repeated for various values of pulse length, from 0.5us to 20us.

## Data

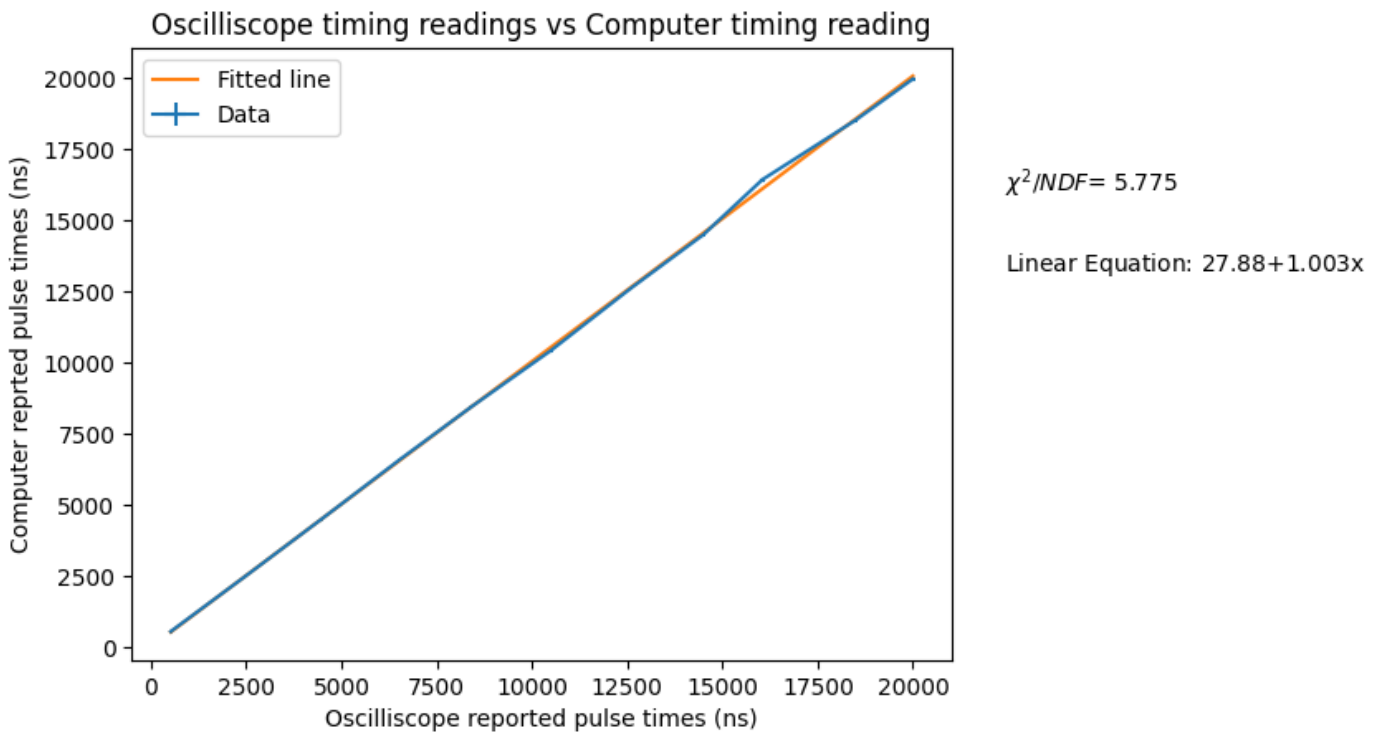


Figure 8: Oscilloscope timings vs Computer timings with a linear curve fit

All points greater than 30us were trimmed out to remove sources of interference. The detector appears to reach a timeout at 40us.

## Analysis:

The maximum event time appears to be 40100 ns, or 40.1 us. This would correspond to the timeout time mentioned in the literature. The minimum event time is shorter than the first data point 0.5us, as linearity was preserved throughout the range of pulse times tested.

## Conclusions:

The FPGA's response range was satisfactory. All muon events relayed to the FPGA should be able to be recorded via the computer interface. The response compared to the oscilloscope is highly linear, with a slope of 1.003. The error of 0.003 was more likely due to oscilloscope cursor error, as it was difficult to align the oscilloscope cursors at the exact same point each time.

## Section 5: Muon Physics



The objective of this portion of the lab was to measure the lifespan of a muon on earth. These results would then be used to calculate the Fermi constant as well as the ratio of positive to negative muons.

## Equipment

- Oscilloscope
- Muon electronics box
- Scintillator
- Desktop with Muon software
- Digital Multimeter

## Procedure

- Configured the discriminator of the muon electronics box to 210mV
- Adjusted the HV input of the scintillator to 700V
- Connected the Scintillator, Muon box, and computer.

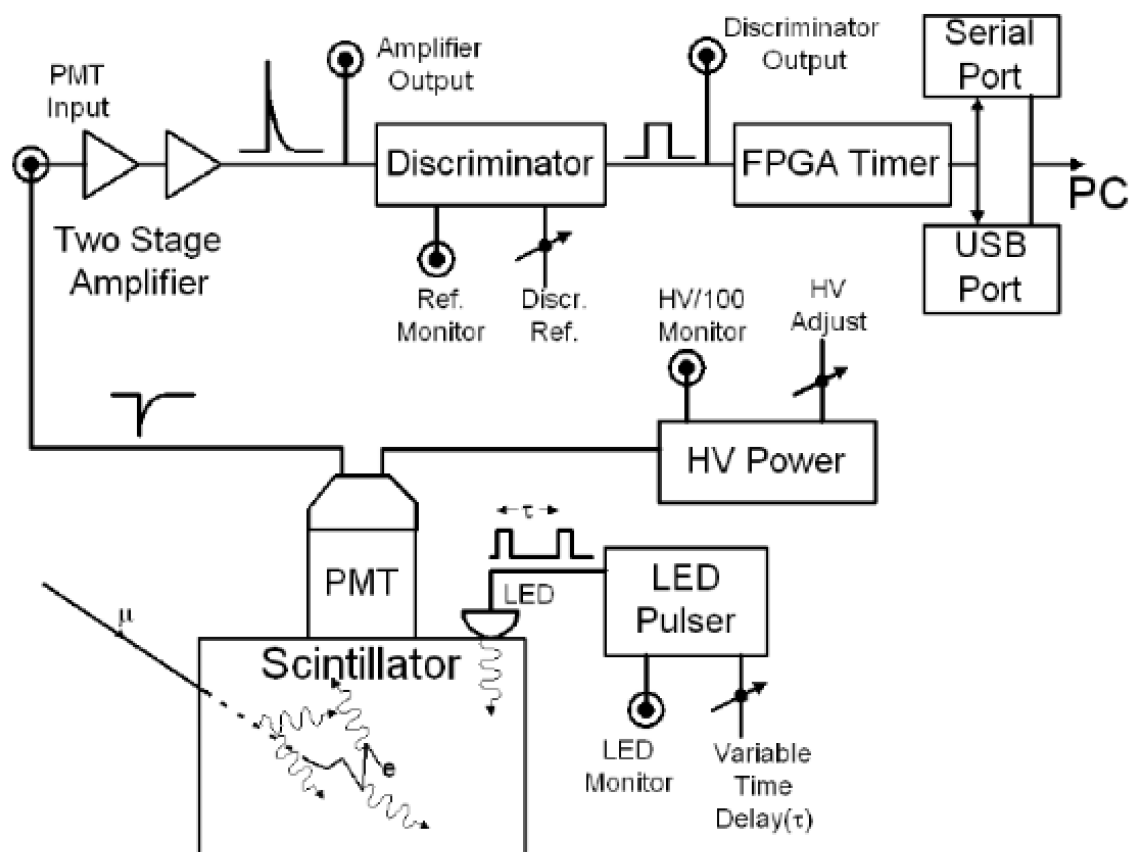


Figure 9: Configuration of Scintillator, Muon electronics box, and Desktop PC

- Began data collection on the computer
- Waited 30 minutes, and verified data collected matched the expected Muon spectrum.
- Waited ~3 days for data to be collected, returned and uploaded data onto personal computers.

## Data

Data was collected over a slightly less than 3 day period.

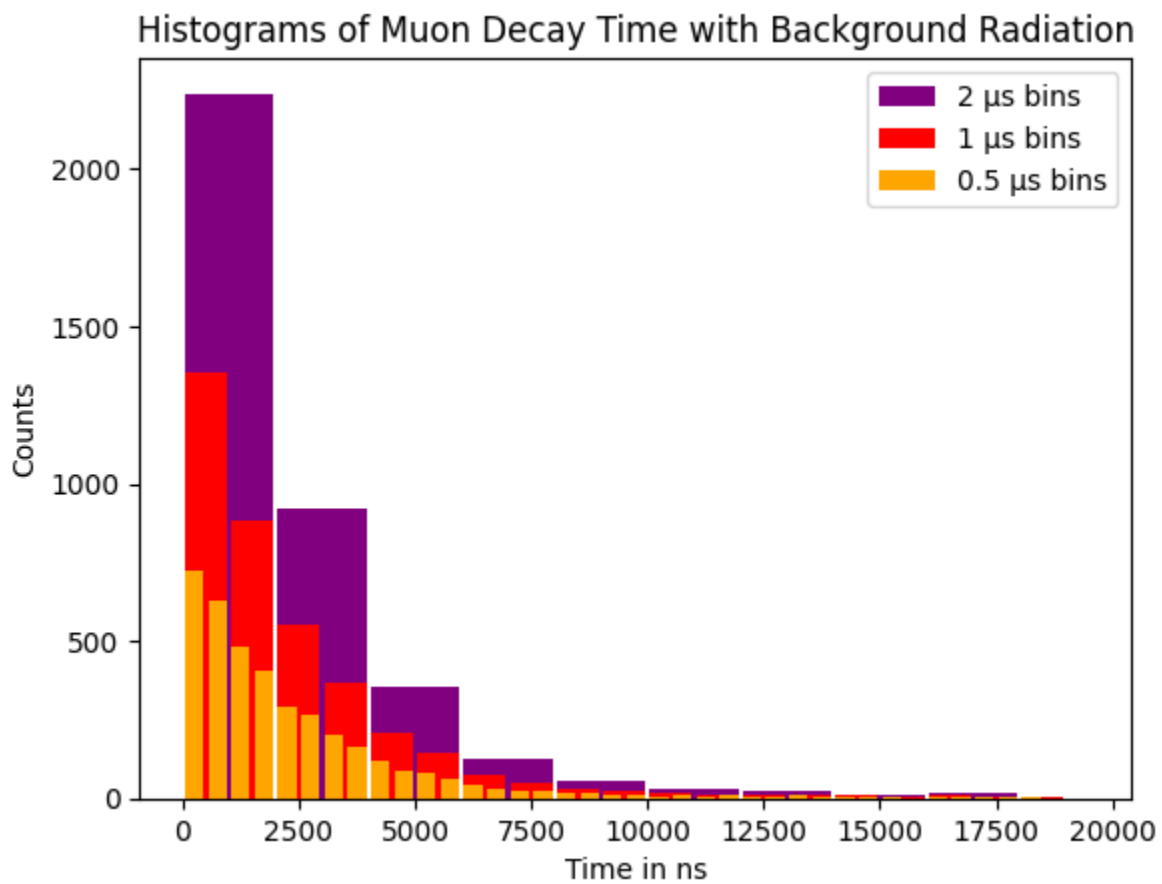
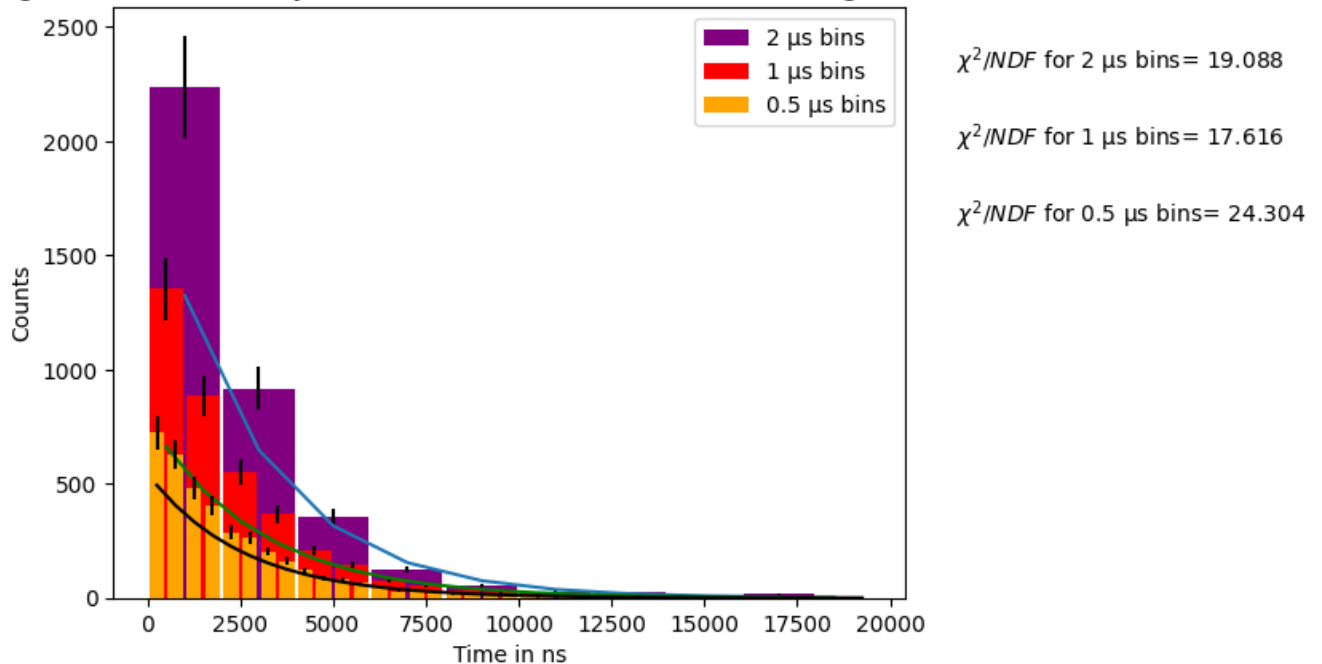


Figure 9.1: Raw Muon lifetime data sorted into 2, 1 and 0.5 us bins

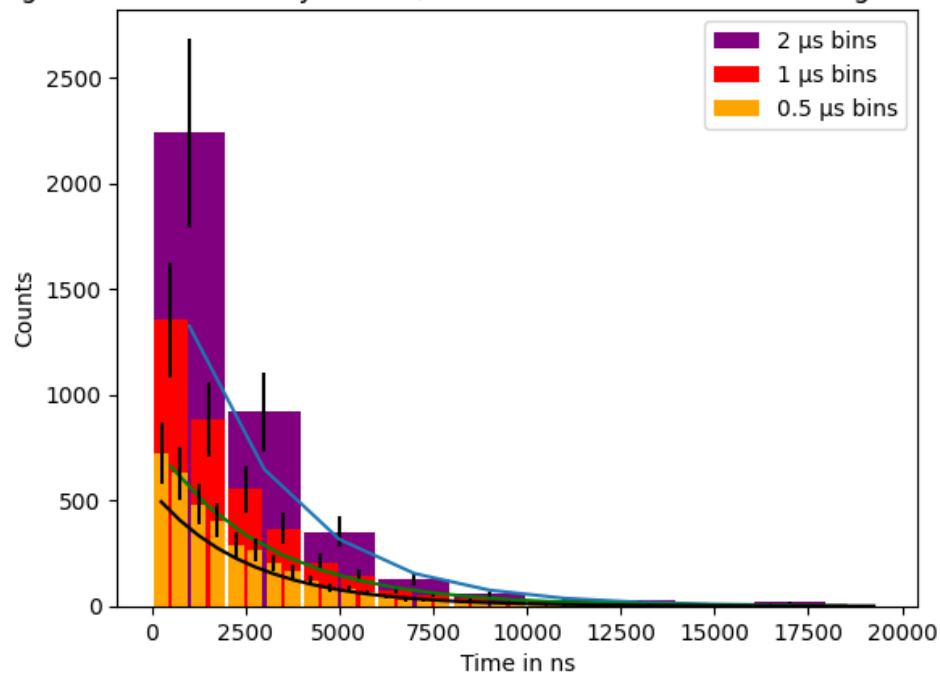
## Analysis

Using Method 1: Error of 10,20 and 30 percent of bin size.

### Histograms of Muon Decay Time w/ 10.0 % bin size error with Background Radiation



Histograms of Muon Decay Time w/ 20.0 % bin size error with Background Radiation

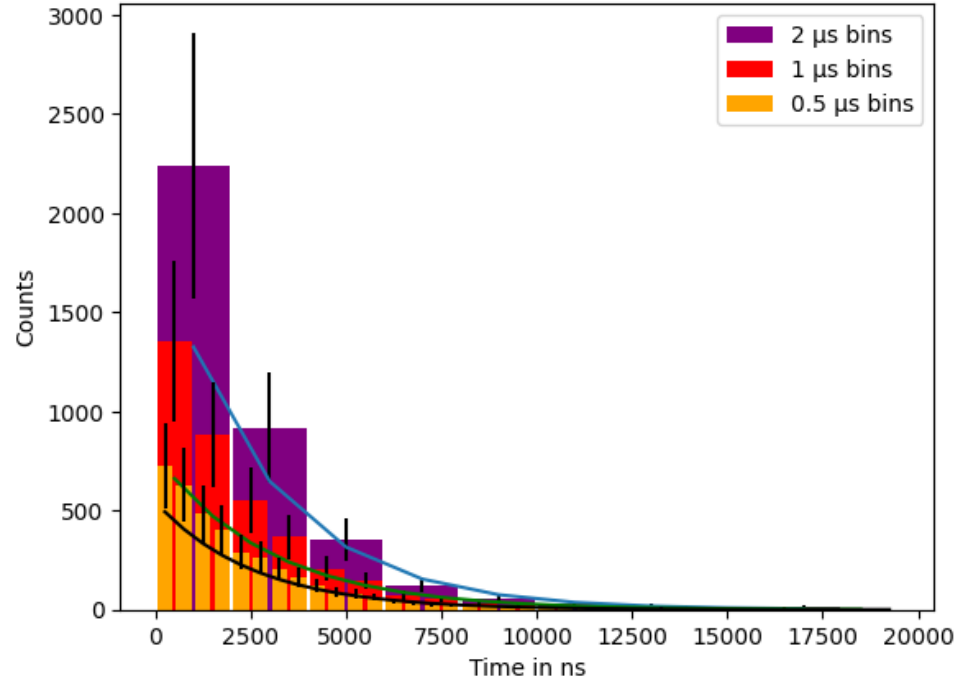


$$\chi^2/NDF \text{ for } 2 \mu\text{s bins} = 4.772$$

$$\chi^2/NDF \text{ for } 1 \mu\text{s bins} = 4.404$$

$$\chi^2/NDF \text{ for } 0.5 \mu\text{s bins} = 6.076$$

Histograms of Muon Decay Time w/ 30.0 % bin size error with Background Radiation



$$\chi^2/NDF \text{ for } 2 \mu\text{s bins} = 2.121$$

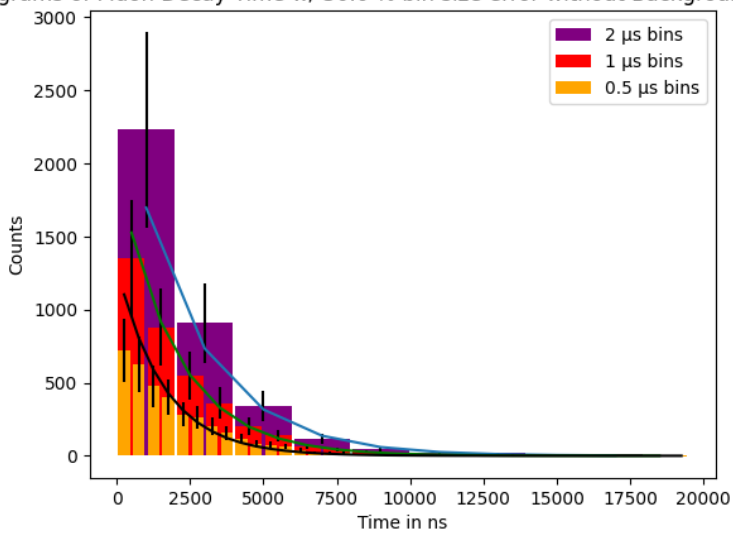
$$\chi^2/NDF \text{ for } 1 \mu\text{s bins} = 1.957$$

$$\chi^2/NDF \text{ for } 0.5 \mu\text{s bins} = 2.7$$

Figures 10, 11, 12: Histograms of muon data with 10, 20 and 30 percent error for each bin respectively

Removing Background Radiation

Histograms of Muon Decay Time w/ 30.0 % bin size error without Background Radiation



$$\chi^2/NDF \text{ for } 2 \mu\text{s bins} = 1.614$$

$$\chi^2/NDF \text{ for } 1 \mu\text{s bins} = 3.906$$

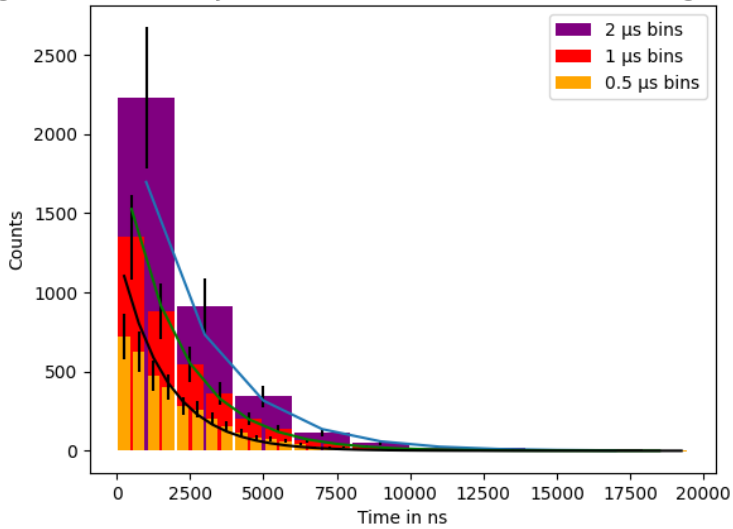
$$\chi^2/NDF \text{ for } 0.5 \mu\text{s bins} = 6.838$$

$$\text{Calculated Decay time for } 2 \mu\text{s bins } \pm = [2382.887 \ 196.217]$$

$$\text{Calculated Decay time for } 1 \mu\text{s bins } \pm = [1962.826 \ 149.962]$$

$$\text{Calculated Decay time for } 0.5 \mu\text{s bins } \pm = [1586.865 \ 134.652]$$

Histograms of Muon Decay Time w/ 20.0 % bin size error without Background Radiation



$$\chi^2/NDF \text{ for } 2 \mu\text{s bins} = 3.63$$

$$\chi^2/NDF \text{ for } 1 \mu\text{s bins} = 8.788$$

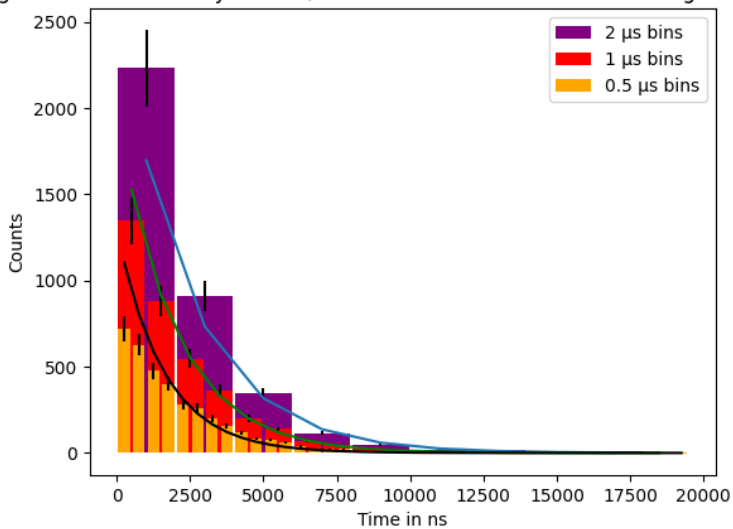
$$\chi^2/NDF \text{ for } 0.5 \mu\text{s bins} = 15.384$$

$$\text{Calculated Decay time for } 2 \mu\text{s bins } \pm = [2382.887 \ 196.217]$$

$$\text{Calculated Decay time for } 1 \mu\text{s bins } \pm = [1962.826 \ 149.962]$$

$$\text{Calculated Decay time for } 0.5 \mu\text{s bins } \pm = [1586.866 \ 134.652]$$

Histograms of Muon Decay Time w/ 10.0 % bin size error without Background Radiation



$$\chi^2/NDF \text{ for } 2 \mu\text{s bins} = 14.521$$

$$\chi^2/NDF \text{ for } 1 \mu\text{s bins} = 35.152$$

$$\chi^2/NDF \text{ for } 0.5 \mu\text{s bins} = 61.537$$

$$\text{Calculated Decay time for } 2 \mu\text{s bins } \pm = [2382.888 \ 196.216]$$

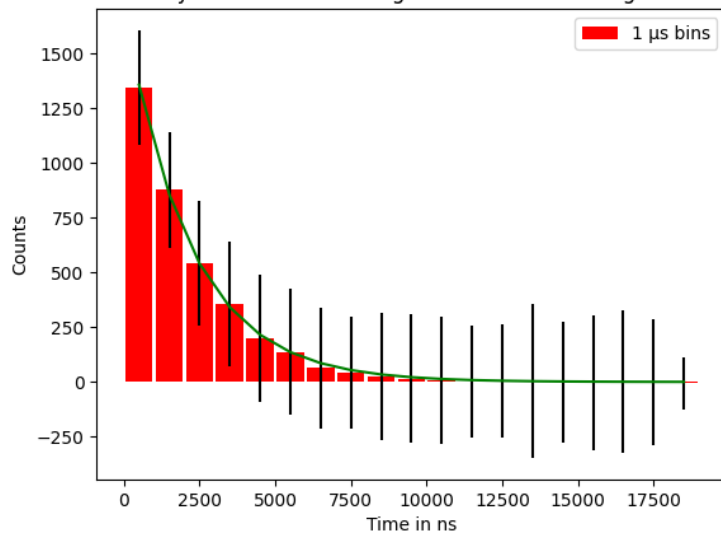
$$\text{Calculated Decay time for } 1 \mu\text{s bins } \pm = [1962.829 \ 149.964]$$

$$\text{Calculated Decay time for } 0.5 \mu\text{s bins } \pm = [1586.867 \ 134.654]$$

Figures 13,14,15: Histograms of muon data with background radiation removed, at error levels 30,20,10 respectively

Method 2: Using bin size to calculate Standard Deviation and using this as error size. Picking 1us bin size:

Histograms of Muon Decay Time without Background Radiation using Standard Deviation of Bins



$\chi^2/NDF$  for 1 μs bins = 0.002

Calculated Decay time for 1 μs bins =  $2173.951 \pm 29.538$

(2173.9509939065156, 29.538065682663856)

Figure 16: Muon histogram using 1us bins and standard deviation as bin error

Making table for all error values and bin sizes for Method 1

Out[47]:

	0.5 us bins	1 us bins	2 us bins
10% error	$1586.8169 \pm 134.6546$	$1962.8274 \pm 149.9635$	$2382.9180 \pm 196.2218$
20% error	$1586.8158 \pm 134.6532$	$1962.8248 \pm 149.9623$	$2382.9166 \pm 196.2220$
30% error	$1586.8156 \pm 134.6530$	$1962.8243 \pm 149.9621$	$2382.9163 \pm 196.2220$

Figure 17: Table of muon lifetimes for 0.5, 1 and 2us bins, at error level 10,20 and 30%

Out[14]:

	0.5 us bins	1 us bins	2 us bins
10% error	61.54	35.15	14.52
20% error	15.38	8.79	3.63
30% error	6.84	3.91	1.61

Figure 18: Table of chi squared values

## Analysis

Next, we are to investigate the Fermi Coupling constant using the muon decay time we derived using method 2. The relation is given below

$$\tau = \frac{192\pi^3\hbar^7}{G_F^2 m^5 c^4}$$

Where  $\tau$  is the lifetime of the muon and  $G_F$  is the Fermi Coupling constant. Rearranging this to solve for the Fermi constant we get the following:

$$G_F^2 = \frac{192\pi^3 \hbar^7}{\tau m^5 c^4}$$

Converting this into the desired final form:

$$\frac{G_F}{(\hbar c)^3} = \sqrt{\frac{1}{\tau}} \sqrt{\frac{\hbar 192 \pi^3}{(m c^2)^5}}$$

$$\text{Mass of muon: } 0.105 \frac{\text{GeV}}{c^2} \quad \hbar = 6.582E - 25$$

Solving constant term:

$$\sqrt{\frac{\hbar 192 \pi^3}{(m_\mu c^2)^5}} = 1.75219 \times 10^{-8}$$

Now we can solve this with uncertainty:

$$\begin{aligned} \Delta G_F &= \sqrt{\left(\frac{\partial G_F}{\partial t} \Delta t\right)^2} \\ \frac{G_F}{(\hbar c)^3} &= 0.0000118838 = \sqrt{\left((-1.75219e - 8 / (2 * \text{sqrt}(1/t)) / t^2\right) \cdot \Delta t\right)^2} \\ &= 8.073e - 7 \\ &= 8.073 \times 10^{-7} \\ &= 8 \times 10^{-7} \\ \frac{G_F}{(\hbar c)^3} &= (1.19 \pm 0.08) \times 10^{-5} \end{aligned}$$

\*credit to <https://pdg.lbl.gov/2020/listings/rpp2020-list-muon.pdf> particle data group for muon mass

Next, we are to solve for  $\rho$ , the ratio of positive to negative muons. This ratio is given by:

$$\rho = -\frac{\tau^+}{\tau^-} \left( \frac{\tau^- - \tau_{\text{obs}}}{\tau^+ - \tau_{\text{obs}}} \right)$$

Where  $\tau^+$  is the decay rate of positively charged muons, identical to the free-space decay rate  $\tau = 2.197 \pm 0.00004 \mu s$ .  $\tau^-$  is the decay time of negatively charged muons, which decay faster in matter. The decay rate of these particles in matter is given by  $\tau_c = 2.043 \pm 0.003 \mu s$

Using our observed muon decay time of

$$\tau_{\text{obs}} = 2.173 \pm 0.029$$

We can solve for rho:

$$\begin{aligned}
\Delta\rho &= \sqrt{\left(\frac{\partial\rho}{\partial t}\Delta t\right)^2 + \left(\frac{\partial\rho}{\partial tp}\Delta tp\right)^2 + \left(\frac{\partial\rho}{\partial tm}\Delta tm\right)^2} \\
&= \sqrt{\left(\left(-tp * 1/tm/(tp-t)^2 * (tm-tp)\right) \cdot \Delta t\right)^2 \\
&= 5.8249714472 \quad \left(+ \left((tp * 1/tm/(tp-t)^2 * (tm-t) - (tm-t) * 1/tm/(tp-t)^1\right) \cdot \Delta tp\right)^2\right. \\
&\quad \left.+ \left((tp * 1/(tp-t)/tm^2 * (tm-t) - tp * 1/(tp-t)/tm^1\right) \cdot \Delta tm\right)^2} \\
&= 8.3391551635 \\
&= 8.3391551635 \times 10^0 \\
&= 8 \times 10^0
\end{aligned}$$

$$\rho = 6 \pm 8$$

## Conclusion:

Through this section we calculated the lifetime of a muon, the Fermi Constant, and the ratio of positive to negative muons that landed within our collector. Our values for muon lifetime were slightly lower than expected but this can also be attributed to negative muons decaying in matter quicker. If we were able to detect muons in free space, we would be able to derive the free space Muon decay time. Our best fitting muon decay time using method 1 was  $2382.9163 \pm 196.2220$  nanoseconds, with a reduced chi squared of 1.61. This is within uncertainty of the literature value of muon lifetime, however, knowing that the detected muon lifetime should be slightly less, this is slightly off. An explanation for this is the approach utilized a flat rate error, which heavily weighted bins with fewer data-points. In an exponentially decaying model like this one, this means that the higher-value bins were much more heavily weighted, increasing the overall calculated muon lifetime. Using a much better approach, taking the standard deviation of each bin, we derived a muon decay time of  $2173.951 \pm 29.538$  nanoseconds, slightly below the free space lifetime as predicted. However, despite this, we had a highly over-constrained graph due to the large errors in the latter part of the spectrum. This is evidenced by the derived reduced chi squared of 0.002. Using this second value, we calculated the Fermi constant and muon ratio  $\rho$ . These values were respectively calculated at  $\frac{G_F}{(\hbar c)^3} = (1.19 \pm 0.08) \times 10^{-5}$  and  $\rho = 6 \pm 8$ , both falling within literature estimated values. The Muon ratio had a large error due to all three muon decay times, both given and derived, having non-negligible errors, resulting in a relatively large propagated error.