Homework due Nov 2

1) Observer O fires a light pulse in the +y-direction. The pulse is observed by another observer who is traveling with velocity u in the x-direction. Use the Lorentz velocity transformations to find v_x' and v_y' and show that O' also measures the speed of the pulse to be *c*.

Observer O measures velocities of $v_x = 0 \; and \; v_y = c$. According to O, the pulse leaves the origin (0,0) and arrives at position $x = 0 \; and \; y = ct$ at time t. According to observer O', the pulse leaves position (0,0) and has spacetime coordinates as follows:

$y' = y$

$x' = \gamma(x - ut)$

$t' = \gamma\left(t - \frac{ux}{c^2}\right)$

The velocity along the y' direction is measured to be $v_y' = \frac{y'}{t'} = \frac{ct}{\gamma t} = \frac{c}{\gamma}$.

The velocity along the x' direction is $v_x' = \frac{x'}{t'} = \frac{\gamma(x-ut)}{\gamma\left(t - \frac{ux}{c^2}\right)} = -\frac{ut}{t} = u$.

The total velocity in x' is therefore

$$v'_T = \left(v_x'^2 + v_y'^2\right)^{\frac{1}{2}} = \sqrt{\frac{c^2}{\gamma^2} + u^2} = \sqrt{c^2\left(1 - \frac{u^2}{c^2}\right) + u^2} = \sqrt{(c^2 - u^2 + u^2)} \;\; = c$$

2) A neutral K meson at rest decays into two π mesons which travel in opposite directions along the x-axis with speeds of 0.815c.

   a. What speed is observed for one of the pi mesons as observed from the frame of the other pi meson?

   In the frame of the K meson (lab frame), the two velocities are $\pm 0.815c$. In the frame of the pion traveling in the negative x direction in the K meson frame, the speed of the other particle is the speed of the K meson frame + the speed of the +x particle in the K meson frame.

   The velocity addition formula from page 14 of the Lorentz Transformation notes, yields

   $$v' = \frac{u+v}{1+\frac{uv}{c^2}} = \frac{0.815c+0.815c}{1+\frac{0.815^2 c^2}{c^2}} = \frac{1.63c}{1+0.815^2} = \frac{1.63c}{1..664} = 0.98c$$

   b. If instead the K meson were moving in the positive x direction at 0.453c with respect to the lab, what speeds would the pi mesons be observed to have in the lab frame?

If the K meson is moving at speed 0.453c with respect to the lab the velocity of the -x particle is:

$$v'(leftparticle) = \frac{v+u}{1+\frac{uv}{c^2}} = \frac{-0.453 - 0.815}{1 + 0.815 \cdot 0.453}c = -0.93c$$

$$v'(right\ particle) = \frac{v+u}{1-\frac{uv}{c^2}} = \frac{-0.453+0.815}{1-0.815\cdot 0.453}c = 0.57c$$

3) Two particles in an accelerator approach each other directly. Each particle measures a speed of 0.85c for the other particle. Both particles are observed to have a speed of *v* as observed in a frame in which they have the same speed. Find *v*.

$0.85c = \frac{v+u}{1+\frac{uv}{c^2}} = \frac{2v}{1+\frac{v^2}{c^2}}$. This can be solved as a quadratic equation.

4) In reference frame S, observer O observes event B to occur 4×10^{-6} s after event A. Events A and B are separated by 1.5 km along the x axis in frame O. With what speed must observer O' be moving in order for the two events to occur simultaneously for O?

Take event A to occur at (x,t)=(0,0) in the lab frame. Event B is ($1500, 4 \times 10^{-6}$) in the O frame.

$x' = \gamma(x - ut)$

$t' = \gamma\left(t - \frac{ux}{c^2}\right)$ (Set to zero). This means that $t = \gamma\frac{vx}{c^2}$. Solve the quadratic.

5) Observer S assigns the following spacetime coordinates to an event:
x=100 km, y=10 km, z=55 km, t=200 microseconds.
What are the coordinates of this event in frame S', which moves in the direction of increasing x with speed 0.90c? (The two coordinate systems share the following coordinates x=0 and t=0 when x'=0, and t'=0.)

This is a straightforward Lorentz transformation problem.

$x' = \gamma(x - ut)$

$t' = \gamma\left(t - \frac{ux}{c^2}\right)$

Coordinates perpendicular to the motion do not change.

$x' = 138\ km$, y'=10 km, z'=55 km, t'=-374 μs.