Rohlf

CHAPTER 6

RUTHERFORD SCATTERING

Attention was drawn to the remarkable fact, first observed by Geiger and Marsden, that a small fraction of the swift α particles from radioactive substances were able to be deflected through an angle of more than 90° as the result of an encounter with a single atom... . In order to account for this large angle scattering of α particles, I supposed that the atom consisted of a positively charged nucleus of small dimensions in which practically all of the mass of the atom was concentrated. The nucleus was supposed to be surrounded by a distribution of electrons to make the atom electrically neutral, and extending to distances from the nucleus comparable to the ordinary accepted radius of the atom. Some of the swift α particles passed through the atoms in their path and entered the intense electric field in the neighborhood of the nucleus and were deflected from their rectilinear path. In order to suffer a deflection of more than a few degrees, the α particle has to pass very close to the nucleus, and it was assumed that the field of force in this region was not appreciably affected by the external electronic distribution. Supposing that the forces between the nucleus and the α particle are repulsive and follow the law of inverse squares, the α particle describes a hyperbolic orbit round the nucleus and its deflection can be simply calculated.

Ernest Rutherford

6-6 PROBING THE STRUCTURE OF THE QUARK

6-7 SUMMARY OF THE SCATTERING EXPERIMENTS

In 1909, Hans Geiger and Ernst Marsden observed that α particles from radioactive decays occasionally scatter at large angles (>90°) when passing through a thin layer of material. Scattering at large angles is possible only if a sufficiently large force is exerted on the α particle. Such a force can arise only if the α particle passes very close to the particle that causes the scattering, much closer than the size of an atom. This led Ernest Rutherford in 1912 to deduce that the positive charge of the atom must be concentrated in a nucleus. With that simple assumption, Rutherford was able to derive a detailed formula for the scattering, which was subsequently verified by more detailed experiments. This marked the beginning of an era of particle-scattering experiments that have led us to our present understanding of the fundamental structure of matter.

6-1 MEASURING STRUCTURE BY PARTICLE SCATTERING

The structure of matter is determined by scattering experiments. These scattering experiments have taken on a wide variety of forms. When we view a cell through a microscope, we scatter photons from the cell and observe the photons with our eye. Measurement of the scattered photons gives us an image of the cell. When light passes by an object, the light "bends" or diffracts. The amount of diffraction depends on the wavelength of the light. Therefore, the resolution of the image has a fundamental limitation that is set by the wavelength of the photons. Any type of particle may be used to probe the structure of matter. The key to achieving good resolution is to use particles of small wavelength.

At the beginning of the twentieth century, Ernest Rutherford designed an experiment to measure the structure of the atom using the α particle as a probe. The α particles were produced in the decay of radon. The α particle has a mass of about 3.7 GeV/c^2 and an electric charge of $+2e$. Rutherford determined that the α particle was a helium atom minus 2 electrons (doubly ionized helium). When Rutherford designed the experiment, he did not know the structure of an α particle, but he assumed that it was extremely small compared to the size of an atom. As Rutherford and his associates soon discovered, the α particle is just the nucleus of the helium atom.

EXAMPLE 6-1

The α particles used in the original Rutherford experiment had a kinetic energy of $E_k = 5.5$ MeV, a typical energy of a nuclear decay. Estimate the spatial resolution that may be achieved by scattering of these α particles.

SOLUTION:

The spatial resolution limit of this experiment is determined by the wavelength of the α particle. The α particle mass energy is

$$m_\alpha c^2 = 3730\,\text{MeV}\,.$$

The α particle momentum is given by the expression

$$\begin{aligned} pc &= \sqrt{\left(E_k + m_\alpha c^2\right)^2 - \left(m_\alpha c^2\right)^2} \\ &= \sqrt{E_k^2 + 2m_\alpha c^2 E_k} \approx \sqrt{2m_\alpha c^2 E_k}\,. \end{aligned}$$

The α particle wavelength (5.7) is

$$\begin{aligned} \lambda &= \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{\sqrt{2m_\alpha c^2 E_k}} \\ &\approx \frac{1240\ \text{MeV}\cdot\text{fm}}{\sqrt{(2)(3730\ \text{MeV})(5.5\ \text{MeV})}} \\ &\approx 6\ \text{fm} = 6\times 10^{-15}\ \text{m}. \end{aligned}$$

The α particle wavelength is much smaller than the size of the atom (about 10^{-10} m); it is about equal to the size of the nucleus. Nature had provided physicists with a tool well matched to the task at hand! ■

The conceptual design of the Rutherford scattering experiment is indicated in Figure 6-1. A thin metal foil is bombarded with α particles and the angle θ of the scattered α particles is measured. The angular distribution, $dN/d\theta$, from the scattering of a large number of α particles gives us important details about the structure of the atom. At the time of the experiment, atoms were known to be electri-

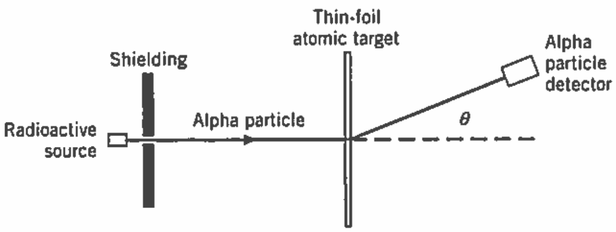

FIGURE 6-1 The Rutherford experiment.
Energetic α particles are passed through a thin metal foil and the angular scattering angle θ is measured.

cally neutral and contain electrons. Therefore, it was known that the atoms also contained positive charge, but there was no knowledge of how the positive charge was distributed in atoms. Consider the collision of an α particle with a single atom as it passes through the thin foil. We refer to the positive charge, however it is distributed in the atom, as the atomic scattering center. We shall neglect collisions of the α particle with electrons because the α particle mass (m_α) is so much greater than the electron mass (m),

$$m_\alpha >> m. \tag{6.1}$$

EXAMPLE 6-2

Estimate the maximum kinetic energy that can be transferred to an electron from a collision with a 6-MeV α particle.

SOLUTION:

The maximum kinetic energy is transferred in a head-on collision. Let m_α be the mass of the α and m be the mass of the electron. Let v_α be the initial speed of the α and v be the recoil speed of the electron. The initial speed of the α particle is given by

$$\frac{1}{2} m_\alpha v_\alpha^2 = E_k,$$

or

$$v_\alpha = \sqrt{\frac{2E_k}{m_\alpha}}.$$

We know from momentum conservation that the speed of the electron after the collision is

$$v \approx 2v_\alpha = 2\sqrt{\frac{2E_k}{m_\alpha}},$$

as may be verified by looking in the frame where the α is at rest. The increase in the kinetic energy of the electron (ΔE) is

$$\Delta E = \frac{1}{2}mv^2 = \frac{1}{2}m\left[2\sqrt{\frac{2E_k}{m_\alpha}}\right]^2$$
$$= \frac{4mE_k}{m_\alpha} \approx \frac{(4)(0.511\,\text{MeV})(6\,\text{MeV})}{3730\,\text{MeV}} \approx 3\,\text{keV}.$$

Note that this energy is much larger than the typical energy of an outer electron in the atom (a few eV) but much smaller than the kinetic energy of the α particle (a few MeV). ■

We now estimate the scattering angle of the α particle. Let the α particle be traveling with a speed v_α and come within a distance r of the atomic scattering center. Let q_1 be the charge of the α particle ($q_1 = 2e$) and let q_2 be the positive charge of the atom ($q_2 = Ze$). The change in momentum (Δp) of the α particle when it passes the atomic scattering center is the electric force acting for a time (Δt), or

$$\Delta p = F\Delta t \approx \left(\frac{kq_1q_2}{r^2}\right)\left(\frac{2r}{v_\alpha}\right), \tag{6.2}$$

where we have estimated Δt to be equal to $2r/v_\alpha$, the time that it takes the α particle to pass by the atomic scattering center. The scattering angle (θ_s) due to this single collision is approximately

$$\theta_s \approx \frac{\Delta p}{p} = \left(\frac{2kq_1q_2}{rv_\alpha}\right)\left(\frac{1}{m_\alpha v_\alpha}\right)$$
$$= \frac{kq_1q_2}{r\left(\frac{1}{2}m_\alpha v_\alpha^2\right)} = \frac{kq_1q_2}{rE_k}, \tag{6.3}$$

where E_k is the kinetic energy of the α particle. The important result of this estimate is that the scattering angle is inversely proportional to the distance (r) between the α and the atomic scattering center. If the scattering center is small in size, as indicated in Figure 6-2a, then r can be very small and large scattering angles are possible. If the atomic scattering center has a finite size (R), then there is a possibility that the α particle can penetrate the atomic scattering center, as indicated in Figure 6-2b. For a penetrating α particle, the force exerted is that due to a smaller charge. For a fixed value of r, the scattering angle is

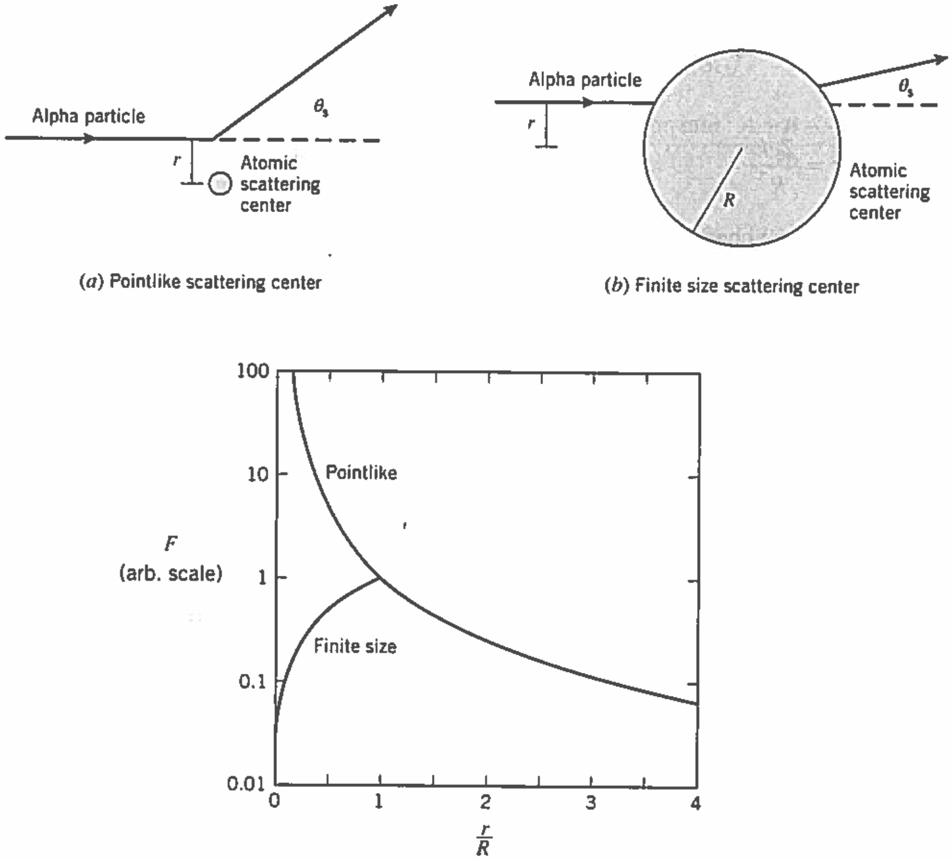

FIGURE 6-2 Force on an α particle as a function of distance (r) from the positive charge inside an atom.
(a) The α particle does not penetrate the positive charge of the atom. The α particle feels the full positive charge (Ze) of the atom. (b) The positive charge of the atom has a finite size and the α particle is able to penetrate the charge. The effective charge that scatters the α particle is only the fraction $(r/R)^3$, where R is the size of the charge. This results in a weaker force compared to (a) and the scattering angle is smaller at fixed r.

smaller for a penetrating α particle than for the case where the atomic scattering center is concentrated at a point.

EXAMPLE 6-3

Consider the charge Ze spread out uniformly in a sphere of radius R. Calculate the electric force at a distance r from the center of the sphere.

SOLUTION:

Gauss's law (B.1), is

$$\oint\!\!\oint d\mathbf{a} \cdot \mathbf{E} = 4\pi k q_{\text{tot}} .$$

For $r > R$, we have

$$4\pi r^2 E = 4\pi k Ze,$$

or

$$E = \frac{Zke}{r^2}.$$

The electric force ($F_{r>R}$) on the charge e is

$$F_{r>R} = eE = \frac{Zke^2}{r^2}.$$

Now consider the case where $r < R$. Applying Gauss's law, we have

$$4\pi r^2 E = 4\pi k Ze\left(\frac{r}{R}\right)^3,$$

or

$$E = \frac{Zekr}{R^3}.$$

The electric force ($F_{r<R}$) on the charge e is

$$F_{r<R} = eE = \frac{Zke^2 r}{R^3}.$$

The force is maximum (F_{max}) when $r = R$:

$$F_{max} = \frac{Zke^2}{R^2}.$$ ■

If the positive charge in an atom were uniformly distributed, then the angular distribution of α particles should be sharply peaked in the forward direction.

EXAMPLE 6-4

Estimate the scattering angle for a single collision of a 6-MeV α particle with a platinum atom ($Z = 78$) if the positive charge was uniformly distributed in the atom.

SOLUTION:

The minimum value of R is the size of the atom. The scattering angle for a single collision is

$$\theta_s \approx \frac{kq_1q_2}{RE_k} = \frac{(2)(78)(1.44\ \text{eV}\cdot\text{nm})}{(0.1\ \text{nm})(6\times10^6\ \text{eV})} = 4\times10^{-4}.$$

The scattering angle for a single collision is very small if the charge is spread out over the entire atom. Even a relatively large number of these collisions will not make a large net scattering angle. ■

A thin target foil contains a very large number of atomic layers, which each scatter the α particle through a small angle. The scattering from each layer can occur in any direction. The resulting distribution will be a Gaussian centered at zero. We can make an order of magnitude estimate of the width of the Gaussian.

EXAMPLE 6-5

Estimate the root-mean-square scattering angle of an α particle passing through a platinum foil of thickness 1 μm.

SOLUTION:

The α particle scatters a small amount as it passes each atom. Each scattering produces a random small change in the direction (angle) of the α particle. The resulting distribution is a Gaussian (see Chapter 2). The average scattering angle is zero and the root-mean-square is proportional to the square root of the number of scatters. The number of scatters is the foil thickness divided by the size of the atom:

$$N \approx \frac{10^{-6}\ \text{m}}{10^{-10}\ \text{m}} = 10^4.$$

Using θ_s from the previous example, the root-mean-square scattering angle (in radians) is

$$\theta_{rms} \approx \sqrt{10^4}\,\theta_s = (100)(4\times10^{-4}) = 0.04,$$

which corresponds to about 2°. Note that we have calculated the root-mean-square scattering angle by considering a very thin foil. The root-mean-square scattering angle increases with the square root of the thickness of the foil. If the foil is thick, the α particle also loses energy to the electrons (a small amount of energy times a large number of collisions). For these two reasons, Geiger and Marsden chose the thinnest foils they could obtain. ■

If the positive charge of the atom is concentrated in a tiny nucleus, the angular distribution of the scattered α particles is much different from the case of a uniform charge distribution. The reason for this is simple: the α particle can come much closer to this charge without penetrating the charge, and the force is much stronger at a shorter distance. A careful measurement of α particle scattering gives us detailed information about the structure of the atom.

Before discussing the results of the Rutherford experiment in more detail, we develop an important tool for the description of scattering experiments, the cross section.

6-2 DEFINITION OF CROSS SECTION

Consider the scattering of two hard spheres, a projectile bouncing off a target of radius R. A stream of projectile spheres is directed toward the target, as shown in Figure 6-3. The probability that a given projectile hits the target is called the scattering probability (P). This scattering probability is proportional to the cross-sectional area (σ) of the target:

$$P \propto \sigma = \pi R^2. \tag{6.4}$$

The area σ is called the *total cross section* for the scattering process. Let the projectile particles cover an area a, where

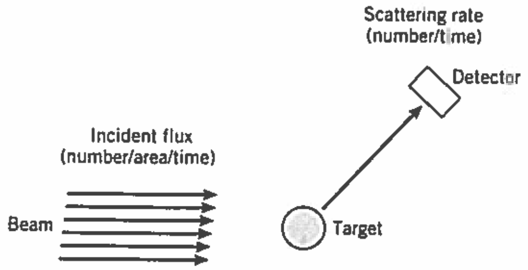

FIGURE 6-3 Definition of cross section.
Projectile particles are fired toward a target. The flux of projectile particles (number per area per time) covers an area larger than the target. The number of projectile particles that scatter per time is called the scattering rate. The cross section is the scattering rate divided by the incident flux. The cross section has units of area and its geometric interpretation is the cross-sectional area of the target that produces the scattering.

$a >> R^2$, and be incident at the rate $\Delta N/\Delta t$. The incident flux (Φ_i) is

$$\Phi_i = \frac{\Delta N}{a\Delta t}. \tag{6.5}$$

and the scattering rate (R_s) is

$$R_s = \left(\frac{\pi R^2}{a}\right)\left(\frac{\Delta N}{\Delta t}\right). \tag{6.6}$$

The cross section is the scattering rate (6.6) divided by the incident flux (6.5),

$$\sigma = \frac{R_s}{\Phi_i} = \frac{\left(\frac{\pi R^2}{a}\right)\left(\frac{\Delta N}{\Delta t}\right)}{\left(\frac{\Delta N}{A\Delta t}\right)} = \pi R^2. \tag{6.7}$$

Now consider the more general case of incoming particles of type A incident on target particles of type B with the reaction,

$$A + B \rightarrow \text{FINAL STATE}. \tag{6.8}$$

The final state stands for any possible outcome that we wish to define. For example, the final state could be particle A having a momentum in some specified range, or it could be a state with one or more new particles created. The rate (number per time) that the final state is produced is called the *transition rate*. The probability for $A + B$ to produce a certain final state or group of final states is specified by the interaction cross section σ which is defined to be

$$\sigma = \frac{\text{transition rate} : A + B \rightarrow \text{FINAL STATE}}{\Phi_i}. \tag{6.9}$$

Cross section has the units of area. One barn (b), as in "hitting the broadside of a barn," is defined to be

$$1\,\text{b} \equiv 10^{-28}\ \text{m}^2. \tag{6.10}$$

The cross section has the geometrical interpretation as the effective cross-sectional area of the target particle as seen by the incoming particle. Cross sections are a function of energy because the wavelengths of the interacting particles are energy-dependent.

> The cross section for any process, $A + B \rightarrow$ FINAL STATE, is a unit of area that is proportional to its probability of occurrence and is defined to be the transition rate divided by the incident flux.

Consider the case of incoming protons striking a liquid hydrogen target. The target is made up of many protons and the fundamental interaction is a proton–proton interaction (not a proton–"target" interaction). Each incident proton has a chance to interact with target protons along the whole length of the target. In this case it is useful to calculate the cross section *per target proton*. The corresponding incident flux is equal to the rate at which the protons strike the target times the number of protons per area in the target. The number of protons per area in the target (N_p/a) depends on the length of the target (L),

$$\frac{N_p}{a} = \frac{N_A L\rho}{10^{-3}\ \text{kg}}, \tag{6.11}$$

where N_A is Avogadro's number and ρ is the density of liquid hydrogen.

EXAMPLE 6-6

The total proton–proton strong interaction cross section is about 40 mb. Calculate the fraction of protons that scatter when a collimated beam of protons is sent through a liquid hydrogen target of length 0.3 meters. The density of liquid hydrogen is 70 kg/m³.

SOLUTION:

Let R_i be the rate of protons incident on the target and R_s be the scattering rate. The incident flux of protons is the incident rate times the number of protons per area in the target. The number of protons per area (6.11) in the target is

$$\frac{N_p}{a} = \frac{N_A L\rho}{10^{-3}\ \text{kg}}.$$

The incident flux is

$$\Phi_i = \frac{R_i N_p}{a} = \frac{R_i N_A L\rho}{10^{-3}\ \text{kg}}.$$

The expression for the cross section *per proton* is the scattering rate divided by the incident flux:

$$\sigma = \frac{R_s}{\Phi_i} = \frac{R_s}{\left[\dfrac{R_i N_A L\rho}{10^{-3}\ \text{kg}}\right]}.$$

The fraction of protons scattered is

$$\begin{aligned}\frac{R_s}{R_i} &= \frac{N_A \sigma L\rho}{10^{-3}\ \text{kg}} \\ &= \frac{\left(6\times10^{23}\right)\left(40\times10^{-31}\ \text{m}^2\right)\left(0.3\ \text{m}\right)\left(70\ \text{kg/m}^3\right)}{10^{-3}\ \text{kg}} \\ &\approx 0.05\,.\end{aligned}$$

Note that the incident flux does not depend on the area of the beam, as long as it is smaller than the cross-sectional area of the target. The incident flux depends on the length of the target because each incident proton encounters target protons along the entire length of the target. ■

Table 6-1 lists some cross sections for various processes. The sizes of the cross sections vary by many orders of magnitude because the interactions listed are governed by different forces. The first is a strong interaction, the second two are electromagnetic interactions, and the last one is a weak interaction.

TABLE 6-1
EXAMPLES OF SOME PARTICLE CROSS SECTIONS AT A CENTER-OF-MASS ENERGY OF 10 GEV.

Process	σ (Approximate)
$p + p \rightarrow$ anything, by strong interaction	40 mb
$\gamma + p \rightarrow$ anything	100 μb
$e^+ + e^- \rightarrow \mu^+ + \mu^-$	1 nb
$\nu + N \rightarrow \mu^- +$ anything	1 pb

6-3 PROBING THE STRUCTURE OF THE ATOM

Discovery of the Nucleus

We now return to a discussion of the great discovery of Geiger and Marsden. When they scattered α particles from a thin platinum foil, they observed that an unexpectedly large fraction of the α particles were scattered at large angles. Even more surprising was the fact that α particles were occasionally scattered through angles greater than 90°. Some α particles were even observed to be scattered backwards! Rutherford was very excited about these experimental results which he described as:

> *It was quite the most incredible event that has ever happened to me in my life. It was almost as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.*

What Rutherford did not know before the experiment, and what he discovered from the experiment was that the "15-inch shell" (the α) was hitting another "15-inch shell" (the nucleus). In 1911, Rutherford explained these data by correctly deducing that a large angle scatter could only be the result of a single collision and therefore that the atom has a dense charged nucleus.

To explain the large angle scattering data, Rutherford hypothesized that all the positive electric charge of the atom was concentrated in a very small volume. This was a revolutionary idea at the time. From his hypothesis, Rutherford applied Coulomb's law for the electric force and derived the form of the angular distribution of the scattered α particles. The angular distribution is called the *differential cross section* ($d\sigma/d\cos\theta$). (The integral of $d\sigma/d\cos\theta$ over all values of $\cos\theta$ gives the total cross section, σ. See problem 12.)

The form of the differential cross section is

$$\frac{d\sigma}{d\cos\theta} \propto \left(\frac{2Zke^2}{E_k}\right)^2 \frac{1}{\left(1-\cos\theta\right)^2}, \tag{6.12}$$

where $2e$ is the charge of the α particle, Ze is the charge of the nucleus and E_k is the α particle kinetic energy.

Note that $(ke^2/E_k)^2$ has dimensions of area. We can rewrite this result in terms of the dimensionless electromagnetic coupling ($\alpha = ke^2/\hbar c$) by multiplying and dividing by $(\hbar c)^2$. The result is

$$\frac{d\sigma}{d\cos\theta} \propto \alpha^2 \left(\frac{\hbar c}{E_k}\right)^2 \frac{1}{(1-\cos\theta)^2}. \quad (6.13)$$

We shall derive this important formula later in the chapter.

> The differential cross section for the scattering of pointlike particles is proportional to the square of the fundamental interaction strength, inversely proportional to the kinetic energy squared and inversely proportional to $(1-\cos\theta)^2$,
>
> $$\frac{d\sigma}{d\cos\theta} \propto \alpha^2 \left(\frac{\hbar c}{E_k}\right)^2 \frac{1}{(1-\cos\theta)^2}.$$

The theory of Rutherford scattering assumes that the nucleus is concentrated at a point, that is, the α particle does not penetrate the nucleus. If the α particle is energetic enough to penetrate the nucleus, then the Rutherford scattering formula does not hold true.

EXAMPLE 6-7

Estimate the kinetic energy of an α particle needed to penetrate the silver nucleus ($Z = 47$). The radius of the silver nucleus is about 5 fm.

SOLUTION:

The α particle can penetrate the nucleus when it has enough energy to overcome the Coulomb repulsion. This happens when

$$E_k = \frac{kq_1q_2}{R},$$

where $q_1 = 2e$, the α electric charge, $q_2 = 47e$, the silver nucleus electric charge, and $R = 5$ fm, the nuclear radius. Therefore,

$$E_k = \frac{(2)(47)(1.44\ \text{MeV}\cdot\text{fm})}{5\ \text{fm}} = 27\ \text{MeV}.$$

Such an α particle is not relativistic. ■

In 1913, Geiger and Marsden carried out a second experiment to quantitatively test the validity of the Rutherford model. The objectives of the experiment were clearly stated in the introduction to the paper by Geiger and Marsden:

> *At the suggestion of Prof. Rutherford, we have carried out experiments to test the main conclusions of the above theory. The following points were investigated:*
>
> *(1) Variation with scattering angle.*
>
> *(2) Variation with thickness of the scattering material.*
>
> *(3) Variation with atomic weight of the scattering material.*
>
> *(4) Variation with velocity of incident α particles.*
>
> *(5) The fraction of particles scattered through a definite angle.*
>
> *The main difficulty of the experiments has arisen from the necessity of using a very intense and narrow source of α particles owing to the smallness of the scattering effect. All the measurements have been carried out by observing the scintillations due to the scattered α particles on a zinc-sulfide screen, and during the course of the experiments over 100,000 scintillations have been counted. It may be mentioned in anticipation that all the results of our investigation are in good agreement with the theoretical deductions of Prof. Rutherford, and afford strong evidence of the correctness of the underlying assumption that an atom contains a strong charge at the centre of all dimensions, small compared with the diameter of the atom.*

The Apparatus

The source of α particles was purified radium contained in a thin-walled 1-mm diameter glass tube. The strength of the α source was huge by laboratory standards, about 0.1 Curie, or about 4 billion nuclear decays per second. Radium has a long decay chain. These decays are discussed in more detail in Chapter 11. The time scale of each decay is characterized by the half-life, the time it takes one-half of the particles to decay. The source was shielded with lead to reduce the background from electrons and photons, which are also produced in the chain of radioactive decays. The α particles were allowed to pass through a small diaphragm and were directed toward a thin foil that served as the nuclear target. Several different foils were mounted on a wheel so that they could be used as targets in successive measurements. The α particle detector was a small (10^{-6} m^2) zinc-sulfide screen mounted a few centimeters away from the target. On passing through the detector, an α particle induces the emission of light. The screen was viewed with a microscope so that a flash of light was visible to the viewer when an α particle struck the screen. In order to observe scattered α particles at various angles, the screen and microscope assembly were allowed to rotate through the range from 5 to 150 degrees with respect to the α particle direction. The whole apparatus was placed in an evacuated container to eliminate the scattering and absorption of the α particles in air.

Angular Dependence

For the angular measurements, the main source of α particles was the decay of radon:

$$^{222}\mathrm{Rn} \rightarrow {}^{218}\mathrm{Po} + \alpha . \qquad (6.14)$$

The energy of the α particle is 5.5 MeV and the half-life of the decay is 3.82 days. A typical set of angular measurements took about two days to complete, so the experimenters needed to make a correction for the decay of the source over the time in which the data were taken. The angular dependence of the α particle scattering was measured by rotating the detection screen plus microscope assembly. The scattering rate was by far the greatest at small angles. The maximum rate that could be accurately measured was roughly two scintillations per second. When the foil was removed, a small number of scintillations per time were observed due to scattering of the α particles from the walls of the container. The experimenters measured the scintillation rate with no foil present and subtracted this background to get the scattering rate in the foil. The background imposed a practical limit on the smallest scattering rate that could be accurately measured. This limit was determined to be about 0.1 scintillation per second. Thus, the experiment was sensitive to scattering rates in the range 0.1 to 2 per second. The rates could be adjusted to be in the sensitive range by choosing the distance from the foil to the detection screen. Since the lifetime of the source was 3.82 days, the large angle data were taken first when the radioactive source was "hottest" and the small angle data were taken after the source had substantially decayed.

Data of Geiger and Marsden for the angular dependence of α particles scattering from a silver foil are shown in Figure 6-4. The number N scattered at a given angle is proportional to the differential cross section. These data provide convincing proof that the angular dependence ($d\sigma/d\cos\theta$) of the scattering cross section has the form

$$\frac{d\sigma}{d\cos\theta} \propto \frac{1}{(1-\cos\theta)^2} . \qquad (6.15)$$

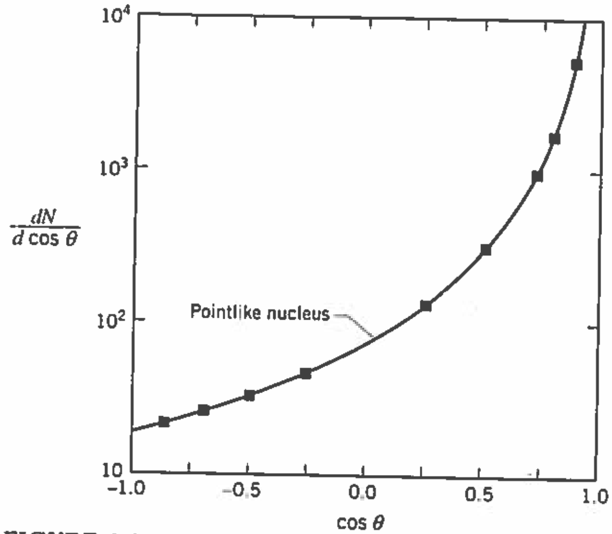

FIGURE 6-4 Angular distribution of α particles after passing through a thin silver foil.
The measured angular distribution is of the form $dN/d\cos\theta \propto 1/(1-\cos\theta)^2$, which can occur only if the scattering center is a "point." The data are from H. Geiger and E. Marsden, *Phil. Mag.* 25, 604 (1913).

This result can hold true only if the scattering is due to a pointlike object. These data prove that the atom has a nucleus.

Variation with Foil Thickness

To study the variation of the scattering cross section with thickness, the experimenters had to take extra care to prepare a monoenergetic α source. For this set of measurements they did not need a long half-life and they used the decay

$$^{214}\mathrm{Bi} \rightarrow {}^{210}\mathrm{Th} + \alpha . \qquad (6.16)$$

This radioactive material (^{214}Bi) was known at the time as *Radium C*. The α particle has an energy of 5.5 MeV and a half-life of about 20 minutes. If the large angle scatters are the result of a single close encounter of the α particle with the nucleus, then the scattering rate should be directly proportional to the target thickness. This would not be true for multiple scattering, where the rate would vary as the square-root of the thickness. Geiger and Marsden measured the scattering rate at a fixed angle near 25 degrees with targets composed of multiple foils. The results are shown in Figure 6-5. The data show that the scattering rate depends linearly on the target thickness, indicating that α particles emerge at large angles as the result of a single scatter.

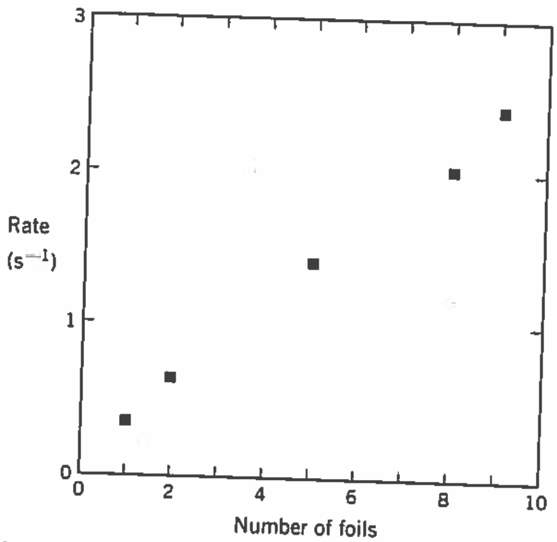

FIGURE 6-5 Alpha particle scattering rate at 25 degrees as a function of target thickness.
The target is made up of a number of gold foils. The scattering rate is proportional to the number of foils or to the thickness of the target. These data indicate that α particles emerge at large angles as the result of a single scatter. From H. Geiger and E. Marsden, *Phil. Mag.* **25**, 604 (1913).

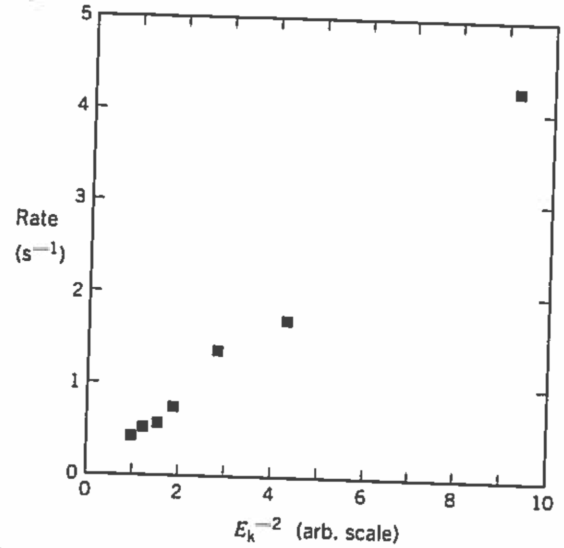

FIGURE 6-6 Energy dependence of the scattering.
From H. Geiger and E. Marsden, *Phil. Mag.* **25**, 604 (1913).

Energy Dependence

To measure the energy dependence of the α scattering, the monoenergetic α source was used. The apparatus was modified so that sheets of mica could be inserted between the source and the target. When α particles pass through the mica, they lose energy by electromagnetic interaction with the electrons in the mica. The distance that an α particle can travel in a material before it loses all its kinetic energy and comes to rest is called the *range* of the particle. The range of α particles in mica had been measured by Geiger in a previous experiment. He found that the range was proportional to the cube of the speed of the α particle, or proportional to the kinetic energy to the 3/2 power. Geiger and Marsden slowed down the α particles with mica sheets to do their large angle scattering experiment at lower energies. They then determined the relative kinetic energies of the α particles by measuring the α particle ranges in mica. The results are shown in Figure 6-6. The energy units are arbitrary since the experimenters did not know the exact kinetic energy of the α particle from the radon decay (5.5 MeV). The data indicate that the scattering rate is inversely proportional to the square of the α particle kinetic energy,

$$\sigma \propto \frac{1}{E_k^{\,2}}. \tag{6.17}$$

Z Dependence

Geiger and Marsden did not know the value of Z for the foils, but they did know the atomic mass numbers of the foils. The atomic mass number is the number of grams corresponding to one mole of atoms. The atomic mass number is also equal to the number of neutrons plus protons in the nucleus. The data showed that the scattering cross section varied approximately as the square of the atomic mass number (A). The reason the data show this effect is that the charge of the nucleus is roughly proportional to the atomic weight. In reality, Z is not exactly proportional to A because the fraction of neutrons increases with increasing A.

Geiger and Marsden determined the approximate value of Z by measuring the fraction of particles that were scattered at large angles. By making this measurement they could compare the theoretical cross section (which contains a factor Z^2) with their measured cross section. This comparison is shown in Figure 6-7.

Derivation of the Rutherford Cross Section

Consider an energetic α particle incident on a nucleus at rest. The trajectory of the α particle is indicated in Figure 6-8. If the nucleus would not scatter the α particle, the α particle would have a distance of closest approach equal to b. The variable b is called the *impact parameter*. The scattering angle θ is defined to be the angle of the outgoing

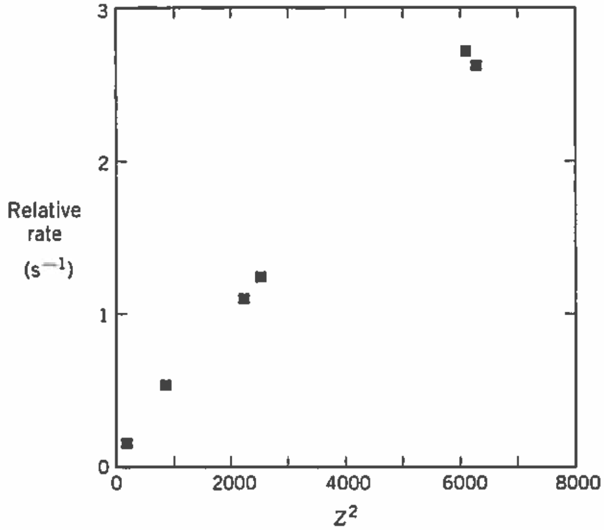

FIGURE 6-7 The Z dependence of the scattering cross section.
From H. Geiger and E. Marsden, *Phil. Mag.* **25**, 604 (1913).

α particle with respect to the incoming α particle. When b is large, the force between the α and the nucleus is small and the scattering angle is small. Conversely, a small impact parameter produces a large force and a large scattering angle. In terms of the impact parameter, the differential of the cross section ($d\sigma$) is equal to the area of a ring of radius b and thickness db:

$$d\sigma = 2\pi b db. \tag{6.18}$$

The differential cross section ($d\sigma/db$) is

$$\frac{d\sigma}{db} = 2\pi b. \tag{6.19}$$

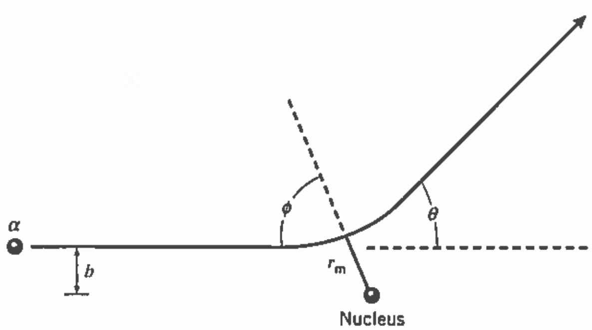

FIGURE 6-8 Definition of the variables for description of the Rutherford scattering experiment.
The incident α particle has an impact parameter b. The trajectory of the α particle is a hyperbola. The α particle has a distance of closest approach r_m with the nucleus and emerges at an angle θ.

For a hard sphere, the range of b is from 0 to R, the radius of the sphere. The cross section for this case is

$$\sigma = 2\pi \int_0^R dbb = \pi R^2. \tag{6.20}$$

The cross section for α scattering is much more interesting. A specific impact parameter will make a specific scattering angle. This is illustrated in Figure 6-9. When the impact parameter is between b and $b + \Delta b$, then the scattering angle is between θ and $\theta - \Delta\theta$. The differential cross section may be written

$$\frac{d\sigma}{d\cos\theta} = \frac{d\sigma}{db}\frac{db}{d\cos\theta} = 2\pi b\frac{db}{d\cos\theta}. \tag{6.21}$$

We need to determine how b depends on $\cos\theta$ to arrive at an expression for the angular distribution of the scattered α particles.

We shall derive an expression for b by finding two independent expressions for the change in momentum Δp of the scattered α particle that involve b and θ. Let $\mathbf{p}_1$ be the momentum of the α particle before the scatter and $\mathbf{p}_2$ be the momentum after the scatter (see Figure 6-10). The square of the change in momentum is

$$\begin{aligned}(\Delta p)^2 &= (\mathbf{p}_2 - \mathbf{p}_1)^2 \\ &= p_2^2 + p_1^2 - 2p_1p_2\cos\theta. \end{aligned} \tag{6.22}$$

Only the direction of the momentum changes, not its magnitude, provided that the mass of the nucleus is much

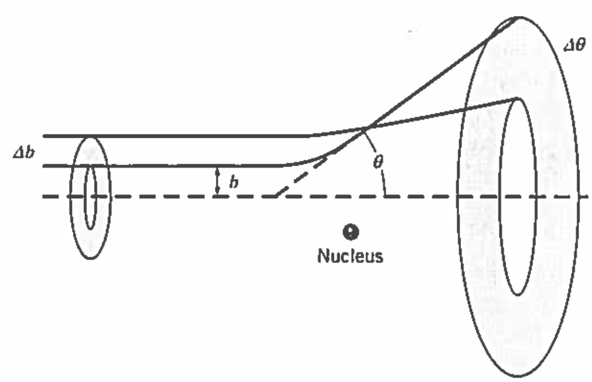

FIGURE 6-9 Relationship between impact parameter b and scattering angle θ.
When the impact parameter is between b and $b + \Delta b$, then the scattering angle is between θ and $\theta - \Delta\theta$. The cross section $\Delta\sigma$ for a particle detected between the rings defined by θ and $\theta - \Delta\theta$ is equal to the area between the rings defined by b and $b + \Delta b$, $\Delta\sigma = 2\pi b\Delta b$.

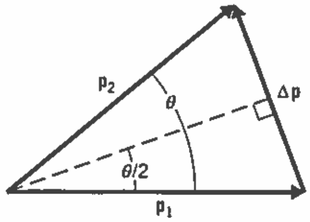

FIGURE 6-10 Momentum vector diagram of the scattered α particle.

larger than the mass of the α particle. The nucleus has momentum (Δp) transferred to it, but very little energy, like a ball bouncing off a brick wall. Since the magnitude of the momentum does not change, we write

$$p_2 = p_1 = p = m_\alpha v, \tag{6.23}$$

where m_α is the mass of the α particle and v is the speed of the α particle. Therefore,

$$\begin{aligned}(\Delta p)^2 &= 2p^2 - 2p^2\cos\theta \\ &= 2p^2(1-\cos\theta)\ ,\end{aligned} \tag{6.24}$$

and we have one of our expressions for the momentum transfer,

$$\Delta p = m_\alpha v\sqrt{2(1-\cos\theta)}. \tag{6.25}$$

The momentum transfer is along a line that bisects the angle ($\pi - \theta$) as indicated in Figure 6-8. The magnitude of the force (F) on the α particle is

$$F = \frac{kq_1q_2}{r^2}, \tag{6.26}$$

where q_1 is the charge of the α particle ($2e$) and q_2 is the charge of the nucleus (Ze). The component of force in the direction of the momentum transfer is $F\cos\phi$. Therefore, the momentum transfer may be written as the time (t) integral of the force:

$$\Delta p = \int_{t_1}^{t_2} dt\,F\cos\phi = kq_1q_2\int_{t_1}^{t_2} dt\frac{\cos\phi}{r^2}. \tag{6.27}$$

In evaluating the integral we must keep in mind that both ϕ and r depend on t. At first sight the integral looks formidable, however, we shall be able to reduce it to an easy form by making use of angular momentum conservation. At any point along the path of the α particle, the component of velocity perpendicular to the direction of the force (v_T) is

$$v_T = r\frac{d\phi}{dt}. \tag{6.28}$$

The angular momentum (L) of the α particle about the nucleus is

$$L = m_\alpha\left(r\frac{d\phi}{dt}\right)r = m_\alpha r^2\frac{d\phi}{dt}. \tag{6.29}$$

The angular momentum is a conserved quantity. When the α particle is a long distance from the nucleus, before the scatter, we have by definition of the impact parameter

$$L = m_\alpha vb. \tag{6.30}$$

By conservation of angular momentum,

$$m_\alpha vb = m_\alpha r^2\frac{d\phi}{dt}. \tag{6.31}$$

So that

$$\frac{dt}{r^2} = \frac{d\phi}{vb}. \tag{6.32}$$

Therefore, our expression for the momentum transfer (6.27) becomes

$$\begin{aligned}\Delta p &= \frac{kq_1q_2}{vb}\int_{-\phi_0}^{\phi_0} d\phi\cos\phi \\ &= \frac{kq_1q_2}{vb}[\sin\phi_0 - \sin(-\phi_0)] \\ &= \left(\frac{2kq_1q_2}{vb}\right)\sin\phi_0\,.\end{aligned} \tag{6.33}$$

Now we must convert the angle ϕ_0, which was a convenient variable for integrating the force, back to the scattering angle θ. The two angles are related by $2\phi_0 + \theta = \pi$, or

$$\phi_0 = \frac{\pi}{2} - \frac{\theta}{2}. \tag{6.34}$$

The momentum transfer (6.33) is

$$\begin{aligned}\Delta p &= \frac{2kq_1q_2}{vb}\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right) \\ &= \frac{2kq_1q_2}{vb}\cos\left(\frac{\theta}{2}\right).\end{aligned} \tag{6.35}$$

Using the trigonometric identity

$$\cos\left(\frac{\theta}{2}\right)=\sqrt{\frac{1+\cos\theta}{2}}, \tag{6.36}$$

we have

$$\Delta p=\frac{\sqrt{2}kq_1q_2}{vb}\sqrt{1+\cos\theta}. \tag{6.37}$$

Now we equate the two expressions for the momentum transfer (6.25 and 6.37):

$$\begin{aligned}\Delta p &= m_\alpha v\sqrt{2(1-\cos\theta)}\\ &=\frac{\sqrt{2}kq_1q_2}{vb}\sqrt{1+\cos\theta}.\end{aligned} \tag{6.38}$$

This is our sought-after expression relating the impact parameter and the scattering angle. Solving for the impact parameter, we have

$$b=\frac{kq_1q_2}{m_\alpha v^2}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}. \tag{6.39}$$

The square of the impact parameter is

$$b^2=\left(\frac{kq_1q_2}{m_\alpha v^2}\right)^2\left(\frac{1+\cos\theta}{1-\cos\theta}\right). \tag{6.40}$$

To calculate the cross section, we make the change of variables:

$$x\equiv\cos\theta, \tag{6.41}$$

and the square of the impact parameter (6.40) becomes

$$b^2=\left(\frac{kq_1q_2}{m_\alpha v^2}\right)^2\left(\frac{1+x}{1-x}\right). \tag{6.42}$$

Differentiating b^2, we have

$$\begin{aligned}2b\,db &=\left(\frac{kq_1q_2}{m_\alpha v^2}\right)^2\frac{(1-x+1+x)}{(1-x)^2}dx\\ &=2\left(\frac{kq_1q_2}{m_\alpha v^2}\right)^2\frac{dx}{(1-x)^2}.\end{aligned} \tag{6.43}$$

The differential cross section as a function of scattering angle is

$$d\sigma=2\pi b\,db=2\pi\left(\frac{kq_1q_2}{m_\alpha v^2}\right)^2\frac{dx}{(1-x)^2}, \tag{6.44}$$

or

$$\frac{d\sigma}{dx}=2\pi\left(\frac{kq_1q_2}{m_\alpha v^2}\right)^2\frac{1}{(1-x)^2}. \tag{6.45}$$

Switching back to $\cos\theta$,

$$\frac{d\sigma}{d\cos\theta}=2\pi\left(\frac{kq_1q_2}{m_\alpha v^2}\right)^2\frac{1}{(1-\cos\theta)^2}. \tag{6.46}$$

We may write the kinetic energy of the incoming particle as $E_k=m_\alpha v^2/2$ to get

$$\frac{d\sigma}{d\cos\theta}=\frac{\pi}{2}\left(\frac{kq_1q_2}{E_k}\right)^2\frac{1}{(1-\cos\theta)^2}. \tag{6.47}$$

If the electric charge of the projectile is $q_1=ze$ and the electric charge of the nucleus is Ze, we have

$$\frac{d\sigma}{d\cos\theta}=\frac{\pi}{2}\left(\frac{zZke^2}{E_k}\right)^2\frac{1}{(1-\cos\theta)^2}. \tag{6.48}$$

In terms of the electromagnetic coupling strength (α), we have

$$\frac{d\sigma}{d\cos\theta}=\frac{\pi}{2}z^2Z^2\alpha^2\left(\frac{\hbar c}{E_k}\right)^2\frac{1}{(1-\cos\theta)^2}. \tag{6.49}$$

This is the *Rutherford formula*. It specifies the scattering rate as a function of angle for the electromagnetic interaction of two charged particles. For the special case of an α particle ($z=2$) scattering from a nucleus, we get

$$\frac{d\sigma}{d\cos\theta}=2\pi Z^2\alpha^2\left(\frac{\hbar c}{E_k}\right)^2\frac{1}{(1-\cos\theta)^2}. \tag{6.50}$$

A Feynman diagram for alpha-nucleus scattering is shown in Figure 6-11. We examine each of the factors appearing the Rutherford formula with the aid of Figure 6-11:

1. The cross section is proportional to the electromagnetic coupling constant (α) squared. The electromagnetic force is mediated by photon exchange. The photon couples to the α particle and to the nucleus. Each coupling has a strength of α.
2. The cross section is inversely proportional to the square of the kinetic energy of the α particle. When

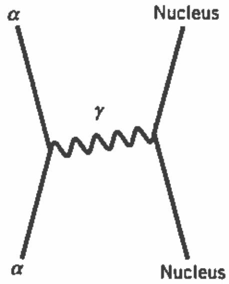

FIGURE 6-11 Feynman diagram for alpha-nucleus scattering.

the kinetic energy is small, the speed of the α particle is small and it spends more time near the nucleus experiencing the electric force. The constant ($\hbar c$) divided by energy has units of length. This quantity squared has units of area. There is no other length scale in the physical process. Thus, we could have predicted on dimensional grounds that the cross section is proportional to $(\hbar c/E_k)^2$.

3. The angular dependence is $(1-\cos\theta)^{-2}$. This angular factor comes from the fact that the force varies as r^{-2}. (Recall the elimination of the factor dt/r^2 in the expression for Δp.) This in turn comes from the fact that the quantum origin of the force is due to the exchange of massless particles (photons).

There is a singularity in the differential cross section at $\theta = 0$, where the cross section is infinite. This means that there is an infinitely large area (πb^2) that the α particle can be in and still get scattered. The total cross section is infinite because the electromagnetic force has an infinite range. Of course, the momentum transfer is very small when b is large.

The expression for the impact parameter is very useful for evaluating the cross section.

EXAMPLE 6-8

Calculate the cross section for a 12-MeV α particle scattering from a silver nucleus ($Z = 47$) at angles greater than (a) 90 degrees and (b) 10 degrees.

SOLUTION:

(a) We find the impact parameter (b_{max}) that corresponds to scattering at 90 degrees. Then impact parameters less than b_{max} produce scatters at angles greater than 90 degrees. The value of b_{max} is

$$b_{max} = \frac{kq_1q_2}{m_\alpha v^2}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{kq_1q_2}{m_\alpha v^2}.$$

The cross section is

$$\sigma = \int_0^{b_{max}} db\frac{d\sigma}{db} = 2\pi\int_0^{b_{max}} dbb$$

$$= \pi b_{max}^{\ 2} = \pi\left(\frac{kq_1q_2}{m_\alpha v^2}\right)^2.$$

Since the charge of the α is $2e$, we have

$$\sigma = \pi Z^2\left(\frac{ke^2}{E_k}\right)^2$$

$$= (3.14)(47)^2\left(\frac{1.44\ \text{Mev}\cdot\text{fm}}{12\ \text{MeV}}\right)^2$$

$$= 100(\text{fm})^2.$$

Since 1 $(\text{fm})^2 = 10^{-30}\ \text{m}^2 = 10^{-2}$ b,

$$\sigma = 1.0\ \text{b}.$$

The silver nucleus is as big as a barn for this process.
(b) For scattering angles greater than 10 degrees, the cross section is larger by the factor

$$\frac{1+\cos\theta}{1-\cos\theta} \approx 130,$$

or

$$\sigma = 130\ \text{b}.$$ ■

So far we have been the analyzing the scattering of a single particle from a single nucleus. In the Rutherford experiment the α particles were made to pass through a thin foil. For a given α particle, we cannot "choose" the impact parameter. When the α particle passes through the foil, it will have random sampling of impact parameters with many nuclei as it passes through the foil. If the α particle happens to have a small impact parameter, then a large angle scattering occurs. A good way to picture the α particle passing though the foil is to think of the nuclei in the foil as an incident flux of particles streaming in the general direction of the α. Consider a foil made of an element with atomic mass A and density ρ. The number of target nuclei per volume is

$$\frac{n}{V} = \frac{N_A\rho}{A(10^{-3}\ \text{kg})}. \tag{6.51}$$

The number of target nuclei per cross-sectional area is the thickness (L) times n/V. It the incident rate of α particles is R_i, then the incident flux is

$$\Phi_i = \frac{R_i L n}{V} = \frac{R_i L N_A \rho}{A(10^{-3}\ \text{kg})}. \tag{6.52}$$

The scattering cross section is

$$\sigma = \frac{R_s}{\Phi_i} = \frac{R_s}{\left[\dfrac{R_i L N_A \rho}{A(10^{-3}\ \text{kg})}\right]}$$

$$= \frac{R_s A(10^{-3}\ \text{kg})}{R_i L N_A \rho}. \tag{6.53}$$

A measurement of the scattering rate is a measurement of the cross section. If the cross section is known, the scattering rate can be predicted.

EXAMPLE 6-9

A stream of 6-MeV α particles passes through a gold foil that has a thickness of 1 µm at a rate of 10^3 per second. Calculate the rate at which α particles are scattered at angles greater than 0.1 radian. The density of gold is 1.93 × 10^4 kg/m^3.

SOLUTION:

The scattering rate (R_s) is

$$R_s = \frac{\sigma R_i N_A L \rho}{(197)(10^{-3}\ \text{kg})}.$$

We need to know the cross section that is given by the Rutherford formula (see Example 6-8):

$$\sigma = \pi Z^2 \left(\frac{ke^2}{E_k}\right)^2 \frac{1+\cos\theta}{1-\cos\theta}.$$

The cosine of 0.1 radian is

$$\cos(0.1) \approx 1 - \frac{(0.1)^2}{2} = 0.995\ .$$

The angular factor is

$$\frac{1+\cos\theta}{1-\cos\theta} = \frac{1.995}{0.005} = 3.99\times10^2.$$

The cross section is

$$\sigma = (3.14)(79)^2\left(\frac{1.44\ \text{Mev}\cdot\text{fm}}{6\,\text{MeV}}\right)^2(3.99\times10^2)$$

$$= 4.50\times10^5\ (\text{fm})^2 = 4.50\times10^{-25}\ \text{m}^2\ .$$

The scattering rate is

$$R_s = \frac{(4.50\times10^{-25}\ \text{m}^2)(10^3\ \text{s}^{-1})(6\times10^{23})(10^{-6}\ \text{m})}{0.197\,\text{kg}}$$

$$\times(1.93\times10^4\ \text{kg/m}^3)$$

$$= 26\ \text{s}^{-1}.$$

The fraction (f) scattered at angles greater than 0.1 radian is

$$f = \frac{(26\ \text{s}^{-1})}{(1000\ \text{s}^{-1})} = 0.026\ . \qquad ■$$

6-4 PROBING THE STRUCTURE OF THE NUCLEUS

The distance of closest approach that the α particle makes with the nucleus depends on both the impact parameter and the kinetic energy of the α For example, if the impact parameter is very small and the energy is small, the α particle gets repelled backwards and the distance of closest approach is much larger than the impact parameter. If the α particle kinetic energy is very large and the impact parameter is small, then the α particle can actually "strike" the nucleus if the nucleus has a finite size. By finite size, we mean that the charge distribution is not a point, but is spread out in space. In this case the α particle has a distance of closest approach that is smaller than the radial extent of the charge of the nucleus. The Rutherford formula for the angular distribution of the scattered α particle is no longer valid because the force is no longer given by kq_1q_2/r^2. The force is smaller because the effective charge that scatters the α particle is smaller than the total charge of the nucleus. The nucleus no longer appears as a pointlike particle to the incoming α particle. The breakdown of the Rutherford formula provides a powerful experimental technique for measuring the size of the nucleus.

We now derive an expression for the distance of closest approach for a non-relativistic α particle. Let the distance of closest approach be r_m and let the corresponding speed of the α particle be v_m. By conservation of energy (neglecting the nuclear recoil), we have

$$E_k = \frac{1}{2}mv_m^2 + \frac{2Zke^2}{r_m}. \tag{6.54}$$

Conservation of angular momentum relates the impact parameter b with r_m:

$$mvb = mv_m r_m. \tag{6.55}$$

Therefore, the kinetic energy of the αparticle (6.54) becomes

$$\begin{aligned} E_k &= \frac{1}{2}m\left(\frac{vb}{r_m}\right)^2 + \frac{2Zke^2}{r_m} \\ &= \frac{E_k b^2}{r_m^2} + \frac{2Zke^2}{r_m}, \end{aligned} \tag{6.56}$$

or

$$r_m^2 - \left(\frac{2Zke^2}{E_k}\right)r_m - b^2 = 0. \tag{6.57}$$

The solution for r_m is

$$r_m = \frac{Zke^2}{E_k} + \frac{1}{2}\sqrt{\left(\frac{2Zke^2}{E_k}\right)^2 + 4b^2}. \tag{6.58}$$

For $b = 0$, the distance of closest approach (6.58) is

$$r_m = \frac{2Zke^2}{E_k}. \tag{6.59}$$

EXAMPLE 6-10

Estimate the minimum kinetic energy of an αparticle that can cause a deviation from the Rutherford formula in scattering from a gold nucleus ($Z = 79$). Approximate the gold nucleus as a closely packed group of 197 neutrons and protons, where the radius of each proton and neutron is equal to 1 fm.

SOLUTION:

The kinetic energy of the αparticle must be greater than the potential energy at the surface (r_n) of the gold nucleus:

$$E_k = \frac{2Zke^2}{r_n}.$$

We may estimate the size of the gold nucleus from

$$\frac{4}{3}\pi r_n^3 = (197)\left(\frac{4}{3}\pi\right)(1\text{ fm})^3,$$

or

$$r_n = (197)^{1/3}\text{ fm} \approx 6\text{ fm}.$$

The αkinetic energy needed to penetrate the nucleus is

$$E_k \approx \frac{(2)(79)(1.44\text{ MeV}\cdot\text{fm})}{6\text{ fm}} \approx 40\text{ MeV}.$$

Note that this kinetic energy is much smaller than the mass energy of the α, so that the use of the nonrelativistic expression for kinetic energy is justified. ■

Measuring the Size of the Nucleus

Electrons make excellent probes to study the electric charge distribution of the nucleus. The electron makes a better probe than the αparticle for determining the charge distribution inside the nucleus because the electron behaves as a point charge (i.e., the electron has no detectable structure). A schematic of an electron scattering experiment is shown in Figure 6-12. A beam of electrons is incident on a target containing the nuclei to be studied. The electrons are scattered in all directions as a result of their electromagnetic interactions with the nuclei in the target. A small fraction of the scattered electrons enter a spectrometer, which is positioned at a scattering angle θ. The momentum of the scattered electron is measured in the spectrometer. The purpose of the spectrometer is to count the number of electrons that are elastically scattered at the angle θ. (Elastic scattering means that the energy of the electron does not change.) The spectrometer is mounted on a circular track so that a wide range of scattering angles may be studied. Since small scattering angles are the result of large impact parameter collisions, the nucleus appears as a pointlike object and the angular distribution, $d\sigma/d\cos\theta$, is to a good approximation given by the Rutherford formula with $z = 1$, $(d\sigma/d\cos\theta)_R$,

$$\begin{aligned} \frac{d\sigma}{d\cos\theta} &\approx \left(\frac{d\sigma}{d\cos\theta}\right)_R \\ &= \frac{\pi}{2}Z^2\alpha^2\left(\frac{\hbar c}{E_k}\right)^2\frac{1}{(1-\cos\theta)^2}. \end{aligned} \tag{6.60}$$

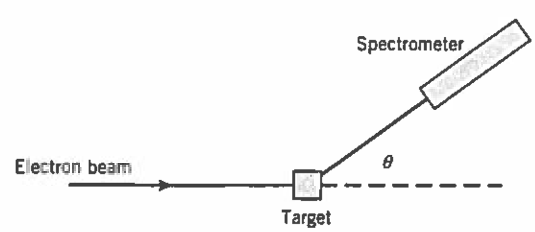

FIGURE 6-12 Electron–proton scattering experiment.

To get a more accurate prediction of the angular distribution, we would need to take into account relativistic and quantum effects and the recoil of the nucleus of mass (M). The result is

$$\left(\frac{d\sigma}{d\cos\theta}\right)_M = \left(\frac{d\sigma}{d\cos\theta}\right)_R \frac{1+\cos\theta}{2\left[1+\dfrac{(1-\cos\theta)E_k}{Mc^2}\right]}. \quad (6.61)$$

This expression is called the Mott cross section (after Nevill Mott). The important part of this cross section is the Rutherford part, which contains the angular factor, $(1-\cos\theta)^{-2}$, and electron kinetic energy factor, E_k^{-2}. The correction to the Rutherford formula is of order unity except when the scattering angle is close to 180 degrees or when the incident electron has a very large energy. The factor of $(1+\cos\theta)/2$ is due to the effect of the electron magnetic moment which is a quantum mechanical effect. The electron magnetic moment is discussed further in Chapter 8. The other portion of the correction factor is due to the recoil of the nucleus.

When electrons are scattered from a pointlike particle, the cross section is given by the Mott formula. If the target particle has a charge distribution that is not a point, then the impact parameter may be so small that the electron penetrates the charge of the nucleus. In this case the force on the electron is smaller than the force due to a point charge and fewer electrons are scattered at large angles compared to the Mott formula. The amount of deviation from the Mott formula as a function of scattering angle or impact parameter gives us information on how the charge is distributed inside the nucleus. In the 1950s, electrons were used as probes to make high-resolution measurements of the nuclear size. The leader of this effort was Robert Hofstadter. In 1953, Hofstadter's group scattered electrons of 125 MeV from several different nuclei. Some of their data on electron–gold scattering is shown in Figure 6-13. For a pointlike nucleus, the angular distribution of scattered electrons would follow the Mott formula (6.61),

$$\frac{dN}{d\cos\theta} \propto \left(\frac{d\sigma}{d\cos\theta}\right)_M, \quad (6.62)$$

The data show a marked deviation from "pointlike" behavior of the nucleus. Since a given scattering angle is a given impact parameter, the amount of deviation from the pointlike cross section tells us the effective nuclear charge that scattered the electron, and therefore, how much the electron was able to penetrate the nucleus. The results of a large number of scattering measurements, like those shown in Figure 6-13, indicate that the charge distribution of the nucleus is spherical with an approximately constant density in the core. This is pictured in Figure 6-14. The size of the nuclear charge (R) is determined from the electron scattering experiments to be

$$R \approx (1.2\ \text{fm})A^{1/3}, \quad (6.63)$$

where A is the atomic mass number of the nucleus. The factor of $A^{1/3}$ is expected in a model, called the Fermi model (after Enrico Fermi), where the nucleus is considered to be a closely packed array of protons and neutrons. The neutrons have no electric charge, but their presence is part of the reason that the charges of the protons are spread out. The charge density distribution of the nucleus does not have a perfectly sharp edge. The details of the shape of the nuclear charge distributions are determined from the electron scattering experiments.

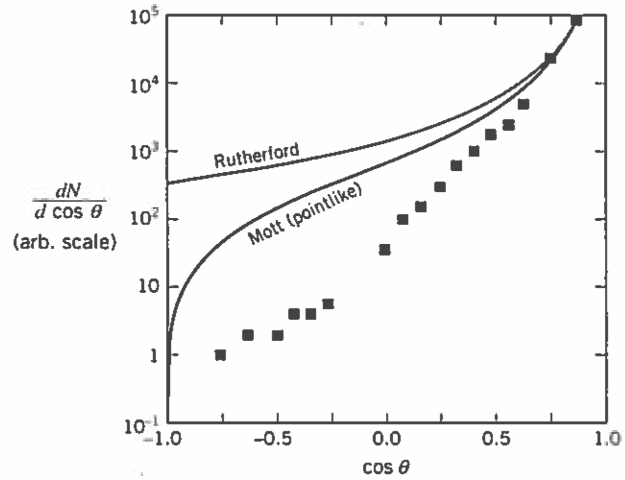

FIGURE 6-13 Determining the size of the nucleus. Electrons with a kinetic energy of 125 MeV are scattered from gold and the angular distribution of scattered electrons ($dN/d\cos\theta$) is measured. The data are inconsistent with a pointlike nucleus. The observed scattering rate is smaller than the rate expected from a pointlike nucleus because the electron penetrates the nucleus and experiences the force of a reduced charge. The amount of reduction from the pointlike scattering rate gives us the size of the gold nucleus, about 3×10^{-15} m. From R. Hofstadter et al., *Phys. Rev.* **92**, 978 (1953).

EXAMPLE 6-11

Electrons of 125 MeV are scattered from a gold nucleus. Calculate the deviation from the Rutherford formula at a

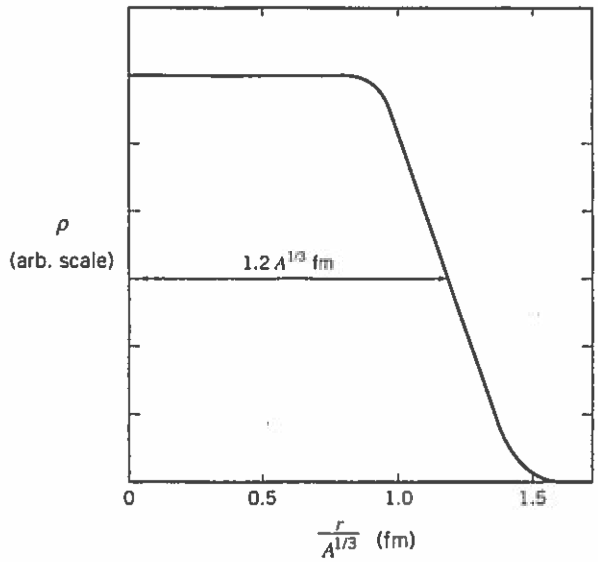

FIGURE 6-14 Fermi model of the nucleus.
The electron–nucleus scattering data determine the charge distribution inside the nucleus. The data from many scattering experiments show that the charge density is spherically symmetric and has a characteristic size of $(1.2 \text{ fm})A^{1/3}$. The edges of the nuclear charge distribution are not perfectly sharp.

scattering angle of 90 degrees due to the electron magnetic moment and nuclear recoil.

SOLUTION:

The effect of the electron magnetic moment is to introduce a factor of

$$\frac{1+\cos\theta}{2} = \frac{1}{2}.$$

The effect of the proton recoil is to introduce a factor of

$$\frac{1}{\left[1+\dfrac{(1-\cos\theta)E_k}{Mc^2}\right]} \approx 1,$$

because

$$Mc^2 >> 125\,\text{MeV}.$$

The net effect is a modification of the Rutherford formula by a factor of 2. This factor is much smaller than the deviation from the pointlike cross section that is observed in the data of Hofstadter. The deviation in the data is caused by a finite size of the nucleus. ■

The Structure of the Nucleus

The nucleus is a bound state of neutrons and protons; the binding force is the strong interaction. (Chapter 11 is devoted to the properties of the nucleus.) It is possible to remove a neutron or a proton from a nucleus in a collision. The strong force has a short range. In a rough approximation, we may think of the nucleus as a closely packed "blob" of neutrons and protons. The neutrons and protons are very close together so that their wave-packets overlap. The nucleus is a complicated object when examined in detail! We proceed now to look at the simplest nucleus, that of hydrogen.

6-5 PROBING THE STRUCTURE OF THE PROTON

The proton does not behave like a pointlike object at distances shorter than about 1 femtometer. This was first observed by scattering electrons from a liquid hydrogen target. We have already remarked that the incoming particle must have sufficiently large energy in order to penetrate the target nucleus. For relativistic electrons, energy is inversely proportional to wavelength. The electron wavelength is a measure of the minimum possible distance of closest approach in a collision. Some diagrams for electron–proton scattering are shown in Figure 6-15. Electron–proton scattering is qualitatively different for three different sizes of the electron wavelength. When the electron wavelength is much larger than 1 femtometer (see Figure 6-15*a*), the proton appears as a point charge. When the wavelength is comparable to 1 femtometer (see Figure 6-15*b*), the electron is observed to penetrate the charge distribution of the proton. Finally, at wavelengths much smaller than 1 fm, the proton charge is resolved into individual quarks (see Figure 6-15*c*). We now discuss this in further detail.

Measuring the Proton Size

The structure of the proton was discovered and measured in great detail by electron scattering. Electron scattering is a powerful technique for determining the electric charge distribution inside the proton. If the proton were a point charge, then the cross section would be that given by the Mott formula (6.61).

If the proton is not a point charge, the structure of the proton charge distribution causes a deviation in the cross section. When an electron penetrates the proton, the force

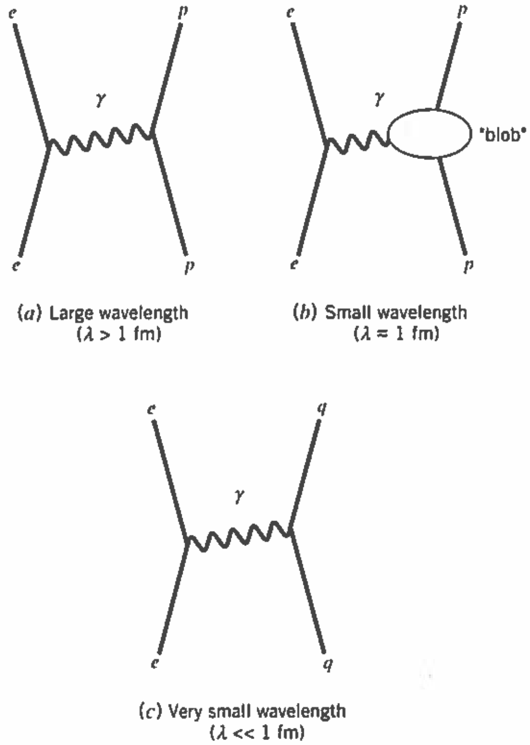

FIGURE 6-15 Feynman diagrams for electron–proton scattering.
(a) When the electron wavelength is much larger than the proton size, the proton appears like a "point." (b) When the wavelength is comparable to the proton size, then the proton appears as a "blob" of charge. The spatial extent of the charge of the proton is observed to be finite with an exponential form. (c) When the wavelength is much smaller than the proton size, the proton is resolved into quarks. The quarks appear as point charges that carry a fraction x of the proton momentum.

on the electron is smaller, and the resulting scattering angle is smaller compared to the case of no penetration (see Figure 6-2). This cross section for a penetrating electron may be written

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{\text{proton}} = \left(\frac{d\sigma}{d\cos\theta}\right)_{\text{M}} [F(\theta)]^2, \quad (6.64)$$

where the factor $[F(\theta)]^2$ is less than unity. The function F depends on the scattering angle because the impact parameter, which determines the value of the effective charge causing the scattering, depends on the scattering angle. The function F is called the proton *form factor*; it contains all the information about how the charge distribution inside the proton differs from a "point." For small-angle scattering,

$$F \approx 1. \quad (6.65)$$

In small-angle electron scattering, the proton size cannot be resolved, and the cross section follows the Mott formula (6.61). At slightly larger angles, the proton form factor differs from unity by an amount that is proportional to the average radius squared of the proton charge distribution:

$$F \approx 1 - C\left\langle r^2 \right\rangle. \quad (6.66)$$

The form factor is therefore related to the average of the radius squared of the charge distribution. The photon exchanged in the scattering has a wavelength small enough to resolve the proton size but too large to resolve the proton structure. The proton form factor was first measured by Robert Hofstadter. Data from Hofstadter is shown in Figure 6-16. Electrons with a kinetic energy of 550 MeV were scattered from protons and the angular distribution of scattered electrons ($dN/d\cos\theta$) was measured. The wavelength of a 550 MeV electron is

$$\lambda = \frac{hc}{pc} = \frac{1240\ \text{MeV} \cdot \text{fm}}{550\ \text{MeV}} \approx 2\ \text{fm}. \quad (6.67)$$

The data are inconsistent with a pointlike proton according to the Mott formula. At large angles the measured

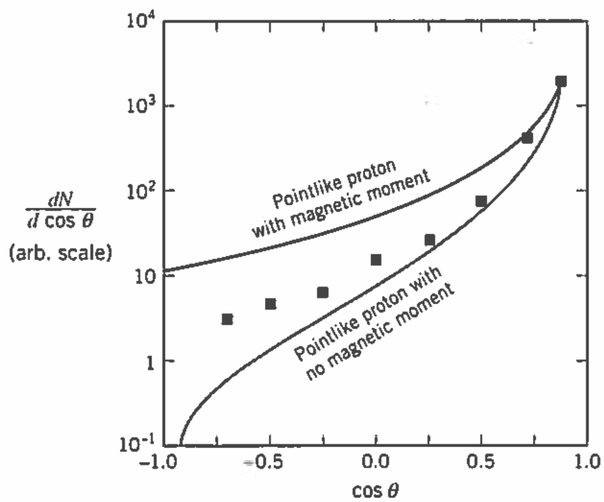

FIGURE 6-16 Determining the size of the proton.
Electrons with a kinetic energy of 550 MeV are scattered from protons and the angular distribution of scattered electrons ($dN/d\cos\theta$) is measured. The data are inconsistent with a pointlike proton. From E. E. Chambers and R. Hofstadter, *Phys. Rev.* **103**, 1454 (1956).

cross section is *larger* than that given by the Mott formula. *How can the cross section be larger than the Mott formula?*

Effect of the Proton Magnetic Moment

Since the measured electron–proton cross section at large angles is larger than the Mott formula, there must be a contribution to the electron–proton cross section that is not contained in the Mott formula. This contribution is due to the proton *magnetic moment*. The magnetic moment does not affect the angular distribution of particles with small speeds because the magnetic force is proportional to the speed. For relativistic electrons the effect of the proton magnetic moment is an important contribution to the scattering cross section. The effect of a magnetic moment is indicated in Figure 6-16. When the contribution to the magnetic moment is included, then the pointlike cross section is much larger than the measured rate. The data lie between that expected for a pointlike proton with no magnetic moment and a pointlike proton with the magnetic moment included. The observed scattering rate is *smaller* than the rate expected from a pointlike proton including magnetic moment because the electron penetrates the proton and experiences the force of a reduced charge and magnetic moment. Both the electric charge and the magnetic moment of the proton are spread out in space. The proton has a finite size. The amount of reduction from the pointlike scattering rate tells us how the charge and magnetic moment are distributed within the proton. The data indicate that both the charge and magnetic moment of the proton are exponentially distributed:

$$\rho = \rho_0 e^{-r/\delta}, \qquad (6.68)$$

where $\delta = 0.8 \times 10^{-15}$ m. In this context we say the size of the proton is about 1 fm.

Measuring the Proton Structure

We have so far only seen the tip of the iceberg concerning the physics contained in electron–proton scattering! The details of the proton structure are revealed by repeating the electron scattering experiment with higher resolution. This requires using a beam of electrons with higher momentum. A series of detailed measurements were made in the 1960s by a team of physicists led by Jerome Friedman, Henry Kendall and Richard Taylor. The experimenters used a beam of electrons with a momentum of 10 GeV/c. (The spectrometers used for these measurements are shown in color plate 15.) The wavelength of a 10-GeV electron is

$$\lambda = \frac{hc}{pc} = \frac{1240 \text{ MeV-fm}}{10^4 \text{ MeV}} \approx 0.1 \text{ fm}. \qquad (6.69)$$

Thus, the experimenters had available to them a probe that was about 20 times finer than the electron probe of Hofstadter.

When a small wavelength electron collides with a proton, the charge distribution of the proton is measured with high resolution. The experimental result is remarkable. The charge distribution is found to be that of a number of pointlike objects. These pointlike particles are the constituent quarks. The proton charge is made up of a number of pointlike quarks that each carry a portion of the proton momentum. (The mass of the quarks is much smaller than the proton mass.) In the center-of-mass of the electron and quark, the scattering process is an elastic collision. The angular distribution of the scattered electrons when viewed in this frame is that due to the scattering of pointlike particles!

Electron–quark scattering is called "deep-inelastic" scattering and is written

$$e^- + p \to e^- + X, \qquad (6.70)$$

where X represents the target proton after it has been struck. The hadronic state X is in general complicated because energy can be converted to matter, but there remains one great simplicity of the interaction, that the electron remains an electron. The energy of the scattered electron E' and the scattering angle θ are measured (see Figure 6-17*a*). This is enough to uniquely determine the fraction of proton momentum (x) carried by the colliding quark. We now show how to calculate the momentum fraction (x) from the measured values of E' and θ. In the center-of-mass system (see Figure 6-17*b*), the electron and quark have momenta of equal magnitudes and opposite directions. Let β be the speed divided by c of the proton in the center-of-mass frame and as usual let $\gamma = (1-\beta^2)^{-1/2}$. The proton momentum (times c) in the center-of-mass frame is given by the Lorentz transformation

$$p_p{}^* c = \gamma\beta Mc^2, \qquad (6.71)$$

where M is the mass of the proton. The quark momentum (times c) is smaller than the proton momentum by the factor x:

$$p_q{}^* c = p_p{}^* cx = \gamma\beta Mc^2 x. \qquad (6.72)$$

For energetic electrons we may neglect the electron mass. Then the electron momentum (times c) in the center-of-

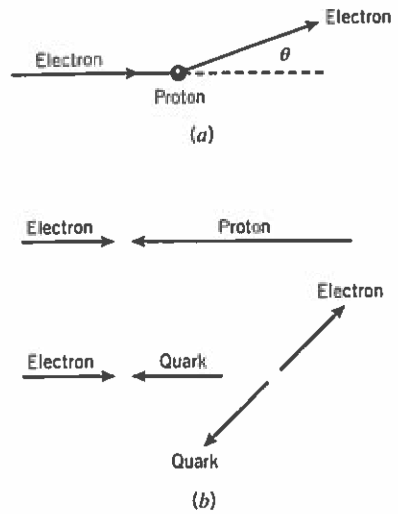

FIGURE 6-17 Electron–proton scattering.
(*a*) Laboratory system and (*b*) electron–quark center-of-mass system.

mass frame is

$$p_e^* c = \gamma E - \gamma\beta E. \qquad (6.73)$$

By definition of the center-of-mass frame, the electron and the quark have equal magnitudes of momentum:

$$p_e^* = p_q^*. \qquad (6.74)$$

Therefore,

$$\gamma E - \gamma\beta E = \gamma\beta Mc^2 x. \qquad (6.75)$$

Solving for x, we get

$$x = \frac{E(1-\beta)}{\beta Mc^2}. \qquad (6.76)$$

In the center-of-mass frame the electron energy is not changed after the scatter. The energy of the scattered electron in the center-of-mass frame is

$$E'^* = \gamma E' - \beta\gamma E' \cos\theta. \qquad (6.77)$$

Conservation of energy (neglecting the electron mass) gives

$$\gamma E - \gamma\beta E = \gamma E' - \beta\gamma E' \cos\theta. \qquad (6.78)$$

We may solve this expression for β,

$$\beta = \frac{E - E'}{E - E'\cos\theta}. \qquad (6.79)$$

We substitute β (6.79) into our expression for the proton momentum fraction (6.76) to get

$$x = \frac{E(1-\beta)}{\beta Mc^2} = \frac{EE'(1-\cos\theta)}{(E-E')Mc^2}. \qquad (6.80)$$

Thus, knowledge of the initial electron energy (E) and the proton mass (M) plus measurement of the electron scattering angle (θ) and the scattered electron energy (E') gives us the proton momentum fraction (x) carried by the quark that collided with the electron. Remarkable! The measurement of the scattered electron energies at a fixed scattering angle gives us the momentum distribution of quarks inside the proton. To an excellent approximation, all scattering angles and incident electron energies give the same x distribution of the quarks. (This last observation is called *scaling*. We shall return to the discussion of scaling in Chapter 17.)

The collective results of many scattering measurements are shown in Figure 6-18. The measured distribution, dP/dx, gives the probability that a single quark

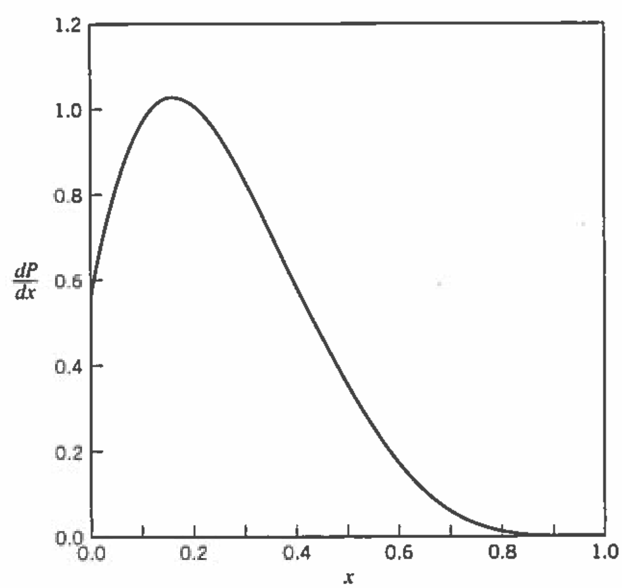

FIGURE 6-18 Structure of the proton.
When the impact parameter is much smaller than 1 femtometer, which is possible for very energetic electrons, then the proton charge is resolved into a distribution of pointlike charges that are called quarks. The scattering is inelastic and momentum is transferred from the electron to the quark that it collides with. The momentum and angular distributions of the scattered electrons gives us the momentum distributions of the quarks inside the proton.

carries a fraction x of the proton momentum. The physical interpretation of the electron–proton scattering is that the electron interacts with the proton by exchanging a photon with a quark inside the proton. The photon exchange occurs only if the quark has just the right momentum fraction x. The angular distribution of the scattered electrons in the electron–quark frame fits that of pointlike particles.

The Difference Between Size and Structure

It is important to make the distinction between the *size* of a particle and the *structure* of a particle. An electron with a momentum p has an approximate size R given by its deBroglie wavelength (5.7):

$$R \approx \lambda = \frac{h}{p}. \tag{6.81}$$

By this we mean that the wavepacket of the electron actually has a characteristic size of h/p. The size of a particle is inversely proportional to its momentum as long as it is not a composite object. Scattering experiments show that the electron is not composite down to a distance of 10^{-18} meters. The electron has no structure that has been detected by experiment; it is a "pointlike" particle. We may picture an electron with momentum p as a "cloud" characterized by the approximate size h/p.

A proton behaves like a pointlike particle down to distances of about 1 femtometer. At low momentum, the approximate size of the proton is h/p. The momentum at which the proton wavelength is 1 fm is

$$\begin{aligned} p &= \frac{h}{\lambda} = \frac{hc}{\lambda c} = \frac{1.24\ \text{GeV}\cdot\text{fm}}{(1\,\text{fm})c} \\ &\approx 1\,\text{GeV}/c. \end{aligned} \tag{6.82}$$

At momenta larger than 1 GeV/c the proton has a smaller wavelength; however, its size remains about 1 fm. This is due to the fact that the proton is a composite object and its size is determined by the wavelengths of the quarks and gluons inside of it. A proton of high momentum will have some energetic quarks with small wavelengths, but it will also have some quarks that have a wavelength of about 1 fm. The proton size is given by the wavelengths of the least energetic quarks. We may make an analogy with a multielectron atom. The atom contains some electrons with small wavelengths (the inner ones), but it also has some electrons (the outer ones) that have a wavelength of about 0.1 nm. The atomic size is determined by the wavelengths of the electrons with the smallest kinetic energy.

* *Challenging*

6-6 PROBING THE STRUCTURE OF THE QUARK

The deep-inelastic electron scattering data show that the quarks inside a proton appear as pointlike objects to an electron with a kinetic energy of 10 GeV. If we have a source of higher energy particles, then we can search for structure of the quarks by searching for a deviation from the pointlike cross section. If the energetic particle is another proton, then the fundamental scattering process is quark–quark scattering,

$$q+q \rightarrow q+q, \tag{6.83}$$

and the quarks themselves may be used to probe the structure of quarks. In the 1980s, a detailed experiment was performed to search for possible structure within the quarks. The experiment was performed not by colliding two energetic protons, but rather by colliding protons and *antiprotons*. The antiproton is the *antiparticle* of the proton. The antiproton has the same mass as the proton, an electric charge of $(-e)$, and a structure identical to the proton except that it is made of pointlike *antiquarks* instead of quarks. Antiparticles are discussed in more detail in Chapter 17. The scattering experiment was performed with protons and antiprotons because, due to their opposite electric charge, they could be made to circulate in opposite directions with a single ring of magnets. In the experiment, the energy of the protons and antiprotons was chosen to be 315 GeV each. The process studied is quark–antiquark elastic scattering:

$$q+\bar{q} \rightarrow q+\bar{q}. \tag{6.84}$$

The experiment is directly analogous to the original Rutherford experiment. The study of this process is greatly complicated by the fact that the quarks are confined inside the proton. When we try to pull a quark out of a proton, for example by striking the quark with another energetic particle, the quark experiences a potential energy barrier from the strong interaction that increases with distance. The quark can never penetrate this barrier, and what is observed experimentally is a sort of strong interaction "spark" in which several hadrons are created. The quark–quark scattering process is indicated in Figure 6-19. This strong interaction spark is somewhat analogous to an electric spark that is created if we make an extremely large electric field between two conductors. The process of a struck energetic quark (or antiquark) materializing as free hadrons is called quark *fragmentation*, which we may write as

$$q \rightarrow \text{hadrons}. \qquad (6.85)$$

The above reaction as we have written it violates all sorts of conservation laws including that of energy and momentum! Energy and momentum are conserved in the event as a whole because the recoiling quarks in the rest of the proton arrange themselves to accomplish just that. Recall that when an energetic photon converts itself into an electron–positron pair, energy and momentum are not exactly conserved in this subprocess and the nucleus must have some recoil momentum; however, essentially all of the photon energy appears in the electron and positron,

$$\gamma \rightarrow e^{+} + e^{-}, \qquad (6.86)$$

and the vector sum of the electron plus positron momenta is very nearly equal to the photon momentum,

$$\mathbf{p}_{\text{photon}} \approx \mathbf{p}_{\text{electron}} + \mathbf{p}_{\text{positron}}. \qquad (6.87)$$

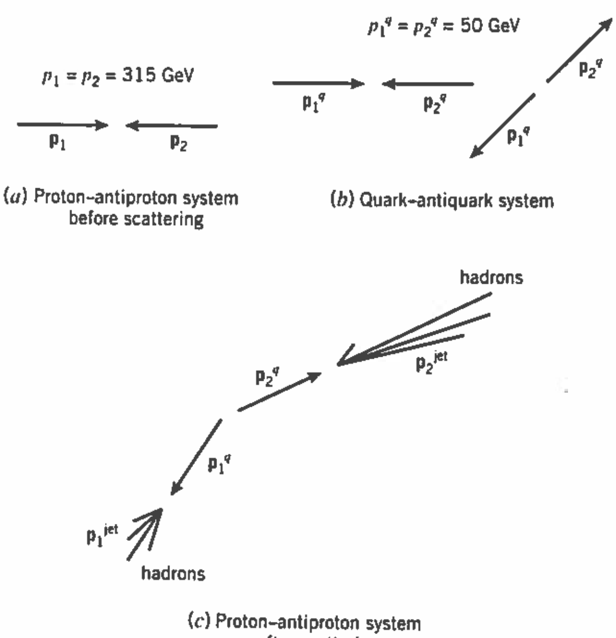

FIGURE 6-19 The quark–quark scattering process.

In the case of the struck quark, the situation is similar: Most of the energy of the quark appears in the hadrons and the vector sum of the momenta of the hadrons, is nearly equal to the quark momentum. The hadrons that arise from a struck quark are called a *jet*.

When quarks scatter with sufficiently large momentum transfer, then the jet of hadrons that they leave in a particle spectrometer becomes a measure of the quark momentum after the scatter ($\mathbf{p}_{\text{quark}}$),

$$q \rightarrow \text{jet}, \qquad (6.88)$$

$$\mathbf{p}_{\text{quark}} \approx \mathbf{p}_{\text{jet}}. \qquad (6.89)$$

Figure 6-20 shows the observed energy deposits in the detector from a large momentum transfer collision. The jet provides an experimental method for determination of the momentum of a scattered quark.

We have derived the Rutherford scattering cross section in the frame where the target particle is at rest. In the center of mass frame, the Rutherford cross section has the same form provided that we replace the kinetic energy squared by the total center-of-mass energy squared

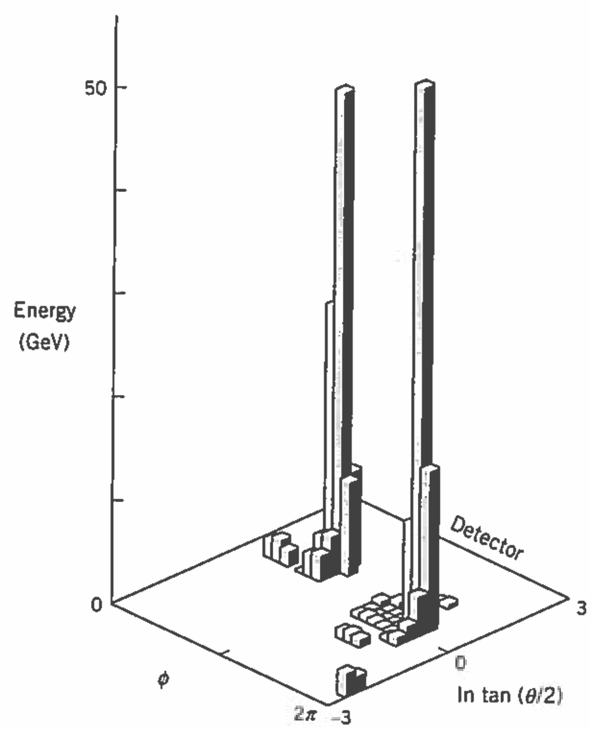

FIGURE 6-20 Measuring the angles of the scattered quarks.
From the UA1 experiment.

(s). The Rutherford cross section becomes

$$\frac{d\sigma}{d\cos\theta} \sim \alpha^2\left(\frac{1}{s}\right)\frac{1}{(1-\cos\theta)^2}. \qquad (6.90)$$

Now we are ready to analyze the angular distribution of scattered quarks. The scattering cross section for two quarks is specified by the rules of the strong interaction. The fundamental physics for quarks scattering by strong interaction does not differ from the fundamental physics of charged particles scattering by the electromagnetic interaction. In each case the force is governed by the exchange of massless quanta by structureless particles. Figure 6-21 shows a Feynman diagram for quark–quark scattering. The two quarks exchange a massless particle called a *gluon*. The probability for the gluon exchange is proportional to the square of the strong interaction coupling parameter (α_s). The angular distribution for quark–quark scattering is given by

$$\frac{d\sigma}{d\cos\theta} \sim \alpha_s^2\left(\frac{1}{s}\right)\frac{1}{(1-\cos\theta)^2}. \qquad (6.91)$$

The form of the strong interaction cross section (6.91) is identical to the form of the electromagnetic interaction cross section (6.90), except that we have replaced α by α_s. The numerical value of the strong interaction coupling parameter for the scattering of 100-GeV quarks is

$$\alpha_s \approx 0.1, \qquad (6.92)$$

compared to

$$\alpha = \frac{1}{137}. \qquad (6.93)$$

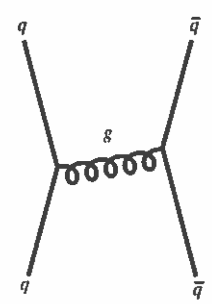

FIGURE 6-21 Feynman diagram for quark–antiquark scattering.
The quark and antiquark exchange a gluon. The probability for the exchange is proportional to the square of the strong interaction coupling parameter (α_s).

Let us examine the origin of each factor in cross section (6.91):

1. The strength of the coupling of a gluon to a quark is α_s for the amplitude squared, and this factor appears as the square because there are two such couplings, one to each of the scattered quarks.
2. The quarks have a size that is specified by their wavelength, and this wavelength is inversely proportional to the quark momentum, and energy and momentum are equivalent ($E \approx pc$) since the quarks are relativistic. The cross section is proportional to the square of the wavelength or inversely proportional to the square of the energy.
3. The angular distribution varies inversely as the square of the momentum transfer squared, and the momentum transfer squared is proportional to ($1-\cos\theta$).

The quark–quark scattering experiment was performed to test the strong interaction version of the Rutherford formula. Agreement with the Rutherford formula is expected if quarks interact by the exchange of massless particles that mediate the strong interaction. A deviation from the Rutherford formula is expected if the quarks have internal structure when probed at short distances. The first high-energy quark–quark scattering data are shown in Figure 6-22. These data are from the UA1 experiment led by Carlo Rubbia. In this experiment, the UA1 physicists observed roughly the same number of quarks scattered at large angles as Geiger and Marsden observed α particles. The quark–quark scattering data agree beautifully with the pointlike cross section formula! These data show that the strong interaction of two quarks at short distances has the same fundamental quantum behavior as the electromagnetic interaction.

Whether or not a particle has structure also is not related to the mass of the particle. This may seem counterintuitive. For example, the bottom quark is pointlike in spite of the fact that it has a mass of about five times the proton mass! The question of why some pointlike particles are so much more massive than others is one of the most profound unanswered physics questions.

6-7 SUMMARY OF THE SCATTERING EXPERIMENTS

The results of the scattering experiments are summarized in Figure 6-23. The nucleus appears as a point like

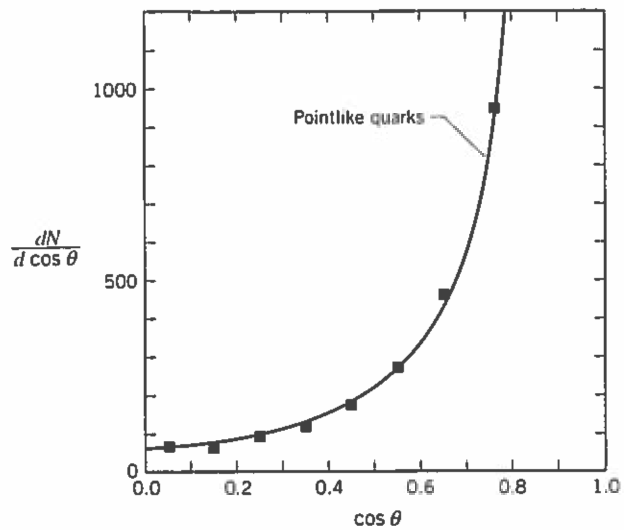

FIGURE 6-22 Angular distribution of scattered quarks and gluons from high energy proton–antiproton collisions measured in the center-of-mass frame of the colliding quarks and gluons.
The distribution fits the strong-interaction (QCD) Rutherford law, demonstrating that the quarks have no structure down to a distance scale of 10^{-18} m. From G. Arnison et al., *Phys. Lett.* **177B**, 244 (1986).

object to an incident charged particle with a kinetic energy of a few MeV. The angular distribution follows the Rutherford formula. When the energy of a charged particle is raised to 100 MeV, then the charged particle can penetrate the nucleus and the angular distribution deviates from the Rutherford formula. The measurement of this deviation determines the structure of the nucleus. The charge of the nucleus is made up of protons. Small angle electron scattering from a single proton follows the pointlike scattering cross section. Large angle "high-energy" electron scattering from a single proton shows a deviation from the pointlike scattering cross section, signaling that the proton is a composite object. Electron scattering at higher energies resolves the proton into its constituent quarks. The electron–quark scattering follows the pointlike scattering cross section. At even much higher energies, quark–antiquark scattering still is observed to follow the pointlike scattering cross section.

The use of the experimental technique pioneered by Rutherford to discover new structure has not been exhausted. Someday, quark–quark scattering experiments will be performed with sensitivity for detection of quark substructure well below the current limit of 10^{-18} meters. For this next exciting experiment, smaller wavelength, higher energy quarks are needed than are currently available in the protons of today's accelerators.

CHAPTER 6: PHYSICS SUMMARY

- The nucleus was discovered by observing that α particles were occasionally scattered at very large angles (even backwards) when they passed through a thin foil. To explain this experimental fact, Rutherford deduced that the positive electric charge of the atom must be concentrated at a "point."
- The scattering cross section of a pointlike projectile particle of kinetic energy E_k and charge (ze) with a pointlike target particle with charge (Ze) by the electromagnetic interaction is given by the Rutherford formula

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi}{2} z^2 Z^2 \alpha^2 \left(\frac{\hbar c}{E_k}\right)^2 \frac{1}{(1-\cos\theta)^2}.$$

  The Rutherford formula has been thoroughly tested by experiment.
- Rutherford scattering breaks down when the projectile particle is energetic enough to penetrate the nucleus. The nucleus is observed to be made up of a closely packed sphere of neutrons and protons held together by the strong interaction. The radius of the nucleus is determined from scattering experiments to be

$$R \approx (1.2\,\text{fm})A^{1/3}.$$

- When electrons with a momentum of about 1 GeV are scattered from protons or neutrons, a deviation in the Rutherford cross section is observed because the proton and neutron are not pointlike particles, but have a finite size. A quantitative measurement of the deviation from the Rutherford cross section shows that the size of the proton and neutron are about 1 fm.
- When electrons with a momentum of about 10 GeV are scattered from protons or neutrons, the detailed structure of the constituents is revealed. The scattering is due to the electromagnetic collision of the electron and a quark. The momentum distribution of the quarks inside the proton or neutron is measured by measuring the momentum transfer between the electron and the quark. In the electron–

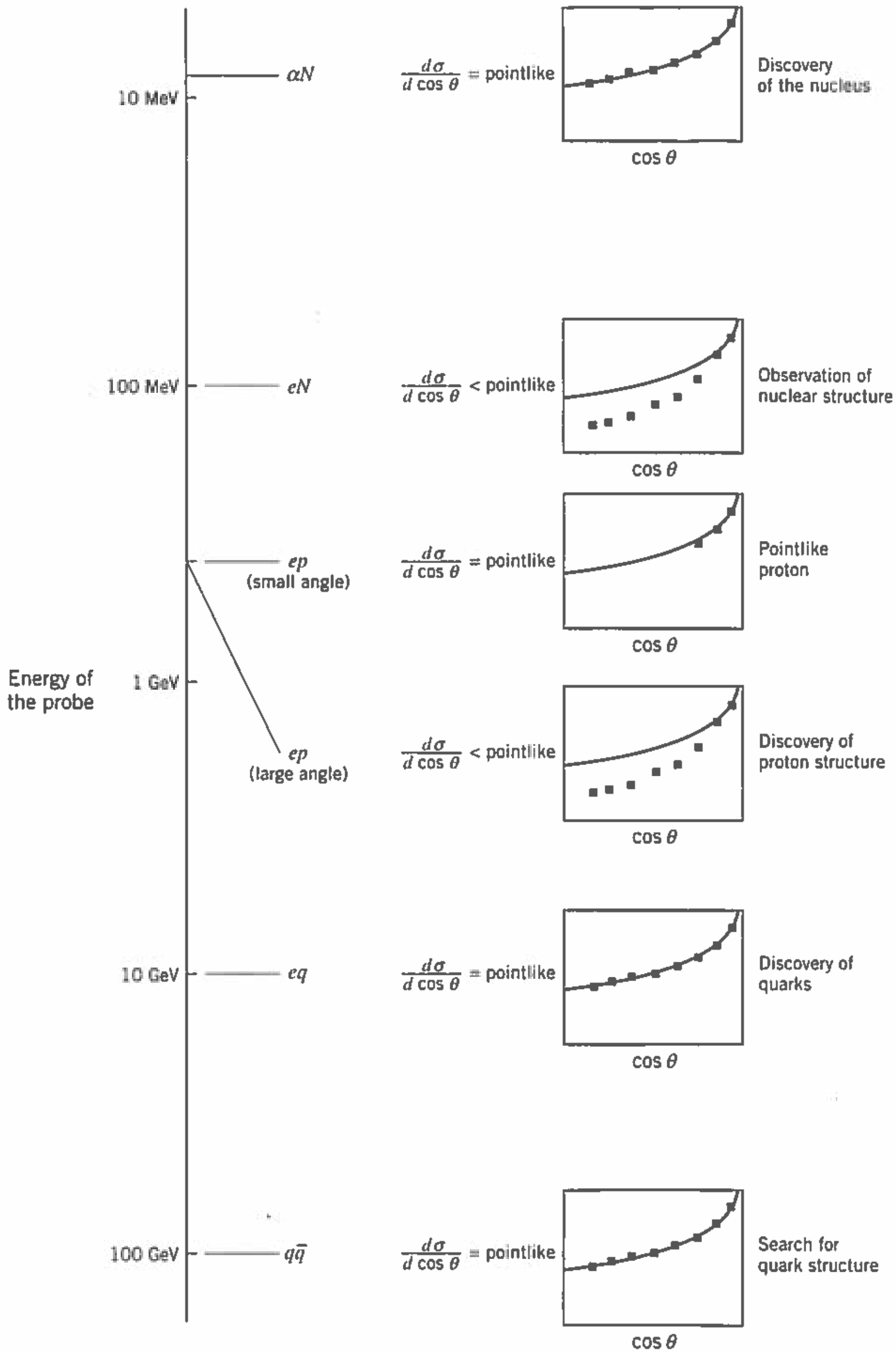

FIGURE 6-23 Summary of the scattering experiments.

quark center-of-mass system, the collision is elastic, and the angular distribution follows that expected for pointlike particles.

- When quarks with a momentum of about 50–100 GeV are scattered from each other, the quarks behave as free particles with regard to the protons that contain them. The quarks interact by the strong interaction, by the exchange of massless particles, analogous to the electromagnetic interaction. The angular distribution of the scattered quarks is expected to be of the Rutherford form, $(1-\cos\theta)^{-2}$, provided that the quarks have no internal structure. Quark–quark scattering data show that the quarks are pointlike down to a distance of 10^{-18} meters.

REFERENCES AND SUGGESTIONS FOR FURTHER READING

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QUESTIONS AND PROBLEMS

Measuring structure by particle scattering

1. Why is the wavelength of an αparticle that comes from a nuclear decay about equal to the size of a nucleus?
2. Explain how is it possible for an αparticle to be scattered directly backwards ($\theta = \pi$) in the Rutherford experiment.
3. In the Rutherford experiment, is it possible to choose the impact parameter? Explain.
4. Macroscopic objects have extremely small wavelengths. Why do they not make good probes for measuring the structure of matter?
5. Could x rays have been used to discover the nucleus?

Definition of cross section

6. If the cross section is infinite, can the scattering rate also be infinite? Explain.
7. Experimenters wish to study the properties of a particle that has a production cross section in proton–proton collisions of 1 nb. They design a cylindrical liquid hydrogen target that has a diameter of 0.05 m and a length of 0.5 m. The proton beam has an intensity of 10^8 per second. If the apparatus can detect the rare particle with an efficiency of 10%, how long does it take to collect 10^6 events?
8. A collimated proton beam is sent into a liquid deuterium bubble chamber which has a thickness of 0.5 m. The density of liquid deuterium is 162 kg/m^3. A sample of 10^5 pictures are analyzed and three rare events are found. (a) Calculate the production cross section for the process. (b) What is the 95% confidence level upper limit for the cross section?
9. The strong interaction has a short range, approximately 1 fm. Use this fact to estimate the cross section for the strong interaction of two energetic protons ($E >> mc^2$). Compare your answer to 40 mb.

Probing the structure of the atom

10. When a particle has structure, why does a deviation from the Rutherford scattering formula at a fixed energy show up at large scattering angles rather than at small angles?
11. In the analysis of the Rutherford experiment, why can we treat the α as a particle scattering from a single nucleus and ignore the wave properties of the α? Make a comparison with the Davisson-Germer experiment.
12. Why is it convenient to write the differential cross section (6.12) as $d\sigma/d\cos\theta$ rather than $d\sigma/d\theta$? Show that if we write the differential cross section as $d\sigma/d\theta$ and integrate over all angles to get the total cross section that we get the same result as integrating $d\sigma/d\cos\theta$ over all values of $\cos\theta$.

13. Consider a beam of protons with momentum of 200 MeV/c scattering from aluminum nuclei. (a) Calculate the cross section for electromagnetic scattering at angles greater than 10°. (b) If the proton beam is sent into an aluminum foil 1 μm thick, what fraction are scattered to angles greater than 10°?

14. How important was it for Geiger and Marsden to evacuate their chamber? For α particles estimate the thickness of air that would have the same cross section as scattering in a gold foil of thickness 0.2 μm. (The density of gold is 1.9×10^4 kg/m^3 and the density of air is 1.2 kg/m^3 at atmospheric pressure and room temperature.)

15. A 10-MeV α particle scatters from a silver nucleus at an angle of 90°. (a) Calculate the impact parameter. (b) Calculate the distance of closest approach.

16. Calculate the kinetic energy of an α particle if the distance of closest approach to a gold nucleus is 10 fm when scattered at 90°.

17. Alpha particles of 5 MeV are scattered from a silver target 1 μm thick. If the scattering rate between 60 and 90 degrees is one per minute, what is the rate at which α particles strike the target?

18. Calculate the cross section for a 5-MeV α particle to be scattered from platinum at an angle between 5 and 10°.

19. A beam of 5-MeV α particles is directed into a silver foil of thickness 1 μm. The fraction of α particles scattered at angles greater than a certain angle (θ) is measured to 10^{-3}. (a) What is the scattering cross section for this process? (b) What is the value of θ? (The density of silver is 1.05×10^4 kg/m^3.)

Probing the structure of the nucleus

20. Protons of energy E_k are incident on a thin sheet of iron atoms at rest. Estimate the largest value of E_k for which the angular distribution of the scattered protons would be expected to follow the Rutherford angular distribution.

21. Alpha particles are scattered from a thin copper target. Estimate the minimum α particle kinetic energy that will cause a deviation from the Rutherford formula.

22. Consider a radioactive source that provides alpha particles with a kinetic energy of 6 MeV. Determine the approximate value of Z such that elements with nuclear charges greater than Z obey the Rutherford scattering law, and elements with nuclear charges smaller than Z show deviations.

Probing the structure of the proton

23. In an electron–proton scattering experiment, what factors limit the number of large angle scatters?

24. Electrons with an energy of 10 GeV are scattered from protons at rest at an angle of 30°. What is the maximum energy of the scattered electron?

25. If an electron with an energy of 10 GeV is scattered from a proton at rest and emerges with an energy of 5 GeV, what is the maximum scattering angle?

*26. An electron with an energy of 10 GeV scatters from a proton at rest and emerges at an angle of 5° with an energy of 5 GeV. What is the speed of the proton in the quark–electron center-of-mass frame?

Probing the structure of the quark

*27. The number of large angle scattering events observed by Geiger and Marsden in the discovery of the nucleus (Figure 6-4) was the same order of magnitude as the number of events observed by the UA1 Collaboration in the search for possible structure of quarks (Figure 6-22). Make an estimate of the total number of collisions needed to make the plots of (a) Figure 6-4 and (b) Figure 6-22.

*28. Consider the scattering of quarks at a center-of-mass energy of 100 GeV. Make an order of magnitude estimate of the probability that the quarks scatter at an angle between 45° and 90°.

Additional problems

29. (a) Calculate the maximum kinetic energy that can be transferred to a gold nucleus from a collision with a 6-MeV α particle. (b) Calculate the maximum kinetic energy that can be transferred to an electron from a collision with a 6-MeV α particle.

30. A beam of relativistic electrons with energy E is directed into a sheet of iron 0.01 meter thick. The fraction of scattered electrons with energy between E_1 and E_2 and angles between θ_1 and θ_2 is measured to be 10^{-6}. Calculate the scattering cross section per nucleon. (The density of iron is 7.87×10^3 kg/m^3.)

31. A beam of α particles is directed into a thin gold target. Scattered α particles are detected in a small detector of cross-sectional area (a) that is placed a distance (R) facing the target at an angle (θ), ($a \ll R^2$). (a) Find an expression for the rate at which scattered α particles strike the detector in terms of the rate at which the incident α particles strike the foil (R_i), the kinetic energy of the α particle (E_k), the thickness of the target (L), and the density (ρ), atomic number (Z), and atomic mass number (A) of the target. (b) Calcu-