A-3. Point O, which is on that line, could represent the coincidence of the clock hand with a fiducial marker on the clock face, corresponding to zero time. Point P_2, whose time coordinate in the S' frame gives unit time ($w' = 1$) on that resting clock, is also on that line. The event represented by P_2 might correspond to a second coincidence of the clock hand with the fiducial marker. In frame S, however, the clock would be seen as a moving clock. We have seen above that $w' = 1$ in the S' frame corresponds to $w = \gamma$ in the S frame. Thus, by S-frame clocks, the unit time interval of the S' clock would be recorded as γ, corresponding exactly to the time dilation effect described by Eq. 2-14b.

In Fig. A-4 we show the calibration of the axes of the frames S and S', the unit time interval along w' being a longer line segment than the unit time interval along w and the unit length interval along x' being a longer line segment than the unit length interval along x. The first thing we must be able to do is to determine the spacetime coordinates of an event such as P directly from the Minkowski diagram. To find the space coordinate of the event, we simply draw a line parallel to the time axis from P to the space axis. The time coordinate is given similarly by a line parallel to the space axis from P to the time axis. The rules hold equally well for the primed frame as for the unprimed frame. In Fig. A-4, for example, the event P has the spacetime coordinates $x = 3.0$ and $w = 2.5$ in S (long dashed lines) and spacetime coordinates $x' = 2.0$ and $w' = 1.2$ in S' (short dashed lines). Figure A-4 was drawn assuming that $\beta = 0.50$, which yields $\gamma = 1.15$. Using these values for β and γ, you can readily derive the S-frame coordinates from the S'-frame coordinates—or conversely—by means of the Lorentz transformation equations (Eq. A-1), thus verifying the graphical relationships displayed in the Minkowski diagram.

In using the Minkowski diagram it is almost as if the rectangular grid of coordinate lines of S (Fig. A-5a) became squashed toward the 45° bisecting line when the coordinate lines of S' are put on the same graph (Fig. A-5b). In more formal language, we say that the Lorentz transformation equations transform an orthogonal (perpendicular) reference frame into a nonorthogonal one. Note that as $\beta \to 1$, corresponding to $v \to c$, the angle ϕ in Fig. A-5b ($= \tan^{-1} \beta$) approaches

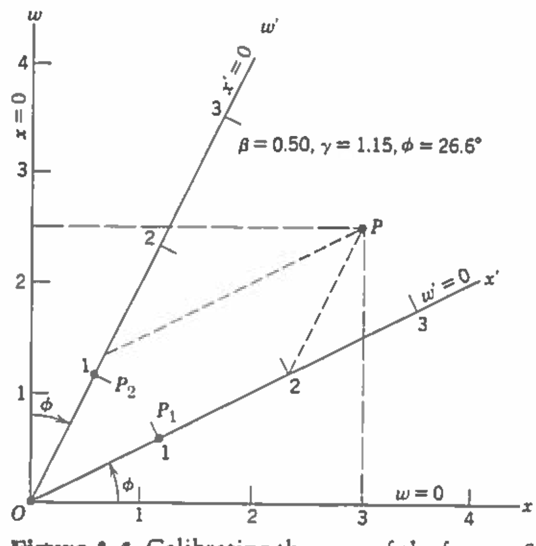

Figure A-4. Calibrating the axes of the frames S and S'.

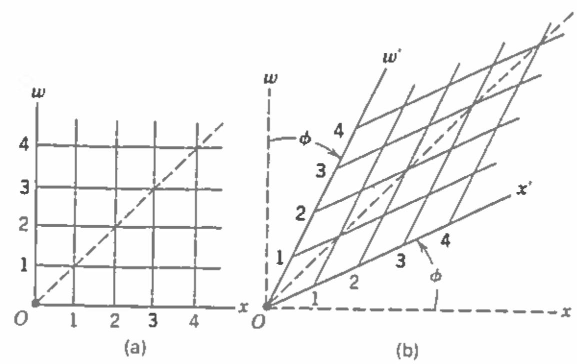

Figure A-5. An orthogonal reference frame, (a), transforms into a nonorthogonal one, (b).

45°, thus compressing the S'-frame coordinate space into a thinner and thinner wedge of the S-frame coordinate space. Alternatively, as $\beta \rightarrow 0$, corresponding to an approach to classical conditions, the angle ϕ between corresponding S and S' axes becomes very small. Even for a speed as high as that of a typical earth satellite (~17,000 mi/h), we note that $\beta = 2.5 \times 10^{-5}$, which yields a value of only 0.0015° for ϕ; relativistic mechanics is not much different from classical mechanics in these circumstances.

A-3 SIMULTANEITY, CONTRACTION, AND DILATION

Now we can easily show the relativity of simultaneity. As measured in S', two events will be simultaneous if they have the same time coordinate w'. Hence, if the events lie on a line parallel to the x' axis, they are simultaneous to S'. In Fig. A-6, for example, events Q_1 and Q_2 are simultaneous in S'; they obviously are not simultaneous in S, occurring at different times w_1 and w_2 there. Similarly, two events R_1 and R_2, which are simultaneous in S, are separated in time in S'.

As for the space contraction, consider Fig. A-7a. Let a meter stick be at rest in the S frame, its end points being at $x = 3$ and $x = 4$, for example. As time goes on, the world line of each end point traces out a vertical line parallel to the w axis.

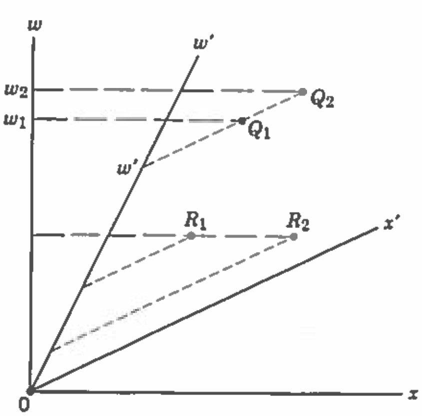

Figure A-6. Showing the relativity of simultaneity.