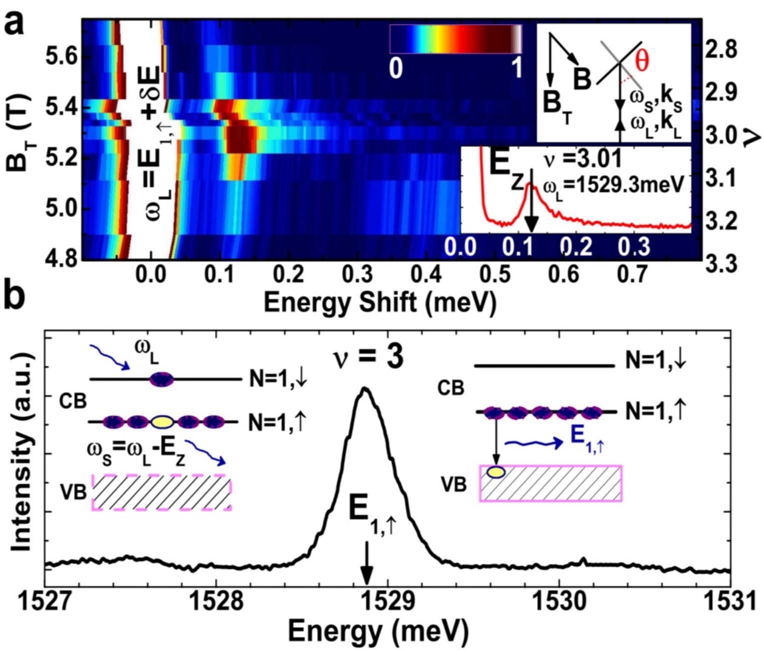
Quantum Physics 1
Class 25

Exam Stats :

https://share.streamlit.io/
trevorguru/quantum-physics

Spin :

Quantum Hall Effect :



Rapid collapse of spin waves in nonuniform phas'o@es of _ -' Yan, Y Gallais, A Pinczuk, L Pfeiffer, K West
Physical review letters 106 (19), 196805 (2011) —--

Class 25

Intrinsic Spin

Last Time:

- Zeeman Effect
  - → circulating charge in a $\vec{B}$ field
  - → orbital angular momentum.
- Potential Energy:

$$U = -\vec{\mu}\cdot\vec{B} \quad ; \quad \vec{\mu} = \text{magnetic dipole}$$

Recall, $\mu = AI$

$$\vec{\mu} = \frac{q}{2m_0}\vec{L}$$

$$U = -\frac{q}{2m_0}\vec{L}\cdot\vec{B}$$

$$= -\frac{q}{2m_0} B L_z$$

$$= -\frac{q}{2m_0} B\hbar m \quad ; \quad m = 0, \pm 1, \pm 2, \ldots$$

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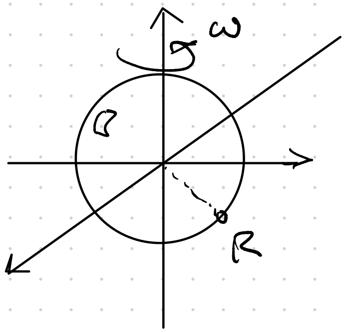
e's are known to have spin (i.e. they behave like tiny magnets).

What is the origin of this spin?

Classical picture: e' is a rotating ball of charge?

Consider:

Solid sphere m_0, charge q, uniformly distributed at the surface.



$$\mu = \frac{q}{2m_0} L$$

$$L = I\omega$$

$$= \frac{2}{5} m_0 R^2 \omega \quad ; \quad I = \frac{2}{5} m_0 R^2$$

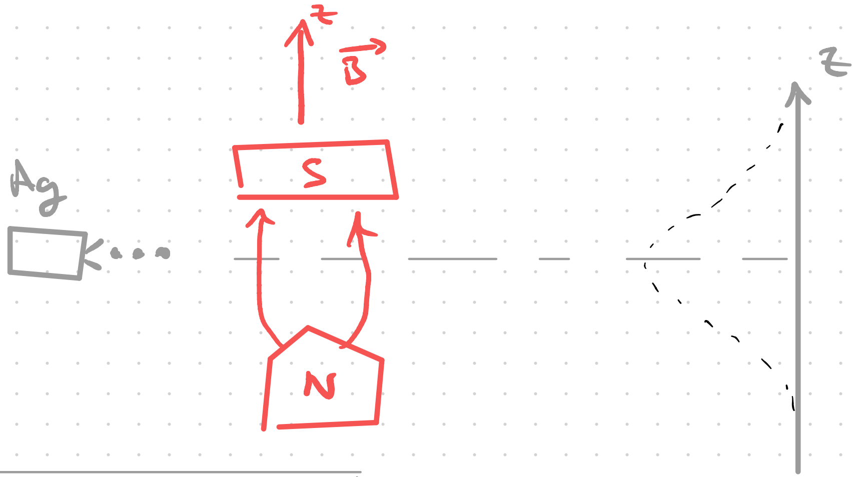
$$\mu = \frac{q}{3} R^2 \omega$$ (integrate, $\mu \sim AI$ over sphere)

$$= \frac{q}{3} R^2 \omega \cdot \frac{L}{\frac{2}{5} m_0 R^2 \omega}$$

$$\mu = \frac{5}{6} \frac{q}{m_0} \vec{L}$$

Stern-Gerlach Experiment

- Ag atoms in the ground state
- $47 e^-$, one e^- in the valence shell
- B_z not uniform

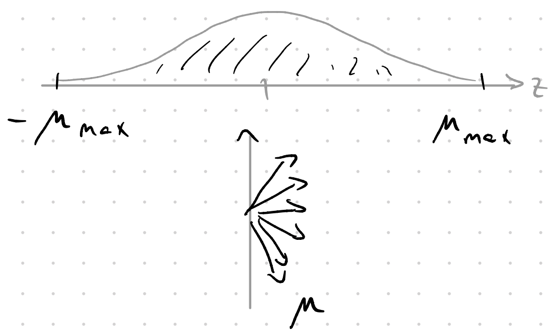


$$\boxed{\vec{\mu} = g\,\frac{\mu_B}{\hbar}\,\vec{S}}$$

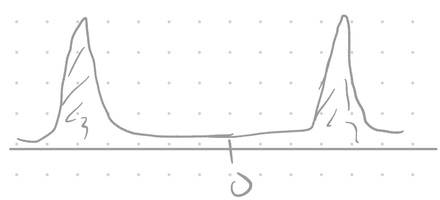
$$\Rightarrow \quad F_z = -\frac{\partial}{\partial z}\left(-\vec{\mu}\cdot\vec{B}\right)$$

$$= g\,\frac{\mu_B}{\hbar}\,\frac{\partial}{\partial z}\,\vec{S}\cdot\vec{B} = \frac{g\,\mu_B}{\hbar}\,S_z\,\frac{\partial B_z}{\partial z}$$

Classically :



Quantum :



$$F_z = g \frac{\mu_B}{\hbar}(m\hbar)\frac{\partial B_z}{\partial z}$$

$$= \mu_B g m \frac{\partial B_z}{\partial z}$$; Recall $\frac{\partial B_z}{\partial z}$, μ_B are known

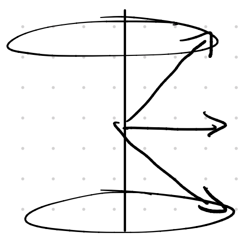
$\Rightarrow \; gm = \pm 1$

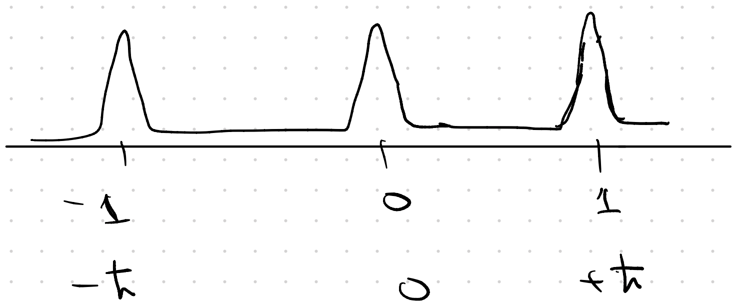
for e^- $\;\; \underline{\underline{g = 2}}$ $\quad \therefore \; \underline{\underline{m = \pm 1/2}}$

$\therefore \; S = 1/2$

Compare with

$l = 1$





(ie. if $\vec{\mu}$ due to orbital motion of charge of e^-)

VIP Observations: (surprising + unfamiliar)

- $S = 1/2$; $S_z = \hbar/2, -\hbar/2$
- $|\vec{S}| = \sqrt{\frac{1}{2}(\frac{1}{2}+1)}\,\hbar$
- $g = 2$, for e^-
- intrinsic property (Not dependent on an "internal structure" or orbital motion of charge.)
- S.E. $\Psi(x,t)$ cannot be used to describe spin states.

  * need to introduce the spin eigenfunction χ

i.e. $\Psi(x,t) \longrightarrow \Psi(x,t)\chi(t)$

& $H_0\Psi(x,t) = i\hbar \dfrac{\partial \Psi(x,t)}{\partial t}$; $H'\chi(t) = i\hbar \dfrac{\partial \chi(t)}{\partial t}$

** No x dependence.

$$\chi = \alpha \chi_+ + \beta \chi_-$$

$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Recall: For rotating ball of charge picture:

$$\vec{\mu} = \frac{5}{6}\frac{q}{m_0}\vec{L} \; ; \quad \mu_B \equiv \frac{q\hbar}{2m_0} \text{, Bohr magneton}$$

$$= \frac{5}{6}\frac{q}{m_0}\cdot\frac{\mu_B}{\mu_B}\cdot\vec{L}$$

$$= \frac{5}{3}\frac{\mu_B}{\hbar}\vec{L} \equiv g\frac{\mu_B}{\hbar}\vec{L} \; ; \quad g \equiv g\text{-factor}$$

for e^- moving in orbit

$$\vec{\mu} = \frac{q}{2m_0}\vec{L} = \frac{\mu_B}{\hbar}\vec{L} \; ; \quad g = 1$$

Now make a BIG leap of faith:

$\vec{L} \longrightarrow \vec{S}$; define $\hat{S}$ operator which behaves like $\hat{L}$.

$$\Rightarrow \quad \vec{\mu} = g\frac{\mu_B}{\hbar}\vec{S}$$

from experiments:

e^-: $g = 2(\times 1.0011596521\ldots)$

p^+: $g = -5.6$

n: $g = +3.8$

& NB $\quad |\vec{S}| = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)\hbar^2}$; $S = 1/2$

In-class 25.1
25.2

NB: $\hat{H}$ for stationary spin particle in $\vec{B}$-field:

$$\hat{H} = -\vec{\mu}_s \cdot \vec{B}$$

for e^-: $\hat{H} = g\frac{\mu_B}{\hbar}\vec{S}\cdot\vec{B}$

$$= g\frac{\mu_B}{\hbar}B_0 S_z \quad ; \quad g = 2$$

eigenfunctions of $\hat{S}_z$: $\hat{S}_z \chi_{\frac{1}{2},m_s} = m_s\hbar\,\chi_{\frac{1}{2},m_s}$

where $m_s = \pm 1/2$

Matrix representation:

$$\chi_{\frac{1}{2},\frac{1}{2}} \equiv \begin{pmatrix}1\\0\end{pmatrix} \quad ; \quad \chi_{\frac{1}{2},-\frac{1}{2}} \equiv \begin{pmatrix}0\\1\end{pmatrix}$$

eg) For a proton:

$$\vec{\mu} = g \frac{\mu_B}{\hbar} \vec{S} \; ; \quad \hat{H} = -g_p \frac{\mu_B}{\hbar} B_0 \hat{S}_z$$

5.6

In-class 25-3

Assume (Big assumption):

that the following commutation relations work.

$$[\hat{S}_x, \hat{S}_y] = i\hbar \hat{S}_z$$

$\vdots$

Consider the matrix (S_z)

$$\chi_{\frac{1}{2},\frac{1}{2}} \equiv \chi_+ \qquad \chi_{\frac{1}{2},-\frac{1}{2}} = \chi_-$$

$$\chi_+ \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_- \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{S}_z \chi_+ = \frac{\hbar}{2}\chi_+$$

$$\hat{S}_z \chi_- = -\frac{\hbar}{2}\chi_-$$

NB χ_+ & χ_- are orthogonal

$(1\ 0)\begin{pmatrix}0\\1\end{pmatrix} = 0$

Matrix: $$(S_z) = \begin{pmatrix} (\chi_+ \hat{S}_z \chi_+) & \underbrace{(\chi_+ \hat{S}_z \chi_-)}_{} \\ (\chi_- S_z \chi_+) & (\chi_- \hat{S}_z \chi_-) \end{pmatrix}$$

eg) $$(\chi_+ \hat{S}_z \chi_+) = (\chi_+ \tfrac{\hbar}{2} \chi_+)$$

$$= \frac{\hbar}{2}(1\ 0)\begin{pmatrix}1\\0\end{pmatrix} = \frac{\hbar}{2}$$

⋮

$$\therefore (S_z) = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

◎ with no proof:

$$(S^2) = (S_x^2 + S_y^2 + S_z^2)$$

$$= \frac{3}{4}\hbar^2\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

determine $\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ eigenstates

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In-class 25.4

<u>NB</u>:

$$\hat{S}^2 \chi_{s,m_s} = s(s+1)\hbar^2 \chi_{s,m_s}$$

like $Y_{l,m}(\theta,\phi)$

$$\hat{S}_z \chi_{s,m_s} = m_s \hbar \chi_{s,m_s}$$