PHYS2210Spring2023_HomeworkSet05_Problems16to20

16) Assume that $\psi(x)$ is an arbitrary normalized real function. a) Calculate $\langle p_x \rangle$ for the wavefunction $e^{i(kx-\omega t)}\psi(x)$. b) Calculate $\langle p_x \rangle$ for the wavefunction $e^{-i\omega t}\psi(x)$.

17) Townsend 3.1

3.1. Show there is no solution to the time-independent Schrödinger equation for a particle in the infinite square well

$$V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{elsewhere} \end{cases}$$

for $E = 0$. *Suggestion*: Start with the differential equation for ψ within the well for $E = 0$:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = 0$$

What is the most general solution to this second-order differential equation? Show that the requirement that the wave function vanish at the boundaries of the well leads to $\psi = 0$.

18) Townsend 3.3

3.3. The wave function for a particle in a box is

$$\Psi(x) = \begin{cases} N & 0 < x < L \\ 0 & \text{elsewhere} \end{cases}$$

where N is a constant. (*a*) Determine a value for N so that the wave function is appropriately normalized. *Note*: In reality, the wave function $\Psi(x)$ does not drop discontinuously to zero at the ends of the well. Assume the change in the wave function occurs over such a small distance that you can neglect this effect in your calculations. (*b*) Calculate the uncertainty Δx in the particle's position.

19) (20 pts) The two state functions,

$$\Psi_1(x,t) = A_1 \cos\left(\frac{\pi x}{L}\right) e^{-i\omega_1 t}$$

$$\Psi_2(x,t) = A_2 \sin\left(\frac{2\pi x}{L}\right) e^{-i\omega_2 t}$$

where the state function is zero for x<-L/2 and x>L/2. Are solutions for the particle in a box problem.

a) Find the A values that normalize the functions by performing the integral of the probability density.

b) Find the expectation value for x for each function. (Write down any integrals one would need to calculate. You may not need to do any integrals if you make a good logical argument. Hint: Making a variable substitution can help you to visualize whether functions are even or odd about the center of the region.)

c) Is the expectation value for each function equal to the most probable value for each function? If not, explain.

d) Find the expectation value of x^2 for each function. (You'll have to do the integrals here.)

e) Find the uncertainty in position ($\Delta x \equiv \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$) for each function.

20) a) Find the state function in the wavevector representation, $A(k)$, for the wavefunction $\Psi_1(x,t) = A_1 \cos\left(\frac{\pi x}{L}\right) e^{-\omega_1 t}$ where the wave function is zero for x<-L/2 and x>L/2 (outside the box).

b) Find the uncertainty in k by completing appropriate integrals.

c) Make an argument for why your calculated uncertainty in k is reasonable.