Quantum Physics 1
Class 7

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Wave packets

Last Time:

Free Space: $$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \psi(x,t)}{\partial t}$$

w/ Potential: $V(x)$
$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi = i\hbar \frac{\partial \psi}{\partial t}$$

classically: $$\frac{p^2}{2m} + V(x) = E$$

$$p \rightarrow \frac{\hbar}{i}\frac{\partial}{\partial x} \quad ; \quad E \rightarrow i\hbar \frac{\partial}{\partial t}$$

Classical: $$\underbrace{F = -\frac{\partial V}{\partial x} = m\frac{d^2 x}{dt^2}}_{\text{if } F=0\text{, no effect on } x(t)} \quad ; \quad x = x(t)$$

Quantum: if $F=0$, $V \neq 0$ $\psi(x,t)$ will change.

Recall also : $\Psi^*(x,t)\,\Psi(x,t)$: probability density.

$\Psi^*(x,t)\,\Psi(x,t)\,dx$: probability of finding particle between x & $x + dx$.

- probability interpretation :

Intensity $\propto$ |wave amplitude|^2

→ double slit interference pattern

⇒ intensity measure by detector & probability interpretation.

- $\Psi(x,t)$ is the observable (measured quantity) or dynamical variable.

__NB__ : { Classically ; x is a dynamically variable, t is parameter.
Quantum : x & t are parameters }

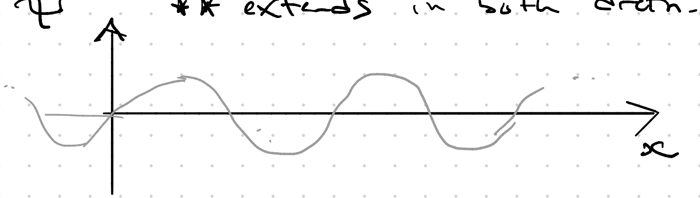
- Recall : $\int_{-\infty}^{+\infty} \Psi^* \Psi \, dx = 1$ Normalization condition.

w/t $\Psi \sim e^{i(kx-\omega t)}$

Normalizable ?

① Consider wave :

Ψ ** extends in both drctn.



② $\int |\psi|^2 dx = \int_{-\infty}^{\infty} |1| \, dx = \infty \neq 1$

Solution to normalization problem?

$\Rightarrow$ Consider linear superposition of waves!

- Superposition of waves $\Rightarrow$ wave packet.
- Superposition of waves with dif. λ can interfere destructively for x far away from the origin.

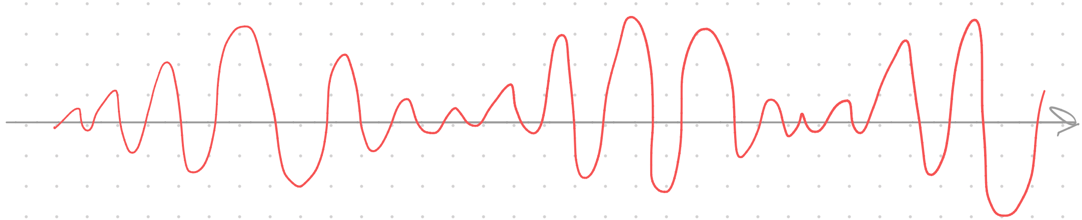
Now, consider two waves with slightly different λ:

$$\Rightarrow A \sin\left[\left(k - \frac{\Delta k}{2}\right)x - \left(\omega - \frac{\Delta \omega}{2}\right)t\right] + A \sin\left[\left(k + \frac{\Delta k}{2}\right)x - \left(\omega + \frac{\Delta \omega}{2}\right)t\right]$$

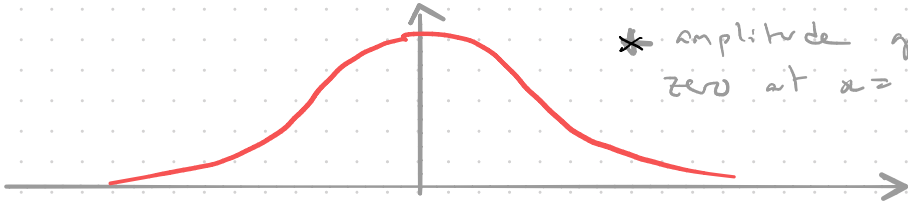
$$= 2A \sin(kx - \omega t) \cos\left[(\Delta k/2)x + (\Delta \omega/2)t\right]$$

trick: $\sin\alpha + \sin\beta = 2\cos\left(\frac{\alpha - \beta}{2}\right) \sin\left(\frac{\alpha + \beta}{2}\right)$

Result: we have wave with lower "beat" frequency.



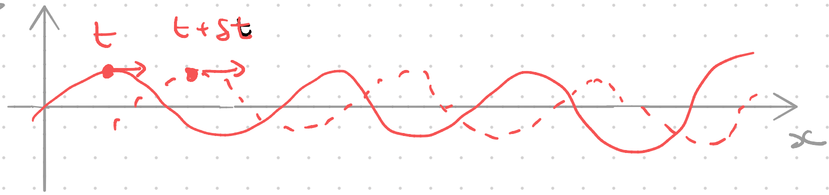
if add ∞ # of waves together with continuously k, you get a wave packet:



* amplitude goes to zero at $x = \pm\infty$

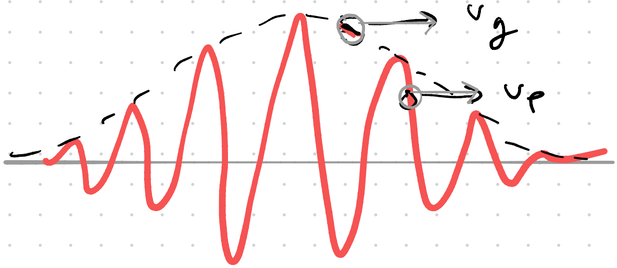
in this case $\int \psi^* \psi \, dx = 1$ is normalizable !

Consider: ψ



track ψ_{crest} = constant

$\Rightarrow kx - \omega t = \text{constant} \quad \therefore \quad \boxed{\frac{dx}{dt} = \frac{\omega}{k}}$ phase velocity



$$= \frac{\hbar\omega}{\hbar k}$$

$$= E/p$$

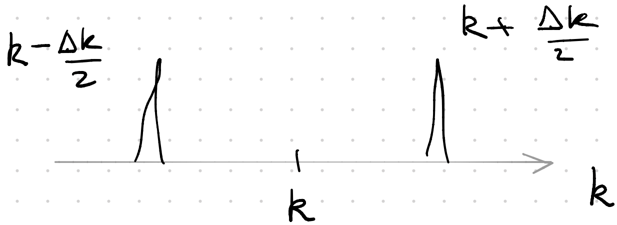
$$= \frac{\frac{1}{2} m v_g^2}{m v_g}$$

$$= \frac{1}{2} v_g$$

* $v_g \equiv$ group velocity (classical velocity)

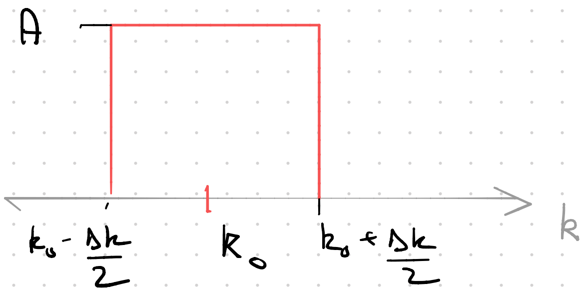
reconsider adding two waves:

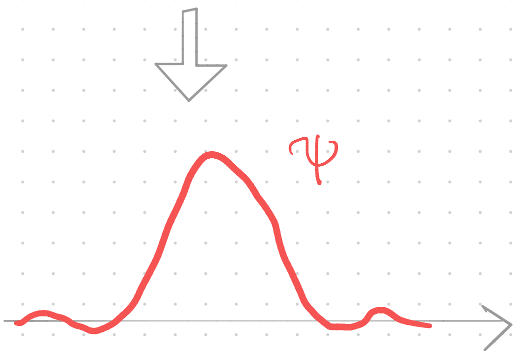
$$e^{i\left((k+\Delta k/2)x-(\omega+\Delta\omega/2)t\right)}+e^{i\left((k-\frac{\Delta k}{2})-(\omega-\frac{\Delta\omega}{2})t\right)}$$



& for continuous distribution of k:

$$A(k)=A \quad \text{for} \quad k_0-\left(\frac{\Delta k}{2}\right)<k<k_0+\left(\frac{\Delta k}{2}\right)$$

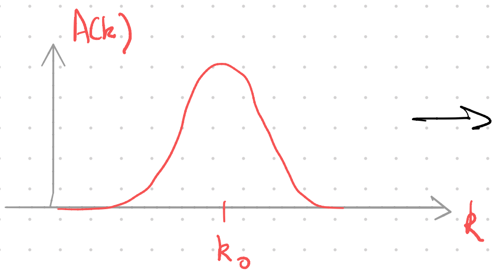




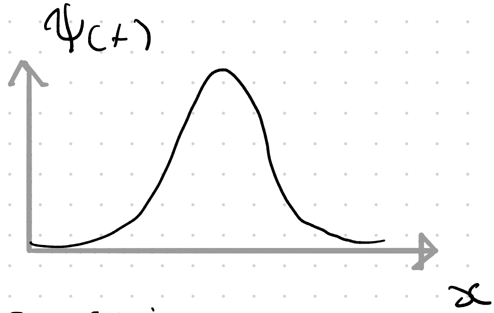
In-class 7-1, 7-2

Next :

Consider :



$\longrightarrow$



gaussian distribution.

$$\Psi \sim \int A(k)\, e^{i(kx-\omega t)}\, dx$$

$\boxed{\text{In-class } 7.3, 7.4}$