

ECON4570/6560 : Problem Set 1

Due by 11:59pm on Sep 13, 2024 <sup>1</sup>

1. Below is the joint probability table for the discrete random variables  $X$  and  $Y$ . Suppose that the conditional probability  $P(X = 1|Y = 10) = 0.5$ .

	$Y = 6$	$Y = 8$	$Y = 10$	
$X = 1$	$A$	$B$	$C$	$G$
$X = 2$	0	$D$	$E$	0.2
$X = 3$	0.2	$F$	0.2	$I$
	0.4	$H$	0.4	1

- Compute the values for  $A, B, C, D, E, F, G, H$  and  $I$ ? Are  $X$  and  $Y$  independent? Why or why not?
  - Compute mean  $\mu_X (= E(X))$  and variance  $\sigma_X^2$  of  $X$ ?
  - Compute conditional mean of  $X$  given  $Y = 6, 8, 10$ , that is, compute  $E(X|Y = 6), E(X|Y = 8), E(X|Y = 10)$ . Are  $X$  and  $Y$  mean independent?
  - Compute the covariance between  $X$  and  $Y$ ? Are  $X$  and  $Y$  correlated?
2. You are having guests over for a mussel feast. In the morning you are at Joes Not-so-Fresh Fish Market trying to decide how many mussels to buy. From experience you know that about one out of every ten will not open (dead and hence not fresh) when cooked and must be thrown away. If you buy 60 mussels, what is the approximate probability that your feast will consist of less than 6 mussels?
3. There are  $N$  children in total in a small town. Someone wants to know what fraction or proportion  $p$  of the children have cavities. They take a random sample of  $n$  children.  $n_1$  out of  $n$  children had cavities.
- Is the estimator  $\frac{n_1}{n}$  unbiased for  $p$ ? is it also consistent? please explain.
  - Assume  $N = 200, n = 80, n_1 = 50$ , and suppose that, in fact, 150 children in the town have cavities. Find the probability that more than 55 children have cavities in a random sample of 80 children.
  - Assume  $N = 200, n = 80, n_1 = 50$ , obtain a 90% confidence interval for the proportion of children in the town with cavities.
  - Assume  $N = 200, n = 80, n_1 = 50$ , and suppose that you want to test the null hypothesis that the proportion of children in the town with cavities is 0.75 against the alternative hypothesis that it is not

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<sup>1</sup>Please submit your problem set via the link at LMS.

equal to 0.75. Using critical value, do you reject the test at 10% significance level? Next compute the p-value for this test, this time do you get the same test result at 10% significance level?