Analog Computer Procedure

Objective: Solve differential equations using circuits.

The circuits you build will consist of the following building blocks: operational amplifiers (op-amps), resistors, and capacitors. From these three elements, you will be able to build inverting amplifiers, differentiating and integrating circuits, and adding (summing) circuits. Be sure to document your measured resistor and capacitor values, you will need these for your analysis.

You will be using a breadboard (Figure 0), which has conducting strips that allow for a consistent voltage to run vertically along neighboring holes. The columns are not attached across the white strip in the middle. The top two and bottom two rows are attached horizontally, but not vertically. This lab is meant to grow your skills in assembling circuits. Circuits can be tricky to debug. In order to help the TAs help you, keep your circuits as organized as possible. You will be graded on the clarity of the wiring of your analog computers.
Power the breadboard in a clear, understandable way, using the rechargeable 9V batteries. The batteries need to be connected to a custom connector where the red lead is for positive DC Voltage, black for negative DC voltage, and green for ground. Use the same color jumper wires to power the op-amps as in the example below (Figure 0) for clarity. Use consistent color coding: in our example, red (+ve) and black (-ve) are used to power op-amps, green for grounding components, white or yellow will be used for the actual signal that is being observed. Avoid adding “risky components” such as exposed wires that could touch each other to your circuit. These are sources of error that will significantly increase the time it takes you to narrow down why the circuit isn’t working. If your signal is unstable, the first places to check are your ground leads. Make sure these connections are tight, and that all grounded components go to the common ground. You will be held accountable for putting away all circuit components once you have completed the lab at the end of your rotation. Prior to this, you can store your breadboard with your name/section # in one of the cabinets. Points will be taken off if the components are not removed at the end of your rotation.. Account for the time this will take at the end.

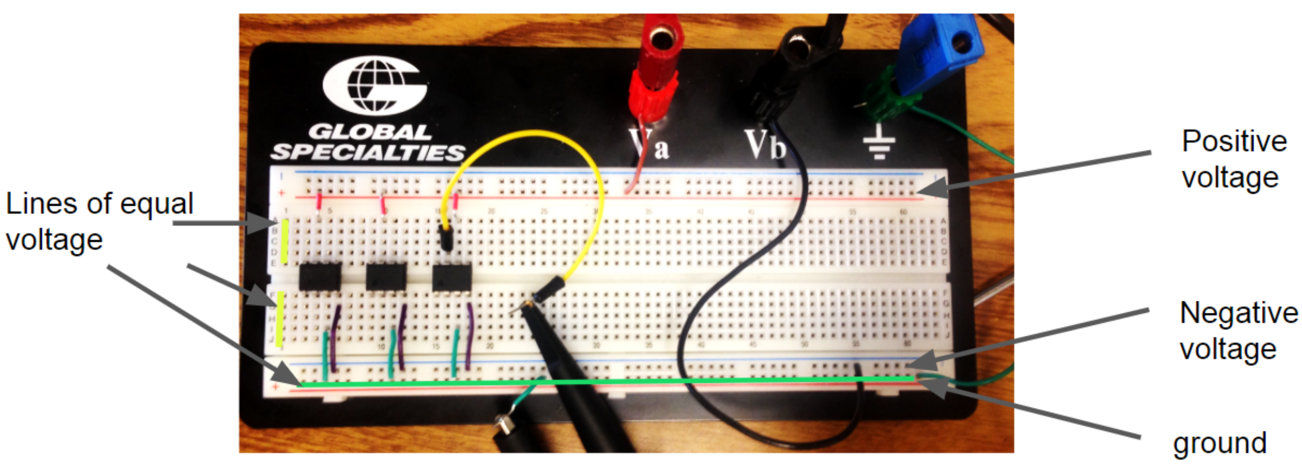

Figure 0: Circuit Guidelines with op-amps

Equipment:
- Breadboard
- Leads, jumper cables, op-amps, resistors, and capacitors
- 2 function generators
- 2 Rechargeable 9V batteries and associated connector
- Oscilloscope
- USB flash-drive

Section I: Getting familiar with the components

In this section, you will build an inverting, an integrating, a differentiating, and an adder circuit. Refer to the theoretical section for an explanation of how the formulas for the components were derived. You are expected to test each components with different input signals from function generators, to save the output from the oscilloscope, and to compare the scope traces with your expectations.
- Using the same breadboard for each component, place the op-amps next to each other straddling the center horizontal line of the breadboard. Following Figures 2, 3, and 4, wire an inverting, integrating, and differentiating circuit.
    - The pinout diagram of the op-amps used in this lab is shown in Figure 1. Be mindful of the orientation of the op-amp which is determined by the circle in the top left corner.

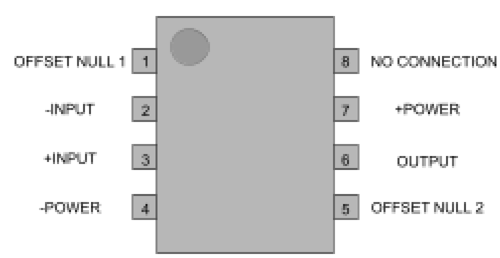

Figure 1: Pinout diagram of the TLE2071CP operational amplifier

- If the op-amp part # differs from TLE2071CP, look for the relevant part on the internet and ensure the correct connections are made. There might be slight differences between models of op-amps

Section I.1: Inverting Amplifier

Figure 2 shows an inverting circuit configured using two resistors on an op-amp. Refer to the notes from the Electronics Lab documentation to understand how this circuit works. The gain of the circuit is from the ratio *R_f/R_in* where *R_f* is the feedback resistor and *R_in* is the input resistor. Note that the signal is inverted at the output so the final gain is - *R_f/R_in*. Using resistors between 1kΩ - 5kΩ, come up with 4 combinations where two of them result in a reduction in output signal and the other two result in a gain in signal. Make sure the resulting output does not come close to the supply voltage of +/- 9V (from the rechargeable batteries). As input signals use (i) DC voltage (use a battery and voltage divider to generate this) and (ii) Sinusoidal waveform (choose one frequency between 1kHz -5kHz). Capture waveforms to show signal inversion (*Note: Both scope channels should be used - one for the input signal, V_in, and the second channel for the output signal, V_out)*

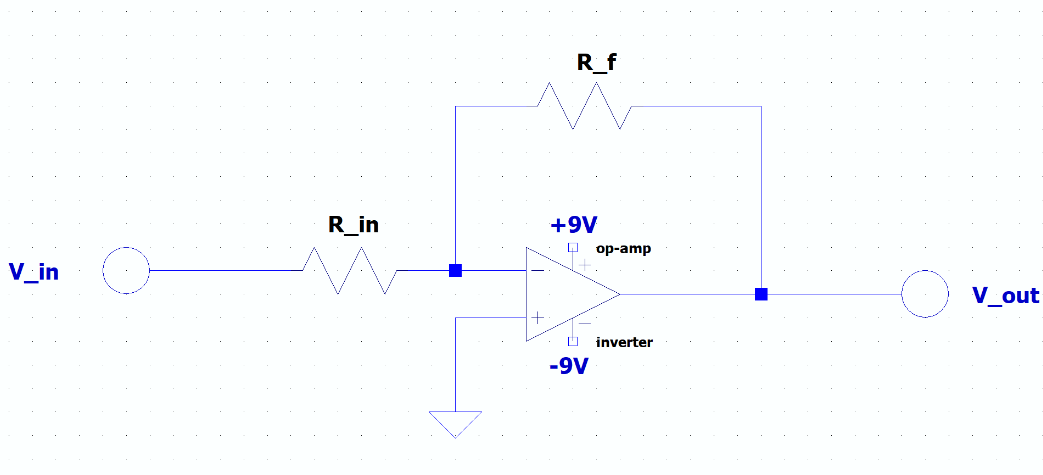

Figure 2: Inverting circuit

Section I.2: Integrating Amplifier

The output voltage for an integrating circuit is $V\_out = V_o(0) - \frac{1}{RC}\int V\_in\,dt$ where $V_0(0)$ is an initial condition (this term can be ignored for systems which are continuously integrating). All integration (and differentiation) is with respect to time, because the electrical properties of our circuit are time dependent. **Pay close attention to the change of sign after the integration.** Figure 3 shows how an op-amp is configured to perform integration. To show circuit performance, choose either a capacitor of 0.01 or 0.1μF and a resistor of either 0.1kΩ, 1kΩ or 10kΩ. Compute the expected time constant (RC) for the chosen combination. Demonstrate the operation of the integrator by using the following signals: (i) Square wave (+/-1V at 1kHz) and (ii) Triangular wave (+/- 1V at 1kHz). Capture waveforms on both channels (input & output) to show that your circuit is functioning properly.

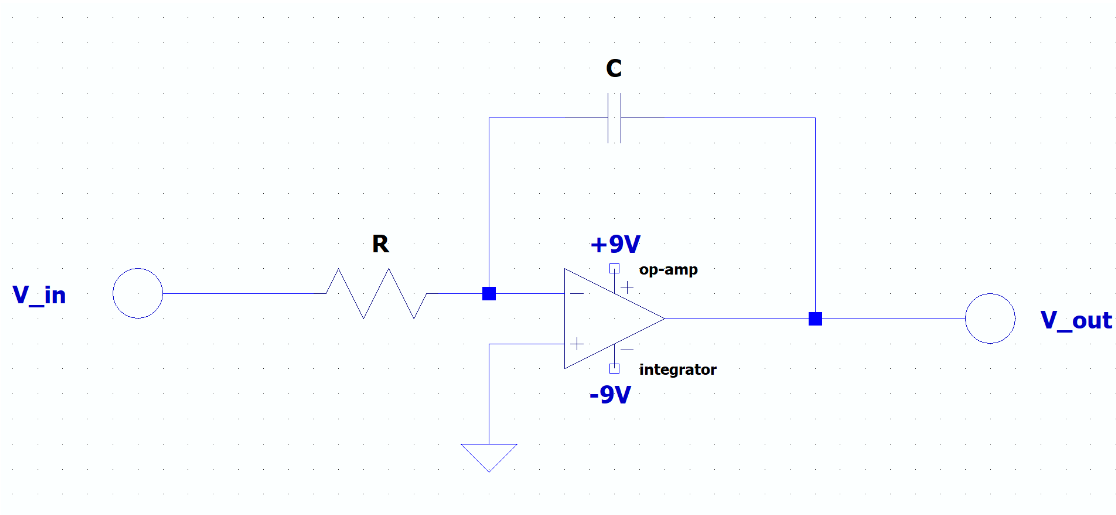

Figure 3: Integrating circuit

Section I.3: Differentiating Amplifier

The output voltage for a differentiating circuit is $V\_out(t) = V_0(0) - RC\frac{dV\_in}{dt}$ where $V_0(0)$ is an initial condition (this term can be ignored when the system is continuously differentiating). **Pay close attention to the change of sign after the differentiation.**

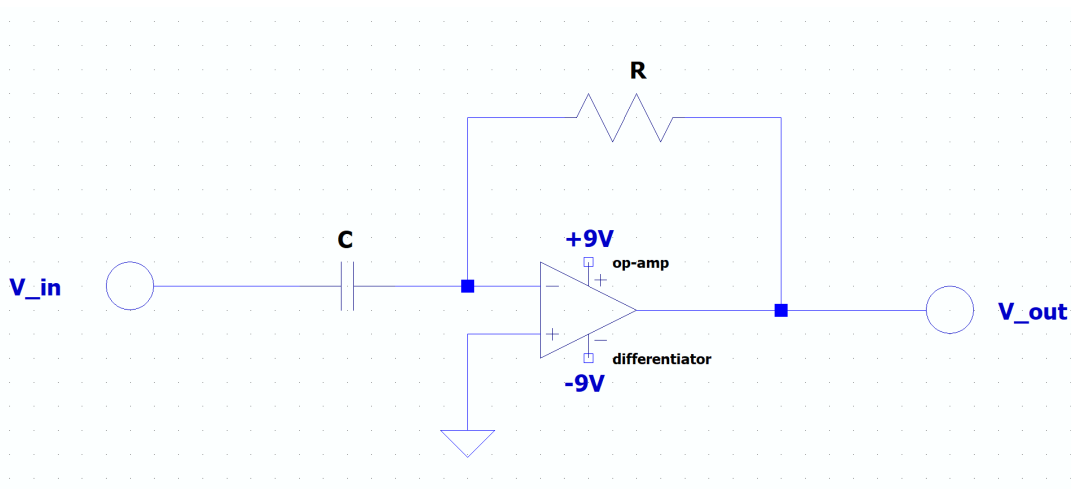

Figure 4: Differentiating circuit

To show circuit performance, choose either a capacitor of 0.01 or 0.1μF and a resistor of either 0.1kΩ, 1kΩ or 10kΩ value. Compute the expected time constant (RC) for the chosen combination. Demonstrate the operation of the differentiator by using the following signals: (i) Square wave (+/-1V at 1kHz) and (ii) Triangular wave (+/- 1V at 1kHz). Capture waveforms on both channels (input & output) to show that your circuit is functioning properly.

Section I.4: Summing amplifier

Change your inverting circuit into a summing circuit following Figure 5 by adding a second resistor (*R_2*) to the feedback resistor (*R_f*). Note that in Figure 5, *R_in* (from Figure 2) is now designated as *R_1*. Use resistors between 1kΩ - 5kΩ.

- Figure 5 shows addition of two voltages. The circuit can easily expanded to add more signals by having more resistors in parallel with *R_1* and *R_2*.
- The voltage in the addition (or summing) circuit is given by

$$-\frac{R\_f}{R\_1}\cdot V\_1-\frac{R\_f}{R\_2}\cdot V\_2\cdots-\frac{R\_f}{R\_N}\cdot V\_N=V\_out$$

- For simplicity choose *R_1=R_2=2xR_f*, so that the gain for each input signal is 0.5. Use the outputs from both function generators to generate the signals for *V_1* and *V_2*. Keep the magnitude of the peak values below 1V. Choose a sinusoidal signal for *V_1* (at about 10kHz) and a square wave for *V_2* (at about 500Hz).

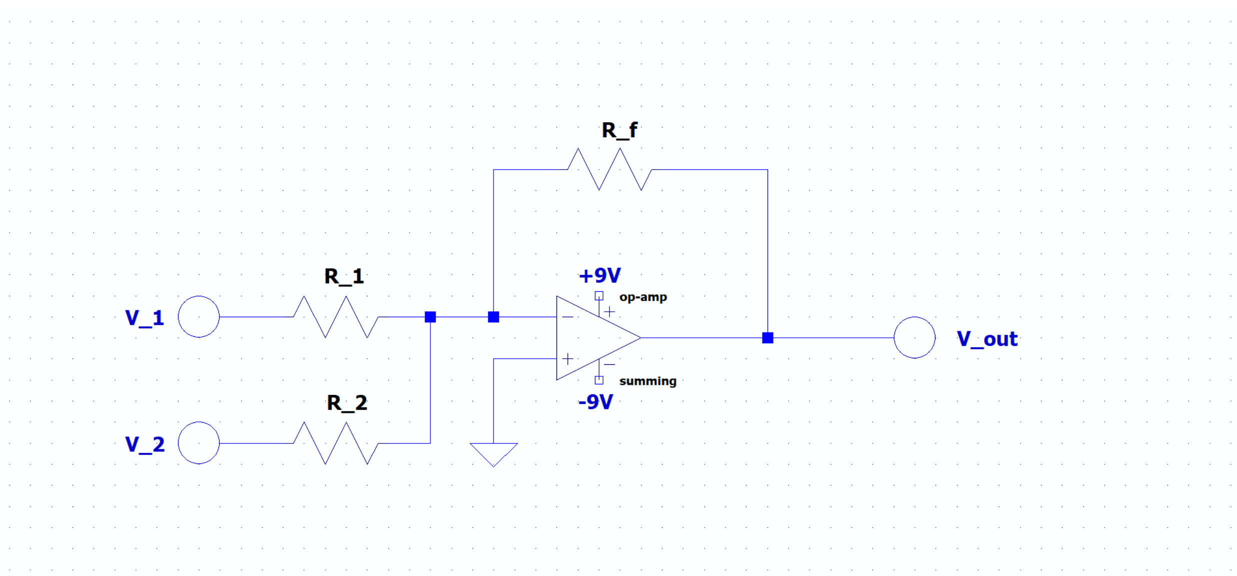

Figure 5: Summing circuit

- Capture the waveforms of the input (the slower square wave) and output signals (the final added waveforms) to verify that your adder is working properly.

- Change the value of *R_1=R_F* and verify that the new output signal is consistent with the change by capturing the relevant waveforms.
- Don't quite take everything apart, as you might be able to reuse components for the next section.

Section II: Solving simple differential equations - the Simple Harmonic Oscillator

As we saw in Section I, we can use a multitude of circuit elements to modify a signal from a function generator. Using what you built in Section I, you will solve a simple harmonic oscillator. A simple harmonic oscillator is modeled by some force $F = - kx$ where $F = m\ddot{x}$ ($\ddot{x}$ is the second derivative with respect to time of x, *k* is the spring constant and *m* is the mass attached to the spring). In order for us to be able to look at the signal, we add a driving function *f(t)* that will be used as a trigger on the oscilloscope as well as to drive the otherwise dampened signal from unavoidable energy losses in the circuit. Therefore, we will be solving the equation of motion for the simple harmonic oscillator with a source term:
$\ddot{x} = -\frac{k}{m}x + f(t)$.

- To set up the circuit, think about $\ddot{x}$ as the sum of its components, $-\frac{k}{m}x$ and $f(t)$. We know $f(t)$ is the signal from the function generator, but how do we generate *x* ? Hint: If $\ddot{x}$ is the second time derivative of x, then x must be the second integration of $\ddot{x}$ with respect to time.
- Note on the values of the resistors on the integrators: the frequency of the oscillator is linked to these values and the gain of the adder circuit. Use the same combination of R & C for both integrators to simplify the analysis.When using 0.01μF capacitors, use a resistance of about 1kΩ. If you decide to use a 0.1μF, use a resistance of about 0.1kΩ. This will give you a range of integration up to about 35 kHz which is quite broad for this analog computer. In addition to this combination of R and C, and this is optional, we suggest you include a resistor that you will connect in parallel with the capacitor to stabilize the gain at low frequencies. We suggest a value of 100kΩ. Use this for all the integrators.
- Before building your circuit for the simple harmonic oscillator, sketch your circuit and have it reviewed by either the Instructor or the TA. (*Note: Make sure the gain of the summing amplifier when combined with the input voltages generates signals that are lower than +/-5V to prevent saturation of the op-amp. Recall the voltages the supply power to the op-amp are only +/-9V.*)
- Setup your circuit, paying close attention to the signs of the addends of $\ddot{x}$. Hint: You will need 3 op-amps: one for summing, two for integrating. (*Note: The summing amplifier will invert the signal so that one ends up with a signal proportional to -f(x). This does not matter as the input signal, f(x), is symmetrical and thus the output will have a phase change but will still look very similar to f(x) and can still be used as a trigger and an additional driver to sustain the oscillations.*)
- Include a drawing of your circuit in your lab notebook. Include the values of capacitance and resistance measured with the multimeter.

- Review the Analog Computer Theory document and understand how the summing circuit plays a critical role in defining the "*k/m*" constant and hence the oscillating frequency. Try different gains for "*x(t)*" from the summing circuit and see if you can get sustained oscillations without the use of *f(t)*.
- Test the behavior of your circuit where $f(t)$ is a low frequency square pulse (50-300Hz), making sure to trigger on the edge of the square pulse. Think about the general solution to a simple harmonic oscillator.
    - Display both $f(t)$ and $x(t)$ on the oscilloscope and save the data.
- Use the "measure" function on the oscilloscope to record an estimation of the amplitude and frequency of your signal. These will be used as initial guesses in the analysis to get a better curve fit.
- Because the circuit could have a certain amount of energy losses, there is a natural decay of amplitude in the oscillations. You will be expected to fit a curve to your scope data to model this natural decay. If you are successful in generating a signal with no decay, then you will not need to perform this step.

Section III: Damped or driven harmonic oscillator

- Modify your circuit from Section II to include a forcing or damping term that is dependent on $\dot{x}$ where the updated equation of motion is $\ddot{x} = \frac{\gamma}{m}\dot{x} - \frac{k}{m}x + f(t)$. The first order term is responsible for forcing or damping. The sign of γ determines whether the oscillator is driven or damped. For this experiment, we will simulate a damped oscillator which requires $\gamma < 0$. Sketch the circuit and show it to the Instructor or TA for verification. (*Hint*: *You will need an inverter amplifier connected to the summing amplifier to generate the damped oscillation. For the inverter, choose a combination of resistors such that the gain is between 0.3-0.7*).
- Note on resistor values: the input resistor choice for the damping term on the addition op-amp should be weighting the addition such that the damping isn't too strong. You don't want to damp your signal instantly since you want to be able to watch the signal decay.
- Hint: account for the natural damping in your expectations of the forced harmonic oscillator. **It is recommended that before you make the final connection to complete the forced damping effect, run the circuit again and check to see if the circuit generates the same signal as in Section III. If the signal has changed from the perspective of the natural damping term, you will need to compare the effect of the forced damping term to this new setup as additional unaccounted dissipative terms due to the additional components have contributed to the change in natural damping**.
- Include a drawing of your updated circuit in your lab notebook with the resistor values.
- Display both the driving force $f(t)$, still a square wave, and $x(t)$ on the oscilloscope and save the data.

Section IV : Nonhomogeneous boundary beats

- Take out the damping term (proportional to $\frac{\gamma}{m}\dot{x}$) from Section III.
- Determine the frequency of your simple harmonic oscillator using the scope. It should not have changed from what you measured in Section II.
- Connect a second function generator to your circuit by separately adding it to $f(t)$ using your summing op-amp. This will now be a third input to the op-amp. Choose a reasonable gain for this input signal, using a range between 0.4-0.8.
- Produce a sine function with the new function generator. Minimize the amplitude of the signal by having the "output" knob on its lowest possible setting.
- The differential equation you're solving now is $\ddot{x} + k/m\,x = sin(\omega_1 t) + f(t)$ where ω_1 is the frequency of the sine term you're adding.
- Adjust the frequency of the nonhomogeneous term ω_1 until you produce a signal that yields an observable beat frequency. Hint: this frequency should be close to the frequency of the harmonic oscillator ω_0.
- How well this signal can be displayed on the oscilloscope now depends on three frequencies: the harmonic oscillator frequency, the nonhomogeneous sine frequency, and the frequency of the slow square pulse that the oscilloscope is triggered on. The square pulse is useful to look at the beginning part of your signal, but it's frequency basically determines the "frame rate" of the oscilloscope and doesn't necessarily match the beat frequency of ω_0 and ω_1 , which is why your signal can look like it's moving. You can use the run/stop button in the upper right corner of the scope to eliminate that factor by just looking at one pulse at a time.
- Save the oscilloscope data on the flash drive and record changes you made to your circuit from the previous parts.

To include in the Report:

Section I:

Provide labelled scope traces of each of the circuit configurations described in the Analog Computer Instructions document. The labels should briefly describe the particular configuration and the resistor/capacitor combinations used for the measurement. The Op-Amp configurations are: Inverting , Integrating, Differentiating and Summing amplifiers.

Section II:

- Show a circuit diagram of your simple harmonic oscillator with numerical values
- Use the formulas of integrating and adding circuits to numerically describe your circuit: what is the value of the factor *k/m*?

- Fit a sine curve to your simple harmonic oscillator. Determine the frequency of the oscillation and the damping constant.
- Compare the frequency of the circuit with the mathematically expected frequency according to your computed value of *k/m*.

Section III:

- Determine the damping constants of both oscillators and their ratio from the stored waveforms. Apply a correction to this, taking into consideration the natural damping of the circuit
- Compare the ratio of the constants to what you would expect based on the gain of the circuit, that is determined by the choice of components that drive the damping term in the circuit.

Section IV:

- Show the scope trace of the beat pattern. If necessary, you can take a video of the beat pattern and include that with your report.
- Provide the general solution of the mathematical problem. You do not need to determine any numerical solutions.
- Based on the general solution, explain what beat frequency you would expect and why. Base this expectation on the frequency of the sine wave from the function generator and on the frequency from part II.
- Compare the calculated beat frequency to your scope trace.