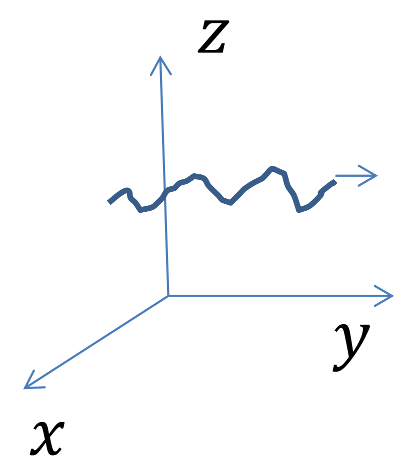
Inclass 19.1. (a) Show that in the 3D free particle problem in the Cartesian coordinate system, separation of variables can be achieved by writing the wavefunction as $\psi(x, y, z) = \phi_x(x)\phi_y(y)\phi_z(z)$ where ϕ_x, ϕ_y, ϕ_z are eigenfunctions of the energy operators $\frac{\hat{p}_x{}^2}{2m}, \frac{\hat{p}_y{}^2}{2m}, \frac{\hat{p}_z{}^2}{2m}$, respectively, with eigenenergies $E_{x,}$ $E_{y,}$ E_z.

(b) Determine $\psi(x, y, z)$.



Inclass 19.2. In the spherical coordinate system, the SE of the central field potential (V(r)) problem can be written as $\left(\frac{\hat{p}_r{}^2}{2m} + \frac{\hat{L}^2}{2mr^2}\right)\psi(r,\theta,\phi) = (E - V(r))\psi(r,\theta,\phi)$.

Show that separation of variables can be achieved by assuming $\psi(r,\theta,\phi) = R(r)Y(\theta,\phi)$. [Hint: multiply the equation by $\frac{2mr^2}{\hbar^2}$.]

Inclass 19.3. Show that the separation of variables for $\hat{L}^2 Y(\theta,\phi) = \lambda\hbar^2 Y(\theta,\phi)$ is possible if we assume $Y(\theta,\phi) = \Theta(\theta)\Phi(\phi)$.

Recall: $\hat{L}^2 = -\hbar^2[\frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\,\frac{\partial}{\partial\theta}\right) + \frac{1}{sin^2\theta}\frac{\partial^2}{\partial\phi^2}]$. Multiply the equation by $sin^2\theta$.

Inclass 19.4. Solve the eigenvalue problem: $\frac{\partial^2 \Phi}{\partial \phi^2} = -m^2 \Phi$

(Hint: the eigenfunction should be the same at ϕ and $\phi+2\pi$.)