Lecture 10: Magnetic forces and fields

In the early 1800s, "electricity" meant electrostatics; "galvanism" referred to the effects produced by continuous currents from batteries; and "magnetism" dealt with the field of lodestones, compass needles, and the terrestrial magnetic field. While it seemed clear to some scientists that there must be a relationship between galvanic currents and electric charge, electricity and magnetism appeared to have nothing to do with one another. Nevertheless, as Hans Christian Oersted was lecturing on these topics to advanced students at the university of Copenhagen in the winter of 1819-1820, he tried the experiment of passing current through a wire above, and at right angles to a compass needle. It had no effect. But after the lecture, he tried the experiment again with a wire running parallel to the compass needle: the needle swung wide. When the current was reversed, it swung the other way!

This led Ampère, Faraday, and others to work out a description of the magnetic action of electric currents culminating in Maxwell's formulation in the early 1860s

Currents exert forces on each other that cannot be explained by electrostatics: parallel currents attract each other and opposite currents repel. This is the magnetic force, associated with the magnetic field.

This force is empirically determined for a point charge to be:

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

SI units: Tesla - T

T = N.s/(C.m)

with $q \equiv$ charge of particle (including sign)

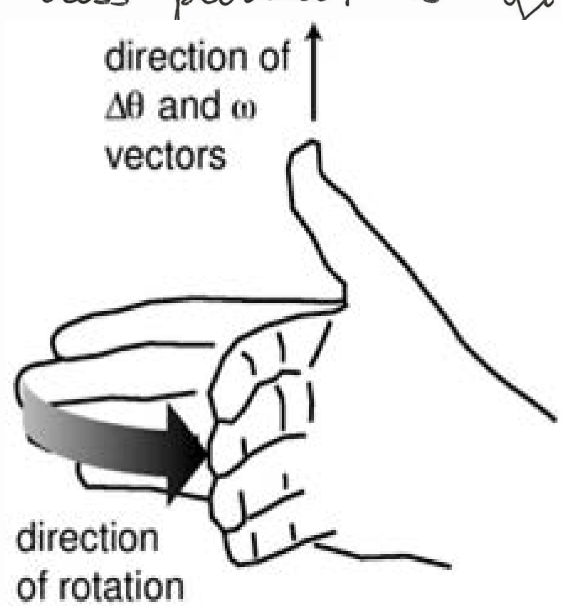
$\vec{v} \equiv$ velocity (including direction)

$\vec{B} \equiv$ magnetic field

The direction of the vector cross product is given by the right hand rule.

$\vec{c} = \vec{a} \times \vec{b}$; $|\vec{c}| = |\vec{a}||\vec{b}|\sin(\phi)$

The direction of $\vec{c}$ is at a right angle to the plane formed by $\vec{a}$ and $\vec{b}$. (3D thinking required!)



In terms of vector components, the vector product $\vec{c} = \vec{a} \times \vec{b}$ is given by:

$$\begin{cases} c_x = a_y b_z - a_z b_y \\ c_y = a_z b_x - a_x b_z \\ c_z = a_x b_y - a_y b_x \end{cases} \quad \text{where} \quad \begin{cases} \vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \\ \vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k} \\ \vec{c} = c_x \hat{i} + c_y \hat{j} + c_z \hat{k} \end{cases}$$

If you are familiar with the use of determinants, you can use the expression:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Think 10.1: A charged particle is initially moving with velocity v_x in a magnetic field pointing in the y direction. What is the subsequent motion of the particle?

A) It slows down

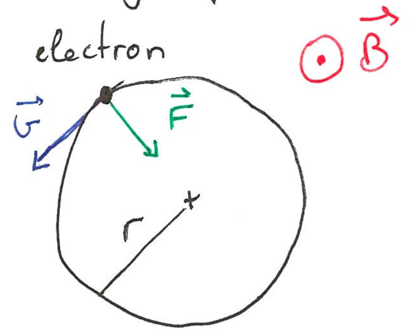
B) It moves at constant speed in a circle in the x - z plane

C) It curves at constant speed until it travels along the field

D) It just travels in a straight line at cst speed.

Motion of a charged particle in a uniform field.

If the speed remains constant, $|\vec{F}| = |q\vec{v} \times \vec{B}|$ also remains constant. Since $|\vec{F}|$ is constant and always perpendicular to $\vec{v}$, the motion is circular.



<u>Note</u>: the electrostatic force can do work: $W_e = -\int_a^b \vec{F} . d\vec{l} = q\Delta V$ but the magnetic force <u>cannot</u> because it is always perpendicular to the motion of the charges:

$$W_m = -\int_a^b \vec{F} . d\vec{l} = -q\int_a^b (\vec{v} \times \vec{B}) \cdot \vec{v}\, dt = 0$$

The orbital radius can easily be found:

$$ma = m\frac{v^2}{r} \text{ for a circular orbit}$$

we know $F_B = |q| v B$, therefore, $r = \frac{mv}{|q| B}$

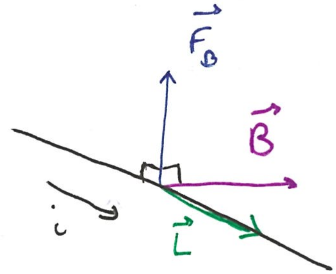
If now we have a large number of charges drifting in the same direction (i.e., a current), each charge experiences a magnetic force. Since charges can't leave the side of the wire, they transfer the force to it

$$\vec{F}_B = -Ne\vec{v}_d \times \vec{B}$$

drift velocity

number of electrons

charge



The number of electrons can be written as $N = nAL$ where n is the electron density. Substituting $-nALe\vec{v}_d$ as $i\vec{L}$, we get:

$$\boxed{\vec{F}_B = i\vec{L} \times \vec{B}}$$ for a straight wire in cst field

For any wire in a nonuniform field,

$$\boxed{\vec{F}_B = \int i\,d\vec{l} \times \vec{B}}$$

Example: A long rigid conductor, lying along the direction $2\hat{i} + 2\hat{j}$, carrying a current i passes through a region of field $\vec{B} = (3\,mT)\hat{i} + x(5\,mT/m)\hat{j}$. Calculate the force on the 1 m segment that lies between $x = 0$ and $x = 0.7$ m.

$$\vec{F} = \int i\,d\vec{l} \times \vec{B} = \int i\,(dx\,\hat{i} + dy\,\hat{j}) \times (3\hat{i} + 5x\hat{j})$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & dy & 0 \\ 3 & 5x & 0 \end{vmatrix} = \hat{k}(5x\,dx - 3dy) \quad \text{and} \quad dx = dy$$

$$\vec{F} = \hat{k}\int_0^{0.7} (5x-3)\,dx$$

Think 10.2: A rectangular loop of wire carries a current I in a plane perpendicular to a magnetic field. What is the net force on the whole wire loop? ($\vec{B}$ into the page)

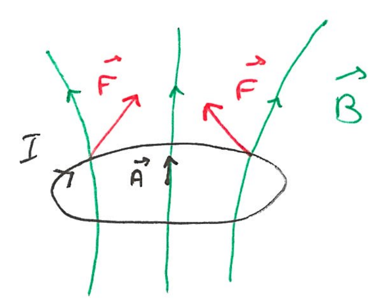
A) zero
D) Into the page
B) Up
E) Out of the page
C) Down
F) left
G) Right

Force on a loop of current.

If the field is uniform then the net force on the entire loop is zero.
If the field diverges then there is a net force along the direction of divergence.

$$\boxed{\vec{F} = \vec{\nabla}(\vec{\mu}\cdot\vec{B})} \quad \text{with } \vec{\mu} = I\vec{A}$$

magnetic moment



A uniform magnetic field will exert a torque on a coil that has N loops and carries a current i. Defining $\vec{\mu}$ as the vector associated with the coil we have:

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}} \qquad \vec{\mu} = Ni\vec{A}$$

- The magnitude of the magnetic dipole moment is $\mu = NiA$
- Its direction is perpendicular to the plane of the coil as given by the right-hand rule.
- The potential energy of the coil is

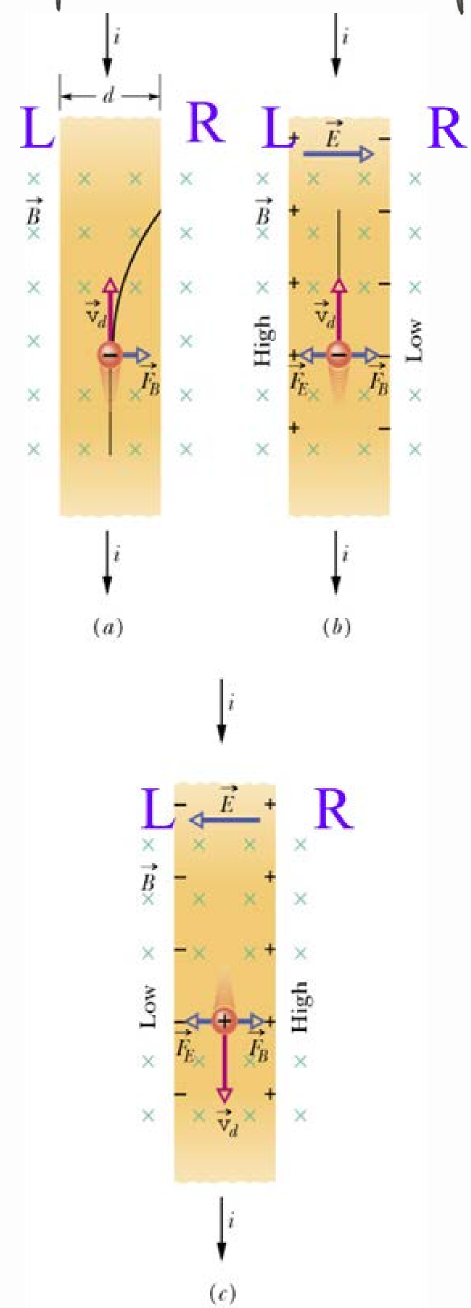
$$\boxed{U = -\vec{\mu} \cdot \vec{B}}$$

- $U = -\mu B \cos\theta$ has a minimum value of $-\mu B$ for $\theta = 0$ (position of <u>stable</u> equilibrium)
- U has a maximum value of $+\mu B$ for $\theta = 180°$ (position of <u>unstable</u> equilibrium)
- The torque is in a direction to align the dipole with the field.

The Hall effect

Consider electrons flowing to give current i and drift velocity $\vec{v}_d$ in a strip of conductor in a $\vec{B}$-field.

A magnetic force will act on the electrons, pushing them to one side of the strip. Electrons will collect on that side until an electric field builds up to oppose the collection



$$\vec{F}_{mag} + \vec{F}_e = 0$$

$$= q\vec{v}_d \times \vec{B} + q\vec{E}_H$$

$$E_H = v_d B \Rightarrow \Delta V_H = w E_H$$

and, using $v_d = \dfrac{J}{ne}$:

$$\boxed{n = \frac{iB}{et V_H}}$$

current

$\leftarrow$ magnetic field

charge density

$\leftarrow$ potential difference

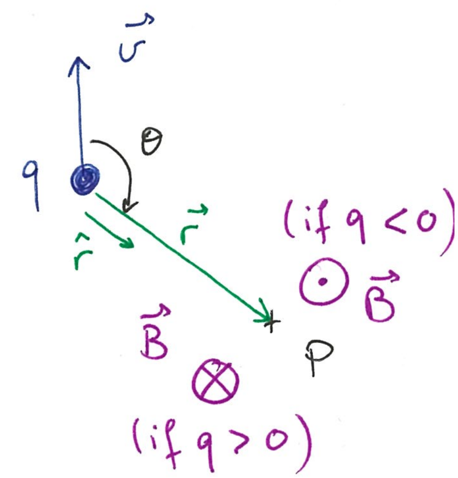
thickness

electron charge

Just like electrostatic descends from Coulomb's law, magnetostatics derives from the empirical law of Biot-Savart.

For a uniform (constant velocity) motion of a point charge, the field is given by:

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}}$$



- $\mu_0 = 4\pi \times 10^{-7}\ \frac{Tm}{A}$ is the "permeability of free space" (a.k.a. the magnetic constant).
- q = charge (sign included)
- $\vec{v}$ = velocity of point charge
- $\vec{r}$ = position vector of field point P, relative to the location of the charge. Points from q to P.
- $\hat{r}$ = unit vector along $\vec{r}$.

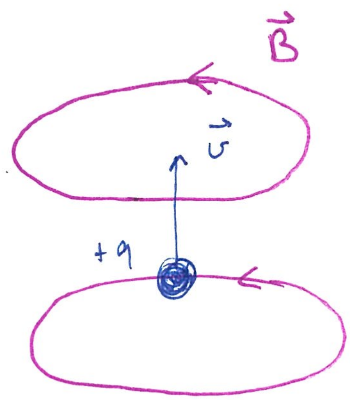
$$B = \frac{\mu_0}{4\pi}\frac{|q|\,v|\sin\theta|}{r^2}$$ with θ angle between $\vec{v}$ and $\vec{r}$.

Biot-Savart Law vs. Coulomb's law

Biot-Savart

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} \; ; \; \vec{F} = q\vec{v} \times \vec{B}$$

- Field dependent on charge moving.
- Field perpendicular to velocity of charge and line between charge location & point of observation.
- Field lines form closed circles.



Coulomb

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}\hat{r} \; ; \; \vec{F} = q\vec{E}$$

- Field independent of charge motion. Only needs charge.
- Field along line between charge and point of observat
- Field lines radially outward for positive charges, inward for negative charges.