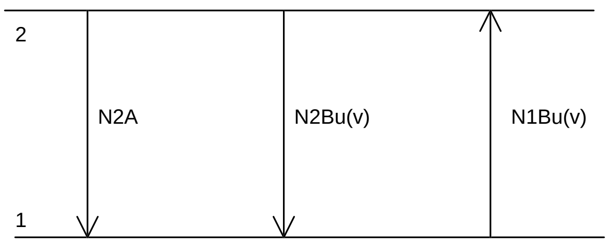
Laser Amplifiers!

Absorption and stimulated emission on a homogeneously broadened atomic transition

N_u, N_l = number of atoms in upper and lower states in a unit volume of material



$$\frac{dN_2}{dt} = -N_2 A - N_2 B \frac{I}{c} + N_1 B \frac{I}{c}$$

Where I is the intensity of light in the volume (assuming no change in I across the volume)

Considering a collimated beam traveling through a region of thickness dz, we can ignore spontaneous emission.

$$\frac{dI}{dz} = [N_2 - N_1] B \frac{h\nu}{c} I$$

$$I(z) = I_0 e^{[N_2 - N_1] B \frac{h\nu}{c} z} = I_0 e^{gz}$$

If $N_2 < N_1$ then gain coefficient in negative (absorption)

If $N_2 > N_1$ then gain coefficient is positive (gain)

How do we make $N_2 > N_1$?

Let's look at rate equations for different level schemes.

The two level scheme in steady state:

$$0=-\frac{N_2}{\tau_2}-N_2\sigma\phi+N_1\sigma\phi$$

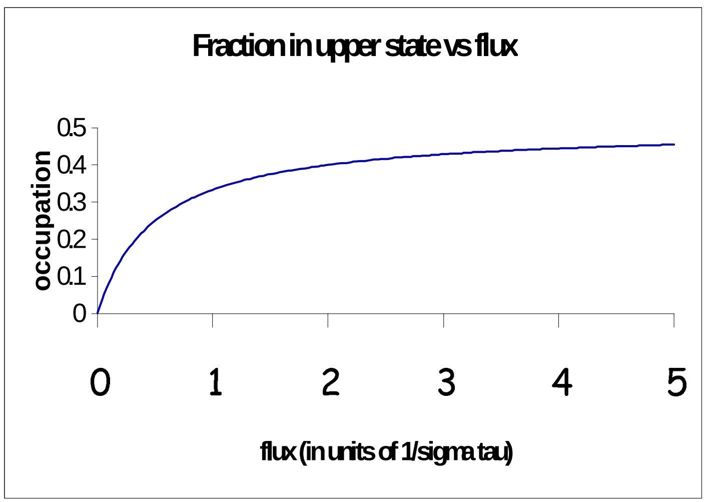
$$N_2=(N_1-N_2)\phi\sigma\tau_2$$

$$\text{and using } N=N_1+N_2$$

$$\frac{N_2}{N}=\frac{1}{2}\left(\frac{1}{1+\frac{1}{2\phi\sigma\tau_2}}\right)$$

or

$$\frac{N_2-N_1}{N}=-\frac{1}{1+2\phi\sigma\tau_2}$$

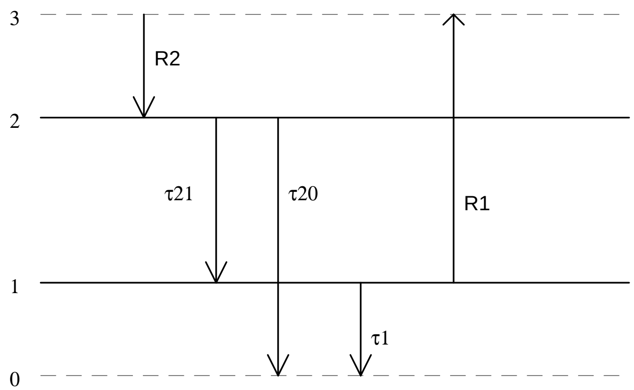


The fraction of atoms in level 2 increases linearly with flux for low flux until the flux approaches $1/\sigma\tau_2$.

The fraction in the upper state saturates at 1/2 for high flux.

<u>With only two levels it is not possible to invert the population by optical pumping</u>.

Rate equations for more than two levels



We consider first the rate equations for two levels (1 and 2) with states above (3) and below (0) that feed them and into which they decay. We do not consider the details of states 0 and 3 yet.

- The external pump processes are R_2, which is the rate at which atoms enter state 2, and R_1, which is the rate at which atoms leave state 1.

Solving in the absence of stimulated emission or absorption:

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2}$$

where

$$\tau_2^{-1} = \tau_{21}^{-1} + \tau_{20}^{-1}$$

and

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}}$$

Under steady-state conditions: $dN_i/dt=0$

Solving for the population difference $\Delta N=N_2-N_1$ between states 1 and 2:

$$\Delta N_{SS0} = R_2\tau_2\left(1 - \frac{\tau_1}{\tau_{21}}\right) + R_1\tau_1$$

Positive values of ΔN_{ss} mean positive gain.

- Large R_1 and R_2
- Long τ_2
- Short τ_1 if $R_1<(\tau_2/\tau_{21})R_2$

- Upper level should be pumped strongly and have a long lifetime in order increase its population.
- Lower level should depump strongly.
- Ideally it is desirable to have:

  $\tau_{21} \cong t_{sp} << \tau_{20}$ so that $\tau_2 \cong t_{sp}$ and $\tau_1 << t_{sp}$

Under these conditions, we have the simplified result:

$\Delta N_{ss0} \cong R_2 t_{sp} + R_1 \tau_1$

Now let's add amplifier radiation to the problem:

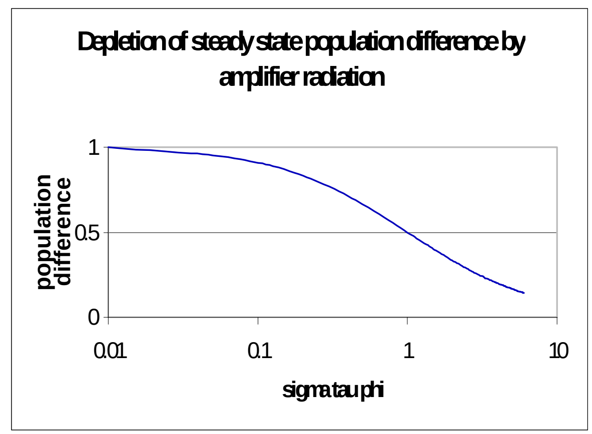
$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - N_2 \phi\sigma + N_1 \phi\sigma$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_2} + N_2 \phi\sigma - N_1 \phi\sigma$$

Under steady state conditions,

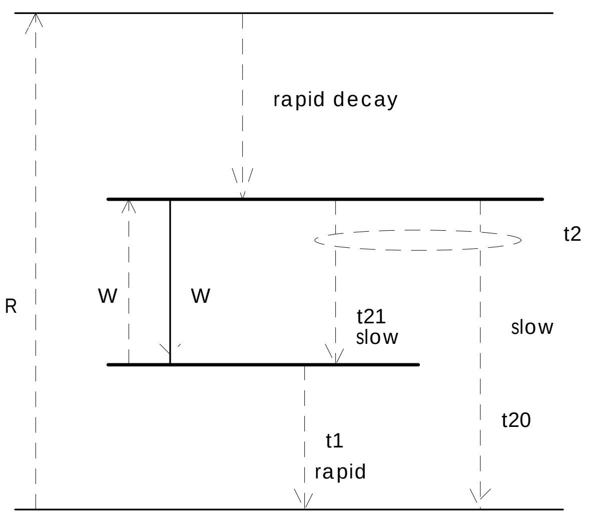
$$\Delta N_{ss} = \frac{\Delta N_{ss0}}{1 + \tau_s \phi\sigma}$$

$$\tau_s = \tau_2 + \tau_1 \left(1 - \frac{\tau_2}{\tau_{21}}\right)$$



Two Special Cases: the 3-level and 4-level systems

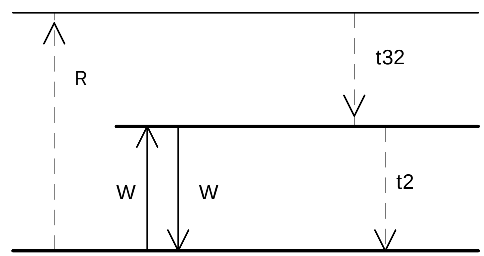
4-level pumping schemes:



$N_0 = R\tau_2\left(1 - \frac{\tau_1}{\tau_{21}}\right)$ gives the steady state population difference in the absence of amplifier radiation.

- It is relatively easy to get inversion in such a system.
- simple for low pump rates - at high rates, we may have to make corrections for saturation of R.

Three level scheme:



Because τ_{32} is fast, N_1 and N_2 can be easily related to the total number of atoms.

$N_1+N_2=N_T$

and solving the rate equations again:

$$N = \frac{N_0}{1+\tau_s W}$$
$$= \frac{2R\tau_{21} - N_T}{1+2\tau_{21}W}$$

In the event that non-radiative losses are small,

then $\tau_{21}=t_{sp}$

in order to get inversion here, requires a pumping

rate:

$$R > \frac{N_T}{2t_{sp}}$$

The large population in the ground state poses an

inherent obstacle to inversion.

Note that the pumping rate must depend on the

degree of inversion because inversion takes

electrons out of level 1.

Pumping techniques:

- electrical gas discharge
- optical pumping
  - flashlamp
  - another laser
- charge injection in a semiconductor laser
- chemical reaction
- energy transfer from another excited system

(see Fig. 13.2-8 in Photonics)

Specific laser amplifier systems:

- Ruby laser
- Nd:YAG
- Er: glass (erbium fiber)
- organic dye
- Ti:Al_2O_3 (ti-sapphire)

Ruby Laser: 3 level amplifier

- λ=694.3 nm
- Cr^{+3} ions replace Al ions in alumina.
- pump electrons from the ground state of the Cr ions to upper levels 2.3 to 3.3 eV above (400 nm to 550 nm)
- normally we pump with a flashlamp but can be pumped by another laser (Ar ion or HeCd)
- τ_{32}~picoseconds; t_{sp}~ 3 ms; phonon broadened ~60 GHz

(see Fig. 5-13 in Silfvast)

Neodymium Yttrium Aluminum Garnet amplifier (Nd:YAG)

- λ=1064 nm
- 4 level laser
- pumping bands at 810, 750, 585, 525 nm
- τ_{32}~100 ns; t_{sp}~ 1.2 ms; τ_1~ 30 ns; phonon broadened ~120 GHz
- pumped by flashlamp or semiconductor laser

see Fig. 5.14 in Silfvast

Er^{+3}: glass fiber amplifier

- λ=1550 nm
- $\Delta\nu$~4000 GHz
- pumped with GaAlAs semiconductor laser (800-1000 nm) or by 1480 nm InGaAsP laser
- acts as a 3-level laser at 300K and as a 4-level laser at 77K.

Linewidth and the Stimulated Emission Rate:

The rate at a particular frequency depends on the population difference, transition or oscillator strength S, the homogeneous linewidth, and the inhomogeneous linewidth.

1. If the homogeneous linewidth is increased then the cross section at the center frequency must be decreased inversely.

$$\sigma_{ul}^{H}(\nu_0) = \frac{\lambda_{ul}^2 A_{ul}}{4\pi^2 \Delta\nu_{ul}^{H}}$$

2. If the inhomogeneous linewidth is increased then the number of atoms that are in resonance with the stimulation radiation decreases.

$$N_{u,l}(\nu) = 2\sqrt{\frac{\ln 2}{\pi}} \frac{N_{u,l}}{\Delta\nu_D} \exp\left\{-\frac{4\ln 2(\nu-\nu_0)^2}{\Delta\nu_D^2}\right\}$$

Gain Saturation

The gain coefficient depends on ΔN, and ΔN is a function of amplifier radiation:

$$\Delta N = \frac{N_0}{1+\phi/\phi_S}$$

$$\frac{1}{\phi_S} = \tau_S \sigma(\nu)$$

so the gain g is given by:

$$g(\nu) = \frac{N_0 \sigma(\nu)}{1+\phi/\phi_S(\nu)}$$

when $\phi = \phi_s$ the gain has dropped to 1/2 of its small-signal value.

ϕ_s is called the saturation flux density.

This corresponds to the situation where the stimulated emission rate equals the normal (spontaneous + collisional + nonradiative) decay rate.

Gain in a laser amplifier of finite length

The gain coefficient above is a "local gain" where the flux does not change as a function of position.

Consider now the gain for a beam traversing a gain medium of length d:

$$d\phi = g(\nu)\phi dz$$

$$\frac{d\phi}{dz} = \frac{g_0\phi}{1+\phi/\phi_S}$$

$$or$$

$$\left(\frac{1}{\phi}+\frac{1}{\phi_S}\right)d\phi = g_0 dz$$

where g_0 is the gain in the absence of amplifier radiation.

Integrating from 0 to z:

$$\ln\left(\frac{\phi(z)}{\phi(0)}\right)+\frac{\phi(z)-\phi(0)}{\phi_S} = g_0 z$$

$\phi(0)$ is the input flux and *$\phi(z)$* is the flux at *z*.

Approximate gain solutions:

- *When $\phi(0)$* and *$\phi(z)$* are both small compared to *ϕ_S*:

$$\ln\left(\frac{\phi(z)}{\phi(0)}\right) \cong g_0 z$$

$$or$$

$$\phi(z) = \phi(0)e^{g_0 z}$$

(The flux grows exponentially.)

- When *$\phi(0)$* is large compared to *ϕ_S*:

$$\frac{\phi(z)-\phi(0)}{\phi_S} \cong g_0 z$$

$$or$$

$$\phi(z) \cong \phi(0) + g_0\phi_S z$$

(The flux grows linearly with distance at a rate that is independent of the input flux.)

For intermediate values, or for long gain regions, we must solve numerically.

Saturable absorbers:

The same arguments and arithmetic apply when the population distribution is normal:

As before:

$$g(\nu) = \frac{N_0 \sigma(\nu)}{1 + \phi / \phi_S(\nu)} \text{ and } \frac{1}{\phi_S} = \tau_S \sigma(\nu)$$

except that N_0 is negative.

For small flux: $\phi(z) = \phi(0) e^{g_0 z} = \phi(0) e^{-\alpha z}$

For large flux: $\phi(z) \cong \phi(0) + g_0 \phi_S z = \phi(0) - \alpha \phi_S z$

- For a cell of fixed length, the ratio of exit to entrance flux will increase from $e^{-\alpha d}$ to 1 as the flux is increased.

Gain of Inhomogeneously Broadened Amplifiers

- Because the small-signal gain coefficient $g_0(\nu)$ is proportional to $\sigma(\nu)$, different subsets of atoms, β, have different gain coefficients, $g_{0\beta}(\nu)$. The average gain is then just the average of the gain coefficient times the number of atoms with that gain coefficient for a given frequency.
- The gain under saturation conditions is very different because some subsets of atoms will saturate before others.

$$g_\beta(\nu) = \frac{g_{0\beta}(\nu)}{1 + \phi / \phi_{S\beta}(\nu)}$$

We then need to average this quantity over all the β's to find the gain profile.

(see Fig. 13.3-6 in Photonics.)

Basic considerations for a laser amplifier

We want the <u>stimulated transition cross section to be large</u>. It is determined by material and wavelength. Once a material is chosen, reducing linewidth increases gain inversely (e.g. by lowering temperature and or pressure)

We want a <u>large population difference</u> and a <u>high density</u> of atoms (but remember that high density increases linewidth (bad)). We address this by choosing a material system with the right lifetimes for 3 or 4 levels. Long transition lifetime for amplifier levels, short lifetimes for lower and pump levels.

We want to have a gain region that is as long as is practical.

A practical amplifier or laser:

Saturation occurs in an amplifier medium when the stimulated emission rate exceeds the spontaneous emission rate. We must have a significant portion of the laser contributing above the saturation level in order to have an efficient laser. (Consider spontaneous radiation to be loss. It is not coherent.)

If the stimulation radiation is generated at one end of the laser then an approximate expression for the length L and diameter d is necessary to achieve saturation at the output is given by:

$$ {}_e\sigma_u^H(\nu)N_u L = 16\left(\frac{L}{d}\right)^2 $$

Typical real lasers require

$$ \sigma_u^H(\nu)N_u L = 12 \pm 5. $$

Practical Laser Lengths and Shapes

Generally $\sigma_u^H(\nu)N_u$ is fixed for a medium so we can only change L and L/d.

For a laser with no mirrors the length required to achieve I_{sat} (and thus reasonable efficiency) corresponds to:

$$\sigma_u^H(\nu)N_u L = 12 \pm 5$$

For a He-Ne gas laser medium:

$$\sigma_u^H(\nu)N_u = 0.15m^{-1}$$

so the length required to achieve saturation is:

L~100 m.

This is not practical.

We can build a laser with mirrors so that we increase the effective length of the laser.

We must also account for loss.

Counting reflection at each mirror of R (<1) then the condition for lasing threshold is that the gain per 2L round trip must exceed the loss:

$$R^2 e^{g2L} \geq 1$$

or

$$g > \frac{1}{L}\ln R$$

The laser then will eventually build up to the saturation intensity at which the gain will drop and gain will equal loss.

We can also have scattering loss within the laser medium and scattering losses at the mirrors, and escape of radiation around the edges of the mirrors