Quantum physics 1

Class 2

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Math Background II

Review:

$$A(x)\frac{d^2y}{dx^2} + B(x)\frac{dy}{dx} + C(x)y = D(x)$$

- $B = D = 0$, $A = 1$, $C = $ constant ; classical oscillator

$\frac{d^2y}{dt^2} + Cy(t) = 0$; with solutions $\underline{\cos(\sqrt{c}\,t), \sin(\sqrt{c}\,t)}$

NB:

- $e^{\pm i\omega t}$ also a solution
- $e^{+i\omega t} + e^{-i\omega t}$, is also a solution.

  $\sim 2\cos(\omega t)$
- $e^{+i\omega t} - e^{-i\omega t} \sim 2i\sin\omega t$, another solu.

⑥ consider $A = D = 0$, $B = C = $ constant

In-class 2.1

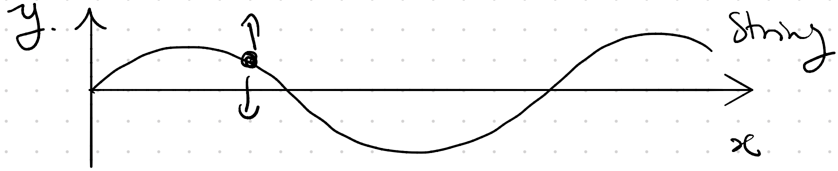
$A = D = 0$; $B = C = $ const. ; $y \longrightarrow \varphi$

$$B\frac{d\varphi}{dx} + C\varphi = 0 \;;\; B = \frac{\hbar}{i} \;;\; C = -p = \text{const.}$$

Partial Differential Eqns (PDE)

- x, or t are both variables. ; $\frac{d}{dx} \rightarrow \frac{\partial}{\partial x}$

example: vibrating string:



$$F = ma$$

$$F \sim \frac{\partial^2 y}{\partial x^2} \; ; \; a \sim \frac{\partial^2 y}{\partial t^2} \; ; \; \frac{\partial^2 y}{\partial x^2} - \frac{1}{v}\frac{\partial^2 y}{\partial t^2} = 0$$

$$\boxed{y(x,t) = y_0 \sin(kx - \omega t)}$$

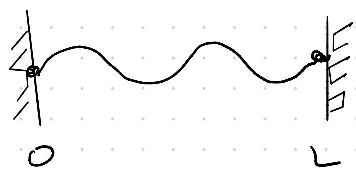
$$v = \sqrt{\frac{\tau}{\mu}}$$

$$v = \omega / k$$

Standing Waves

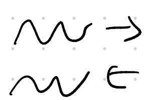
$$y(x,t) = y_0 \sin(kx - \omega t) + y_0 \sin(kx + \omega t)$$

$$= 2y_0 \sin kx \cos \omega t$$



Boundary conditions:

$y = 0$, $x = 0, L$



$\Rightarrow \quad kL = n\pi$

$$\boxed{k_n = \frac{n\pi}{L}}$$

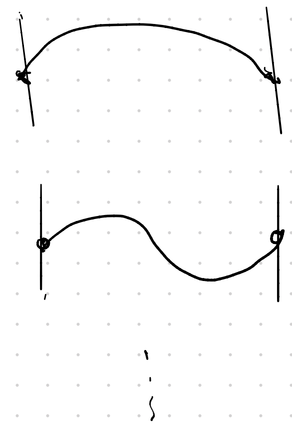
$$k_n = \frac{2\pi}{\lambda_n}$$

$$y \approx e^{\pm i(kx - \omega t)}$$

$0 = (\sin kx) \cos \omega t$

$kL = \pi n \,; \quad n = 1, 2, 3, \ldots$

$$k_n = \frac{\pi n}{L}$$



Maxwell's Eqns:

without sources $\rho = 0, \; j = 0$

$$\boxed{\begin{array}{ll} \nabla \cdot E = 0 \quad \ldots (1) & \nabla \cdot B = 0 \quad \ldots (2) \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \quad \ldots (3) & \nabla \times \vec{B} = \dfrac{\partial E}{\partial t} \quad \ldots (4) \end{array}}$$

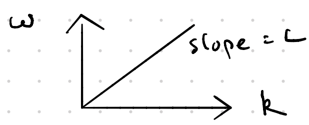
from (3): $\dfrac{\partial^2 E}{\partial x^2} - \dfrac{1}{c^2}\dfrac{\partial^2 E}{\partial t^2} = 0 \quad \ldots (5) \quad v \to c$

$\boxed{\omega = kc}$

using: curl of curl identity:

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$$

to go from (3) to (5)



Inclass 2.2

Separation of variables in PDE

$$\frac{\partial^2 E(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E(x,t)}{\partial t^2} = 0 \; ; \quad E(x,t) = Q(x)\phi(t)$$

$$\Rightarrow \frac{\partial^2}{\partial x^2}\left(Q(x)\phi(t)\right) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(Q(x)\phi(t)\right)$$

$$\Rightarrow \phi(t)\frac{\partial^2 Q(x)}{\partial x^2} = \frac{1}{c^2} Q(x)\frac{\partial^2 \phi(t)}{\partial t^2}$$

$$\Rightarrow \frac{1}{Q(x)}\frac{\partial^2 Q(x)}{\partial x^2} = \frac{1}{c^2}\frac{1}{\phi(t)}\frac{\partial^2 \phi(t)}{\partial t^2}$$

$$= \text{const} = s$$

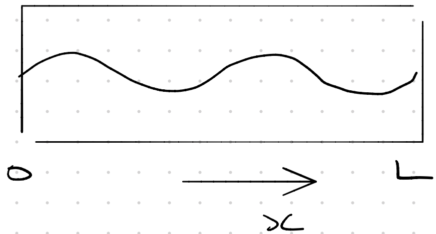
$$\begin{cases} \dfrac{\partial^2 Q(x)}{\partial x^2} = s\,Q(x) \\[2ex] \dfrac{\partial^2 \phi(t)}{\partial t^2} = c^2 s\,\phi(t) \end{cases}$$

In-class 2-3

$$E = [Ae^{ikx} + Be^{-ikx}]e^{-i\omega t}$$

if $x = 0, L$; $E = 0$

assumption : $A = -B$



$E = 0$

@ $x = 0, L$

In-class 2-4