PHYS 2210 Spring 2024 – Homework Problems 32-35

32) Townsend Problem 4.24

(a) Verify that the wave function (see Example 4.3) in the region $x > 0$ for the step potential of Section 4.6 leads to zero probability current in this region.
(b) Use the conservation of probability equation $\frac{\partial \Psi^* \Psi}{\partial t} = -\frac{\partial j_x}{\partial x}$ to argue that the probability current must also vanish in the region x< 0 as well for this energy eigenfunction. What can you therefore conclude about the magnitude of the reflection coefficient?

33) Townsend Problem 4.25

4.25. Solve the time-independent Schrödinger equation for a particle of mass m and energy $E > V_0$ incident from the left on the step potential

$$V(x) = \begin{cases} V_0 & x < 0 \\ 0 & x > 0 \end{cases}$$

See Fig. 4.37. Determine the reflection coefficient R and the transmission coefficient T. Verify that probability is conserved.

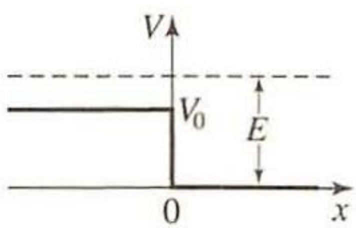

Figure 4.37 A particle with energy $E > V_0$ is incident from the left on the step potential.

34) Townsend Problem 4.26 (There is a significant amount of algebra necessary for this one.)

4.26. Solve equations (4.128) through (4.131) for the ratio A/C and verify that the transmission coefficient T for tunneling through a square barrier is given by (4.133), namely

$$T = \left[1 + \frac{\left(k^2+\kappa^2\right)^2}{4k^2\kappa^2}\sinh^2 \kappa a\right]^{-1}$$

where

$$k = \frac{\sqrt{2mE}}{\hbar} \qquad \text{and} \qquad \kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

35) A proton and a deuteron (a particle with the same charge as a proton, but twice the mass) attempt to penetrate a rectangular potential barrier of height 10 Me V and thickness 10^{-14} m. Both particles have total energies of 3 MeV. (a) Use qualitative arguments to predict which particle has the highest probability of succeeding. (b) Evaluate quantitatively the probability of success for both particles.