# Quantum Physics 1
# Class 8

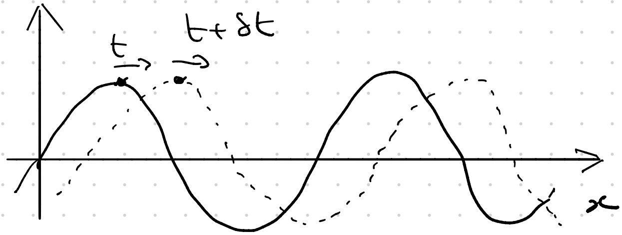
- Exam 1 review session coming up → LMS announcement
  → Room TBD
  → Respond to poll.
- Mock Exam posted to LMS.
- Friday DEI discussion.

# Class 8

## Expectation Values

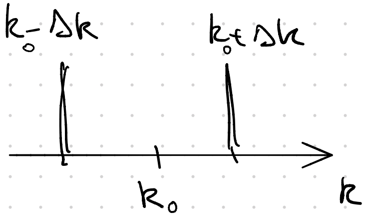
Review:

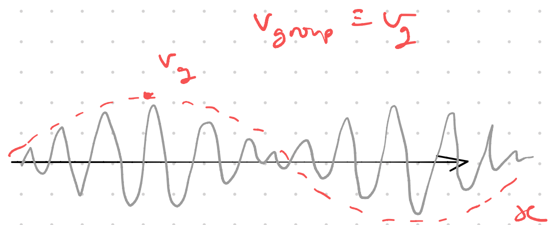
- Free Particle $\sim e^{i(kx-\omega t)}$




$v_{phase} = \frac{1}{2} v_g$

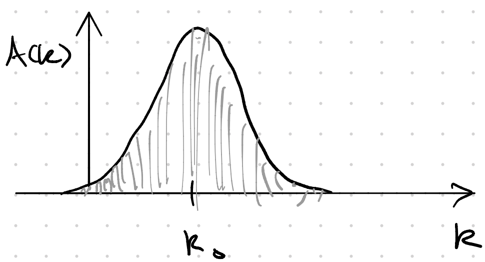
w/t group velocity → velocity of envelope of wave packet.




More generally:

Consider distribution of $A(k)$

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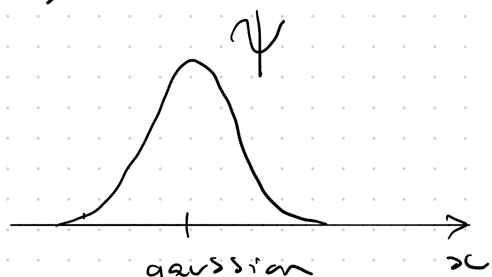
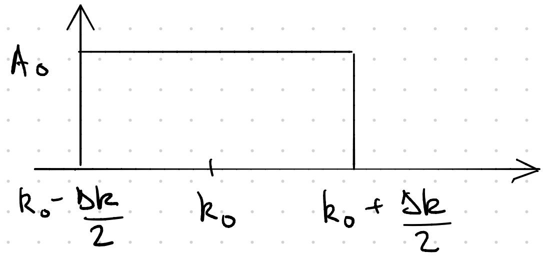



gaussian wave packet

$$A(k) \approx e^{-(k-k_0)^2/\sigma^2}$$

eg)




$$\Psi(x) = \int_{-\infty}^{\infty} A(k)\, e^{ikx}\, dk$$

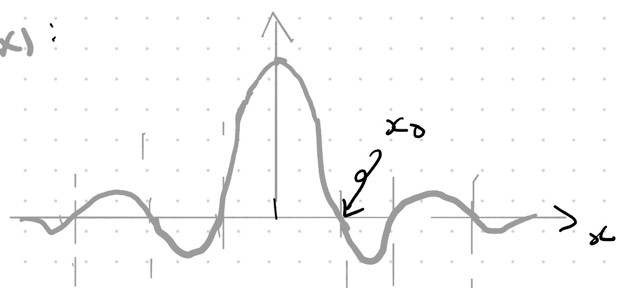
$$= \int_{k_0-\Delta k/2}^{k_0+\Delta k/2} A_0\, e^{ikx}\, dx$$

$$= A_0 \frac{1}{ix} e^{ikx} \Big|_{k_0-\Delta k/2}^{k_0+\Delta k/2}$$

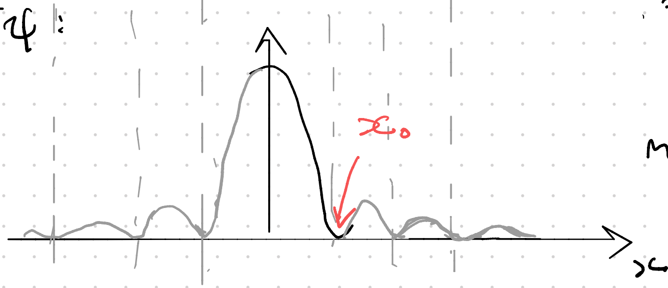
$$= \frac{A}{ix}\left[e^{i(k_0+\frac{\Delta k}{2})x} - e^{i(k_0-\frac{\Delta k}{2})x}\right]$$

$$\approx e^{ik_0x} \cdot \frac{\sin\left(\frac{\Delta k}{2}x\right)}{x}$$

$\Psi(x)$ :




$\Psi^*\Psi$ :




Minimum @

$$\frac{\Delta k}{2} x_0 = \pi$$

$$x_0 = \frac{2\pi}{\Delta k}$$

Recall $\Psi^*\Psi$ : probability density of finding particle at $x, t$

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Recall : Mean / Average

Average of a value = Sum of values × fraction of "occurrence" of value

$$= \sum \frac{\text{value} \times N(\text{value})}{N_{\text{Total}}}$$

$$= \sum \text{value} \cdot \underbrace{\left(\frac{N(\text{value})}{N}\right)}_{P_r}$$

ie. for a value $x$:

$$\langle x \rangle = \sum_i x_i Pr_i \qquad (Pr \equiv \text{probability})$$

In quantum mechanics:

$$\langle x \rangle = \int \Psi^* \hat{x} \Psi \, dx \equiv \text{Expectation value of } x$$

$$\langle x \rangle = \int x \, |\Psi|^2 dx$$

What about $p$? ( From Ehrenfest theorem: $\frac{d}{dt}\langle x \rangle = \frac{\langle p \rangle}{m}$ )

$$\langle p \rangle = \int \Psi^* \underbrace{\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)}_{\hat{p} \equiv \text{momentum operator}} \Psi \, dx$$

$$\langle p \rangle = \int \Psi^* \hat{p} \Psi \, dx$$

What about uncertainty? If have the mean, we can estimate uncertainty w/ the standard deviation:

Recall: $\sigma^2 = \sum_i (x_i - \bar{x})^2 \cdot \frac{1}{N}$ ; $\sigma \equiv$ standard deviation.

$$= \sum_i (x_i - \langle x \rangle)^2 \cdot Pr_i$$

$$= \sum_i (x^2 - 2x\langle x \rangle + \langle x \rangle^2) Pr_i$$

$$= \langle x^2 \rangle - 2\langle x \rangle\langle x \rangle + \langle x \rangle^2$$

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$

Now, $\sigma^2 \rightarrow \Delta A$, uncertainty.

In general,

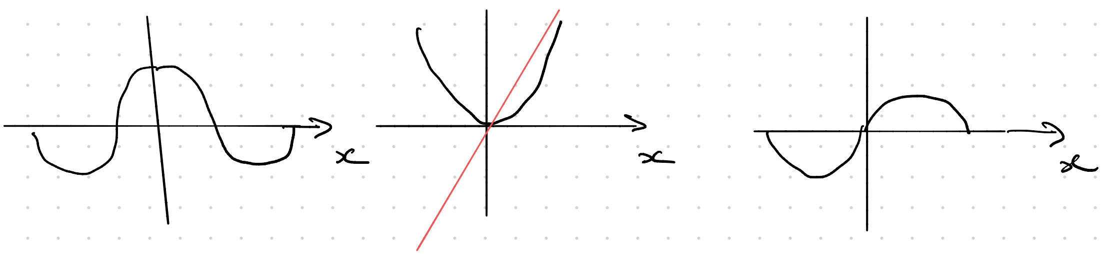
$$\boxed{\Delta A^2 = \langle A^2 \rangle - \langle A \rangle^2}$$

& for QM : $\Delta x^2 = \langle x^2 \rangle - \langle x \rangle^2$

$$\boxed{\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}}$$

$$\boxed{\text{In-class } 8.1,\ 8.2}$$

Hint: Integrating even & odd functions

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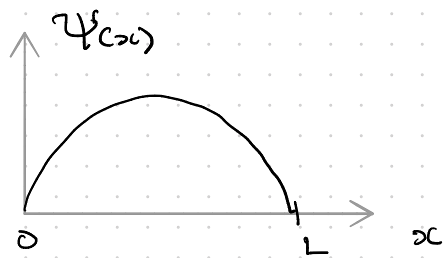
Example: $\psi(x) = \begin{cases} \frac{\sqrt{30}}{L^3}\, x(L-x); & 0 < x < L \\ 0 & , \text{ elsewhere} \end{cases}$

$$\langle x \rangle = \int_0^L \sqrt{\frac{30}{L^3}}\, x(L-x)\; x \sqrt{\frac{30}{L^3}}\, x(L-x)\, dx$$

$$= L/2$$



$\therefore \quad \Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \quad ; \quad \langle x^2 \rangle = \frac{2L^2}{7}$

$\Delta x = \frac{L}{2\sqrt{7}}$

& $\langle p \rangle = \int_0^L \psi^* \frac{\hbar}{i} \frac{\partial}{\partial x} \psi \, dx \quad ; \quad \langle p^2 \rangle = \int_0^L \psi^* \left(\frac{\hbar}{i} \frac{\partial}{\partial x}\right)\left(\frac{\hbar}{i} \frac{\partial}{\partial x}\right) \psi \, dx$

$= 0 \qquad\qquad = \frac{10\hbar^2}{L^2}$

$\therefore \quad \Delta p = \frac{\sqrt{10}\,\hbar}{L}$

$\therefore \quad \Delta x \, \Delta p = \left(\frac{L}{2\sqrt{7}}\right)\left(\frac{\sqrt{10} \cdot \hbar}{L}\right)$

$= \underbrace{\frac{1}{2}\sqrt{\frac{10}{7}} \cdot \hbar}_{\geq \hbar/2}$

Recall: Heisenberg Uncertainty $\Delta x \, \Delta p \geq \hbar/2$

In-class 8.3, 8.4