

1. Assume you know the energy eigenfunctions $\Psi_1(x, t)$ of the Hamiltonian operator and you find that $\int \psi^*(x) \Psi_2 dx = 0.1$ What is the probability of observing a particle energy of $\hbar\omega_2$ at any time?

$$\Psi(x) = \sum_i c_i \psi_i$$

$$C_i = \int \psi(x) \Psi_i(x) dx$$

$$P(E_1) = C_1^* C_1 = C_1^2$$

$$P(E_1) = 0.01$$

2. A particle in a state given by $\Psi(x, t) = A(\psi_1 e^{(iE_1 t/\hbar)} + 3\psi_2 e^{iE_2 t/\hbar})$ where $E_1 = 1eV$ and $E_2 = 5eV$ and both ψ_1 and ψ_2 are normalized
 - a. What is the expectation value of a large number of energy measurements of this system? Explain.

$$A = \frac{1}{\sqrt{10}} \rightarrow C_1 = \frac{1}{\sqrt{10}}, \quad C_3 = \frac{3}{\sqrt{10}}$$

$$\frac{C_1^2 E_1 + C_2^2 E_2}{2}$$

$$\frac{\frac{1}{10}(1eV) + \frac{9}{10}(5eV)}{2}$$

$$\langle E \rangle = \frac{46}{10} \approx 4.6eV$$

The average of energy levels weighted by the constants of how "much" of each energy level there is yields the expectation value.

- b. The most likely measurement is 5, because its a 90 percent chance

3. The spatial wavefunction of a particle decays monotonically to zero in a particular region of space. What can you say about the eigenenergy of the state relative to the potential energy in that region?

$$\frac{\partial^2 \psi}{\partial x^2} \approx -(E - V)\psi$$

