Inclass 22.1. Show that $Y_{1,1}(\theta, \phi) = -\sqrt{\frac{3}{8\pi}} sin\theta e^{i\phi}$ is normalized:

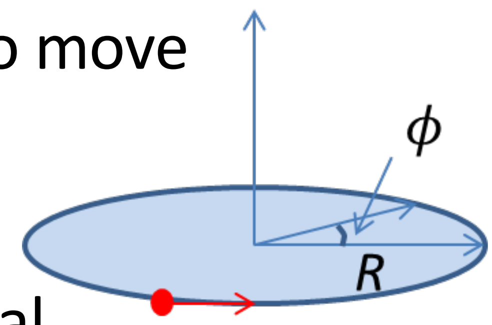
$$\int Y_{1,1}{}^{*} Y_{1,1}\, d\Omega = \int Y_{1,1}{}^{*} Y_{1,1}\, sin\theta d\theta d\phi = 1.$$

Given integral: $\int sin^3 x dx = -cosx + \frac{cos^3 x}{3} + C$

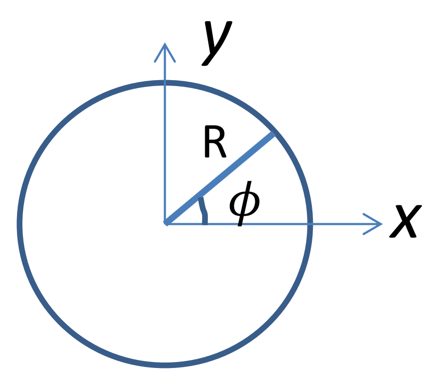
Inclass 22.2. (a) Consider the $l = 1$ spherical harmonic basis: $Y_{1,1}(\theta, \phi)$, $Y_{1,0}(\theta, \phi)$, and $Y_{1,-1}(\theta, \phi)$. Determine the matrix representation of the $\hat{L}_z$ operator, (L_z).

(b) Given $\Psi(\theta, \phi) = \frac{1}{\sqrt{2}} Y_{1,1}(\theta, \phi) + \frac{1}{\sqrt{2}} Y_{1,-1}(\theta, \phi)$, determine the expectation value of L_z using the matrix method.

Inclass 22.3. Consider a bead of mass m_0 confined to move In a circle in the x-y plane with radius R.



(a) Determine the angular momentum and energy operators of the system, in both Cartesian and spherical (polar) coordinate systems.



(b) Determine the eigenfunctions and eigenvalues of the angular momentum and energy operators in spherical coordinate system.

Inclass 22.4. Consider a bead of mass m_0 which is confined to move in a spherical shell with radius R.

(a) Determine the energy operator of the system, in both Cartesian and spherical (polar) coordinate systems.

(b) Determine the eigenfunction and eigenenergy of the energy operator.