Quantum Physics 1
Class 26.

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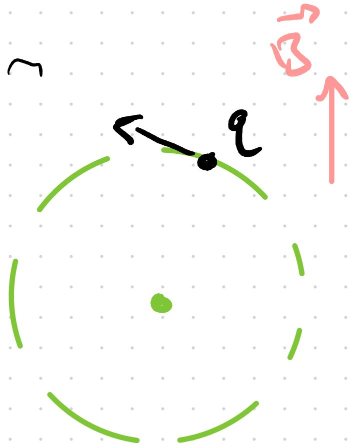
More on quantum physics

Review :

I) Orbital angular momentum



$$H = H_0 - \vec{\mu} \cdot \vec{B}$$

H-atom : $\dfrac{p^2}{2m} + V(r)$

$|\vec{L}| = \sqrt{\ell(\ell+1)}\,\hbar$; $\ell = 0, 1, 2, \ldots$

$-\vec{\mu} \cdot \vec{B} = -g \dfrac{q B_z}{2m_0} L_z$; $g = 1$ for e^-

$\therefore$ for H-atom

$$H \Rightarrow \boxed{E_n - \frac{q B_z}{2m_0} m\hbar}$$

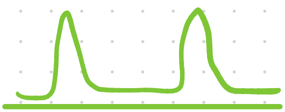
II) Spin ($\vec{L} = 0$, no orbital motion)

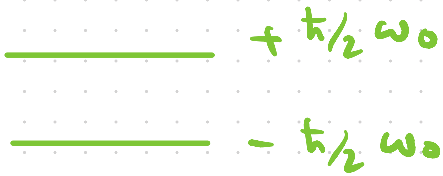
$$H = -\vec{\mu} \cdot \vec{B} = -g \frac{q B_z}{2 m_0} \cdot s\hbar \; ; \quad s = \pm 1/2$$

For e^-, $g = 2$

For p, $g = 5.6$

- Zeeman splitting $\longrightarrow$ Stern Gerlach Experiment





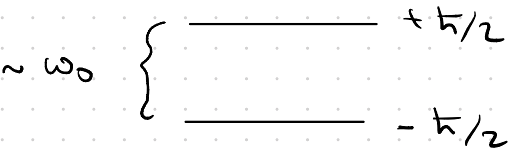
where, $\omega_0 = \dfrac{|e| B_z}{m_e}$, $g = -2$

Applications of Spin:

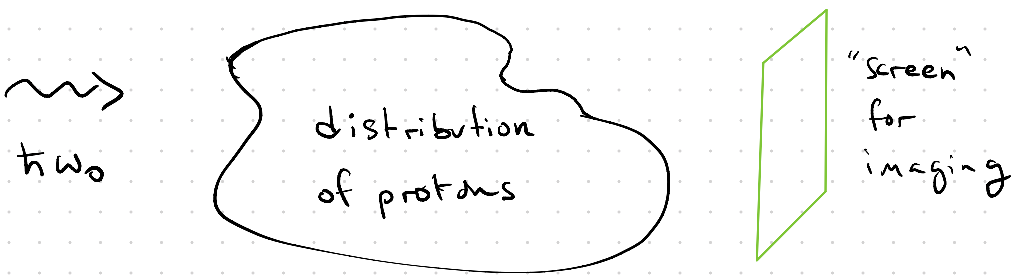
- Nuclear magnetic resonance (NMR), MRI
- Giant magnetoresistance (GMR, hard drives)
- Future: Quantum Computing.

$$H = -\vec{\mu}_s \cdot \vec{B}$$
$$= \omega_0 S_z$$

where $\omega_0 = -\dfrac{g|e|B_z}{2M_0}$; $g = +5.6$



$$\Delta E = \hbar\omega_0$$

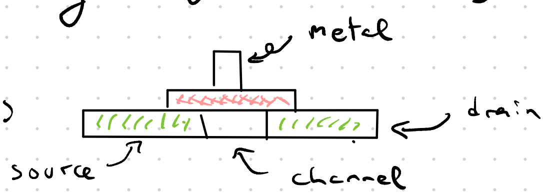


In-class 26.1

Computing

- Conventional: Binary digits (0, 1) Computing

Transistors:



Quantum Computing:

- Two-level system (Qubits)

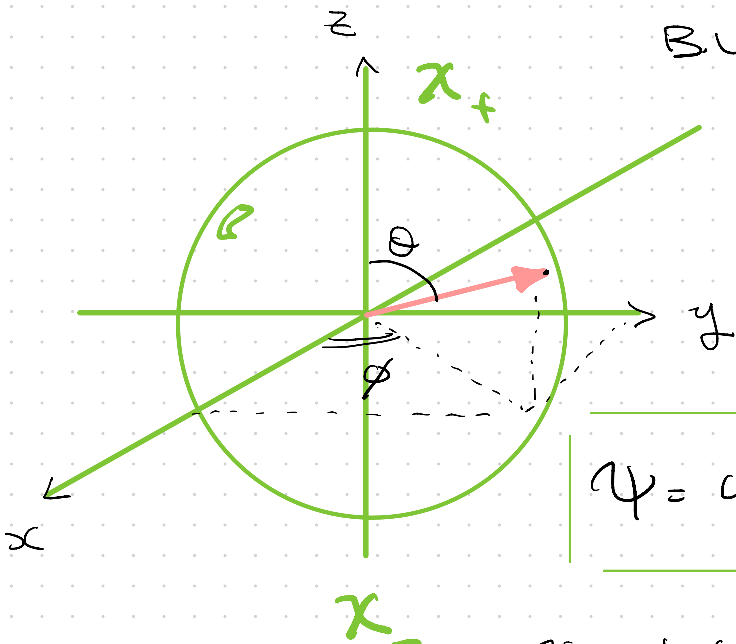
$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} ; \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

recall: $\chi_+ \equiv \chi_{\frac{1}{2}, \frac{1}{2}} ; \chi_- \equiv \chi_{\frac{1}{2}, -\frac{1}{2}}$

in general ; $\Psi = \alpha \chi_+ + \beta \chi_-$

α, β complex ; 4 variables exist.

<u>NB</u> $\Psi = \sum c_n \varphi_n(x)$; & $|\alpha|^2 + |\beta|^2 = 1$

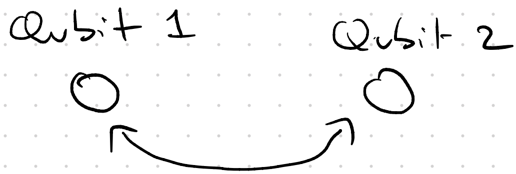


BLOCH SPHERE

$$\boxed{\Psi = \cos\frac{\theta}{2} + e^{i\phi} \sin\frac{\theta}{2}}$$

$$\Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

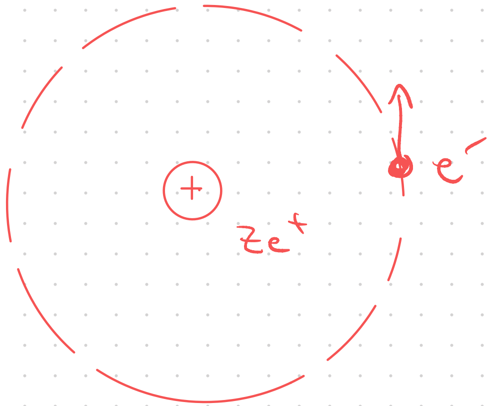
Entanglement:



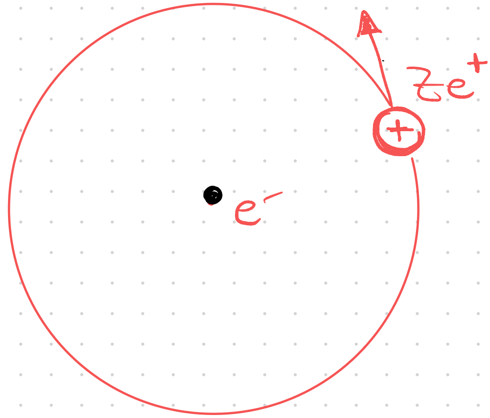
In-class 26.2
26.3

Not on the exam. 3

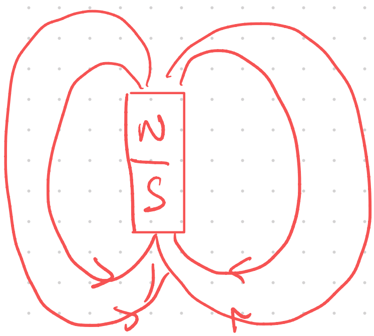
Spin-orbit coupling & relativistic effects in the hydrogen atom.

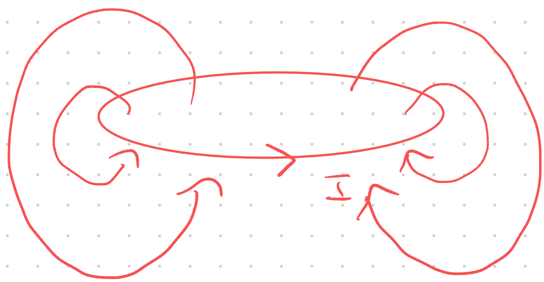


nucleus rest frame



e- rest frame.





$$V_{SL} = -\vec{\mu} \cdot \vec{B} \sim \vec{S} \cdot \vec{L}$$

"spin-orbit interaction"

What about special relativity ?

Special relativity & QM

$\Rightarrow$ Relativistic quantum mechanics

$$E = \sqrt{p^2c^2 + m^2c^4} \quad ; \quad \hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

Dirac Eqn: $E = c\vec{\alpha}\cdot\vec{p} + \beta m_0 c^4$;

α, β : 4x4 matrices .

eg) $s = 1/2$ particle ; $g = 2$

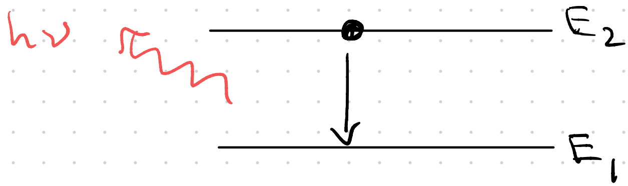
$$\psi_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \psi_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\psi_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \psi_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

In-class 26-4

H-atom:

spontaneous emission:



③ Quantum field theory