# Exam 1 Part B

Graded

**Student**

Paul Lea

**Total Points**

52 / 80 pts

**Question 1**

## Method of Images

3 / 5 pts

- ✔ **+ 2 pts** Recognizing a method of images problem
- **+ 1 pt** $\sigma = \epsilon_0 E|_{z=0}$ or $\sigma = -\partial V/\partial r|_{z=0}$
- ✔ **+ 1 pt** $\vec{E}(0,0,0) = -\frac{2q}{4\pi\epsilon_0 d^2}\hat{k}$
- **+ 1 pt** $\sigma = -6.37 \times 10^{-7}\, C/m^2$

**Question 2**

## Gauss' Law

5 / 10 pts

- ✔ **+ 2 pts** Gauss' law or Coulomb's law
- **+ 2 pts** $r < R$: $E(4\pi r^2) = \frac{1}{\epsilon_0}\frac{4}{3}\pi r^3\rho$
- **+ 2 pts** $\vec{E} = \frac{\rho r}{3\epsilon_0}\hat{r}$
- ✔ **+ 2 pts** $r > R$: $E(4\pi r^2) = \frac{1}{\epsilon_0}\frac{4}{3}\pi R^3\rho$
- ✔ **+ 2 pts** $\vec{E} = \frac{\rho R^3}{3\epsilon_0 r^2}\hat{r}$
- **− 1 pt** Point adjustment

1 direction?

**Question 3**

## Potential from field

2 / 8 pts

- ✔ **+ 2 pts** $V(r) = -\int \vec{E}\cdot d\vec{l}$
- **+ 2 pts** $-\int_{\infty}^{R} \frac{\rho R^3}{3\epsilon_0 r^2} dr$
- **+ 2 pts** $-\int_{R}^{0} \frac{\rho r}{3\epsilon_0} dr$
- **+ 2 pts** $V(0) = \frac{\rho R^2}{2\epsilon_0}$
- **+ 0 pts** No answer

2 The field and thus the potential is different inside and outside the sphere.

Question 4

## Series circuit potential difference

4 / 8 pts

- ✔ **+ 2 pts** Switch is open so circuit is in series and all devices have the same current
- ✔ **+ 2 pts** Ohm's law: $I = (40.0V)/(175\Omega) = 0.229A$

**+ 2 pts** Loop rule: $V_b - V_a =$ emf - $V_{75\Omega}$

**+ 2 pts** $V_b - V_a = -2.14V$

**+ 0 pts** No answer

Question 5

## P=VI

4 / 4 pts

- ✔ **+ 2 pts** Correct power formula
- ✔ **+ 2 pts** $P = 5.73W$

**+ 0 pts** No answer

Question 6

## Parallel circuit current

7 / 7 pts

- ✔ **+ 1 pt** Switch is closed so we now have two circuits in parallel
- ✔ **+ 2 pts** Ohm's law in left loop:
  $I_{left} = (25V)/(100\Omega) = 0.250A$
- ✔ **+ 2 pts** Ohm's law in right loop: $I_{right} = (15V)/(75\Omega) = 0.200A$
- ✔ **+ 2 pts** Kirchhoff junction rule:
  $I_{am} = I_{left} - I_{right} = 0.050A$

**+ 0 pts** No answer

3 Please explain what you're doing or I will not give you the points next time

Question 7

## Equivalent capacitance

6 / 6 pts

- ✔ **+ 2 pts** middle branch: 3 capacitors in series so $\frac{1}{C_{middle}} = \sum \frac{1}{C_i}$
- ✔ **+ 1 pt** $C_{middle} = 5.29\,nF$
- ✔ **+ 2 pts** Full circuit: 3 capacitors in parallel so $C_{eq} = \sum C_i$
- ✔ **+ 1 pt** $C_{eq} = 19.3\,nF$

**+ 0 pts** No answer

Question 8

## Total system charge

**5** / 5 pts

- ✔ **+ 2 pts** $Q = C\Delta V$
- ✔ **+ 2 pts** $Q_{tot} = C_{eq}V$
- ✔ **+ 1 pt** $Q_{tot} = 482\,nC$
- **+ 0 pts** Click here to replace this description.

Question 9

## Capacitor charge

**6** / 6 pts

- ✔ **+ 2 pts** Capacitors are in parallel so they are at the same potential
- ✔ **+ 2 pts** $Q_{6.5} = C_{6.5}V$
- ✔ **+ 2 pts** $Q_{6.5} = 162\,nC$
- **+ 0 pts** Click here to replace this description.

Question 10

## Potential difference across capacitor in series

**2** / 6 pts

- **+ 2 pts** Capacitors in series have the same charge
- ✔ **+ 2 pts** $Q = C_{eq}V = (5.29\ nF)(25V) = 132.25\ nC$
- **+ 2 pts** $V_{10} = \frac{Q}{C_{10}} = 13.2V$
- **+ 0 pts** Click here to replace this description.

Question 11

## Field from potential

**5** / 5 pts

- ✔ **+ 2 pts** $\vec{E} = -\vec{\nabla}V$
- ✔ **+ 3 pts** $\vec{E} = -z\hat{i} - 2y\hat{j} - x\hat{k}$
- **+ 0 pts** No answer

Question 12

## Gauss' law in differential from

**3** / 5 pts

- ✔ **+ 3 pts** $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$
- **+ 2 pts** $\vec{\nabla} \cdot \vec{E} = -2\epsilon_0$
- **+ 0 pts** No answer

Question 13

## Work

**0** / 5 pts

**+ 2 pts** $W = Q\Delta V$

**+ 1 pt** $\Delta V = V(1,2,3) - V(\infty) = V(1,2,3)$
$(V(\infty) = 0)$

**+ 2 pts** $W = 7Q$

✔ **+ 0 pts** Click here to replace this description.

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Part II: Problems - Include a logical statement or relevant equation for full credit

**Problem 1.** A charge of +10.0 nC is placed a distance of 0.050 meters above a very large, flat, grounded, conducting sheet. Calculate the surface charge density on the sheet at a position directly below the charge.

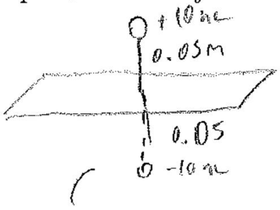


Image Charge

$$\vec{E}_t = \vec{E}_+ + \vec{E}_-$$

$$= \frac{20E-9}{4\pi\epsilon_0 (0.05)^2}$$

$$\boxed{= 7.19E3 \text{ N/C } \hat{k}}$$

**Problem 2.** A solid ball of charge with uniform volume charge density $\rho$ and radius $R$ is centered at the origin.

a) Calculate the electric field inside and outside the ball.

$\boxed{\text{Inside } = 0}$

b/c no flux through surface inside sphere

Outside ($R<r$)

$$q_{enc} = \rho \frac{4}{3}\pi R^3$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \rho \tfrac{4}{3}\pi R^3$$

$$\boxed{\vec{E} = \frac{\rho 4\pi R^3}{12\pi\epsilon_0 r^2}}$$

r - distance from center of ball/origin

b) Calculate the electric potential at the origin using a reference at infinity.

$$V = k\frac{q}{r} = \boxed{k\frac{\rho \tfrac{4}{3}\pi R^3}{r}}$$

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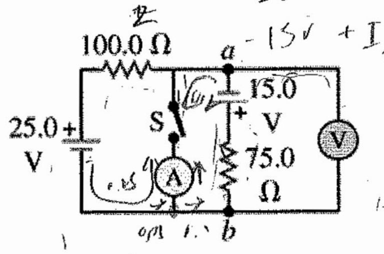
$25V - I_1(100) = 0$

$-15V + I_2(75) = 0$

$I_1 = 0.25$

$I_2 = 0.2$

**Problem 3.** The figure displays an electrical circuit. All components are ideal.




a) Find the potential difference measured by the voltmeter with the switch open as shown.

$\sum_{i=1}^{N} \Delta V_i = 0$

$\sum_{i=1}^{N} I_i = 0$

$25 - 75 I_1 + 15 + 100 I_2 = 0$

$V_{ab} = 75(0.2) - 15 = 0$

$\boxed{V_{ab} = 0}$

b) Find the power supplied by the 25 V battery to the circuit when the switch is open.

$P = IV$

$P = 0.25(25)$

$\boxed{P = 6.25 \text{ W}}$

c) Find the current measured by the ammeter with the switch closed.

$0.25 - 0.2 = \boxed{0.05 \text{ A}}$

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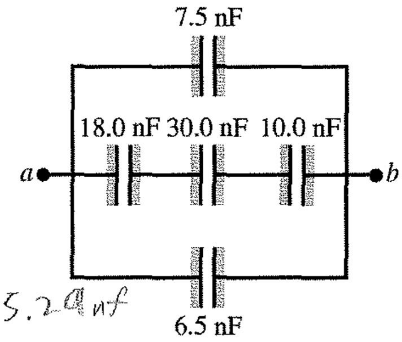
**Problem 4.** For the system of capacitors shown in the figure, a potential difference of 25 V is maintained across $ab$




a) What is the equivalent capacitance of this system between $a$ and $b$?

Series $\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$\frac{1}{C_t} = \frac{1}{18} + \frac{1}{30} + \frac{1}{10} = \frac{17}{90} = \frac{1}{C_t} \rightarrow C_t = \frac{90}{17} = 5.29\,nf$

Parallel

$C_t = C_1 + C_2 + \ldots$

$C_t = 5.29 + 7.5 + 6.5 = \boxed{19.29\,nF}$

b) How much charge is stored by this system?

$Q = C\Delta V$

$19.29(25) = 482.25$

$\boxed{482.25\,nC}$

c) How much charge does the 6.5 nF capacitor store?

$Q = C\Delta V$

$6.5(25)$

$Q = \boxed{162.5\,nf}$

d) What is the potential difference across the 10 nF capacitor?

$V_t = V_1 + V_2 + \ldots$ $\qquad Q_t = 5.29 \cdot 25V$

$\frac{132.25\,nC}{5.29\,nf} = \frac{Q}{18} + \frac{Q}{30} + \frac{Q}{10}$

$Q = 132.25/3$

$V_3 = \frac{44.083\,nC}{10\,nf} = \boxed{4.408\,V}$

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**Problem 5.** The electric potential with respect to infinity inside a sphere with radius $R = 10$ m is given by the function $V(x, y, z) = xz + y^2$

a) What is the electric field inside the sphere?

$$\vec{E} = -\nabla V$$

$$\boxed{\vec{E} = -\langle z, 2y, x \rangle}$$

b) What is the volume charge density inside the sphere?

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\epsilon_0 (\vec{\nabla} \cdot \vec{E}) = \rho$$

$$z^2 + (2y)^2 + x^2$$

$$\boxed{z^2 + 4y + x^2 = \rho}$$

c) How much work would be required to bring a point charge $+Q$ from $\infty$ to $(1, 2, 3)$?