Fast Globally Optimal and Geometrically Consistent 3D Shape Matching

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Code&Paper

Geometrically Consistent Shape Matching

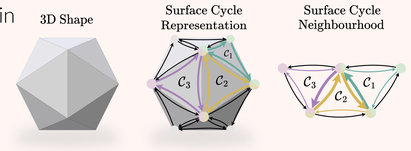
Goal: find correspondences between two non-rigidly deformed **3D shapes** $\mathcal{X}$ and $\mathcal{Y}$ while preserving neighbourhood relations of surface elements.

Source Geometric Consistency ✗ Geometric Consistency ✓

Applications: Texture transfer, protein alignment, shape completion , ...

Our Contributions

- A novel **globally geometrically consistent** formalism for 3D shape matching.
- A **novel 3D shape representation** in which we represent a 3D shape using a *collection of surface cycles*.



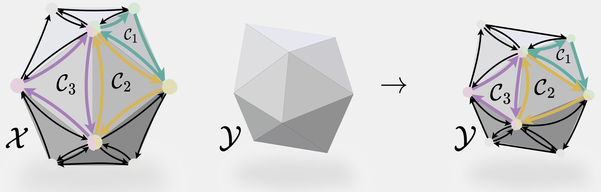
- Fast computation of **high quality** and geometrically consistent matchings.
- Our formalism **generalises to other matching problems**, e.g. image-image matching, or planar-graph matching.

Comparison to Previous Works

Method	Globally Optimal	Geometrically Consistent	Scalable
MINA [1]	(✓)	✗	✗
SIGMA [4]	(✓)	✗	✗
PMSDP [5]	✗	✗	✓
Windheuser et al. [8]	✗	✓	✗
SpiderMatch [7]	✓	(✓)	(✓)
Ours	✓	✓	✓

⇒ Our approach is the **first to combine the three desirable properties**: (i) global optimality, (ii) neighbourhood preservation, and (iii) scalability.

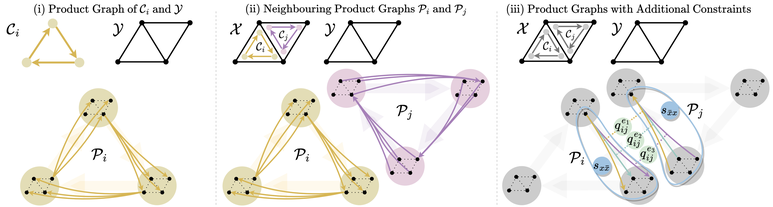
Key Idea



We represent shape $\mathcal{X}$ using n **surface cycles** $\mathcal{C}_1, \dots, \mathcal{C}_n$ and then consistently match these to shape $\mathcal{Y}$ while preserving neighbourhood relations between the cycles.

Details

Hyper Product Graph

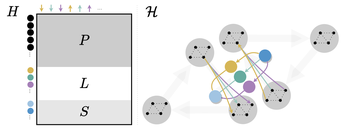


(i) product graph $\mathcal{P}_i$ between i-th surface cylce $\mathcal{C}_i$ and the target shape $\mathcal{Y}$
(ii) neighbourhood of surfaces cycles determines neighbourhood of product graphs
(iii) coupling of neighbouring product graphs at opposite edges
→ geometric consistency

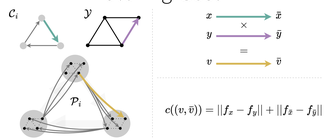
Resulting Optimisation Problem

$$\min_{x \in \{0,1\}^m} c^T x \quad \text{s.t.} \quad Hx = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Constraint Matrix



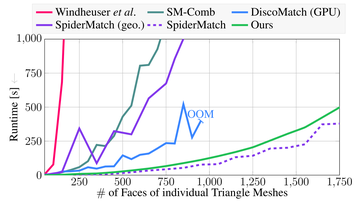
Matching Cost



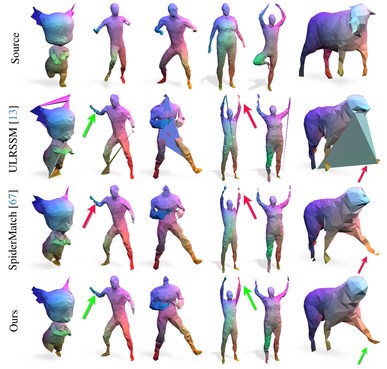
We compute features $f_x, f_{\bar{x}}, f_y, f_{\bar{y}}$ using [2].

Results

Runtime



3D Shape Matching



Partial-Full 3D Shape Matching

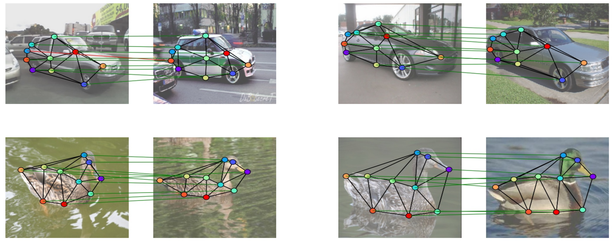
Matching Errors

Method	FAUST	SMAL	DT4D Intra	DT4D Inter	BeCoS
ULRSSM [2]	0.031	0.048	0.033	0.041	0.057
SmoothShells [3]	0.379	0.376	0.373	0.420	0.370
DiscreteOpt [6]	0.110	0.268	0.075	0.170	0.270
SpiderMatch [7]	0.029	**0.044**	0.027	0.041	0.057
Ours	**0.027**	**0.044**	**0.024**	**0.039**	**0.056**

Smoothness Errors

Method	FAUST	SMAL	DT4D Intra	DT4D Inter	BeCoS
ULRSSM [2]	2.1	2.6	3.4	2.6	4.1
SmoothShells [3]	1.9	2.8	1.6	2.3	5.7
DiscreteOpt [6]	8.9	13.6	7.5	10.7	1.1
SpiderMatch [7]	1.7	1.8	1.7	2.1	0.81
Ours	**0.46**	**0.53**	**0.48**	**0.62**	**0.77**

Image-Image Matching (Proof-of-Concept)



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[2] Dongliang Cao, Paul Roetzer, and Florian Bernard. Unsupervised learning of robust spectral shape matching. *TOG*, 2023.
[3] Marvin Eisenberger, Zorah Lähner, and Daniel Cremers. Smooth shells: Multi-scale shape registration with functional maps. In *CVPR*, 2020.
[4] Maolin Gao, Paul Roetzer, Marvin Eisenberger, Zorah Lähner, Michael Moeller, Daniel Cremers, and Florian Bernard. Sigma: Scale-invariant global sparse shape matching. In *ICCV*, 2023.
[5] Haggai Maron, Nadav Dym, Itay Kezurer, Shahar Kovalsky, and Yaron Lipman. Point registration via efficient convex relaxation. *TOG*, 2016.
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[7] Paul Roetzer and Florian Bernard. Spidermatch: 3d shape matching with global optimality and geometric consistency. In *CVPR*, 2024.
[8] Thomas Windheuser, Ulrich Schlickewei, Frank R Schmidt, and Daniel Cremers. Geometrically consistent elastic matching of 3d shapes: A linear programming solution. In *ICCV*, 2011.

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