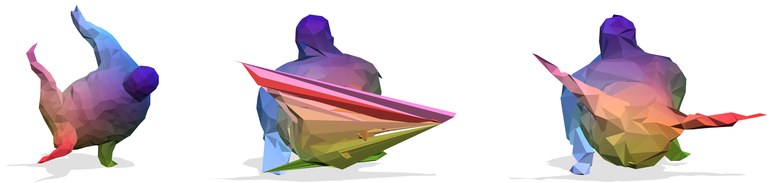
SpiderMatch: 3D Shape Matching with Global Optimality and Geometric Consistency

Paul Roetzer Florian Bernard
University of Bonn

Project Page

3D Shape Matching

Finding correspondences between two non-rigidly (and potentially non-isometrically) deformed 3D shapes. Geometric consistency is essential for good quality matching:

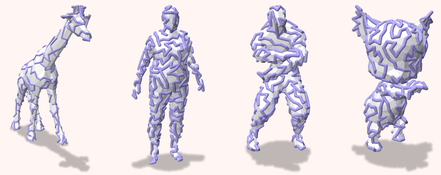


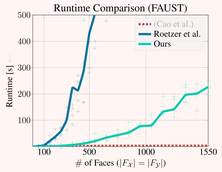
Source Geometric Consistency ✗ Geometric Consistency ✓

Applications: texture transfer, protein matching, statistical shape analysis in medical imaging, 3D reconstruction, shape completion

Our Contributions

- Globally optimal solutions for geometrically consistent 3D shape matching
- Novel integer linear program (ILP) formulation (based on our SpiderCurves)
- Efficiently finding solutions for problems with practically relevant sizes
- SpiderCurve 3D shape representation (closed curve visiting every vertex)





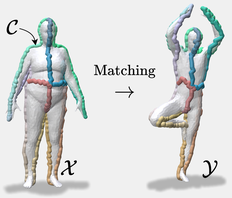
Related Work

Our work is the first to establish **geometrically consistent** 3D shape matchings with **global optimality** and **without** relying on **keypoints** or **pruning** strategies.

Method	Geometrically Consistent	Globally Optimal	Keypoint Free	Pruning Free
SIGMA [4]	✗	(✓)	✗	✗
MINA [1]	✗	(✓)	✗	✗
PMSDP [6]	✗	✗	✓	✗
SM-comb [8]	(✓)	✗	✓	✓
Ours	✓	✓	✓	✓

SpiderMatch

High-Level Idea



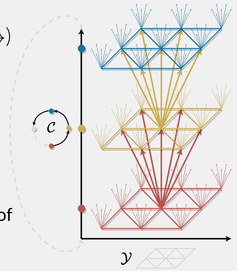
We extract a cyclic path $\mathcal{C}$ on the surface of 3D shape $\mathcal{X}$, and match $\mathcal{C}$ to the 3D target shape $\mathcal{Y}$.

ILP-Formulation

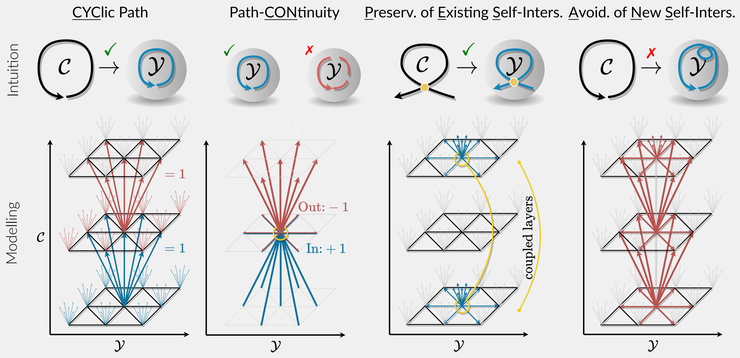
- Discretisation: $\mathcal{C} = (V^{\mathcal{C}}, E^{\mathcal{C}})$ (directed, cyclic graph)
  $\mathcal{Y} = (V^{\mathcal{Y}}, E^{\mathcal{Y}})$ (mesh/undir. graph)
- Search space: edges $E^{\mathcal{P}}$ in product graph $\mathcal{P}$ (inset →)
  size: $|E^{\mathcal{P}}| \approx |E^{\mathcal{C}}||E^{\mathcal{Y}}|$
- Our optimisation problem:

$$\min_{x \in \{0,1\}^{|E^{\mathcal{P}}|}} c^T x \quad \text{s.t} \quad \underbrace{\text{(CYC), (CON),}}_{\text{Dijsktra} \to \text{polyn. time}} \quad \underbrace{\text{(PES), (ANS)}}_{\text{ILP-Solver} \to \text{still fast}}$$

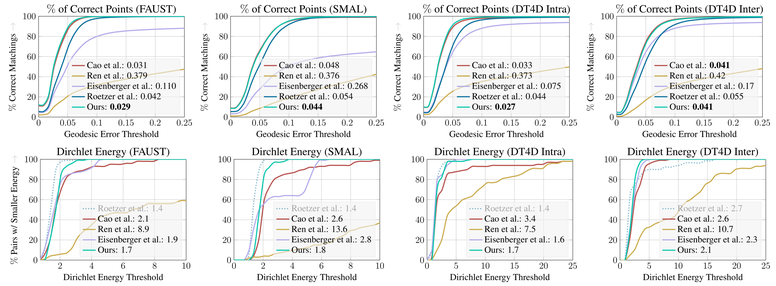
  - $x \in \{0,1\}^{|E^{\mathcal{P}}|}$: binary encoding of $E^{\mathcal{P}}$, i.e. edge i either **part of solution** $x_i = 1$ or **not** $x_i = 0$
  - $c \in \mathbb{R}_+^{|E^{\mathcal{P}}|}$: edge-cost (difference of deep features)



- Constraints:



Results



References

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