$\Sigma IGMA$: Scale-Invariant Global Sparse Shape Matching

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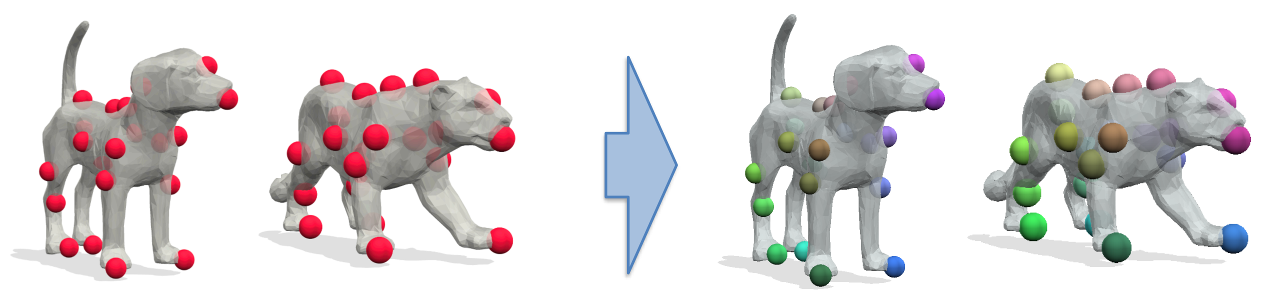
Overview

Problem of interest: **Non-rigid shape matching**

Given a pair of shapes $(\mathcal{X}, \mathcal{Y})$ and their sparse keypoints $(\mathcal{I}, \mathcal{J})$,

we seek for accurate correspondences $\mathbf{P}$, s.t. $\mathcal{I} \sim \mathbf{P}\mathcal{J}$



Projected LBO

+ Combines intrinsic and extrinsic information
+ Used to quantify the quality of deformation
+ useful for *geometry reconstruction*
+ Provably invariant to $E(3)$ action (c.f. Lemma 1)

stiffness matrix

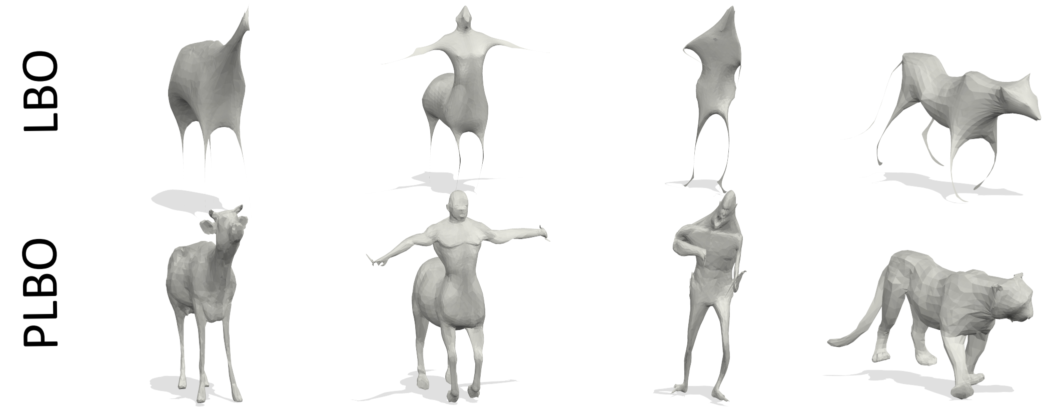
$$\Delta^{(\mathcal{X})}_{\text{proj}} := (\mathbf{\Pi}^{(\mathcal{X})})^\top \Delta^{(\mathcal{X})}_{\text{stiff}} \mathbf{\Pi}^{(\mathcal{X})}$$

all 1 vector

$$\mathbf{\Pi}^{(\mathcal{X})} := \mathbf{I} - \tilde{\mathbf{X}}(\tilde{\mathbf{X}}^\top\tilde{\mathbf{X}})^{-1}\tilde{\mathbf{X}}^\top, \quad \text{with } \tilde{\mathbf{X}} := (\mathbf{X} \quad \mathbf{1})$$

Euclidean coord.

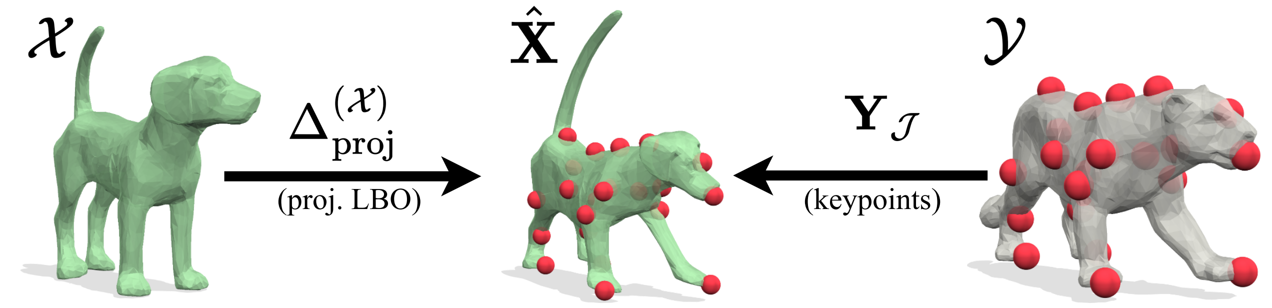
Lemma 1: The PLBO is invariant under $E(3)$ action, i.e. $\Delta^{(\mathcal{X})}_{\text{proj}}(\mathbf{X}) = \Delta^{(\mathcal{X})}_{\text{proj}}(\mathbf{X}\mathbf{R}^\top + \mathbf{1}\mathbf{t}^\top)$ for $\mathbf{R} \in \mathrm{O}(3), \mathbf{t} \in \mathbb{R}^3$



Global Sparse Matching

ΣIGMA

+ Mix-Integer Program (MIP) formalism
+ global optimality guarantee
+ initialisation-free
+ provably invariant to global scaling & SE(3)



reconstruction term *deformation term*

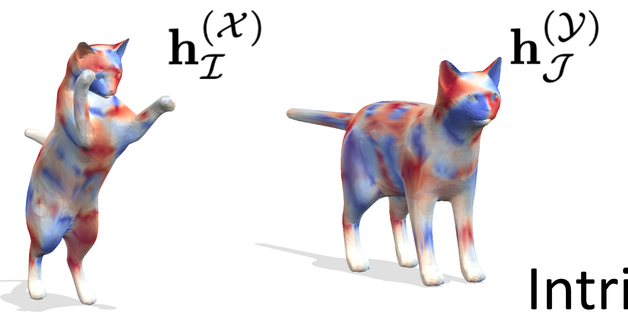
$$\min_{\mathbf{P},\hat{\mathbf{X}}} \frac{1}{n}\|\hat{\mathbf{X}}_{\mathcal{I}} - \mathbf{P}\mathbf{Y}_{\mathcal{J}}\|_F + \frac{\lambda}{n_v^{(\mathcal{X})}}\|\Delta^{(\mathcal{X})}_{\text{proj}}\hat{\mathbf{X}}\|_F$$

$$\text{s.t.} \quad \mathbf{P} \in \{0,1\}^{n\times n},\ \mathbf{P}^\top\mathbf{1}_n = \mathbf{1}_n,\ \mathbf{P}\mathbf{1}_n = \mathbf{1}_n$$

Optionally for shape with intrinsic symmetry, an *orientation-aware term* can be added:

Orientation features

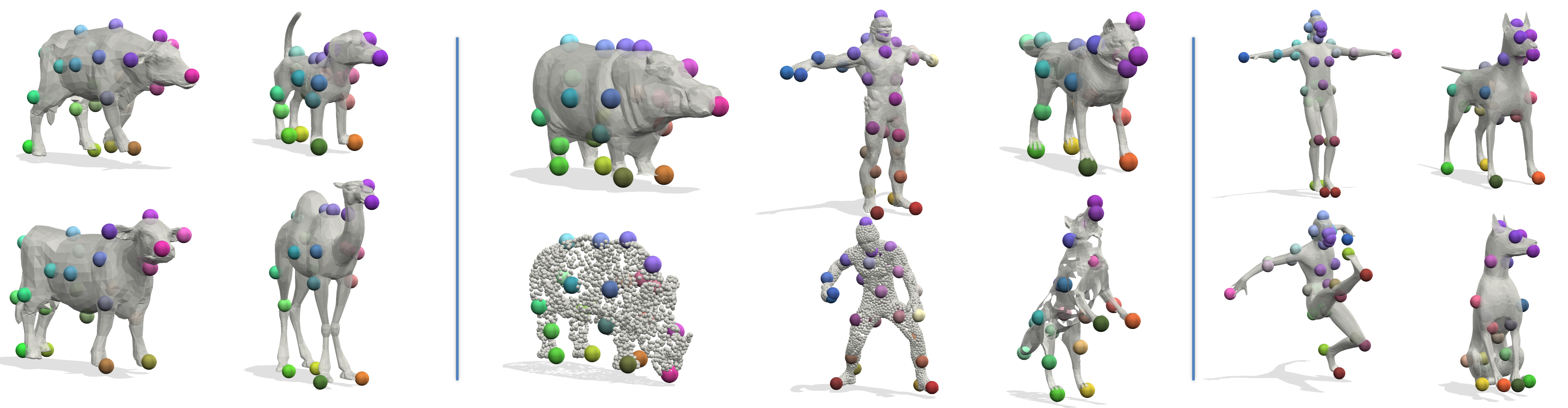
$$\frac{1}{n}\|\mathbf{h}^{(\mathcal{X})}_{\mathcal{I}} - \mathbf{P}\mathbf{h}^{(\mathcal{Y})}_{\mathcal{J}}\|_F$$



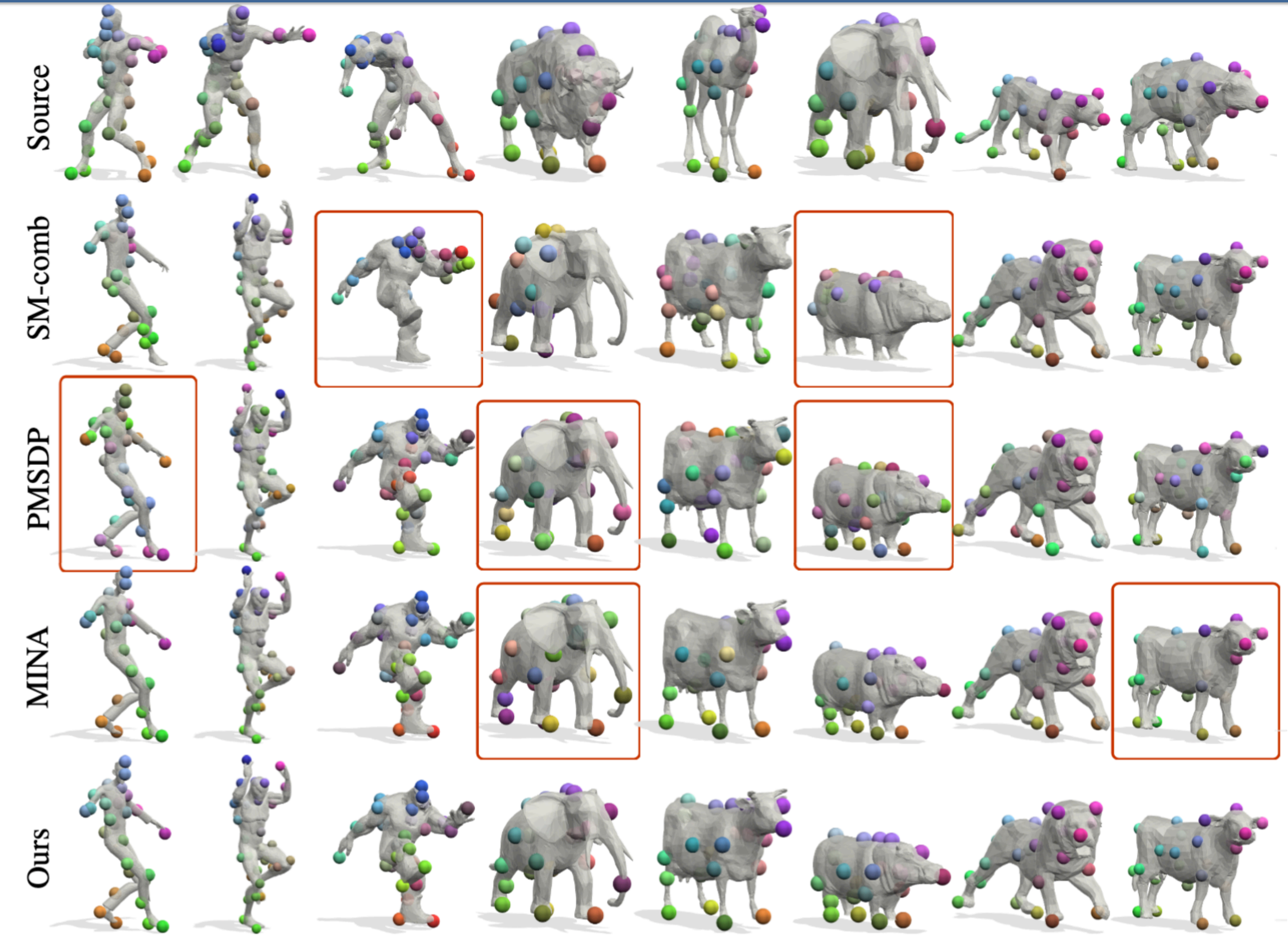
$$\mathbf{h}^{(\mathcal{X})} = \begin{pmatrix} \langle(\nabla\mathbf{f}^{(\mathcal{X})})_1 \times (\nabla\mathbf{g}^{(\mathcal{X})})_1, \mathbf{n}^{(\mathcal{X})}_1\rangle \\ \vdots \\ \langle(\nabla\mathbf{f}^{(\mathcal{X})})_{|\mathbf{X}|} \times (\nabla\mathbf{g}^{(\mathcal{X})})_{|\mathbf{X}|}, \mathbf{n}^{(\mathcal{X})}_{|\mathbf{X}|}\rangle \end{pmatrix}$$

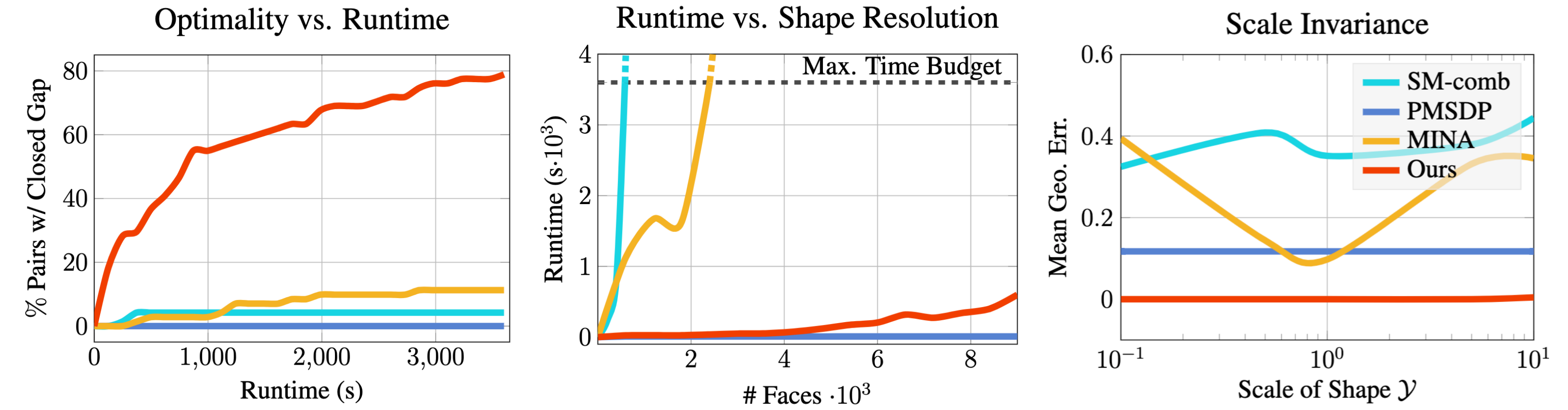
Intrinsic gradients outer normals

Lemma 2: The global optimum of $\Sigma IGMA$ formalism is invariant under $SE(3)$ transformation of the input shapes $\mathcal{X}, \mathcal{Y}$.



Experiments





References

[1] Bernard, Suri, Theobald. Convex Mixed-Integer Programming for Non-Rigid Shape Alignment. CVPR, 2020.
[2] Roetzer, Swoboda, Cremers, Bernard. A Scalable Combinatorial Solver for Elastic Geometrically Consistent 3D Shape Matching. *CVPR,* 2022.
[3] Maron, Nym, Kezurer, Kovalsky, Lipman. Point Registration via Efficient Convex Relaxation. *SIGGRAPH,* 2016.
[4] Ren, Poulenard, Wonka, Ovsjanikov. Continuous and Orientation-preserving Correspondence via Functional Maps. *SIGGRAPH,* 2018.
[5] Dyke, Lai, Rosin, Zappalà, Dykes, Guo, Li, Marin, Melzi, Yang. SHREC'20. *Computers & Graphics,* 2020.
[6] Magnet, Ren, Sorkine-Hornung, Ovsjanikov. Smooth Non-Rigid Shape Matching via Effective Dirichlet Energy Optimization. *3DV,* 2022.
[7] Zuffi, Kanazawa, Jacobs, Black, *3D Menagerie: Modeling the 3D Shape and Pose of Animals. CVPR,* 2017.