Analysis of an A340 Takeoff Video

I came across a video of an A340 taking off from Viracopos International Airport in South America where the aircraft airborne only just before before the end of the runway. You can see the video here, it starts part way through the take off:

It seemed an unusual take off to me and I was curious about what objective information such as speed and distance data could be extracted from the video by looking at individual frames.

Data

The aircraft was identified on youtube as an A340-300 registration 9H-BIG operated by Air-X. It was taking off from runway 15. I used Wikipedia to get some drawings of the A340-300. And of course OpenStreetMap and Google Earth and Google Street View.

Unknowns

I don't know the following:

- Anything about A340 operations, reference speeds, limits, screen heights etc.
- The date and time of this video.
- The metrological conditions existed at the take off time.
- The terrain, especially runway slope.
- Any movement of the observer (I have assumed that the observer is stationary).
- Any details of the video camera or operator.

- Any editing of the video that alters its fidelity (the video is taken in good faith).

If any of these assumptions are wrong it will affect this analysis greatly.

Measurements

Three measurements seemed practical and useful:

- Ground speed; the aircraft transits multiple fixed objects such as lampposts, lighting towers and so on. Taking transit points such as the aircraft nose and tail cone and knowing the length of the fuselage gives the ground speed. Some of the transit points can be identified on publicly available aerial photography. These transits can be used to determine the aircraft position once the observers position is known.
- Aspect; this is the relative bearing of the observer from the axis of the aircraft. The measurement is made by observing when parts of the aircraft line up (for example the nose with number 3 engine). A more accurate method is to measure the apparent span and length of the aircraft in a single frame.
- Pitch; The pitch of the aircraft was measured by comparing the line of cabin windows relative to the video frame. This is not a very reliable measurement as it is vulnerable to camera roll, which is unknown. Still, some conclusions can be drawn.

An estimate of the error was made for each measurement.

Terminology

- ‘Time’ is video time in seconds, unless specified otherwise.
- ‘Distance’ is in metres from either the start of the video or the start of the runway (as specified).
- ‘X axis’ is along runway 15 with x=0 at the threshold of runway 15. Values in metres.
- ‘Y axis’ is at right angles to runway 15 +ve to the right, -ve to the left. Values in metres.

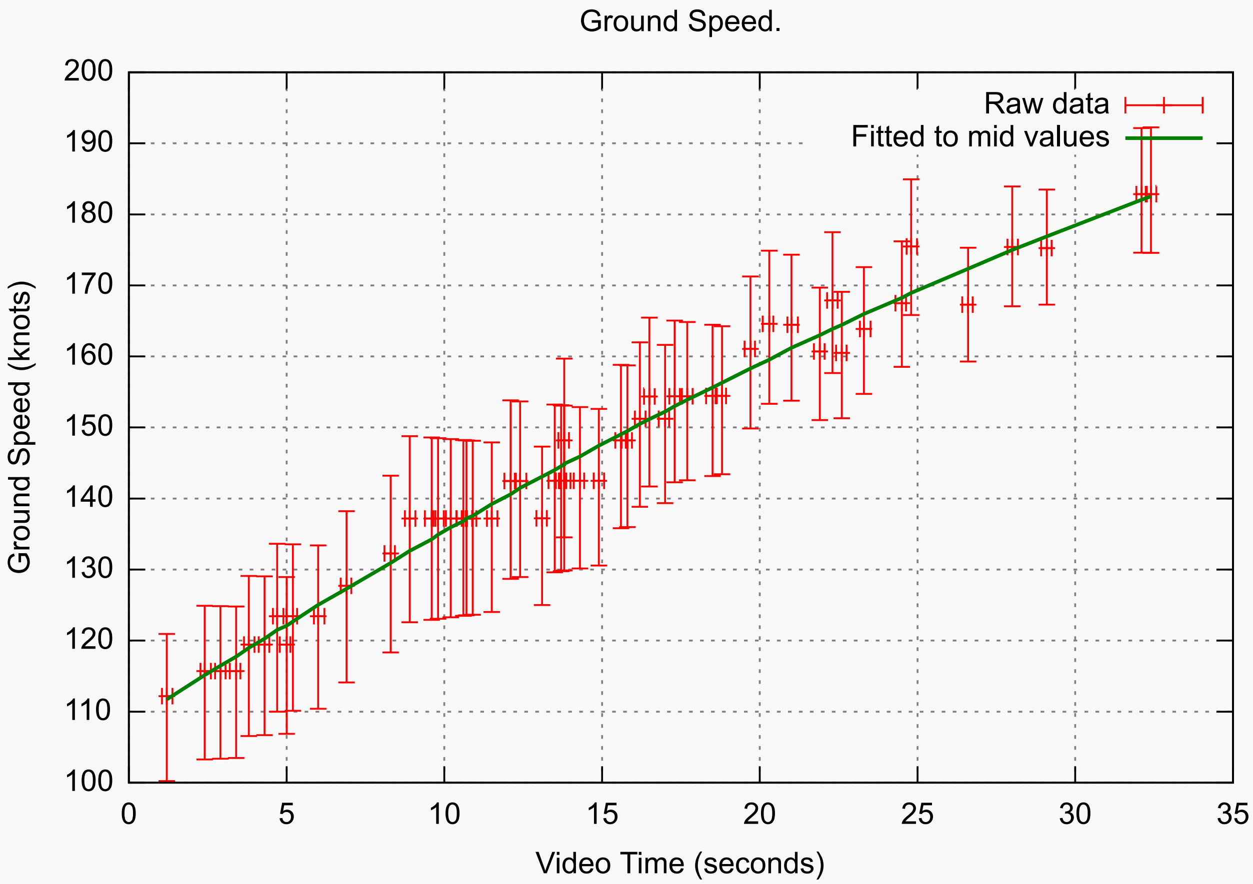
Analysis

Ground Speed

52 transits of fixed objects were measured during the video and the ground speed and likely error were calculated. The error is twofold:

- Error in assessing the time of the event, this is assumed to be ±5 frames or ±1/6 of a second.
- Transit error, this is assumed to be ±1 frame (±1/30 second). However the error calculation is exaggerated by the aspect of the aircraft so when the observer is at right angles to the path of flight it is at a minimum but increases as the observer approaches the for and aft aircraft axis (where the error would be infinite). See ‘Aspect’ below.

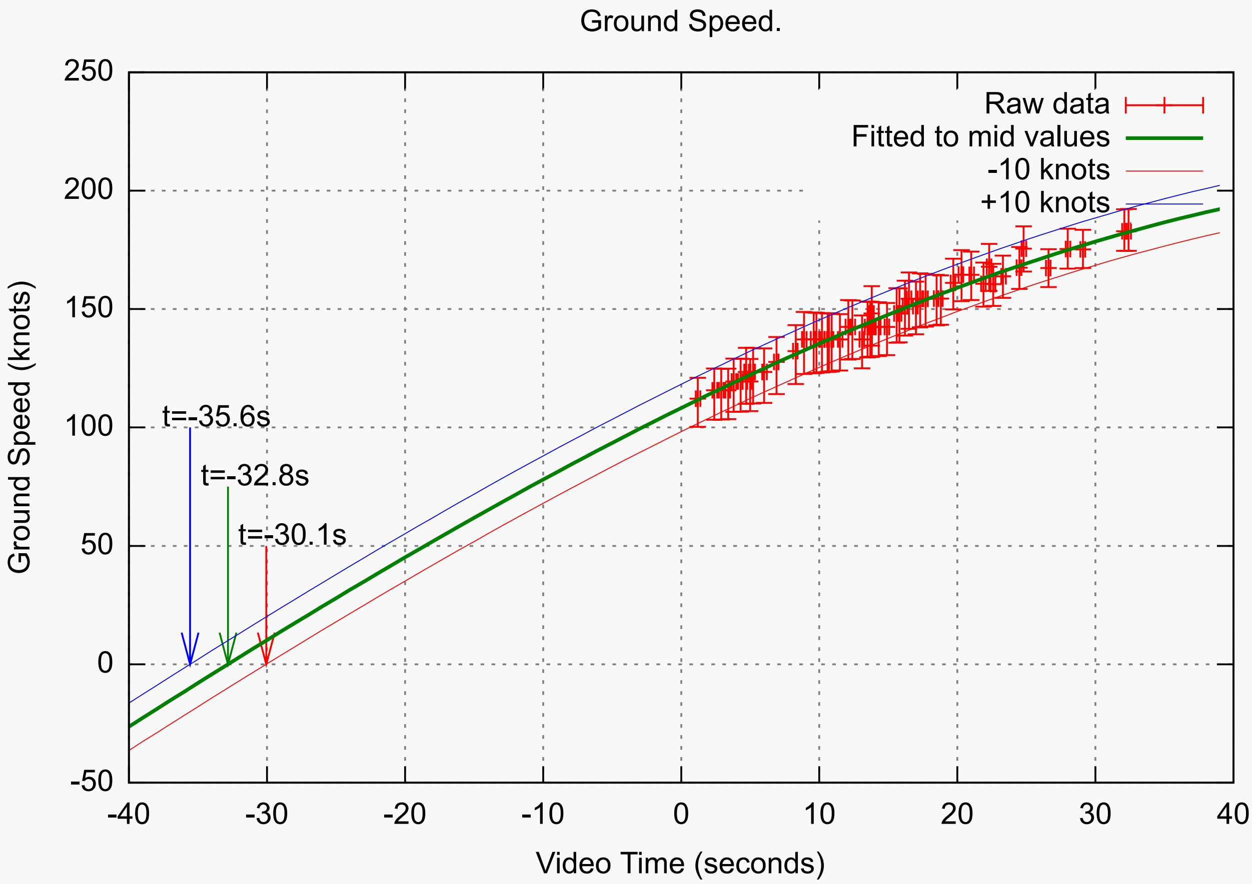
Here are the calculated ground speed in knots with the errors and a best fit to the mid value:



The ground speed error is ±10 knots.

Extrapolating Before the Video Starts

The best fit curve can be extrapolated back to the start of the take off:



This is a large extrapolation, with its consequent dangers, but the data seems reasonable. This extrapolation gives a start of take off (video) time of -33 ±3 seconds.

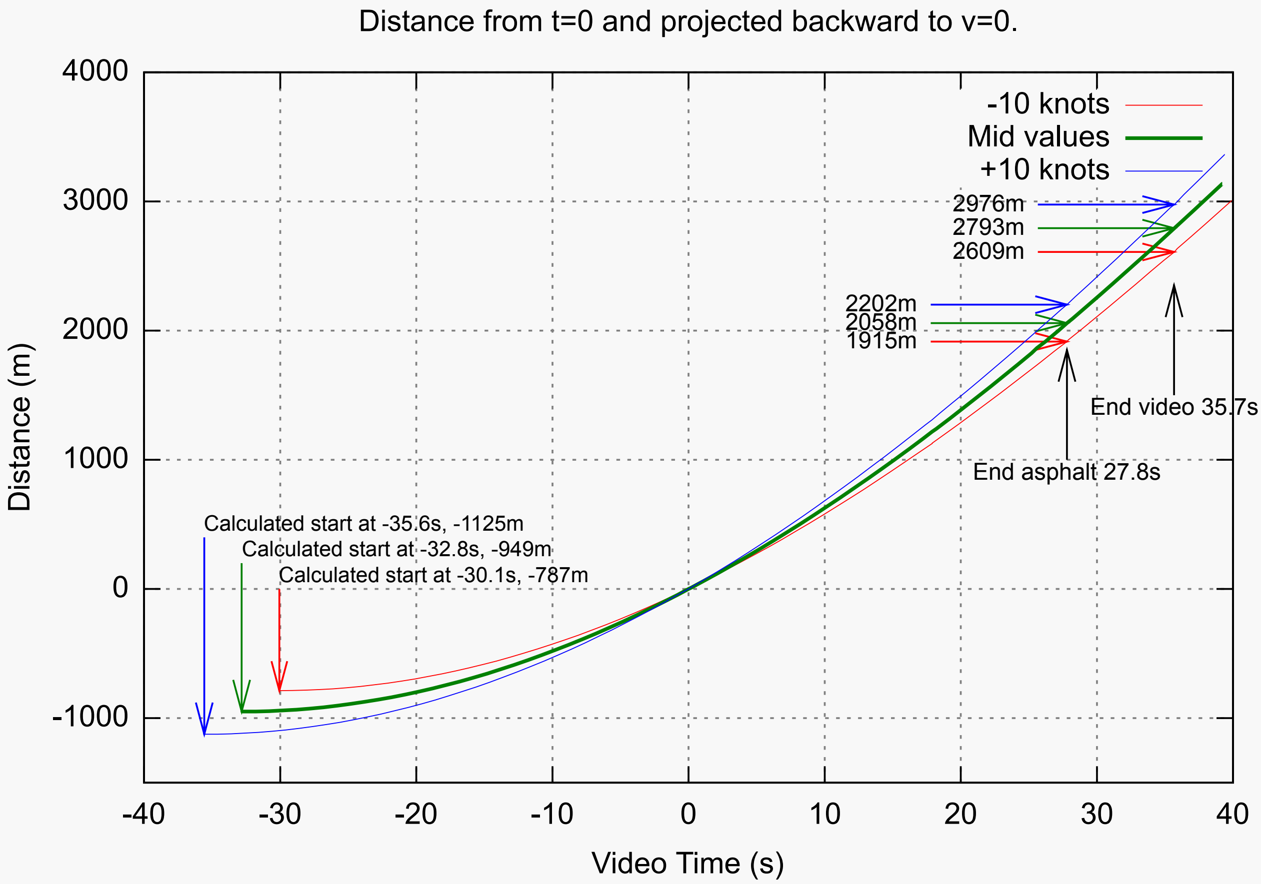
Distance

Distance from video start can be estimated by integrating the ground speed data. This calculation will be affected by errors in ground speed estimation proportional to the speed error multiplied by time.

This graph shows the distance traveled for:

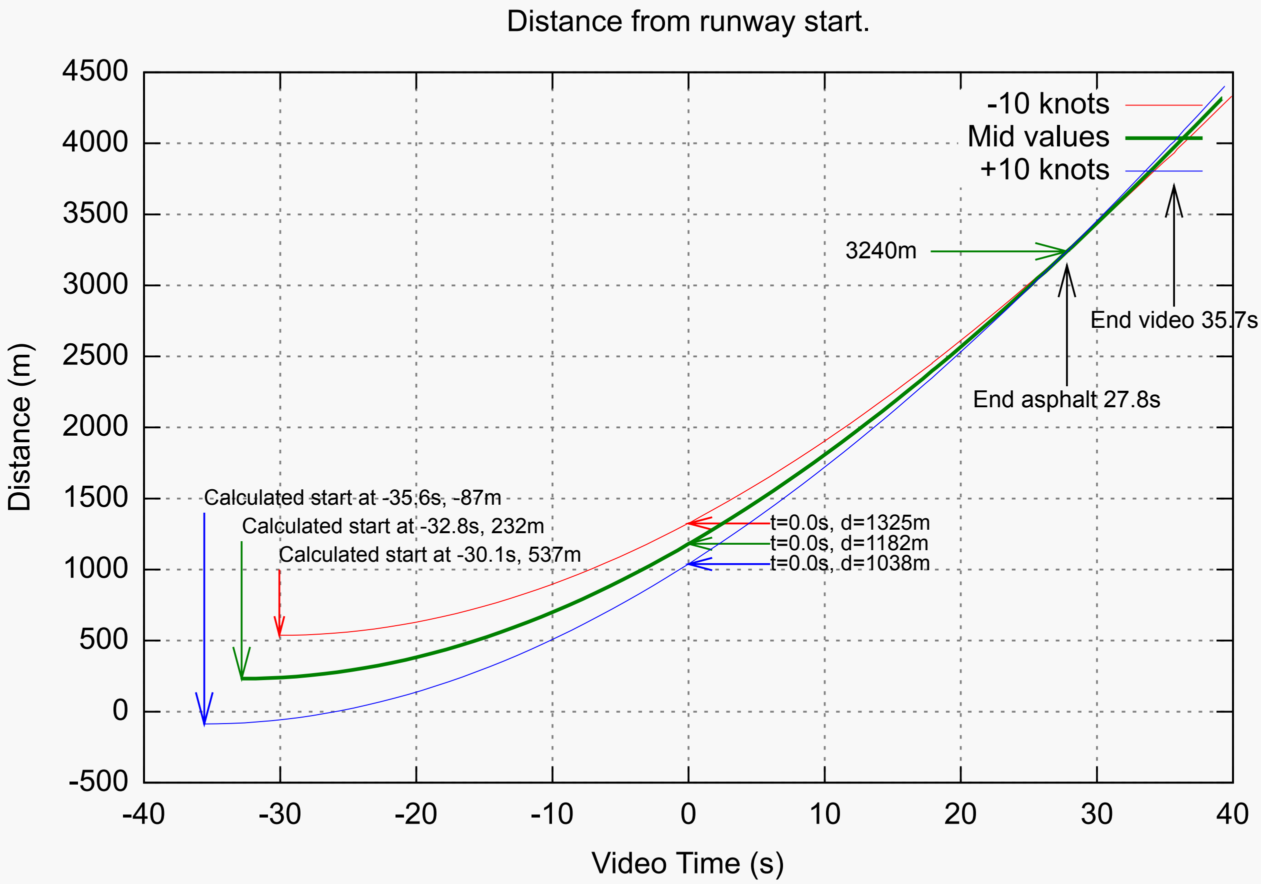
- Ground speed from mid values -10 knots.
- Ground speed from mid values.
- Ground speed from mid values +10 knots.

The graph is centered around t=0 and distance from start of video = 0.



Calculating the Location of the Start of Take Off

The distance curves can be shifted such that the estimates intersect when the aircraft crosses the end of the asphalt at t=27.8s. This is d=3240m from the start of the runway. Now the graph shows video time on the x axis and the estimated distance from the runway start on the y axis.



This graph shows:

Calculation	Start time (s, video time)	Distance from start of Runway (m)	Distance from start of Runway t=0 (m)
Mid speed -10 knots	-30.1	537	1325
Mid speed	-32.8	232	1182
Mid speed +10 knots	-35.6	-87	1038
Range, worst error case	**-33 ± 3**	**232 ± 319**	**1182 ± 144**

The mid position looks entirely plausible as from OpenStreetMap there are two entry points to runway 15:

- Taxiway D which is at the runway start.
- Taxiway H which is about 200m from runway start.

Either could have been used, the accuracy of distance calculation is too poor to say which at the moment. However further confidence and accuracy can be gained in this ground speed / distance calculation by looking at the calculation of observer position.

Bearings to the Observer

As the aircraft passes down the runway it presents a difference *aspect* to the observer and this can give valuable clues to the observers relative position (assumed fixed). The aspect can be measured by seeing when parts of the aircraft line up with each other, for example the nose and the centre of engine number three.

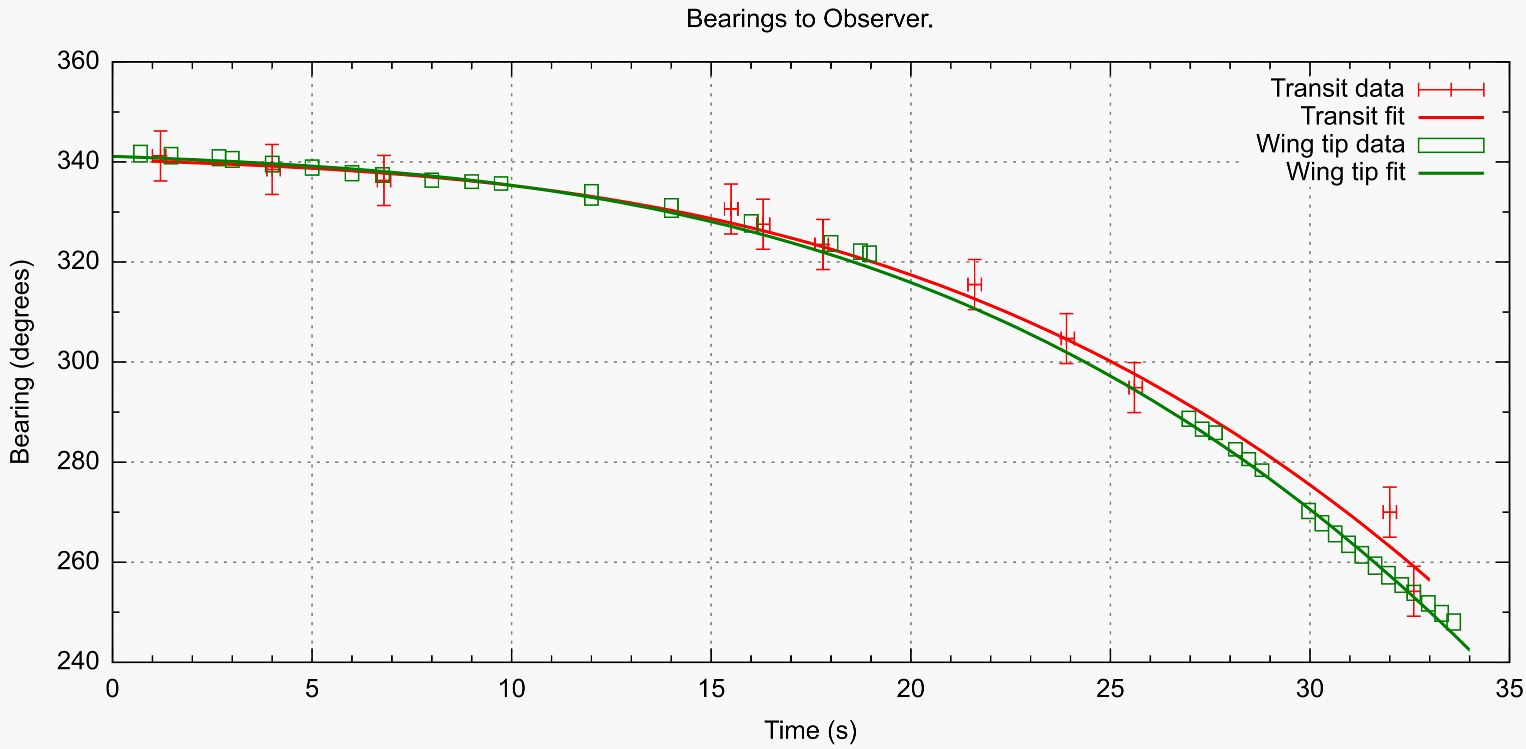
The value of this measurement is it contributes to an error estimate for ground speed and can identify the observer location (assumed fixed) and so provide a check on the ground speed/distance calculation.

11 measurements of aspect were made and the Wikipedia plan drawing of the A340-300 was used to estimate the bearing of the observer from the fore and aft axis. The error in aspect/bearing is assumed to be ±3 degrees.

Another set of measurements were made by comparing the relative size of the apparent span and apparent length. This is believed to be less sensitive to errors due to the larger baseline involved in the measurement.

Initially we assume that the aircraft has a constant heading with no yaw but once we have identified the observers position we can examine any yaw that developed, for example to compensate for a crosswind.

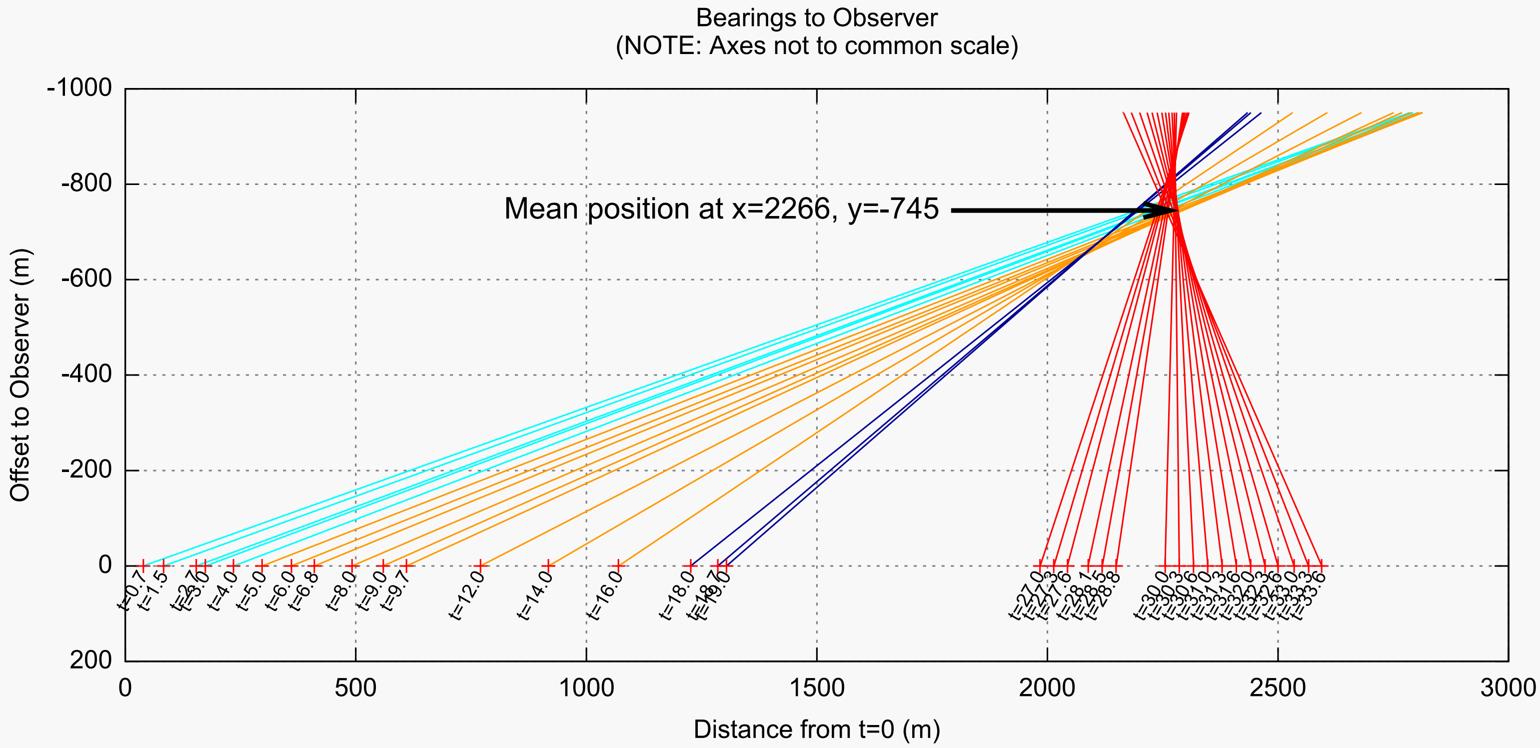
Here is a graph of those bearings with the estimated errors and best fit of the data:



The wing tip data is more sensitive than the transit data and will be used from here on.

Combining Bearings and Distance Data

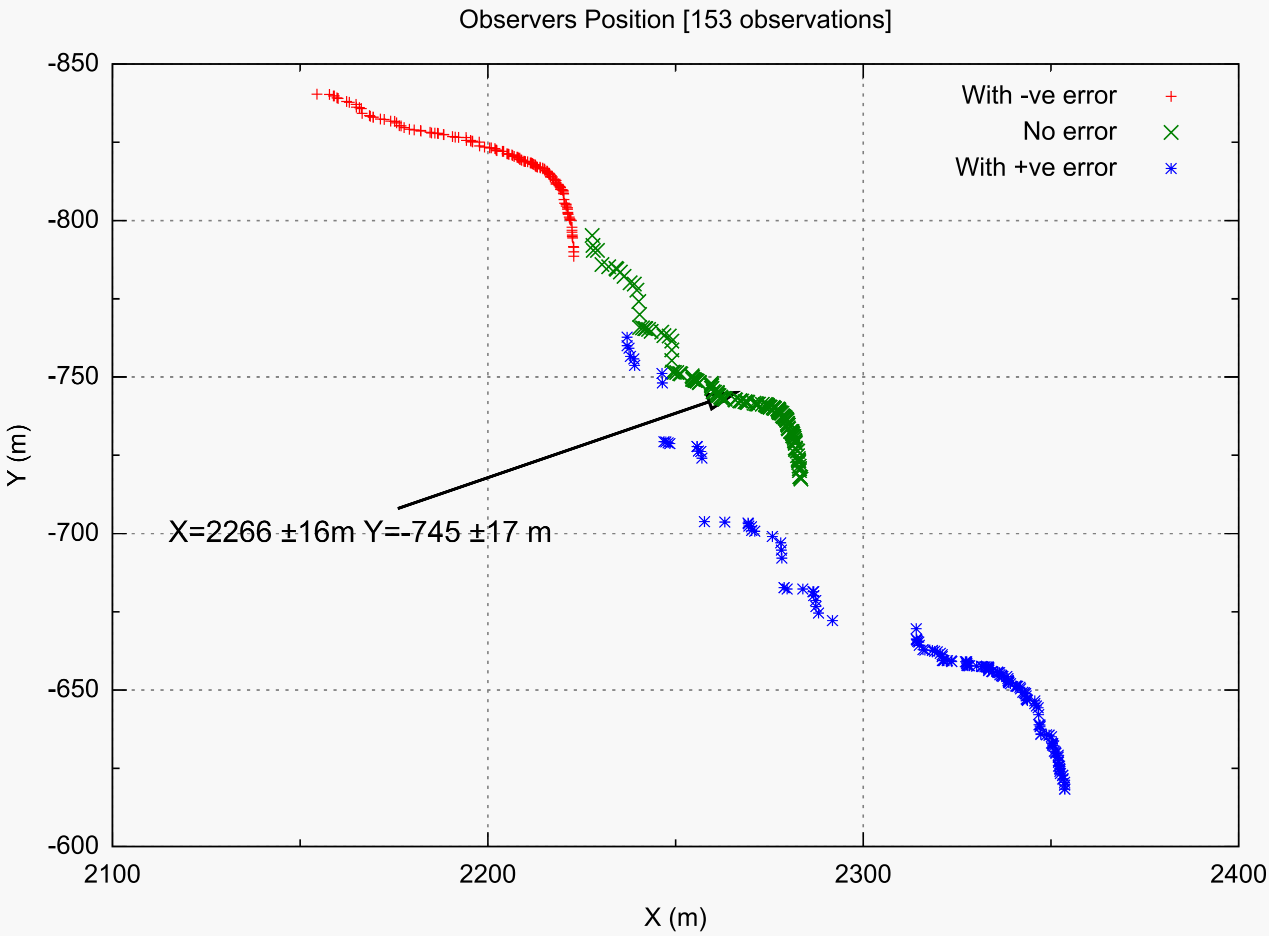
The bearings to the observer are plotted here:



These bearings are filtered:

- The Cyan bearings are ignored as the angle is too small to be accurate.
- Pairs of bearings are ignored unless the baseline between them is >1250m

Selecting all the combinations of remaining bearing pairs gives the following observer positions. X is distance from start of video, Y is the distance to the left or right of the aircraft axis +ve right and -ve left.



The observers position is given above, given our estimate of the position of the aircraft at t=27.8s at the end of the asphalt.

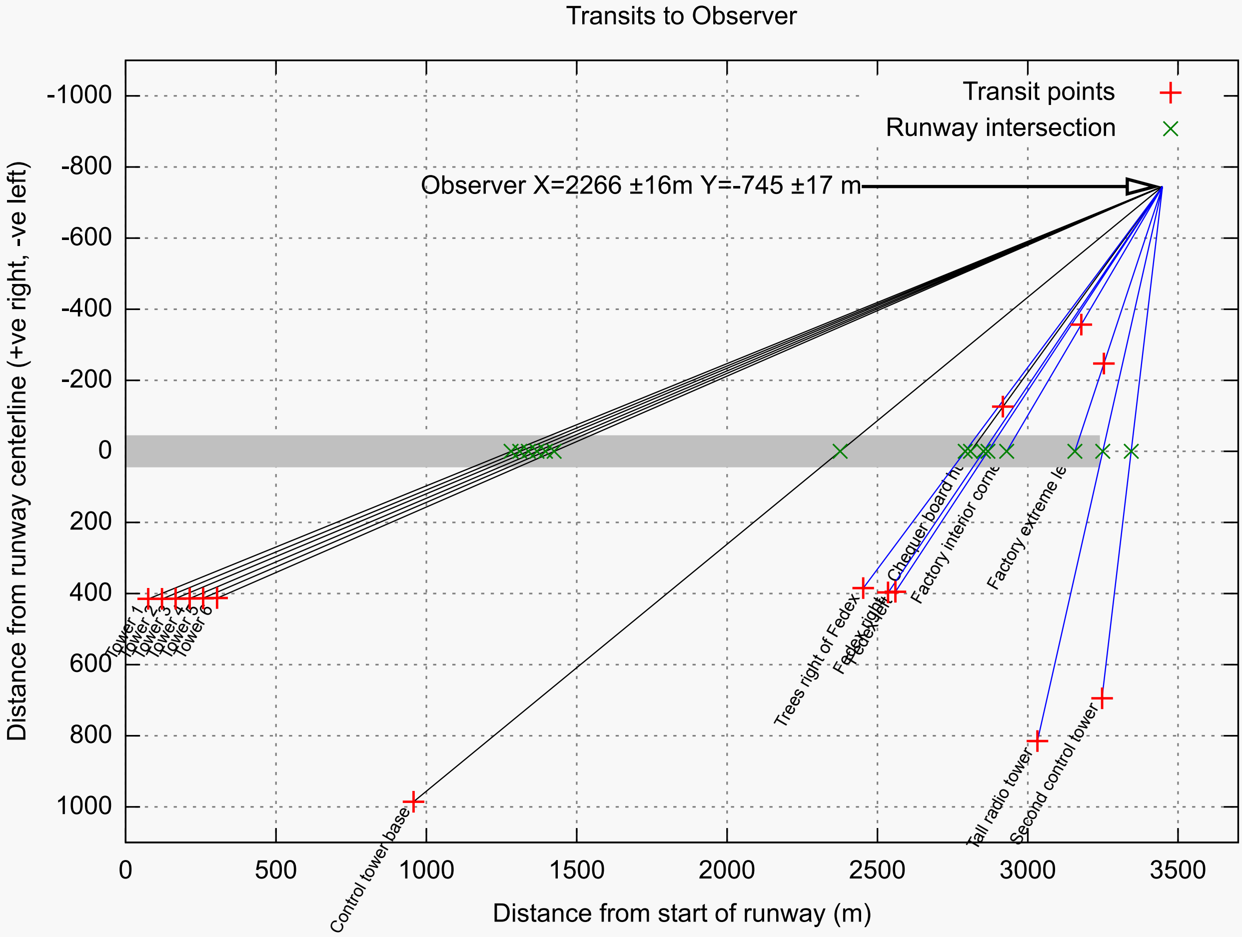
Here is a Google StreetView image of that area facing towards the extended centreline. Features that match are:

- The tower in the far distance on the right which appears in the video at t=29.
- The flyover mid right in the far distance.
- The power cables on the left that are seen at t=32 where they are extremely foreshortened as they are in line with the camera.

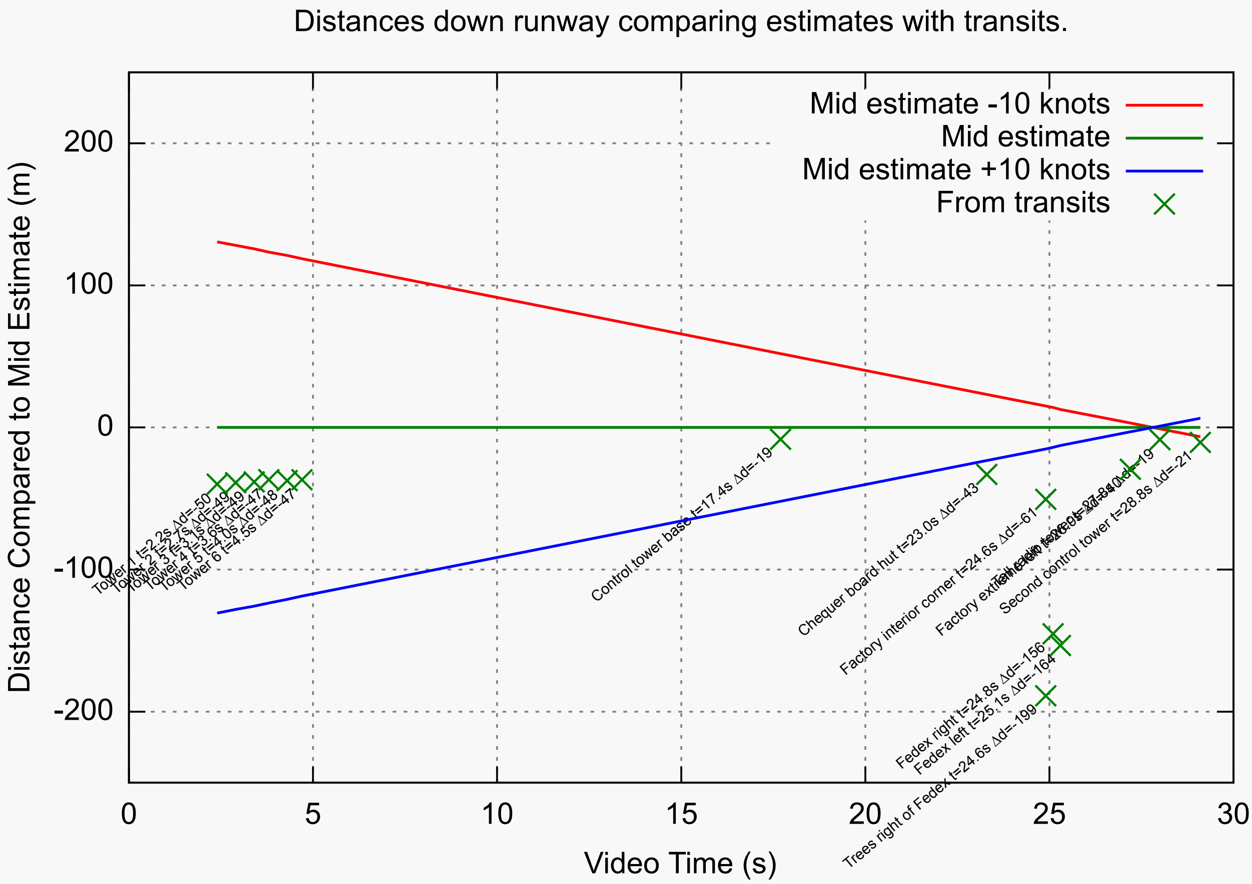
(Copyright Google)

The observer is to the left of this Google StreetView image and at an elevation about half way up the power poles to be able to see the runway.

Using Transits to Improve the Distance Calculation



Runway transit distances:



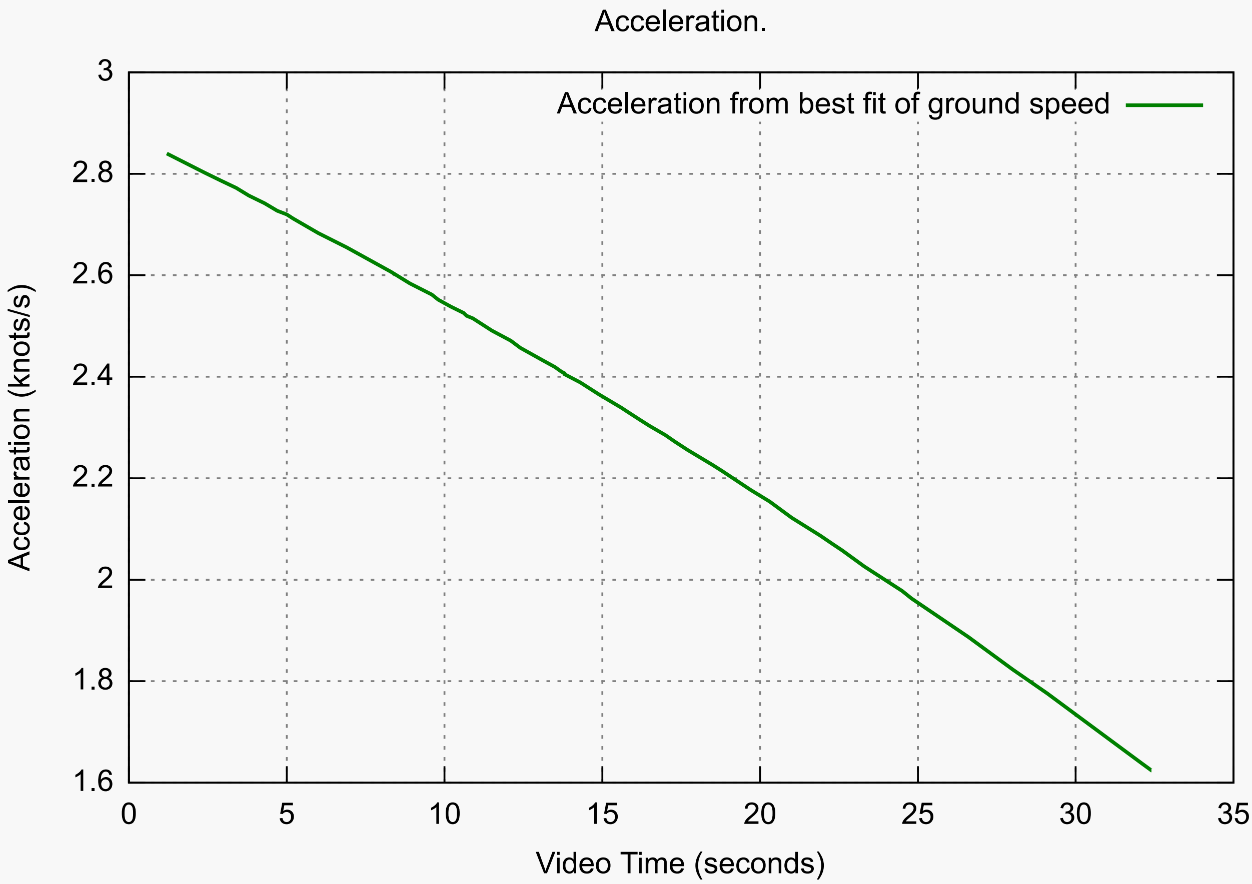
This additional calculation suggests that our ground speed estimate is low by 3 knots but the accuracy has improved to ±5 knots. This also suggests that the distance estimate is correct ±25m for the duration of the video.

Acceleration

This can be calculated by taking the derivative of the ground speed. Given constant thrust the acceleration would be expected to decline in several stages:

- As parasitic drag increases: 0 <= t < 18
- Additionally it falls further as induced drag is added as the nose wheel comes off: 18 <= t < 25.5
- Additionally it falls even further as the aircraft starts to climb: 25.5 <= t < 33.7

None of these specific events are visible in the analysis, likely due to smoothing of the ground speed data but the calculated acceleration shows the expected (general) decline.



Errors in acceleration

The acceleration from our ground speed model is identical whether a -10 or +10 knot error is assumed. However an error estimate can be made by looking at the time and distance error estimates.

Speed Error	Time from start take off to end asphalt (s)	Final Speed (knots)	Mean Acceleration (knots/s)	Mean Acceleration Error (knots/s)	Distance from start take off to end asphalt (m)
-10 knots	57.9	165	2.85	-0.04	2703
0 knots	60.6	175	2.89	0	3008
+10 knots	63.4	185	2.92	+0.03	3327

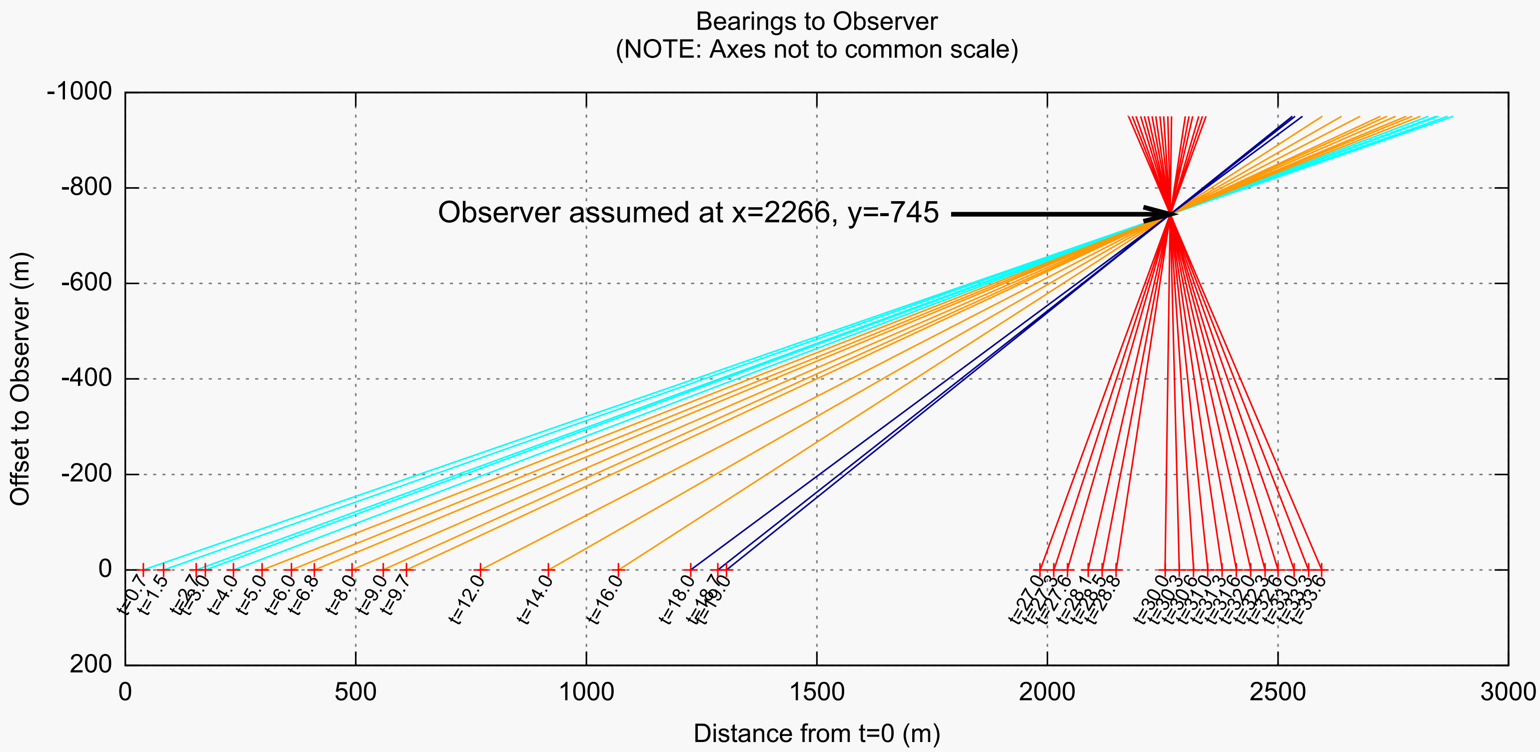
This suggests that the acceleration figures are likely correct to ±0.04 knots/second.

The extreme worst case is when the time estimate and speed estimate are reversed:

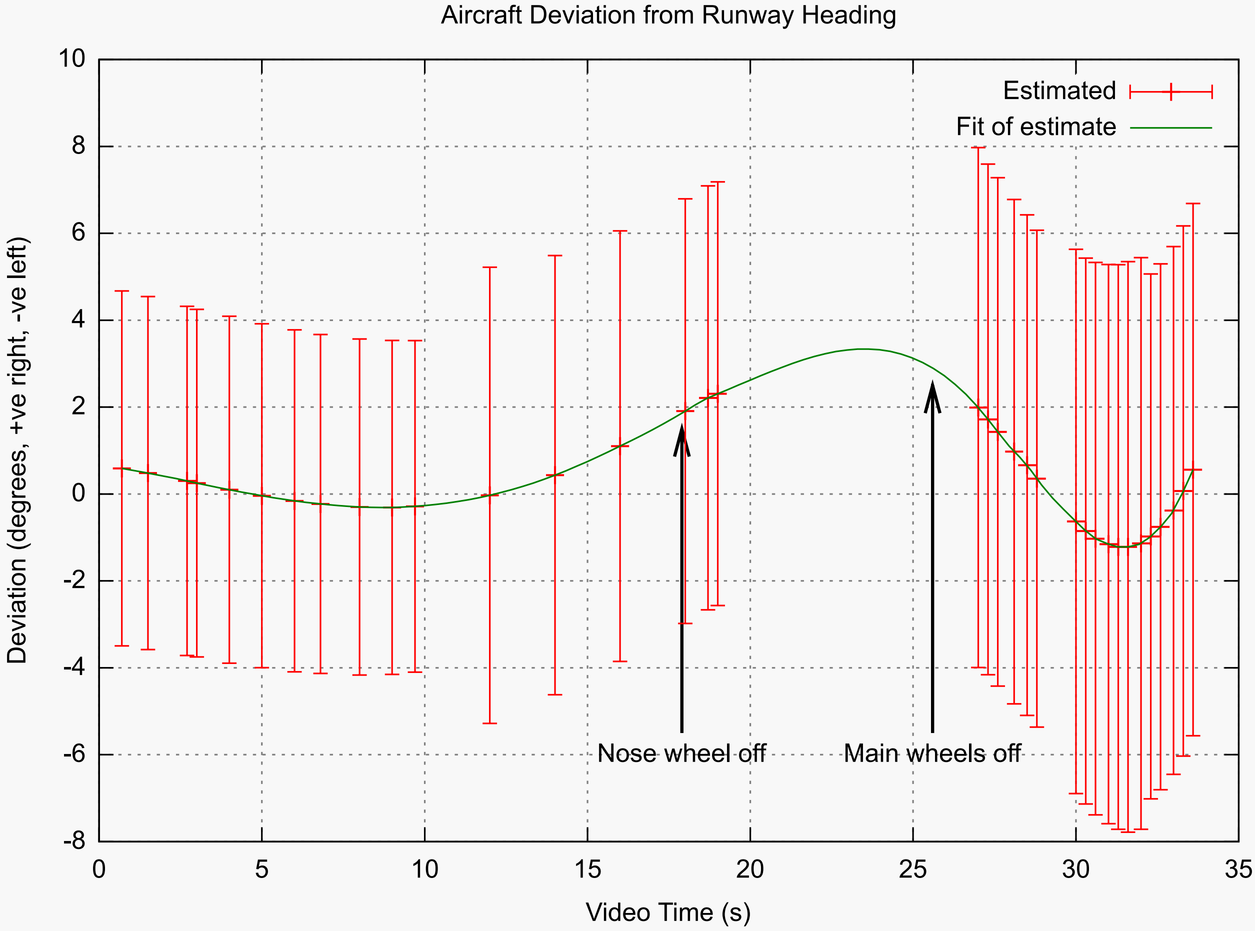
Speed Error	Time from start take off to end asphalt (s)	Final Speed (knots)	Mean Acceleration (knots/s)	Mean Acceleration Error (knots/s)	Distance from start take off to end asphalt (m)
-10 knots	63.4	165	2.60	-0.29	2692
0 knots	60.6	175	2.89	0	2729
+10 knots	57.9	185	3.20	+0.31	2756

Aircraft Yaw

Assuming now we know the observers position we can re-plot the bearings through the observers position and calculate the yaw of the aircraft. The bearings now look like this:



And the calculated yaw is the adjustment made to the bearing data to make it coincide with the observers position:



The error term vastly exceeds the calculated data but, even so, there seems an interesting, but tentative, story in the estimated values.

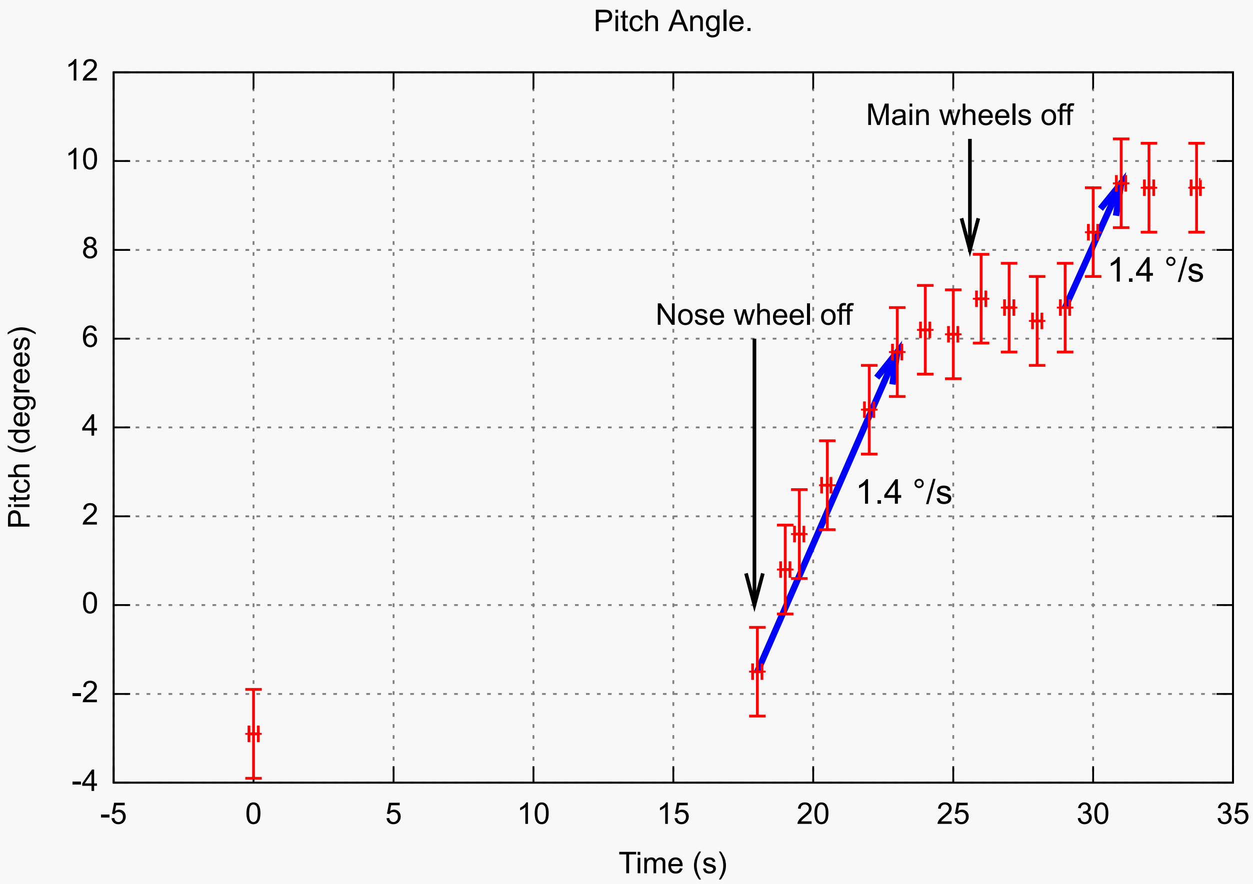
- In the early part of the take off whilst all wheels are on the ground the yaw is within a small range (<1 degree).
- As the nose wheel comes off the asphalt a yaw of about 3 degrees to the right develops.
- As the aircraft becomes fully airborne the yaw returns initially to 1 degree left and then around zero. Maximum yaw rate is around 1.5 degrees per second.

These conclusions are tentative because the error terms vastly exceed the trend.

Aircraft Pitch

Pitch is regarded as the least reliable measurement as it is directly affected by camera roll angle of which we know nothing.

The graph below shows the estimated pitch. At t=17.9 rotation starts at 1.4 degrees/second until a pitch angle of +8 degrees (relative to the nose wheel on the runway). A further increase to +12 degrees relative is observed at t=29 to 31.



Angle of View

TODO

Conclusions

Table of Events

Event	Video Time (s)	Time from Start Take Off (s)	Ground Speed (knots)	Acceleration (knots/s)	From Start of Runway (m)	To end Asphalt (m)	Notes
Start of take off	-32.8 ± 2.8	0.0 ± 2.8	0	3.6	232 ± 312	3008 ± 312	Estimated
Video starts	0.0	32.8	108 ± 10	2.9	1182 ± 143	2058 ± 143	
Nose wheel off	17.9	50.7	154 ± 10	2.2	2400 ± 51	840 ± 51	Rotation of ~1.4 °/s to t=23
Main wheels off	25.6	58.5	171 ± 10	1.9	3048 ± 11	192 ± 11	
End asphalt	27.8	60.6	175 ± 10	1.8	3240 ± 0	0 ± 0	Defined datum
Video ends	35.7	68.5	188 ± 10	1.5	3974 ± 40	-734 ± 40	

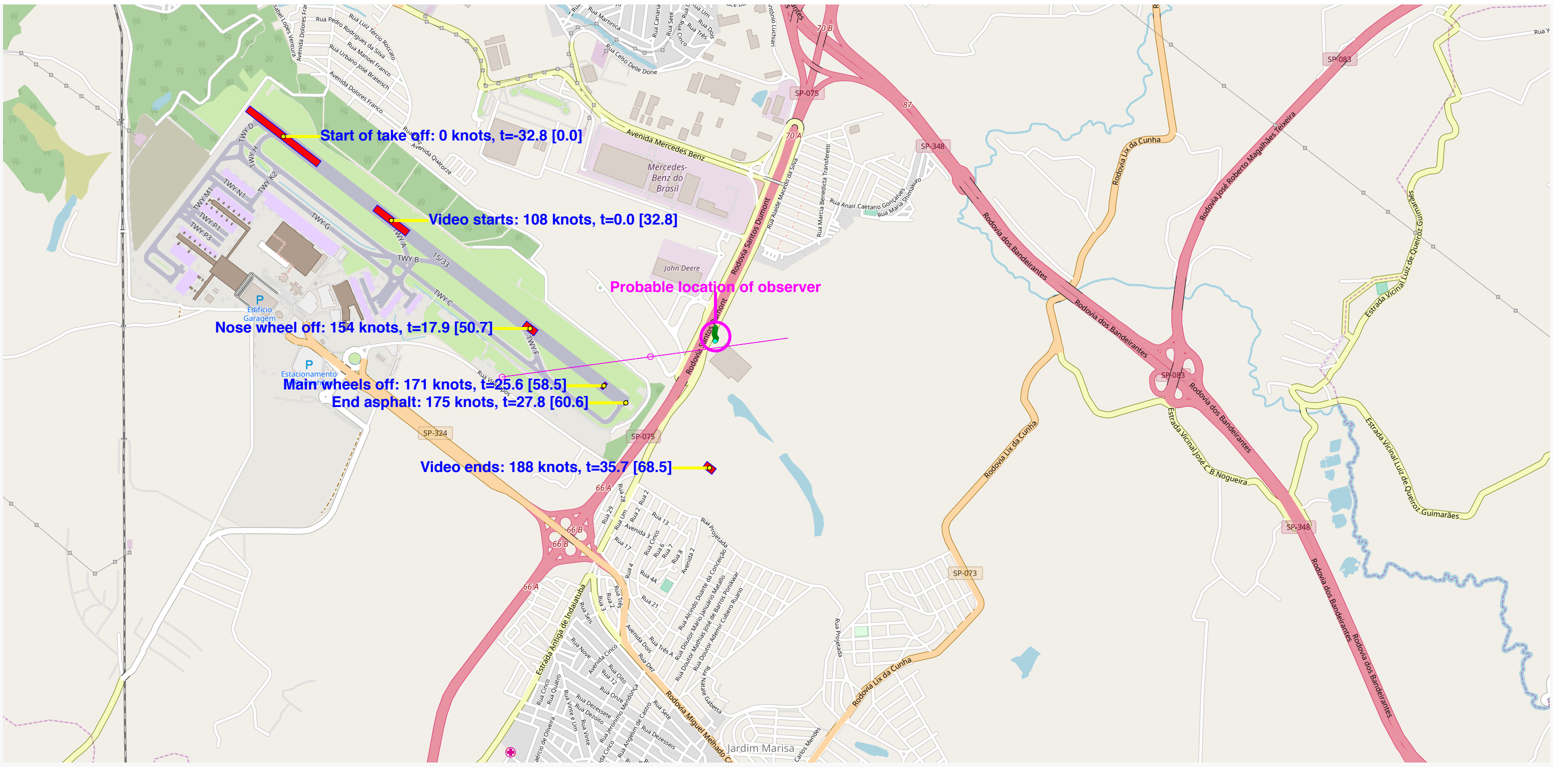
Mapping

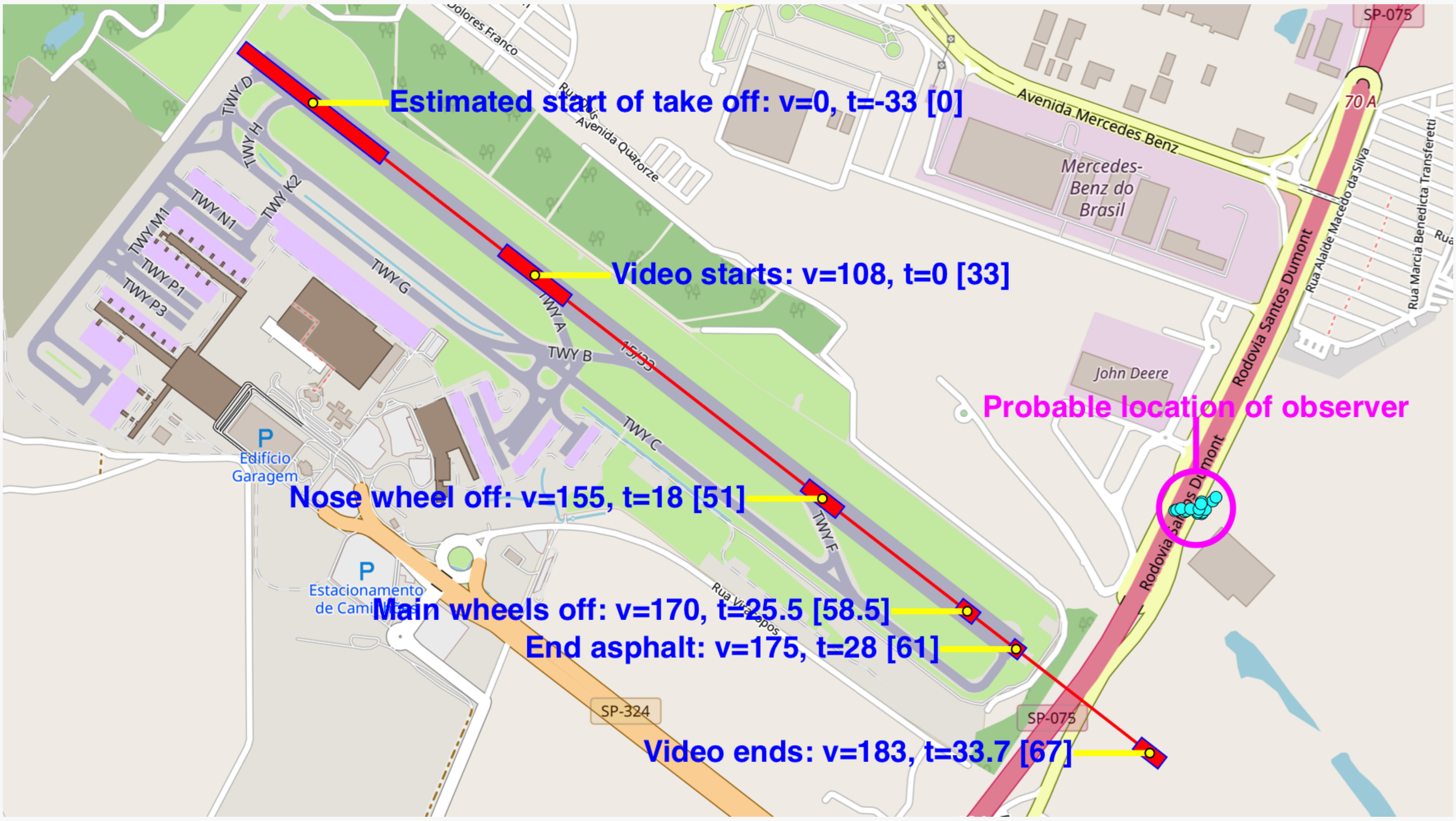
Here is the data plotted on an image of Viracopos International Airport from Open Street Map. The estimated ground speed (knots) for each event is shown with the t=video time followed by the estimated time since start of take off in square brackets. The red boxes illustrate the accuracy of the position estimate.

The probable location of the observer is also shown.

The annotations in blue contain:

- v= The ground speed in knots.
- t= The time as video time in seconds.
- […] The time as estimated time from start of take off in seconds.





Resources

Software

- `ffmpeg` video extraction software.
- Graphic Converter - Pixel perfect frame measurement.