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Decision Support

A hierarchical consensus reaching process for group decision making with noncooperative behaviors

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ABSTRACT

With the development of technological and societal paradigms, we witness a trend where a large number of experts participate in decision-making processes, and large-scale group decision making has become a much researched topic. A large-scale group decision-making problem usually involves many experts with various backgrounds and experiences. In these cases, an effective consensus reaching process is essential to guarantee the support of all experts, especially in large-scale group decision-making settings. This study proposes a hierarchical consensus model that allows the number of adjusted opinions to vary depending on the specific level of consensus in each iterative round. Furthermore, this study also introduces a method to detect and manage noncooperative behaviors by means of the hierarchical consensus model. The minimum spanning tree clustering algorithm is used to classify experts. A weight determination method combining the size of the subgroup and the sum of squared errors is developed for subgroups. Finally, an illustrative example is provided to demonstrate the practicality of the proposed model.

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1. Introduction

With the rapid development of technological and social paradigms, such as Web 2.0 (Simeonova, 2018), e-democracy (Sundberg, 2019), and social networks (Peng et al., 2018), there is a tendency for a large number of experts to participate in decision-making processes. Therefore, large-scale group decision-making (LSGDM) has become a timely topic and has attracted widespread attention (Palomares, Martínez & Herrera, 2014; Shi, Wang, Palomares, Guo & Ding, 2018; Xu, Du & Chen, 2015). An LSGDM problem can be defined as a situation in which at least 20 experts (Tang et al., 2021) participate to select the best option from a set of possible alternatives.

An LSGDM problem usually involves many experts with various backgrounds, professional skills and levels of experience. At the beginning of the decision-making process, experts' opinions may differ significantly. As a result, the group decision result does not usually guarantee the support of all experts because they may think that their opinions are not fully considered. Therefore, the consensus reaching process (CRP) is a necessity in LSGDM. A CRP is a dynamic and iterative process aiming to improve the level of agree-

ment. Normally, the CRP is implemented by a moderator who is in charge of improving the level of consensus until it achieves a pre-determined threshold (Mata, Martínez & Herrera-Viedma, 2009). The moderator needs to calculate the level of agreement in each iterative round. If the level of agreement is not acceptable, then the moderator applies feedback strategies to improve it; and if the level of agreement is acceptable, the moderator adopts the selection process to obtain the final decision result.

There are different feedback patterns to improve the level of agreement such as the fully supervised (interactive) strategy, semisupervised strategy and automatic strategy (Wu & Xu, 2018). The automatic strategy can reduce the time consumption since it modifies opinions automatically without experts' participation. Conversely, the interactive feedback strategy needs discussion rounds between the moderator and experts. If the level of agreement is low, then many changes are required to reach an acceptable level. In such cases, the automatic strategy without interactions may cause the updated preferences to be contrary to the original experts' willingness. As a result, the derived consensus is only a computed consensus. Therefore, the interactive strategy can ensure that experts feel part of the decision-making process. However, one problem of the interactive strategy is that if the level of agreement is close to the predefined threshold, the value of an interactive iteration is not as good as that at a low level of agreement. Another important issue in the interactive strategy is

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that experts may engage in noncooperative behaviors and refuse to make changes when receiving adjusted recommendations.

To address this issue, this study proposes a hierarchical CRP model that combines the aforementioned interactive strategy and automatic strategy. It depends on the specific level of consensus in each iterative round. When the level of consensus is low, this model uses the interactive strategy; and when the level of consensus is close to the predefined consensus threshold, then the experts' preference values will be adjusted automatically. The proposed model can reduce the time consumption and ensure experts' participation. This model is applicable to some practical situations. For instance, in the manuscript review process, reviewers may make many comments in the first round. In such a case, the editor of the journal needs to return the revisions of the authors to the reviewers. In the final round, reviewers may only have limited comments such as suggesting grammatical corrections or updated references. In this case, there is no need to return the author's revisions to the reviewers. Furthermore, based on the hierarchical consensus model, this study proposes two strategies to manage the noncooperative behaviors of experts with respect to their degrees of cooperation. The first strategy addresses situations in which experts modify a part of the suggested modifications. If an expert does not make any modifications or does make opposing modifications (fully noncooperative behavior), then the second strategy will be adopted. If an expert of a subgroup engages in fully noncooperative behavior, then the other members of this subgroup will be asked if they agree that this member's modification is included in automatic mechanism. This mechanism leaves space for experts to compensate for their behaviors. The weight penalty mechanism will be introduced only when other members do not reach a majority agreement. The contributions of this study can be highlighted as follows:

- (1) We propose a hierarchical consensus model combining the interactive strategy and automatic strategy. Different strategies will be implemented depending on the specific level of consensus in each round.
- (2) To address the noncooperative behaviors of experts, we introduce a cooperativity index to detect the degrees of cooperation of experts. Then, by means of the hierarchical consensus model, two strategies are proposed to manage these behaviors based on different degrees of cooperation. The second strategy incorporates a discussion session for subgroup members before entering their assessments in the weight penalty mechanism.
- (3) A weight determination method for subgroups is developed. This method considers two indicators: the size of a subgroup and the sum of squared errors (SSE). The SSE reflects the cohesiveness of a subgroup. Hence, a cohesive subgroup will exert considerable influence in the global group.

These contributions matter in LSGDM problems. Cost management and uncooperativeness management are two critical issues in LSGDM (Tang & Liao, 2021). The hierarchical consensus model can reduce time cost because of the automatic strategy and consider the willingness of experts because of the interactive strategy. Furthermore, this study adopts the conversion between the interactive strategy and the automatic strategy to manage noncooperative behaviors. This conversion can leave space for experts to further compensate for their behaviors and may reduce noncooperative behaviors to some extent.

The rest of this paper is structured as follows. In Section 2, the preliminaries of this study are presented. Section 3 introduces the hierarchical consensus model. Section 4 detects and manages noncooperative behaviors. An illustrative example is provided in Section 5. Concluding remarks are given in Section 6.

2. Preliminaries

In this section, descriptions of LSGDM problems and the reciprocal comparison matrices (RCMs) used to construct the proposed model are presented.

An LSGDM problem is defined as a situation in which a large group of experts participate in a problem that requires selecting the optimal alternative from a set of possible solutions (Wu & Xu, 2018). There are two widely used standards regarding the number of experts in LSGDM problems: one is 11 experts (Xu, Zhong, Chen, & Zhou, 2015) and the other is 20 experts (Tang & Liao, 2021; Wu & Xu, 2018). In this study, we use the latter standard. It is not a huge issue if the number of experts is no more than 50. It is suitable to use a statistical method if the size of a large group is above 50 or the size of a subgroup is above 30 (Zhong, Xu, Chen & Goh, 2020).

The main elements of an LSGDM problem are as follows:

- (1) A discrete set of feasible alternatives, which represent possible solutions to the LSGDM problem:

$$X = \{x_1, x_2, \dots, x_n\} \quad (n \geq 2)$$

- (2) A set of experts who are invited to evaluate alternatives:

$$E = \{e_1, e_2, \dots, e_m\} \quad (m \geq 20)$$

As mentioned above, the group decision result does not guarantee the support of all subgroups. Therefore, the CRP needs to be applied in the LSGDM. The CRP in LSGDM is an iterative process containing some iterative rounds. A CRP consists of the following steps:

Step 1: Consensus measurement. After gathering the preferences of experts, we need to compute the consensus level of the group using a consensus measure and check whether the current level of consensus achieves the predefined threshold. If the threshold has not been met, then a discussion round should be conducted as Step 2; otherwise, the group proceeds to the selection process.

Step 2: Consensus improvement process. This step identifies which subgroup, alternative or pair of alternatives contributes less than others in reaching a high level of agreement. Then, some suggestions for adjustments are generated to modify the preferences of the subgroup, alternative or pair of alternatives. The interactive strategy returns the preferences and the modified suggestions to experts while the automatic strategy makes modifications without a discussion among experts.

In this study, experts use RCMs to express their preferences over alternatives. The definition of an RCM is given as follows:

Definition 1. (Wu, Xu & Hipel, 2019). Let $e_k \in E$. $p^k = (p_{ij}^k)_{n \times n}$ is e_k 's RCM over X for which $p_{ij}^k + p_{ji}^k = 1$, and $0 \leq p_{ij}^k \leq 1$, where $i, j \in \{1, 2, \dots, n\}$.

p_{ij}^k is interpreted as the degree of preference of x_i over x_j . The inequality $p_{ij}^k > 0.5$ indicates that x_i is preferred to x_j , $p_{ij}^k = 1$ denotes that x_i is absolutely preferred to x_j , and $p_{ij}^k = 0.5$ indicates that x_i and x_j are indifferent.

3. Hierarchical consensus reaching process

As mentioned before, an LSGDM problem usually contains a large group of experts with various backgrounds. Therefore, this would constitute a challenge to reach an agreement because of the time cost or noncooperative behaviors of experts. This study adopts the adaptive consensus (Tang, Liao, Xu, Streimikiene & Zheng, 2020) to design the hierarchical CRP. When the level of consensus is low, it is reasonable that many experts' preferences should be changed given that a large number of modifications and

many iterative rounds might be necessary. In such cases, the interactive feedback strategy will be used to ensure experts' participation. When the level of consensus is high but not high enough, a small number of changes are required. In such situations, it is time consuming to adopt the feedback strategy, but it is reasonable to implement the automatic strategy to make the small number of modifications. The consensus model consists of four steps: 1) subgroup clustering, 2) weight determination for subgroups, 3) the consensus measure, and 4) the adaptive feedback mechanism.

3.1. Subgroup clustering

Clustering is the process of partitioning a set of objects into clusters so that the objects within the same cluster are similar to each other but are dissimilar to objects in different clusters (Saxena, Prasad & Gupta, 2017). Different clustering algorithms such as k-means clustering (Wu & Xu, 2018), fuzzy c-means clustering (Tang et al., 2020) and agglomerative hierarchical clustering (Wang, Xu, Huang & Cai, 2018) have been used in LSGDM problems. Following Zhao, Xu, Liu and Wang (2012), this study uses the minimum spanning tree (MST) clustering algorithm to classify experts.

A graph $G = (E, L)$ is composed of a set of nodes E and a set of edges L that connect the nodes. The set L can be binary relations taking values 0 or 1 or relations using degrees of trust varying from 0 to 1. In this study, we use the Euclidean distance to define the set L over $E \times E$. A path in a graph is a sequence of edges that link the nodes, such as $A-B-C-D$. A circuit is a closed path such as $A-B-C-D-A$. A connected graph has paths between any pair of nodes. A connected acyclic graph containing all nodes is called a spanning tree. If a spanning tree's weight is the lowest among all possible spanning trees of G , then this spanning tree is called the MST.

The main steps of the RCM-based MST clustering algorithm are as follows:

Step 1: Construct the distance matrix $D = (d_{kh})_{m \times m}$ for experts, where:

$$d_{kh} = d(P_k, P_h) = \sqrt{\frac{2}{n(n-1)} \sum_{i < j} (p_{ij}^k - p_{ij}^h)^2} \quad (k, h = 1, 2, \dots, m, k \neq h) \quad (1)$$

Step 2: Construct the graph $G = (E, L)$ regarding all nodes e_k , where $k = 1, 2, \dots, m$. The edge l_{kh} between nodes e_k and e_h has weight d_{kh} .

Step 3: Arrange all edges in ascending order according to their weights and select the edge with the smallest weight. Then, select the edge with the smallest weight from the rest of all edges that do not form a closed loop with those edges that have been chosen. Repeat this process until $m - 1$ edges are selected.

Step 4: Group all nodes into clusters by eliminating all edges with weights greater than a threshold λ . Then, a certain number of subgroups can be obtained.

Based on this clustering method, we can choose suitable a number of clusters for an LSGDM problem. Hereby, we denote the clustering results as $\{C_1, C_2, \dots, C_H\}$, where H is the number of clusters. Note that $C_1 \cup C_2 \cup \dots \cup C_H = E$ and $C_h \cap C_q = \emptyset$ for any $h, q = 1, 2, \dots, H, h \neq q$.

3.2. Weight determination for subgroups

After classifying experts into several subgroups, the next step is to determine the weight of each subgroup. Many studies used the size (number of experts) of a subgroup as a criterion to determine the weight (Liu, Xu, Montes, Ding & Herrera, 2019; Shi et al., 2018; Wu & Xu, 2018; Zhang, Dong & Herrera-Viedma, 2018). However, this method may have limitations. For instance, subgroups with

different inner characteristics but with the same number of experts will be treated equally in the decision-making process. Considering this, in our study, the size of a subgroup and the evaluation criterion for the effectiveness of the clustering results, i.e., SSE (Wan, Wong & Prusinkiewicz, 1988), are integrated to determine the weights of subgroups.

Let $P_{C_h} = (p_{ij}^{C_h})_{n \times n}$ be the collective RCM of subgroup C_h , which can be obtained by aggregating the average RCMs of the experts in C_h . We can use Eq. (1) to calculate the distance $d(P_{hr}, P_{C_h})$ between the RCM $P_r^{C_h}$ of $e_r^{C_h}$ in C_h and P_{C_h} , where $e_r^{C_h} \in C_h$. Then, the SSE of C_h is calculated as $\sum_{e_r^{C_h} \in C_h} d(P_r^{C_h}, P_{C_h})$. In this study, we use the average SSE as an indicator to determine the weight of a subgroup:

$$AS(C_h) = \frac{1}{\#C_h} \sum_{e_r^{C_h} \in C_h} d(P_r^{C_h}, P_{C_h})$$

where $\#C_h$ is the number of experts in C_h .

In addition, if more experts are grouped into a subgroup, then a higher weight should be assigned to this subgroup (Xu, Zhong, Chen, & Zhou, 2015). We integrate these two indicators to determine the weight of a subgroup:

$$\varphi(C_h) = \frac{e^{(\max AS - AS(C_h))\beta}}{\sum_{u=1}^H e^{(\max AS - AS(C_u))\beta}} \cdot \frac{\#C_h}{m}, \quad \forall h \in \{1, 2, \dots, H\}$$

where $\max AS$ is the maximum value of AS for all subgroups and β is the parameter that controls the impact of AS in the weight calculation process. Then, the weight of a subgroup can be defined as:

$$\omega(C_h) = \frac{\varphi(C_h)}{\sum_{u=1}^H \varphi(C_u)}, \quad \forall h \in \{1, 2, \dots, H\} \quad (2)$$

3.3. Consensus measure

After obtaining the weights of subgroups, we calculate the degree of consensus among all subgroups. There are two methods to compute the level of consensus: one is constructing a consensus matrix by aggregating each pair of subgroups' similarity matrix and then obtaining the degree of consensus associated with positions, alternatives and experts (Xu, Du, Chen & Cai, 2019); and the other is constructing a global collective matrix by aggregating all subgroups' preference relations and then comparing subgroup's solutions with the collective one (Dong, Zhang & Herrera-Viedma, 2016). This study uses the latter method.

Initially, the preference relation of a subgroup is obtained by aggregating all individual RCMs of the experts in the subgroup using the weighted average operator. Then, the collective RCM $P_c = (p_{ij}^c)_{n \times n}$ is obtained, where:

$$p_{ij}^c = \sum_{h=1}^H \omega(C_h) \cdot p_{ij}^{C_h}$$

Then, the degree of consensus $Con(C_h)$ of subgroup C_h is defined as:

$$Con(C_h) = 1 - d(P_{C_h}, P_c)$$

The global level of consensus $GCon$ is obtained as:

$$GCon = \frac{1}{H} \sum_{h=1}^H Con(C_h) \quad (3)$$

The degree of consensus on a position (pair of alternatives) can be defined as:

$$cp_{ij}^{C_h} = 1 - |p_{ij}^{C_h} - p_{ij}^c|$$

The degree of consensus on an alternative is:

$$ca_i^{C_h} = \frac{\sum_{j=1, j \neq i}^n cp_{ij}^{C_h}}{n-1}$$

All these consensus measures are in the interval $[0, 1]$. In the CRP, a predefined threshold ζ should be set in advance. If $GCon \geq \zeta$, the whole group reaches an accepted consensus, and then the selection process is applied. Otherwise, another iterative round should be implemented. The value of ζ depends on the specific problem that we are addressing. When the problem is significantly important, a high ζ such as 0.9 or 0.85 is required. When the problem is not very important or the decision has to be made quickly for emergency problems, a low ζ is required.

3.4. Adaptive feedback mechanism

As mentioned before, the interactive strategy can ensure the participation of experts in the decision-making process. However, this strategy is usually time consuming due to multiround negotiations and discussions. The automatic strategy is efficient, while the willingness of experts is not fully considered. Therefore, it is relevant to develop a feedback mechanism considering the willingness of experts and the time cost simultaneously.

First, we distinguish the global level of consensus $GCon$ into four categories: 1) low, 2) medium, 3) high, and 4) high enough. When the level of consensus is low, a large number of changes are required. Then, the interactive strategy is necessary because experts' opinions can be fully considered. As the level of consensus increases, the number of preference values that need to be modified decreases. When the consensus level is high, then a very small number of changes are required. To reduce the time consumption, it is reasonable to use the automatic strategy in such situations. To better balance the willingness of experts and the time consumption, we define three different adjusted strategies corresponding to the specific global level of consensus. Different strategies for searching the preference values to change are described in detail as follows.

Before using the adaptive searching strategies, three parameters θ_1 , θ_2 and the global consensus threshold ζ should be fixed at the start of the CRP to differentiate four consensus situations: low, medium, high and high enough. The different consensus situations are identified as follows:

- (1) Low consensus level: $GCon < \theta_1$;
- (2) Medium level of consensus: $\theta_1 \leq GCon < \theta_2$;
- (3) High level of consensus: $\theta_2 \leq GCon < \zeta$; and
- (4) High enough level of consensus level: $GCon \geq \zeta$.

The fourth situation does not need to adopt any improvement action. The other three preference search strategies are elaborated as follows:

1) *Preference searching for a low consensus (PS-Low)*. Usually, at the start of a CRP, experts' preferences are quite different from each other, and therefore the level of agreement is low. In this situation, a large number of changes are required so as to accelerate the convergence process. To maintain the enthusiasm for participation and real willingness, this situation adopts the interactive feedback strategy. To accomplish this, all subgroups are required to make modifications to all pairs of alternatives that have low levels of agreement. Therefore, with this strategy, all subgroups will be willing to share the final solution because their opinions will be fully considered.

The search strategy for the PS-Low is as follows: a) For all subgroups, the pairs of alternatives with a level of agreement lower than a threshold, γ_1 , are identified as

$$PAL_1 = \{(i, j) | cp_{ij}^{C_h} < \gamma_1\} \quad (4)$$

The value of γ_1 can be fixed and static before the CRP or be determined according to the specific level of consensus in each iterative round. The selection of γ_1 plays a critical role in the CRP.

A high γ_1 may require many changes while a low γ_1 may increase the number of iterative rounds. Here, a dynamic value of γ_1 that changes during the CRP is considered. We use the average degree of consensus at the alternative level as γ_1 such that $\sum_{i=1}^n (\sum_{j=1, i \neq j}^n cp_{ij}^{C_h}) / (n^2 - n)$.

Generation of advice. Once the pairs of alternatives have been identified, the direction rules are generated by the moderator to provide suggestions. Note that in each iterative round, each preference value changes by 0.1. The direction rules are as follows.

DR. 11: If $p_{ij}^{C_h} < p_{ij}^c$, then all experts in subgroup C_h should increase their assessment associated with the pair of alternatives (x_i, x_j) ;

DR. 12: If $p_{ij}^{C_h} > p_{ij}^c$, then all experts in subgroup C_h should decrease their assessment associated with the pair of alternatives (x_i, x_j) ; and

DR. 13: If $p_{ij}^{C_h} = p_{ij}^c$, then all experts in subgroup C_h should not change their assessment associated with the pair of alternatives (x_i, x_j) .

2) *Preference searching for a medium consensus (PS-Med)*. After several iterative rounds, the level of agreement among subgroups will be higher than that at the start of the CRP. In this situation, it is reasonable to reduce the number of experts who need to make modifications. In the former situation, all experts in a low agreement subgroup are required to make modifications. In this situation, only the experts whose level of consensus is lower than the average level of consensus of the experts of this subgroup to the global group are considered.

Another important issue about *PS-Med* is the introduction of the automatic mechanism. If an expert only has a small number of preference values that need to be modified, then it is logical to use the automatic strategy to adjust the preference values because it is time consuming to adopt the interactive strategy. Switching from the interactive strategy to the automatic strategy requires experts' consent. In this study, we assume that if the proportion of preference values in the upper triangle of an RCM that need to be changed is less than 20%, then the automatic strategy will be implemented. For instance, an RCM composed of pairwise comparisons over 4 alternatives has 6 preference values in the upper triangle. If there is only one preference value that needs to be modified, then the automatic strategy will be implemented.

The search strategy for *PS-Med* is as follows:

- (a) First, the subgroups whose level of agreement is lower than the global level of consensus are identified as

$$SUB_2 = \{C_h | Con(C_h) < GCon\} \quad (5)$$

- (b) For an identified subgroup, the experts whose degree of consensus is lower than the average level of consensus of all experts in this subgroup of the global group are identified as

$$EXP_2 = \{e_r^{C_h} | Con_r^{C_h} < \overline{Con}_r^{C_h} | e_r^{C_h} \in C_h\} \quad (6)$$

where $Con_r^{C_h}$ is the degree of consensus of $e_r^{C_h}$. $Con_r^{C_h}$ has the same calculation method as that of $Con(C_h)$. $\overline{Con}_r^{C_h} = \frac{1}{\#C_h} \sum_{e_r^{C_h} \in C_h} Con_r^{C_h}$.

- (c) The alternatives for which the degree of consensus is lower than a threshold, γ_2 , are identified as

$$ALT_2 = \{x_i | ca_i^r < \gamma_2 \wedge e_r^{C_h} \in EXP_2\} \quad (7)$$

where γ_2 can be calculated analogous to γ_1 . For convenience, we use ca_i^r to denote the degree of consensus of x_i for $e_r^{C_h}$.

- (d) The pairs of alternatives that need to be modified are identified as

$$PAL_2 = \{(i, j) | cp_{ij}^r < \gamma_2 \wedge x_i \in ALT_2\} \quad (8)$$

For convenience, we use cp_{ij}^r to denote the degree of consensus of the position (i, j) for $e_r^{C_h}$.

Generation of advice. Once the experts and pairs of alternatives have been identified, the direction rules will be generated. For convenience, we use p_{ij}^r to denote the preference value of $e_r^{C_h}$ in the position (i, j) .

DR. 21. If $p_{ij}^r < p_{ij}^c$ and no less than 20% of the preference values need to be modified, then expert $e_r^{C_h}$ in subgroup C_h should increase the assessment associated with the pair of alternatives (x_i, x_j) ; otherwise, if less than 20% of preference values need to be modified, then the preference value p_{ij}^r will increase by 0.1 automatically.

DR. 22. If $p_{ij}^r > p_{ij}^c$ and no less than 20% of the preference values need to be modified, then expert $e_r^{C_h}$ in subgroup C_h should decrease the assessment associated with the pair of alternatives (x_i, x_j) ; otherwise, if less than 20% of the preference values need to be modified, then the preference value p_{ij}^r will decrease by 0.1 automatically.

3) *Preference searching for a high consensus (PS-High).* In the last several iterative rounds, the level of agreement may be very close to the predefined threshold ζ . In this situation, only a small number of changes are required. It seems logical to adopt the automatic strategy to improve the level of agreement. This can reduce the time consumption and supervision cost of the moderator. Furthermore, fewer changes than in the previous two situations are required. Only the experts whose level of agreement is lower than the average degree of agreement of all experts in one subgroup are identified. Furthermore, for those identified experts, only less than 20% of their preference values are modified. This ratio can guarantee a certain amount of modifications, which is helpful for avoiding overadjustment.

The search strategy for *PS-High* is as follows:

- (a) For a subgroup C_h , the experts whose degree of consensus is lower than that of this subgroup of the global group are identified as

$$EXP_3 = \{e_r^{C_h} | \text{Con}_r^{C_h} < \overline{\text{Con}_r^{C_h}} | e_r^{C_h} \in C_h\} \quad (9)$$

- (b) The alternatives for which the degree of consensus is lower than a threshold, γ_3 , are identified as

$$ALT_3 = \{x_i | ca_i^r < \gamma_3 \wedge e_r^{C_h} \in EXP_3\} \quad (10)$$

where γ_3 can be calculated analogous to γ_1 .

- (c) The pairs of alternatives that need to be modified are identified as

$$PAL_3 = \{(i, j) | cp_{ij}^r < \gamma_3 \wedge x_i \in ALT_3\} \quad (11)$$

For these identified pairs of alternatives, we select one pair of alternatives to modify randomly.

Generation of advice. Once the experts and pairs of alternatives have been identified, the direction rules will be generated:

DR. 31. If $p_{ij}^r < p_{ij}^c$, then the preference value p_{ij}^r will increase by 0.1 automatically;

DR. 32. If $p_{ij}^r > p_{ij}^c$, then the preference value p_{ij}^r will decrease by 0.1 automatically; and

DR. 33. If $p_{ij}^r = p_{ij}^c$, no change is demanded.

This adaptive feedback mechanism can manage issues including the time cost and the willingness of experts in LSGDM through combining the automatic strategy and interactive strategy. In initial rounds, the interactive strategy can maintain the enthusiasm of the group evaluation body of experts. In last rounds, the automatic mechanism can reduce time cost and avoid overadjustment.

4. Noncooperative behavior detection and management

In the CRP, one limitation of the interactive feedback strategy is that experts may refuse to make modifications or only change a small fraction. In some cases, some experts may even change preference values in the opposite direction. In this section, we discuss how to detect and manage noncooperative behaviors in LSGDM.

4.1. Noncooperative behavior detection

In the decision-making process, experts who are reluctant to adjust their preference values or who make changes in the opposite direction are regarded as noncooperative members. In this section, we propose a method to measure the degree of cooperation of each expert in each iterative round. To do this, a cooperative index is introduced. Then, the degree of cooperation based on the cooperative index is obtained and used to identify different types of noncooperative behaviors.

Definition 2: Let TPS_k^t be the total number of adjusted preference values in the upper triangle of the RCM that expert e_k has been suggested to modify and TDM_k^t be the total differences between the number of suggested modifications and the number of modifications made by expert e_k in the t th round. The cooperative index of e_k in round t , CI_k^t , is defined as:

$$CI_k^t = \begin{cases} 1, & \text{if } TPS_k^t = 0; \\ 1 - \frac{TDM_k^t}{TPS_k^t}, & \text{otherwise.} \end{cases} \quad (12)$$

It is obvious that $CI_k^t \in [-1, 1]$. A higher CI_k^t implies that e_k 's behavior is more cooperative in the t th round. If $CI_k^t = 1$, there is no difference between the suggested modifications and the modifications that have been made by e_k in the t th round. $CI_k^t = 0$ denotes that e_k makes no modification in the t th round. If $CI_k^t = -1$, then all adjusted preference values made by e_k are in the entirely opposite direction.

To better understand the calculations of TPS , TDM and CI , we provide a simple example.

Example 1: Consider a CRP in round 1 in an LSGDM problem with four alternatives $\{x_1, x_2, x_3, x_4\}$. The original RCMs of two experts are

$$P_1 = \begin{pmatrix} 0.5 & 0.9 & 0.9 & 0.8 \\ 0.1 & 0.5 & 0.7 & 0.8 \\ 0.1 & 0.3 & 0.5 & 0.4 \\ 0.2 & 0.2 & 0.6 & 0.5 \end{pmatrix}, \text{ and } P_2 = \begin{pmatrix} 0.5 & 0.3 & 0.7 & 0.8 \\ 0.7 & 0.5 & 0.3 & 0.6 \\ 0.3 & 0.7 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.7 & 0.5 \end{pmatrix}$$

The moderator provides suggested modifications for these two experts, which are shown as:

$$\bar{P}_1^1 = \begin{pmatrix} 0.5 & 0.9 & 0.9 & 0.7 \\ 0.1 & 0.5 & 0.6 & 0.7 \\ 0.1 & 0.4 & 0.5 & 0.4 \\ 0.3 & 0.3 & 0.6 & 0.5 \end{pmatrix}, \text{ and } \bar{P}_2^1 = \begin{pmatrix} 0.5 & 0.3 & 0.7 & 0.7 \\ 0.7 & 0.5 & 0.4 & 0.5 \\ 0.3 & 0.6 & 0.5 & 0.3 \\ 0.3 & 0.5 & 0.7 & 0.5 \end{pmatrix}$$

Thus, $TPS_1^1 = (0.8 - 0.7) + (0.7 - 0.6) + (0.8 - 0.7) = 0.3$. Similarly, $TPS_2^1 = 0.3$.

We assume that these two experts provide their modified RCMs as follows:

$$P_1^1 = \begin{pmatrix} 0.5 & 0.9 & 0.9 & 0.7 \\ 0.1 & 0.5 & 0.7 & 0.8 \\ 0.1 & 0.3 & 0.5 & 0.4 \\ 0.3 & 0.2 & 0.6 & 0.5 \end{pmatrix}, \text{ and } P_2^1 = \begin{pmatrix} 0.5 & 0.3 & 0.7 & 0.9 \\ 0.7 & 0.5 & 0.3 & 0.6 \\ 0.3 & 0.7 & 0.5 & 0.3 \\ 0.1 & 0.4 & 0.7 & 0.5 \end{pmatrix}$$

The first expert only modifies a single position. Therefore, $TDM_1^1 = 0.2$. Then, we have $CI_1^1 = 0.333$. Similarly, $CI_2^1 = -0.333$.

Next, we classify different noncooperative behaviors according to the value of CI .

- (a) $CI_k^t = 1$ denotes that e_k has received all suggestions and made the corresponding modifications. We call e_k fully cooperative.

- (b) $CI_k^t \in (0, 1)$ denotes that e_k only has received suggestions and made some of the corresponding modifications. We call e_k semicooperative.
- (c) $CI_k^t \in [-1, 0]$ denotes that expert e_k does not make any modification or does make some modifications in the opposite direction. We call e_k fully noncooperative.

4.2. Noncooperative behavior management

We have classified all experts into three categories. The next step is to manage semicooperative and fully noncooperative behaviors. Most existing studies (Palomares, Martínez, & Herrera, 2014; Palomares, Quesada & Martínez, 2014; Quesada, Palomares & Martínez, 2015a, Quesada, Palomares & Martínez, 2015b; Xu et al., 2019; Zhang, Ignatius & Zhao, 2015) used the weight penalty mechanism to manage noncooperative behaviors. This mechanism reduces the weights of noncooperative experts or subgroups so that they have a small influence on the group.

In this study, we use both the hierarchical consensus model and weight penalty mechanism to manage noncooperative behaviors. Two strategies are described below.

Strategy 1: If an expert engages in semicooperative behavior, then this expert makes some of the suggested modifications. The cooperative index of this expert is in the interval $(0, 1)$. As stated earlier, this study assumes that an expert agrees that 20% of all preference values can be modified using the automatic mechanism. In this situation, no more than but close to 20% of the preference values will be changed automatically.

Strategy 2: If an expert engages in fully noncooperative behavior, then this expert makes no suggested modifications or makes opposite modifications. The cooperative index of this expert is in the interval $[-1, 0]$. The management strategy for fully noncooperative experts is as follows. If an expert $e_r^{C_h, t}$ in subgroup C_h^t engages in noncooperative behavior, then all other members $e_s^{C_h, t}$ ($e_s^{C_h, t} \in C_h^t, s \neq r$) of the subgroup will be asked whether they agree that the noncooperative member's modifications enter the automatic mechanism or not. If at least two-thirds of the other members agree, then the noncooperative member's preference values will be modified automatically; otherwise, the decision weight of this subgroup will be penalized.

Here, a weight updating function is developed. The cooperative index is divided into 4 parts, each of which corresponds to a weight adjustment coefficient, as displayed in Fig. 1. The updated weight $\omega(C_h)^{t+1}$ of C_h^{t+1} in the following round is computed as:

$$\omega(C_h)^{t+1} = \begin{cases} 0.9 \times \omega(C_h)^t, & CI_r^{C_h, t} \in [-0.25, 0] \\ 0.7 \times \omega(C_h)^t, & CI_r^{C_h, t} \in [-0.5, -0.25] \\ 0.5 \times \omega(C_h)^t, & CI_r^{C_h, t} \in [-0.75, -0.5] \\ 0.3 \times \omega(C_h)^t, & CI_r^{C_h, t} \in [-1, -0.75] \end{cases} \quad (13)$$

where $\omega(C_h)^t$ is the weight of subgroup C_h in the t th discussion round and $\omega(C_h)^{t+1}$ is the adjusted weight of subgroup C_h in round $t+1$. For instance, if $\omega(C_h)^t = 0.4$, $CI_r^{C_h, t} = -0.6$, and we have $\omega(C_h)^{t+1} = 0.4 \times 0.5 = 0.2$.

Normalizing the weights, we have

$$\hat{\omega}(C_h)^{t+1} = \frac{\omega(C_h)^{t+1}}{\sum_{h=1}^H \omega(C_h)^{t+1}} \quad (14)$$

where $\hat{\omega}(C_h)^{t+1} \in [0, 1]$ and $\sum_{h=1}^H \hat{\omega}(C_h)^{t+1} = 1$. Once the weights of the subgroups are normalized, they are considered in the $t+1$ th iterative round to calculate the collective RCM.

Unlike previous studies, in our proposed model, if a subgroup has noncooperative experts, the weight penalty mechanism will

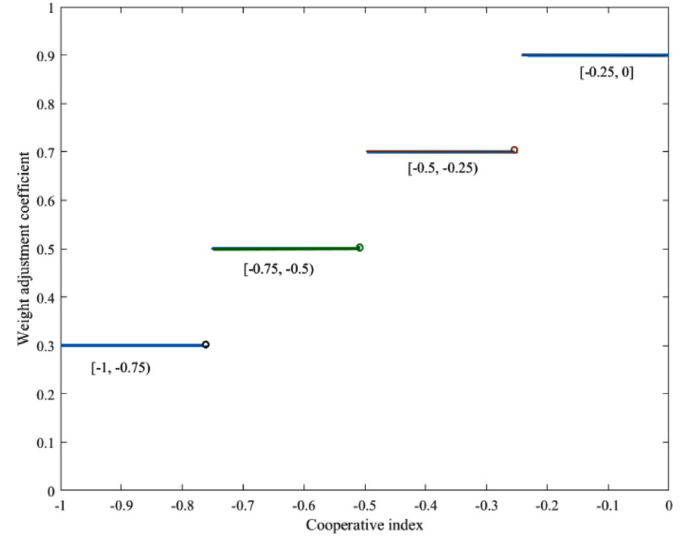


Fig. 1. Relationship between the cooperative index and weight adjustment coefficient.

not be implemented directly. If at least two-thirds of other members of the subgroup do not agree with the automatic adjustment strategy, then the weight penalty mechanism will be implemented. Therefore, this model leaves space for experts to further compensate for their behaviors. Furthermore, this model can reduce noncooperative behaviors to some extent because experts know that their behaviors may lead to a reduction in the weight of the subgroup to which they belong.

4.3. The procedure of the proposed method

To facilitate the calculation and analysis, the steps of the proposed model are given as follows.

Step 1: Classify all experts into H subgroups $\{C_1, C_2, \dots, C_H\}$ using the MST clustering algorithm.

Step 2: Determine the weights of the subgroups using Eq. (2).

Step 3: Calculate the global level of consensus $GCon$ using Eq. (3).

Step 4: Consider the following four situations: if $GCon < \theta_1$, then apply the "PS-Low" strategy; if $\theta_1 \leq GCon < \theta_2$, then apply the "PS-Med" strategy; if $\theta_2 \leq GCon < \zeta$, then apply the "PS-High" strategy and return to Step 3; and if $GCon \geq \zeta$, then go to Step 7.

Step 5: Calculate the degree of cooperation degree CI_k^t for e_k who received suggestions using Eq. (12).

Step 6: Consider the following two situations: if $CI_k^t \in (0, 1)$, then apply Strategy 1 and return to Step 3; and if $CI_k^t \in [-1, 0]$, then apply Strategy 2. In such cases, if more than two-thirds of all other members $e_s^{C_h, t}$ ($e_s^{C_h, t} \in C_h^t, s \neq r$) agree that the noncooperative member's ($e_r^{C_h, t}$) modifications enter the automatic mechanism, then return to Step 3. Otherwise, use Eqs. (13) and (14) to update the weights of the subgroups, and return to Step 2.

Step 7: Derive the priorities and rank all alternatives using the weighted average operator.

Step 8: End.

The proposed model is illustrated in Fig. 2.

5. Illustrative example

In this section, an illustrative example is provided to show the applicability and usefulness of the proposed model. Then, comparative analyses with existing studies are given.

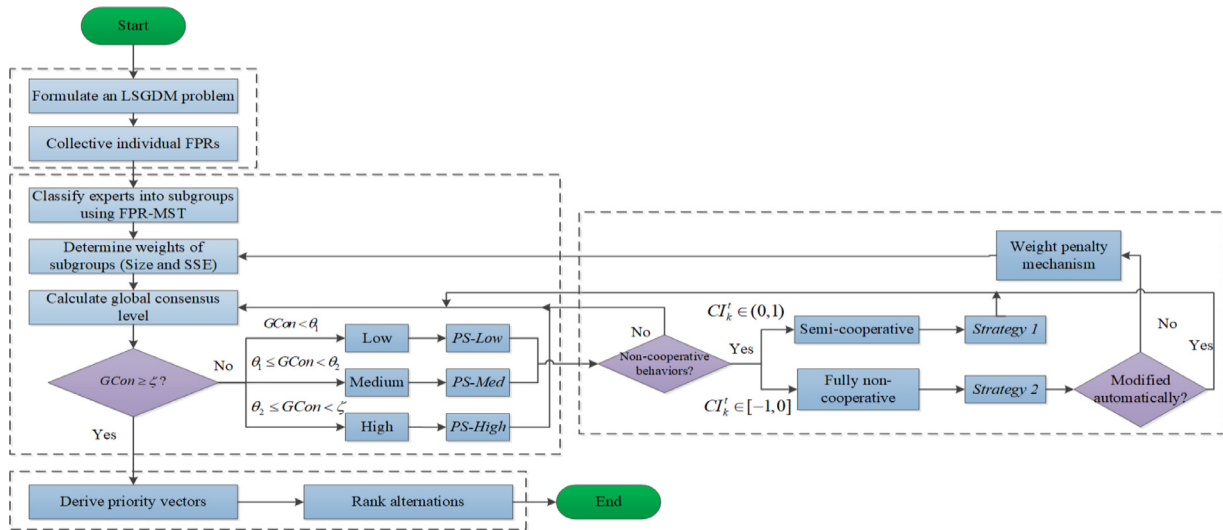


Fig. 2. Flowchart of the proposed model.

Table 1
The structure of subgroups.

Subgroup C_h	Number of experts	Experts	RCM of the subgroup
C_1	2	e_1, e_{14}	$\begin{pmatrix} 0.5000 & 0.9000 & 0.8000 & 0.8000 \\ 0.1000 & 0.5000 & 0.8000 & 0.7500 \\ 0.2000 & 0.2000 & 0.5000 & 0.2000 \\ 0.2000 & 0.2500 & 0.8000 & 0.5000 \end{pmatrix}$
C_2	9	$e_2, e_3, e_4, e_6, e_7, e_8, e_9, e_{10}, e_{12}$	$\begin{pmatrix} 0.5000 & 0.3111 & 0.6778 & 0.6778 \\ 0.6889 & 0.5000 & 0.6222 & 0.7111 \\ 0.3222 & 0.3778 & 0.5000 & 0.6000 \\ 0.3222 & 0.2889 & 0.4000 & 0.5000 \end{pmatrix}$
C_3	9	$e_5, e_{11}, e_{13}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}$	$\begin{pmatrix} 0.5000 & 0.5222 & 0.3444 & 0.2778 \\ 0.4778 & 0.5000 & 0.2889 & 0.3556 \\ 0.6556 & 0.7111 & 0.5000 & 0.5111 \\ 0.7222 & 0.6444 & 0.4889 & 0.5000 \end{pmatrix}$

5.1. Problem formulation

Suppose that the municipal government of Chengdu, China decides to invest in a new metro route. A group of 20 experts $\{e_1, e_2, \dots, e_{20}\}$ are invited to evaluate four possible alternatives $\{x_1, x_2, x_3, x_4\}$. All experts evaluate these four alternatives using RCMs. To save space, we only present a subset of these RCMs here. Refer to the Appendix for the data of all 20 experts (Table A1).

$$P_1 = \begin{pmatrix} 0.5 & 0.9 & 0.9 & 0.8 \\ 0.1 & 0.5 & 0.8 & 0.8 \\ 0.1 & 0.2 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.7 & 0.5 \end{pmatrix}, P_2 = \begin{pmatrix} 0.5 & 0.2 & 0.7 & 0.9 \\ 0.8 & 0.5 & 0.3 & 0.7 \\ 0.3 & 0.7 & 0.5 & 0.3 \\ 0.1 & 0.3 & 0.7 & 0.5 \end{pmatrix},$$

$$P_{20} = \begin{pmatrix} 0.5 & 0.7 & 0.4 & 0.1 \\ 0.3 & 0.5 & 0.3 & 0.4 \\ 0.6 & 0.7 & 0.5 & 0.7 \\ 0.9 & 0.6 & 0.3 & 0.5 \end{pmatrix}$$

Some parameters are set as follows: $\zeta = 0.87$, $\theta_1 = 0.8$, $\theta_2 = 0.85$, $N = 3$, and $\beta = 5$. In the following, we use the proposed model to solve this problem.

5.2. Solution to the problem

First, we use the MST clustering algorithm to classify all experts into 3 subgroups. To save space, we do not present the clustering process here. The detailed process can be found in the Appendix. The clustering results are presented in Table 1.

Table 2
The weight of each subgroup.

Subgroup C_h	$\#G_h$	$AS(C_h)$	$\omega(C_h)$
C_1	2	0.0612	0.1686
C_2	9	0.1935	0.3915
C_3	9	0.1703	0.4399

After the clustering process, next, we need to determine the weights of the subgroups, which are shown in Table 2.

We can obtain the collective RCM of the whole group:

$$P_c = \begin{pmatrix} 0.5000 & 0.5033 & 0.5517 & 0.5224 \\ 0.4967 & 0.5000 & 0.5056 & 0.5615 \\ 0.4483 & 0.4944 & 0.5000 & 0.4935 \\ 0.4776 & 0.4385 & 0.5065 & 0.5000 \end{pmatrix}$$

Then, we use Eq. (3) to compute the levels of consensus: $Con(C_1) = 0.7100$, $Con(C_2) = 0.8561$, and $Con(C_3) = 0.8207$. We have $GCon = 0.7956 < \theta_1 = 0.8$. Next, we use the proposed model to improve the level of agreement.

First Round.

Since $GCon < \theta_1$, PS-Low is adopted. The degrees of consensus on the alternative levels for the three subgroups are provided in Fig. 3.

According to Eq. (4), pairs of alternatives that should be modified are the following: $C_1 : (1, 2), (1, 3), (1, 4), (2, 3), (3, 4)$; and $C_3 : (1, 4), (2, 3), (2, 4)$.

Generation of advice.

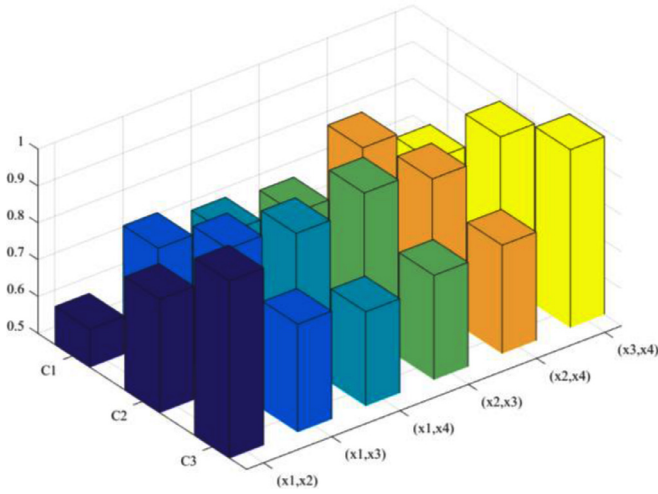


Fig. 3. The degrees of consensus on the levels of pairs of alternatives.

According to DR. 11 and DR. 12, all experts in subgroup C_1 should increase the assessments associated with the pair of alternatives (3, 4) and decrease the assessments associated with pairs of alternatives (1, 2), (1, 3), (1, 4) and (2, 3); all experts in subgroup C_3 should increase the assessments associated with pairs of alternatives (1, 4), (2, 3) and (2, 4).

Suppose that the new RCMs provided by the experts are:

$$\begin{aligned}
 P_1^1 &= \begin{pmatrix} 0.5 & 0.8 & 0.8 & 0.7 \\ 0.2 & 0.5 & 0.7 & 0.8 \\ 0.2 & 0.3 & 0.5 & 0.4 \\ 0.3 & 0.2 & 0.6 & 0.5 \end{pmatrix}, P_5^1 = \begin{pmatrix} 0.5 & 0.3 & 0.4 & 0.5 \\ 0.7 & 0.5 & 0.2 & 0.6 \\ 0.6 & 0.8 & 0.5 & 0.2 \\ 0.5 & 0.4 & 0.8 & 0.5 \end{pmatrix}, \\
 P_{11}^1 &= \begin{pmatrix} 0.5 & 0.3 & 0.4 & 0.6 \\ 0.7 & 0.5 & 0.4 & 0.5 \\ 0.6 & 0.6 & 0.5 & 0.7 \\ 0.4 & 0.5 & 0.3 & 0.5 \end{pmatrix}, P_{13}^1 = \begin{pmatrix} 0.5 & 0.6 & 0.1 & 0.4 \\ 0.4 & 0.5 & 0.5 & 0.3 \\ 0.9 & 0.5 & 0.5 & 0.4 \\ 0.6 & 0.7 & 0.6 & 0.5 \end{pmatrix}, \\
 P_{14}^1 &= \begin{pmatrix} 0.5 & 0.8 & 0.6 & 0.7 \\ 0.2 & 0.5 & 0.7 & 0.7 \\ 0.4 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.3 & 0.8 & 0.5 \end{pmatrix}, P_{15}^1 = \begin{pmatrix} 0.5 & 0.8 & 0.4 & 0.5 \\ 0.2 & 0.5 & 0.2 & 0.2 \\ 0.6 & 0.8 & 0.5 & 0.3 \\ 0.5 & 0.8 & 0.7 & 0.5 \end{pmatrix}, \\
 P_{16}^1 &= \begin{pmatrix} 0.5 & 0.4 & 0.4 & 0.2 \\ 0.6 & 0.5 & 0.2 & 0.3 \\ 0.6 & 0.8 & 0.5 & 0.3 \\ 0.8 & 0.7 & 0.7 & 0.5 \end{pmatrix}, P_{17}^1 = \begin{pmatrix} 0.5 & 0.4 & 0.4 & 0.2 \\ 0.6 & 0.5 & 0.7 & 0.4 \\ 0.6 & 0.3 & 0.5 & 0.8 \\ 0.8 & 0.6 & 0.2 & 0.5 \end{pmatrix}, \\
 P_{18}^1 &= \begin{pmatrix} 0.5 & 0.6 & 0.4 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.8 \\ 0.6 & 0.6 & 0.5 & 0.8 \\ 0.8 & 0.2 & 0.2 & 0.5 \end{pmatrix}, P_{19}^1 = \begin{pmatrix} 0.5 & 0.6 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.4 & 0.2 \\ 0.8 & 0.6 & 0.5 & 0.4 \\ 0.7 & 0.8 & 0.6 & 0.5 \end{pmatrix}, \\
 P_{20}^1 &= \begin{pmatrix} 0.5 & 0.7 & 0.4 & 0.2 \\ 0.3 & 0.5 & 0.4 & 0.5 \\ 0.6 & 0.6 & 0.5 & 0.7 \\ 0.8 & 0.5 & 0.3 & 0.5 \end{pmatrix}
 \end{aligned}$$

Next, we need to detect whether noncooperative behaviors exist. The degrees of cooperation of those experts who received modified suggestions are shown in Fig. 4. As we can see from Fig. 4, three experts engage in noncooperative behaviors. e_{11} and e_{15} are semicooperative experts. Thus, Strategy 1 is used to manage these two experts. Since e_{19} is a fully noncooperative expert, Strategy 2 will be adopted to manage this expert.

First, the moderator seeks advice from other members of subgroup C_3 . Suppose that 4 experts agree and 4 experts do not agree that the modifications of e_{19} 's RCM enters the automatic strategy. The proportion of experts who support the automatic mechanism

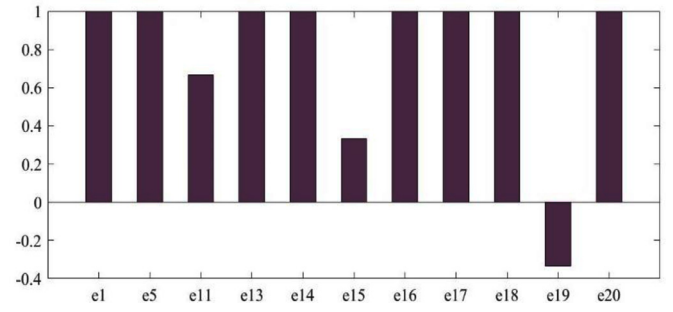


Fig. 4. Degree of cooperation of experts who need to make modifications.

is less than $2/3$. Therefore, the weight penalty mechanism is introduced. Using Eqs. (13) and (14), the updated weights of these three subgroups are $(0.194, 0.451, 0.355)^T$.

Second Round

We can obtain the updated RCMs of the three subgroups and the collective RCM of the whole group:

Then, we use Eq. (3) to compute the levels of consensus: $Con(C_1) = 0.808$, $Con(C_2) = 0.888$, and $Con(C_3) = 0.838$. We have $\theta_1 = 0.8 < GCon = 0.845 < \theta_2 = 0.85$. Next, we use PS-Med to improve the level of agreement. According to Eq. (5), the subgroups that should make modifications are C_1 and C_3 ; and according to Eq. (6), the experts that should make modifications are e_{13} , e_{14} , e_{15} , e_{16} , e_{17} , and e_{19} . According to Eqs. (7) and (8), the positions that should be modified are $PAL_2^{13} = \{(1, 3), (2, 4)\}$, $PAL_2^{14} = \{(1, 2), (3, 4)\}$, $PAL_2^{15} = \{(1, 2), (2, 3), (2, 4), (3, 4)\}$, $PAL_2^{16} = \{(1, 4), (2, 3), (2, 4), (3, 4)\}$, $PAL_2^{17} = \{(1, 4), (2, 4), (3, 4)\}$, and $PAL_2^{19} = \{(1, 3), (1, 4), (2, 4)\}$.

Generation of advice.

According to DR. 21, the assessments that should be increased are $PAL_2^{13} = \{(1, 3), (2, 4)\}$, $PAL_2^{14} = \{(3, 4)\}$, $PAL_2^{15} = \{(2, 3), (2, 4), (3, 4)\}$, $PAL_2^{16} = \{(1, 4), (2, 3), (2, 4), (3, 4)\}$, $PAL_2^{17} = \{(1, 4), (2, 4)\}$, and $PAL_2^{19} = \{(1, 3), (1, 4), (2, 4)\}$. According to DR. 22, the assessments that should be decreased are $PAL_2^{14} = \{(1, 2)\}$, $PAL_2^{15} = \{(1, 2)\}$, and $PAL_2^{17} = \{(3, 4)\}$.

Third Round

Suppose that all experts have received suggestions and made corresponding modifications.

The levels of consensus are the following: $Con(C_1) = 0.831$, $Con(C_2) = 0.900$, $Con(C_3) = 0.855$, and $GCon = 0.862$. Since $0.85 < GCon < 0.87$, next, we use the PS-High to improve the level of consensus. According to Eqs. (9)–(11), the preference values that should be modified are the following: $PAL_3^1 = (1, 2)$, $PAL_3^2 = (1, 2)$, $PAL_3^3 = (1, 2)$, $PAL_3^4 = (1, 2)$, $PAL_3^5 = (2, 3)$, $PAL_3^6 = (1, 2)$, $PAL_3^9 = (3, 4)$, $PAL_3^{10} = (3, 4)$, $PAL_3^{13} = (1, 3)$, $PAL_3^{16} = (1, 4)$, $PAL_3^{18} = (1, 4)$, and $PAL_3^{20} = (1, 4)$. According to DR. 31 and DR. 32, the preference values that should be increased by 0.1 are $PAL_3^2 = (1, 2)$, $PAL_3^3 = (1, 2)$, $PAL_3^4 = (1, 2)$, $PAL_3^5 = (2, 3)$, $PAL_3^6 = (1, 2)$, $PAL_3^{13} = (1, 3)$, $PAL_3^{16} = (1, 4)$, $PAL_3^{18} = (1, 4)$, and $PAL_3^{20} = (1, 4)$; and the preference values that should be decreased by 0.1 are $PAL_3^1 = (1, 2)$, $PAL_3^9 = (3, 4)$, and $PAL_3^{10} = (3, 4)$.

Fourth Round

In this round, the levels of consensus for three subgroups are the following: $Con(C_1) = 0.844$, $Con(C_2) = 0.9134$, and $Con(C_3) = 0.87$. Then, we have $GCon = 0.8734 > 0.87$. Thus, the CRP ends and the selection is applied. Using the weighted average operator, we obtain the final alternative ranking as $x_2 > x_1 > x_3 > x_4$.

5.3. Discussion and comparison

After 4 iterations, the level of agreement of the global group reaches the predefined threshold. Our proposed consensus model can generate suggestions depending on the specific level of con-

sensus. In the first iterative round, 37 adjustments are generated. In the second and third rounds, there are 18 and 12 adjustments, respectively. In the first round, because of the noncooperative behaviors of e_{11} and e_{15} , two preference values are modified automatically, and the other 35 preference values are modified using the interactive feedback strategy. In the second round, because every expert has at least two modifications, all 18 modifications are implemented using the interactive strategy. In the third round, all 12 preference values are modified automatically.

The differences between our study and the existing methods are as follows:

- (1) Comparison with the adaptive CRP. [Rodríguez, Labella, Tré and Martínez \(2018\)](#) applied the adaptive consensus to solve LS-GDM problems. They set two parameters to classify the level of consensus. One is the level of consensus for advice generation, $\delta = 0.7$; and the other is the global consensus threshold, $J = 0.85$. If the level of consensus is lower than 0.7, then all experts with a degree of proximity lower than the average degree of proximity are selected to modify identified pairs of alternatives. If $0.7 < GCon < 0.85$, then the level of consensus is high but not sufficient. In this situation, the selection of subgroups is the same as the former situation. The difference is that the number of involved experts to modify a specific pair of alternatives is reduced. Only those experts whose opinions are furthest from the collective opinion will be selected. In the first round, if using our model, all experts (100%) in C_1 and C_3 are required to make modifications in a total of 37 preference values. $GCon$ is improved from 0.7956 to 0.845. If we use [Rodríguez et al. \(2018\)](#)'s method, a total of 12 preference values are identified to make modifications in the first round. $GCon$ is improved from 0.7956 to 0.828, which is lower than 0.845. The principle reason is that only a part of experts make modifications on the identified preference values using [Rodríguez et al. \(2018\)](#)'s method. Thus, our method has a higher iterative efficiency in initial iterations because the adaptive mechanism covers a large scope of the group when $GCon$ is far from ζ . Note that using two methods, we can obtain the same final ranking. Usually, our proposed adaptive CRP covers a wide range of subgroups when the value of $GCon$ is low and a narrow range of subgroups when $GCon$ is high. Using our method, the number of preference values in RCMs that are required to be modified in three iterations are 37, 18, and 12, respectively, and the number of experts that are required to make modifications in three iterations are 11, 6, and 0, respectively. The level of consensus on each pair of

alternatives of C_2 is high in the first round (see [Fig. 3](#)), and thus C_2 does not participate in the revision. Furthermore, another important difference is the introduction of the automatic mechanism in our model. It can consider the time cost and expert willingness simultaneously.

- (2) Comparison with noncooperative detection and management. Most existing papers ([Palomares, Quesada, & Martínez, 2014](#); [Quesada, Palomares & Martínez, 2015a](#); [Xu et al., 2019](#)) studying noncooperative behaviors in LS-GDM adopted the cooperative coefficient and weight penalty mechanism. [Zhang, Palomares, Dong & Wang \(2018\)](#) analyzed this topic in social network environments. Different from these studies, the proposed method first adopts the conversion between the interactive strategy and automatic strategy to manage noncooperative behaviors. This difference is important because our method can make a part of suggested modifications for stubborn noncooperative experts. The time taken to reach consensus can also be reduced. Furthermore, as discussed in [Section 4.2](#), the proposed method can leave space for experts and may reduce noncooperative behaviors to some extent.

This study has some limitations. The weights of subgroups can only be updated when a subgroup with fully noncooperative experts does not pass the discussion session. In other situations, the weights of subgroups will be static. Furthermore, this study does not consider the situation in which a whole subgroup engages in noncooperative behaviors. Each method has its own focal points. No study performs better in all situations since each study has its advantages and disadvantages.

6. Conclusions

In this study, we have proposed a hierarchical CRP model combining the interactive strategy and the automatic strategy to LS-GDM problems. The interactive strategy considers experts' real willingness, and the automatic strategy can reduce the time consumption. When the level of consensus is low, the interactive strategy will be implemented; and when the consensus level is close to the predefined threshold, then the automatic strategy is adopted. Therefore, this model can combine the advantages of both strategies. Moreover, this study introduced a method to detect and manage noncooperative behaviors. This method has the following essential features: 1) it managed noncooperative behaviors via the conversion between the interactive mechanism and automatic mechanism, and the conversion principle obtains experts' awareness and consent; and 2) it did not introduce the weight

Table 3
FRPs of the subgroups and the global group (Round 2).

C_1				C_2				C_3				Global group			
0.5	0.8	0.7	0.7	0.5000	0.3111	0.6778	0.6778	0.5000	0.5222	0.3444	0.3556	0.5000	0.4809	0.5638	0.5678
0.2	0.5	0.7	0.75	0.6889	0.5000	0.6222	0.7111	0.4778	0.5000	0.3778	0.4444	0.5191	0.5000	0.5506	0.6240
0.3	0.3	0.5	0.3	0.3222	0.3778	0.5000	0.6000	0.6556	0.6222	0.5000	0.5111	0.4362	0.4494	0.5000	0.5102
0.3	0.25	0.7	0.5	0.3222	0.2889	0.4000	0.5000	0.6444	0.5556	0.4889	0.5000	0.4322	0.3760	0.4898	0.5000

Table 4
FRPs of subgroups and the global group (Round 3).

C_1				C_2				C_3				Global group			
0.5	0.75	0.7	0.7	0.5000	0.3111	0.6778	0.6778	0.5000	0.5111	0.3667	0.3889	0.5000	0.4673	0.5717	0.5796
0.25	0.5	0.7	0.75	0.6889	0.5000	0.6222	0.7111	0.4889	0.5000	0.4000	0.5000	0.5327	0.5000	0.5585	0.6438
0.3	0.3	0.5	0.35	0.3222	0.3778	0.5000	0.6000	0.6333	0.6000	0.5000	0.5222	0.4283	0.4415	0.5000	0.5238
0.3	0.25	0.65	0.5	0.3222	0.2889	0.4000	0.5000	0.6111	0.5000	0.4778	0.5000	0.4204	0.3562	0.4762	0.5000

penalty mechanism directly. Other members of the subgroup play critical roles in the process of managing noncooperative behaviors. Tang and Liao (2021) summarized five critical issues in LSGDM: 1) dimension reduction; 2) weighting and aggregating decision information; 3) cost management; 4) behavior management; and 5) knowledge distribution. In this study, we used the MST clustering algorithm to reduce the dimensionality of the problem and introduced a weight determination method. The hierarchical consensus model and noncooperative behavior management method provided two viewpoints to address the third and fourth issues.

In the future, we will investigate how to address the situation in which a whole subgroup engages in noncooperative behaviors.

Furthermore, the minimum adjustment cost in the LSGDM is also an interesting topic that deserves further research.

Tables 3 and 4

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Appendix

Tables A1, A2 and A3

Table A1
RCMs of 20 experts.

$P_1 = \begin{pmatrix} 0.5 & 0.9 & 0.9 & 0.8 \\ 0.1 & 0.5 & 0.8 & 0.8 \\ 0.1 & 0.2 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.7 & 0.5 \end{pmatrix}$	$P_2 = \begin{pmatrix} 0.5 & 0.2 & 0.7 & 0.9 \\ 0.8 & 0.5 & 0.3 & 0.7 \\ 0.3 & 0.7 & 0.5 & 0.3 \\ 0.1 & 0.3 & 0.7 & 0.5 \end{pmatrix}$	$P_3 = \begin{pmatrix} 0.5 & 0.1 & 0.6 & 0.8 \\ 0.9 & 0.5 & 0.6 & 0.3 \\ 0.4 & 0.4 & 0.5 & 0.2 \\ 0.2 & 0.7 & 0.8 & 0.5 \end{pmatrix}$	$P_4 = \begin{pmatrix} 0.5 & 0.1 & 0.9 & 0.3 \\ 0.9 & 0.5 & 0.6 & 0.6 \\ 0.1 & 0.4 & 0.5 & 0.2 \\ 0.7 & 0.4 & 0.8 & 0.5 \end{pmatrix}$	$P_5 = \begin{pmatrix} 0.5 & 0.3 & 0.4 & 0.4 \\ 0.7 & 0.5 & 0.1 & 0.5 \\ 0.6 & 0.9 & 0.5 & 0.2 \\ 0.6 & 0.5 & 0.8 & 0.5 \end{pmatrix}$
$P_6 = \begin{pmatrix} 0.5 & 0.2 & 0.6 & 0.8 \\ 0.8 & 0.5 & 0.8 & 0.9 \\ 0.4 & 0.2 & 0.5 & 0.9 \\ 0.2 & 0.1 & 0.1 & 0.5 \end{pmatrix}$	$P_7 = \begin{pmatrix} 0.5 & 0.1 & 0.6 & 0.7 \\ 0.9 & 0.5 & 0.8 & 0.8 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.2 & 0.4 & 0.5 \end{pmatrix}$	$P_8 = \begin{pmatrix} 0.5 & 0.4 & 0.8 & 0.6 \\ 0.6 & 0.5 & 0.4 & 0.8 \\ 0.2 & 0.6 & 0.5 & 0.7 \\ 0.4 & 0.2 & 0.3 & 0.5 \end{pmatrix}$	$P_9 = \begin{pmatrix} 0.5 & 0.6 & 0.6 & 0.8 \\ 0.4 & 0.5 & 0.9 & 0.9 \\ 0.4 & 0.1 & 0.5 & 0.9 \\ 0.2 & 0.1 & 0.1 & 0.5 \end{pmatrix}$	$P_{10} = \begin{pmatrix} 0.5 & 0.8 & 0.6 & 0.8 \\ 0.2 & 0.5 & 0.6 & 0.7 \\ 0.4 & 0.4 & 0.5 & 0.9 \\ 0.2 & 0.3 & 0.1 & 0.5 \end{pmatrix}$
$P_{11} = \begin{pmatrix} 0.5 & 0.3 & 0.4 & 0.6 \\ 0.7 & 0.5 & 0.3 & 0.4 \\ 0.6 & 0.7 & 0.5 & 0.7 \\ 0.4 & 0.6 & 0.3 & 0.5 \end{pmatrix}$	$P_{12} = \begin{pmatrix} 0.5 & 0.3 & 0.7 & 0.4 \\ 0.7 & 0.5 & 0.6 & 0.7 \\ 0.3 & 0.4 & 0.5 & 0.7 \\ 0.6 & 0.3 & 0.3 & 0.5 \end{pmatrix}$			
$P_{13} = \begin{pmatrix} 0.5 & 0.6 & 0.1 & 0.3 \\ 0.4 & 0.5 & 0.4 & 0.2 \\ 0.9 & 0.6 & 0.5 & 0.4 \\ 0.7 & 0.8 & 0.6 & 0.5 \end{pmatrix}$	$P_{14} = \begin{pmatrix} 0.5 & 0.9 & 0.7 & 0.8 \\ 0.1 & 0.5 & 0.8 & 0.7 \\ 0.3 & 0.2 & 0.5 & 0.1 \\ 0.2 & 0.3 & 0.9 & 0.5 \end{pmatrix}$	$P_{15} = \begin{pmatrix} 0.5 & 0.8 & 0.4 & 0.5 \\ 0.2 & 0.5 & 0.1 & 0.2 \\ 0.6 & 0.9 & 0.5 & 0.3 \\ 0.5 & 0.8 & 0.7 & 0.5 \end{pmatrix}$	$P_{16} = \begin{pmatrix} 0.5 & 0.4 & 0.4 & 0.1 \\ 0.6 & 0.5 & 0.1 & 0.2 \\ 0.6 & 0.9 & 0.5 & 0.3 \\ 0.9 & 0.8 & 0.7 & 0.5 \end{pmatrix}$	
$P_{17} = \begin{pmatrix} 0.5 & 0.4 & 0.4 & 0.1 \\ 0.6 & 0.5 & 0.6 & 0.3 \\ 0.6 & 0.4 & 0.5 & 0.8 \\ 0.9 & 0.7 & 0.2 & 0.5 \end{pmatrix}$	$P_{18} = \begin{pmatrix} 0.5 & 0.6 & 0.4 & 0.1 \\ 0.4 & 0.5 & 0.3 & 0.7 \\ 0.6 & 0.7 & 0.5 & 0.8 \\ 0.9 & 0.3 & 0.2 & 0.5 \end{pmatrix}$	$P_{19} = \begin{pmatrix} 0.5 & 0.6 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.4 & 0.3 \\ 0.8 & 0.6 & 0.5 & 0.4 \\ 0.7 & 0.7 & 0.6 & 0.5 \end{pmatrix}$	$P_{20} = \begin{pmatrix} 0.5 & 0.7 & 0.4 & 0.1 \\ 0.3 & 0.5 & 0.3 & 0.4 \\ 0.6 & 0.7 & 0.5 & 0.7 \\ 0.9 & 0.6 & 0.3 & 0.5 \end{pmatrix}$	

Table A2
The distance matrix.

0	0.3651	0.4143	0.4041	0.4761	0.3797	0.3719	0.3215	0.3055	0.2915	0.4509	0.3582	0.5017	0.1225	0.4082	0.6055	0.505	0.4726	0.4564	0.4743
0.3651	0	0.2198	0.2944	0.2708	0.3342	0.2614	0.2309	0.3958	0.3719	0.2708	0.2915	0.4378	0.3629	0.3512	0.4865	0.4601	0.4359	0.3979	0.4528
0.4143	0.2198	0	0.2677	0.2972	0.3873	0.2769	0.344	0.4453	0.4359	0.2799	0.3215	0.3742	0.3786	0.3851	0.4778	0.4041	0.4813	0.3512	0.4546
0.4041	0.2944	0.2677	0	0.3055	0.4021	0.2858	0.2944	0.4583	0.4708	0.3559	0.2415	0.434	0.4062	0.4243	0.4619	0.3719	0.4082	0.3894	0.4143
0.4761	0.2708	0.2972	0.3055	0	0.4743	0.3894	0.3266	0.5132	0.4528	0.238	0.324	0.2614	0.4378	0.216	0.3367	0.3536	0.3215	0.2273	0.3028
0.3797	0.3342	0.3873	0.4021	0.4743	0	0.1414	0.2345	0.1683	0.2708	0.324	0.2236	0.5099	0.4435	0.5148	0.5083	0.4041	0.4062	0.4762	0.469
0.3719	0.2614	0.2769	0.2858	0.3894	0.1414	0	0.2273	0.2483	0.3266	0.2915	0.1826	0.4509	0.3916	0.4673	0.4882	0.3697	0.3979	0.4123	0.4435
0.3215	0.2309	0.344	0.2944	0.3266	0.2345	0.2273	0	0.2646	0.2345	0.238	0.1354	0.4223	0.3719	0.3512	0.3873	0.344	0.2828	0.3719	0.3342
0.3055	0.3958	0.4453	0.4583	0.5132	0.1683	0.2483	0.2646	0	0.1683	0.3697	0.2677	0.4983	0.3629	0.4865	0.5323	0.4143	0.3958	0.4601	0.4453
0.2915	0.3719	0.4359	0.4708	0.4528	0.2708	0.3266	0.2345	0.1683	0	0.3028	0.2769	0.4243	0.3416	0.3719	0.4453	0.3786	0.3342	0.3873	0.3559
0.4509	0.2708	0.2799	0.3559	0.238	0.324	0.2915	0.238	0.3697	0.3028	0	0.2273	0.2614	0.4453	0.2769	0.2517	0.2483	0.2708	0.2345	0.2614
0.3582	0.2915	0.3215	0.2415	0.324	0.2236	0.1826	0.1354	0.2677	0.2769	0.2273	0	0.3742	0.3916	0.3764	0.3488	0.2449	0.2483	0.3266	0.2944
0.5017	0.4378	0.3742	0.434	0.2614	0.5099	0.4509	0.4223	0.4983	0.4243	0.2614	0.3742	0	0.4472	0.2273	0.2915	0.2517	0.3028	0.0577	0.216
0.1225	0.3629	0.3786	0.4062	0.4378	0.4435	0.3916	0.3719	0.3629	0.3416	0.4453	0.3916	0.4472	0	0.3674	0.6069	0.5053	0.4848	0.4082	0.469
0.4082	0.3512	0.3851	0.4243	0.216	0.5148	0.4673	0.3512	0.4865	0.3719	0.2769	0.3764	0.2273	0.3674	0	0.3464	0.3719	0.3109	0.1958	0.2483
0.6055	0.4865	0.4778	0.4619	0.3367	0.5083	0.4882	0.3873	0.5323	0.4453	0.2517	0.3488	0.2915	0.6069	0.3464	0	0.2121	0.238	0.2799	0.1871
0.505	0.4601	0.4041	0.3719	0.3536	0.4041	0.3697	0.344	0.4143	0.3786	0.2483	0.2449	0.2517	0.5053	0.3719	0.2121	0	0.2198	0.2309	0.1826
0.4726	0.4359	0.4813	0.4082	0.3215	0.4062	0.3979	0.2828	0.3958	0.3342	0.2708	0.2483	0.3028	0.4848	0.3109	0.238	0.2198	0	0.2614	0.1354
0.4564	0.3979	0.3512	0.3894	0.2273	0.4762	0.4123	0.3719	0.4601	0.3873	0.2345	0.3266	0.0577	0.4082	0.1958	0.2799	0.2309	0.2614	0	0.1826
0.4743	0.4528	0.4546	0.4143	0.3028	0.469	0.4435	0.3342	0.4453	0.3559	0.2614	0.2944	0.216	0.469	0.2483	0.1871	0.1826	0.1354	0.1826	0

Table A3The clustering results with different values of λ .

λ	Corresponding clustering results
$\lambda = 0$	$\{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_6\}, \{e_7\}, \{e_8\}, \{e_9\}, \{e_{10}\}, \{e_{11}\}, \{e_{12}\}, \{e_{13}\}, \{e_{14}\}, \{e_{15}\}, \{e_{16}\}, \{e_{17}\}, \{e_{18}\}, \{e_{19}\}, \{e_{20}\}$
$\lambda = 0.0577$	$\{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_6\}, \{e_7\}, \{e_8\}, \{e_9\}, \{e_{10}\}, \{e_{11}\}, \{e_{12}\}, \{e_{13}, e_{19}\}, \{e_{14}\}, \{e_{15}\}, \{e_{16}\}, \{e_{17}\}, \{e_{18}\}, \{e_{20}\}$
$\lambda = 0.1225$	$\{e_1, e_{14}\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_6\}, \{e_7\}, \{e_8\}, \{e_9\}, \{e_{10}\}, \{e_{11}\}, \{e_{12}\}, \{e_{13}, e_{19}\}, \{e_{14}\}, \{e_{15}\}, \{e_{16}\}, \{e_{17}\}, \{e_{18}\}, \{e_{20}\}$
$\lambda = 0.1354$	$\{e_1, e_{14}\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_6\}, \{e_7\}, \{e_8, e_{12}\}, \{e_9\}, \{e_{10}\}, \{e_{11}\}, \{e_{13}, e_{19}\}, \{e_{14}\}, \{e_{15}\}, \{e_{16}\}, \{e_{17}\}, \{e_{18}, e_{20}\}$
$\lambda = 0.1414$	$\{e_1, e_{14}\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_6, e_7\}, \{e_8, e_{12}\}, \{e_9\}, \{e_{10}\}, \{e_{11}\}, \{e_{13}, e_{19}\}, \{e_{14}\}, \{e_{15}\}, \{e_{16}\}, \{e_{17}\}, \{e_{18}, e_{20}\}$
$\lambda = 0.1683$	$\{e_1, e_{14}\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_6, e_7, e_9, e_{10}\}, \{e_8, e_{12}\}, \{e_{11}\}, \{e_{13}, e_{19}\}, \{e_{14}\}, \{e_{15}\}, \{e_{16}\}, \{e_{17}\}, \{e_{18}, e_{20}\}$
$\lambda = 0.1826$	$\{e_1, e_{14}\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_6, e_7, e_8, e_9, e_{10}, e_{12}\}, \{e_{11}\}, \{e_{13}, e_{19}\}, \{e_{14}\}, \{e_{15}\}, \{e_{16}\}, \{e_{17}, e_{18}, e_{19}, e_{20}\}$
$\lambda = 0.1871$	$\{e_1, e_{14}\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_6, e_7, e_8, e_9, e_{10}, e_{12}\}, \{e_{11}\}, \{e_{13}, e_{19}\}, \{e_{14}\}, \{e_{15}\}, \{e_{16}, e_{17}, e_{18}, e_{19}, e_{20}\}$
$\lambda = 0.1958$	$\{e_1, e_{14}\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_6, e_7, e_8, e_9, e_{10}, e_{12}\}, \{e_{11}\}, \{e_{13}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}\}$
$\lambda = 0.216$	$\{e_1, e_{14}\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_6, e_7, e_8, e_9, e_{10}, e_{12}\}, \{e_{11}\}, \{e_{13}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}\}$
$\lambda = 0.2198$	$\{e_1, e_{14}\}, \{e_2, e_3\}, \{e_4\}, \{e_6, e_7, e_8, e_9, e_{10}, e_{12}\}, \{e_{11}\}, \{e_{13}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}\}$
$\lambda = 0.2309$	$\{e_1, e_{14}\}, \{e_4\}, \{e_2, e_3, e_6, e_7, e_8, e_9, e_{10}, e_{12}\}, \{e_{11}\}, \{e_{13}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}\}$
$\lambda = 0.2345$	$\{e_1, e_{14}\}, \{e_4\}, \{e_2, e_3, e_6, e_7, e_8, e_9, e_{10}, e_{12}\}, \{e_{11}\}, \{e_{13}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}\}$
$\lambda = 0.2415$	$\{e_1, e_{14}\}, \{e_2, e_3, e_4, e_6, e_7, e_8, e_9, e_{10}, e_{12}\}, \{e_{11}\}, \{e_{13}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}\}$

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