Neurocomputing 325 (2019) 159–171

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Finite-time leaderless consensus of uncertain multi-agent systems against time-varying actuator faults

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ARTICLE INFO

Article history:
Received 18 March 2018
Revised 16 September 2018
Accepted 7 October 2018
Available online 15 October 2018

Communicated by Dr. Derui Ding

Keywords:
Leaderless consensus
Multi-agent systems
Fault-tolerant control
Time-varying actuator faults

ABSTRACT

This paper addresses the finite-time leaderless consensus problem for a class of continuous-time multi-agent systems subject to linear fractional transformation uncertain parameters using an observer-based fault-tolerant controller. Here, it is considered that the network of the system is described by an undirected graph subject to fixed topology and the aforementioned controller is impacted by time-varying actuator faults. Then, the desired consensus protocols are proposed in such a way that the effects of possible uncertainties and actuator faults are compensated efficiently within a prescribed finite-time period. More precisely, the leaderless consensus analysis is carried out in the framework of Lyapunov-Krasovskii functional and the required conditions for the existence of proposed fault-tolerant controller are derived in terms of linear matrix inequalities. Moreover, the proposed consensus design parameters can be computed by solving a set of linear matrix inequality constraints. Finally, two examples including a formation flying satellites model are provided to show the efficiency and usefulness of the proposed control scheme.

1. Introduction

In the past few decades, the consensus analysis of multi-agent systems has been an active research and has attracted much of attention of control communities because of its practical applications in diverse areas, such as the formation of mobile robots, distributed sensor networks, control of electric power grids, and multiple spacecraft attitude alignment [1–3]. In general, consensus means that the states or outputs of a group of agents converge to some common value with the aid of an appropriate control algorithm. So far in the literature, different control methods have been applied to design the consensus protocols including impulsive control [4], sampled-data control [5], sliding mode control [6], adaptive control [7] and event-triggered control [8]. It should be mentioned that the study of consensus problems can be mainly classified into two categories, namely, leaderless consensus [9,10] and leader-follower consensus (also known as distributed tracking) [11,12]. More specifically, in leaderless consensus problems, the final consensus value may not be available all the time because it depends on the system dynamics and the agents' initial conditions, whereas in leader-follower consensus problems, the final consensus value is known prior as the trajectory of the leader. However, in recent years, the focus on solving leaderless consensus problems has been increased enormously and numerous interesting results have been proposed under different scenarios subsequently [13–15].

Note that most of the works mentioned above on multi-agent systems are based on the state feedback approach. Nevertheless, in some practical cases, it is very difficult to collect full state information of an agent, while the relative outputs of its neighbors are easy to acquire [16]. In such situations, the investigation on consensus of multi-agent systems via feedback of relative outputs is more appropriate compared with the traditional state feedback control method. Over the past couple of decades, the observer-based consensus algorithms have gained a considerable amount of attention and many important results have been reported. For instance, see [17–20] and the references cited therein. On the other hand, it should be mentioned that the existence of uncertainties in the dynamics of multi-agent systems may lead to loss of connectivity, collision among agents and deterioration of consensus performance. Therefore, it is of foremost importance to discuss the consensus of multi-agent systems in the presence of uncertainties. Related references can be seen in [21–23].

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https://doi.org/10.1016/j.neucom.2018.10.020
0925-2312/

Though most of researches on multi-agent systems concentrated about asymptotic consensus protocols (convergence time is infinite), in many practical engineering systems, it is required that consensus be achieved within a finite-time period. To deal with such situations, finite-time consensus protocols have been proposed based on the theory of finite-time stability [24,25]. In contrast to the usual finite-time stability concept, some literature have shown that it is possible to speed up the convergence rate to a desired finite time by designing an appropriate nonlinear activation function, see [26–28] and the references cited therein. However, the main feature of finite-time consensus protocols is not only providing faster convergence but also having better robustness against uncertainties over other consensus protocols. Hence, the analysis of finite-time consensus of multi-agent systems has profited extensive concern among research communities in the last few years [29–33]. Interestingly, the problems of finite-time consensus subject to strongly connected network and fixed-time consensus under leader-following network for a class of switched multi-agent systems have been solved in [30]. By designing a novel discontinuous disturbed observer, the problem of finite-time robust consensus for leader-following nonlinear multi-agent systems has been discussed in [31].

When the number of agents in a multi-agent system is very large, it is often difficult to reach the desired consensus since the exact information could not be received by the agents due to communication issues. To overcome this shortcoming, it is more significant to apply control schemes to achieve the consensus. In the context of multi-agent systems, a variety of control schemes have been employed to design valid consensus protocols. For example, see [34–37] and references therein. During the controller implementation, it is quite nature that faults frequently occur in the actuators, which may yield unacceptable system performance [38]. In such situation, the fault-tolerant control scheme is more suitable since it has the capability of preserving consensus in the presence of possible actuator faults. During the past decades, constant progress has been made on the fault-tolerant control design problems for consensus of multi-agent systems [39–41]. However, as far as our knowledge, the problem of finite-time fault-tolerant consensus of multi-agent systems has not yet been investigated completely, which motivates this present study.

Based on the existing results and motivated by the above observation, this paper discusses the issue of finite-time leaderless consensus of uncertain multi-agent systems by using the observer-based fault-tolerant controller in the presence of time-varying actuator faults. First, by constructing Luenberger state observer for each agent, the agents unknown states are estimated over a prescribed finite-time interval. Second, a robust fault-tolerant control that takes the effect of time-varying actuator faults is designed to ensure the desired finite-time consensus. Third, sufficient conditions guaranteeing the finite-time leaderless consensus of considered multi-agent systems are established based on the Lyapunov-Krasovskii stability theory and the finite-time stability. Subsequently, a design method of the proposed observer-based fault-tolerant control is presented. At last, the effectiveness and applicability of the proposed control design are illustrated by numerical simulations.

2. Graph theory preliminaries and model description

In this section, we first provide some notions from algebraic graph theory and then formulate the leaderless fault-tolerant consensus problem of multi-agent systems.

In order to represent the interactions among a finite number of agents, say N, in an environment, an undirected graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ is composed by a non-empty vertex set of nodes $\mathcal{V}=\{v_1,v_2,\dots,v_N\}$, an edge set $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$, and an adjacency matrix $\mathcal{A}=[a_{ij}]_{N\times N}$ with non-negative elements a_{ij}. The adjacency element $a_{ij}>0$ is associated with the fact that there exists an edge between node v_i and node v_j, otherwise, $a_{ij}=0$. The set of neighbors of node v_i is denoted by $\mathcal{N}_i=\{v_j\in\mathcal{V}:(v_i,v_j)\in\mathcal{E}\}$. A graph $\mathcal{G}$ is connected if for any pair of distinct nodes v_i and v_j in $\mathcal{G}$, there exists a path between v_i and v_j $(i,j=1,2,\dots,N)$. The Laplacian matrix $\mathcal{L}=[l_{ij}]\in\mathbb{R}^{N\times N}$ is defined by $l_{ii}=\sum_{j=1,j\neq i}a_{ij}$ and $l_{ij}=-a_{ij}$ for $i\neq j$, which satisfies zero row sum property.

Consider a group of N autonomous agents with identical uncertainties and actuator faulty input, whose dynamic equations are expressed as

$$\begin{aligned}\dot{x}_i(t)&=(A+\Delta A(t))x_i(t)+Bu_i^f(t),\\ y_i(t)&=Cx_i(t),\quad i=1,2,\dots,N,\end{aligned}\tag{1}$$

where $x_i(t)\in\mathbb{R}^n$ denotes the state of the ith agent; $u_i^f(t)\in\mathbb{R}^m$ is the consensus protocol of the ith agent, which consists of time-varying actuator faults; $y_i(t)\in\mathbb{R}^p$ is the measured output of the ith agent; A, B and C are given constant matrices with appropriate dimensions; $\Delta A(t)$ is the uncertain matrix taking of the linear fractional transformation parameter uncertainty structure $\Delta A(t)=H\Delta(t)E$, where H and E are constant matrices with appropriate dimensions and the time-varying function $\Delta(t)$ satisfies $\Delta(t)=[I-F(t)J]^{-1}F(t)$ in which J is a known matrix satisfying $I-JJ^T>0$ and $F(t)$ is an unknown time-varying matrix with Lebesgue measurable elements bounded by $F^T(t)F(t)\leq I$.

In this study, as full-state measurements are not available in many real applications, it is assumed that an agent is not able to access full state information but only output information of its neighbors. Therefore, to achieve the desired consensus, an observer is adopted for the agent to estimate those unaccessible state information. Thus, the following observer dynamics for the system (1) is considered:

$$\begin{aligned}\dot{\hat{x}}_i(t)&=A\hat{x}_i(t)+Bu_i^f(t)+L_0[y_i(t)-\hat{y}_i(t)],\\ \hat{y}_i(t)&=C\hat{x}_i(t),\quad i=1,2,\dots,N,\end{aligned}\tag{2}$$

where $\hat{x}_i(t)\in\mathbb{R}^n$ is the observer state of the ith agent; $\hat{y}_i(t)\in\mathbb{R}^p$ denotes the output estimation vector of the ith agent; $L_0\in\mathbb{R}^{n\times p}$ is the gain matrix of the observer. Further, the fault-tolerant controller consisting of time-varying actuator faults is adopted as $u_{i,q}^f(t)=(1-\lambda_{i,q}(t))u_{i,q}(t)$, $0\leq\lambda_{i,q}(t)\leq\bar{\lambda}_{*,q}<1$ for $q=1,2,\dots,m$, where $u_{i,\,q}(t)$ and $u_{i,q}^f(t)$ represent the input signal and the output signal of the ith actuator, respectively; $\lambda_{i,\,q}(t)$ is the actuator failure factor, which is assumed to be unknown and piecewise continuous bounded; $\bar{\lambda}_{*,q}$ is a known constant representing the upper bound of $\lambda_{i,\,q}(t)$. Let us define $u_i^f(t)=[u_{i,1}^f(t),u_{i,2}^f(t),\dots,u_{i,m}^f(t)]^T$ and $\lambda_i(t)=\text{diag}\{\lambda_{i,1}(t),\lambda_{i,2}(t),\dots,\lambda_{i,m}(t)\}$. Then, the overall actuator fault model can be expressed as

$$u_i^f(t)=(I_m-\lambda_i(t))u_i(t),\quad i=1,2,\dots,N,\tag{3}$$

where $u_i(t)$ is defined by $u_i(t)=K\sum_{j\in\mathcal{N}_i}a_{ij}[\hat{x}_i(t-\tau(t))-\hat{x}_j(t-\tau(t))]$ in which $K\in\mathbb{R}^{p\times n}$ is the feedback gain matrix to be determined and $\tau(t)$ is the time-varying delay function that satisfies $0\leq\tau_1\leq\tau(t)\leq\tau_2$ and $0\leq\dot{\tau}(t)\leq\mu<1$, where τ_1 and τ_2 are known lower and upper bounds of $\tau(t)$, respectively, and μ is the derivative limit of $\tau(t)$.

Now, by substituting the control term (3) into (2), the following set of equations can be obtained:

$$\begin{aligned}\dot{\hat{x}}_i(t)&=A\hat{x}_i(t)+B(I_m-\lambda_i(t))K\sum_{j\in\mathcal{N}_i}a_{ij}[\hat{x}_i(t-\tau(t))-\hat{x}_j(t-\tau(t))]\\ &\quad+L_0[y_i(t)-\hat{y}_i(t)],\\ &i=1,2,\dots,N.\end{aligned}\tag{4}$$

After connecting the observer dynamics (4) and the system dynamics (1) together, and setting $e_i(t) = x_i(t) - \hat{x}_i(t)$, we can get the error system as

$$\dot{e}_i(t) = (A + \Delta A(t) - L_0 C)e_i(t) + \Delta A(t)\hat{x}_i(t). \quad (5)$$

By utilizing properties of the Kronecker product and simple manipulation, the above set of observer systems and error systems can be rewritten in the following forms:

$$\begin{aligned}\dot{\hat{x}}(t) &= (I \otimes A)\hat{x}(t) + (\mathcal{L} \otimes BK)\hat{x}(t - \tau(t)) \\ &\quad - (\mathcal{L} \otimes BK)\lambda(t)\hat{x}(t - \tau(t)) + (I \otimes L_0 C)e(t),\end{aligned} \quad (6)$$

$$\dot{e}(t) = [(I \otimes A) + (I \otimes \Delta A(t)) - (I \otimes L_0 C)]e(t) + (I \otimes \Delta A(t))\hat{x}(t), \quad (7)$$

where $\hat{x}(t) = [\hat{x}_1^T(t), \hat{x}_2^T(t), \ldots, \hat{x}_N^T(t)]^T$, $e(t) = [e_1^T(t), e_2^T(t), \ldots, e_N^T(t)]^T$ and $\lambda(t) = \mathrm{diag}\{\lambda_1(t), \lambda_2(t), \ldots, \lambda_N(t)\}$ with $0 \le \lambda(t) \le \bar{\lambda}_* < I$ and $\bar{\lambda}_* = \mathrm{diag}\{\bar{\lambda}_{*,1}, \bar{\lambda}_{*,2}, \ldots, \bar{\lambda}_{*,N}\}$. Further, by defining $\xi(t) = [\hat{x}^T(t) \;\; e^T(t)]^T$, and combining (6) and (7), the following augmented system can be obtained:

$$\dot{\xi}(t) = (\bar{A} + \Delta\bar{A}(t))\xi(t) + \bar{B}\xi(t - \tau(t)), \quad (8)$$

where

$$\bar{A} = \begin{bmatrix} (I \otimes A) & (I \otimes L_0 C) \\ 0 & (I \otimes A) - (I \otimes L_0 C) \end{bmatrix},$$

$$\Delta\bar{A}(t) = \begin{bmatrix} 0 & 0 \\ (I \otimes \Delta A(t)) & (I \otimes \Delta A(t)) \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} (\mathcal{L} \otimes BK) - (\mathcal{L} \otimes BK)\lambda(t) & 0 \\ 0 & 0 \end{bmatrix}.$$

This study aims at designing a consensus protocol in the form of (3) to keep the leaderless consensus in the system (1) over a desired finite time. To achieve this goal, it is sufficient to show that the augmented system (8) is finite-time stable. Before presenting the main results, we provide some preliminaries, which are more indispensable for the later development.

Assumption 1. The undirected communication graph $\mathcal{G}$ is connected, and there is only one agent that can access its own output information and can be chosen arbitrarily.

Assumption 2. The matrix pair (A, B) is stabilizable and (A, C) is detectable.

Definition 1. The augmented system (8) is finite-time stable with respect to $(c_1, c_2, \tau_2, T^*, S)$, where S is a positive definite matrix and $c_2 > c_1 > 0$ are scalars, if $\sup_{-\tau_2 \le t_0 \le 0} \{\xi^T(t_0)(I \otimes S)\xi(t_0), \dot{\xi}^T(t_0)(I \otimes S)\dot{\xi}(t_0)\} \le c_1 \Rightarrow \xi^T(t)(I \otimes S)\xi(t) < c_2$, $t \in [0, T^*]$.

Lemma 1. *[42] For any full row rank matrix $\Pi \in \mathbb{R}^{q \times n}$ with $q < n$, there exists a singular-value decomposition of Π as $\Pi = U[\mathbb{S} \;\; 0]V^T$, where $U \in \mathbb{R}^{q \times q}$ and $V \in \mathbb{R}^{n \times n}$ are unitary matrices and $\mathbb{S} \in \mathbb{R}^{n \times n}$ is a diagonal matrix with non-negative diagonal elements in decreasing order.*

Lemma 2. *[42] For a given matrix $\Pi \in \mathbb{R}^{q \times n}$ with $q < n$ and full row rank, there exists a matrix $\bar{X} \in \mathbb{R}^{q \times q}$ satisfying $\Pi X = \bar{X}\Pi$ for any matrix $X \in \mathbb{R}^{n \times n}$, if and only if X can be described as $X = V\begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix}V^T$, where $X_{11} \in \mathbb{R}^{q \times q}$, $X_{22} \in \mathbb{R}^{(n-q) \times (n-q)}$ and $V \in \mathbb{R}^{n \times n}$ is the unitary matrix of singular-value decomposition of Π.*

Lemma 3. *[43] Let $\mathcal{R}_1, \mathcal{R}_2 \in \mathbb{R}^{n \times n}$ be positive definite matrices, $\mathcal{M}_1, \mathcal{M}_2 \in \mathbb{R}^n$ and $\alpha \in (0, 1)$. Then, for any $\mathcal{Q}_1, \mathcal{Q}_2 \in \mathbb{R}^{n \times n}$, the following inequality holds:*

$$\begin{aligned}\Omega(\alpha) &\ge \mathcal{M}_1^T[\mathcal{R}_1 + (1 - \alpha)(\mathcal{R}_1 - \mathcal{Q}_1\mathcal{R}_1^{-1}\mathcal{Q}_1^T)]\mathcal{M}_1 \\ &\quad + \mathcal{M}_2^T[\mathcal{R}_2 + \alpha(\mathcal{R}_2 - \mathcal{Q}_2^T\mathcal{R}_2^{-1}\mathcal{Q}_2)]\mathcal{M}_2 \\ &\quad + 2\mathcal{M}_1^T[\alpha\mathcal{Q}_1 + (1 - \alpha)\mathcal{Q}_2]\mathcal{M}_2,\end{aligned} \quad (9)$$

where $\Omega(\alpha) = \frac{1}{\alpha}\mathcal{M}_1^T\mathcal{R}_1\mathcal{M}_1 + \frac{1}{1-\alpha}\mathcal{M}_2^T\mathcal{R}_2\mathcal{M}_2$.

3. Main results

This section starts with the finite-time stability analysis of the augmented system (8) in the absence of uncertain term $(\Delta\bar{A}(t) = 0)$. Then, it will be extended to the case with uncertain term and the explicit expression of feedback control gain and observer gain ensuring the robust finite-time stability of the augmented system (8) will be presented subsequently. It should be mentioned that the aforementioned processes are equivalent to indirectly solve the desired leaderless consensus problem of the multi-agent system (1). Here, for representation convenience, we denote $P = \mathrm{diag}\{P_1, P_1\}$, $Q_p = \mathrm{diag}\{Q_{1p}, Q_{2p}\}$ and $R_q = \mathrm{diag}\{R_{1q}, R_{2q}\}$, $(p = 1, 2, 3, q = 1, 2)$.

Theorem 1. *Let Assumptions 1 and 2 be true. For given positive scalars c_1, c_2, T^*, $\alpha \in (0, 1)$, μ, τ_1 and τ_2, and positive definite matrix S, the augmented system (8) with $\Delta\bar{A}(t) = 0$ is finite-time stable with respect to $(c_1, c_2, \tau_2, T^*, S)$ if there exist symmetric matrices $P > 0$, $Q_p > 0$ $(p = 1, 2, 3)$ and $R_q > 0$ $(q = 1, 2)$, any appropriate dimensioned matrices Z_1, Z_2 and positive scalars λ_g $(g = 1, 2, \ldots, 7)$ such that the following constraints are satisfied:*

$$\begin{bmatrix} [\Lambda_{a,b}]_{11 \times 11} & \Gamma_1^l & \Gamma_2^l & \Gamma_3^l \\ * & -(I \otimes R_2) & 0 & 0 \\ * & * & -3(I \otimes R_2) & 0 \\ * & * & * & -5(I \otimes R_2) \end{bmatrix} < 0, \quad l = 1, 2, \quad (10)$$

$$c_1 e^{\delta T^*}\left(\lambda_1 + \tau_1\lambda_2 + \tau_2\lambda_3 + \tau_2\lambda_4 + \frac{\tau_1^3}{2}\lambda_5 + \frac{(\tau_2 - \tau_1)^3}{2}\lambda_6\right) - \lambda_7 c_2 < 0, \quad (11)$$

where

$$\Lambda_{1,1} = (I \otimes P)\bar{A} + \bar{A}^T(I \otimes P) + \sum_{k=1}^{3}(I \otimes Q_k) - 4(I \otimes R_1),$$

$$\Lambda_{1,2} = -2(I \otimes R_1), \quad \Lambda_{1,3} = (I \otimes P)\bar{B},$$

$$\Lambda_{1,5} = 6(I \otimes R_1), \quad \Lambda_{1,10} = \tau_1\bar{A},$$

$$\Lambda_{1,11} = (\tau_2 - \tau_1)\bar{A}, \quad \Lambda_{2,2} = -(I \otimes Q_1) - 4(I \otimes R_1) - 64(2 - \alpha)(I \otimes R_2),$$

$$\Lambda_{2,3} = 24(2 - \alpha)(I \otimes R_2) + 9\alpha(I \otimes Z_1) + 9(1 - \alpha)(I \otimes Z_2),$$

$$\Lambda_{2,4} = -3\alpha(I \otimes Z_1) - 3(1 - \alpha)(I \otimes Z_2),$$

$$\Lambda_{2,5} = 6(I \otimes R_1), \quad \Lambda_{2,6} = 288(2 - \alpha)(I \otimes R_2),$$

$$\Lambda_{2,7} = -480(2 - \alpha)(I \otimes R_2),$$

$$\Lambda_{2,8} = -24\alpha(I \otimes Z_1) - 24(1 - \alpha)(I \otimes Z_2),$$

$$\Lambda_{2,9} = 36\alpha(I \otimes Z_1) + 36(1 - \alpha)(I \otimes Z_2),$$

$$\Lambda_{3,3} = -(1 - \mu)(I \otimes Q_2) - 9(2 - \alpha)(I \otimes R_2) - 64(1 + \alpha)(I \otimes R_2) - 3\alpha(I \otimes Z_1) - 3(1 - \alpha)(I \otimes Z_2),$$

$$\Lambda_{3,4} = 24(1 + \alpha)(I \otimes R_2), \quad \Lambda_{3,6} = -108(2 - \alpha)(I \otimes R_2) - 24\alpha(I \otimes Z_1)^T - 24(1 - \alpha)(I \otimes Z_2)^T,$$

$$\Lambda_{3,7} = 180(2 - \alpha)(I \otimes R_2) + 36\alpha(I \otimes Z_1)^T + 36(1 - \alpha)(I \otimes Z_2)^T,$$

$$\Lambda_{3,8} = 288(1 + \alpha)(I \otimes R_2) + 8\alpha(I \otimes Z_1) + 8(1 - \alpha)(I \otimes Z_2),$$

$$\Lambda_{3,9} = -480(1 + \alpha)(I \otimes R_2) - 12\alpha(I \otimes Z_1) - 12(1 - \alpha)(I \otimes Z_2),$$

$$\Lambda_{3,10} = \tau_1\bar{B},$$

$$\Lambda_{3,11} = (\tau_2 - \tau_1)\bar{B}, \quad \Lambda_{4,4} = -(I \otimes Q_3) - 9(1+\alpha)(I \otimes R_2),$$
$$\Lambda_{4,6} = 8\alpha(I \otimes Z_1)^T + 8(1-\alpha)(I \otimes Z_2)^T,$$
$$\Lambda_{4,7} = -12\alpha(I \otimes Z_1)^T - 12(1-\alpha)(I \otimes Z_2)^T,$$
$$\Lambda_{4,8} = -108(1+\alpha)(I \otimes R_2), \quad \Lambda_{4,9} = 180(1+\alpha)(I \otimes R_2),$$
$$\Lambda_{5,5} = -12(I \otimes R_1), \quad \Lambda_{6,6} = -1296(2-\alpha)(I \otimes R_2),$$
$$\Lambda_{6,7} = 2160(2-\alpha)(I \otimes R_2),$$
$$\Lambda_{6,8} = 64\alpha(I \otimes Z_1) + 64(1-\alpha)(I \otimes Z_2),$$
$$\Lambda_{6,9} = -96\alpha(I \otimes Z_1) - 96(1-\alpha)(I \otimes Z_2),$$
$$\Lambda_{7,7} = -3600(2-\alpha)(I \otimes R_2),$$
$$\Lambda_{7,8} = -96\alpha(I \otimes Z_1) - 96(1-\alpha)(I \otimes Z_2),$$
$$\Lambda_{7,9} = 144\alpha(I \otimes Z_1) + 144(1-\alpha)(I \otimes Z_2),$$
$$\Lambda_{8,8} = -1296(1+\alpha)(I \otimes R_2), \quad \Lambda_{8,9} = 2160(1+\alpha)(I \otimes R_2),$$
$$\Lambda_{9,9} = -3600(1+\alpha)(I \otimes R_2), \quad \Lambda_{10,10} = -(I \otimes R_1)^{-1},$$
$$\Lambda_{11,11} = -(I \otimes R_2)^{-1},$$
$$\Gamma_1^1 = [0 \ (I \otimes Z_1) \ -(I \otimes Z_1) \ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T,$$
$$\Gamma_2^1 = [0 \ (I \otimes Z_1) \ (I \otimes Z_1) \ 0\ 0 \ -2(I \otimes Z_1) \ 0\ 0\ 0\ 0\ 0]^T,$$
$$\Gamma_3^1 = [0 \ (I \otimes Z_1) \ -(I \otimes Z_1) \ 0\ 0 \ -6(I \otimes Z_1) \ 12(I \otimes Z_1) \ 0\ 0\ 0\ 0]^T,$$
$$\Gamma_1^2 = [0\ 0 \ (I \otimes Z_2) \ -(I \otimes Z_2) \ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T,$$
$$\Gamma_2^2 = [0\ 0 \ (I \otimes Z_2) \ (I \otimes Z_2) \ 0\ 0\ 0 \ -2(I \otimes Z_2) \ 0\ 0\ 0]^T,$$
$$\Gamma_3^2 = [0\ 0 \ (I \otimes Z_2) \ -(I \otimes Z_2) \ 0\ 0\ 0 \ -6(I \otimes Z_2) \ 12(I \otimes Z_2) \ 0\ 0]^T$$

and the remaining parameters are zero.

Proof. To establish the finite-time stability criterion for the augmented system (8) with $\Delta\bar{A}(t) = 0$, the Lyapunov-Krasovskii functional candidate is selected in the following form:

$$V(\xi(t)) = \sum_{l=1}^{3} V_l(\xi(t)), \tag{12}$$

where

$$V_1(\xi(t)) = \xi^T(t)(I \otimes P)\xi(t),$$
$$\begin{aligned} V_2(\xi(t)) = & \int_{t-\tau_1}^{t} \xi^T(s)(I \otimes Q_1)\xi(s)\mathrm{d}s \\ & + \int_{t-\tau(t)}^{t} \xi^T(s)(I \otimes Q_2)\xi(s)\mathrm{d}s \\ & + \int_{t-\tau_2}^{t} \xi^T(s)(I \otimes Q_3)\xi(s)\mathrm{d}s, \end{aligned}$$
$$\begin{aligned} V_3(\xi(t)) = & \tau_1 \int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{\xi}^T(s)(I \otimes R_1)\dot{\xi}(s)\mathrm{d}s\mathrm{d}\theta \\ & +(\tau_2 - \tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{\xi}^T(s)(I \otimes R_2)\dot{\xi}(s)\mathrm{d}s\mathrm{d}\theta. \end{aligned}$$

Then, by calculating the time derivative of $V(\xi(t))$ along the solution of the augmented system (8) without uncertain term, we can easily get that

$$\dot{V}_1(\xi(t)) = \xi^T(t)(I \otimes P)\dot{\xi}(t) + \dot{\xi}^T(t)(I \otimes P)\xi(t), \tag{13}$$

$$\begin{aligned} \dot{V}_2(\xi(t)) \le & \, \xi^T(t)\big[(I \otimes Q_1) + (I \otimes Q_2) + (I \otimes Q_3)\big]\xi(t) \\ & - \xi^T(t-\tau_1)(I \otimes Q_1)\xi(t-\tau_1) \\ & - (1-\mu)\xi^T(t-\tau(t))(I \otimes Q_2)\xi(t-\tau(t)) \\ & - \xi^T(t-\tau_2)(I \otimes Q_3)\xi(t-\tau_2), \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{V}_3(\xi(t)) = & \, \dot{\xi}^T(t)\big[\tau_1^2(I \otimes R_1) + (\tau_2-\tau_1)^2(I \otimes R_2)\big]\dot{\xi}(t) \\ & - \tau_1 \int_{t-\tau_1}^{t} \dot{\xi}^T(s)(I \otimes R_1)\dot{\xi}(s)\mathrm{d}s \\ & - (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1} \dot{\xi}^T(s)(I \otimes R_2)\dot{\xi}(s)\mathrm{d}s. \end{aligned} \tag{15}$$

By applying Lemma 1 in [44] to the integral term involving $(I \otimes R_1)$ in Eq. (15), it can be rewritten as

$$\begin{aligned} & - \tau_1 \int_{t-\tau_1}^{t} \dot{\xi}^T(s)(I \otimes R_1)\dot{\xi}(s)\mathrm{d}s \\ & \le \begin{bmatrix} \xi(t) \\ \xi(t-\tau_1) \\ \frac{1}{\tau_1}\int_{t-\tau_1}^{t}\xi(s)\mathrm{d}s \end{bmatrix}^T \\ & \times \begin{bmatrix} -4(I \otimes R_1) & -2(I \otimes R_1) & 6(I \otimes R_1) \\ * & -4(I \otimes R_1) & 6(I \otimes R_1) \\ * & * & -12(I \otimes R_1) \end{bmatrix} \\ & \times \begin{bmatrix} \xi(t) \\ \xi(t-\tau_1) \\ \frac{1}{\tau_1}\int_{t-\tau_1}^{t}\xi(s)\mathrm{d}s \end{bmatrix}. \end{aligned} \tag{16}$$

Moreover, the last integral term in (15) can be split as

$$\begin{aligned} & - (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1} \dot{\xi}^T(s)(I \otimes R_2)\dot{\xi}(s)\mathrm{d}s \\ & \le -(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau(t)} \dot{\xi}^T(s)(I \otimes R_2)\dot{\xi}(s)\mathrm{d}s \\ & - (\tau_2-\tau_1)\int_{t-\tau(t)}^{t-\tau_1} \dot{\xi}^T(s)(I \otimes R_2)\dot{\xi}(s)\mathrm{d}s. \end{aligned} \tag{17}$$

By applying Lemma 1 in [43] and Lemma 3 to the above integral terms, we can get

$$\begin{aligned} & - (\tau_2-\tau_1)\int_{t-\tau(t)}^{t-\tau_1} \dot{\xi}^T(s)(I \otimes R_2)\dot{\xi}(s)\mathrm{d}s \\ & - (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau(t)} \dot{\xi}^T(s)(I \otimes R_2)\dot{\xi}(s)\mathrm{d}s \\ & \le -\frac{(\tau_2-\tau_1)}{\tau(t)-\tau_1}\zeta_1^T(t)\mathcal{R}_2\zeta_1(t) \\ & - \frac{(\tau_2-\tau_1)}{\tau_2-\tau(t)}\zeta_2^T(t)\mathcal{R}_2\zeta_2(t) \\ & \le (1-\alpha)\zeta_1^T(t)Z_1\mathcal{R}_2^{-1}Z_1^T\zeta_1(t) + \alpha\zeta_2^T(t)Z_2^T\mathcal{R}_2^{-1}Z_2\zeta_2(t) \\ & - (2-\alpha)\zeta_1^T(t)\mathcal{R}_2\zeta_1(t) - (1+\alpha)\zeta_2^T(t)\mathcal{R}_2\zeta_2(t) \\ & - 2\alpha\zeta_1^T(t)Z_1\zeta_1(t) - 2(1-\alpha)\zeta_2^T(t)Z_2\zeta_2(t), \end{aligned} \tag{18}$$

where $\zeta_1^T(t) = [\xi(t-\tau_1) - \xi(t-\tau(t))\xi(t-\tau_1) + \xi(t-\tau(t)) - \frac{2}{\tau_2-\tau_1}\int_{t-\tau(t)}^{t-\tau_1}\xi(s)\mathrm{d}s\, \xi(t-\tau_1) - \xi(t-\tau(t)) - \frac{6}{\tau_2-\tau_1}\int_{t-\tau(t)}^{t-\tau_1}\xi(s)\mathrm{d}s + \frac{12}{(\tau_2-\tau_1)^2}\int_{t-\tau(t)}^{t-\tau_1}(t-s)\xi(s)\mathrm{d}s]$, $\zeta_2^T(t) = [\xi(t-\tau(t)) - \xi(t-\tau_2)\xi(t-\tau(t)) + \xi(t-\tau_1) - \frac{2}{\tau_2-\tau(t)}\int_{t-\tau_2}^{t-\tau(t)}\xi(s)\mathrm{d}s\,\xi(t-\tau(t)) - \xi(t-\tau_2) - \frac{6}{\tau_2-\tau(t)}\int_{t-\tau_2}^{t-\tau(t)}\xi(s)\mathrm{d}s + \frac{12}{(\tau_2-\tau(t))^2}\int_{t-\tau_2}^{t-\tau(t)}(t-\tau(t)-s)\xi(s)\mathrm{d}s]$ and $\mathcal{R}_2 = diag\{(I \otimes R_2), 3(I \otimes R_2), 5(I \otimes R_2)\}$.

By considering (13)–(18) together, we eventually have

$$\begin{aligned} \dot{V}(\xi(t)) \le & \, \zeta^T(t)[\Omega^0]_{9\times 9}\zeta(t) + (1-\alpha)\zeta_1^T(t)Z_1^T\mathcal{R}_2^{-1}Z_1\zeta_1(t) \\ & + \alpha\zeta_2^T(t)Z_2^T\mathcal{R}_2^{-1}Z_2\zeta_2(t) \\ & + \dot{\xi}^T(t)\big[\tau_1^2(I \otimes R_1) + (\tau_2-\tau_1)^2(I \otimes R_2)\big]\dot{\xi}(t), \end{aligned} \tag{19}$$

where $\zeta(t) = [\xi^T(t)\xi^T(t-\tau_1)\xi^T(t-\tau(t))\xi^T(t-\tau_2)\frac{1}{\tau_1}\int_{t-\tau_1}^{t}\xi^T(s)\mathrm{d}s\frac{1}{\tau_2-\tau_1}\int_{t-\tau(t)}^{t-\tau_1}\xi^T(s)\mathrm{d}s\frac{1}{(\tau_2-\tau_1)^2}\int_{t-\tau(t)}^{t-\tau_1}(t-s)\xi^T(s)\mathrm{d}s\frac{1}{\tau_2-\tau(t)}\int_{t-\tau_2}^{t-\tau(t)}\xi^T(s)\mathrm{d}s\frac{1}{(\tau_2-\tau(t))^2}\int_{t-\tau_2}^{t-\tau(t)}(t-\tau(t)-s)\xi^T(s)\mathrm{d}s]^T$ and the elements of $[\Omega^0]_{9\times 9}$ are same as in the first nine rows and the first nine columns of (10).

Define $\eta_1 = [0 \ I \ I \ I \ 0 \ I \ I \ 0 \ 0]$, $\eta_2 = [0 \ I \ I \ I \ 0 \ 0 \ 0 \ I \ I]$ and $\Gamma = [\bar{A} \ 0 \ \bar{B} \ 0\ 0\ 0\ 0\ 0\ 0]$. Then, by the virtue of Schur complement, the inequality (19) can be equivalently written as

$$\dot{V}(\xi(t)) \leq \zeta^T(t)\Lambda\zeta(t), \tag{20}$$

where $\Lambda = [\Omega^0]_{9\times 9} + (1-\alpha)\eta_1^T Z_1^T \mathcal{R}_2^{-1} Z_1\eta_1 + \alpha\eta_2^T Z_2^T \mathcal{R}_2^{-1} Z_2\eta_2 + \Gamma^T[\tau_1^2(I\otimes R_1) + (\tau_2-\tau_1)^2(I\otimes R_2)]\Gamma$. It is noted from (20) that, if $\Lambda < 0$ then, it is obvious that $\dot{V}(\xi(t)) < 0$, if the conditions in (10) are satisfied. Furthermore, if there exists a constant $\delta > 0$, we can have $\dot{V}(\xi(t)) < \delta V(\xi(t))$ which implies that $e^{-\delta t}V(\xi(t)) < V(\xi(0))$. Now, introduce some new parameters: $\tilde{P} = S^{-\frac{1}{2}}PS^{-\frac{1}{2}}$, $\tilde{Q}_1 = S^{-\frac{1}{2}}Q_1S^{-\frac{1}{2}}$, $\tilde{Q}_2 = S^{-\frac{1}{2}}Q_2S^{-\frac{1}{2}}$, $\tilde{Q}_3 = S^{-\frac{1}{2}}Q_3S^{-\frac{1}{2}}$, $\tilde{R}_1 = S^{-\frac{1}{2}}R_1S^{-\frac{1}{2}}$ and $\tilde{R}_2 = S^{-\frac{1}{2}}R_2S^{-\frac{1}{2}}$. Then, by using condition (12) and $0 \leq t \leq T^*$, we can get

$$\begin{aligned}
V(\xi(t)) &< e^{\delta t}\{V(\xi(0))\}\\
&\leq e^{\delta t}\Bigg\{\xi^T(0)(I\otimes P)\xi(0) + \int_{-\tau_1}^{0}\xi^T(s)(I\otimes Q_1)\xi(s)\mathrm{d}s\\
&\quad + \int_{-\tau(0)}^{0}\xi^T(s)(I\otimes Q_2)\xi(s)\mathrm{d}s\\
&\quad + \int_{-\tau_2}^{0}\xi^T(s)(I\otimes Q_3)\xi(s)\mathrm{d}s\\
&\quad + \tau_1\int_{-\tau_1}^{0}\int_{\theta}^{0}\dot{\xi}^T(s)(I\otimes R_1)\dot{\xi}(s)\mathrm{d}s\mathrm{d}\theta\\
&\quad + (\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}\int_{\theta}^{0}\dot{\xi}^T(s)(I\otimes R_2)\dot{\xi}(s)\mathrm{d}s\mathrm{d}\theta\Bigg\}\\
&\leq e^{\delta T^*}\Bigg[\lambda_{\max}(I\otimes\tilde{P}) + \tau_1\lambda_{\max}(I\otimes\tilde{Q}_1) + \tau_2\lambda_{\max}(I\otimes\tilde{Q}_2)\\
&\quad + \tau_2\lambda_{\max}(I\otimes\tilde{Q}_3) + \frac{\tau_1^3}{2}\lambda_{\max}(I\otimes\tilde{R}_1)\\
&\quad + \frac{(\tau_2-\tau_1)^3}{2}\lambda_{\max}(I\otimes\tilde{R}_2)\Bigg]\sup_{-\tau_2\leq t_0\leq 0}\\
&\quad\times\left\{\xi^T(t_0)(I\otimes S)\xi(t_0), \dot{\xi}^T(t_0)(I\otimes S)\dot{\xi}(t_0)\right\}\\
&\leq c_1e^{\delta T^*}\Bigg(\lambda_1 + \tau_1\lambda_2 + \tau_2\lambda_3 + \tau_2\lambda_4\\
&\quad + \frac{\tau_1^3}{2}\lambda_5 + \frac{(\tau_2-\tau_1)^3}{2}\lambda_6\Bigg),
\end{aligned} \tag{21}$$

where $\lambda_1 = \lambda_{\max}(I\otimes\tilde{P})$, $\lambda_2 = \lambda_{\max}(I\otimes\tilde{Q}_1)$, $\lambda_3 = \lambda_{\max}(I\otimes\tilde{Q}_2)$, $\lambda_4 = \lambda_{\max}(I\otimes\tilde{Q}_3)$, $\lambda_5 = \lambda_{\max}(I\otimes\tilde{R}_1)$, $\lambda_6 = \lambda_{\max}(I\otimes\tilde{R}_2)$ and c_1 is given in Definition 1. Moreover, it also follows from (12) that

$$\begin{aligned}
V(\xi(t)) &\geq \xi^T(t)(I\otimes P)\xi(t)\\
&\geq \lambda_{\min}(I\otimes\tilde{P})\xi^T(t)(I\otimes S)\xi(t)\\
&\geq \lambda_7\xi^T(t)(I\otimes S)\xi(t),
\end{aligned} \tag{22}$$

where $\lambda_7 = \lambda_{\min}(I\otimes\tilde{P})$. Now, by combining the inequalities (21) and (22), we can get

$$\begin{aligned}
&\xi^T(t)(I\otimes S)\xi(t)\\
&\leq \frac{c_1(\lambda_1 + \tau_1\lambda_2 + \tau_2\lambda_3 + \tau_2\lambda_4 + \frac{\tau_1^3}{2}\lambda_5 + \frac{(\tau_2-\tau_1)^3}{2}\lambda_6)}{e^{-\delta T^*}\lambda_7} < c_2.
\end{aligned} \tag{23}$$

Therefore, if the condition (11) holds, then the augmented system (8) without uncertain term is finite-time stable according to Definition 1. This implies that the multi-agent system (1) without uncertain term is reached the desired consensus in finite-time, which completes the proof of this theorem. □

It should be noted that the obtained conditions in Theorem 1 are not in the form of linear matrix inequality (LMI) because the matrix elements consist of multiplication of two unknown parameters. Thus, to convert those conditions into LMI format, the following theorem is presented.

Theorem 2. *Suppose that Assumptions 1 and 2 hold. The augmented system (8) without uncertain term is finite-time stable with respect to $(c_1, c_2, \tau_2, T^*, S)$ for given positive scalars c_1, c_2, T^*, $\alpha\in(0,1)$, μ, τ_1, τ_2 and γ_l $(l=1,2)$ and positive definite matrix S, if there exist symmetric matrices $X>0$, $\hat{Q}_p > 0$ $(p=1,2,3)$ and $\hat{R}_q > 0$ $(q=1,2)$, any appropriate dimensioned matrices $\hat{Z}_1$, $\hat{Z}_2$, Y and W, and positive scalars r, α_p $(p=1,2,3)$ and β_q $(q=1,2)$ such that the following LMIs are satisfied:*

$$\begin{bmatrix} [\hat{\Lambda}_{a,b}]_{11\times 11} & \hat{\Gamma}_1^l & \hat{\Gamma}_2^l & \hat{\Gamma}_3^l\\ * & -(I\otimes\hat{R}_2) & 0 & 0\\ * & * & -3(I\otimes\hat{R}_2) & 0\\ * & * & * & -5(I\otimes\hat{R}_2)\end{bmatrix} < 0,$$
$$l = 1, 2, \tag{24}$$

$$\hat{R}_l < \gamma_l X, \quad l = 1, 2, \tag{25}$$

$$rS^{-1} < X < S^{-1}, \quad 0 < \hat{Q}_p < \alpha_p S^{-1}, \quad 0 < \hat{R}_q < \beta_q S^{-1}, \tag{26}$$

$$\begin{bmatrix} c_1(\tau_1\alpha_1 + \tau_2\alpha_2 + \tau_2\alpha_3 + \tau_1^3\beta_1 + (\tau_2-\tau_1)^3\beta_2) - e^{\delta T^*}c_2 & \sqrt{c_1}\\ * & -r\end{bmatrix}$$
$$< 0, \tag{27}$$

where

$$\begin{aligned}
\hat{\Lambda}_{1,1} &= \hat{\Theta}_1 + \hat{\Theta}_1^T + \sum_{k=1}^{3}(I\otimes\hat{Q}_k) - 4(I\otimes\hat{R}_1), \quad \hat{\Lambda}_{1,2} = -2(I\otimes\hat{R}_1),\\
\hat{\Lambda}_{1,3} &= \hat{\Theta}_2, \quad \hat{\Lambda}_{1,5} = 6\gamma_1(I\otimes X), \quad \hat{\Lambda}_{1,10} = \tau_1\hat{\Theta}_1^T,\\
\hat{\Lambda}_{1,11} &= (\tau_2-\tau_1)\hat{\Theta}_1^T,\\
\hat{\Lambda}_{2,2} &= -(I\otimes\hat{Q}_1) - 4(I\otimes\hat{R}_1) - 64(2-\alpha)(I\otimes\hat{R}_2),\\
\hat{\Lambda}_{2,3} &= -24(2-\alpha)(I\otimes\hat{R}_2) + 9\alpha(I\otimes\hat{Z}_1) + 9(1-\alpha)(I\otimes\hat{Z}_2),\\
\hat{\Lambda}_{2,4} &= -3\alpha(I\otimes\hat{Z}_1) - 3(1-\alpha)(I\otimes\hat{Z}_2),\\
\hat{\Lambda}_{2,5} &= 6\gamma_1(I\otimes X),\\
\hat{\Lambda}_{2,6} &= 288(2-\alpha)\gamma_2(I\otimes X),\\
\hat{\Lambda}_{2,7} &= -480(2-\alpha)(I\otimes\hat{R}_2),\\
\hat{\Lambda}_{2,8} &= -24\alpha(I\otimes\hat{Z}_1) - 24(1-\alpha)(I\otimes\hat{Z}_2),\\
\hat{\Lambda}_{2,9} &= 36\alpha(I\otimes\hat{Z}_1) + 36(1-\alpha)(I\otimes\hat{Z}_2),\\
\hat{\Lambda}_{3,3} &= -(1-\mu)(I\otimes\hat{Q}_2) - 9(2-\alpha)(I\otimes\hat{R}_2) - 64(1+\alpha)(I\otimes\hat{R}_2)\\
&\quad - 3\alpha(I\otimes\hat{Z}_1) - 3(1-\alpha)(I\otimes\hat{Z}_2),\\
\hat{\Lambda}_{3,4} &= 24(1+\alpha)\gamma_2(I\otimes X),\\
\hat{\Lambda}_{3,6} &= -108(2-\alpha)(I\otimes\hat{R}_2) - 24\alpha(I\otimes\hat{Z}_1)^T - 24(1-\alpha)(I\otimes\hat{Z}_2)^T,\\
\hat{\Lambda}_{3,7} &= 180(2-\alpha)\gamma_2(I\otimes X) + 36\alpha(I\otimes\hat{Z}_1)^T + 36(1-\alpha)(I\otimes\hat{Z}_2)^T,\\
\hat{\Lambda}_{3,8} &= 288(1+\alpha)\gamma_2(I\otimes X) + 8\alpha(I\otimes\hat{Z}_1)^T + 8(1-\alpha)(I\otimes\hat{Z}_2)^T,\\
\hat{\Lambda}_{3,9} &= -480(1+\alpha)(I\otimes\hat{R}_2) - 12\alpha(I\otimes\hat{Z}_1) - 12(1-\alpha)(I\otimes\hat{Z}_2),\\
\hat{\Lambda}_{3,10} &= \tau_1\hat{\Theta}_2^T,\\
\hat{\Lambda}_{3,11} &= (\tau_2-\tau_1)\hat{\Theta}_2^T,\\
\hat{\Lambda}_{4,4} &= -(I\otimes\hat{Q}_3) - 9(1+\alpha)(I\otimes\hat{R}_2),\\
\hat{\Lambda}_{4,6} &= 8\alpha(I\otimes\hat{Z}_1)^T + 8(1-\alpha)(I\otimes\hat{Z}_2)^T,\\
\hat{\Lambda}_{4,7} &= -12\alpha(I\otimes\hat{Z}_1)^T - 12(1-\alpha)(I\otimes\hat{Z}_2)^T,
\end{aligned}$$

$\hat{\Lambda}_{4,8} = -108(1+\alpha)(I \otimes \hat{R}_2),$

$\hat{\Lambda}_{4,9} = 180(1+\alpha)\gamma_2(I \otimes X),$

$\hat{\Lambda}_{5,5} = -12(I \otimes \hat{R}_1),$

$\hat{\Lambda}_{6,6} = -1296(2-\alpha)(I \otimes \hat{R}_2),$

$\hat{\Lambda}_{6,7} = 2160(2-\alpha)\gamma_2(I \otimes X),$

$\hat{\Lambda}_{6,8} = 64\alpha(I \otimes \hat{Z}_1) + 64(1-\alpha)(I \otimes \hat{Z}_2),$

$\hat{\Lambda}_{6,9} = -96\alpha(I \otimes \hat{Z}_1) - 96(1-\alpha)(I \otimes \hat{Z}_2),$

$\hat{\Lambda}_{7,7} = -3600(2-\alpha)(I \otimes \hat{R}_2),$

$\hat{\Lambda}_{7,8} = -96\alpha(I \otimes \hat{Z}_1) - 96(1-\alpha)(I \otimes \hat{Z}_2),$

$\hat{\Lambda}_{7,9} = 144\alpha(I \otimes \hat{Z}_1) + 144(1-\alpha)(I \otimes \hat{Z}_2),$

$\hat{\Lambda}_{8,8} = -1296(1+\alpha)(I \otimes \hat{R}_2),$

$\hat{\Lambda}_{8,9} = 2160(1+\alpha)\gamma_2(I \otimes X),$

$\hat{\Lambda}_{9,9} = -3600(1+\alpha)(I \otimes \hat{R}_2),$

$\hat{\Lambda}_{10,10} = -\dfrac{(I \otimes X)}{\gamma_1},$

$\hat{\Lambda}_{11,11} = -\dfrac{(I \otimes X)}{\gamma_2},$

with $\hat{\Theta}_1 = \begin{bmatrix} (I \otimes AX_1) & (I \otimes WC) \\ 0 & (I \otimes AX_2) - (I \otimes WC) \end{bmatrix}$, $\hat{\Theta}_2 = \begin{bmatrix} (\mathcal{L} \otimes BY) - (\mathcal{L} \otimes BY)\bar{\lambda}_* & 0 \\ 0 & 0 \end{bmatrix}$ *and the remaining parameters are zero. Moreover, the control design parameters can be obtained by the following relations:* $K = YX_1^{-1}$ *and* $L_0 = WU\mathbb{S}X_{11}^{-1}\mathbb{S}^{-1}U^T$.

Proof. In order to prove this theorem, first, consider the following linear congruence transformations: $X_1 = P_1^{-1}$, $X = \text{diag}\{X_1, X_1\}$, $\hat{Q}_k = XQ_kX$ $(k = 1, 2, 3)$, $\hat{R}_l = XR_lX$ $(l = 1, 2)$ and $\hat{Z}_l = XZ_lX$ $(l = 1, 2)$. Then, pre- and post-multiplying the constraint (10) in Theorem 1 by $\text{diag}\{\underbrace{(I \otimes X), \ldots, (I \otimes X)}_{9 \text{ times}}, (I \otimes I), (I \otimes I), (I \otimes X), (I \otimes X), (I \otimes X)\}$ and its transpose, respectively, and using Schur complement, it is easy to obtain the LMIs in (24), where the term CX_1 is written as $\bar{X}_1C$ with the aid of Lemmas 1 and 2 in which $\bar{X}_1 = U\mathbb{S}X_{11}^{-1}\mathbb{S}^{-1}U^T$.

Further, define $\tilde{X} = S^{\frac{1}{2}}XS^{\frac{1}{2}}$, $1 < \tilde{\lambda}_7 = \lambda_{\min}(\tilde{X})$, $\tilde{\lambda}_1 = \lambda_{\max}(\tilde{X}) > r$ and consider the relation $\lambda_{\max}(\tilde{X}) = \frac{1}{\lambda_{\min}(\tilde{P})}$. Besides, it is noted from Theorem 1 that $\lambda_7 < (I \otimes S^{\frac{-1}{2}}PS^{\frac{-1}{2}}) < \lambda_1$, $0 < (I \otimes S^{\frac{-1}{2}}Q_1S^{\frac{-1}{2}}) < \lambda_2$, $0 < (I \otimes S^{\frac{-1}{2}}Q_2S^{\frac{-1}{2}}) < \lambda_3$, $0 < (I \otimes S^{\frac{-1}{2}}Q_3S^{\frac{-1}{2}}) < \lambda_4$, $0 < (I \otimes S^{\frac{-1}{2}}R_1S^{\frac{-1}{2}}) < \lambda_5$ and $0 < (I \otimes S^{\frac{-1}{2}}R_2S^{\frac{-1}{2}}) < \lambda_6$. According to the above congruence transformation, these relations can be changed into $\tilde{\lambda}_1^{-1}(I \otimes S^{-1}) < (I \otimes \tilde{X}) < \tilde{\lambda}_7^{-1}(I \otimes S^{-1})$, $0 < (I \otimes \hat{Q}_{l-1}) < \tilde{\lambda}_7^{-2}\tilde{\lambda}_l(I \otimes S^{-1})$ $(l = 2, 3, 4)$ and $0 < (I \otimes \hat{R}_{z-4}) < \tilde{\lambda}_7^{-2}\tilde{\lambda}_z(I \otimes S^{-1})$ $(z = 5, 6)$. Now, if we set $\tilde{\lambda}_l \le \alpha_{l-1}$ $(l = 2, 3, 4)$ and $\tilde{\lambda}_z \le \beta_{z-4}$ $(z = 5, 6)$, then the constraint (26) can easily be obtained. From (26), the condition (11) can be equivalently viewed as (27), which is the desired condition. This completes the proof. □

Next, by taking the uncertain term into account, we will design the observer-based fault-tolerant control protocol that guarantees the robust finite-time stability of the augmented system (8). To do this, we again consider the same Lyapunov-Krasovskii functional as in (12) and follow the similar lines in the proof of Theorem 2, we can obtain the desired result, which is given in the following theorem.

Theorem 3. *For given positive scalars* c_1, c_2, T^*, $\alpha \in (0, 1)$, μ, τ_1, τ_2 *and* γ_l $(l = 1, 2)$, *and symmetric matrix S, the augmented system (8) is robustly finite-time stable under Assumptions 1 and 2, if there exist real constant matrices* $X > 0$, $\hat{Q}_p > 0$ $(p = 1, 2, 3)$, $\hat{R}_q > 0$ $(l = 1, 2)$, *any appropriate dimensioned matrices* $\hat{Z}_1$, $\hat{Z}_2$, *Y and W, and positive scalars r,* α_p $(p = 1, 2, 3)$, β_q *and* ϵ_q $(q = 1, 2)$ *such that the LMIs (25)-(27) and the following conditions hold:*

$$\hat{\Phi}_l = \begin{bmatrix} [\hat{\Lambda}_{a,b}]_{11\times 11} & \hat{\Gamma}_1^l & \hat{\Gamma}_2^l & \hat{\Gamma}_3^l & \bar{\epsilon}_1\nabla_1 & \nabla_2^T & \bar{\epsilon}_2\tau_2\nabla_1 & \nabla_2^T \\ * & -(I \otimes \hat{R}_2) & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -3(I \otimes \hat{R}_2) & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -5(I \otimes \hat{R}_2) & 0 & 0 & 0 & 0 \\ * & * & * & * & -\bar{\epsilon}_1 I & \bar{\epsilon}_1 J & 0 & 0 \\ * & * & * & * & * & -\bar{\epsilon}_1 I & 0 & 0 \\ * & * & * & * & * & * & -\bar{\epsilon}_2 I & \bar{\epsilon}_2 J \\ * & * & * & * & * & * & * & -\bar{\epsilon}_2 I \end{bmatrix} < 0, l = 1, 2, \tag{28}$$

where $\nabla_1 = \begin{bmatrix} \bar{H}^T & \underbrace{0 \cdots 0}_{13} \end{bmatrix}^T$, $\nabla_2 = \begin{bmatrix} \bar{E}X & \underbrace{0 \cdots 0}_{13} \end{bmatrix}$, $\bar{H} = \begin{bmatrix} 0 & 0 \\ H & H \end{bmatrix}$, $\bar{E} = \begin{bmatrix} 0 & 0 \\ E & E \end{bmatrix}$, $\bar{\epsilon}_1 = \epsilon_1^{-2}$ *and* $\bar{\epsilon}_2 = \epsilon_2^{-2}$.

Proof. By replacing the matrices $\bar{A}$ with $\bar{A} + \Delta\bar{A}(t)$ in Theorem 2, we can have

$$\begin{aligned} \tilde{\Phi}_1 &= \hat{\Lambda} + \nabla_1\Delta(t)\nabla_2 + [\nabla_1\Delta(t)\nabla_2]^T + \tau_1\nabla_1\Delta(t)\nabla_2 \\ &\quad + \tau_1[\nabla_1\Delta(t)\nabla_2]^T + (\tau_2 - \tau_1)\nabla_1\Delta(t)\nabla_2 \\ &\quad + (\tau_2 - \tau_1)[\nabla_1\Delta(t)\nabla_2]^T \\ &= \hat{\Lambda} + \nabla_1\Delta(t)\nabla_2 + [\nabla_1\Delta(t)\nabla_2]^T + \tau_2\nabla_1\Delta(t)\nabla_2 \\ &\quad + \tau_2[\nabla_1\Delta(t)\nabla_2]^T, \end{aligned}$$

where ∇_1 and ∇_2 are defined in the theorem statement. With the aid of Lemma 2.5 in [45], the above expression can be further written as

$$\begin{aligned} \tilde{\Phi}_1 &= \hat{\Lambda} + \begin{bmatrix} \epsilon_1^{-1}\nabla_2^T \\ \epsilon_1\nabla_1 \end{bmatrix}^T \begin{bmatrix} I & -J \\ -J^T & I \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_1^{-1}\nabla_2 \\ \epsilon_1\nabla_1^T \end{bmatrix} \\ &\quad + \tau_2 \begin{bmatrix} \epsilon_2^{-1}\nabla_2^T \\ \epsilon_2\nabla_1 \end{bmatrix}^T \begin{bmatrix} I & -J \\ -J^T & I \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_2^{-1}\nabla_2 \\ \epsilon_2\nabla_1^T \end{bmatrix}. \end{aligned}$$

Here, by applying Schur complement, we can get $\hat{\Phi}_1$. Thus, if the LMIs in (28) hold, then the augmented system (8) is robustly finite-time stable, which completes the proof. □

Remark 1. So far, many of the researchers have investigated the consensus of multi-agent system through various control approaches, for instance see [1–6]. However, very few researchers only have concentrated on the design of fault-tolerant controller for achieving consensus of multi-agent systems, see [38–40] and references therein. In particular, they have designed the fault-tolerant controllers by considering actuator faults to be time-invariant. In practice, time-varying actuator faults are more suitable than time-invariant ones for reflecting the real evolution of component failures. Based on this fact, the problem of fault-tolerant consensus of multi-agent systems with time-varying actuator faults has been reported [41]. In many practical engineering systems, it is required that consensus be achieved within a finite-time period. Thus, the study of finite-time consensus of multi-agent systems has become a very hot research topic in recent days [29–32]. Nevertheless, the design of fault-tolerant controller for attaining finite-time consensus in multi-agent systems with time-varying actuator faults has not been fully considered. The main aim of this paper, therefore, is to make the first attempt to cope with this issue by constructing an observer-based fault-tolerant controller subject to time-varying actuator faults, the problem of finite-time consensus of uncertain multi-agent systems is investigated.

Remark 2. In proof of Theorem 1, we employ an improved reciprocally convex inequality to deal with the integral term

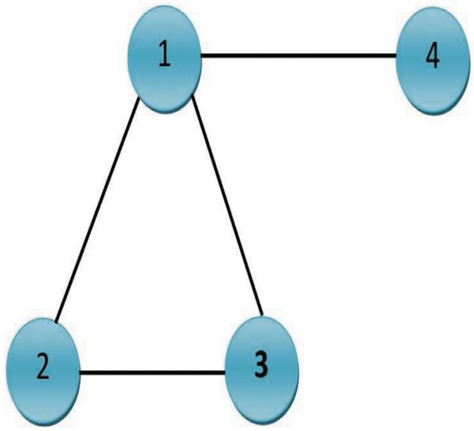

Fig. 1. Undirected interconnection topology.

$(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{\xi}^T(s)(I\otimes R_2)\dot{\xi}(s)\mathrm{d}s$. The peculiar reason for employing this inequality is that it could provide a maximum lower bound with less slack matrix variables compared to the reciprocally convex inequality and the extended reciprocally convex inequality, which has recently been proved in [43]. Though the obtained LMI-based consensus conditions are given in terms of complicated forms and induce the computational complexity, it is convenient to check their feasibility without tuning parameters significantly by resorting to MATLAB LMI control toolbox.

Remark 3. Recently, some significant and efficient methods on the stochastic nonlinear systems have been proposed in [46,47]. More specifically, the boundedness of stochastic systems has been discussed in [46] by implementing round-Robin protocols for effectively mitigating data congestions and saving energies. In [47], the security control problem for stochastic nonlinear systems in the presence of deception attacks has been investigated by using a dynamic output feedback controller. It would be an interesting issue to extend the proposed work into the adaptive fault-tolerant control for consensus of stochastic multi-agent systems based on the aforementioned scenarios.

4. Numerical examples

In this section, two numerical examples are provided to illustrate the validity and effectiveness of the proposed control scheme.

Example 1. Consider a multi-agent system in the form of (1) with four agent systems ($N=4$) under a fixed communication topology shown in Fig. 1. The parameter matrices of system (1) are given as

$$A=\begin{bmatrix}-0.18 & 0.19\\ 0.16 & -0.30\end{bmatrix},\quad B=\begin{bmatrix}0.95\\ 0.22\end{bmatrix},\quad C=\begin{bmatrix}0.25 & 0.75\end{bmatrix},$$

$$H=\begin{bmatrix}0.05 & 0.02\\ 0.07 & 0.05\end{bmatrix}\text{ and } E=\begin{bmatrix}0.01 & 0.03\\ 0.04 & 0.02\end{bmatrix}.$$

Here, the Laplacian matrix and the time-varying delay are chosen as $\mathcal{L}=\begin{bmatrix}3 & -1 & -1 & -1\\ -1 & 2 & -1 & 0\\ -1 & -1 & 2 & 0\\ -1 & 0 & 0 & 1\end{bmatrix}$ and $\tau(t)=0.5+0.5\sin(t)$, respectively. Furthermore, the rest of parameters are taken as follows: $\tau_1=0$, $\tau_2=1$, $\mu=0.5$, $J=0.5I$, $\alpha=0.1$, $\gamma_1=0.5$, $\gamma_2=0.4$, $\bar{\lambda}_*=0.9$, $c_1=1$, $c_2=2.3$, $S=I$, $\delta=0.001$ and $T^*=100$. With these parameter values, by solving the LMI constraints in the statement of Theorem 3, we can get the feasible solution from which the feedback control gain matrix and observer gain matrix can be computed as $K=[-1.0340\quad -0.4664]$ and $L_0=[0.1375\quad 0.1486]^T$, respectively.

Table 1
Calculated minimum optimum value of c_2.

c_1	1	2	3	4	5
c_2 (when $T^*=100$)	2.3125	3.1205	5.7652	8.2168	9.9122
c_2 (when $T^*=110$)	2.5426	4.7871	7.1258	8.8761	10.7271
c_2 (when $T^*=120$)	2.8217	5.5250	7.9521	9.5520	11.2723

Table 2
Calculated maximum allowable upper bound of τ_2 for different values of τ_1.

τ_1	0.1	0.15	0.2	0.25	0.3
τ_2(when $\mu=0.5$)	2.5125	2.5746	2.5768	2.6011	2.6040

For simulation purposes, we choose the initial conditions as $x_1(0)=[0.5\quad -0.35]^T$, $x_2(0)=[0.55\quad 0.4]^T$, $x_3(0)=[1.23\quad 1.20]^T$, $x_4(0)=[-0.4\quad 0.45]^T$, $\hat{x}_1(0)=[0.3\quad 0.6]^T$, $\hat{x}_2(0)=[-0.9\quad 0.3]^T$, $\hat{x}_3(0)=[0.42\quad 0.66]^T$ and $\hat{x}_4(0)=[0.8\quad 0.5]^T$. With these initial conditions, the corresponding response curves of systems (1) and (3) under the obtained control parameters are plotted in Figs. 2–5. In brief, the state trajectories of all agents along with their observers are displayed in Fig. 2, wherein the dotted lines represent the observer states and the dashed lines denote the four agents states. It can easily be observed from Fig. 2 that the states of the nodes are exactly synchronized with the observer states within the prescribed time period, which shows the efficiency of the proposed control strategy. Moreover, the corresponding error state responses and the control response curves are given in Figs. 3 and 4, respectively. In addition, the time evolution of $e_i^T(t)Se_i(t)$ $(i=1,2,3,4)$ is depicted in Fig. 5, where it is clearly visible that the error states of four agents taking the initial values within the optimal bound value of c_1 do not exceed the optimal bound value of $c_2=2.3$, which means that the required consensus of considered multi-agent system (1) is achieved within a given finite-time interval.

In addition, the optimal bound value of c_2 is computed for various values of c_1 and T^*, and are listed in Table 1. Moreover, Table 2 displays the maximum allowable upper bound of τ_2 for different values of τ_1, where it can be seen that the maximum allowable upper bound value of τ_2 is increased when the value of τ_1 increases. Thus, it can be concluded from these simulations that the designed fault-tolerant consensus algorithm effectively works for the considered multi-agent system.

Example 2. Consider the formation flying of a group of four satellites in low Earth orbit as mentioned in [20]. In this example, it is assumed that the communication topology of four satellites is described by an undirected graph shown in Fig. 1. Further, the dynamic of each satellite is governed by the following differential equation:

$$\dot{x}_i(t)=\begin{bmatrix}0 & I_3\\ A_1 & A_2\end{bmatrix}x_i(t)+\begin{bmatrix}0\\ I_3\end{bmatrix}u_i^f(t),$$
$$y_i(t)=Cx_i(t),\quad i=1,2,3,4, \tag{29}$$

where $A_1=\begin{bmatrix}0 & 0 & 0\\ 0 & 3\omega^2 & 0\\ 0 & 0 & -\omega^2\end{bmatrix}$, $A_2=\begin{bmatrix}0 & 2\omega & 0\\ -2\omega & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$, and $C=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\end{bmatrix}$.

Moreover, the rest of parameters are chosen as $\tau_1=0$, $\tau_2=0.2$, $\mu=0.1$, $\gamma_1=32$, $\gamma_2=12$, $\bar{\lambda}_*=0.9I_3$, $c_1=1$, $c_2=5$, $\delta=0.001$, $T^*=120$ and $\omega=0.001$. Now, by solving the LMI constraints in Theorem 1, we can get the following

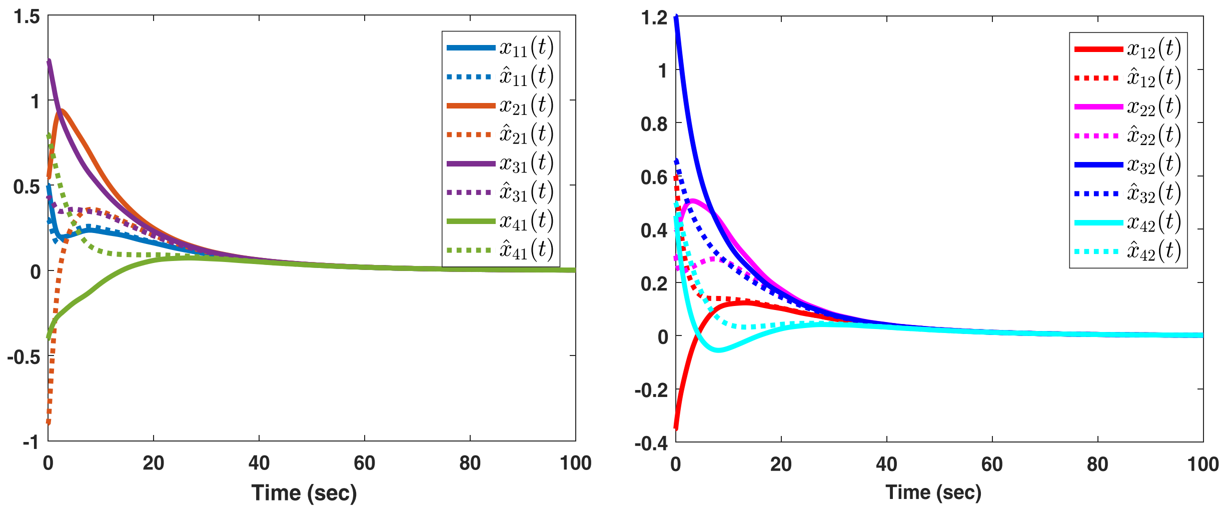

Fig. 2. Actual and observer state trajectories with controller (3).

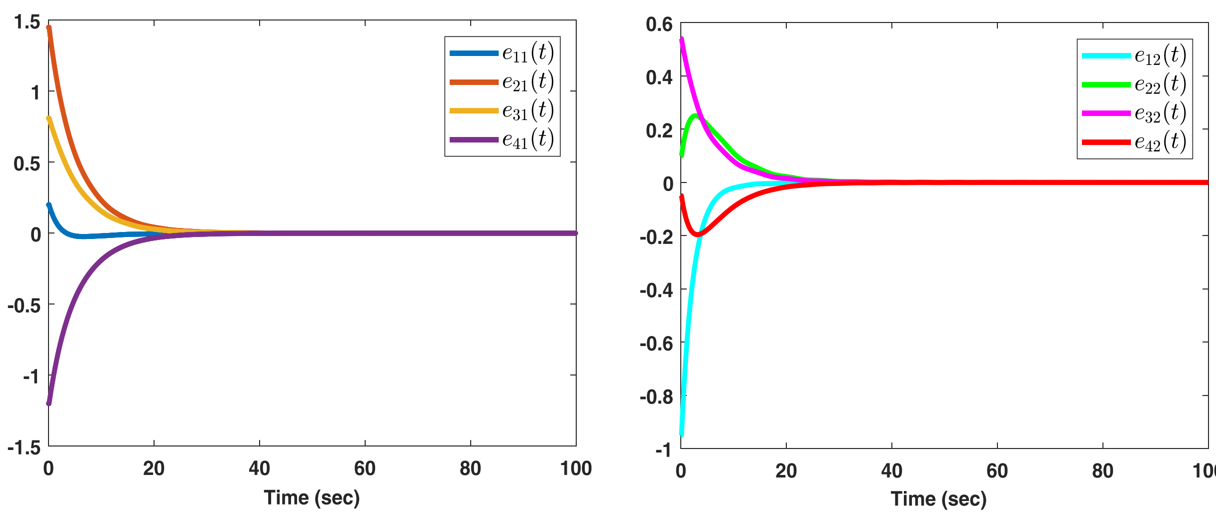

Fig. 3. Responses of error system (5).

feedback control gain matrix and observer gain matrix: $K = \begin{bmatrix} -0.2186 & -0.0001 & 0 & -1.3279 & -0.0008 & 0 \\ 0.0001 & -0.2186 & 0 & 0.0008 & -1.3279 & 0 \\ 0 & 0 & -0.2186 & 0 & 0 & -1.3279 \end{bmatrix}$ and $L_0 = \begin{bmatrix} 0.4429 & 0.0000 & 0 \\ -0.0000 & 0.4429 & 0 \\ 0 & 0 & 0.4429 \\ 0.1025 & -0.0002 & 0 \\ 0.0002 & 0.1025 & 0 \\ 0 & 0 & 0.1025 \end{bmatrix}$, respectively.

Based on the above gain values and the initial conditions $x_1(0) = [100\ \ 100\ \ 0\ \ 0\ \ 0\ \ 0]^T$, $x_2(0) = [-100\ \ 100\ \ 0\ \ 0\ \ 0\ \ 0]^T$, $x_3(0) = [100\ \ 0\ \ 110\ \ 0\ \ 0\ \ 0]^T$, $x_4(0) = [-100\ \ 10\ \ 120\ \ 0\ \ 0\ \ 0]^T$, $\hat{x}_1(0) = [-90\ \ 110\ \ 40\ \ -60\ \ 120\ \ 80]^T$, $\hat{x}_2(0) = [60\ \ 70\ \ 50\ \ 90\ \ 75\ \ -80]^T$, $\hat{x}_3(0) = [85\ \ -25\ \ -70\ \ 20\ \ 45\ \ 160]^T$ and $\hat{x}_4(0) = [-10\ \ 20\ \ 30\ \ 40\ \ -50\ \ 60]^T$, simulations are depicted in Figs. 6–20. Specifically, the state trajectories of $x_{ia}(t)$ $(i = 1, 2, 3, 4;\ a = 1, 2, 3, 4, 5, 6)$ along with their observer states are displayed in Figs. 6–11. In all these figures, the actual states are perfectly estimated by the observer states, which illustrates the effectiveness of the designed consensus protocol. Moreover, the corresponding error state responses and the control response curves are given in Figs. 12–17 and in Figs. 18–20, respectively.

From this simulation results, it can be concluded that the considered system model (29) can achieve formation flying under the fault-tolerant control strategy. Therefore, the proposed consensus design is very much suitable for the formation flying control problems, which shows the usefulness and applicability of the obtained theoretical results.

5. Conclusion

In this paper, the problem of leaderless fault-tolerant consensus of an uncertain multi-agent systems over a finite domain has been investigated via the observer-based approach. A distributed Luenberger state observer has been proposed to make the agents reach consensus in finite-time when there exist uncertainties in the system model and time-varying actuator faults in the control design. Sufficient conditions ensuring the finite-time leaderless consensus of the system under study have been obtained by employing the Lyapunov-Krasovskii stability theory and finite-time stability theory together with the linear matrix inequality approach. Based on the obtained conditions, the explicit expression of the desired fault-tolerant consensus control has been presented subsequently. Finally, simulations have been carried out to verify the effectiveness of the proposed consensus control design

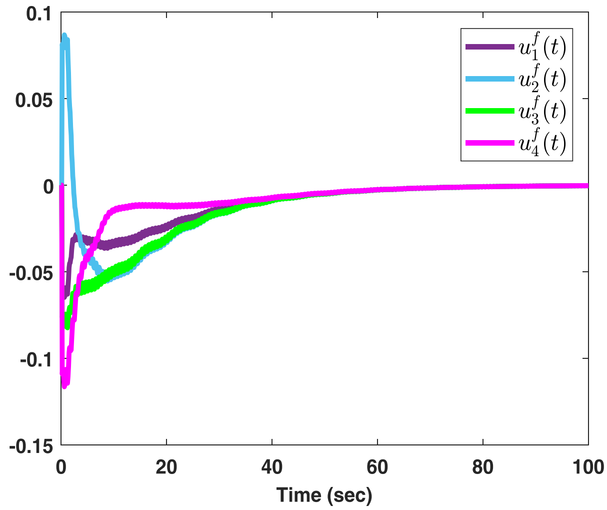

Fig. 4. Control responses.

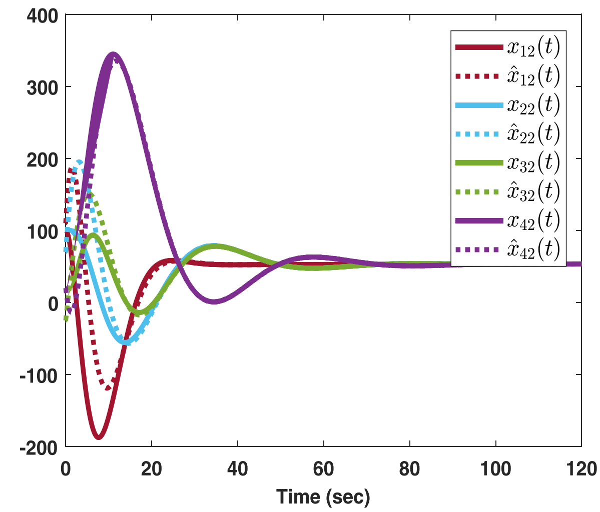

Fig. 7. State trajectories of $x_{i2}(t)$ and $\hat{x}_{i2}(t)$.

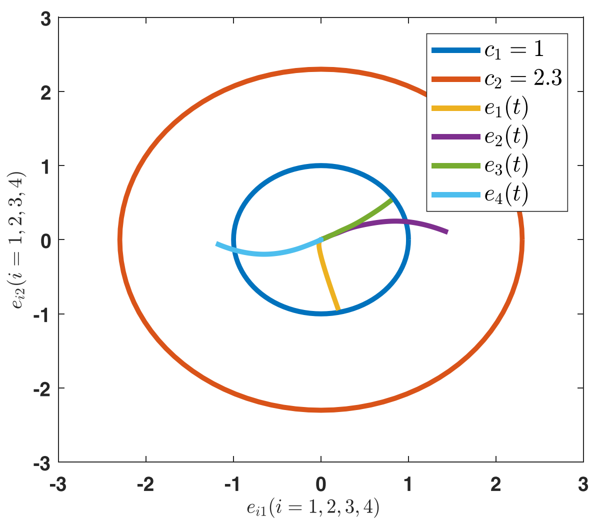

Fig. 5. Evolution of $e_i^T(t)Se_i(t)$.

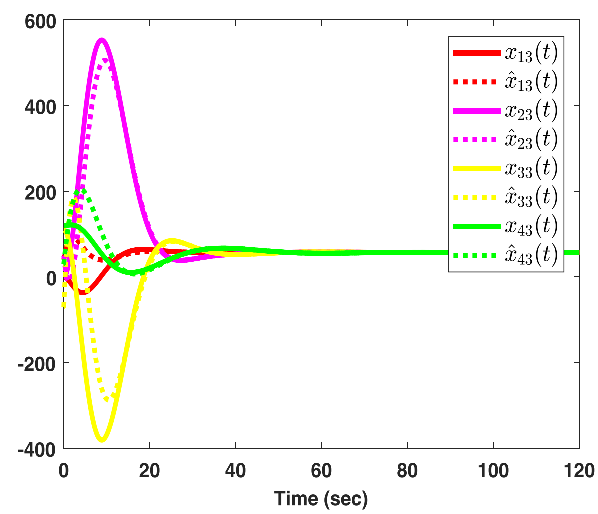

Fig. 8. State trajectories of $x_{i3}(t)$ and $\hat{x}_{i3}(t)$.

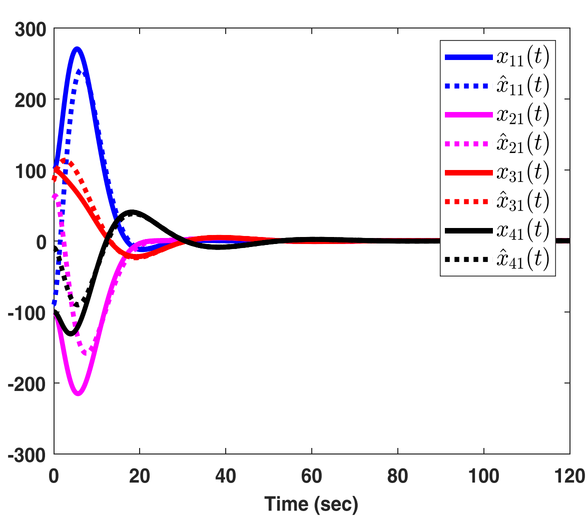

Fig. 6. State trajectories of $x_{i1}(t)$ and $\hat{x}_{i1}(t)$.

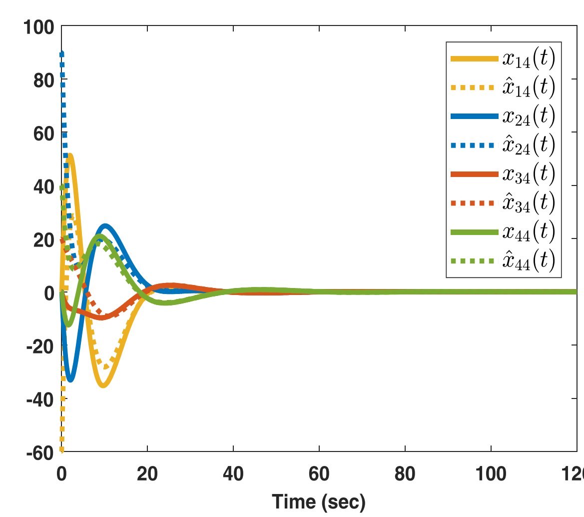

Fig. 9. State trajectories of $x_{i4}(t)$ and $\hat{x}_{i4}(t)$.

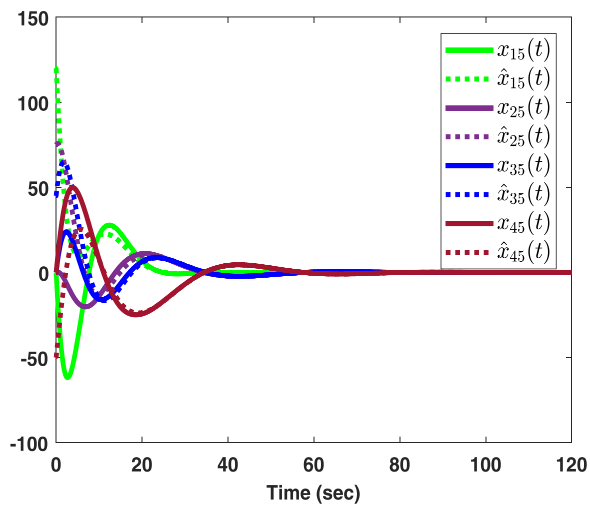

Fig. 10. State trajectories of $x_{i5}(t)$ and $\hat{x}_{i5}(t)$.

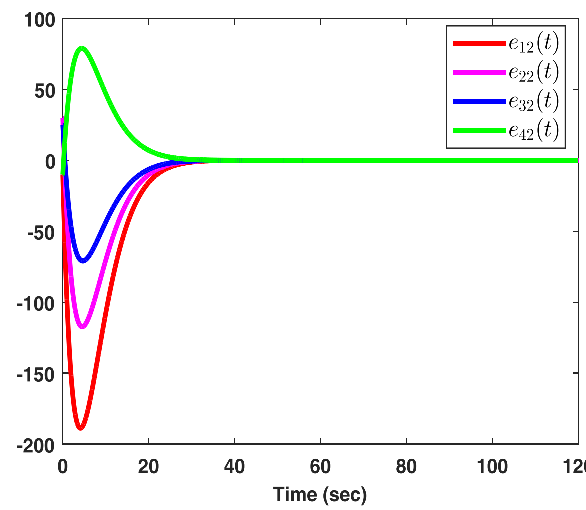

Fig. 13. Error state trajectories of $e_{i2}(t)$.

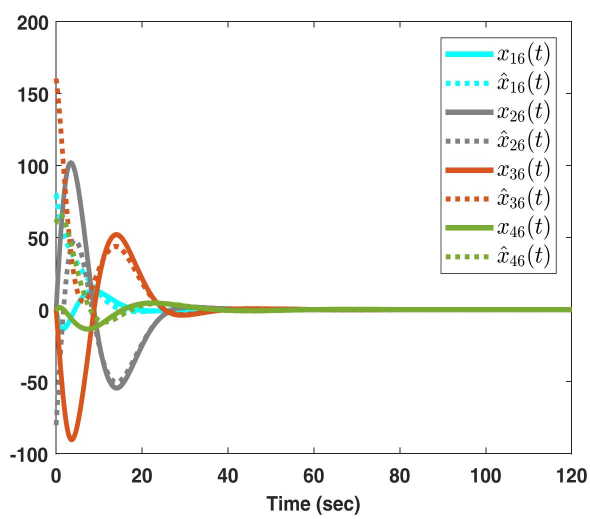

Fig. 11. State trajectories of $x_{i6}(t)$ and $\hat{x}_{i6}(t)$.

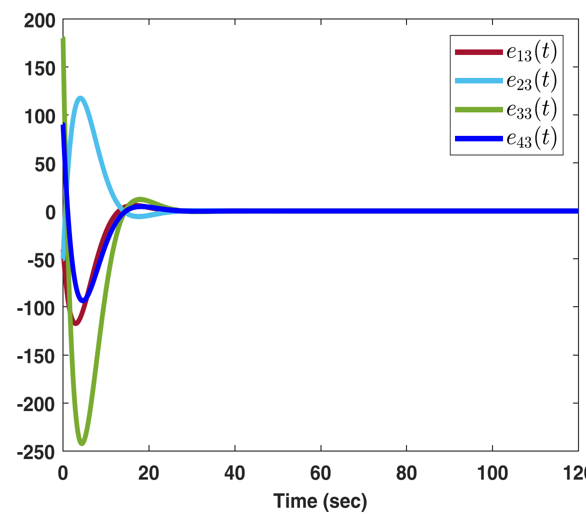

Fig. 14. Error state trajectories of $e_{i3}(t)$.

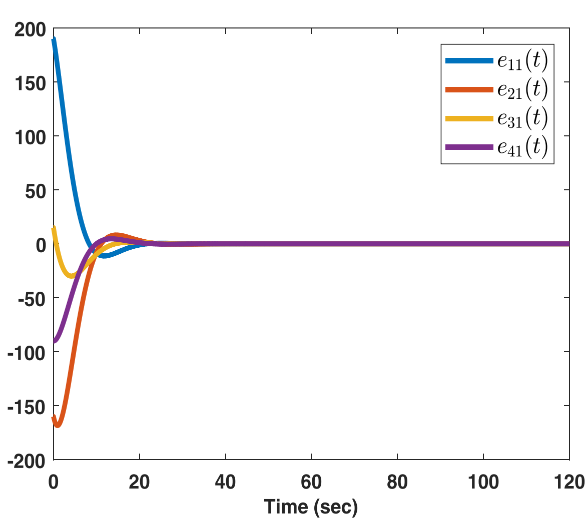

Fig. 12. Error state trajectories of $e_{i1}(t)$.

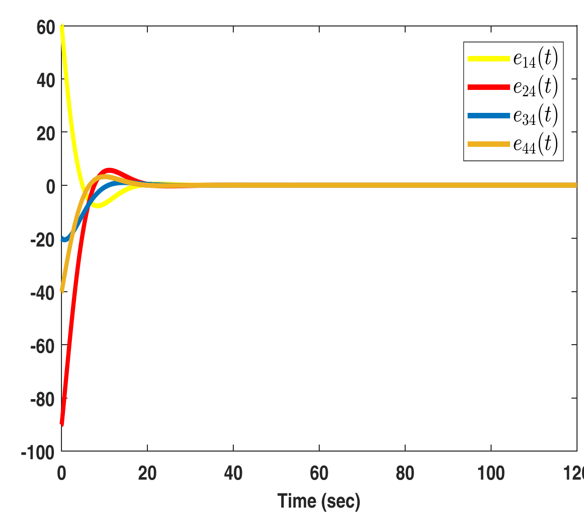

Fig. 15. Error state trajectories of $e_{i4}(t)$.

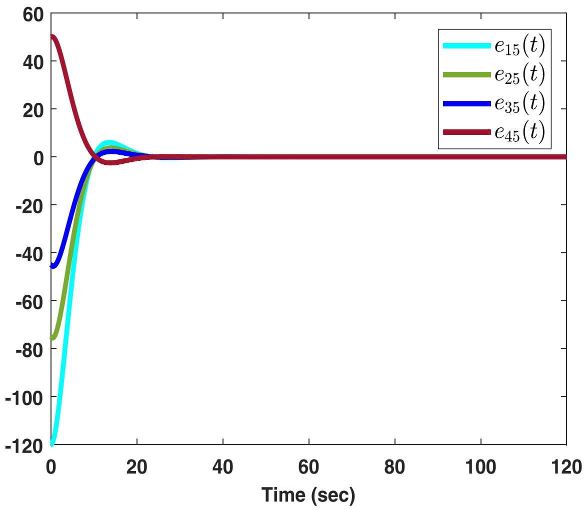

Fig. 16. Error state trajectories of $e_{i5}(t)$.

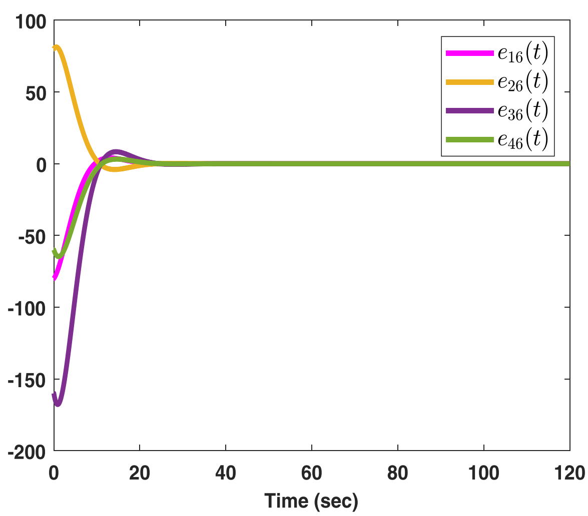

Fig. 17. Error state trajectories of $e_{i6}(t)$.

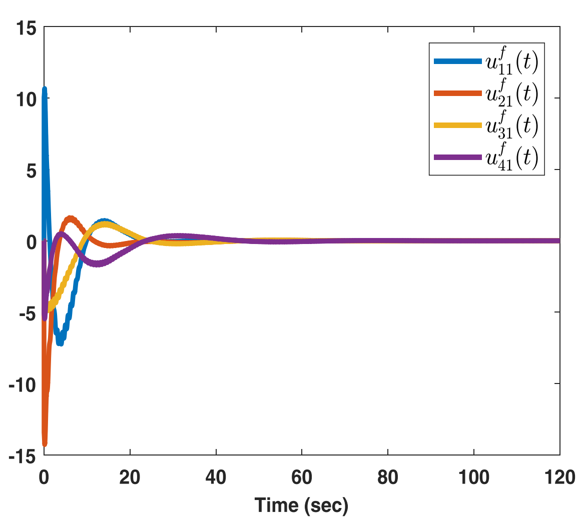

Fig. 18. Control response of $u^f_{i1}(t)$.

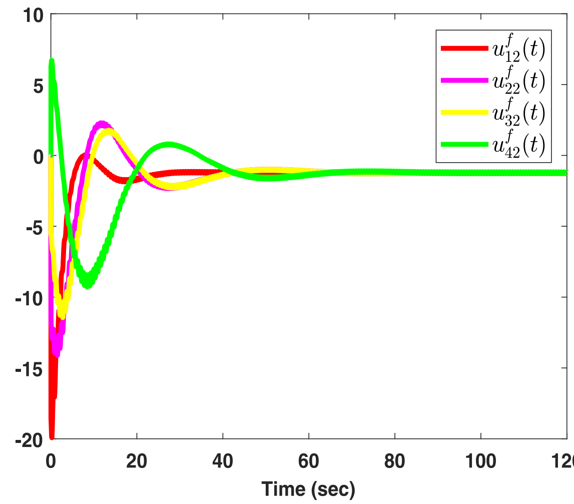

Fig. 19. Control response of $u^f_{i2}(t)$.

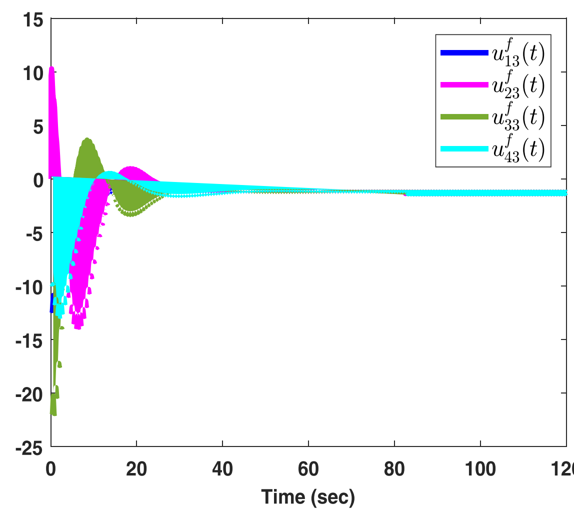

Fig. 20. Control response of $u^f_{i3}(t)$.

method. It is worth noting that the developed results can be extended to stochastic multi-agent systems with distributed time delays based on the optimal control approach [48–50]. This will be a direction of future research.

Acknowledgment

The work of Ramalingam Sakthivel was supported by the NBHM/DAE under grant No. 2/48(4)/2013/ NBHM(R.P)/R&D II/687 and the work of Yongdo Lim was supported by the National Research Foundation of Korea (NRF) grant founded by the Korea government (MEST) (No. 2015R1A3A2031159) and (2016R1A5A1008055).

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