

A Hierarchical Model for Fast Distributed Consensus in Dynamic Networks

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Abstract—We present two new consensus algorithms for dynamic networks. The first, Fast Raft, is a variation on the Raft consensus algorithm that reduces the number of message rounds in typical operation. Fast Raft is ideal for fast-paced distributed systems where membership changes over time and where sites must reach consensus quickly. The second, C-Raft, is targeted for distributed systems where sites are grouped into clusters, with fast communication within clusters and slower communication between clusters. C-Raft uses Fast Raft as a building block and defines a hierarchical model of consensus to improve upon throughput in globally distributed systems. We prove the safety and liveness properties of each algorithm. Finally, we present an experimental evaluation of both algorithms in AWS.

I. INTRODUCTION

State machine replication is a foundational tool that is used to provide availability and fault tolerance in distributed systems. Sites use consensus algorithms as a method to achieve agreement on the order of updates. Such algorithms appear in many of today’s distributed systems. Chubby [1] utilizes consensus for the Google File System to elect a master server that coordinates replication of files. Autopilot [2] creates fault-tolerant replicas in Microsoft’s data centers around the world using consensus. Quorum [3], a permissioned blockchain technology based on Ethereum, achieves data consistency through consensus.

Modern distributed systems are often large-scale, globally distributed, and dynamic. Data centers used by Google, Amazon, and Microsoft are spread globally to reach people across the world [4]. Cellular networks face latency across continents and are highly dynamic with mobile participants [5]. IoT/edge networks often face failures and unreliable messaging, requiring fault-tolerant systems [6].

Two of the most widely adopted consensus algorithms, Paxos [7] and Raft [8], are not designed with such systems in mind. These algorithms specify methods to maintain a replicated log in an asynchronous system with message loss and crash failures. Both require several rounds of messaging between a leader and a majority of sites. Such an operation can lead to high latency for a client’s value to be published. This can be problematic in modern systems that service millions of clients every day. Fast Paxos [9] is a variation on Paxos that reduces the number of message rounds in typical operation to mitigate this problem.

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Raft specifies methods for dealing with membership changes, but with the assumption that sites are added and removed by a system administrator, such that membership changes are never unexpected by the cluster. Vertical Paxos [10], another Paxos variation, similarly depends on an auxiliary configuration master for reconfiguration. However, in many distributed systems, membership changes may be sudden, and may occur silently. Many variations on Paxos variations have been proposed that improve consensus for modern systems, such as Dynamic Paxos [11], EPaxos [12], and Delegator Paxos [13]. These algorithms either do not handle dynamic membership explicitly, or require the same number message rounds as Paxos and Raft do.

To address the limitations of previous works, we propose Fast Raft, with the goal of improving consensus speed in dynamic and globally distributed systems. Fast Raft is based on Fast Paxos, and speeds up typical consensus operation by paying a slightly higher penalty for recovery in the face of failures. Fast Raft specifies details omitted from Fast Paxos, such as leader election and unexpected membership changes.

While Fast Raft addresses some of the limitations of consensus algorithms in fast-paced dynamic networks, it still requires communication between a leader and all sites in the system to reach consensus. This communication paradigm may not scale to global networks, where such all-to-one communication is both time and bandwidth consuming. In such systems, a hierarchical model can be beneficial. Here, sites are grouped into clusters. The bulk of the computation is performed within each cluster, with results combined globally at a lower frequency. Such a hierarchy can be either physical or logical. Physical hierarchical models have been utilized in database systems [14], sensor networks [15] and edge computing [16]. Federated learning [17] is an example of a logical hierarchy used to achieve both privacy and improved model training. Such a logical hierarchy can be applied to blockchain as well. Desu et al. [18] proposed a partitioning of blockchain into a hierarchy of sub-chains to improve scalability and reduce energy consumption.

To complement Fast Raft, we propose a hierarchical consensus algorithm that we call *Clustered Raft* or *C-Raft*. C-Raft is designed to improve throughput in globally distributed systems by utilizing a hierarchical structure. Rather than all sites taking part in consensus, sites in a cluster run local consensus, then cluster leaders replicate entries in batches to other clusters. C-Raft is defined with Fast Raft as a building block, further improving the speed of consensus in typical operation. We provide a specification for C-Raft and prove that it satisfies safety and liveness properties. We also provide

experimental results of the performance of Fast Raft and C-Raft against classic Raft in AWS. On average, Fast Raft, is twice as fast as classic Raft if message loss is below 5% while C-Raft achieves 5x the throughput of Raft in a globally distributed system.

The rest of the paper is organized as follows. Section II formulates the problem and states the properties we wish to satisfy. In Section III, we provide an overview of the classic Raft and Fast Raft algorithms. Section IV presents detailed pseudocode for Fast Raft, as well as proves the properties it satisfies. We present the C-Raft algorithm and prove its safety and liveness in Section V. Section VI presents experiments comparing Fast Raft and C-Raft with classic Raft. We discuss related works in Section VII, and conclude in Section VIII.

II. SYSTEM MODEL

We consider a system that consists of a set of sites. Sites can crash and may recover, and we assume each site has a means of stable storage that can be read from upon recovery. Messaging between sites is asynchronous with potential message loss. Sites may join or leave the system over time. It is assumed joining sites have a means of contacting sites already in the system.

A *global log* is replicated at every site. The log is an infinite array of log entries, indexed using the natural numbers. Sites *propose* entries to be inserted in the global log. Sites reach consensus on the entry, at which point the entry is *committed*.

Our goal is to satisfy the following properties.

Definition 2.1: Safety If a site commits an entry at some index in the log, no site can commit a different entry at the same index.

Definition 2.2: Liveness For any *proposed* entry v , it will eventually be the case that v is stored in the log at all sites.

By satisfying safety, every site will have the same entries in each index of the logs, thus defining a *total order* for every entry committed. By the findings of Fischer, Lynch, and Paterson [19], we cannot guarantee liveness in a fully asynchronous system with crash failures without additional conditions. Along with the specification of Fast Raft and C-Raft, we prove they satisfy safety, and identify conditions for liveness.

III. BACKGROUND AND ALGORITHM OVERVIEW

In this section, we provide an overview of classic Raft, and the modifications made to construct Fast Raft.

A. Classic Raft

In classic Raft, time is split into *terms* numbered in a monotonically increasing manner. Sites take on one or more roles in a term. Proposers propose new log entries to be appended to the log. A unique leader gathers proposals of log entries and coordinates consensus on the entries. There are followers that participate in consensus on new log entries. Finally, there are candidates that attempt to be elected as a new leader if they suspect the current leader of having failed. In a typical term, one candidate is elected as a leader, then

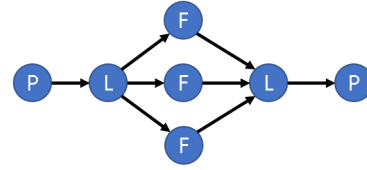


Fig. 1: Message flow of appending an entry in classic Raft. P is the proposer, L is the leader, F are the followers.

consensus on log entries can begin. We describe the means of consensus and election below.

The log can contain both proposed entries and committed entries. The *commitIndex* is a value stored at each site that determines which values in the site's log are committed. Entries in indices at or before *commitIndex* are considered committed by the site. Proposers will send proposed entries to the term's leader. The leader appends proposed entries to its log and it sends an *AppendEntries* message to followers to also append the entry. When the leader receives acknowledgement that a majority, or *classic quorum*, has appended the entry, the leader's *commitIndex* is updated, indicating that the value has been committed. The leader then notifies the proposer. On subsequent *AppendEntries*, the leader will include its *commitIndex* and followers can update their own *commitIndex* accordingly.

In our system model, it is possible for messages to be lost, or sites to crash. If a leader fails, then a new leader must be elected. As a means of failure detection, the leader maintains a heartbeat with followers. If a follower does not receive a heartbeat message within some *election timeout*, the follower converts to a *candidate*, increments its term number, and begins an election. The candidate sends *RequestVote* messages to all sites, who are notified of the term change. Messages from leaders of lower terms will be rejected and notified that a new term has begun. Upon receiving a *RequestVote* message, a site sends a vote to the candidate if the candidate's log is at least as *up-to-date* as its own. A log with the most log entries from the most recent term is considered the most up-to-date. If a majority of sites send votes to the candidate, it becomes the leader for that term.

It is possible that multiple followers time out and become candidates for the same term. This may lead to no candidate being elected in a term. Candidates that fail to receive enough votes wait for a *randomized timeout* before incrementing the term number again and retrying an election. These randomized timeouts ensure that a leader is eventually elected with high probability. A Candidate converts back to a follower if it receives an *AppendEntries* message from a newly elected leader at the same or higher term number.

To allow for dynamic membership, Raft defines a special type of log entry called a *configuration* entry. This entry contains a list of all *voting members* of the system, the sites can take part in consensus. If a site receives a consensus-related message from a site that is not in the configuration, the message is ignored. Each site considers the last appended configuration entry to be its current configuration. Raft as-

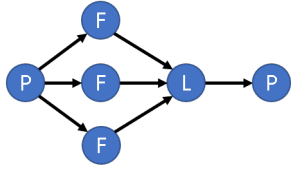


Fig. 2: Message flow of inserting a new entry in Fast Raft. P is the proposer, L is the leader, F are the followers.

sumes a system administrator proposes configuration changes to the leader, and that only one site is added or removed from the configuration at a time. Without this restriction, it may be possible that messages are lost in consensus on reconfiguration, and there is a group of sites with the old configuration and quorum size. If the old quorum does not overlap with every new quorum, two leaders may be elected, violating safety.

When a site is proposed to be added to a configuration, the leader first catches up the site on all entries in the leader's log. During this period, the joining site is considered a *non-voting member* of the system, and it cannot take part in consensus yet. Once caught up, the new configuration is appended to the leader's log and consensus is run. Once a majority has appended the new configuration entry, the joining site is considered a voting member. A similar protocol is followed for a site leaving the configuration.

B. Fast Raft

When there are no concurrent proposals, Fast Raft decreases the number of message rounds from three to two before an entry is committed. To achieve this without violating safety, Fast Raft requires a recovery mechanism in the face of failures.

Fast Raft has the same roles as classic Raft, but consensus works differently. Fast Raft provides two methods of committing values to the log: a *fast track* and a *classic track*. The fast track, as the name indicates, requires fewer rounds to commit a value. The classic track is followed if the fast track fails due to message loss or concurrent proposals. We describe these tracks below.

When a proposer has a new entry e to be inserted in the log at an index i , rather than sending the entry to the leader, the proposer sends the entry to all sites. Upon receiving the new entry, a site a inserts the proposed entry to its log at index i . This contrasts with classic Raft, where the log is treated as a growing list that is always appended to. In the case of Fast Raft, site a may miss a proposal for an entry at index $j < i$. Follower a will insert the entry at i , leaving index j empty. After inserting, the follower forwards the message to the leader, indicating that a has voted for entry e . At this point, entry e in site a 's log is marked as *self-approved*, as site a inserted the entry itself.

The leader gathers these votes for entries at index i . After receiving votes, the leader makes a decision on the entry that should be inserted into its own log. Let M be the number of sites in the configuration. If $\lceil \frac{3M}{4} \rceil$ (or more) sites, a *fast*

quorum, have voted for the same entry e , then the leader commits e . The message flow of the fast track in Fast Raft is shown in Figure 2.

With this method, the leader may receive votes for different entries, and may be missing votes if messages are lost. The leader will wait for at least a classic quorum of votes. There are only two scenarios: either a fast quorum has appended the same entry or not. In the first scenario, the leader needs to insert this entry. Otherwise, the leader can insert any entry. Due to the chosen quorum sizes, if a fast quorum has inserted an entry, it will be the entry with the most votes in any classic quorum. We provide intuition as to why this is in the following example. Consider a scenario with five sites. For the same index, proposals e and f were proposed. Four sites, a fast quorum, insert e , and one site inserts f . Vote messages are lost and the leader only receives from three sites, a classic quorum. No matter which three sites the leader receives votes from, two of the sites must have voted for e . Thus, if a entry has been inserted by a fast quorum, the leader will always insert that entry.

If the leader inserts an entry but does not commit it, we revert to the classic track, which is identical to classic Raft. The leader sends *AppendEntries* messages to have a classic quorum insert the entry that had the most votes. Entries inserted on the classic track are marked as *leader-approved*. If a follower receives an entry from the leader that it already inserted, it will update it to be leader-approved. Note, the classic track only occurs after attempting the fast track, and thus, in this situation, we suffer the penalty of an extra message round compared to classic Raft.

Leader election follows the same flow as in classic Raft with some alterations. Since proposers can send directly to followers, the definition of up-to-date is modified to only include leader-approved entries. Self-approved entries cannot be considered in this check, as proposers can send an arbitrarily large number of proposals to a follower that ultimately may not have been agreed upon. Once the most up-to-date candidate is elected, Fast Raft runs a recovery algorithm. Self-approved entries were not considered in the election, and need to be evaluated to ensure safety. All followers resend their self-approved entries to the newly elected leader. If a leader from a previous term committed any of these entries, then there will be a fast quorum that has inserted the entry, and the new leader will make the same decision as previous leaders and commit the entry.

Fast Raft deals with dynamic membership similarly to classic Raft. However, we do not assume a system administrator proposes configuration changes. Instead, joining and leaving sites send join or leave requests to the leader. It is now the role of the leader to ensure that only one site joins at a time by processing join requests sequentially. Any sites with pending joins are considered non-voting members of the system. Unlike Raft, Fast Raft has specification for when a site leaves the system silently, as sites may not always propose a leave request before leaving the system.

IV. FAST RAFT

We now dive into the details and pseudocode for the Fast Raft algorithm and provide a proof of safety and discussion of liveness. Some parts of the Fast Raft specification are the same as in classic Raft. We mark items that are new to Fast Raft in blue and with a † symbol.

A. Site State

Each site stores some state that is persistent in stable storage, as well as volatile state.

Persistent state on all sites:

- *currentTerm* : latest term sent to site (initialized to 0)
- *votedFor* : candidateId that site voted for in current term (initialized to null)
- *log*[] : global log to be replicated
- *lastLogIndex* : last index appended to log
- *lastLeaderIndex* : last index appended to log by the leader.†
- *configuration* : last configuration appended to the log.
- *commitIndex* : index of highest log entry known to be committed (initialized to 0)

The variables *currentTerm* and *votedFor* are used for leader election to help determine which site will be the new leader. This is discussed more in Section IV-C. The *lastLogIndex*, *lastLeaderIndex*, and *configuration* can be determined based on the log, but are separate variables here for notation convenience.

As discussed in the overview, the *commitIndex* indicates the entries in the log that are committed. Any entry in an index larger than *commitIndex* is not yet committed. As *commitIndex* is in volatile state, if a site crashes and recovers, it will need to relearn which log entries are committed from the current leader.

Contents of a log entry:

- *data* : data to be replicated.
- *term* : term number when the entry was appended.
- *insertedBy* : either self or leader.†

Each log entry contains some data to be replicated and the term number in which it was added. This information is necessary for deciding which site is most up-to-date and should be elected leader. The only change from classic Raft here is the *insertedBy* value, indicating if the entry was self-approved or leader-approved.

Volatile state on the leader:

- *nextIndex*[] : for each site, index of the next log entry to send to that site (initialized to leader's last committed† log entry + 1).
- *matchIndex*[] : for each site, index of highest log entry known to be replicated on site (initialized to 0).
- *fastMatchIndex*[] : for each site, index of highest log entry known to be sent to the leader that matches the leader's choice (initialized to 0)†.

- *possibleEntries*[] : array of sets of possible entries for the index in the leader's log. Each index corresponds to a log index. Each index holds a set of pairs, each consisting of a proposed entry and number of votes for that entry†.

The leader keeps track of *nextIndex*[*i*] to determine what log entries need to be sent to site *i*. Rather than the leader sending all entries, it only sends entries starting from *nextIndex*. *matchIndex*[*i*] indicates the latest log index that site *i* matches with the leader. Once a classic quorum is at the same or higher *matchIndex*, the leader commits the entry at that index.

Fast Raft has an additional array, *fastMatchIndex*, separate from *matchIndex* for determining if an entry can be committed on the fast track or classic track. Each of these arrays contain an index for a site in the configuration, and thus are subject to size changes. When a site joins, a new slot in each array must be added. For leaving sites, it may be beneficial for the values associated with them to be stored in case they return to the system at a later time.

Unique to Fast Raft, the *possibleEntries* structure plays an important role. A leader in classic Raft would immediately append entries proposed to it. However, Fast Raft needs a method by which to keep track of the votes of followers for a log index. The leader makes its decision on what entry to insert or commit based on the contents of *possibleEntries*. This decision is explained in the next subsection.

B. Inserting Entries

Log entries are proposed to be inserted into the replicated log at specific indices. The leader gather votes for log entries at an index *i* and determines which entries to commit. In this section, we describe the flow of entries from proposed to committed on both the fast track and the classic track.

To propose an entry:

- 1) Send log entry to *leaderId* all members in configuration†.
- 2) If log entry not committed after proposal timeout, resend log entry†.

In Fast Raft, proposers send directly to all members, rather than going through the leader. In the case of concurrent proposals, a proposer's entry may be overwritten by another proposed entry that garnered more votes. As such, a proposer has a *proposal timeout*, a period of time to wait until reproposing if its entry is not committed.

When follower receives a proposed entry *e* for index *i*†:

- 1) If entry is duplicate and committed, notify proposer.
- 2) If there is no entry at index *i*, insert entry to log at index *i*.
- 3) Set *e.insertedBy* = *self*
- 4) Send *log*[*i*] and *commitIndex* to *leaderId*.

Sites insert entries sent to them and then send their vote to the leader. If an entry already exists at index *i*, the follower does not overwrite it. This contrasts with Raft, in that sites do

not always necessarily append entries to their end of their log, meaning log entries of higher indices can be inserted before lower indices if messages are lost. Sites need to be aware of duplicate entries, as a proposer may resend its entry if it is not committed. The leader is treated as a follower in this scenario, and follows the same process.

When leader receives an entry e for index k from site i [†]:

- 1) If $e \in possibleEntries[k]$, increment count for e . Otherwise, add e to $possibleEntries[k]$ with $count = 1$.
- 2) Set $nextIndex[i] = sentCommitIndex$.

The leader receives votes from followers for entries at an index k , and tracks the votes in $possibleEntries[k]$. The leader also sets the $nextIndex$ for agent i to the last committed index. In Fast Raft, this is necessary for ensuring sites stay consistent with a newly elected leader. We discuss this in more detail in Section IV-C.

Periodically run by the leader[†]:

- 1) While there exists a $k = commitIndex + 1$ for which at least a classic quorum of votes has been received:
 - a) Insert entry e from $possibleEntries[k]$ with highest number of votes. Break ties arbitrarily.
 - b) Set $e.insertedBy = leader$
 - c) Update $fastMatchIndex[i]$ for all i that voted for entry e .
 - d) If e is elsewhere in $possibleEntries$, set to a null vote to avoid inserting a duplicate entry.
 - e) If there is a fast quorum that exists such that $fastMatchIndex[i] \geq k$, and $log[k].term = currentTerm$: set $commitIndex = k$

New to Fast Raft, the leader periodically checks if an entry can be committed on the fast track. As discussed in the overview, if a fast quorum has voted for an entry, it can be committed. Otherwise, if at least a classic quorum has voted for an index, the leader chooses the entry with the most votes to insert into its log. The leader then switches to the classic track, sending this entry to the follower to insert.

The fast track can only be taken here if the last index was committed. This restriction is necessary since $commitIndex$ indicates that all entries at or before the index are committed. If any entry was able to take the fast track, then $commitIndex$ could be updated prematurely, committing log entries that should not be.

Periodically run by the leader:

- 1) **Create AppendEntries message for each follower containing:**
 - $term$: leader's term
 - $leaderId$: for followers to redirect sites
 - $entries[]$: log entries to insert
- 2) For each follower i , if $lastLeaderIndex^{\dagger} \geq nextIndex[i]$, include all log entries in $entries[]$ starting at $nextIndex[i]$ and ending at $lastLeaderIndex^{\dagger}$.
- 3) Send *AppendEntries* messages to followers.

Periodically, the leader sends *AppendEntries* to all fol-

lowers. The message contains entries the follower may not have inserted yet. Even if there are no new entries for a site, the leader sends an empty message to followers. This heartbeat lets sites know the leader is still active.

When a follower receives AppendEntries message:

- 1) Reset election timer.
- 2) **Create response to AppendEntries message:**
 - $term$: currentTerm, for leader to update itself
 - $success$: true if follower contained entry matching $prevLogIndex$ and $prevLogTerm$
- 3) If $term < currentTerm$, set $success = false$ and respond.
- 4) If an existing entry conflicts with a new one **overwrite the existing entry[†]**.
- 5) If $leaderCommit > commitIndex$ set index to the minimum of $leaderCommit$ and $lastLogIndex$

Sites check if their term number is higher than the one sent. If it is, this means the sender is no longer the leader. The follower will send back $success = false$ to the old leader. Otherwise, the site continues to inserting new log entries. In Fast Raft, sites overwrite any entries at the same indices as the entries the leader sent to them. Classic Raft would remove any entries inconsistent with the current leader here. However, since proposers send to followers first, the leader may be unaware of entries for some indices. Overwriting these could violate safety. Thus, the leader only overwrites entries it has made safe decisions about. At this point, the entries are leader-approved.

Note that sites receive the leader's $commitIndex$ with new entries. Leaders are always the first to commit an entry, and followers only update their own $commitIndex$ after receiving from the leader.

When the leader receives AppendEntries message response from follower i :

- 1) If $term > currentTerm$, set $currentTerm = term$ and convert to a follower.
- 2) If $success = true$, update $nextIndex[i]$ and $matchIndex[i]$.
- 3) If an index k exists such that $k > commitIndex$, a classic quorum of $matchIndex[i] \geq k$, and $log[k].term = currentTerm$, then set $commitIndex = k$

The leader updates $nextIndex$ and $matchIndex$ based on the value of $success$. If $matchIndex$ indicates a majority has inserted an entry, the leader commits the entry.

C. Leader Election

In contrast to Paxos, leader election in classic and Fast Raft is worked into the algorithm. The heartbeat message from the leader acts as a failure detector for the followers.

When election timeout occurs:

- 1) Convert to a candidate.
- 2) Increment *currentTerm*.
- 3) Set *votedFor* = *self*.
- 4) **Create RequestVote message containing:**
 - *term* : candidate's term
 - *candidateId* : candidate requesting vote
 - *candLastLogIndex* : index of candidate's last leader-approved[†] log entry
 - *candLastLogTerm* : term of candidate's last leader-approved[†] log entry
- 5) Send *RequestVote* message to all other sites.
- 6) Reset election timer.

If a follower reaches an *election timeout*, it begins leader election by increasing its term number and requests votes by sending *RequestVote* messages to all other sites. It is important that the election timeout cannot be shorter than message delays, otherwise it is possible for followers to continuously start elections, and no progress can be made.

When receiving a RequestVote message from a candidate:

- 1) **Create response to RequestVote message:**
 - *term* : currentTerm, for candidate to update itself
 - *voteGranted* : true means candidate received vote
 - *selfApprovedEntries*[] : all self-approved entries in log[†]
- 2) If *term* < *currentTerm* set *voteGranted* = *false* and respond.
- 3) If *votedFor* = *null* or *votedFor* = *candidateId*, *candLastLogIndex* ≥ *lastLeaderIndex* and *candLastLogTerm* ≥ *log[lastLeaderIndex].term*, or *candLastLogTerm* > *lastLeaderIndex.term*[†], set *voteGranted* = *true*, add all self-approved entries in log to *selfApprovedEntries*[†] and respond.
- 4) Otherwise, set *voteGranted* = *false* and respond.

Sites that receive the *RequestVote* message immediately move to the new term. This means leaders from previous terms are no longer leaders. A site votes for a candidate if the candidate is as *up-to-date* as it is, or more. This ensures that the most up-to-date live site will be elected.

In classic Raft, the most up-to-date site has the most log entries from the most recent term. However, in Fast Raft, proposers can send directly to followers to insert into their log. Instead, the most up-to-date site has the most recent leader-approved log entry. This alone does not guarantee safety, as a failed site may have committed an entry on the fast-track. Once elected, the new leader needs to run a recovery algorithm (discussed below). The recovery requires knowledge of self-approved entries in a classic quorum of sites. The recovery ensures that if it is possible that an entry was committed, then it will be committed by the new leader. As such, sites that vote for the candidate will also include their self-approved entries in their *RequestVote* response message.

When a candidate receives any message:

- 1) If message is a response to *RequestVote* and *voteGranted* = *true*, increment votes.
- 2) If *AppendEntries* received from new leader: convert to follower.
- 3) If votes received from majority of sites: become leader, copy all self-approved entries received to *possibleEntries*[†], and send an initial *AppendEntries* heartbeat.

If the candidate is considered up-to-date by a majority, meaning it has received their votes, then it converts to a leader. Otherwise, it loses the election, and will retry after a randomized timeout. Other followers may timeout and attempt to be elected as well. Randomized timeouts are used to help ensure a leader is eventually elected.

In Fast Raft, when the new leader is elected, the leader fills its *possibleEntries* structure with the received self-approved entries. As specified in Section IV-B, for each index where a classic quorum has sent their votes for self-approved entries, the leader chooses the entry with the most votes to insert or commit to its log. Leader-approved entries are treated the same as they are treated in classic Raft, but self-approved entries require this resending of log entries to ensure safety. We discuss more in Section IV-E.

D. Membership

Similar to how leader election is tied into consensus using term numbers, the membership configuration is a special log entry. When a site wishes to join or leave the system, sites first runs consensus on a new log entry that defines the change in configuration. Sites follow the configuration that was last inserted into their log to determine quorum sizes and which sites to communicate with. Messages from sites not listed in the configuration are ignored.

When a site wants to join the system[†]:

- 1) Send a join request to sites in the system.
- 2) If join not accepted in join timeout, resend request.

Upon receiving a join request:

- 1) If not the leader, redirect to the leader[†].
- 2) If a duplicate request, ignore[†].
- 3) Catch up joining site.

Upon catching up site:

- 1) Create a log entry for the new configuration including the new site.
- 2) Start consensus on new log entry.
- 3) When consensus reached (the entry is committed), notify the new site.

Upon receiving a leave request:

- 1) Create a log entry for the new configuration excluding the site.
- 2) Start consensus on new log entry.

The original Raft paper assumes there is a system administrator that proposes configuration changes. In Fast Raft,

sites send join or leave requests to voting members. Before a site joins, the leader will catch-up the site by sending all its log entries up until this point. During this period, the site is considered a non-voting member. The leader sends *AppendEntries* messages to the joining site, but does not consider its vote in consensus of log entries. After the joining site is caught up, the leader will create a new configuration entry to include it, and run consensus on it. Once the configuration entry is committed, the joining site can take part in consensus.

As discussed previously, to satisfy safety, only one site may join at a time, which is managed by the leader. For example, if a leader receives join requests for two sites, it will process the join requests sequentially. It will create a configuration entry for one site, wait until the entry is committed, then create an entry for the other site.

If site leaves silently[†]:

- 1) If the leader left the system, a new leader is elected to detect the failure.
- 2) Leader detects silent leave if a follower does not respond to enough *AppendEntries* messages, specified by the member timeout.
- 3) Send leave request for the missing site and start consensus on new configuration.

As we consider systems where sites can enter or leave at any time, we have to modify our membership mechanism from the original specification in classic Raft. Multiple sites entering the system can be buffered by the current leader, only allowing one to join the configuration one at a time. However, this mechanism cannot be employed for sites leaving the system. It is not reasonable to expect that a site requesting to leave will stay in contact, or resend its request if lost. Without some centralized system administrator to ensure sites update configurations before a site leaves, we must specify how our system will handle a site leaving silently.

Just as the heartbeat message is used for followers to determine if the leader has failed, the response to the heartbeat message can be used by the leader to determine if a site has left the system silently. We introduce a tunable *member timeout*, the number of missed *AppendEntries* responses before the leader initiates consensus on a new configuration that excludes the unresponsive follower. The follower may still be in the system, in which case it will need to send a join request to return to the configuration.

E. Safety

First, we prove safety in Fast Raft ignoring membership changes. We then show that membership changes do not affect the proof. To ensure the safety property is satisfied, we introduce some invariants that will build up to satisfy it.

Invariant 1: If a follower commits an entry at some index, then the leader has committed the same entry in the same index.

Invariant 2: If a leader has committed an entry at some index, then no leader in a previous term has committed a different entry at the same index.

Theorem 1: If Invariants 1 and 2 hold, then the safety property holds.

Proof: By Invariant 1, followers only commit once notified that the leader has committed. By Invariant 2, leaders will always commit the same entries as leaders from previous terms. A leader will never overwrite a committed entry. As every leader and follower of every term will always commit the same entries at the same indices, safety is satisfied. ■

It is left to prove that Fast Raft satisfies the invariants.

Lemma 1: Invariant 1 holds.

Proof: A follower only updates its *commitIndex* by taking the minimum of the leader's *commitIndex* and its own *log*'s length. It will never set *commitIndex* to an index greater than the leader's *commitIndex*. Therefore, the follower will only commit entries if the leader has committed them. ■

Lemma 2: Invariant 2 holds.

Proof: We define a log entry to be *chosen* for an index if one of the following two conditions hold:

- 1) there is a fast quorum with the same self-approved or leader-approved entry, or
- 2) there is a classic quorum with the same leader-approved entry.

To prove Lemma 2, we must prove that once an entry is chosen, no other entry can be chosen, and that a leader of a term will only commit chosen entries.

First, we note that no entries for an index can be chosen simultaneously as every classic quorum and fast quorum intersect. A follower may overwrite its entry for an index with an entry sent from the leader. Suppose an entry v was chosen by a fast quorum R . As proven by Zhao in [20], any classic quorum of votes that the leader could have received on the fast track must have a majority overlap with R . Entry v will have the most votes in any classic quorum, and the leader always inserts the entry with the most votes. Thus, when a leader sends an entry to followers, either the entry is chosen, or no entry is chosen for the index. So, a follower never overwrites a chosen entry.

It is left to prove that a leader always commits a chosen entry. If the leader of a term t successfully gathers votes of a fast quorum for the same entry, it commits the chosen entry. Otherwise, the leader inserts an entry for the classic track. Either this is a chosen entry, or no entry was previously chosen. The proof of safety for the classic track follows directly from the proof for classic Raft [8].

Suppose the leader of term t fails before committing. Leaders of terms $s > t$, upon election, will first run Fast Raft's recovery algorithm, gathering the votes of all self-approved entries. If an entry was chosen in term t , then it is included in the resent self-approved entries because a classic quorum is required to elect a leader. The newly elected leader also inserts the entry with the most votes, the chosen value, just as the leader in term t did. Thus, any entry committed by a leader in term t must also be committed by all leaders in terms $s > t$. ■

We now consider membership changes. When a site joins or leave the system, a new configuration entry is created and



committed. As with other log entries, not all sites necessarily have the new configuration entry inserted. These sites will not update their quorum sizes. If such a site loses contact with the current leader, it will start an election, and could wrongfully think its the new leader due to making decisions on smaller quorum sizes. This can lead to two leaders being elected in a term, allowing safety to be violated.

As proven in Ongaro’s thesis [8], safety in classic Raft is preserved if only one site joins or leaves at a time. A majority of the sites reach consensus on membership changes one at a time, never changing quorum sizes in a such a way to allow two leaders to be elected. In Fast Raft, when sites announce their joins and leaves, only one site can join or leave at a time, as the leader will only process join or leave requests one at a time. For each request, consensus on the new configuration must occur, and thus at least a majority of sites in the system know about the quorum size changes. As Fast Raft’s leader election only differs in the recovery algorithm when a leader is elected, safety is preserved for announced joins and leaves, just as in classic Raft.

If sites leave the system silently, the true quorum sizes may decrease. Thus, consensus or election decisions may be based on quorum sizes that are larger than necessary. However, using larger quorums cannot lead to two leaders being elected and violating safety. It may affect liveness, as discussed in the next subsection.

F. Liveness

In order to guarantee liveness, we require no concurrent proposals, otherwise it is possible proposed entries will be overwritten. Further, we require at least a classic quorum of messages are delivered to the leader to complete consensus.

It is also required that if a majority of sites leave the system silently, that there be a leader that remains active long enough to detect and commit the configuration change. If a majority of sites leave, consensus on new log entries cannot be completed on the fast or classic tracks. After the leader has a member timeout with the sites, the leader can insert a new configuration and decrease the leader’s perception of quorum sizes, and commit a new configuration entry. However, if the leader fails before committing the new configuration entry, or was part of the sites that silently left the system, the remaining sites will be unable to elect a new leader. In such a scenario, the system is deadlocked.

V. C-RAFT

The goal of C-Raft is improve the throughput of consensus in globally distributed systems where latency between distant sites can be very large. C-Raft mitigates this by performing two levels of consensus: local consensus with nearby sites, and thus have lower message latency, and then global consensus on batches of locally committed entries.

A. System Model

As in Fast Raft, the system has asynchronous communication between all sites. Sites form a set of clusters. The membership of each cluster can change over time, and each

This allows aircraft to be owners of one or more airspaces

site can be a member of one or more clusters. We assume sites are aware of which cluster they are a member of, but not necessarily the membership of other sites. Further, the number of clusters may change over time. We separate the means of communication within clusters, *intra-cluster*, and across clusters, *inter-cluster*. Both are asynchronous with message loss, but we assume that intra-cluster communication is “easier”, whether this is due to communication cost, bandwidth, or minimum message propagation time.

In addition to the global log, each cluster also replicates its own *local log*. The local log serves two purposes: buffering of log entries for the global log, and state replication for inter-cluster consensus. Within each cluster, sites propose entries to be placed in the local log of that cluster. Sites of the cluster reach consensus on the entry, at which point the entry is committed to the local log. Periodically, the leader of a cluster proposes a batch of local entries to be committed to the global log. Batches may be created and proposed based on how many entries have been placed in the local log, an amount of time passing, or a user-defined need.

During inter-cluster consensus, it is possible for a cluster leader to fail and a new one be elected. The local log of the cluster is used here to ensure a leader’s state for the global log is passed on to new leaders if there is a failure during at any step of inter-cluster consensus. This is achieved through consensus on special log entries for inter-cluster consensus state replication.

The hierarchical structure allows proposers to have their entries replicated locally, with the guarantee that the entries will eventually be replicated to other clusters and totally-ordered in the global log. C-Raft utilizes this model to improve throughput of consensus at the global scale.

B. Algorithm

C-Raft is defined with two levels of Fast Raft: one for intra-cluster consensus, and one for inter-cluster consensus. The intra-cluster consensus algorithm is simply the execution of Fast Raft over the members of the cluster. In inter-cluster consensus, the leaders of each cluster act as members in a configuration. The members of this configuration are followers of inter-cluster consensus, and *local leaders* for intra-cluster consensus. A *global leader* is elected by the local leaders, and Fast Raft is executed the same as specified in Section IV with some small, but important, changes. Inter-cluster consensus pseudocode is presented below, and the changes to Fast Raft are marked in blue and by a † symbol. Much of Fast Raft remains the same in inter-cluster consensus, and is left out of the pseudocode here.

Sites now contain state for both cluster levels, intra-cluster state and inter-cluster state, with the same types of variables. Variables associated with intra-cluster consensus are prefixed with *local*, and they are prefixed with *global* for inter-cluster consensus. Sites part of intra-cluster consensus propose entries for the local log, and local leaders propose batches of local *log* entries to be inserted in the global *log*. Note, that as a local leader is a member of the global

configuration, if a new local leader is elected, it must send a join request to enter the global configuration.

During inter-cluster consensus, local leaders receive entries to insert into their global log either from proposers or the global leader. The important difference from Fast Raft here is that the local leader must first run intra-cluster consensus before inserting the entry into its global log. The local leader proposes a special log entry for the local log. We call this a *global state entry*. The purpose of this log entry is to replicate the local leader's state in inter-cluster consensus. The global state entry contains in it the global entries that the local leader has inserted into the global log, and their indices. Sites within the cluster run Fast Raft on this special log entry. Once the entry is committed to the local log, the local leader inserts the entry into its global log. The purpose of the global state entry is to ensure that if the local leader of that cluster fails, the global entry that was inserted is still available for future local leaders.

It is possible that the local leader overwrites its global log entry if the global leader sends a different entry at the same index. In this case, the local leader proposes a global state entry to its cluster that overrides a previous one. Similar to configuration entries, a site determines its global state based on the global state entry last inserted in the local log.

When follower receives a proposed entry e for index i :

- 1) If entry is duplicate and committed, notify proposer.
- 2) If there is no entry at index i , insert entry to log at index i .
- 3) Set $e.insertedBy = self$
- 4) Run intra-cluster consensus on global state entry for e at k^\dagger .
- 5) Once entry is committed to the local log[†], send entry and $commitIndex$ to $leaderId$.

Periodically run by the leader:

- 1) While there exists a $k = commitIndex + 1$ for which at least a classic quorum of votes has been received for the global log[†]:
 - 2) If there is an entry e with $e.insertedBy = leader$, then insert to log.
 - 3) Else, insert entry e from $possibleEntries[k]$ with highest number of votes. Break ties arbitrarily.
 - 4) Run intra-cluster consensus on global state entry for e at k^\dagger .
 - 5) Once entry is committed to the local log[†], update $fastMatchIndex[i]$ for all i that voted for entry e .
- ...

When a follower receives AppendEntries message:

- ...
- 4) If an existing entry conflicts with a new one, overwrite the existing entry.
 - 5) Set all new entries $e.insertedBy = leader$
 - 6) Run intra-cluster consensus on global state entry for e at k^\dagger .

- 6) For both local and global variables[†]: if $leaderCommit > commitIndex$, set $commitIndex$ to the minimum of $leaderCommit$ and $lastLogIndex$

In inter-cluster consensus, a global state entry is run through intra-cluster consensus when the local leader receives a proposal for a global entry, when the global leader chooses an entry to be placed in the global log, and when a local leader receives a global *AppendEntries* message.

Sites now contain a global *commitIndex* indicating what entries are committed in the global log. The global leader lets local leaders know when entries are committed through heartbeat messages, just as in Fast Raft. Local leaders now need to include their global *commitIndex* in the *AppendEntries* message to let followers at the local level know which global entries are committed.

C. Cluster Membership

Similar to how Fast Raft's configuration defines the membership of sites in the cluster, C-Raft's local leaders are defined by a global configuration. A new cluster can be formed if a new local leader is added to the global configuration.

When a site wants to make a new cluster[†]:

- 1) Send a join request to local leaders[†].
- 2) If join not accepted in join timeout, resend request.

Upon receiving a join request:

- 1) If not the leader, redirect to the leader.
- 2) If a duplicate request, ignore.
- 3) Catch up joining site.

Upon catching up site:

- 1) Create a global[†] log entry for the new configuration including the new site.
- 2) Start consensus at global level on new log entry.
- 3) When consensus reached (the entry is committed), notify the new site.

To form a new cluster, a site sends a global join request a member of the global configuration. The global leader catches up the site on global log entries if necessary, then insert the new configuration to the log. It then proposes the new global configuration entry for global consensus. If the configuration change is committed, the requesting site becomes the local leader of the new cluster. Sites that enter this new cluster may now send join requests to the local leader to start proposing entries for intra-cluster consensus.

D. Safety

We prove that C-Raft satisfies safety.

Theorem 2: C-Raft satisfies safety in Definition 2.1.

Proof: Let us consider the global level of C-Raft. This is defined the same as Fast Raft, with the exception of the recursive call of Fast Raft at the local level. Before an entry is inserted into the global log of any site, Fast Raft is run on global state entry at the local level. As proven in Section IV, Fast Raft satisfies safety, thus we can be certain that the entry will be committed to the local log of any future local leaders.

Once an entry is inserted into the local leader’s global log, the cluster maintains the state of the local leader in inter-cluster consensus, and the global level of Fast Raft acts no differently than its local version. Fast Raft is proven to satisfy safety, and thus C-Raft satisfies safety. ■

E. Liveness

As the global level of C-Raft is defined with a modified Fast Raft, similar liveness conditions apply. Instead of liveness conditions being based on sites failing, the conditions require clusters to not fail. If the conditions for liveness in Fast Raft do not hold within a cluster, we consider this to mean the cluster has failed. For example, if a majority of the cluster has failed, then the local leader cannot insert global log entries and will block on consensus step for the global state entry. For liveness to be guaranteed at the global level, liveness must be guaranteed for intra-cluster consensus in enough clusters for inter-cluster consensus to continue.

VI. EXPERIMENTS

To showcase the performance of Fast Raft and C-Raft, we performed experiments on Amazon Web Services (AWS). Each site was set up as a separate EC2 AWS instance running Ubuntu 18.04 LTS. To simulate clusters, instances were started in different regions around the world including North America, South America, Europe, and Asia. Sites assigned to the same AWS region were considered part of the same cluster. Roundtrip message latency varies between 10 to 300 ms between AWS regions and is less than 1 ms within regions. To force certain percentages of message loss, we changed traffic control settings in Linux using the *tc* command.

We implemented classic Raft, Fast Raft, and C-Raft in Python 3.6 using UDP sockets for communication. The leader’s heartbeat timer was set to 100 ms for intra-cluster consensus, and 500 ms for inter-cluster consensus. To measure latency of committing an entry, the proposer started a timer when first proposing an entry and stopped the timer when the leader notified it that the entry was committed. The proposer only proposed a new entry after the previous entry was committed.

A. Classic Raft vs. Fast Raft

First, we compared the commit latency of classic Raft and Fast Raft in a single cluster. We chose a site at random to be the proposer and measured the average latency for entries committed over 100 trials when using classic Raft and Fast Raft. In the experiments, we had five sites in one region and varied message loss between 0% and 10%.

Figure 3 shows the results of the experiment. When message loss was low, Fast Raft achieved about half the latency as classic Raft. However, as message loss increased, Fast Raft started to degrade in performance while classic Raft maintained similar latency. As more messages were dropped, the classic-track was used more in Fast Raft, causing it to face an extra message round more often. This reinforces the observation that Fast Raft is best used when message loss is not common.

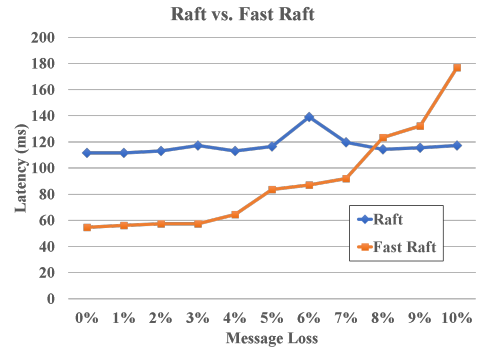


Fig. 3: The average latency of committing entries in classic Raft and Fast Raft, with five sites and different percentages of message loss.

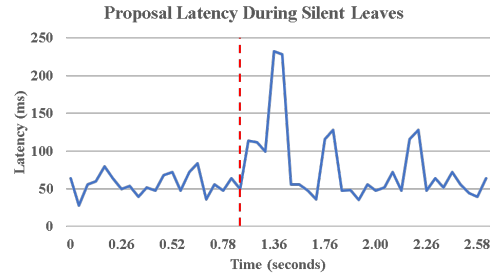


Fig. 4: Latency on proposed entries being committed in Fast Raft in a cluster of five sites. The vertical red line indicates when two sites silently leave the cluster.

B. Silently Leaving a Cluster

Next, we studied the effect of sites silently leaving a cluster on commit latency in Fast Raft. We started with a cluster of five sites, had two of the sites silently leave the cluster, and measured the latency on committing entries during this period. The fast quorum size before leaves was four. As such, during the period before the leader detected the leaves, proposed entries used the classic track. Message loss was set to 5% and the leader’s member timeout occurred after five missed heartbeat responses.

Figure 4 shows the results of this experiment. The vertical red line indicates when the sites left the cluster. Before this point, the proposer was typically able to use the fast track. The variability in latency of proposals here can be due to proposals occurring closer or further from the heartbeat timeout of the leader, or due to message loss causing the leader to use the classic track. Once the leave occurred, there was a brief period where the fast track was no longer available to the proposer. The very large spike to above 200 ms in this section was likely due to the concurrent proposals with the leader for a configuration change. After this point, proposal latency returned to a range of 50 to 100 ms.

C. Classic Raft vs. C-Raft

Finally, we compared the throughput of Raft and C-Raft. Sites in the same AWS region formed a cluster. We chose one proposer at random per cluster. Each proposer waited

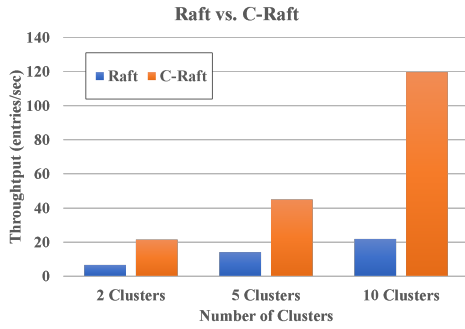


Fig. 5: Average throughput of committing entries in classic Raft versus C-Raft. Experiment run for with 20 sites total. Each cluster is in a different AWS region.

until its last proposed entry is committed before proposing another. We compared throughput based on how many entries were committed to the global log in classic Raft and C-Raft, averaged over five 3-minute trials.

For the C-Raft implementation, a cluster proposed a batch of entries to the global log after ten entries were committed in the local log. We tested with 20 sites total, split evenly over a varying number of clusters. Note, there were more proposers in the system as the number of clusters increased for both C-Raft and Raft. The results are shown in Figure 5. C-Raft showed significant improvements over classic Raft in these experiments, with a $5\times$ throughput increase for 10 clusters. In addition to using Fast Raft for intra-cluster and inter-cluster consensus, C-Raft was able to run consensus quickly within clusters where latency was low, then only run slower consensus at the global level between leaders (rather than all sites) periodically. Batching entries for consensus has a large impact when sites are distributed geographically in this manner.

VII. RELATED WORK

As discussed before, the Paxos algorithm [7] is a popular consensus algorithm. Fast Paxos was presented in [9] with the goal of removing a round of messaging from the classic Paxos in the absence of concurrent proposals. A common critique of these algorithms is how much of the implementation is left up to the reader. Some users looked for consensus protocols that were more well-defined, and more ready for implementation. Raft [8] is a consensus protocol that filled this need. Fast Raft takes the same approach to speeding up Raft as Fast Paxos does for Paxos, while maintaining the understandability design philosophy of Raft.

Many Paxos variations have been proposed to improve upon the original algorithm. Generalized Paxos [21] aims to mitigate the shortcomings of Fast Paxos by defining a partial ordering of committed entries. This change allows for concurrent proposals of non-conflicting entries to occur without slowdowns. Egalitarian Paxos [12] notes that the leader creates a potential bottleneck in the system, and proposes an algorithm that does not rely on a fixed stable leader. Flexible

Paxos [22] makes a simple but strong observation that leader election and replication phases can have different quorum sizes, allowing for smaller replication quorum sizes without violating safety. Dynamic Paxos [11] builds on Flexible Paxos, applies it to a geographically distributed setting, and further reduces the leader election quorum size. Applying the improvements of these algorithms to Raft would be an interesting future direction.

Delegator Paxos [13] is a hierarchical form of Paxos. Similar to C-Raft, in Delegator Paxos, Paxos is run in separate clusters for local replication. Consensus inside a cluster runs for a specified k user requests. After this point, a global leader is chosen, and leaders of each cluster take part in Paxos at a global level. Batches of entries are replicated to other clusters. The major difference from classic Paxos is that before a follower can accept a global leader’s proposed batch, it must replicate to a majority inside its cluster to ensure safety in the case of failures. C-Raft takes inspiration from this model to define a hierarchical model with additional benefits of dynamic clusters and faster consensus.

Another relevant Paxos variant is Institutionalised Paxos [23]. This work also considers a system that is divided into clusters. To deal with dynamic cluster membership, the leader keeps track of the number of sites in the cluster and how the quorum size grows/shrinks. It is assumed that the leader does not fail or leave the cluster without handing off the value for the number of sites. This is impractical in many systems where the leader may fail or leave the cluster unexpectedly. Raft and C-Raft, in contrast, deal with membership changes by keeping track of the number of sites through reaching consensus on a configuration.

VIII. CONCLUSION

We presented two new consensus algorithms: Fast Raft, a variation on the Raft consensus algorithm that speeds up consensus in typical operation, and C-Raft, which defines a hierarchical model of Fast Raft consensus. Both algorithms deal with membership changes in dynamic networks. We proved safety for both algorithms and discussed their liveness requirements. Finally, we presented an experimental evaluation of both algorithms in AWS. Our experiments show that Fast Raft can achieve half the latency of classic Raft when message loss is low, and C-Raft can achieve a $5\times$ throughput improvement over classic Raft in a globally distributed scenario. In future work, we plan to explore extending C-Raft to support partially-ordered log entries, similar to Generalized Paxos [21].

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