Enabling a Transactive Distribution System via Real-Time Distributed Optimization

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***Abstract*—Increasing penetration of distributed generators and flexible loads in a distribution power system triggers the need for transactive mechanisms. This paper proposes a transactive mechanism and model that allow noncontrollable resources to trade their power deviations from schedules with elastic ones. The proposed mechanism can reveal the cost of uncertainty and the scarcity of flexibility. The proposed model is optimized in a fully distributed manner in which the welfare of each prosumer is maximized individually through information exchange between neighbors. A distributed neurodynamic algorithm is applied to handle the global constraints on supply-demand balance and limits on branch flows and voltages with guaranteed optimality and fast convergence. A case is studied on a 118-bus distribution system to demonstrate the effectiveness and efficiency of the proposed approach.**

***Index Terms*—Transactive system, distributed generation, distributed optimization, neurodynamics.**

I. Introduction

A. Motivation

DISTRIBUTED generators and intelligent loads are increasingly penetrating distribution power systems (DPSs). Traditionally distributed generators are treated as negative loads and their uncertainties are omitted, whereas the flexibility of intelligent loads is exploited via demand response programs. However, this becomes problematic as the penetration of distributed generators and intelligent loads goes higher. On the one hand, high variability of distributed generators may trigger considerable uncertainties to distribution power flows and the need of fast-ramping reserves [1]. On the other hand, present demand response programs have not fully unlocked the ramping capabilities of flexible resources [2], [3]. Innovative approaches, such as transactive mechanisms, are needed to better coordinate noncontrollable resources and flexible resources [4].

With the increasing penetration of prosumers, i.e., owners of distributed generation and load assets, the traditional centralized DPS management may also become intractable in terms of real-time computation and communication. Plus, due to privacy and autonomy concerns, prosumers may hesitate to share all their personal information with a centralized operation center. A fully-distributed coordination algorithm becomes favorable in this context [5].

The paper aims to address the following problems. How can elastic prosumers be incentivized to absorb noncontrollable prosumers' uncertainties in a transactive manner? How can this be achieved via prosumers' distributed coordination?

B. Related Work

A few pilot programs and studies have been carried out in the field of transactive mechanisms. The Pacific Northwest Smart Grid Demonstration project allowed nodes to exchange their costs and electricity demand, until they reach a consensus on the amount of energy exchange among them [6]. The Olympic Peninsula project allowed distributed energy resources to bid in the wholesale market via retailers [7]. Study [8] proposed a transactive scheme where flexible and less flexible resources can negotiate to reduce demand during a high-price event. Study [9] envisioned a scheme where individual households communicate with neighbors to smooth their aggregate load curves. Study [10] demonstrated a residential transactive scheme that runs a retail electricity market for distributed resources on a distribution feeder every 5 minutes. Study [11] proposed a local trading center who runs a local trading market for end prosumers and provides bidirectional energy trading opportunities.

However, none of the above transactive mechanisms are designed from the perspective of driving flexible distributed resources to absorb the uncertainties of noncontrollable resources. Nor do they generate uncertainty-oriented price signals, which can be used to reveal the cost of uncertainty and the scarcity of flexibility.

In the past few years, distributed optimization algorithms emerged as an efficient framework, especially in smart gird area [12]. Alternating Direction Method of Multipliers

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(ADMM) [13] or its variants are one kind of prevailing methods. Traditional ADMM requires a coordinator to update the dual variable [14]. The proximal message passing technique is one special case of ADMM that optimizes an agent's local objective as well as the mismatch between the agents and neighbors [15]. Some other variant techniques include analytical target cascading [16], auxiliary problem principle [17], optimality condition decomposition [18]. Recent ADMM-based distributed algorithms [19]–[21] can handle both local and global constraints by introducing auxiliary variables. However, the performance of ADMM-like algorithm is generally problem dependent. The step size parameter should be adjusted to balance the convergence rate and accuracy [22]. Continuous-time gradient flow dynamic algorithms also have been widely employed for solving distributed optimization problems, see [23]–[25]. These algorithms transform an optimization problem into a dynamic system, whose equilibria correspond to the Karush-Kuhn-Tucker (KKT) points for the original problem. The neurodynamic approach based on neural networks (NNs), as one kind of gradient-based algorithms, is a promising approach that can be implemented physically in programmable circuits [26] and thus delivers real-time solutions. Recent work has studied on constrained optimization in various contexts [27]–[32]. However, little work on gradient-based distributed optimization has supported more complex formulation, such as local and globally-coupled constraints.

In summary, few studies have proposed distributed algorithms that are fast enough to be implemented in practice and can cope with global constraints such as the power flow constraints in power systems.

C. Contributions

The contributions of this paper include the following:

1) It proposes a distributed transactive DPS mechanism that allows noncontrollable prosumers to trade their power deviations from schedules with elastic ones. This mechanism respects prosumers' autonomy and frees the distribution system operator (DSO) from real-time balancing of supply and demand. The mechanism can also determine the charge on uncertainties and the reward of the ability to absorb uncertainties.
2) It proposes a transactive DPS model where elastic prosumers determine their generation/load adjustments to absorb the uncertainties of noncontrollable prosumers. The model considers constraints on supply-demand balance, power flow, generation and load limits, and voltage magnitudes.
3) It proposes a fully-distributed algorithm based on neurodynamics. With the capability of parallel computation and hardware implementation, the neurodynamic algorithm can ensure fast convergence, real-time solution, and global optimum. The algorithm is distributed in that prosumers only communicate limited information with their neighbors and thus maintain privacy.
4) It gives proof of the convergence and optimality of the proposed algorithm.

D. Organization

The rest of the paper is organized as follows. Section II describes the proposed transactive mechanism. Section III proposes the real-time transactive model. Sections IV and V present the proposed distributed algorithm and analyze its convergence. Section VI gives a case study. Section VII concludes the paper.

II. Proposed Transactive Mechanism

In this paper, it is assumed that every resource needs to follow a generation/consumption schedule that is determined and finalized one day ahead (day-ahead scheduling methods are presented in studies such as [33]). Due to forecast errors, the actual generation/consumption of a noncontrollable resource may deviate from its day-ahead schedule. In this case, spinning reserves would be deployed to offset this deviation and restore the dynamic balance between the supply and demand. As a result, this resource will be charged for deploying reserves. However, this noncontrollable resource can also opt to transact with flexible resources, asking them to help offset the deviation. For example, if a noncontrollable resource generates less or consumes more than its day-ahead schedule, it can buy energy to offset the deviation and avoid deviation charges. Flexible resources can sell energy to profit. If a noncontrollable resource generates more or consumes less than its day-ahead schedule, it can sell energy to offset the deviation. Flexible resources can buy energy at a cheaper price to increase their utility.

This mechanism has the following features.

(i) It makes Pareto improvement to all participating parties. It is assumed that the transactive market is perfectly competitive while market participants have perfect information. Below is an example. Assume that the energy price in the day-ahead market is \$30/MWh and the capacity price is \$20/MWh. A noncontrollable load realizes that its actual usage will be 8MWh more than its awarded energy. Without the transactive mechanism, this load will be charged \$50/MWh for the 8MWh deviation. With the transactive mechanism, however, the noncontrollable load can buy energy from flexible resources to offset the deviation. As long as the clearing price for the transaction is between \$30/MWh and \$50/MWh, both the buyer and sellers can benefit. In Fig. 1, the clearing price is \$42/MWh. The consumer and producer surpluses are positive, as shown in the light blue area and light green area, respectively.

(ii) It respects prosumers' autonomy and frees the DSO from real-time balancing of supply and demand.

(iii) It can also determine the charge on uncertainties and the reward of the ability to absorb uncertainties. These prices are useful for deploying uncontrollable or flexible resources in a distribution system. A low charge on uncertainties can incentivize the deployment of uncontrollable resources, whereas a high reward of the ability to absorb uncertainties can incentivize the deployment of flexible resources.

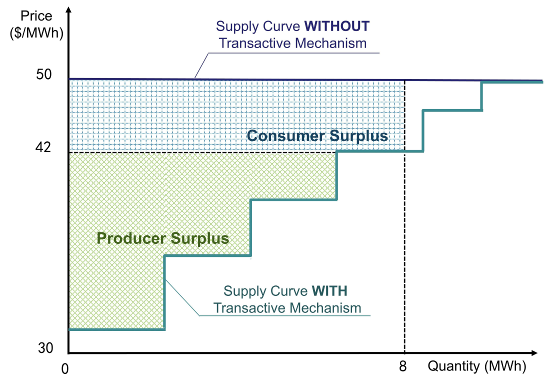

Fig. 1. Illustrative surpluses of buyers and sellers in the transactive mechanism.

(iv) It lowers the required MW quantity of spinning reserve because flexible resources can also serve as reserves when properly rewarded. This can further postpone generation and transmission investment.

III. Real-Time Transactive Model

A. Notations

In this paper, all variables are *Italic* whereas all parameters are upright.

1) Sets:

$\mathrm{S_{bus}}$ Set of buses
$\mathrm{S_{brn}}$ Set of branches
$\mathrm{S_{prs}}$ Set of prosumers
$\mathrm{S_{sell}}, \mathrm{S_{buy}}$ Sets of energy sellers and buyers
$\mathrm{S_{non}}, \mathrm{S_{ela}}$ Sets of noncontrollable and elastic prosumers
$\mathrm{S_{gen}}, \mathrm{S_{ld}}$ Sets of generation and load resources
$\mathrm{S_{sell}} \cup S_{buy} = S_{non} \cup \mathrm{S_{ela}} = S_{gen} \cup S_{ld} = S_{prs}$.
$\mathrm{S_{sell}} \cap S_{buy} = \mathrm{S_{non}} \cap S_{ela} = S_{gen} \cap S_{ld} = \emptyset$.

2) Parameters:

$\pi_{\mathrm{R_{up}}}$ Charge per MWh for reserve ramping up. It is the price of energy plus capacity.
$\pi_{\mathrm{R_{down}}}$ Payment per MW for reserve ramping down. It is the price of energy minus capacity.
$\Delta\mathrm{P_i^d}$ Amount of prosumer i's deviation
$\mathrm{a_i}, \mathrm{b_i}$ Resource-specific positive constants
$\Delta\mathrm{P_i^m}$ Maximum generation/load the elastic prosumer i can adjust
$\Gamma_{\mathrm{ji}}^{\mathrm{PV}}, \Gamma_{\mathrm{ji}}^{\mathrm{QV}}$ Sensitivity of the voltage magnitude of bus j with respect to the P/Q power injection of prosumer i
$\mathrm{V_j^0}$ Scheduled nodal voltage magnitude at bus j
$\mathrm{V^{m+}}, \mathrm{V^{m-}}$ Upper/lower limits of nodal voltage magnitudes
$\Gamma_{\mathrm{li}}^{\mathrm{PI}}, \Gamma_{\mathrm{li}}^{\mathrm{QI}}$ Sensitivity of the current magnitude of branch l with respect to the P/Q power injection of prosumer i
$\mathrm{I_l^0}$ Scheduled current magnitude of branch l
$\mathrm{I_l^{max}}$ Current limit of branch l
χ_i Power factor of prosumer i
$\Delta\mathrm{Q_i^{m+}}$ Upper limit of reactive power adjustment of prosumer i
$\Delta\mathrm{Q_i^{m-}}$ Lower limit of reactive power adjustment of prosumer i.

3) Variables:

ΔP_i Amount of active power purchase/sell of prosumer i with peers. ΔP_i is positive/negative if prosumer i sells/buys energy.
ΔQ_i Adjustment of reactive power injection of prosumer i relative to her schedule. ΔQ_i is positive/negative if prosumer i increases/decreases reactive power injection.
ΔV_j Nodal voltage magnitude deviation at bus j.
ΔI_l Current magnitude deviation of branch l.

4) Functions:

$U(\Delta P_i)$ Change of utility of prosumer i
$C(\Delta P_i)$ Change of cost of prosumer i.

B. Objective Function

All prosumers will self-determine their optimal generation/load adjustments so as to maximize social welfare within the distribution system.

$$\max \sum_{i \in \mathrm{S_{buy}}} U(\Delta P_i) - \sum_{i \in \mathrm{S_{sell}}} C(\Delta P_i) \tag{1}$$

If prosumer i is a noncontrollable resource that generates/consumes less/more than its schedule by $\Delta\mathrm{P_i^d}$ ($\Delta\mathrm{P_i^d} \le 0$) and she does not buy additional energy from other prosumers, fast-ramping reserves would offset her deviation and charge her at a high price $\pi_{\mathrm{R_{up}}}$. Hence, prosumer i is willing to buy energy from peers at a price of at most $\pi_{\mathrm{R_{up}}}$. In another word, $\pi_{\mathrm{R_{up}}}$ is her bid as a buyer. Prosumer i can benefit from the transaction as long as the clearing price is lower than $\pi_{\mathrm{R_{up}}}$.

$$\begin{aligned} &U(\Delta P_i) = \pi_{\mathrm{R_{up}}}(-\Delta P_i) \\ &\Delta\mathrm{P_i^d} \le \Delta P_i \le 0, i \in \mathrm{S_{buy}} \cap \mathrm{S_{non}} \end{aligned} \tag{2}$$

If prosumer i is a noncontrollable resource that generates/consumes more/less than its schedule by $\Delta\mathrm{P_i^d}$ ($\Delta\mathrm{P_i^d} \ge 0$) and she does not sell additional energy to other prosumers, fast-ramping reserves would offset her deviation and pay her at a low price $\pi_{\mathrm{R_{down}}}$. Hence, prosumer i is willing to sell energy to peers at a price of no less than $\pi_{\mathrm{R_{down}}}$.$\pi_{\mathrm{R_{down}}}$ is her per-unit opportunity cost of selling energy to peers and thus serves as her asking price as a seller. Prosumer i can benefit from the transaction as long as the clearing price is higher than $\pi_{\mathrm{R_{down}}}$.

$$\begin{aligned} &C(\Delta P_i) = \pi_{\mathrm{R_{down}}} \Delta P_i \\ &0 \le \Delta P_i \le \Delta\mathrm{P_i^d}, i \in \mathrm{S_{sell}} \cap \mathrm{S_{non}} \end{aligned} \tag{3}$$

In a power undersupply scenario (noncontrollable resources underestimate/overestimate their actual load/generation), if prosumer i is an elastic resource, she can sell energy to absorb others' deviations at the following cost.

$$\begin{aligned} &C(\Delta P_i) = \mathrm{a_i} \Delta P_i^2 + \mathrm{b_i} \Delta P_i \\ &0 \le \Delta P_i \le \Delta\mathrm{P_i^m}, i \in \mathrm{S_{sell}} \cap \mathrm{S_{ela}} \end{aligned} \tag{4}$$

In a power oversupply scenario (noncontrollable resources underestimate/overestimate their actual generation/load), if prosumer i is an elastic resource, she can buy energy to absorb others' deviations for the following utility. Because a flexible resource's utility should increase as it buys energy, $-\mathrm{b_i}/2\mathrm{a_i} < \Delta \mathrm{P_i^m} < 0$ holds for elastic buyers.

$$\begin{aligned} U(\Delta P_i) &= -\mathrm{a_i}\Delta P_i^2 - \mathrm{b_i}\Delta P_i \\ \Delta \mathrm{P_i^m} &\le \Delta P_i \le 0, i \in \mathrm{S_{buy}} \cap \mathrm{S_{ela}}. \end{aligned} \tag{5}$$

C. Constraints

The distribution system in this paper can include loops and are not limited only to radial structure. The constraints include one equality constraint on supply-demand balance, and a set of inequality constraints on branch flows and voltage magnitudes. Because of the limited scale of power deviations of prosumers, we can linearize all these constraints based on the scheduled operation point.

1) Supply-Demand Balance: In the proposed transactive mechanism, the energy sold should equal the energy bought.

$$\sum_{i \in \mathrm{S_{prs}}} \Delta P_i = 0. \tag{6}$$

2) Limits on Branch Flows and Voltages: Deviations of uncontrollable resources and resulting transactions among peers can alter the power flows in a distribution system. While such transactions can increase social welfare, security constraints must be met.

$$\begin{cases} \Delta V_j = \sum_{i \in \mathrm{S_{non}}} \Gamma_{\mathrm{ji}}^{\mathrm{PV}} \Delta \mathrm{P_i^d} + \sum_{i \in \mathrm{S_{ela}}} \Gamma_{\mathrm{ji}}^{\mathrm{PV}} \Delta P_i + \sum_{i \in \mathrm{S_{prs}}} \Gamma_{\mathrm{ji}}^{\mathrm{QV}} \Delta Q_i, \\ \quad -\mathrm{V_j^0} + \mathrm{V^{m-}} \le \Delta V_j \le \mathrm{V^{m+}} - \mathrm{V_j^0},\ j \in \mathrm{S_{bus}} \end{cases} \tag{7}$$

$$\begin{cases} \Delta I_l = \sum_{i \in \mathrm{S_{non}}} \Gamma_{\mathrm{li}}^{\mathrm{PI}} \Delta \mathrm{P_i^d} + \sum_{i \in \mathrm{S_{ela}}} \Gamma_{\mathrm{li}}^{\mathrm{PI}} \Delta P_i + \sum_{i \in \mathrm{S_{prs}}} \Gamma_{\mathrm{li}}^{\mathrm{QI}} \Delta Q_i, \\ \Delta I_l \le \mathrm{I_l^{max}} - \mathrm{I_l^0},\ l \in \mathrm{S_{brn}} \end{cases} \tag{8}$$

Equation (7) describes the voltage change relative to the day-ahead schedule. The change is driven by three terms. The first driver is the active power deviation of noncontrollable resources. It is a constant in the model. The second driver is the active power adjustment of flexible resources as a result of the transactive mechanism. The third driver is the reactive power adjustment, which comes from all prosumers to avoid voltage violations. Note that all sensitivity coefficients are derived at the scheduled operation point.

Equation (8) describes the current change in the distribution system, with a structure analogous to (7).

3) Limits on Resources: Equations (2)-(5) place active power adjustment limits on prosumers, and (9)-(10) place reactive power adjustment limits.

$$\Delta Q_i = \chi_\mathrm{i} \Delta P_i, i \in \mathrm{S_{ld}} \tag{9}$$

$$\Delta \mathrm{Q_i^{m-}} \le \Delta Q_i \le \Delta \mathrm{Q_i^{m+}}, i \in \mathrm{S_{gen}}. \tag{10}$$

D. Market Clearing

By solving (1)-(10), each prosumer obtains her active and reactive power adjustments $(\Delta P_i, \Delta Q_i)$. The above model also yields distribution-system locational marginal prices for power adjustments (DLMPA), at which buyers are charged and sellers get paid. Similar to the transmission-level locational marginal price, the DLMPA on bus j indicates the marginal cost for a 1-MWh electricity usage adjustment at bus j. The formula for the DLMPA is also analogous to that in the transmission side [34]:

$$p_j = u - \sum_{i \in \mathrm{S_{bus}}} \Gamma_{\mathrm{ij}}^{\mathrm{PV}} \big(\lambda_i - \tilde{\lambda}_i\big) - \sum_{l \in \mathrm{S_{brn}}} \Gamma_{\mathrm{lj}}^{\mathrm{PI}} \eta_l \tag{11}$$

where

p_j	DLMPA on bus j
u	Lagrange multiplier of the supply-demand balance constraint. $u > 0$.
λ_i	Lagrange multiplier of the upper voltage constraint on bus i. $\lambda_i \ge 0$.
$\tilde{\lambda}_i$	Lagrange multiplier of the lower voltage constraint on bus i. $\tilde{\lambda}_i \ge 0$.
η_l	Lagrange multiplier of the power flow constraint on branch l. $\eta_l \ge 0$.

Remark 1: All prosumers can benefit by joining the transactive market. The producer surplus for an uncontrollable seller-type prosumer at bus j is $(p_j - \pi_{\mathrm{R_{down}}})\Delta P_i$ (i.e., energy sell revenue $p_j \Delta P_i$ minus cost $\pi_{\mathrm{R_{down}}} \Delta P_i$). The consumer surplus for an uncontrollable buyer-type prosumer at bus j is $(\pi_{\mathrm{R_{up}}} - p_j)(-\Delta P_i)$ (i.e., utility $\pi_{\mathrm{R_{up}}}(-\Delta P_i)$ minus energy purchase cost $p_j(-\Delta P_i)$). Note that $p_j \in [\pi_{\mathrm{R_{down}}}, \pi_{\mathrm{R_{up}}}]$ always holds, because producer/consumer surpluses must be nonnegative so that prosumers are willing to transact. Similarly, the producer/consumer surpluses of elastic prosumers are also nonnegative, otherwise they would choose not to transact.

p_j reflects the charge on uncertainties and the reward on absorbing uncertainties at bus j. Let $\pi_\mathrm{E}^{\mathrm{DA}}$ be the day-ahead energy price in the distribution system. $\pi_\mathrm{E}^{\mathrm{DA}} \in [\pi_{\mathrm{R_{down}}}, \pi_{\mathrm{R_{up}}}]$ holds. If p_j is close to $\pi_\mathrm{E}^{\mathrm{DA}}$, it implies that uncontrollable prosumers can sell/buy power deviations at almost the day-ahead energy price. That is, charges on uncertainties are low, probably due to sufficient flexibility or low penetration of uncontrollable resources in the distribution system. If p_j is close to $\pi_{\mathrm{R_{down}}}$ or $\pi_{\mathrm{R_{up}}}$, it implies that the transactive market does not contribute much to lowing uncertainty charges, probably due to insufficient flexibility or a significant level of uncertainties in the distribution system.

The total power injection adjustment from all elastic prosumers is $\Delta P_i^{\mathrm{ela}} = \sum_{i \in \mathrm{S_{ela}}} \Delta P_i$. Thanks to the transactive mechanism, elastic prosumers are able to eliminate $\Delta P_i^{\mathrm{ela}}$-MW supply-demand imbalance, shouldering part of the spinning reserve burden for the DSO.

IV. Proposed Distributed Algorithm

A. Outline

The proposed distributed algorithm works by letting each prosumer know its DLMPA and thus self-determine its optimal active and reactive power adjustments. Assuming that each prosumer is rational and aims to maximize its producer/consumer surplus, its decision about optimal $(\Delta P_i, \Delta Q_i)$ would be aligned with that determined by a centralized algorithm as long as it obtains its true DLMPA.

In our algorithm, prosumers exchange information until all prosumers reach a consensus about the shadow prices (i.e., Lagrange multipliers) of constraints on supply-demand balance, power flows, and voltages. Specifically, each prosumer communicates with neighbors its estimates on the shadow prices for all constraints and the disagreements between its estimates and neighbors' estimates. A proportional-integral (PI) controller is designed that forces every prosumer's estimates on shadow prices to converge to the true values (detailed in the next subsection). With the consensus of all shadow prices, each prosumer can easily calculate its DLMPA.

The proposed algorithm is fully distributed because each prosumer only needs its personal data (including purchase/sell preferences and operating limits) and shared information (neighbors' estimates of shadow prices). No center is needed to handle all the data or coordinate all the prosumers. Thereby each prosumer has the autonomy and authority to form its own objective and limits, and hence, keeps privacy.

Each prosumer updates its decisions and estimates via a neurodynamic algorithm modeled by a set of differential equations. By designing a circuit to mimic the dynamics of an NN, each prosumer can instantaneously react to input data changes and exchange updated information. It is theoretically proved that the distributed neurodynamic algorithm can converge to a global optimum. Therefore, the algorithm is efficient, scalable and has a plug-and-play feature.

B. Mathematical Formulation

We optimize the welfare objective for each prosumer, each based on an individual NN. Each NN owns a local objective function. By enabling each NN to exchange information with neighbors, the overall welfare across all prosumers can be maximized.

Denoting the objective function (1) and the bound constraints (2)-(5) in a unified form yields:

$$\min \sum_{i\in \mathrm{S_{prs}}} \alpha_i \Delta P_i^2 + \beta_i \Delta P_i,\ \ \Delta P_i \in\ \Omega_{P_i}, \tag{12}$$

where

$$(\alpha_i,\beta_i)=\begin{cases}\alpha_i=0,\beta_i=\pi_{\mathrm{R_{up}}}, & i\in \mathrm{S_{buy}}\cap \mathrm{S_{non}}\\ \alpha_i=0,\beta_i=\pi_{\mathrm{R_{down}}}, & i\in \mathrm{S_{sell}}\cap \mathrm{S_{non}}\\ \alpha_i=\mathrm{a_i},\beta_i=\mathrm{b_i}, & i\in \mathrm{S_{sell}}\cap \mathrm{S_{ela}}\\ \alpha_i=\mathrm{a_i},\beta_i=\mathrm{b_i}, & i\in \mathrm{S_{buy}}\cap \mathrm{S_{ela}}\end{cases}$$

$$\Omega_{P_i}=\left\{\Delta P_i \left| \begin{array}{ll}\Delta \mathrm{P_i^d}\le \Delta P_i\le 0, & i\in \mathrm{S_{buy}}\cap \mathrm{S_{non}}\\ 0\le \Delta P_i \le \Delta\mathrm{P_i^d}, & i\in \mathrm{S_{sell}}\cap \mathrm{S_{non}}\\ 0\le \Delta P_i\le \Delta \mathrm{P_i^m}, & i\in \mathrm{S_{sell}}\cap \mathrm{S_{ela}}\\ \Delta\mathrm{P_i^m}\le \Delta P_i\le 0, & i\in \mathrm{S_{buy}}\cap \mathrm{S_{ela}}\end{array}\right.\right\} \tag{13}$$

Let $\Omega_z := \{z\in \Re^n : l_k \le z_k \le h_k, k=1,\ldots,n\}$. P_{Ω_z} is a projection operator defined as

$$P_{\Omega_z}(z_k)=\begin{cases} l_k, & z_k<l_k;\\ z_k, & l_k\le z_k\le h_k;\\ h_k, & z_k>h_k.\end{cases} \tag{14}$$

The following inequality holds for the projection operator P_{Ω_z} on a convex Ω_z as follows [35]:

$$\left(P_{\Omega_z}(z)-z\right)^T\left(y-P_{\Omega_z}(z)\right)\ge 0,\ \ \forall y\in\Omega_z. \tag{15}$$

According to the KKT conditions for convex objective (12) with constraints (6)-(10), and based on the projection theorem [35], [36], optimal solutions satisfy the following set of equations:

$$\begin{aligned}&-2\alpha_i\Delta P_i-\beta_i+\gamma_i\chi_i+u-\sum_{j\in \mathrm{S_{bus}}}\Gamma_{\mathrm{ji}}^{\mathrm{PV}}\left(\lambda_j-\tilde{\lambda}_j\right)\\ &-\sum_{l\in \mathrm{S_{brn}}}\Gamma_{\mathrm{li}}^{\mathrm{PI}}\eta_l\in N_{\Omega_{P_i}},\ \text{ if } i\in \mathrm{S_{ela}}\\ &-2\alpha_i\Delta P_i-\beta_i+\gamma_i\chi_i+u\in N_{\Omega_{P_i}},\ \text{ if } i\in \mathrm{S_{non}}\\ &-\gamma_i-\sum_{j\in \mathrm{S_{bus}}}\Gamma_{\mathrm{ji}}^{\mathrm{QV}}\left(\lambda_j-\tilde{\lambda}_j\right)-\sum_{l\in \mathrm{S_{brn}}}\Gamma_{\mathrm{li}}^{\mathrm{QI}}\eta_l\in N_{\Omega_{Q_i}}\end{aligned} \tag{16}$$

where $N_{\Omega_{P_i}}$ and $N_{\Omega_{Q_i}}$ are the normal cones of Ω_{P_i} and Ω_{Q_i} at the optimal ΔP_i and ΔQ_i.

Equation (16) holds if and only if the following statements hold:

$$\begin{aligned}&P_{\Omega_{P_i}}\left[\Delta P_i-\left(2\alpha_i\Delta P_i+\beta_i-\gamma_i\chi_i-u+\sum_{j\in \mathrm{S_{bus}}}\Gamma_{\mathrm{ji}}^{\mathrm{PV}}\left(\lambda_j-\widetilde{\lambda}_j\right)\right.\right.\\ &\qquad\left.\left.+\sum_{l\in \mathrm{S_{brn}}}\Gamma_{\mathrm{li}}^{\mathrm{PI}}\eta_l\right)\right]=\Delta P_i, \text{if } i\in \mathrm{S_{ela}}\\ &P_{\Omega_{P_i}}\left[\Delta P_i-(2\alpha_i\Delta P_i+\beta_i-\gamma_i\chi_i-u)\right]=\Delta P_i, \text{if } i\in \mathrm{S_{non}}\\ &P_{\Omega_{Q_i}}\left[\Delta Q_i-\left(\gamma_i+\sum_{j\in \mathrm{S_{bus}}}\Gamma_{\mathrm{ji}}^{\mathrm{QV}}\lambda_j+\sum_{l\in \mathrm{S_{brn}}}\Gamma_{\mathrm{li}}^{\mathrm{QI}}\eta_l\right)\right]=\Delta Q_i\\ &\lambda_j=\left(\lambda_j+\Delta V_j-\left(\mathrm{V^{m+}}-\mathrm{V_j^0}\right)\right)^+\\ &\tilde{\lambda}_j=\left(\tilde{\lambda}_j-\Delta V_j-\left(\mathrm{V_j^0}-\mathrm{V^{m-}}\right)\right)^+\\ &\eta_l=\left(\eta_l+\Delta I_l-\left(\mathrm{I_l^{max}}-\mathrm{I_l^0}\right)\right)^+\\ &\Delta Q_i=\chi_\mathrm{i}\Delta P_i, \text{if } i\in \mathrm{S_{ld}};\ \ \gamma_i=0, \text{if } i\in \mathrm{S_{gen}};\\ &\sum_{i\in \mathrm{S_{prs}}}\Delta P_i=0\end{aligned} \tag{17}$$

where

u,λ_j,$\tilde{\lambda}_j$,η_l	Lagrange multipliers for (6), (7), and (8), same with those in Section III-C.
γ_i	Lagrange multiplier for (9).
i,j,l	$i\in \mathrm{S_{prs}}$, $j\in \mathrm{S_{bus}}$, and $l\in \mathrm{S_{brn}}$ apply to all equations unless specified otherwise.
Ω_{Q_i}	$\{z_i \mid \Delta\mathrm{Q_i^{m-}}\le z_i\le \Delta\mathrm{Q_i^{m+}}\}$, $i\in \mathrm{S_{gen}}, z_i\in\Re$, $i\in \mathrm{S_{ld}}$.

We can then construct a distributed neurodynamics-based algorithm as follows to find the KKT equilibrium in (17).

$$\begin{aligned}\epsilon\frac{d}{dt}\Delta P_i=P_{\Omega_{P_i}}&\left[-2\alpha_i\Delta P_i-\beta_i+\chi_i\gamma_i+u_i-\sum_{l\in \mathrm{S_{brn}}}\Gamma_{\mathrm{li}}^{\mathrm{PI}}\eta_{li}\right.\\ &\left.-\sum_{j\in \mathrm{S_{bus}}}\Gamma_{\mathrm{ji}}^{\mathrm{PV}}\left(\lambda_{ji}-\widetilde{\lambda}_{ji}\right)\right]-\Delta P_i, \text{if } i\in \mathrm{S_{ela}}\end{aligned} \tag{18a}$$

$$\epsilon\frac{d}{dt}\Delta P_i=P_{\Omega_{P_i}}\left[-2\alpha_i\Delta P_i-\beta_i+\chi_i\gamma_i\ +u_i\right]-\Delta P_i,\ \text{ if } i\in \mathrm{S_{non}} \tag{18b}$$

$$\epsilon \frac{d}{dt}\Delta Q_i = P_{\Omega_{Q_i}}\left[\Delta Q_i - \sum_{j\in \mathrm{S_{bus}}} \Gamma^{\mathrm{QV}}_{\mathrm{ji}}\left(\lambda_{ji} - \widetilde{\lambda}_{ji}\right) - \sum_{l\in \mathrm{S_{brn}}} \Gamma^{\mathrm{QI}}_{\mathrm{li}}\eta_{li} - \gamma_i\right] - \Delta Q_i \tag{18c}$$

$$\epsilon \frac{d}{dt}\gamma_i = \Delta Q_i - \chi_{\mathrm{i}}\Delta P_i, \text{ if } i \in \mathrm{S_{ld}};\ \gamma_i = 0, \text{ if } i \in \mathrm{S_{gen}} \tag{18d}$$

$$\epsilon \frac{d}{dt}u_i = -\delta(u_i) - \delta(v_i) - \Delta P_i - \Delta \dot{P}_i \tag{18e}$$

$$\epsilon \frac{d}{dt}\lambda_{ji} = \left(\Delta V_{ji} - \frac{1}{\mathrm{m}}\left(\mathrm{V}^{\mathrm{m}+} - \mathrm{V}^0_{\mathrm{j}}\right) + \lambda_{ji} - \delta\left(\mu_{ji}\right) - \delta\left(\lambda_{ji}\right)\right)^+ - \lambda_{ji} + \Delta \dot{V}_{ji} \tag{18f}$$

$$\epsilon \frac{d}{dt}\widetilde{\lambda}_{ji} = \left(\frac{1}{\mathrm{m}}\left(\mathrm{V}^0_{\mathrm{j}} - \mathrm{V}^{\mathrm{m}-}\right) - \Delta \mathrm{V_{ji}} + \widetilde{\lambda}_{\mathrm{ji}} - \delta\left(\widetilde{\mu}_{\mathrm{ji}}\right) - \delta\left(\widetilde{\lambda}_{\mathrm{ji}}\right)\right)^+ - \widetilde{\lambda}_{ji} - \Delta \dot{V}_{ji} \tag{18g}$$

$$\epsilon \frac{d}{dt}\eta_{li} = \left(\Delta I_{li} - \frac{1}{\mathrm{m}}\left(\mathrm{I}^{\max}_{\mathrm{l}} - \mathrm{I}^0_{\mathrm{l}}\right) + \eta_{li} - \delta(\omega_{li}) - \delta(\eta_{li})\right)^+ - \eta_{li} + \Delta \dot{I}_{li} \tag{18h}$$

$$\epsilon \frac{d}{dt}v_i = \delta(u_i),\ \epsilon \frac{d}{dt}\omega_{li} = \delta(\eta_{li}), \tag{18i}$$

$$\epsilon \frac{d}{dt}\mu_{ji} = \delta\left(\lambda_{ji}\right),\ \epsilon \frac{d}{dt}\widetilde{\mu}_{ji} = \delta\left(\widetilde{\lambda}_{ji}\right) \tag{18j}$$

where

$(\cdot)^+$	$(z)^+ = \max\{z, 0\}$
ϵ	Scaling parameter $\epsilon > 0$
$\mathrm{N_i}$	Set of prosumer i's neighbors.
m	Number of prosumers
$\delta(\cdot)$	$\delta(s_i)$: $\sum_{k\in N_i}(s_i - s_k)$
$u_i, \lambda_{ji}, \widetilde{\lambda}_{ji}, \eta_{li}$	Local estimates for Lagrange multipliers by prosumer i for (6)-(8), $i \in N_i$
$v_i, \mu_{ji}, \tilde{\mu}_{ji}, \omega_{li}$	Estimates for the summations of the disagreements between $u_i, \lambda_{ji}, \tilde{\lambda}_{ji}, \eta_{li}$, $i \in N_i$.
γ_i	Estimates for local Lagrange multipliers by prosumer i for (9) when $i \in \mathrm{S_{ld}}$
ΔV_{ji}	$\begin{cases} \Gamma^{\mathrm{PV}}_{\mathrm{ji}}\Delta P_i + \Gamma^{\mathrm{QV}}_{\mathrm{ji}}\Delta Q_i, & i \in \mathrm{S_{ela}} \\ \Gamma^{\mathrm{PV}}_{\mathrm{ji}}\Delta \mathrm{P^d_i} + \Gamma^{\mathrm{QV}}_{\mathrm{ji}}\Delta Q_i, & i \in \mathrm{S_{non}} \end{cases}$
ΔI_{li}	$\begin{cases} \Gamma^{\mathrm{PI}}_{\mathrm{li}}\Delta P_i + \Gamma^{\mathrm{QI}}_{\mathrm{li}}\Delta Q_i, & i \in \mathrm{S_{ela}} \\ \Gamma^{\mathrm{PI}}_{\mathrm{li}}\Delta \mathrm{P^d_i} + \Gamma^{\mathrm{QI}}_{\mathrm{li}}\Delta Q_i, & i \in \mathrm{S_{non}} \end{cases}$

The intuition behind (18) is as follows. According to (18d) γ_i ($i \in \mathrm{S_{ld}}$) will converge to a stable state when (9) holds. By substituting $v_i = \sum_{k\in N_i}\int(u_i - u_k)$ into (18e), one can see that $(u_i - u_k)$, $\mathrm{k} \in \mathrm{N_i}$ is driven towards zero by a PI controller, with $\sum_{k\in N_i}(u_i - u_k)$ serving as the proportional controller and $\sum_{k\in N_i}\int(u_i - u_k)$ serving as the integral controller. Similarly, $(\lambda_{ji} - \lambda_{jk})$, $(\tilde{\lambda}_{ji} - \tilde{\lambda}_{jk})$ and $(\eta_{li} - \eta_{lk})$, $\mathrm{k} \in \mathrm{N_i}$ would converge to zero.

Therefore, with the help of v_i, μ_{ji}, $\tilde{\mu}_{ji}$, and ω_{li} that sum up the disagreements between u_i, λ_{ji}, $\tilde{\lambda}_{ji}$, and η_{li}, all the local $u_i, \lambda_{ji}, \tilde{\lambda}_{ji}, \eta_{li}$ reach consensuses and converge to the global Lagrange multipliers u, λ_j, $\tilde{\lambda}_j$, and η_l of constraints (6)-(8). The right part of (18a), (18b) and (18c) will also converge to zero. Per KKT conditions (17), ΔP_i and ΔQ_i will converge to the optimal solution to problem (1).

The internal design of the proposed algorithm (18) is depicted in Fig. 2. An NN in (18) is designed for each prosumer. Through parallel computation including addition,

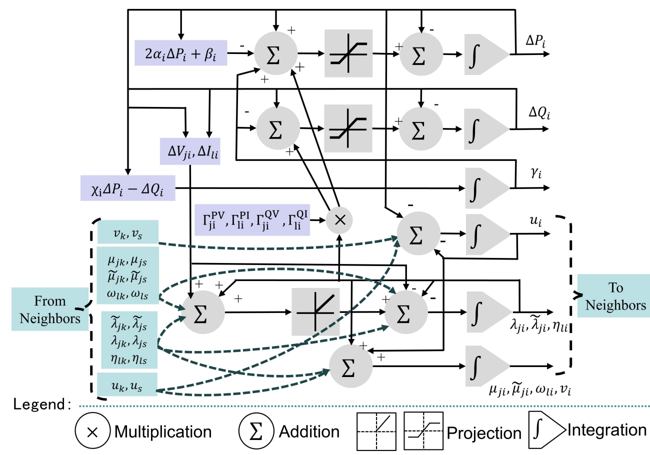

Fig. 2. Internal design of NN i.

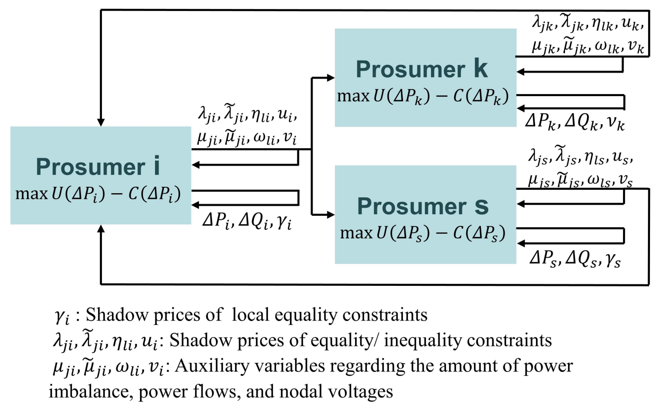

Fig. 3. Information exchange between prosumers.

projection, and integration, the proposed recurrent NNs can serve for real-time optimization.

Fig. 3 shows the information exchange of prosumer i with neighbors k and s. Each NN optimizes its own objective regarding α_i and β_i, updates its local information ΔP_i, ΔQ_i, and γ_i within each NN. Meanwhile, they exchange the local estimates on global shadow prices $u_i, \lambda_{ji}, \tilde{\lambda}_{ji}, \eta_{li}$, and summations of the disagreements $v_i, \mu_{ji}, \tilde{\mu}_{ji}, \omega_{li}$ with neighbors, which guarantees the privacy of each local neural network.

V. Convergence Analysis

In this section, two main theorems will be introduced. In Theorem 1, we will explain that the equilibrium states of (18) are the same as the solution of KKT conditions (17). In Theorem 2, we further prove that from any initial states in Ω, (18) will converge to the equilibrium states, i.e., the optimal ΔP_i and ΔQ_i, and stays thereafter.

Theorem 1: ΔP_i and ΔQ_i are optimal solutions to the problem (1) if and only if there exist $(\Delta P_i, \Delta Q_i, \gamma_i, u_i, \lambda_{ji}, \tilde{\lambda}_{ji}, \eta_{li}, v_i, \mu_{ji}, \tilde{\mu}_{ji}, \omega_{li})$ such that they are the equilibrium states of (18).

Proof: See Appendix A. ■

TABLE I
PARAMETERS OF NONELASTIC RESOURCES IN THE 118-BUS SYSTEM

Type	Bus	ΔP_i^d(MWh)	$\Delta Q_i^{m\pm}$ (MVarh)	χ_i
$S_{buy} \cap S_{ld}$	74	-0.059485	/	0.403
$S_{buy} \cap S_{gen}$	79	-0.019239	±0.024486	/
	116	-0.091803	±0.17971	/
$S_{sell} \cap S_{gen}$	53	0.08103	±0.189332	/

TABLE II
PARAMETERS OF ELASTIC RESOURCES IN THE 118-BUS SYSTEM

Type	Bus	a_i	b_i	ΔP_i^m(MWh)	$\Delta Q_i^{m\pm}$(MVarh)	χ_i
$S_{buy} \cap S_{ld}$	20	4	52.67	-0.08194	/	0.6432
$S_{sell} \cap S_{ld}$	76	4.2	52.67	0.00790	/	0.4266
$S_{sell} \cap S_{gen}$	44	3.8	52.67	0.08044	±0.04805	/
	73	3.6	52.67	0.07012	±0.07903	/

Let Ω be $\Omega_{P_i, i \in S_{prs}} \times \Omega_{Q_i,\ i \in S_{prs}}$, and define a vector z by stacking all the variables in (18) for $i \in S_{prs}$, $j \in S_{bus}$, $l \in S_{brn}$. We have the following lemma.

Lemma 1: Let $z(t)$ be the state trajectory of NN (18) with initial point $z(t_0)$. If $z(t_0) \in \Omega$, then for all $t \geq t_0$, $z(t) \in \Omega$.

Proof: See Appendix B. ∎

We now show the asymptotic convergence of the proposed distributed NNs to the equilibrium.

Theorem 2: The proposed distributed NNs (18) with initial points such that $\Delta P_i(t_0) \in \Omega_{P_i, i \in S_{prs}}$, $\Delta Q_i(t_0) \in \Omega_{Q_i, i \in S_{prs}}$ are stable in the sense of Lyapunov and converge to the optimal ΔP_i and ΔQ_i.

Proof: See Appendix C. ∎

Remark 2: The proposed neurodynamic model (18) can also be reformulated into a discretized form as follows:

$$z(t+1) = z(t) + \mathbf{s}\mathbf{H}(z(\mathbf{t})) \tag{19}$$

where $\mathbf{s}$ is the step size. By choosing a small enough step size $\mathbf{s}$, the stability and convergence of the discrete-time system in (19) can be preserved.

The proposed discretized approach (19) is also capable of solving the proposed real-time transactive model.

VI. SIMULATION RESULTS

A. Data

Simulation is carried out on a 118-bus distribution system [37]. Some distributed generators and flexible loads are added into the system as shown in Tables I and II. We assume that the asking/bidding prices from flexible resources in the transactive market are higher/lower than the day-ahead energy price, so that flexible resources can profit in the joint day-ahead-transactive market.

The day-ahead energy price is \$52.67/MWh. The capacity prices of both up and down reserves are \$18.21/MWh. Therefore π_{Rdown}=\$34.46/MWh and π_{Rup}=\$70.88/MWh.

B. Results of the Transactive Mechanism

In Table III, the first and second columns record the type and location of each prosumer who has participated in the transactive market. The third column records the quantity of electricity sell (as a positive number) and purchase (as a negative number) for each prosumer. The fourth column shows DLMPAs, which vary across buses because constraints on branches 73-74 and 77-78 are binding. The DLMPAs also price the uncertainty and flexibility in the distribution system. The DLMPAs at most buses are close to the day-ahead energy price, implying that the distribution system has abundant flexible resources to absorb uncertainties. The DLMPA at bus 74, however, are equal to π_{Rup}, implying that flexibility is scarce at this location.

The fifth column shows the charges (as a negative number) or payments (as a positive number) to the noncontrollable resources without the proposed transactive mechanism. In this case, the generation/load deviation of noncontrollable resources needs to be offset by spinning reserves and is settled at the price of π_{Rdown} or π_{Rup}. Per the sixth column, the noncontrollable resources are better off with the transactive mechanism where they are charged less or paid more. The flexible resources also profit from the transactive mechanism where their producer/consumer surpluses are positive.

By comparing ΔP_i and ΔP_i^d for noncontrollable resources, one can also see that the deviation of noncontrollable resources has mostly been offset by flexible resources. Specifically speaking, 0.0784-MWh reserves can be saved by the transactive mechanism.

TABLE III
RESULTS OF THE TRANSACTIVE MECHANISM (TM)

Type	Bus	ΔP_i (MWh)	DLMPA (\$/MWh)	Welfare w/o TM (\$)	Welfare w/ TM (\$)
$S_{non} \cap S_{buy} \cap S_{ld}$	74	-0.0483	70.88	-4.216	-4.091
$S_{non} \cap S_{buy} \cap S_{gen}$	79	-0.0192	53.31	-1.364	-1.026
$S_{non} \cap S_{buy} \cap S_{gen}$	116	-0.0918	52.67	-6.507	-4.836
$S_{non} \cap S_{sell} \cap S_{gen}$	53	0.08103	52.67	2.792	4.268
$S_{ela} \cap S_{buy} \cap S_{ld}$	20	0	52.67	0	0
$S_{ela} \cap S_{sell} \cap S_{ld}$	76	0.0079	68.27	0	0.1230
$S_{ela} \cap S_{sell} \cap S_{gen}$	44	0.0004	52.67	0	0
$S_{ela} \cap S_{sell} \cap S_{gen}$	73	0.0701	53.25	0	0.0232

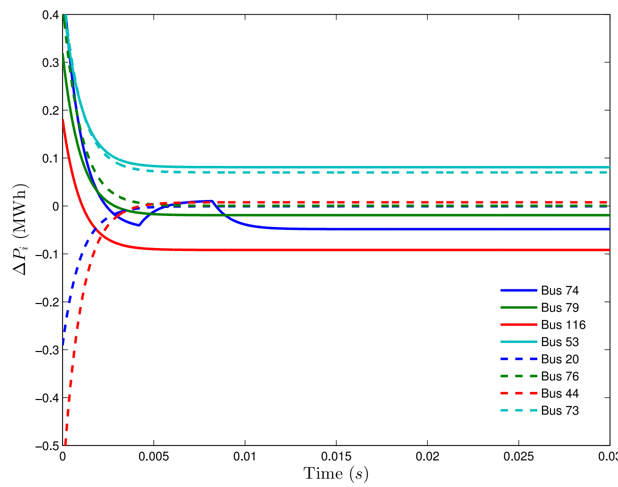

Fig. 4. Transient behaviors of the quantity of electricity purchase/sell ΔP_i ($i \in S_{prs}$).

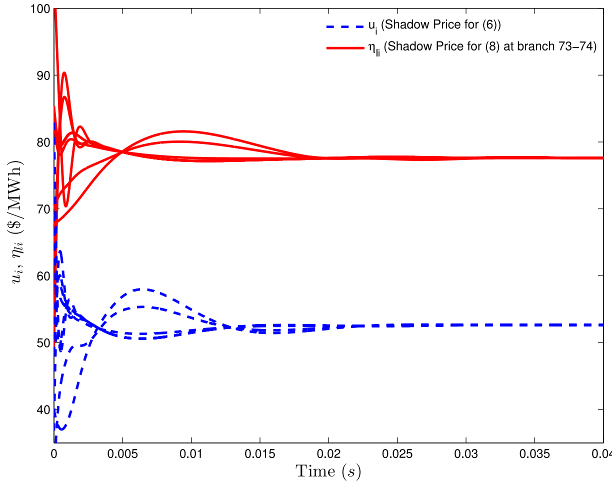

Fig. 5. Consensus behaviors of shadow prices u_i ($i \in \mathrm{S_{prs}}$) for supply-demand balance (6) and shadow prices η_{li} ($i \in \mathrm{S_{prs}}$) for power flow limits (8) on branch 73-74.

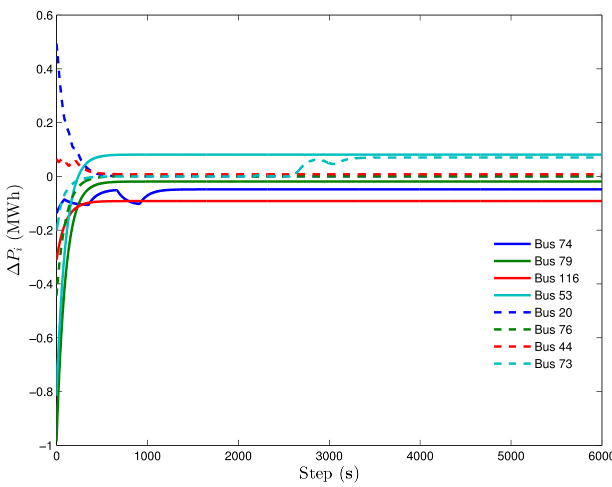

Fig. 6. Transient behaviors of the quantity of electricity purchase/sell ΔP_i ($i \in \mathrm{S_{prs}}$) by the discretized algorithm.

C. Performance of the Proposed Algorithm

We then test performance of the proposed algorithm. The convergence time for distributed NNs (18) is proportional to ϵ. According to circuit design, $\epsilon = 10^{-5} \sim 10^{-8}$ is achievable. By letting $\epsilon = 10^{-5}$, Fig. 4 depicts the transient behaviors of the quantity of electricity purchase/sell ΔP_i for each prosumer from any random initial states in Ω. It only takes the distributed NNs 0.03 seconds to yield the optimal solution, which is the same with the optimum derived by a centralized solver.

Fig. 5 illustrates the movement of each prosumer's estimates on shadow prices u_i (for supply-demand balance constraint (6)) and η_{li} (for power flow constraints (8) on branch 73-74). The local shadow prices $u_i, \lambda_{ji}, \tilde{\lambda}_{ji}, \eta_{li}$ from each prosumer i will reach consensuses and then converge to stable states in about 0.03 seconds, which substantiates real-time solvability of our proposed approach.

Given that most existing distributed algorithms take discretized forms, we also test the performance using the discretized algorithm in (19), Remark 2. The step size in (19) is set as $\mathbf{s} = \mathbf{0.01}$. Fig. 6 depicts the transient behaviors of the quantity of electricity purchase/sell in a discretized way. The discretized algorithm takes about four thousand steps to reach the optimal solution. One can see that the discretized approach is also effective but slow compared to the neurodynamic model.

VII. Conclusion

This paper proposes a transactive mechanism and model that allow elastic prosumers to determine their generation/load adjustments to absorb the uncertainties of noncontrollable prosumers. The proposed approach can make Pareto improvement to all participants, free the DSO from real-time balancing of supply and demand, reveal the cost of uncertainty and the scarcity of flexibility, and reduce system-wide spinning reserve requirements.

The transactive mechanism is solved by a fully-distributed neurodynamic algorithm. Through limited communications with neighbors only, each prosumer, represented by a neural network, is optimized individually to maximize its own welfare while capturing globally-coupled constraints involving all prosumers. The neurodyanmic algorithm is proved to converge to global optimum in real-time via hardware implementation.

Regarding future work, one may develop communication-delay-tolerant algorithms, design distributed algorithms in the context of non-cooperative prosumers, develop bidding strategies for flexible resources, or develop transactive mechanisms that allow flexible resources to provide other ancillary services.

Appendix A
Proof of Theorem 1

(i) On the one hand, the equilibrium states of (18) satisfy

$$\frac{d}{dt}\Big(\Delta P_i, \Delta Q_i, \gamma_i, u_i, \lambda_{ji}, \tilde{\lambda}_{ji}, \eta_{li}, v_i, \mu_{ji}, \tilde{\mu}_{ji}, \omega_{li}\Big) = 0, \;\; i \in \mathrm{S_{prs}}.$$

According to (18j), $\sum_{k\in N_i}(u_i - u_k) = 0$, $\sum_{k\in N_i}(\lambda_{ji} - \lambda_{jk}) = 0$, $\sum_{k\in N_i}(\tilde{\lambda}_{ji} - \tilde{\lambda}_{jk}) \in 0$, $\sum_{k\in N_j}(\eta_{li} - \eta_{lk}) = 0$. For any branch l and bus j, every $u_i, \eta_{li}, \lambda_{ji}, \tilde{\lambda}_{ji}$ reaches a consensus at every prosumer i.

Summing up (18e) for all the prosumers at the equilibrium states yields:

$$\sum_{i\in \mathrm{S_{prs}}}\left[\sum_{k\in N_i}(u_i - u_k + v_i - v_k) - \Delta P_i - \Delta \dot{P}_i\right] = 0 \tag{20}$$

Because $\Delta \dot{P}_i = 0$, $\sum_{i\in \mathrm{S_{prs}}}\sum_{k\in N_i}(u_i - u_k)$ and $\sum_{i\in \mathrm{S_{prs}}}\sum_{k\in N_i}(v_i - v_k)$ are zero, we have $\sum_{i\in \mathrm{S_{prs}}}\Delta P_i = 0$. In addition, as $\Delta \dot{V}_{ji} = 0$, (18f) for all the prosumers is equivalent to:

$$\Delta V_{ji} - \delta\big(\mu_{ji}\big) - \delta\big(\lambda_{ji}\big) - \frac{1}{\mathrm{m}}\Big(\mathrm{V}^{\mathrm{m}+} - \mathrm{V}_{\mathrm{j}}^{0}\Big) \le 0, \lambda_{ji} \ge 0$$

$$\lambda_{ji}\bigg(\Delta V_{ji} - \delta\big(\mu_{ji}\big) - \delta\big(\lambda_{ji}\big) - \frac{1}{\mathrm{m}}\Big(\mathrm{V}^{\mathrm{m}+} - \mathrm{V}_{\mathrm{j}}^{0}\Big)\bigg) = 0$$

Summing up (18f) for all the prosumers at all these three conditions yields:

$$\sum_{i\in \mathrm{S_{non}}} \Gamma_{\mathrm{ji}}^{\mathrm{PV}} \Delta \mathrm{P}_{\mathrm{i}}^{\mathrm{d}} + \sum_{i\in \mathrm{S_{ela}}} \Gamma_{\mathrm{ji}}^{\mathrm{PV}} \Delta P_i + \sum_{i\in \mathrm{S_{prs}}} \Gamma_{\mathrm{ji}}^{\mathrm{QV}} \Delta Q_i \leq \mathrm{V}^{\mathrm{m+}} - \mathrm{V}_{\mathrm{j}}^0$$

Similarly, by summing up (18g) (18h) for all the prosumers, we have

$$\sum_{i\in \mathrm{S_{non}}} \Gamma_{\mathrm{ji}}^{\mathrm{PV}} \Delta \mathrm{P}_{\mathrm{i}}^{\mathrm{d}} + \sum_{i\in \mathrm{S_{ela}}} \Gamma_{\mathrm{ji}}^{\mathrm{PV}} \Delta P_i + \sum_{i\in \mathrm{S_{prs}}} \Gamma_{\mathrm{ji}}^{\mathrm{QV}} \Delta Q_i \geq \mathrm{V}^{\mathrm{m-}} - \mathrm{V}_{\mathrm{j}}^0$$
$$\sum_{i\in \mathrm{S_{non}}} \Gamma_{\mathrm{li}}^{\mathrm{PI}} \Delta \mathrm{P}_{\mathrm{i}}^{\mathrm{d}} + \sum_{i\in \mathrm{S_{ela}}} \Gamma_{\mathrm{li}}^{\mathrm{PI}} \Delta P_i + \sum_{i\in \mathrm{S_{prs}}} \Gamma_{\mathrm{li}}^{\mathrm{QI}} \Delta Q_i \leq \mathrm{I}_{\mathrm{l}}^{\max} - \mathrm{I}_{\mathrm{l}}^0.$$

Denote u, η_l, λ_j, $\tilde{\lambda}_j$ as the consensus states of u_i, η_{li}, λ_{ji}, $\tilde{\lambda}_{ji}$. These equilibrium states satisfy the KKT conditions (17).

(ii) On the other hand, suppose that ΔP_i and ΔQ_i are the optimal solutions of (1). Note that there exist u, η_l, λ_j, $\tilde{\lambda}_j$, and γ_i such that (17) holds. Let $u_i = u$, $\eta_{li} = \eta_l$, $\lambda_{ji} = \lambda_j$, $\tilde{\lambda}_{ji} = \tilde{\lambda}_j$ for every prosumer i, and then (18i), (18j) hold.

We stack up all the (18e) for all the prosumers as

$$\boldsymbol{L}\big[(u_1, u_2, \ldots, u_m)^T + (v_1, v_2, \ldots, v_m)^T\big] = -(\Delta P_1, \ldots, \Delta P_m)^T,$$

where $\boldsymbol{L}$ is a Laplacian matrix, which is defined as the difference of the graph degree matrix and adjacency matrix [38]. When the graph is undirected and connected, $\boldsymbol{L}$ satisfies $\boldsymbol{1^T L = 0}$. In addition, note that $\boldsymbol{1^T}\Delta P_i = 0$. According to orthogonal decomposition [39], there exists v_i such that (18e) holds. Similarly, there exist μ_{ji}, $\tilde{\mu}_{ji}$, and ω_{li} such that (18f), (18g) and (18h) hold.

To sum up, if and only if ΔP_i and ΔQ_i are solutions to problem (1), there exist $(\gamma_i, u_i, \lambda_{ji}, \tilde{\lambda}_{ji}, \eta_{li}, v_i, \mu_{ji}, \tilde{\mu}_{ji}, \omega_{li})$ such that $(\Delta P_i, \Delta Q_i, \gamma_i, u_i, \lambda_{ji}, \tilde{\lambda}_{ji}, \eta_{li}, v_i, \mu_{ji}, \tilde{\mu}_{ji}, \omega_{li})$ are the equilibria of (18).

Appendix B
Proof of Lemma 1

Note that the right terms of (18) are locally Lipschitz continuous. According to the local existence theorem of differential inclusions [40], there exists a continuous solution $z(t)$ of (18) for $[t_0, T)$. Moreover, based on Nagumo's theorem [41] and the proof in [42], $\forall t \geq t_0$, $z(t) \in \Omega$.

Appendix C
Proof of Theorem 2

Define a vector z by stacking all variables in (18) for $i \in \mathrm{S_{prs}}$, $j \in \mathrm{S_{bus}}$, $l \in \mathrm{S_{brn}}$. Define the vector function $H(z)$ as the right part by stacking all the variables. Equation (18) of all the prosumers then can be rewritten as

$$\epsilon \frac{dz}{dt} = H(z). \tag{21}$$

Denote $\{\Delta P_i^*, \Delta Q_i^*, \gamma_i^*, u_i^*, \lambda_{ji}^*, \tilde{\lambda}_{ji}^*, \eta_{li}^*, v_i^*, \mu_{ji}^*, \tilde{\mu}_{ji}^*, \omega_{li}^*, \quad i \in \mathrm{S_{prs}}\}$ as the optimal solutions satisfying that

$$\frac{d}{dt}\Big(\Delta P_i^*, \Delta Q_i^*, \gamma_i^*, u_i^*, \lambda_{ji}^*, \tilde{\lambda}_{ji}^*, \eta_{li}^*, v_i^*, \mu_{ji}^*, \tilde{\mu}_{ji}^*, \omega_{li}^*\Big) = 0, .$$

Define z^* by stacking all optimal states $\{\Delta P_i^*, \Delta Q_i^*, \gamma_i^*, u_i^*, \lambda_{ji}^*, \tilde{\lambda}_{ji}^*, \eta_{li}^*, v_i^*, \mu_{ji}^*, \tilde{\mu}_{ji}^*, \omega_{li}^*\}$.

Inspired by the proof in [27], [43], and [44], we define

$$\begin{aligned}\phi(z) = &\sum_{i\in S_{\mathrm{prs}}} \Big(\alpha_i \Delta P_i^2 + \beta_i \Delta P_i\Big)\\
&+ \sum_{i\in S_{\mathrm{prs}}} \Bigg\{ \sum_{j\in S_{\mathrm{bus}}} \Bigg[\lambda_{ji} - \Big(\Delta V_{ji} - \frac{1}{\mathrm{m}}\Big(\mathrm{V}^{\mathrm{m+}} - \mathrm{V}_{\mathrm{j}}^0\Big)\Big)^+\Bigg]^2\\
&+ \sum_{j\in S_{\mathrm{bus}}} \Bigg[\tilde{\lambda}_{ji} - \Big(-\Delta V_{ji} + \frac{1}{\mathrm{m}}\Big(\mathrm{V}^{\mathrm{m-}} - \mathrm{V}_{\mathrm{j}}^0\Big)\Big)^+\Bigg]^2\\
&+ \sum_{l\in S_{\mathrm{brn}}} \Bigg[\eta_{li} - \Big(\Delta I_{li} - \frac{1}{\mathrm{m}}\Big(\mathrm{I}_{\mathrm{l}}^{\max} - \mathrm{I}_{\mathrm{l}}^0\Big)\Big)^+\Bigg]^2\Bigg\}\\
&+ \sum_{i\in S_{\mathrm{ld}}} \big[\gamma_i - (\Delta Q_i - \chi_i \Delta P_i)\big]^2\end{aligned} \tag{22}$$

A Lyapunov function is defined as follows:

$$V = \phi(z) - \phi\big(z^*\big) + \frac{1}{2}\|z - z^*\|_2^2 - \big(z - z^*\big)^T \partial\phi\big(z^*\big), \tag{23}$$

where ∂ denotes subgradient.

According to [43, Th. 1], we have

$$\forall z,\ \ V \geq \frac{1}{2}\|z - z^*\|_2^2,\ V_{\{z=z^*\}} = 0. \tag{24}$$

Hence, V is positive definite, $V = 0$ if and only if $\boldsymbol{z = z^*}$. Furthermore, $V \to \infty$ as $z \to \infty$ for all $z \in \Omega$.

Let $\boldsymbol{u}, \boldsymbol{\lambda}, \tilde{\boldsymbol{\lambda}}, \boldsymbol{\eta}$ be the vectors by stacking up all possible $u_i, \lambda_{ji}, \tilde{\lambda}_{ji}, \eta_{li}$. Let $\boldsymbol{v}, \boldsymbol{\mu}, \tilde{\boldsymbol{\mu}}, \boldsymbol{\omega}$ be the vectors by stacking up all possible $v_i, \mu_{ji}, \tilde{\mu}_{ji}, \omega_{li}$, then we have

$$\boldsymbol{Lu^* = 0, L\lambda^* = 0, L\tilde{\lambda}^* = 0, L\eta^* = 0} \tag{25}$$

Similar to the proof of DDFA model in [44], the function V along the trajectories of (18) satisfies that

$$\begin{aligned}\mathcal{L}_{\mathcal{H}}V = \Bigg\{ a : a = &\sum_{i\in S_{\mathrm{ela}}} \Bigg(2\alpha_i \Delta P_i + \beta_i - \chi_i\gamma_i - u_i + \sum_{l\in \mathrm{S_{brn}}} \Gamma_{\mathrm{li}}^{\mathrm{PI}} \eta_{li}\\
&+ \sum_{j\in \mathrm{S_{bus}}} \Gamma_{\mathrm{ji}}^{\mathrm{PV}} \Big(\lambda_{ji} - \tilde{\lambda}_{ji}\Big) + \Delta P_i - \Delta P_i^*\Bigg) H(\Delta P_i)\\
&+ \sum_{i\in \mathrm{S_{non}}} \big(\beta_i + 2\alpha_i \Delta P_i - \chi_i \gamma_i - u_i + \Delta P_i - \Delta P_i^*\big) H(\Delta P_i)\\
&+ \sum_{i\in S_{\mathrm{prs}}} \Bigg(\gamma_i + \sum_{j\in \mathrm{S_{bus}}} \Gamma_{\mathrm{ji}}^{\mathrm{QV}} \Big(\lambda_{ji} - \tilde{\lambda}_{ji}\Big) + \sum_{l\in \mathrm{S_{brn}}} \Gamma_{\mathrm{li}}^{\mathrm{QI}} \eta_{li} + \Delta Q_i\\
&- \Delta Q_i^*\Bigg) H(\Delta Q_i) + \nabla_{\boldsymbol{u}} V \dot{\boldsymbol{u}} + \nabla_{\boldsymbol{\lambda}} V \dot{\boldsymbol{\lambda}} + \nabla_{\tilde{\boldsymbol{\lambda}}} V \dot{\tilde{\boldsymbol{\lambda}}}\\
&+ \nabla_{\boldsymbol{\eta}} V \dot{\boldsymbol{\eta}} + \nabla_{\boldsymbol{v}} V \boldsymbol{L u} + \nabla_{\boldsymbol{\mu}} V \boldsymbol{L\lambda} + \nabla_{\tilde{\boldsymbol{\mu}}} V \boldsymbol{L}\tilde{\boldsymbol{\lambda}} + \nabla_{\boldsymbol{\omega}} V \boldsymbol{L\eta} \Bigg\},\end{aligned} \tag{26}$$

where $H(\Delta P_i)$ is the right hand side of (18a) or (18b), and $H(\Delta Q_i)$ is the right hand side of (18c). Followed from variational inequality and KKT conditions, we have

$$\mathcal{L}_{\mathcal{H}}V \le -\boldsymbol{u}^T\boldsymbol{L}\boldsymbol{u} - \tilde{\boldsymbol{\lambda}}^T\boldsymbol{L}\tilde{\boldsymbol{\lambda}} - \boldsymbol{\lambda}^T\boldsymbol{L}\boldsymbol{\lambda} - \boldsymbol{\eta}^T\boldsymbol{L}\boldsymbol{\eta} - \sum_{i\in S_{prs}}\Big[2\alpha_i\|\Delta P_i - \Delta P_i^*\|_2^2 + \|H(\Delta P_i)\|_2^2 + \|H(\Delta Q_i)\|_2^2\Big] - \sum_{i\in S_{ld}}\|\Delta Q_i + \chi_i\Delta P_i\|_2^2 \le 0 \quad (27)$$

First, $\mathcal{L}_{\mathcal{H}}V \le 0$ implies $z(t) \in \{z \in \Omega | V(z(t)) \le V(z(t_0))\}$ and $V(z) \to +\infty$ whenever $\|z\|_2 \to +\infty$. Second, denote the points satisfying $\mathcal{L}_{\mathcal{H}}V = 0$ as $\mathcal{R}$. Then (27) implies that

$$\mathcal{R} = \Big\{z|\ \dot{\Delta P}_i = 0, \dot{\Delta Q}_i = 0, \dot{\gamma} = 0, i \in S_{prs}, \boldsymbol{u}, \boldsymbol{\lambda}, \tilde{\boldsymbol{\lambda}}, \boldsymbol{\eta} \in \mathrm{range}\{\alpha\mathbf{1}\}\Big\},$$

where range$\{\alpha\mathbf{1}\}$ is the null space of $\boldsymbol{L}$. Let $\mathcal{M}$ be the largest invariant subset of $\bar{\mathcal{R}}$. It follows from the invariance principle that $z(t) \to \mathcal{M}$ as $t \to \infty$. Note that $\mathcal{M}$ is invariant. The trajectory $z(t) \in \mathcal{M}$ for all $t \ge 0$ if $z(t_0) \in \mathcal{M}$.

Based on the LaSalle invariance principle, any trajectory of neurodynamic model (18) will converge to its largest weakly invariant set $\mathcal{M}$.

We claim that the maximal invariance subset within the set $\mathcal{R}$ is exactly the equilibrium point of (18). The compactness and convexity of the invariant set imply the existence of equilibrium points of NNs (18). Assume that an optimal solution is $(\Delta P_i^*, \Delta Q_i^*, \gamma_i^*, u_i^*, \lambda_{ji}^*, \tilde{\lambda}_{ji}^*, \eta_{li}^*, v_i^*, \mu_{ji}^*, \tilde{\mu}_{ji}^*\omega_{li}^*)$. According to (18j), $\boldsymbol{u}, \boldsymbol{\lambda}, \tilde{\boldsymbol{\lambda}}, \boldsymbol{\eta} \in \mathrm{range}\{\alpha\mathbf{1}\}$ implies $dv_i/dt = 0, d\mu_{ji}/dt = 0$, $d\tilde{\mu}_{ji}/dt = 0$, $d\omega_{li}/dt = 0$ for all $i \in S_{prs}$, $j \in S_{bus}$, $l \in S_{brn}$. Hence, $du_i/dt = -\delta(u_i) - \delta(v_i) - \Delta P_i$ must be zero; otherwise u_i will go to infinity. As a result, $du_i/dt = 0$, $i \in S_{prs}$. Similarly, the right hand side of (18f) and (18h) must be zero. Then we have $d\lambda_{ji}/dt = 0, d\tilde{\lambda}_{ji}/dt = 0, d\eta_{li}/dt = 0$, where $i \in S_{prs}$, $j \in S_{bus}$, $l \in S_{brn}$. According to $d\Delta P_i/dt = 0, d\Delta Q_i/dt = 0, d\gamma_i/dt = 0$, one can see that ΔP_i and ΔQ_i reach optimum at any point $z \in M$ that satisfies $H(z) = 0$.

In summary, based on the LaSalle invariance principle and Lyapunov stability of the equilibrium point, the neurodynamic model (18) converges to its equilibrium point set and stays thereafter.

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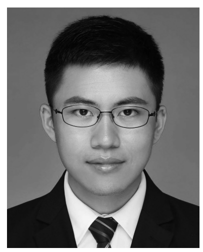

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