Auton Agent Multi-Agent Syst (2007) 15:253–279
DOI 10.1007/s10458-007-9011-z

A formal framework for connective stability of highly decentralized cooperative negotiations

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Published online: 10 April 2007

Abstract Multiagent cooperative negotiation is a promising technique for modeling and controlling complex systems. Effective and flexible cooperative negotiations are especially useful for open complex systems characterized by high decentralization (which implies a low amount of exchanged information) and by dynamic connection and disconnection of agents. Applications include ad hoc network management, vehicle formation, and physiological model combination. To obtain an effective control action, the *stability* of the negotiation, namely the guarantee that an agreement will be eventually reached, is of paramount importance. However, the techniques usually employed for assessing the stability of a negotiation can be hardly applied in open scenarios. In this paper, whose nature is mainly theoretical, we make a first attempt towards engineering stable cooperative negotiations proposing a framework for their analysis and design. Specifically, we present a formal *protocol* for cooperative negotiations between a number of agents and we propose a *criterion* for negotiation stability based on the concept of *connective stability*. This is a form of stability that accounts for the effects of structural changes on the composition of a system and that appears very suitable for multiagent cooperative negotiations. To show its possible uses, we apply our framework for connective stability to some negotiations taken from literature.

Keywords Cooperative negotiation · Connective stability · Lyapunov criterion · Multiagent optimization

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1 Introduction

Multiagent systems can play a prominent role in modeling and controlling complex systems [8,40]. Complex systems are structured in several heterogeneous subsystems and their global behavior depends on the interactions of these subsystems [4,29]. Multiagent systems provide a paradigm that appears "natural" for modeling complex systems [23]. Controlling a complex system implies the determination of a set of coordinated control actions, one for each subsystem. The control actions influence the system's behavior in order to reach the desired effect. Multiagent systems provide an effective technique to tackle this task: *cooperative negotiation* [6,28]. Indeed, controlling complex systems can be viewed as a decentralized multiobjective optimization problem [3] that can be solved by cooperative negotiation [8], where the control actions are produced as the agreement between negotiating agents. Cooperative negotiation is effective in controlling complex systems because usually it is robust with respect to the composition of the subsystems' network, that can change dynamically. Notwithstanding its promising role to control complex systems, multiagent cooperative negotiation lacks a satisfactory and comprehensive theory. Some crucial questions are still open: the *stability* of the negotiation [17], its *optimality* [10,21,22], its *real-time convergence* [6,28], and its *adaptation* both to different applications and to changing conditions in a given application.

In this paper we take a step towards engineering cooperative negotiations. In particular, we focus on the stability of these negotiations. Informally, a negotiation is stable when an agreement among agents will be eventually achieved. The relevance of stable negotiations and of techniques for studying negotiation stability is well-known in literature [33]. Classic techniques for stability analysis [33] can be hardly applied to open systems with high level of decentralization and a potentially large number of agents. The stability techniques proposed in multiagent literature are rather ad hoc and do not always provide formal results [6,28]. Our proposal is instead based on a framework to develop stable cooperative negotiations that is formal and widely applicable, also to reconfigurable open multiagent systems.

The proposed framework comprises a *protocol* that defines the mechanism with which agents negotiate and a *criterion* that, when satisfied, assures that the negotiations carried on under the proposed protocol are stable. The protocol, preliminarily introduced in [17], is an extension of [35] and has been designed to be expressive enough to capture a large class of cooperative negotiations. The stability criterion has been derived exploiting the concept of *connective stability* [36]. Connective stability is a form of stability studied in dynamical systems [25] that naturally adapts to multiagent negotiations, since it studies the stability of dynamical systems that can undergo structural modifications, as in the case of open multiagent systems when agents connect and disconnect [16]. To the best of our knowledge, ours is the first attempt to apply connective stability to multiagent negotiation. The criterion we provide is an initial result that can open the way to a deeper analysis of the connective stability of cooperative negotiations. For example, in this work we assume that agents' knowledge is completely decentralized (agents have not any information concerning other agents). Future extensions could cover situations in which the amount of agents' mutual knowledge is larger. To define the criterion, we have considered a negotiation process as a dynamical system to which we have applied the Lyapunov stability criterion [27]. By virtue of this approach, we have been able to find a set of sufficient stability constraints, each one referring to a single agent independently of the others. In this way, when each agent satisfies its constraint, the cooperative negotiation is guaranteed to be stable regardless of the composition of the agent network. To show the usefulness of the proposed framework, we applied it to the negotiations presented in [12,13] and in [17,19].

To summarize, the main original contributions of this paper are: a formulation of the notion of connective stability in the cooperative negotiation arena, the definition of a protocol for cooperative negotiations, and the definition of a sufficient criterion that guarantees connective stability. Note that in this paper we aim at providing theoretical contributions rather than addressing implementation and practical aspects. For example, we do not discuss problems like the time it takes to negotiate under given conditions.

This paper is structured as follows. In Sect. 2 we review the main results at the intersection of complex system control, decentralized multiobjective optimization, and cooperative negotiation. In Sect. 3 we introduce our cooperative negotiation protocol. In Sect. 4 we define the notion of connective stability for multiagent cooperative negotiations. In Sect. 5 we propose a sufficient criterion for the connective stability of multiagent cooperative negotiations. In Sect. 6 we show how the proposed framework can be applied to some negotiations presented in literature. Section 7 draws some conclusions. Appendix A reports the proofs of some theorems presented in the paper.

2 Complex systems and cooperative negotiation

In what follows we first show how the problem of controlling a complex system can be formulated as a multiobjective optimization problem where objectives are embedded in different agents. Then, we illustrate how cooperative negotiation can be used to solve such a problem and we review the cooperative negotiation literature at the light of complex system control.

2.1 Distributed and dynamic multiobjective optimization problems

A generic distributed and dynamic multiobjective optimization problem with n agents can be formulated as:

$$\begin{cases} \max\limits_{\mathbf{x}} \; [J_1\left(\mathbf{x}, \mathbf{w}_1(\tau)\right), \ldots, J_n\left(\mathbf{x}, \mathbf{w}_n(\tau)\right)] \\ \text{s.t. } [f_1(\mathbf{x}), \ldots, f_n(\mathbf{x})] \leq 0 \end{cases} \tag{1}$$

where $\mathbf{x}$ is a vector of m continuous real-valued variables, each $\mathbf{w}_i(\tau)$ is a vector of parameters dependent on time τ, and J_i and f_i are the objective function and the constraint, respectively, relative to agent i. Given a τ, namely given the $\mathbf{w}_i(\tau)$, the solution of the problem (1) is obtained by finding a vector $\hat{\mathbf{x}}(\tau)$ that maximizes the objective functions under the constraints. Since each J_i depends on the values of the variables in $\mathbf{x}$, each agent i takes part in finding these values. The problem (1) is *distributed* since each agent behaves as a decision maker that embeds an objective function J_i and the corresponding constraint f_i. The problem (1) is *dynamic* since the parameters $\mathbf{w}_i(\tau)$ dynamically change and agents can dynamically connect and disconnect the system, introducing and removing objective functions (and corresponding constraints). Both distribution and dynamicity prevent the use of a centralized decision maker for solving the problem (1). In addition, being modifications happening unpredictably, the optimization process must be repeated at any τ when a modification of the problem occurs (e.g., whenever an agent joins the system, the values of $\mathbf{w}_i(\tau)$ change, or agents' objectives change).

Instances of the problem (1) can be found in several application fields; a particularly prominent one concerns the control of complex systems [8].

2.2 Controlling complex systems

Although several controversial definitions of "complex system" exist, almost all share a common point: the behavior of a complex system depends on the dynamic interaction of several heterogeneous subsystems [4,29]. The control of a complex system is hard to tackle, requiring to determine a set of coordinated control actions for the subsystems. Each control action influences a subsystem's behavior in order to obtain a desired effect on the whole system. Significant examples of complex systems that are difficult to control can be found in ad hoc network management [37], vehicle formation [22], market regulation [5,19], and physiological model combination [2,17].

The basic principle used in traditional control techniques for complex systems is to integrate several control models, each one associated to a different subsystem. For instance, the multimodeling paradigm — introduced in [14,15] — employs a set of models at different abstraction levels. The adoption of multiple models of a phenomenon raises several issues regarding, for example, their coherence and the ways in which they are combined. In particular, the models can be "overlapping", for example because different alternative models can be used to provide the value of a given variable. As a consequence, conflicts can occur. Techniques such as weighted average, model selection (according to confidence indexes of the models), and fuzzy combination are commonly employed [34], but they do not effectively solve the conflicts since they usually do not take into account any inter-effect between the models.

The optimal decentralized control approach [4,32,38] is emerging as a very effective technique to control complex systems by using different models. Optimal decentralized control can be seen as an instance of the multiobjective optimization problem of Sect. 2.1. An agent can control only some of the variables that compose the vector $\mathbf{x}$, however the control action undertaken by an agent i affects the values of the objective functions of other agents, being these functions dependent on $\mathbf{x}$. Thus, if each agent produces singularly its control action taking into account only its objective function, this control action would be obviously inefficient. Instead, the control actions need to be produced taking into account all agents' objective functions, finding the argument $\hat{\mathbf{x}}(\tau)$ that solves the problem (1). In this case, problem (1) is distributed because the complex system to be controlled is composed of several subsystems that are acting as autonomous decision makers and it is dynamic because the subsystems can change over time. Optimal decentralized control is robust: without centralization, it leads to systems that show a graceful degradation of performance when individual subsystems malfunction or fail [21,32]. Optimal decentralized control is also scalable: it can be used effectively for systems with a large number of subsystems.

Let us now discuss some significant examples of complex system control. In an ad hoc network [37], each node can be modeled as an agent embedding an objective function that relates the amount of communication traffic at the node with: the energy available at the node, the amount of communication traffic requested by other nodes, and the energy available at other nodes. The goal is to determine a set of coordinated control actions that define the optimal amount of communication traffic at each node. Similarly, in vehicle formation [22], the goal is to optimally determine the position of each vehicle to keep it in formation according to the other vehicles (e.g., their size, position, speed, and acceleration), to the environment where it moves (e.g., its obstacles, size, and shape), and to the tasks the formation is accomplishing. In market regulation [5,19], the goal is to determine the quantity and the price of the goods produced by the firms in order to maximize the profit. Finally, in controlling physiological phenomena [2,17], the goal is to determine the optimal combination of the control actions produced by a set of partial physiological controllers in order to mimic a physiological function.

In the rest of this section, we describe in more detail how the problem of vehicle formation (actually, a simplified instance) can be formulated as a distributed and dynamic multiobjective optimization problem of the form (1). In the following of this paper, we will use this example to better illustrate some concepts. Consider a number of vehicles with different properties (e.g., different dimensions and locomotion abilities) that should move in formation in a bidimensional environment disseminated with obstacles. The position and the shape of the obstacles are supposed known. The formation leader chooses the path and the velocity. The other vehicles have to decide their control actions (where to move and at what velocity) in order to follow the leader, to keep the formation, and to avoid the obstacles. This problem can be formulated in the form (1). We call $\mathbf{x}_i = [x_i, y_i]$ the vector that expresses the relative position of agent i with respect to the leader (in the bidimensional space where the vehicles move). Note that agent i can control only the variables in $\mathbf{x}_i = [x_i, y_i]$. The vector $\mathbf{x}$ of (1) is defined as the composition of the $\mathbf{x}_i$, namely $\mathbf{x} = [\mathbf{x}_1, \ldots, \mathbf{x}_n]$. In this way, with n vehicles and a leader, the vector $\mathbf{x}$ is composed of $2n$ real-valued variables. Each vehicle (agent) i has an objective function J_i that evaluates the position of the vehicle, according to its distance from the other vehicles of the formation and from the obstacles. In this case, the parameters $\mathbf{w}_i(\tau)$ include the position and the velocity of the formation leader and the position of the obstacles. Note that the objective functions J_is depend on the type of formation (e.g., triangle or diamond formation). Note also that each vehicle i will try to find its optimal position, namely the optimal value $\hat{\mathbf{x}}(\tau)$ of $\mathbf{x}$ that maximizes its objective function J_i (actually, vehicle i will try to find the optimal value $\hat{\mathbf{x}}_i(\tau)$ of the portion of $\mathbf{x}$ it can control). The constraint $f_i(\mathbf{x}) \leq 0$ can express, for example, the minimum distance the vehicle i must keep from the obstacles and from other vehicles. The vehicle formation problem formulated in this way is distributed because the objective functions are distributed over the vehicles, that act as decision makers. The problem is also dynamic because the number of vehicles can change (for example, because a vehicle broke down), the objective functions can change (for example, because the system switches from a triangle formation to a diamond formation), and the parameters $\mathbf{w}_i(\tau)$ can change. When this problem is solved in a decentralized way, each vehicle contributes to the solution by adjusting its position, without requiring any centralized supervisor of the formation. This makes the system robust (there is not any single point of failure) and scalable (the number of vehicles can change dynamically).

2.3 Cooperative negotiation

The peculiarities of the problem formulated in Sect. 2.1 — distribution and dynamicity — make classic operational research techniques hard to apply, pushing towards the development of alternative approaches for its solution [28]. Among the proposed approaches, a promising one is cooperative negotiation, where agents behave as autonomous decision makers and economic concepts are employed to drive agents towards an agreement corresponding to the optimal solution [22,39]. The basic idea behind cooperative negotiation is that the agents attempt to solve the optimization problem by bargaining without any centralized decision maker [9]. Note that the negotiation must be repeated at any τ a modification of the optimization problem occurs.

Cooperative negotiation exploits *competitive mechanisms* to allow agents to reach *cooperative agreements*. Specifically, as in competitive economic situations, agents do not share information and their interaction is ruled by a protocol [26]. However, differently from competitive economic situations, agents do not behave as selfish maximizers, but they try to reach efficient cooperative agreements [41]. Regarding the outcome, agents aim at reaching the agreements prescribed by cooperative game theory (e.g., Nash bargaining solution) and not the agreements prescribed by competitive game theory (i.e., Nash equilibria). The bridge between competitive and cooperative game theory is an issue that has been intensively studied in literature. For instance, a well-known result is that, under some assumptions, the Nash equilibrium outcome converges to the Nash bargaining solution [7]. This shows that reaching cooperative agreements with competitive mechanisms is possible.

The main issues in the study of cooperative negotiations concern the design of negotiation protocols and agents' negotiation strategies able to produce optimal solutions, robustly with respect to the composition of the system. The most appropriate notion of solution of a multiobjective problem is Pareto optimality. Since usually there are more Pareto optimal solutions, in literature several criteria have been proposed to discriminate between them [42]. Some of these criteria are: generalized Nash bargaining solution [31], Kalai-Smorodinsky solution [24], Kalai solution [24], and weighted sum of the objective functions. A negotiation protocol is usually designed with the purpose to allow agents to reach a large number of possible optimal solutions. Conversely, negotiation strategies are usually designed to satisfy one of the optimality criteria mentioned above. These two issues — the definition of an appropriate protocol for cooperative negotiations and the development of negotiation strategies to produce cooperative agreements — have not yet been completely and satisfactorily addressed in literature, where rather ad hoc solutions have been developed for each specific case [6,17].

When considering our example of vehicle formation, the negotiation process brings each vehicle (except the formation leader) to adjust its position relative to the positions of the other vehicles. Each vehicle negotiates on the values of all the variables in the vector $\mathbf{x}$ (we recall that it is formed by the coordinates of the relative positions of all the vehicles with respect to the leader). This amounts to say that the vehicles negotiate on the arguments of their objective functions J_is. The "best" Pareto optimal solutions of this problem should balance the utilities of all vehicles (i.e., the values of the objective functions J_is). The Nash bargaining solution can pick up these solutions. Appropriate negotiation protocol and strategies have to be developed in order to reach these solutions: we will do it in Sect. 3.

2.4 Cooperative negotiation for controlling complex systems

When cooperative negotiation is employed to solve the optimality problems of controlling complex systems, other issues, beyond optimality, must be considered to guarantee an effective and efficient control action. Some of these issues are: the *stability* of the negotiation [17] (does the negotiation always reach an agreement? what are the constraints under which a negotiation is stable?), the *real-time convergence* to an agreement [6,28] (do the agents converge to an agreement within a temporal deadline?), and the *adaptation* of the negotiation to the changes in the context (for example, can the negotiation mechanism adapt to bandwidth modifications?). The study of these issues aims at ascribing some properties — stable, real-time, adaptable — to the negotiation.

In this paper, we focus on stability. The problem of negotiation stability is particularly hard because usually classic stability techniques cannot be applied [33]. Off-line techniques cannot be applied because the composition of the agent network is not known a priori. On-line techniques cannot be applied because there is not any entity having complete information about agents. This has pushed multiagent researchers to explore alternative techniques for assuring stability that can be roughly organized in two main groups. In the first group there are techniques based on implementation aspects. These solutions adopt, at runtime, flags and checks to avoid infinite loops and to force the termination of the negotiation [6,28]. These approaches do not assure that agents converge to an agreement, but only that the negotiation ends. However, our *desideratum* is that the agents converge to an agreement and not that the negotiation is truncated exogenously. We would like to assure that, given the agents enough time, an optimal solution will be found endogenously. The techniques in the second group try to provide theoretical and formal proofs of stability. For example, in [11] a decentralized algorithm for Pareto optimal negotiations is presented and its stability is formally proved in a very simple case. However, as remarked by the authors themselves, it is hard to find a proof of the stability of their algorithm in a general case: for each specific case a new ad hoc proof must be produced. A similar approach is discussed in [21]. In general, the attempts to formally study the stability of a negotiation are limited to very specific settings. Every time an agent's objective changes or an agent joins or leaves, a new proof of stability, if existing, must be produced. In this paper, we try to overcome this limitation by proposing a general framework for stability of cooperative negotiations.

In our example of vehicle formation, stability is essential since without an agreement vehicles can crush each other or with obstacles. The forced termination of a negotiation can result in a danger for a vehicle, if the agreement (i.e., the vector $\mathbf{x}$) reached at that time is disadvantageous for the vehicle. It is therefore important that the vehicles converge to an agreement. In studying the stability of the negotiation, we would like that an agreement be found in any setting. In this case, off-line techniques to study stability cannot be applied since they require to analyze a large number (possibly infinite) of different settings (different formations, different number of vehicles, different obstacles positions, and so on). On-line techniques to study stability can be hardly used since they require to be applied every time the setting changes. Moreover, these techniques usually require a central entity that stores all the information about all the vehicles currently composing the system. The approach we adopt in this paper is to devise a framework to study the stability of a negotiation independently from the specific setting, namely to study connective stability.

3 A protocol for cooperative negotiation

In this section we present a cooperative negotiation protocol that allows agents to reach a number of possible optimal solutions. The protocol has been introduced and used in [18,19], where agents reach generic Pareto optimal agreements and Nash bargaining solutions, and in [1,17], where agents reach optimal agreements with respect to weighted sum of the objective functions.

In the cooperative negotiation setting we consider, n agents bargain over the values of some variables over which they have conflicts. The negotiation is intrinsically *many-to-many*, being many the agents that are in conflict on a variable. Moreover, the negotiation is defined on *multiple issues*, being multiple the variables over which agents have conflicts.

The protocol we present is basically an extension of the classic alternating-offers protocol [35], modified mainly by the introduction of a *mediator*. The negotiation is carried on as follows:

(1) the agents compute their (new) offers;
(2) the agents send these offers to the mediator;
(3) the mediator computes an agreement;
(4) the mediator sends this agreement to the agents as its counter-offer;
(5) the above steps are repeated until all agents agree with the mediator.

Note that these steps define a single negotiation that can be repeated several times (as discussed in Sect. 2.3).

The presence of a mediator enables several interesting properties:

- *multilateralism*, the mediator easily allows to conduct negotiations between more than two agents,
- *symmetry*, since agents make their offers at the same time, the asymmetry of the classic alternating-offers protocol is removed,
- *decentralization*, agents can negotiate via the mediator without interacting with each other explicitly.

The mediator is logically distinct from the agents participating in the negotiation. That said, the mediator can be implemented as an independent entity or an agent involved in the negotiation can play also the role of mediator. In principle, given a set of conflicting agents, a mediator for each conflicting variable could be present. However, in what follows we assume, without losing generality, that a single mediator works over all the conflicting variables.

Agents' preferences are formulated as utility functions $\mathcal{U}_i$s (the economic correspondent of the objective functions J_is of Sect. 2.1). Specifically, they are such that $\mathcal{U}_i(\mathbf{x}) = J_i(\mathbf{x}, \mathbf{w}_i(\tau))$, where the values of the parameters $\mathbf{w}_i(\tau)$ are considered as constants in $\mathcal{U}_i$ and do not appear explicitly. Formally, each agent i embeds a *utility function* $\mathcal{U}_i$ defined on $\mathbb{R}^m$ of m real-valued variables; $\mathcal{U}_i : \mathbb{R}^m \to \mathbb{R}$. m is the number of issues (variables) over which the agents are in conflict. We call *agreement set of agent* i the set $I_i \subseteq \mathbb{R}^m$ such that for all $\mathbf{x} \in \bar{I}_i$ (where $\bar{I}_i$ is the boundary of I_i) it is $\mathcal{U}_i(\mathbf{x}) = 0$ and for all $\mathbf{x} \in I_i/\bar{I}_i$ it is $\mathcal{U}_i(\mathbf{x}) > 0$ ($/$ is set difference). I_i represents the portion of space of the m issues over which the agent i is interested in negotiating.

We call $\mathbf{p}^t_{i\to e} \in I_i$ the *offer* of agent i to the mediator e at time t, and we call $\mathbf{p}^t_{e\to i} \in \mathbb{R}^m$ the *counter-offer* of the mediator e to the agent i at time t. (Note that, in general, the counter-offer produced by the mediator e can be any point in $\mathbb{R}^m$.) Time is discrete, with $t \in \mathbb{N}$. Given the agreement sets I_is relative to all the agents, we call $A \subseteq \mathbb{R}^m$ the *agreement set* defined as the intersection of all the I_is; namely, $A = \bigcap_{i=1}^n I_i$. A represents the portion of the space of the m issues over which all agents are interested in negotiating. Intuitively, if $A = \emptyset$, then no agreement can be reached by the agents. We note in addition that, since the agents can connect and disconnect the system at any time, A can vary over time. For simplicity, in what follows, we assume that changes can occur only between two negotiations (see discussion of Sect. 2.3). Specifically, at a given time τ, we find the solution $\hat{\mathbf{x}}(\tau)$ of problem (1) by a cooperative negotiation that works over time $t = t_0, t_0 + 1, \ldots$, where $t_0 = \tau$. At time $\tau + \Delta\tau$, we find the solution $\hat{\mathbf{x}}(\tau + \Delta\tau)$ of problem (1) by a cooperative negotiation that works over time $t = t_0, t_0 + 1, \ldots$, where $t_0 = \tau + \Delta\tau$. Hence, t denotes the time within a negotiation, while τ denotes time between negotiations. Accordingly, we can say that t changes more quickly that τ.

The next offer $\mathbf{p}^{t+1}_{i\to e}$ of an agent i is produced according to a *negotiation function* $\mathcal{F}_i$ whose arguments are the previous offers made by the agent i, the previous counter-offers made by the mediator e to the agent i, and time t. $\mathcal{F}_i$ depends on t since agent i can have preferences changing over time. We call t_0 the initial time of the negotiation; in this way, defining a generic initial time, we can easily cope with repeated negotiations. We define $\mathcal{F}_i$ as follows: $\mathbf{p}^{t+1}_{i\to e} = \mathcal{F}_i\left(\{\mathbf{p}^{\mathrm{t}}_{i\to e}\}_{\mathrm{t}\in[t_0,t]}, \{\mathbf{p}^{\mathrm{t}}_{e\to i}\}_{\mathrm{t}\in[t_0,t]}, t\right)$, where $\mathcal{F}_i : I_i^{(t-t_0)} \times \mathbb{R}^{m(t-t_0)} \times \mathbb{N} \to I_i$. We simplify $\mathcal{F}_i$ assuming a Markov hypothesis. Hence, the definition of $\mathcal{F}_i$ becomes $\mathbf{p}^{t+1}_{i\to e} = \mathcal{F}_i\left(\mathbf{p}^t_{i\to e}, \mathbf{p}^t_{e\to i}, t\right)$, where $\mathcal{F}_i : I_i \times \mathbb{R}^m \times \mathbb{N} \to I_i$.

The *agreement* at time t, $\mathbf{a}^t$, is calculated by the mediator by evaluating an *agreement function* $\mathcal{A}$ (embedded by the mediator) whose argument is the set of the offers of all the agents, formally, $\mathbf{a}^t = \mathcal{A}\left(\{\mathbf{p}^t_{i\to e}\}_{i\in[1,n]}\right)$ where $\mathcal{A} : I_1 \times I_2 \times \ldots \times I_n \to \mathbb{R}^m$. Note that $\mathcal{A}$ could depend on other quantities (e.g., time t or $\mathbf{a}^{t-1}$), but we do not need such generality in the following of this paper. The counter-offer $\mathbf{p}^t_{e\to i}$ of the mediator e to an agent i is just $\mathbf{a}^t$, namely $\mathbf{p}^t_{e\to i} = \mathbf{a}^t$.

We now show how the above concepts are instantiated in our example of vehicle formation. The utility function $\mathcal{U}_i$ of each vehicle i is defined on the variables composing the vector $\mathbf{x}$, namely on the $2n$ coordinates of the positions of the vehicles with respect to the formation leader (recall that n is the number of vehicles, except the leader). Since the number of vehicles can change over time, the variables in $\mathbf{x}$ can change over time τ. The utility function of agent i, $\mathcal{U}_i$, depends on the parameters of $\mathbf{w}_i(\tau)$; for example, the position and the velocity of the formation leader and the position of the obstacles. The agreement set of agent i, I_i, is the set of values of $\mathbf{x}$ that make $\mathcal{U}_i$ non-negative. When the vectors $\mathbf{w}_i(\tau)s$ change, also the sets I_is change. The negotiation functions and the agreement function appropriate for this application are those reported in Sect. 6.4.

To summarize, each agent i is described by a pair $\langle\mathcal{U}_i, \mathcal{F}_i\rangle$, the mediator is described by a singleton $\langle\mathcal{A}\rangle$, and a negotiation is described by a collection of agents, a mediator, and an initial time: formally, $\mathcal{N} = \left\langle\{\langle\mathcal{U}_i, \mathcal{F}_i\rangle\}_{i\in[1,n]}, \mathcal{A}, t_0\right\rangle$. $\{\langle\mathcal{U}_i, \mathcal{F}_i\rangle\}_{i\in[1,n]}$ is the *agent network* conducting the negotiation.

4 Connective stability for cooperative negotiation

Given an agent network cooperatively negotiating on the basis of the given protocol, the guarantee of eventually achieving an agreement is fundamental to have an effective control action. A stable agreement is an expression of the benevolent cooperation of agents; if each agent operates selfishly a common goal cannot be always achieved. Two aspects must be balanced: on the one hand, the agents are required to adhere to a class of negotiation functions for cooperating, on the other hand, they are required to be free in selecting their negotiation functions for achieving the optimal agreement. In this perspective, we aim at determining some constraints (hopefully, as broad as possible) over agents' negotiation functions that assure the stability of the negotiation.

We define the stability of a negotiation as follows.

Definition 4.1 Given an agent i and a vector $\bar{\mathbf{p}} \in I_i$, a succession of offers $\mathbf{p}^t_{i\to e}$s made by agent i (with each $\mathbf{p}^t_{i\to e} \in I_i$) converges to $\bar{\mathbf{p}}$ when $\exists \bar{t}_i > 0$ such that $\mathbf{p}^t_{i\to e} = \bar{\mathbf{p}}$ for all $t \geq \bar{t}_i$.

Definition 4.2 Given a negotiation $\mathcal{N} = \left\langle\{\langle\mathcal{U}_i, \mathcal{F}_i\rangle\}_{i\in[1,n]}, \mathcal{A}, t_0\right\rangle$, $\mathcal{N}$ is *stable* when there exists a vector $\bar{\mathbf{p}} \in A$ such that the succession of the offers $\mathbf{p}^t_{i\to e}$s converges to $\bar{\mathbf{p}}$ for all agents $i \in [1, n]$.

A negotiation is stable when there is a time $\bar{t}$ (the largest of all $\bar{t}_i$ of Definition 4.1) for which all the agents make the same offer and they keep to make such offer for all $t > \bar{t}$; namely, when $\forall t \geq \bar{t}\ \forall i, j\ \mathbf{p}^t_{i\to e} = \mathbf{p}^t_{j\to e}(= \bar{\mathbf{p}})$. Notice that the concept of negotiation stability does not concern neither the achievement of an optimal agreement (i.e., optimality) nor the achievement of an agreement within a given temporal deadline (i.e., real-time convergence). When a negotiation is stable, the agents are guaranteed to eventually reach an agreement if they are given enough time. In this work we study the negotiation stability, namely, the fact that the offers of the agents get eventually equal.

Considering the protocol described in Sect. 3, the study of the negotiation stability can be formulated as the study of a class of negotiation functions $\{\mathcal{F}_i\}_{i\in[1,n]}$ that assure the stability of the negotiation given the utility functions $\{\mathcal{U}_i\}_{i\in[1,n]}$, the agreement function $\mathcal{A}$, the initial time t_0, and the dynamics of the agent network. Decentralization and runtime reconfigurability of the agent network are the original issues that mostly distinguish our approach from other works concerning stability.

To tackle these issues, we propose to use the concept of *connective stability*, a particular form of stability adopted to study dynamical systems under structural modifications [20,36]. The concept of connective stability is very suitable for multiagent systems since it deals with systems composed of interconnected subsystems (the agents) and subject to structural perturbations (the agents connect and disconnect) [16]. According to a decentralized approach, techniques for studying connective stability in the context of dynamical systems usually associate some stability constraints to each subsystem of the network. In this way, they do not require any centralized verification of the stability constraints every time a network modification occurs. This approach fits well to the idea of cooperative negotiation that, as we have shown before, is characterized by the absence of any centralized decision maker. We formally define the connective stability of a negotiation as follows.

Definition 4.3 A negotiation $\mathcal{N} = \left\langle \{\langle \mathcal{U}_i, \mathcal{F}_i\rangle\}_{i\in[1,n]}, \mathcal{A}, t_0 \right\rangle$ is *connectively stable* when it is stable regardless of the composition of the agent network $\{\langle \mathcal{U}_i, \mathcal{F}_i\rangle\}_{i\in[1,n]}$ taking part into the negotiation.

In particular, connective stability guarantees that agents eventually agree independently of the number n of agents taking part into the negotiation. In a connectively stable negotiation, the agent network can change over time without affecting the achievement of an agreement. Trivially, if a negotiation is connectively stable, then it is also stable. The vice versa is not always true.

As just said, connective stability is a stronger concept than "simple" stability. However, in the case of open systems the concept of connective stability is more appropriate. Indeed, as discussed in Sect. 2.4, stability of negotiations can be studied off-line, requiring the analysis of a large number of settings, or on-line, requiring a centralized entity that has all the information about the system. In both ways, the study of stability is difficult in open systems. This is why the concept of connective stability fits more naturally with open systems. To better explain the difference between stability and connective stability, let us consider our vehicle formation example. In this case, in order to guarantee the stability, we should find the constraints under which, given a specific setting (e.g., number of vehicles, formation, and obstacle positions), the agreement will be eventually reached. Every change of the setting (i.e., every change of $\mathcal{N}$) requires to find new constraints to assure the stability of the negotiation. Connective stability requires to define a single set of constraints under which the vehicles will eventually reach an agreement, in any setting.

In the following section, we introduce a criterion for the connective stability of a cooperative negotiation conducted in a decentralized way.

5 A criterion for connective stability

5.1 Overview of the criterion

The proposed criterion for connective stability is expressed as a set of constraints that are "spread" over the agents. If each agent independently satisfies its "local" constraints, then the connective stability of the negotiation is guaranteed. "Local" constraints ease the analysis and design of cooperative negotiations because they promote decentralization. In particular, it is not required that the negotiation functions be mutually known by the agents. The methodology we follow for defining our criterion is:

(a) we associate a dynamical system to a cooperative negotiation conducted according to the protocol of Sect. 3 such that, when the dynamical system is at equilibrium, the corresponding negotiation is stable;
(b) we determine a set of constraints under which the dynamical system eventually converges to the equilibrium and under which the associated negotiation is connectively stable.

In the context of dynamical systems, the analysis of connective stability of a system is commonly performed employing a vector of Lyapunov functions [27]. In particular, a Lyapunov function is associated to each subsystem and a combination, usually a weighted sum, of these Lyapunov functions is associated to the whole system [20]. In our case, differently from literature on connective stability, we cannot assume any a priori knowledge about what and when subsystems can connect and disconnect, because this assumption would be unrealistic in our setting. For this reason, we could not apply to our dynamical system (equivalent to a negotiation) the classic techniques for connective stability and we had to devise a new technique.

The steps (a) and (b) above are discussed in the following two sections, respectively.

5.2 The dynamical system associated to a negotiation and its equilibrium

In this section, we define a dynamical system that is equivalent to a negotiation conducted according to the protocol of Sect. 3. We recall that a negotiation of this kind is stable when $\forall i, j\ \mathbf{p}^t_{i\to e} = \mathbf{p}^t_{j\to e}$ for all $t \geq \bar{t}$. With this idea in mind, we define the state x^t of the dynamical system as:

$$x^t = \sum_{i=1}^{n}\sum_{j=1}^{n}\left\|\mathbf{p}^t_{i\to e} - \mathbf{p}^t_{j\to e}\right\|^2 \tag{2}$$

where $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^m$. The state x^t is a real-valued number that provides a measure of the distance between the offers of the agents. Then, we define the following dynamical system:

$$\begin{aligned} x^{t+1} &= \sum_{i=1}^{n}\sum_{j=1}^{n}\left\|\mathbf{p}^{t+1}_{i\to e} - \mathbf{p}^{t+1}_{j\to e}\right\|^2 \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n}\left\|\mathcal{F}_i\left(\mathbf{p}^t_{i\to e}, \mathcal{A}\left(\{\mathbf{p}^t_{k\to e}\}_{k\in[1,n]}\right), t\right) - \mathcal{F}_j\left(\mathbf{p}^t_{j\to e}, \mathcal{A}\left(\{\mathbf{p}^t_{k\to e}\}_{k\in[1,n]}\right), t\right)\right\|^2 \\ &= f(x^t, t) \end{aligned} \tag{3}$$

Given how we defined it in (2), the state x^t is equal to 0 if and only if the offers of all the agents are coincident, namely when $\forall\ i, j\ \mathbf{p}^t_{i\to e} = \mathbf{p}^t_{j\to e}$. This means that the negotiation is stable if and only if $x = 0$ is an equilibrium for the dynamical system (3). Note that the initial state x^{t_0} of (3), at time t_0, depends on the initial offers of the agents.

To prove that $x = 0$ can be an equilibrium point for (3), we need the following two definitions.

Definition 5.1 An agreement function $\mathcal{A}$ is said *idempotent* when: if, for a given t, $\forall i\ \mathbf{p}^t_{i\to e} = \mathbf{p}^t$ then it is $\mathcal{A}\left(\{\mathbf{p}^t_{i\to e}\}_{i\in[1,n]}\right) = \mathcal{A}\left(\{\mathbf{p}^t\}_{i\in[1,n]}\right) = \mathbf{p}^t$.

The idempotency property says that, if all the agents agree on an offer at t, then the agreement at t is that offer.

Definition 5.2 A negotiation function $\mathcal{F}_i$ is said *consistent* when: if, for a given t, $\mathbf{p}^t_{i\to e} = \mathbf{p}^t_{e\to i} = \mathbf{p}^t$ then it is $\mathcal{F}_i\left(\mathbf{p}^t_{i\to e}, \mathbf{p}^t_{e\to i}, t\right) = \mathcal{F}_i(\mathbf{p}^t, \mathbf{p}^t, t) = \mathbf{p}^t$.

The consistency property says that, if the counter-offer received by agent i at time t is satisfactory, then agent i does not change its offer. The following theorem holds (proof is reported in Appendix A.1).

Theorem 5.3 *Given the dynamical system* (3)*, if $A \neq \emptyset$, $\mathcal{A}$ is idempotent, and all $\mathcal{F}_i$s are consistent, then $x = 0$ is an equilibrium point for* (3).

If the hypotheses of the above theorem are satisfied, then $x = 0$ is an equilibrium point for the dynamical system (3). This means that if the system (3) goes in $x = 0$, then it stays there forever. Recalling (3), we can reformulate Theorem 5.3 as follows. Under the hypotheses of the theorem, if, at some time instant $\bar{t}$, it is $\mathbf{p}^{\bar{t}}_{i\to e} = \mathbf{p}^{\bar{t}}$ for all i, then it is also $\mathbf{p}^t_{i\to e} = \mathbf{p}^{\bar{t}}$ for all i and for all $t > \bar{t}$ and the (stable) agreement will be $\mathbf{p}^{\bar{t}}$.

We now study the convergence of the dynamical system (3) to $x = 0$.

5.3 The convergence of the dynamical system

Given the dynamical system introduced in the previous section, we study its convergence to the equilibrium point $x = 0$. In this study, we use a notion of stability that is:

- *Asymptotic*: the dynamical system (3) eventually converges to $x = 0$.
- *Uniform*: the dynamical system (3) converges to $x = 0$ independently of initial time t_0.
- *Global*: the dynamical system (3) converges to $x = 0$ from any initial state x^{t_0}.
- *Connective*: the dynamical system (3) converges to $x = 0$ independently of n.

Saying that the dynamical system (3) satisfies the above notion of stability amounts to say that the associated negotiation is connectively stable independently of t_0 and of the initial offers of the agents.

In order to study the asymptotic uniform global stability of the dynamical system (3) we exploit, following a common approach in dynamical systems, an appropriate formulation of the Lyapunov theorem. (The connective stability of (3) will be studied later.) To introduce this formulation of the Lyapunov theorem, we need the following definition [25].

Definition 5.4 A function $\Psi : \mathbb{R}^p \to \mathbb{R}$ is a *global Lyapunov function* for a dynamical system $\mathbf{x}^{t+1} = f(\mathbf{x}^t)$ if for all the admissible states $\mathbf{x} \in \mathbb{R}^p$ it is:

(i) Lipschitz[1],
(ii) positive definite in the equilibrium $\mathbf{x} = \mathbf{0}$[2],
(iii) radially unbounded[3],
(iv) and there exists a $\mathcal{KL}$-function ϕ such that, given $\mathbf{x} \in \mathbb{R}^p$, $\Psi(f(\mathbf{x})) - \Psi(\mathbf{x}) \leq -\phi(\|f(\mathbf{x})\|, t - t_0)$[4].

[1] A function Ψ is Lipschitz, if for each $\mathbf{x}_1$ and $\mathbf{x}_2 \in \mathbb{R}^p$, there exists $k \in \mathbb{R}$, $k > 0$ such that $|\Psi(\mathbf{x}_1) - \Psi(\mathbf{x}_2)| < k \cdot \|\mathbf{x}_1 - \mathbf{x}_2\|$.

[2] A function Ψ is positive definite in $\mathbf{x} = \mathbf{0}$, if $\Psi(\mathbf{0}) = 0$, and for all $\mathbf{x} \in \mathbb{R}^p$, $\mathbf{x} \neq \mathbf{0}$ we have that $\Psi(\mathbf{x}) > 0$.

[3] A function Ψ is radially unbounded, if $\lim_{\|\mathbf{x}\|\to\infty} \Psi(\mathbf{x}) = \infty$.

[4] A $\mathcal{KL}$-function $\phi(r, s)$ is a function defined as $\phi : [0, \infty) \times [0, \infty) \to [0, \infty)$, such that, given s, the mapping $\phi(r, s)$ is positive definite in the origin and strictly monotonically increasing in r and, conversely, given r, the mapping $\phi(r, s)$ is monotonically decreasing in s and $\phi(r, s) \to 0$ as $s \to \infty$.

A global Lyapunov function is strictly monotonically decreasing along the trajectories of the dynamical system. The formulation of the Lyapunov theorem we use is the following [25].

Theorem 5.5 *(Lyapunov stability theorem) If a global Lyapunov function Ψ exists for a discrete dynamical system, then such dynamical system is asymptotically uniformly globally stable.*

We apply this theorem to the dynamical system (3). In our case, $p = 1$. We choose a function $\Psi(x) = x$ and we determine the constraints under which such a function is a global Lyapunov function for (3). We preliminarily need the following definition (where $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^m$).

Definition 5.6 The scalar quantity $\Gamma_i^{t+1} = \frac{\left\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{a}^t\right\|}{\left\|\mathbf{p}_{i\to e}^{t} - \mathbf{a}^t\right\|}$ is called *contraction factor* for the agent i at time $t+1$.

The contraction factor relates the offers of agent i produced at times t and $t+1$ to the agreement computed at time t. More precisely, given an agent i, if its offer at time $t+1$ is closer to the agreement $\mathbf{a}^t$ than its offer at time t, then Γ_i^{t+1} is smaller than 1. Conversely, if the offer of agent i at time $t+1$ is farther from the agreement $\mathbf{a}^t$ than its offer at time t, then Γ_i^{t+1} is larger than 1. Finally, if the distances between the offers of agent i at t and $t+1$ and the agreement $\mathbf{a}^t$ are equal, then Γ_i^{t+1} is 1. Summarily, the contraction factor expresses *if* and *how much* the offers of the agent i get closer to the agreement. It could be said that the contraction factor measures the cooperative attitude of an agent. Note that the distance between an offer and an agreement is computed in $\mathbb{R}^m$. Note also that, rigourously speaking, in the definition of the contraction factor we should have used the counter-offer $\mathbf{p}_{e\to i}^t$ instead of $\mathbf{a}^t$. However, for better clarity and given that we assume $\mathbf{p}_{e\to i}^t = \mathbf{a}^t$ (see Sect. 3), we use $\mathbf{a}^t$.

Considering the dynamical system (3), Theorem 5.5, and $\Psi(x) = x$ we can prove the following theorem (proof is reported in Appendix A.2).

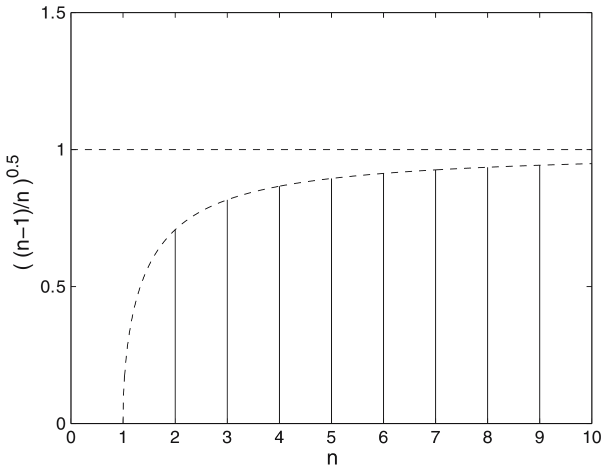
Theorem 5.7 *Given n agents, if $\forall\, t > t_0$ it is $\Gamma_i^t < \sqrt{\frac{n-1}{n}}$ for all i, then the dynamical system (3) is asymptotically uniformly globally stable in $x = 0$ (under the hypotheses of Theorem 5.3).*

Under the hypotheses of the above theorem, the negotiation associated to the dynamical system (3) is stable for all the initial offers of the agents (global stability) and for any initial time t_0 (uniform stability). Notice that the constraints are relative to single agents. In particular, if every agent i independently satisfies the constraint $\Gamma_i^t < \sqrt{\frac{n-1}{n}}$ of Theorem 5.7, the negotiation is stable. The verification of the constraints can be carried out locally by the agents, without requiring that agents share their negotiation functions.

The threshold $\sqrt{\frac{n-1}{n}}$ depends on the number n of agents taking part to the negotiation. This makes the stability of the negotiation of Theorem 5.7 not connective. In Fig. 1, the function $\sqrt{\frac{n-1}{n}}$ is shown. Interestingly, the constraint on Γ_i^t is less strict the larger the number of agents.

The state $x^t = \sum_{i=1}^n \sum_{j=1}^n \left\|\mathbf{p}_{i\to e}^t - \mathbf{p}_{j\to e}^t\right\|^2$ has an interesting geometric interpretation: it is twice the sum of the squared length of the vectors composing the skeleton of the polyhedron whose vertexes are the $\mathbf{p}_{i\to e}^t$s. An example of this skeleton associated to x^t is shown in Fig. 2(a). In this example, $m = 2$, namely the $\mathbf{p}_{i\to e}^t$s are points in a bidimensional space,

Fig. 1 The function $\sqrt{\frac{n-1}{n}}$



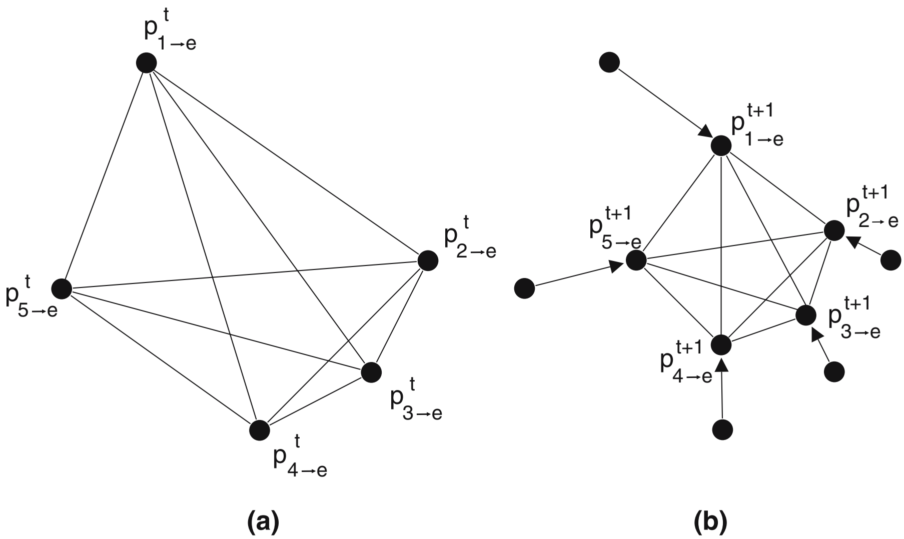


Fig. 2 The skeletons associated to x^t **(a)** and to x^{t+1} **(b)**

and $n = 5$, namely there are five points. The fact that $\Psi(x) = x$ is a global Lyapunov function (under the hypotheses of Theorem 5.7) means that $\Psi(x) = x$ is strictly monotonically decreasing in time. In turn, this means that the skeleton associated to x^t "reduces" at each time step of the negotiation as effect of the new offers of the agents, as shown in the example of Fig. 2b. In this way, if the negotiation is stable, the state of the associated dynamical system reaches the equilibrium point $x = 0$, and the skeleton collapses in a single point.

Different values of the threshold of the constraint on Γ_i^t identify different circular regions, with $\mathbf{a}^t$ as common center, where the new offer $\mathbf{p}_{i\to e}^{t+1}$ of the agent i is allowed to be located in order to keep the negotiation stable (Fig. 3 reports some examples relative to a situation in which $m = 2$). The larger the value of the threshold, the larger the region where the new offer of the agent i can be located. In order to give more freedom to agents in selecting their

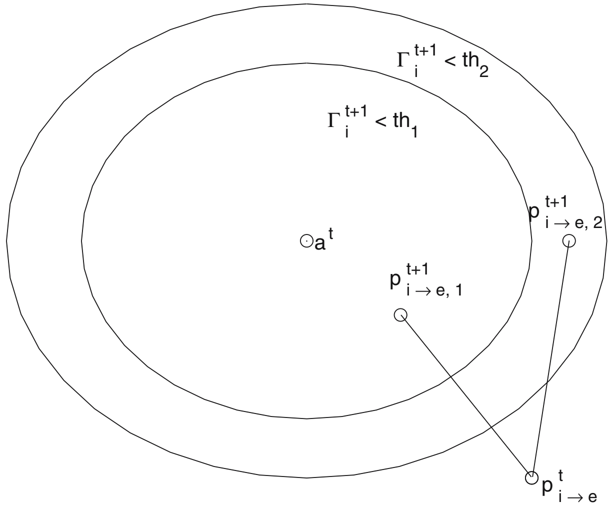


Fig. 3 Examples of offers according to different thresholds of Γ_i^t: $\mathbf{p}_{i\to e,1}^{t+1}$ is offered according to $\Gamma_i^{t+1} < th_1$ with $th_1 < 1$, while $\mathbf{p}_{i\to e,2}^{t+1}$ is offered according to $\Gamma_i^{t+1} < th_2$ with $th_1 < th_2 < 1$

optimal offers (without losing stability), it is desirable to have large values of the threshold of the constraint on Γ_i^t.

Surprisingly, both the connective stability of the negotiation and the relaxation of the constraints on Γ_i^t go in parallel as can be seen in the following theorem that constitutes one of the main results of this paper (proof is reported in Appendix A.3).

Theorem 5.8 *Given a negotiation $\mathcal{N}$ and given any $\epsilon \in \mathbb{R}$, $\epsilon > 0$, if $\forall\, i\ \forall\, t > t_0$ it is $\Gamma_i^t < 1 - \epsilon$, then $\mathcal{N}$ is connectively stable for all the initial offers of the agents and for any initial time t_0 (under the hypotheses of Theorem 5.3).*

Differently from Theorem 5.7, the above theorem determines a set of constraints that are not only locally-verifiable but are also independent of the number of agents. In this sense, Theorem 5.8 provides a sufficient criterion to study the connective stability of a negotiation carried on according to the protocol of Sect. 3. Conversely, if a negotiation $\mathcal{N}$ is designed in order to satisfy the hypotheses of Theorem 5.8, then $\mathcal{N}$ is connectively stable.

We now relax some hypotheses of Theorem 5.8. In particular, a negotiation is connectively stable also when $\Gamma_i^t < 1 - \epsilon$ for all i and for all $t > t_c$, with $t_c > t_0$. This fact is stated in the following theorem (proof trivially follows from the uniform and global stability established by Theorem 5.7).

Theorem 5.9 *Given a negotiation $\mathcal{N}$ and given any $\epsilon \in \mathbb{R}$, $\epsilon > 0$, if $\exists\, t_c$, $t_c > t_0$, such that $\forall i\ \forall\, t > t_c$ it is $\Gamma_i^t < 1 - \epsilon$, then the negotiation $\mathcal{N}$ is connectively stable for all the initial offers of the agents and for any initial time t_0 (under the hypotheses of Theorem 5.3).*

Theorem 5.9 allows agents to be free in selecting their optimal offers at the beginning of the negotiation (from t_0 to t_c), while constraining them to converge to an agreement only in the rest of the negotiation (from t_c on). This fact can be exploited in designing connectively stable negotiations. Note that t_c is the time from which the convergence begins.

According to the above results, we can identify a class of negotiation functions that make the negotiation connectively stable. Preliminarily, we provide the following definition.

Definition 5.10 A negotiation function $\mathcal{F}_i$ is *cooperative* when, given any $\epsilon \in \mathbb{R}$, $\epsilon > 0$, $\mathcal{F}_i$ returns offers that satisfy $\Gamma_i^t < 1 - \epsilon$, for all $t > t_c$.

Then, we define the class of *consistent and cooperative negotiation functions* (CCNFs) that is composed of negotiation functions that are both consistent and cooperative.

Note that, given a negotiation $\mathcal{N} = \langle \{\langle \mathcal{U}_i, \mathcal{F}_i \rangle\}_{i \in [1,n]}, \mathcal{A}, t_0 \rangle$, if all the $\mathcal{F}_i$s are CCNFs (and both $\mathcal{A}$ is idempotent and $A \neq \emptyset$), then $\mathcal{N}$ is connectively stable.

In the following section, we discuss some examples in which the proposed stability framework can be employed.

6 Some examples of application of the proposed framework

In this section, we show some applications of the proposed framework. They are intended to preliminarily validate our approach and to show how it can be employed. In the first three examples, we consider some negotiations and we analyze their connective stability. In particular, we find the conditions under which Theorem 5.8 can be applied. In the fourth example, we show how to design a connectively stable negotiation by defining appropriate negotiation functions. In this way, we cover a range of possible uses of our framework, from analysis to design.

6.1 Application to the negotiation decision functions

We consider a cooperative negotiation setting where agents negotiate according to the negotiation decision functions proposed in [12] and the agreement function $\mathcal{A}$ embedded by the mediator is the average of agents' offers. Note that $\mathcal{A}$ is idempotent. Although the negotiation decision function paradigm has been originally developed for electronic commerce (i.e., a competitive situation), here we cast it in a cooperative negotiation setting in order to study its stability.

In this case, there are two agents ($n = 2$), a buyer b and a seller s, that negotiate over a single issue ($m = 1$) and that have (strictly monotonic) utility functions $\mathcal{U}_i : \mathbb{R} \to \mathbb{R}$ defined as $\mathcal{U}_b(x) = \mathrm{RP}_b - x$ and $\mathcal{U}_s(x) = x - \mathrm{RP}_s$ (where RP_i is the offer with minimal utility for agent i). Called IP_i the initial offer of agent i ($i \in \{b, s\}$), agent i produces its next offer according to the following negotiation function:

$$\mathcal{F}_i(p^t_{e\to i}, t) = \begin{cases} 0 & \text{if } \mathcal{U}_i(p^t_{e\to i}) \geq \mathcal{U}_i(\mathrm{IP}_i + \phi_i(t+1)(\mathrm{RP}_i - \mathrm{IP}_i)) \\ \mathrm{IP}_i + \phi_i(t+1)(\mathrm{RP}_i - \mathrm{IP}_i) & \text{otherwise} \end{cases} \tag{4}$$

where $\phi_i(t)$ is a monotonically increasing function of time that returns values in [0, 1] and is defined as:

$$\phi_i(t) = \kappa_i + (1 - \kappa_i)\left(\frac{\min(t, T_i)}{T_i}\right)^{\frac{1}{\psi_i}} \tag{5}$$

where κ_i, ψ_i are parameters of the agent i and T_i is the temporal deadline of the negotiation. Agents accept the received counter-offer if $\mathcal{U}_i(p^t_{e\to i}) \geq \mathcal{U}_i(p^{t+1}_{i\to e})$.

Given by the monotonicity of (5) for $t_0 \leq t \leq T_i$, it is trivial to see that negotiation functions (4) are CCNFs and that $t_c = t_0$. Hence, since the $\mathcal{F}_i$s are CCNFs and $\mathcal{A}$ is idempotent, the negotiation is guaranteed to be stable and an agreement is eventually reached (provided that T_is are large enough or infinite).

6.2 Application to the yield factor based negotiation functions

We consider a negotiation setting similar to that of the previous section, but with agents using yield factor based negotiation functions [13]. In this case, $\mathcal{U}_i : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$; for example, the utility function of buyer agent b is defined as:

$$\mathcal{U}_b(x, t) = (\delta_b)^t (\mathrm{RP}_b - x),$$

Agent i ($i \in \{b, s\}$) produces its next offer according to the following negotiation function:

$$\mathcal{F}_i(p^t_{e\to i}, t) = \begin{cases} 0 & \text{if } \mathcal{U}_i(p^t_{e\to i}) \geq \mathcal{U}_i\left(\mathrm{RP}_i \cdot \left(1 - \frac{1-y^{t+1}}{(\delta_i)^{t+1}}\right)\right) \\ \mathrm{RP}_i \cdot \left(1 - \frac{1-y^{t+1}}{(\delta_i)^{t+1}}\right) & \text{otherwise} \end{cases} \tag{6}$$

where y^t is the yield factor and δ_i is a parameter of agent i.

When, given any $\epsilon \in \mathbb{R}$, $\epsilon > 0$, it is $y^{t+1} > 1 - \delta_i(1 - y^t) + \epsilon$, negotiation functions (6) are CCNFs. To show it, we firstly note that the $\mathcal{F}_i$s (6) are trivially consistent. Then, we consider the next offer of the buyer agent b, it can be written as:

$$p^{t+1}_{b\to e} = \mathrm{RP}_b \cdot \left(1 - \frac{1 - y^{t+1}}{(\delta_b)^{t+1}}\right) \tag{7}$$

By construction (obviously being $p^t_{b\to e} \leq p^t_{s\to e}$ and $\mathcal{A}$ producing a^t as the average of $p^t_{b\to e}$ and $p^t_{s\to e}$), $a^t \geq p^t_{b\to e}$. Thus we can write: $\left|p^t_{b\to e} - a^t\right| = a^t - p^t_{b\to e}$. In order for $\mathcal{F}_b$ to be CCNF it must be cooperative: $\Gamma^{t+1}_b < 1 - \epsilon$. This amounts to say that $a^t - p^{t+1}_{b\to e} < a^t - p^t_{b\to e} - \epsilon$, then $p^{t+1}_{b\to e} > p^t_{b\to e} + \epsilon$. Using (7):

$$\mathrm{RP}_b \cdot \left(1 - \frac{1 - y^{t+1}}{(\delta_b)^{t+1}}\right) > \mathrm{RP}_b \cdot \left(1 - \frac{1 - y^t}{(\delta_b)^t}\right)$$

$$+ \epsilon y^{t+1} > 1 - \delta_b(1 - y^t) + \epsilon \tag{8}$$

In a similar way it can be shown that a constraint similar to (8) holds for the negotiation function of the seller agent s to be CCNF.

Hence, since the $\mathcal{F}_i$s are CCNFs (under the above constraints) and $\mathcal{A}$ is idempotent, the negotiation is guaranteed to be stable.

6.3 Application to the negotiation functions for combining physiological control models

In previous work, we have proposed cooperative negotiation as a way to combine several control models of a single physiological process. In particular, a cooperative negotiation approach has been used in [2] to combine control models of the glucose-insulin metabolism in a diabetic patient and in [1,17] to combine control models of the heart frequency regulation.

In both cases, there are n agents negotiating over m issues. Each agent i is associated with a control model described as a utility function $\mathcal{U}_i : \mathbb{R}^m \to \mathbb{R}$. An utility function $\mathcal{U}_i$ describes how well the control model of agent i mimics the physiological process.

The agreement function $\mathcal{A}$ is the weighted average of the offers of the agents, formally:

$$\mathbf{a}^t = \frac{\sum_{i=1}^n \mathbf{p}^t_{i\to e} \cdot \omega^t_i}{\sum_{i=1}^n \omega^t_i} \tag{9}$$

Note that $\mathcal{A}$ is idempotent.

To determine the next offer, an agent i considers two components: one selfish and one cooperative. The next offer of agent i is produced according to the following negotiation function:

$$\mathbf{p}_{i\to e}^{t+1} = \mathcal{F}_i(\mathbf{p}_{i\to e}^t, \mathbf{p}_{e\to i}^t) = \mathbf{p}_{i\to e}^t + \left\|\mathbf{p}_{e\to i}^t - \mathbf{p}_{i\to e}^t\right\| \left(\alpha_i \mathbf{u}_i^{t+1} + \beta_i \mathbf{w}_i^{t+1}\right) \tag{10}$$

where α_i and β_i are called *negotiation parameters*, $\|\cdot\|$ is the norm in $\mathbb{R}^m$, and $\mathbf{u}_i^{t+1}$ and $\mathbf{w}_i^{t+1}$ are two versors[5] in $\mathbb{R}^m$. In brief, $\mathbf{u}_i^{t+1}$ identifies the direction of the line connecting $\mathbf{p}_{i\to e}^t$ to $\mathbf{p}_{e\to i}^t$ (and so to $\mathbf{a}^t$), $\mathbf{w}_i^{t+1}$ identifies the direction of the tangent to the level curve $\mathcal{U}_i(\mathbf{x}) = \mathcal{U}_i(\mathbf{p}_{i\to e}^t)$ in $\mathbf{p}_{i\to e}^t$. For the purposes of this paper we need only the formal definition of $\mathbf{u}_i^{t+1}$ (refer to [17] for the formal definition of $\mathbf{w}_i^t$):

$$\mathbf{u}_i^{t+1} = \frac{\mathbf{p}_{e\to i}^t - \mathbf{p}_{i\to e}^t}{\|\mathbf{p}_{e\to i}^t - \mathbf{p}_{i\to e}^t\|} \tag{11}$$

By applying Theorem 5.8, we can determine the ranges of the pairs of values of the negotiation parameters (α_i, β_i) that assure the connective stability of the negotiation.

Theorem 6.1 *The negotiation functions* (10) *are CCNFs if for all i:*

$$\begin{aligned} &(0 < \beta_i < 1 - \epsilon \quad \wedge \quad \epsilon + \beta_i < \alpha_i < 2 - \epsilon - \beta_i) \\ \vee &(-1 + \epsilon < \beta_i < 0 \quad \wedge \quad \epsilon - \beta_i < \alpha_i < 2 - \epsilon + \beta_i) \end{aligned}$$

Proof is reported in Appendix A.4. From the definition of $\mathcal{A}$ in (9), when the constraints of Theorem 6.1 are satisfied, the negotiation is connectively stable (by Theorem 5.8). This means that, when the constraints of Theorem 6.1 are satisfied, our decentralized control system provides a stable control action independently of the agents (and of the control models they embed) connected to the system. This is to say that the control system is both decentralized and reconfigurable.

6.4 Application to the decentralized Pareto optimality negotiation functions

In [19] we have proposed a class of negotiation functions to produce approximate Pareto optimal agreements between two agents (hence, $n = 2$) that operate over m issues. The negotiation functions $\mathcal{F}_i$s are defined as follows:

$$\mathcal{F}_i(\mathbf{p}_{i\to e}^t, \mathbf{p}_{e\to i}^t, t) = \mathbf{p}_{i\to e}^t + \delta \cdot \|\mathbf{p}_{e\to i}^t - \mathbf{p}_{i\to e}^t\| \cdot \frac{\mathbf{o}_i^t - \mathbf{p}_{i\to e}^t}{\|\mathbf{o}_i^t - \mathbf{p}_{i\to e}^t\|}$$

where $\delta \in [0, 1]$ and the versor $\mathbf{o}_i^t$ is defined as:

$$\mathbf{o}_i^t = \left(\left(\mathbf{p}_{e\to i}^t - \mathbf{t}_i^t\right) \cdot \mathbf{v}_i^t\right) \mathbf{v}_i^t + \mathbf{t}_i^t$$

with the versors $\mathbf{t}_i^t$ and $\mathbf{v}_i^t$ defined as:

$$\mathbf{t}_i^t : \begin{cases} \left| \dfrac{\nabla \mathcal{U}_i(\mathbf{t}_i^t)}{\|\nabla \mathcal{U}_i(\mathbf{t}_i^t)\|} \cdot \mathbf{v}_i^t \right| = 1 \\ \mathcal{U}_i(\mathbf{t}_i^t) = \mathcal{U}_i(\mathbf{p}_{i\to e}^t) \end{cases}$$

[5] A versor is a vector with unitary norm.

$$\mathbf{v}_i^t = \frac{\mathbf{p}_{e\to i}^t - \mathbf{p}_{i\to e}^t}{\|\mathbf{p}_{e\to i}^t - \mathbf{p}_{i\to e}^t\|}$$

The agreement function $\mathcal{A}$ is the arithmetic mean of the offers made on each variable.

We now show how to apply our connective stability criterion to make the above $\mathcal{F}_i$s CCNFs. Initially an agent i produces a tentative proposal $\tilde{\mathbf{p}}_{i\to e}^{t+1}$ using $\mathcal{F}_i$ as defined above; if $\tilde{\mathbf{p}}_{i\to e}^{t+1}$ satisfies $\Gamma_i^t < 1-\epsilon$, it will be the proposal $\mathbf{p}_{i\to e}^{t+1}$ of the agent. Otherwise, the agent produces a new proposal $\mathbf{p}_{i\to e}^{t+1}$ according to three constraints: (1) it is the closest one to the tentative proposal $\tilde{\mathbf{p}}_{i\to e}^{t+1}$, (2) it is located at a distance $\delta \cdot \|\mathbf{p}_{e\to i}^t - \mathbf{p}_{i\to e}^t\|$ from the previous proposal $\mathbf{p}_{i\to e}^t$, and (3) it satisfies $\Gamma_i^t < 1-\epsilon$.

Algorithmically, the proposal of agent i, in the case the tentative proposal $\tilde{\mathbf{p}}_{i\to e}^{t+1}$ does not satisfy $\Gamma_i^t < 1-\epsilon$, is produced as follows. We calculate the angle θ_i^t between $\tilde{\mathbf{p}}_{i\to e}^{t+1}$ and the vector $(\mathbf{p}_{e\to i}^t - \mathbf{p}_{i\to e}^t)$ as:

$$\theta_i^t = \arccos\left(\mathbf{v}_i^t \cdot \frac{\mathbf{o}_i^t - \mathbf{p}_{i\to e}^t}{\|\mathbf{o}_i^t - \mathbf{p}_{i\to e}^t\|}\right)$$

Called ψ_i^t the angle between the proposal $\mathbf{p}_{i\to e}^{t+1}$ and the vector $(\mathbf{p}_{e\to i}^t - \mathbf{p}_{i\to e}^t)$, the proposal $\mathbf{p}_{i\to e}^{t+1}$ of the agent has module given by the constraint (2) above and direction given by the solution of the minimization problem $\psi_i^t = \arg\min_{\psi} |\psi - \theta_i^t|$ with the three constraints discussed above. Formally:

$$\psi_i^t = \begin{cases} \arg\min\limits_{\psi} |\psi - \theta_i^t| \\ (1)\ \text{ s.t. } \tilde{\mathbf{o}}_i^t = \tan(\psi) \cdot \|\mathbf{p}_{e\to i}^t - \mathbf{p}_{i\to e}^t\| \cdot \dfrac{\mathbf{o}_i^t - \mathbf{p}_{e\to i}^t}{\|\mathbf{o}_i^t - \mathbf{p}_{e\to i}^t\|} \\ (2)\ \text{ s.t. } \mathbf{p}_{i\to e}^{t+1} = \delta \cdot \|\mathbf{p}_{e\to i}^t - \mathbf{p}_{i\to e}^t\| \cdot \dfrac{\tilde{\mathbf{o}}_i^t - \mathbf{p}_{i\to e}^t}{\|\tilde{\mathbf{o}}_i^t - \mathbf{p}_{i\to e}^t\|} \\ (3)\ \text{ s.t. } \dfrac{\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{p}_{e\to i}^t\|}{\|\mathbf{p}_{i\to e}^t - \mathbf{p}_{e\to i}^t\|} < 1-\epsilon \end{cases}$$

where $\tilde{\mathbf{o}}_i^t$ is a point of the vector $(\mathbf{p}_{e\to i}^t - \mathbf{o}_i^t)$. Practically, we can change the value of ψ from $|\psi - \theta_i^t| = 0$ up, until we find the value of the angle ψ_i^t (the closest to θ_i^t by construction) that satisfies the three given constraints.

Imposing the constraint $\Gamma_i^{t+1} < 1-\epsilon$ generally makes the agreement sub-optimal. However, the optimality can be restored by leaving agents to converge from a time $t_c > t_0$ (Theorem 5.9). The connective stability guaranteed by the negotiation functions we designed allows the reconfigurability of the agent network.

7 Conclusions and future works

The stability of cooperative negotiations is widely considered an important and relevant issue to address [33]. Stability is indeed decisive in controlling complex systems via cooperative negotiation. In particular, the concept of connective stability appears to be very relevant for cooperative negotiations devoted to control complex systems, where the composition of the agent network is usually dynamic. In this paper, we have proposed a formal framework for connectively stable cooperative negotiations. Specifically, we have presented a protocol that describes a class of cooperative negotiations and a criterion to make these negotiations connectively stable. The criterion provides a set of stability constraints that refer individually to each agent.

The study of stability of cooperative negotiations raises several issues that deserve further investigations. In particular, it could be interesting both to enrich the proposed framework and to employ it to address more practical applications. Two theoretical improvements for the framework appear interesting: the first one concerns the relaxation of the connective stability constraints to provide more freedom to the agents in situations in which the amount of mutual knowledge is larger; the second one concerns the study (starting from some initial results presented in [19]) of the optimality and quality of the agreement under temporal constraints (i.e., the real-time convergence to an optimal agreement). Two practical applications that could be addressed by employing our framework are related to ad hoc network management and vehicle formation. In addressing these applications, it will be interesting to investigate some practical aspects like, for example, the time it takes to reach an agreement under given conditions. Finally, a further interesting issue for future works is to study the relation between a cooperative negotiation approach (like the one adopted in this paper) and a distributed constraint optimization approach (like the one presented in [30]), when both approaches can be applied to control a complex system.

Acknowledgement The authors are glad to thank Francesco Di Giunta for the fruitful discussions and the two anonymous reviewers for helping in improving the presentation of the paper.

Appendix

A Proofs of the Theorems

A.1 Proof of the Theorem 5.3

The point $x = 0$ is an equilibrium point for the dynamical system (3) when: if $x^{\bar{t}} = 0$ then $\forall\, t > \bar{t}$, $x^t = 0$. Let us first consider $t = \bar{t}$, we have that $x^{\bar{t}} = 0$ when:

$$0 = \sum_{i=1}^{n} \sum_{j=1}^{n} \left\| \mathbf{p}^{\bar{t}}_{i \to e} - \mathbf{p}^{\bar{t}}_{j \to e} \right\|^2 \tag{12}$$

Notice that (12) holds if and only if there exists some $\mathbf{p}^{\bar{t}} \in A$ such that $\forall\, i, j$ $\mathbf{p}^{\bar{t}}_{i \to e} = \mathbf{p}^{\bar{t}}_{j \to e} = \mathbf{p}^{\bar{t}}$. This can happen, obviously, if the agreement set A is not empty. Let us now consider $t = \bar{t} + 1$. Given that $\mathcal{A}$ is idempotent, $\mathcal{A}\left(\{\mathbf{p}^{\bar{t}}\}_{i \in [1,n]}\right) = \mathbf{p}^{\bar{t}}$, and, given that, for all i, $\mathcal{F}_i$ is consistent, $\mathbf{p}^{\bar{t}+1}_{i \to e} = \mathcal{F}_i\left(\mathbf{p}^{\bar{t}}, \mathbf{p}^{\bar{t}}, \bar{t}\right) = \mathbf{p}^{\bar{t}}$ for all i. Thus, by (3), $x^{\bar{t}+1} = 0$. It is trivial to see that $x^t = 0$ also for all $t > \bar{t}$, and thus $x = 0$ is an equilibrium point of the dynamical system (3). □

A.2 Proof of the Theorem 5.7

We consider the candidate Lyapunov function $\Psi(x) = x$. We look for the constraints under which Ψ is a global Lyapunov function satisfying the conditions (i)–(iv) in Definition 5.4 and thus, by Theorem 5.5, under which the dynamical system (3) is asymptotically uniformly globally stable.

(i) $\Psi(x) = x$ is Lipschitz since $\forall x_1, x_2 \in \mathbb{R}, \exists\, k \in \mathbb{R} : \ |\Psi(x_1) - \Psi(x_2)| = \|x_1 - x_2\| \leq k\|x_1 - x_2\|$. In particular, any $k \geq 1$ satisfies the inequality.

(ii) $\Psi(x) = x$ is positive definite in $x = 0$ by definition of x^t, see (2). In particular, $\Psi(x) = x$ is a sum of positive terms and it is equal to 0 only in $x = 0$.

(iii) $\Psi(x) = x$ is trivially radially unbounded, being $\lim\limits_{\|x\|\to\infty} x = \infty$.

(iv) To verify this condition we (iv.a) firstly study when $\Psi(x) = x$ is strictly monotonically decreasing over time t but $x = 0$; then we (iv.b) exploit this result to study when there exists a $\mathcal{KL}$-function ϕ that satisfies $\Psi(x^{t+1}) - \Psi(x^t) \leq -\phi(\|x^{t+1}\|, t - t_0)$. Note that (iv.a) is a more relaxed condition than (iv.b) but its study helps in the study of (iv.b).

(iv.a) In general, $\Psi(x) = x$ is not strictly monotonically decreasing over time t because its values depend on x^t. In order to guarantee that Ψ is strictly monotonically decreasing (except in $x = 0$), we must determine the constraints under which $\Psi\left(x^{t+1}\right) - \Psi\left(x^t\right) < 0$. We can write the expression of $\Psi\left(x^t\right)$, introducing the agreement $\mathbf{a}^t$:

$$
\begin{aligned}
\Psi\left(x^t\right) = x^t &= \sum_{i=1}^{n}\sum_{j=1}^{n} \left\|\mathbf{p}^t_{i\to e} - \mathbf{p}^t_{j\to e}\right\|^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \left\|\left(\mathbf{p}^t_{i\to e} - \mathbf{a}^t\right) - \left(\mathbf{p}^t_{j\to e} - \mathbf{a}^t\right)\right\|^2 \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} \left(\left\|\mathbf{p}^t_{i\to e} - \mathbf{a}^t\right\|^2 + \left\|\mathbf{p}^t_{j\to e} - \mathbf{a}^t\right\|^2 - 2\left(\mathbf{p}^t_{i\to e} - \mathbf{a}^t\right)\cdot\left(\mathbf{p}^t_{j\to e} - \mathbf{a}^t\right)\right) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} \left\|\mathbf{p}^t_{i\to e} - \mathbf{a}^t\right\|^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} \left\|\mathbf{p}^t_{j\to e} - \mathbf{a}^t\right\|^2 \\
&\quad -2\sum_{i=1}^{n}\sum_{j=1}^{n} \left(\mathbf{p}^t_{i\to e} - \mathbf{a}^t\right)\cdot\left(\mathbf{p}^t_{j\to e} - \mathbf{a}^t\right) \\
&= n\sum_{i=1}^{n} \left\|\mathbf{p}^t_{i\to e} - \mathbf{a}^t\right\|^2 + n\sum_{j=1}^{n} \left\|\mathbf{p}^t_{j\to e} - \mathbf{a}^t\right\|^2 \\
&\quad -2\sum_{i=1}^{n} \left(\mathbf{p}^t_{i\to e} - \mathbf{a}^t\right)\cdot\sum_{j=1}^{n}\left(\mathbf{p}^t_{j\to e} - \mathbf{a}^t\right) \\
&= 2n\sum_{i=1}^{n} \left\|\mathbf{p}^t_{i\to e} - \mathbf{a}^t\right\|^2 - 2\left\|\sum_{i=1}^{n}\left(\mathbf{p}^t_{i\to e} - \mathbf{a}^t\right)\right\|^2
\end{aligned}
\tag{13}
$$

Working in a similar way on the expression of $\Psi\left(x^{t+1}\right)$, we obtain:

$$
\Psi\left(x^{t+1}\right) = 2n\sum_{i=1}^{n} \left\|\mathbf{p}^{t+1}_{i\to e} - \mathbf{a}^t\right\|^2 - 2\left\|\sum_{i=1}^{n}\left(\mathbf{p}^{t+1}_{i\to e} - \mathbf{a}^t\right)\right\|^2 \tag{14}
$$

$\Psi\,(x) = x$ is strictly monotonically decreasing when:

$$
\Psi\left(x^{t+1}\right) - \Psi\left(x^t\right) < 0
$$

From (13) and (14):

$$\Psi\left(x^{t+1}\right)-\Psi\left(x^{t}\right)= 2n\sum_{i=1}^{n}\left(\left\|\mathbf{p}_{i\to e}^{t+1}-\mathbf{a}^{t}\right\|^{2}-\left\|\mathbf{p}_{i\to e}^{t}-\mathbf{a}^{t}\right\|^{2}\right)$$
$$-2\left\|\sum_{i=1}^{n}\left(\mathbf{p}_{i\to e}^{t+1}-\mathbf{a}^{t}\right)\right\|^{2}+2\left\|\sum_{i=1}^{n}\left(\mathbf{p}_{i\to e}^{t}-\mathbf{a}^{t}\right)\right\|^{2}$$

Writing $\left\|\mathbf{p}_{i\to e}^{t+1}-\mathbf{a}^{t}\right\|$ as $\Gamma_i^{t+1}\cdot\left\|\mathbf{p}_{i\to e}^{t}-\mathbf{a}^{t}\right\|$ (recall Definition 5.6), the above expression becomes:

$$\Psi\left(x^{t+1}\right)-\Psi\left(x^{t}\right)= 2n\sum_{i=1}^{n}\left(\left(\left(\Gamma_i^{t+1}\right)^{2}-1\right)\left\|\mathbf{p}_{i\to e}^{t}-\mathbf{a}^{t}\right\|^{2}\right)$$
$$-2\left\|\sum_{i=1}^{n}\left(\mathbf{p}_{i\to e}^{t+1}-\mathbf{a}^{t}\right)\right\|^{2}+2\left\|\sum_{i=1}^{n}\left(\mathbf{p}_{i\to e}^{t}-\mathbf{a}^{t}\right)\right\|^{2}$$

Since $\left\|\sum_{i=1}^{n}\left(\mathbf{p}_{i\to e}^{t+1}-\mathbf{a}^{t}\right)\right\|^{2}$ is, by definition, a positive (or zero) quantity:

$$\Psi\left(x^{t+1}\right)-\Psi\left(x^{t}\right)\le 2n\sum_{i=1}^{n}\left(\left(\left(\Gamma_i^{t+1}\right)^{2}-1\right)\left\|\mathbf{p}_{i\to e}^{t}-\mathbf{a}^{t}\right\|^{2}\right)+2\left\|\sum_{i=1}^{n}\left(\mathbf{p}_{i\to e}^{t}-\mathbf{a}^{t}\right)\right\|^{2}$$

(by applying the triangular inequality to $\left\|\sum_{i=1}^{n}\left(\mathbf{p}_{i\to e}^{t}-\mathbf{a}^{t}\right)\right\|^{2}$)

$$\le 2n\sum_{i=1}^{n}\left(\left(\left(\Gamma_i^{t+1}\right)^{2}-1\right)\left\|\mathbf{p}_{i\to e}^{t}-\mathbf{a}^{t}\right\|^{2}\right)+2\sum_{i=1}^{n}\left\|\mathbf{p}_{i\to e}^{t}-\mathbf{a}^{t}\right\|^{2}$$
$$=2\sum_{i=1}^{n}\left\|\mathbf{p}_{i\to e}^{t}-\mathbf{a}^{t}\right\|^{2}\left(n\left(\left(\Gamma_i^{t+1}\right)^{2}-1\right)+1\right) \tag{15}$$

A sufficient condition for expression (15) to be strictly negative is:

$$n\left(\left(\Gamma_i^{t+1}\right)^{2}-1\right)+1<0 \text{ for all } i$$

More generally, when:

$$\forall\, i, t>t_0\ \ \Gamma_i^{t}<\sqrt{\frac{n-1}{n}} \tag{16}$$

$\Psi(x)=x$ is strictly monotonically decreasing along the system trajectories, satisfying (iv.a).

(iv.b) Finally, we must determine a $\mathcal{KL}$-function ϕ such that:

$$\Psi\left(x^{t+1}\right)-\Psi\left(x^{t}\right)\le-\phi\left(\left\|x^{t+1}\right\|,t-t_0\right)$$

By definition of $\mathcal{KL}$-function [25], $\phi\left(\|x^{t+1}\|,t-t_0\right)$ is a function defined as $\phi:[0,\infty)\times[0,\infty)\to[0,\infty)$, such that given a $t-t_0$ value, the mapping $\phi\left(\|x^{t+1}\|,t-t_0\right)$ is positive definite in the origin and strictly monotonically increasing in $\|x^{t+1}\|$, and, conversely, given $\|x^{t+1}\|$, the mapping $\phi\left(\|x^{t+1}\|,t-t_0\right)$ is monotonically decreasing in t and $\phi\left(\|x^{t+1}\|,t-t_0\right)\to 0$ as $t\to\infty$. Under the constraints (16), $\Psi\left(x^{t}\right)=x^{t}$ strictly monotonically decreases when t increases, or x^{t} strictly monotonically increases when t decreases. Thus, under the constraints (16), the fact that ϕ strictly monotonically increases in $\|x^{t+1}\|$ reduces to the fact that ϕ strictly monotonically decreases in t. In this way, we can study the behavior of ϕ only with respect to t. In particular, when ϕ is a $\mathcal{KL}$-function and constraints (16) are satisfied, it is:

$$\forall\, t > t_0 \;\; \phi\left(\left\|x^{t+1}\right\|, t+1-t_0\right) < \phi\left(\left\|x^{t+1}\right\|, t-t_0\right) < \phi\left(\left\|x^{t}\right\|, t-t_0\right)$$

With this consideration, we need to build a function ϕ such that ($\mathcal{KL}1$) ϕ is monotonically decreasing in t, ($\mathcal{KL}2$) $\phi \to 0$ as $t \to \infty$, and ($\mathcal{KL}3$) ϕ is positive definite in $\|x^{t+1}\| = 0$.

We call $\Delta\Psi = \Psi(x^{t+1}) - \Psi(x^t)$. From (15), we need to build a $\mathcal{KL}$-function ϕ such that:

$$\Delta\Psi \leq 2\sum_{i=1}^{n} \|\mathbf{p}_{i\to e}^{t} - \mathbf{a}^{t}\|^2 \left(n\left(\left(\Gamma_i^{t+1}\right)^2 - 1\right) + 1\right) \leq -\phi\left(\left\|x^{t+1}\right\|, t-t_0\right) \tag{17}$$

We define ϕ as follows:

$$\phi\left(\left\|x^{t+1}\right\|, t-t_0\right) = 2\sum_{i=1}^{n} \phi_i\left(\left\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{a}^{t}\right\|, t-t_0\right)$$

such that, writing (17) with respect to a single agent i:

$$2\,\|\mathbf{p}_{i\to e}^{t} - \mathbf{a}^{t}\|^2 \left(n\left(\left(\Gamma_i^{t+1}\right)^2 - 1\right) + 1\right) \leq -2\phi_i\left(\left\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{a}^{t}\right\|, t-t_0\right) \tag{18}$$

We impose some constraints on ϕ_i to make $\phi = \sum_{i=1}^{n}\phi_i$ to satisfy the conditions ($\mathcal{KL}1$)–($\mathcal{KL}3$).

- ($\mathcal{KL}1_i$) If, for all i, ϕ_i is strictly monotonically decreasing in t, then $\phi = \sum_{i=1}^{n}\phi_i$ is monotonically decreasing in t.
- ($\mathcal{KL}2_i$) If, for all i, $\phi_i \to 0$ as $t \to \infty$, then $\phi = \sum_{i=1}^{n}\phi_i \to 0$ as $t \to \infty$.
- ($\mathcal{KL}3_i$) If, for all i, ϕ_i is positive definite in $\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{a}^{t}\| = 0$, then $\phi = \sum_{i=1}^{n}\phi_i$ is definite positive in $x = 0$ (see (14)).

If we can build a generic function ϕ_i that satisfies (18) and conditions ($\mathcal{KL}1_i$)–($\mathcal{KL}3_i$), then, $\phi = \sum_{i=1}^{n}\phi_i$ will be a $\mathcal{KL}$-function.

We define ϕ_i as:

$$\begin{aligned}\phi_i\left(\left\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{a}^{t}\right\|, t-t_0\right) &= \left\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{a}^{t}\right\|^2 \frac{\varsigma_i^t}{t+1-t_0}\\ &= \left(\Gamma_i^{t+1}\right)^2 \|\mathbf{p}_{i\to e}^{t} - \mathbf{a}^{t}\|^2 \frac{\varsigma_i^t}{t+1-t_0}\end{aligned} \tag{19}$$

where ς_i^t is a time-dependent parameter that assures that ϕ_i satisfies (18) and the conditions ($\mathcal{KL}1_i$)–($\mathcal{KL}3_i$). Inequality (18) is satisfied if:

$$\varsigma_i^t \leq -\frac{n\left(\left(\Gamma_i^{t+1}\right)^2 - 1\right) + 1}{\left(\Gamma_i^{t+1}\right)^2} \cdot (t+1-t_0) \tag{20}$$

Condition ($\mathcal{KL}1_i$) is satisfied if:

$$\left(\Gamma_i^{t+1}\right)^2 \|\mathbf{p}_{i\to e}^{t} - \mathbf{a}^{t}\|^2 \frac{\varsigma_i^t}{t+1-t_0} < \left(\Gamma_i^{t}\right)^2 \left\|\mathbf{p}_{i\to e}^{t-1} - \mathbf{a}^{t-1}\right\|^2 \frac{\varsigma_i^{t-1}}{t-t_0}$$

From which we obtain the following condition on ς_i^t:

$$\varsigma_i^t < \frac{(\Gamma_i^t)^2}{(\Gamma_i^{t+1})^2} \cdot \frac{\left\| \mathbf{p}_{i\to e}^{t-1} - \mathbf{a}^{t-1} \right\|}{\left\| \mathbf{p}_{i\to e}^{t} - \mathbf{a}^{t} \right\|} \cdot \frac{t+1-t_0}{t-t_0} \cdot \varsigma_i^{t-1} \tag{21}$$

Since Γ_i^{t+1} is bounded in t (because we assume that the constraints (16) are satisfied) and $\|\mathbf{p}_{i\to e}^t - \mathbf{a}^t\|$ is bounded in t (because $\mathcal{F}_i$ and $\mathcal{A}$ unbounded in t are not reasonable), condition $(\mathcal{KL}2_i)$ $(\lim_{t\to\infty} \phi_i \to 0)$ is satisfied if ς_i^t is bounded in t. Combining (20), (21), and the boundedness of ς_i^t, we obtain (notice that the right term of inequality (21) is bounded until an agreement is reached):

$$\varsigma_i^t < \min\left(-\frac{n\left(\left(\Gamma_i^{t+1}\right)^2 - 1\right) + 1}{\left(\Gamma_i^{t+1}\right)^2} \cdot (t+1-t_0),\ \frac{\left(\Gamma_i^t\right)^2}{\left(\Gamma_i^{t+1}\right)^2} \frac{\left\| \mathbf{p}_{i\to e}^{t-1} - \mathbf{a}^{t-1} \right\|}{\left\| \mathbf{p}_{i\to e}^{t} - \mathbf{a}^{t} \right\|} \frac{t+1-t_0}{t-t_0} \varsigma_i^{t-1} \right)$$

Condition $(\mathcal{KL}3_i)$ is satisfied if $\phi_i > 0$ and equal to 0 only in $\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{a}^t\| = 0$. This happens, by (19), when $\varsigma_i^t > 0$. Then, considering the initial value $\varsigma_i^{t_0}$, we obtain the following recursive definition of ς_i^t:

$$\begin{cases} t = t_0,\ \varsigma_i^{t_0} < -\frac{n\left((\Gamma_i^{t_0+1})^2 - 1\right)+1}{(\Gamma_i^{t_0+1})^2} \\ t > t_0,\ 0 < \varsigma_i^t < \min\left(-\frac{n\left((\Gamma_i^{t+1})^2-1\right)+1}{(\Gamma_i^{t+1})^2} \cdot (t+1-t_0),\ \frac{(\Gamma_i^t)^2}{(\Gamma_i^{t+1})^2} \frac{\|\mathbf{p}_{i\to e}^{t-1} - \mathbf{a}^{t-1}\|}{\|\mathbf{p}_{i\to e}^{t} - \mathbf{a}^{t}\|} \frac{t+1-t_0}{t-t_0} \varsigma_i^{t-1} \right) \end{cases}$$

In conclusion, we have shown how to build, under the constraints (16), a $\mathcal{KL}$-function ϕ for our problem that satisfy (17). The condition (iv.b) is, thus, satisfied. This means that, under the constraints (16), the candidate Lyapunov function $\Psi(x) = x$ satisfies the hypotheses (i)–(iv) of the definition of global Lyapunov function. Then $\Psi(x) = x$ is a global Lyapunov function for the dynamical system (3).

By Theorem 5.5, the dynamical system (3) is thus asymptotically uniformly globally stable. □

A.3 Proof of the Theorem 5.8

Given a negotiation $\mathcal{N} = \langle \{\langle \mathcal{U}_i, \mathcal{F}_i \rangle\}_{i\in[1,n]}, \mathcal{A}, t_0 \rangle$, we can always build a negotiation $\mathcal{N}^* = \langle \{\mathcal{U}_i^*, \mathcal{F}_i^*\}_{i\in[1,r]}, \mathcal{A}^*, t_0 \rangle$ where $r > n$ and such that:

- $\forall i \in [1,n]\ \mathcal{U}_i^* = \mathcal{U}_i$ and $\mathcal{F}_i^* = \mathcal{F}_i$,
- $\forall i \in [n+1,r]\ \mathcal{U}_i^*$ and $\mathcal{F}_i^*$ are any function, and
- $\mathcal{A}^* = \mathcal{A} + \sum_{i=n+1}^r \omega_i \cdot \mathbf{p}_{i\to e}^t$ with $\omega_i = 0\ \forall i \in [n+1,r]$.

Notice that $\mathcal{N}$ and $\mathcal{N}^*$ are equivalent because they, when subject to the same constraints on Γ_i^t, reach the same agreement, since the $r-n$ "dummy" agents of $\mathcal{N}^*$ that augment $\mathcal{N}$ do not affect the offers and counter-offers produced during the negotiation process. Given a $\epsilon \in \mathbb{R}$, $\epsilon > 0$, there is an r large enough such that $\sqrt{\frac{r-1}{r}} > 1-\epsilon$. Thus, given a negotiation $\mathcal{N}$ and any ϵ, we can always build an equivalent negotiation $\mathcal{N}^*$ by adding a number of "dummy" agents large enough such that $\sqrt{\frac{r-1}{r}} > 1-\epsilon$. For Theorem 5.7, $\mathcal{N}^*$ is stable when

$\Gamma_i^t < \sqrt{\frac{r-1}{r}}$ and, in particular, when $\Gamma_i^t < 1-\epsilon$ ($\forall i$ and $\forall t > t_0$). Being $\mathcal{N}$ equivalent to $\mathcal{N}^*$, $\mathcal{N}$ is stable when $\Gamma_i^t < 1-\epsilon$ ($\forall i$ and $\forall t > t_0$). The constraint $\Gamma_i^t < 1-\epsilon$ is independent of the number of agents taking part into the negotiation. Thus, when $\Gamma_i^t < 1-\epsilon$ ($\forall i$ and $\forall t > t_0$), the negotiation $\mathcal{N}$ is connectively stable. □

A.4 Proof of the Theorem 6.1

Negotiation functions (10) are trivially consistent. We now study when they are cooperative. According to (10) and (11) we can write $\left\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{a}^t\right\|$ as follows (remember that $\mathbf{p}_{e\to i}^t = \mathbf{a}^t$):

$$\begin{aligned}\left\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{a}^t\right\| &= \left\|\mathbf{p}_{i\to e}^t + \alpha_i(\mathbf{a}^t - \mathbf{p}_{i\to e}^t) + \beta_i \mathbf{w}_i^{t+1}\cdot\left\|\mathbf{p}_{i\to e}^t - \mathbf{a}^t\right\| - \mathbf{a}^t\right\| \\ &= \left\|\left(\mathbf{p}_{i\to e}^t - \mathbf{a}^t\right)(1-\alpha_i) + \beta_i \mathbf{w}_i^{t+1}\cdot\left\|\mathbf{p}_{i\to e}^t - \mathbf{a}^t\right\|\right\|\end{aligned}$$

Squaring the previous equation and remembering that, being it a versor, $\mathbf{w}_i^{t+1}\cdot\mathbf{w}_i^{t+1} = 1$, we have:

$$\left\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{a}^t\right\|^2 = \left\|\mathbf{p}_{i\to e}^t - \mathbf{a}^t\right\|^2\left((1-\alpha_i)^2 + \beta_i^2 + 2\beta_i(\alpha_i - 1)\,\mathbf{u}_i^{t+1}\cdot\mathbf{w}_i^{t+1}\right)$$

and, finally:

$$\frac{\left\|\mathbf{p}_{i\to e}^{t+1} - \mathbf{a}^t\right\|^2}{\left\|\mathbf{p}_{i\to e}^t - \mathbf{a}^t\right\|^2} = (\Gamma_i^{t+1})^2 = (1-\alpha_i)^2 + \beta_i^2 + 2\beta_i(\alpha_i - 1)\,\mathbf{u}_i^{t+1}\cdot\mathbf{w}_i^{t+1}$$

The negotiation functions (10) are cooperative when, given any $\epsilon\in\mathbb{R}$, $\epsilon > 0$, it is $\Gamma_i^{t+1} < 1-\epsilon$, namely when:

$$\Gamma_i^{t+1} = \sqrt{(1-\alpha_i)^2 + \beta_i^2 + 2\beta_i(\alpha_i - 1)\mathbf{u}_i^{t+1}\cdot\mathbf{w}_i^{t+1}} < 1-\epsilon$$

We call θ_i^{t+1} the angle between $\mathbf{u}_i^{t+1}$ and $\mathbf{w}_i^{t+1}$, and we split the above inequality according to the sign of $\beta_i\cdot(\alpha_i - 1)$:

$$\begin{array}{ll}\beta_i\cdot(\alpha_i-1) > 0 & \cos(\theta_i^{t+1}) < \dfrac{1-(1-\alpha_i)^2-\beta_i^2}{2\beta_i\cdot(\alpha_i-1)} - \epsilon \\ \beta_i\cdot(\alpha_i-1) < 0 & \cos(\theta_i^{t+1}) > \dfrac{1-(1-\alpha_i)^2-\beta_i^2}{2\beta_i\cdot(\alpha_i-1)} - \epsilon \\ \beta_i = 0 & -1+\epsilon < (1-\alpha_i) < 1-\epsilon \\ \alpha_i = 1 & -1+\epsilon < \beta_i < 1-\epsilon\end{array}$$

Since $\cos(\cdot)$ has upper and lower limits, it is $-1 \le \cos(\theta_i^{t+1}) \le 1$. In such a way, we can determine the ranges of values of α_i and β_i for which the previous inequalities are satisfied independently of θ_i^{t+1}:

$$\begin{array}{ll}\beta_i\cdot(\alpha_i-1) > 0 & \dfrac{1-(1-\alpha_i)^2-\beta_i^2}{2\beta_i\cdot(\alpha_i-1)} > 1+\epsilon \\ \beta_i\cdot(\alpha_i-1) < 0 & \dfrac{1-(1-\alpha_i)^2-\beta_i^2}{2\beta_i\cdot(\alpha_i-1)} < -1+\epsilon\end{array}$$

After some math:

$$\begin{array}{ll} \alpha_i > 1 \ \beta_i > 0 & \alpha_i < 2 - \beta_i - \epsilon \\ \alpha_i < 1 \ \beta_i < 0 & \alpha_i > \epsilon - \beta_i \\ \alpha_i < 1 \ \beta_i > 0 & \alpha_i > \epsilon + \beta_i \\ \alpha_i > 1 \ \beta_i < 0 & \alpha_i < 2 - \epsilon + \beta_i \end{array}$$

Plotting the above inequalities on the Cartesian plane (α_i, β_i), it is easy to see that the regions that constitute the solutions of the above inequalities are:

$$\begin{aligned} &(0 < \beta_i < 1 - \epsilon \quad \wedge \quad \epsilon + \beta_i < \alpha_i < 2 - \epsilon - \beta_i) \\ \vee &(-1 + \epsilon < \beta_i < 0 \quad \wedge \quad \epsilon - \beta_i < \alpha_i < 2 - \epsilon + \beta_i) \end{aligned}$$

Under the above constraints, the negotiation functions (10) are cooperative and thus CCNFs. □

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