Local Delay in Static and Highly Mobile Poisson Networks with ALOHA

Conference Paper · June 2010

DOI: 10.1109/ICC.2010.5502205 · Source: IEEE Xplore

Martin Haenggi
University of Notre Dame

Local Delay in Static and Highly Mobile Poisson Networks with ALOHA

Martin Haenggi
Dept. of Electrical Engineering
University of Notre Dame, IN 46556, USA
mhaenggi@nd.edu

***Abstract*—Communication between two neighboring nodes is the most basic operation in wireless networks. Yet very little research has focused on the *local delay*, defined as the mean time it takes a node to connect to its nearest neighbor. In this paper, we derive the local delay in Poisson networks with ALOHA and find the conditions for which the local delay is finite. It turns out that while the local delay is always finite in highly mobile networks, there is a phase transition in static networks, *i.e.*, there is a maximum transmit probability above which the local delay is infinite.**

I. Introduction

A fundamental necessary condition for a wireless network to provide any useful functionality is that its nodes can connect to their nearest neighbors in a finite amount of time. Consequently, the *local delay*, defined as the mean time, in numbers of time slots, until a packet is successfully received over a link between nearest neighbors, is an important quantity to study. We derive the local delay in interference-limited networks with Poisson distributed nodes and ALOHA and study the conditions for it to be finite. The network nodes are assumed to be either highly mobile, in which case a new realization of the Poisson point process (PPP) is drawn in each time slot, or static, in which case the nodes are fixed for all time. A mathematical framework for the analysis of the local delay in Poisson ALOHA networks is provided in [1]. We build on this framework to obtain concrete results for different cases of nearest-neighbor communication.

II. Network Model

We consider a marked Poisson point process (PPP) $\hat{\Phi} = \{(x_i, t_{x_i})\} \subset \mathbb{R}^2 \times \{0,1\}$, where $\Phi = \{x_i\}$ is a homogeneous PPP of intensity λ and the marks $\{t_{x_i}\}$ are iid Bernoulli with $\mathbb{P}(t=1) = p = 1-q$. A mark of 1 indicates that the node transmits whereas a 0 indicates listening. The large-scale path loss is assumed to be r^α over distance r. A transmission from a node x to a node y is successful if the signal-to-interference ratio (SIR) exceeds a threshold θ. For a transmission from $x \in \Phi$ to $y \in \Phi$, the SIR is

$$\mathrm{SIR}_{xy} \triangleq \frac{S_{xy}}{I_{xy}} ,$$

where $S_{xy} \triangleq t_x h_{xy} \|x-y\|^{-\alpha}$ and

$$I_{xy} \triangleq \sum_{(z,t_z) \in \hat{\Phi} \setminus \{(x,t_x)\}} t_z h_{zy} \|z-y\|^{-\alpha} .$$

This definition implies that the transmit powers are normalized to 1, that $I = \infty$ if $t_y = 1$ (y is itself transmitting), and $\mathrm{SIR} = 0$ if $t_x = 0$. The fading h_{xy} is exponential with mean 1 and iid for all $x, y \in \Phi$ and over time (block Rayleigh fading). Time is slotted, and transmission attempts are synchronized.

Let $\mathcal{S}$ denote the *static elements* of the network, *i.e.*, all the randomness that does not change from time slot to time slot. We study the case $\mathcal{S} = \emptyset$, called the high-mobility case since a new realization of the PPP is drawn in each timeslot), and the static case $\mathcal{S} = \Phi$. Let $\mathcal{C}_\mathcal{S}$ be the event that the typical node situated at the origin $o \triangleq (0,0) \in \mathbb{R}^2$ successfully connects to its nearest neighbor in a single transmission *conditioned on* $\mathcal{S}$. Since all events considered are temporally iid, there is no need to add a time index to this event. Conditioning on Φ having a point at the origin o implies that the expectations that involve the point process are taken with respect to the Palm distribution $\mathbb{P}^o$ of Φ and denoted by $\mathbb{E}^o$ [2]. $\mathcal{S}$ does not include conditioning on the event that o is a transmitter or receiver. The partner node y, will be chosen according to one of the four basic cases of nearest-neighbor communication: Nearest-receiver transmission (NRT), nearest-neighbor transmission (NNT), nearest-transmitter reception (NTR), and nearest-neighbor reception (NNR). $\mathbb{P}^o(\mathcal{C}_\mathcal{S})$ is given by

$$\mathbb{P}^o(\mathcal{C}_\mathcal{S}) = \mathbb{P}^o(\mathrm{SIR}_{uv} > \theta \mid \mathcal{S}) ,$$

where $u = o$, $v = y$ for NRT and NNT, and $u = y$, $v = o$ for NTR and NNR. A packet whose transmission failed will be re-transmitted at subsequent occasions (as per the ALOHA MAC), until successfully received. The local delay is the mean number of slots needed until success. Conditioned on $\mathcal{S}$, the success events are temporally iid, so the conditional local delay is geometric with mean $\mathbb{P}^o(\mathcal{C}_\mathcal{S})^{-1}$. The local delay is then obtained by integration with respect to (w.r.t.) $\mathcal{S}$:

Definition 1 (Local delay) *The local delay is*

$$D \triangleq \mathbb{E}^o_\mathcal{S}\left(\frac{1}{\mathbb{P}^o(\mathcal{C}_\mathcal{S})}\right) .$$

Considering D as a function of p, we also define the *minimum delay* as

$$D_{\min} \triangleq \min_p \{D(p)\} ,$$

the *optimum transmit probability* as

$$p_{\mathrm{opt}} \triangleq \arg\,\min_p \{D(p)\} ,$$

and the *maximum transmit probability* (for finite local delay) as

$$p_{\max} \triangleq \sup\{p \colon D(p) < \infty\} \,.$$

III. High-Mobility Networks

In this case, there are no static elements, so $\mathcal{S} = \emptyset$, and $D = \mathbb{P}^o(\mathcal{C})^{-1}$. We need the following Lemma:

Lemma 1 *Let $\mathcal{H} \subset \mathbb{R}^2$ and*

$$I = \sum_{(x,t)\in\Phi} t h_x \|x\|^{-\alpha} \,. \tag{1}$$

The conditional Laplace transform of I given that $\mathcal{H}$ does not contain any nodes of Φ is $\mathcal{L}_I(s \mid \mathcal{H})$=

$$\mathcal{L}_I(s \mid \mathcal{H} \cap \Phi = \emptyset) = \exp\left(-\lambda p \int_{\mathbb{R}^2\setminus\mathcal{H}} \frac{s}{s + \|x\|^\alpha} \mathrm{d}x\right) \,. \tag{2}$$

Proof: This follows from the probability generating functions for (non-stationary) PPPs [2]. ∎

The distributional properties of the interference I, defined in (1), do not depend on where it is measured. If $\mathcal{H} = \emptyset$, the success probability of a transmission between two nodes at distance R is [3]

$$\mathbb{P}^o(\mathcal{C} \mid R) = pq\mathbb{E}^o(e^{-\theta R^\alpha I}) = pq\mathcal{L}_I(\theta R^\alpha) = pq\exp(-\gamma p \lambda R^2) \,,$$

where

$$\gamma \triangleq \theta^{2/\alpha} C(\alpha) \quad \text{and} \quad C(\alpha) \triangleq 2\pi^2/(\alpha \sin(2\pi/\alpha)) \,. \tag{3}$$

Here γ is the *spatial contention* [4]. It depends on the path loss exponent α, the SIR threshold θ, and the network geometry. Practical values for narrowband transmission range from about 5 or 7dB ($\alpha = 4$, $\theta = 1$), to about 530 or 27dB ($\alpha = 2.5$, $\theta = 20$ dB). As $\alpha \downarrow 2$, $\gamma \to \infty$.

A. Nearest-receiver transmission (NRT)

In this case, R is the distance from the origin to the nearest *receiver* in Φ, which is Rayleigh distributed with mean $1/(2\sqrt{q\lambda})$ [5], *i.e.*,

$$f_R(r) = 2q\lambda\pi r \exp(-q\lambda\pi r^2) \,.$$

Hence

$$\mathbb{P}^o(\mathcal{C}) = \frac{p\pi}{\pi + \gamma p q^{-1}}$$

and

$$D^{\mathrm{NRT}} = \frac{1}{\mathbb{P}^o(\mathcal{C})} = \frac{1}{p} + \frac{\gamma}{\pi q} \,. \tag{4}$$

The optimum transmit probability is

$$p_{\mathrm{opt}}(\gamma) = \frac{\pi - \sqrt{\pi\gamma}}{\pi - \gamma} \,. \tag{5}$$

As expected, $p_{\mathrm{opt}}(\gamma)$ is monotonically decreasing from $p_{\mathrm{opt}}(0) = 1$. Since $\mathbb{P}^o(\mathcal{C})$ is always positive, the local delay is always finite for $p \in (0,1)$, so $p_{\max} = 1$. The minimum delay is

$$D^{\mathrm{NRT}}_{\min}(\gamma) = 1 + 2\sqrt{\frac{\gamma}{\pi}} + \frac{\gamma}{\pi} \,. \tag{6}$$

Fig. 2 shows the local delay as a function of p for $\alpha = 4$ and $\theta = 10$ (high-mobility NRT curve), and Fig. 3 shows $D_{\min}(\gamma)$.

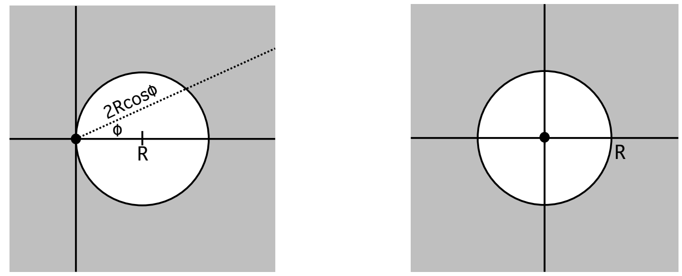

Fig. 1. The shaded area is the integration domain for the interference in the cases of nearest-neighbor transmission (left) and nearest-transmitter reception (right). In both cases, the receiver is situated at the origin.

B. Nearest-neighbor transmission (NNT)

Let y be the typical node's nearest neighbor and $R = \|y\|$. In this case R is distributed as $f_R(r) = 2\lambda\pi r \exp(-\lambda\pi r^2)$, and having the nearest neighbor at distance R implies that there is no interferer in the ball $B_o(R)$ centered at o with radius R. So y sees the conditional interference, conditioned on the disk $B_o(R)$ being empty. By stationarity of Φ, the situation is statistically the same if the transmitter is located at $(R,0)$ and its nearest neighbor at o, as shown in Fig. 1 (left), with the typical receiver at the origin. Hence we can apply Lemma 1 with $\mathcal{H} = B_{(R,0)}(R)$. The left half plane is not affected by the empty disk, so we can write

$$\mathcal{L}_I(s \mid \mathcal{H}) = \exp(-\lambda p C(\alpha) s^{2/\alpha}/2) \exp(-\lambda p A(R,s)) \,, \tag{7}$$

where

$$A(R,s) = \int_{-\pi/2}^{\pi/2} \underbrace{\int_{2R\cos\phi}^{\infty} \frac{rs}{r^\alpha + s} \mathrm{d}r \, \mathrm{d}\phi}_{A'(R,s,\phi)}$$

is the integral over the right half plane with the hole, expressed in polar coordinates (see Fig. 1 (left)).

Finding a general closed-form solution for this double integral (that would also have to be integrable w.r.t. R) seems hopeless. Instead, we focus on the case $\alpha = 4$ and aim at finding an accurate and integrable approximation. Letting $s = \theta R^\alpha = \theta R^4$, the inner integral is

$$A'(R, \theta R^4, \phi) = \frac{1}{2}\sqrt{\theta} R^2 \left(\frac{\pi}{2} - \arctan\left(\frac{4\cos(\phi)^2}{\sqrt{\theta}}\right)\right) \,.$$

For the integration over ϕ, we use $\arctan x \lessapprox x$ for $x \leqslant 1$ (linear regime) and $\arctan x \gtrapprox \pi/2 - 1/x$ for $x > 1$ (saturation regime) as approximations. The transition between the linear and the saturation regime occurs at $\phi = \phi_0$, where $\phi_0 = \arccos(\theta^{1/4}/2)$, valid if $\theta < 16$. For larger θ (high rates), the $\arctan$ is always in the linear regime, hence $\phi_0 = 0$. With this approximation, integrating over ϕ yields $A(R, \theta R^4) \approx$

$$2\int_0^{\phi_0} \frac{\theta R^2}{8\cos(\phi)^2} \mathrm{d}\phi \;+\; 2\int_{\phi_0}^{\pi/2} \frac{\sqrt{\theta}}{2}\left(\frac{\pi}{2} - \frac{4\cos(\phi)^2}{\sqrt{\theta}}\right) \mathrm{d}\phi \,. \tag{8}$$

A general solution exists but would not be integrable w.r.t. R. However, for $\theta \geqslant 16$, we obtain $A(R, \theta R^4) \gtrapprox \pi R^2(\pi\sqrt{\theta}/4 -$

$1) = R^2(\gamma/2-\pi)$. The spatial contention from the left half plane is $\gamma/2$, (see (7)), so at high rates,

$$\mathbb{P}^o(\mathcal{C} \mid R) \lessapprox pq \exp(-\lambda p R^2(\gamma-\pi))\,.$$

This shows that the effect of the hole is an improvement of the spatial contention by π. For $\theta < 16$, where both integrals in (8) are positive, we find that $A(R,\theta R^4) = \Theta(\theta^{3/4})$, $\theta \to 0$, indicating that we can write $A(R,\theta R^4) \approx c\theta^{3/4}$ for small θ. We use this approximation for the range $\theta \in [0,16)$ and choose c such that the approximation is continuous:

$$A(R,\theta R^4) \approx \pi R^2 \left(\frac{\pi-1}{8}\right), \qquad \theta \leqslant 16$$

Inserting this in (7) and deconditioning w.r.t. R gives

$$\mathbb{P}(\mathcal{C}) \begin{cases} \lessapprox \frac{2pq}{\pi p\sqrt{\theta}+2q}\,, & \theta \geqslant 16\,, \\ \approx \frac{8pq}{2\pi p\sqrt{\theta}+p\theta^{3/4}(\pi-1)+8}\,, & \theta \leqslant 16\,. \end{cases} \tag{9}$$

For $\theta \geqslant 16$, This is exactly the same probability that we found in the nearest-receiver case! So for $\alpha = 4$ and when θ is large, we can reach the nearest node with the same success probability as the nearest receiver, even if we do not know whether it is transmitting or receiving. The uncertainty about the transmission state of the nearest neighbor is exactly offset by the certainty of not having an interferer around the transmitter. The local delay for $\alpha = 4$ is

$$D^{\rm NNT} \begin{cases} \gtrapprox \frac{\pi\sqrt{\theta}}{2q} + \frac{1}{p}\,, & \theta \geqslant 16 \\ \approx \frac{1}{pq} + \frac{\pi\sqrt{\theta}}{4q} + \frac{(\pi-1)\theta^{3/4}}{8q}\,, & \theta \leqslant 16 \end{cases} \tag{10}$$

In the high-rate case, the optimum transmit probability and minimum delay are the same as for NRT, see Figs. 2 and 3, while for small θ, there is a small difference in favor of NRT.

When comparing the NRT and NNT schemes, it also needs to be factored in that the distance to the nearest receiver is a factor $q^{-1/2}$ larger than the distance to the nearest neighbor. So the main benefit of NRT is that more distance is covered.

C. Nearest-transmitter reception (NTR)

Next we consider the case where the typical node at o, receives from its nearest transmitter, say y. This implies that there are no interferers in the disk of radius $R = \|y\|$ around the receiver. So in this case, we apply Lemma 1 with $\mathcal{H} = B_o(R)$, see Fig. 1 (right). For $\alpha = 4$, this situation has been analyzed in the context of CSMA in [6, Sec. 3.7]. Taking the Laplace transform [6, Eqn. (3.46)] and replacing s by θR^4 yields

$$\mathbb{P}^o(\mathcal{C} \mid R) = q \exp\left(-\lambda p \pi R^2 \sqrt{\theta}\left[\frac{\pi}{2} - \arctan\left(\frac{1}{\sqrt{\theta}}\right)\right]\right)\,.$$

For $\theta > 1$, $1/\sqrt{\theta} \gtrapprox \arctan(1/\sqrt{\theta})$, and we obtain

$$\begin{aligned}\mathbb{P}^o(\mathcal{C} \mid R) &\lessapprox q \exp\left(-\lambda p \pi R^2\left(\sqrt{\theta}\frac{\pi}{2}-1\right)\right)\\ &= q\exp(-p\lambda R^2(\gamma-\pi))\,, \qquad (\alpha=4,\theta>1)\,.\end{aligned}$$

So, as in the case of high-rate nearest-neighbor transmission, the spatial contention is reduced by π. This can be explained as follows: In the NRT case, given R, the success probability equals the probability that a disk of area A_0 of size γR^2 around the receiver is free of interferers. In NNT, the receiver sits on the boundary of a disk of radius R known to be interferer-free, while in the NTR case, it is in the center of such a disk. If γ is large enough, larger than about 4π, the hole $\mathcal{H}$ is a subset of A_0 for both NTR and NNT and the success probability is the probability that A_0 is empty given that $\mathcal{H}$ is empty. The difference is that in the NTR case, this spatial contention benefit already materializes for small θ since A_0 and $\mathcal{H}$ are concentric.

In the NTR case, R is distributed as $f_R(r) = 2\pi p\lambda r \exp(-p\lambda\pi r^2)$. Integration yields

$$D^{\rm NTR} = \frac{\pi\sqrt{\theta}}{2q}\,, \tag{11}$$

which is minimized at $p = 0$. This result is modeling artefact, since due to the lack of noise, the benefit of reducing the interferer density compensates for the increased transmission distance. See Figs. 2 and 3 again for the plots.

For $\theta < 1$, using $\arctan x \gtrapprox \pi/2 - 1/x$, we find

$$\mathbb{P}^o(\mathcal{C} \mid R) \gtrapprox q\exp(-\lambda p\pi\theta R^2)\,.$$

D. Nearest-neighbor reception (NNR)

This is quite similar to NTR, with the difference that the nearest neighbor is at distance $1/(2\sqrt{\lambda})$ on average and that the delay increases by a factor $1/p$ since the nearest neighbor only transmits with probability p.

IV. Static Networks

Here, $\mathcal{S} = \Phi$, and as before, the typical node attempts to connect to its nearest receiver or nearest neighbor. By Jensen's inequality, we already have a bound from the previous section, namely $D > \mathbb{P}(\mathcal{C})^{-1}$, but as we shall see, this bound is often very loose. The reason is the correlation of the interference in the static case.

The following lemma is the conditional counterpart to Lemma 1:

Lemma 2 *Let I denote the interference as defined in* (1), $\mathcal{H} \subset \mathbb{R}^2$, *and let*

$$\mathcal{L}_I(s \mid \Phi, \mathcal{H}) = \mathbb{E}^o(\exp(-sI \mid \Phi, \Phi\cap\mathcal{H}=\emptyset))$$

be the conditional Laplace transform given Φ and given that there is no transmitter in $\mathcal{H}$. Then

$$\mathbb{E}^o\left(\frac{1}{\mathcal{L}_I(s \mid \Phi,\mathcal{H})}\right) = \exp\left(\lambda\int_{\mathbb{R}^2\setminus\mathcal{H}} \frac{ps}{sq+\|x\|^\alpha}\mathrm{d}x\right)\,, \tag{12}$$

which for $\mathcal{H} = \emptyset$ evaluates to

$$= \exp\left(\frac{p\lambda C(\alpha)s^{2/\alpha}}{q^{1-2/\alpha}}\right)\,, \tag{13}$$

with $C(\alpha)$ as defined in (3). *The local delay conditioned on a link distance R is obtained by replacing s by θR^α.*

Proof: Follows from [1, Lemma 16.6.5]. ■

A. Nearest-receiver transmission with fixed partitioning (NRT)

Here we consider the case where the partitioning into potential transmitters and receivers is fixed[1], *i.e.*, the transmitters are chosen from Φ with probability p, as before, but there exists another, independent PPP of receivers Φ_r of intensity $\lambda_r = q\lambda$. So, in this model, the nodes in Φ that do not transmit are not available as receivers. This assumption maintains the same density of (actual) transmitters and receivers as in the other models.

Proposition 1 *If the nodes are partitioned into a set of potential transmitters Φ of intensity λ and a set of receivers Φ_r of intensity $\lambda_r = q\lambda$, the local delay at the typical transmitter is*

$$D^{\mathrm{NRT}} = \frac{1}{p}\,\frac{\pi}{\pi - \gamma p q^{2/\alpha-2}} \tag{14}$$

if $pq^{2/\alpha-2} < \pi/\gamma$.

Proof: Replacing s in (13) by θR^α yields the success probability given a link distance R. Since $C(\alpha)(\theta R^\alpha)^{2/\alpha} = \gamma R^2$, deconditioning on R yields

$$D^{\mathrm{NRT}} = \frac{1}{p} 2\pi q\lambda \int_0^\infty \exp\left(\frac{\lambda p \gamma r^2}{q^{1-2/\alpha}}\right) r \exp(-\pi q \lambda r^2)\mathrm{d}r\,,$$

which evaluates to (14). ■

Due to the term $q^{2/\alpha-2}$, the expression for the local delay is slightly unwieldy. Since $2/\alpha - 2 \in (-2,-1)$, we obtain upper and lower bounds for the delay by replacing the exponent $2/\alpha - 2$ by -1 and -2, respectively. So we have $\overline{D} \geqslant D \geqslant \underline{D}$ for

$$\overline{D} = \frac{1}{p}\,\frac{\pi}{\pi - \gamma p q^{-2}}\,, \quad \gamma p < q^2\pi \tag{15}$$

$$\underline{D} = \frac{1}{p}\,\frac{\pi}{\pi - \gamma p q^{-1}}\,, \quad \gamma p < q\pi \tag{16}$$

The condition for a finite upper delay bound $\overline{D}$ yields a *lower* bound for $p_{\max}$, and vice versa. Therefore, $\overline{p}_{\max} \geqslant p_{\max} \geqslant \underline{p}_{\max}$ for

$$\overline{p}_{\max} = \frac{\pi}{\gamma+\pi}\,; \qquad \underline{p}_{\max} = 1 + \frac{\gamma}{2\pi}\left(1 - \sqrt{1 + \frac{4\pi}{\gamma}}\right). \tag{17}$$

As $\gamma \to \infty$, the bounds are tight, since

$$\overline{p}_{\max} = \frac{\pi}{\gamma} - \frac{\pi^2}{\gamma^2} + O(\gamma^{-3})$$

$$\underline{p}_{\max} = \frac{\pi}{\gamma} - \frac{2\pi^2}{\gamma^2} + O(\gamma^{-3})\,.$$

The tightness of the bounds is confirmed in Fig. 2, where the local delay $D(p)$ and the two bounds are shown for $\alpha = 4$ and $\theta = 10$, and in Fig. 4, which plots $p_{\max}(\theta)$ and the two bounds.

In the following, we are using this lower bound $\underline{D}$ to obtain closed-form results on the other quantities of interest.

[1]The standard NRT scheme where the marks are iid from time slot to time slot cannot be analyzed in the same way since the transmission success events are not conditionally independent.

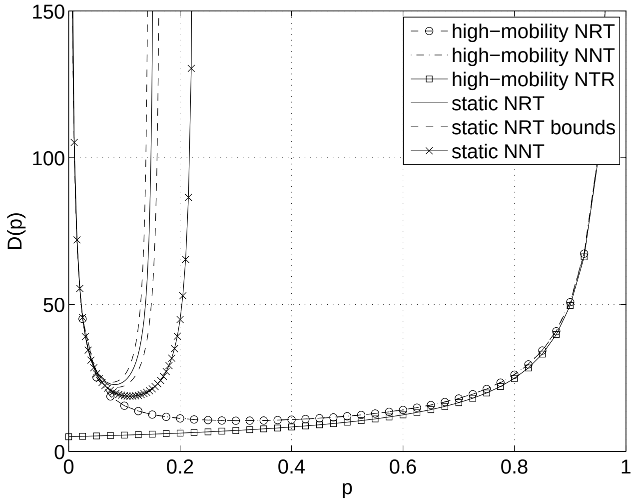

Fig. 2. The local delay for the high-mobility and static cases as a function of the channel access probability p for $\alpha = 4$ and $\theta = 10$. The curves for the two high-mobility cases NRT and NNT are essentially identical and thus not distinguishable in the plot; the difference is only $0.02/q$ for the parameters chosen.

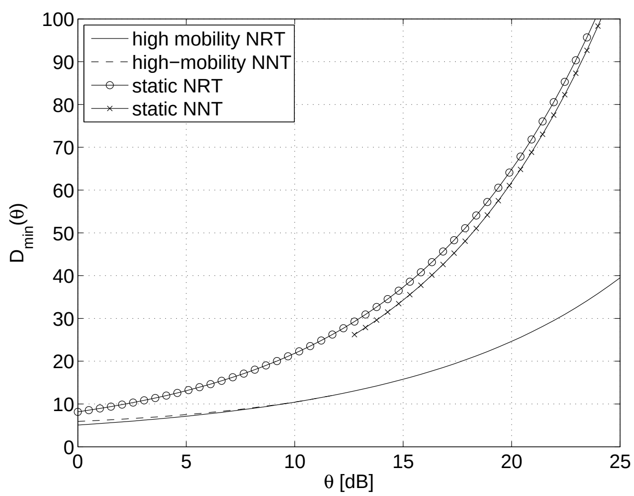

Fig. 3. The minimum achievable delay as a function of θ for the high-mobility and the static cases. The difference between NRT and NNT is rather small in both cases.

For the delay-minimizing channel access probability, we find

$$\overline{p}_{\mathrm{opt}} = 1 - \sqrt{\frac{\gamma}{\gamma+\pi}}\,.$$

Interestingly, letting $q_{\min} = 1 - p_{\max}$, we have $q_{\mathrm{opt}} = \sqrt{q_{\min}}$. The delay diverges as $q \uparrow 1$ and $q \downarrow q_{\min}$, and the optimum q is the geometric mean between these two values.

B. Nearest-neighbor transmission (NNT)

Here $\mathcal{S} = \Phi$ and the transmission occurs to the nearest neighbor. We again focus on the case $\alpha = 4$. Applying Lemma 2 to the case where $\mathcal{H} = B_{(R,0)}(R)$ gives for the local delay given R

$$D_R = \exp\left(\lambda p R^2(\gamma/(2\sqrt{q}) + A(R,\theta,q))\right)\,,$$

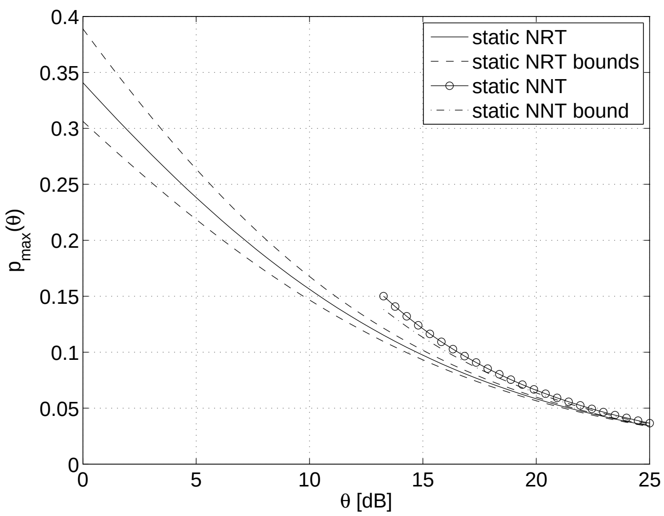

Fig. 4. The maximum transmit probability $p_{\max}$ for finite local delay in the static NRT case (solid line) with the two bounds (17) (dashed lines) and for NNT as a function of θ for $\alpha = 4$.

where $\gamma/(2\sqrt{q}) = \pi^2\sqrt{\theta}/4\sqrt{q}$ stems from the integral over the hole-less left half plane and $A(R, \theta, q) =$

$$\frac{\sqrt{\theta}}{2\sqrt{q}} \int_{-\pi/2}^{\pi/2} \int_{2R\cos\phi}^{\infty} \left(\frac{\pi}{2} - \arctan\left(\frac{4\cos(\phi)^2}{\sqrt{\theta q}} \right) \right) \mathrm{d}r\mathrm{d}\phi \,.$$

Similarly to the high-mobility NNT case, we approximate the double integral in the high-rate regime as

$$A(R, \theta, q) \gtrapprox \pi \left(\frac{\pi\sqrt{\theta}}{4\sqrt{q}} - \frac{1}{q} \right) , \qquad \theta q \geqslant 16 \,. \tag{18}$$

So in this case, the high-rate regime starts at $\theta \geqslant 16/q$. It follows that

$$D_R \lessapprox \frac{1}{pq} \exp\left(\frac{\lambda p \pi R^2}{\sqrt{q}} \left(\frac{\pi\sqrt{\theta}}{2} - \frac{1}{\sqrt{q}} \right) \right)$$

and by deconditioning on R we get the result:

Proposition 2 *In the NNT model with static nodes, the local delay for $\alpha = 4$ and $\theta \geqslant 16/q$ is*

$$D^{\mathrm{NNT}} \lessapprox \frac{1}{p} \frac{\pi}{\pi - \gamma p \sqrt{q}} \tag{19}$$

for $p < \pi/(\gamma\sqrt{q})$.

It follows that $p < \pi/\gamma$ is a conservative condition for finite local delay, see the dash-dotted curve in Fig. 4.

V. Comparison of the Asymptotics

Fig. 3 shows the minimum achievable delay as a function of θ. In the static case, both upper and lower bounds on the minimum delay have an asymptotic slope of $4/\pi$; in the high-mobility case, the slope is $1/\pi$. So we have as $\gamma \to \infty$:

$$\text{high-mobility: } D_{\min}(\gamma) \sim \frac{\gamma}{\pi} = \Theta(\theta^{2/\alpha})$$
$$\text{static: } D_{\min}(\gamma) \sim \frac{4\gamma}{\pi} = \Theta(\theta^{2/\alpha})$$

$\gamma \to \infty$ occurs as $\alpha \downarrow 2$ or $\theta \to \infty$, hence for fixed α these asymptotics provide a high-SIR *rate-delay trade-off*: Assuming the rate of transmission is $\rho \triangleq \log_2(\theta)$, valid for large θ, then in all cases

$$D_{\min}(\rho) = \Theta(2^{2\rho/\alpha}) \,, \qquad \theta \to \infty \,.$$

We conclude that the delay increases exponentially in the rate of transmission, and that the effect of the mobility is hidden in the pre-constant.

VI. Conclusions

We have analyzed the local delay in mobile and static Poisson ALOHA networks for different cases of nearest-neighbor pairs. A finite local delay means that the fraction of nodes that cannot connect to their nearest neighbor in finite time is negligible.

Highly mobile network provide enough time diversity to keep the local delay finite in all cases, and the local delay is quite robust against deviations from the optimum. We calculate the local delay for the extreme case, where a new realization of the point process is drawn in each time slot. The situation is more challenging to analyze when the nodes do not move, and only fading and the transmitter/receiver state of the nodes remain as sources of randomness. In this case, the local delay becomes infinite when the transmit probability p exceeds a certain threshold, *i.e.*, there is a phase transition, and deviations from the optimum p can be costly.

An asymptotic comparison of highly mobile and fully static networks (with fixed transmitter and receiver sets) reveals that the difference in the minimum local delays is a factor of four as the spatial contention γ goes to infinity. For finite γ, the difference is smaller. This gives upper and lower bounds for all practical networks.

Acknowledgments

The partial support of NSF (grants CNS 04-47869, CCF 728763) and the DARPA/IPTO IT-MANET program (grant W911NF-07-1-0028) is gratefully acknowledged.

References

[1] F. Baccelli and B. Blaszczyszyn, “Stochastic Geometry and Wireless Networks: Theory and Applications”, *Foundations and Trends in Networking (NOW Publishers)*, vol. 3-4, 2009.

[2] D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic Geometry and its Applications*. John Wiley & Sons, 1995. 2nd Ed.

[3] F. Baccelli, B. Blaszczyszyn, and P. Mühlethaler, “An ALOHA Protocol for Multihop Mobile Wireless Networks,” *IEEE Transactions on Information Theory*, vol. 52, pp. 421–436, Feb. 2006.

[4] M. Haenggi, “Outage, Local Throughput, and Capacity of Random Wireless Networks,” *IEEE Transactions on Wireless Communications*, vol. 8, pp. 4350–4359, Aug. 2009.

[5] M. Haenggi, “On Distances in Uniformly Random Networks,” *IEEE Trans. on Information Theory*, vol. 51, pp. 3584–3586, Oct. 2005.

[6] M. Haenggi and R. K. Ganti, *Interference in Large Wireless Networks*. *Foundations and Trends in Networking (NOW Publishers)*, vol. 3, no. 2, pp. 127–248, 2008.