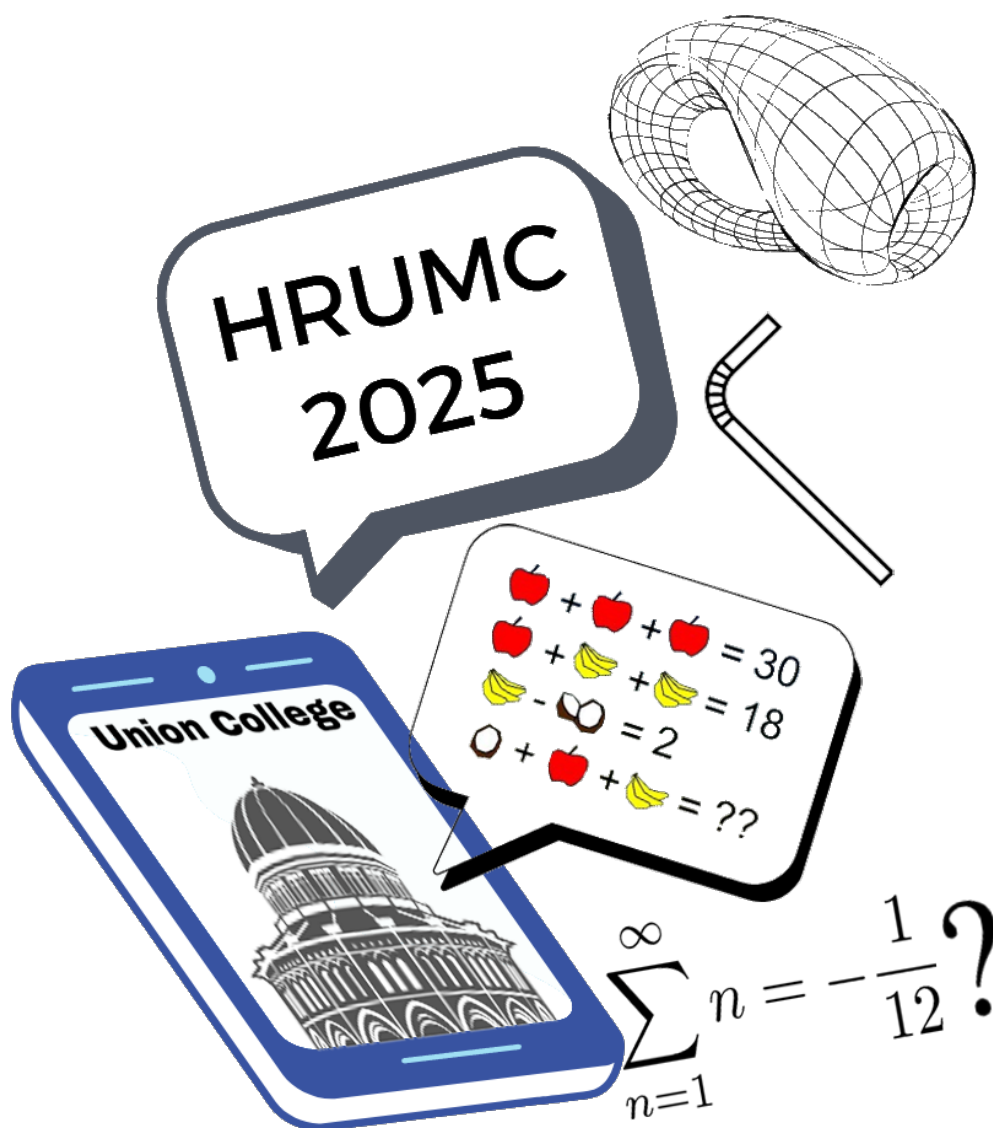


Hudson River Undergraduate Mathematics Conference

Union College
April 5th, 2025



Support for the 2025 Hudson River Undergraduate Mathematics Conference was provided by the National Science Foundation (Award #2429253).

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Lunch Panel Information

During the lunch break, please attend one of the following panels (where lunch will be provided):

- *Making the most of one's undergraduate mathematics experience* (Olin Center 115)

Panelists:

- Audrey Benson (Union College)
- Rebekkah Clauson (Siena College)
- Soleil Demick (Skidmore College)
- Amy Geng (Mount Holyoke College)

- *Careers/Life after a math major* (Reamer Auditorium)

Panelists:

- Dan Anderson (Math/CS Teacher, Shenendehowa High School)
- Mark Mortensen (Retired actuary)
- Radu Neagu (Director, Enterprise Risk Management, Santander)
- Katie Schuff (Associate Attorney, Nagy Wolfe Appleton LLP)

- *Lunch with discussion topics* (Hale House)

Other Information

Wifi

Use the network “Union-Guest” with password UNION1795 . Alternatively, eduroam is available.

Conference T-Shirt

The conference T-shirt will be sold at registration, the lunch panels, and the afternoon coffee break, while supplies last, for \$15, cash only.

Exhibit

Feel free to check out the Olivier models exhibit at the bottom of the stairs in the Wold Center. The models were made in Paris in the 1830s for French mathematician Théodore Olivier, who devised them to represent the intersections of 3D objects in his teaching of descriptive geometry. Additional models are on display on the second floor of Bailey Hall.

Local organizing committee

Audrey Benson
Paul Friedman (chair)
Jeffrey Hatley
Jeffrey Jauregui
Brenda Johnson
Francesca Morone
Tremaine Richardson
Christina Tønnesen-Friedman
Fanhui Xu

HRUMC steering committee

Kariane Calta, Vassar College
Tim Chumley, Mount Holyoke College
Vincent Ferlini, Keene State College
Paul Friedman, Union College
Scott Greenhalgh, Siena College
Jeffrey Hatley, Union College
Ben Lotto, Vassar College
Greg Malen, Skidmore College
Steve Miller, Williams College
Lucy Oremland, Skidmore College
Kursad Tosun, Siena College

Welcome and Keynote Address

Welcoming Remarks and Introduction of the Vice President:

Dr. Paul Friedman, Union College

Welcoming Remarks from Vice President Angrist:

Dr. Michele Angrist, Union College

Introduction of the Keynote Speaker:

Audrey Benson, Francesca Morone, and Tremaine Richardson

Keynote Address

Dr. Álvaro Lozano-Robledo, Professor of Mathematics, University of Connecticut

Keynote Address

Dr. Álvaro Lozano-Robledo
University of Connecticut

Math in the Age of Social Media

Abstract

In this talk we will talk about the role that social media plays in the public perception of mathematics and, in particular, whether one can use social media platforms, such as Instagram, TikTok or YouTube, to move the needle and create engaging mathematical content that has a math-positive spin to counter the prevalent math phobia in society.

Specifically, we will discuss certain math questions that seem to generate tremendous amounts of engagement, and we will use these “hot topics” to talk about the very important math concepts behind them. For instance, some of these questions include:

- Why do we teach Pythagoras’ theorem in school? What is it good for and will we ever use it in real life?
- Why do we need proofs anyway?
- Why would a new proof of Pythagoras’ theorem (such as the wonderful proof recently discovered by two New Orleans teens) be exciting?
- Is Math discovered or invented?
- What is an axiom? How and why do we build mathematics out of axioms?
- What did Gödel prove in his “incompleteness theorem”? Should we give up since Math is “incomplete”?
- Does a straw have one or two holes?
- What is a Klein bottle and what is topology?
- Does the sum of all the positive integers $1 + 2 + 3 + \dots$ really equal $-1/12$?
- What is the Riemann zeta function, what is the Riemann hypothesis, and why is there a \$1M prize to whoever solves this esoteric math problem?

... among many other questions. As examples, the presentation will also include some of the speaker’s videos that have been viewed millions of times.

Biography

Álvaro Lozano-Robledo is a professor of mathematics at the University of Connecticut. He received his PhD from Boston University in 2004. After temporary positions at Colby College and Cornell University, Álvaro has worked at UConn since 2008. His research interests are in the area of arithmetic geometry (the crossroads of number theory and algebraic geometry). He has published two books, “Elliptic Curves, Modular Forms, and their L-Functions”, and “Number Theory and Geometry” Álvaro’s blog, A Field Guide to Mathematics, contains other short stories and also other pieces of interest to mathematicians. In his spare time, he shares math on social media (@mathandcobb on Instagram, YouTube, and TikTok) with mathematical content and some comedy bits.

List of Sessions

Morning Session (9:45–10:40am)

Abstract Algebra IA	Olin 106
Abstract Algebra IB	Olin 107
AI and Machine Learning I	Wold 225
Applied Math I	ISEC 222
Combinatorics and Recursion	ISEC 120
Graph Theory I	ISEC 150
Image Analysis	ISEC 218
Math Throughout History	Olin 206
Mathematical Physics	Olin 306
Number Theory I	ISEC 124
Statistics I	Wold 128
Topology	Olin 204

Early Afternoon Session (2:00–2:55pm)

Abstract Algebra II	Olin 106
AI and Machine Learning II	Wold 225
Analysis I	Olin 107
Applications of Topology and Networks	Olin 204
Applied Math II	ISEC 222
Combinatorics and Games	ISEC 120
Graph Theory II	ISEC 150
Mathematical Modeling I	ISEC 218
Number Theory II	ISEC 124
Statistics II	Wold 128
Tilings and Fractals	Olin 206

Late Afternoon Session (3:30–4:25)

Analysis II	Olin 107
Differential Equations	ISEC 222
Geometry	Olin 204
Graph Theory III	ISEC 150
Math Education	Olin 206
Mathematical Modeling II	ISEC 218
Number Theory IIIA	ISEC 124
Number Theory IIIB	ISEC 120
Probability	Wold 225
Statistics III	Wold 128

Note: talks listed as “Level 1” are generally beginner level, while talks listed as “Level 2” are typically more advanced.

Parallel Sessions – Morning

Abstract Algebra IA

Olin 106

Chair: Jeff Hatley (Union College)

9:45 - 10:00 *A Glimpse into Algebraic Geometry: Cubics and Their Group Law* (Level 1)

Nicholas J. Padilla (Fordham University)

Algebraic geometry is an expansive field that uses commutative algebra to study geometric creatures. In this visual-oriented talk, we will study cubic forms (i.e., homogeneous polynomials of degree three) and show that you can define a group structure over them in most cases. We will explore two main variations of group laws over cubics and investigate how allowing the identity in the group law (i.e., the neutral element) to be the cubic's inflection point leads to a simplified group structure. Our examples will also make accessible some basics of projective geometry (e.g., points at infinity). The objective of the talk is to introduce undergraduates who are studying or have studied abstract algebra to one of the subject's intriguing geometric applications.

10:05 - 10:20 *Spectral Equivalence of Solvable Lie Algebras with Heisenberg Nilradical* (Level 2)

Gary Hu (Williams College)

We classify a family of low-dimensional solvable Lie algebras through an equivalence relation known as spectral equivalence, which is a weaker notion than isomorphism and is inspired by ideas from spectral theory. Building on Libor Snobl's work, we focus on solvable Lie algebras \mathfrak{s} with Heisenberg nilradicals $\mathfrak{h}(m)$ and f -dimensional extension. Such Lie algebras with 3D and 5D nilradicals have been fully classified. In 2020, Rongwei Yang introduced an analog of the characteristic polynomial for solvable Lie algebras. This naturally leads to a mysterious invariant $k(L)$, the number of distinct linear factors of the polynomial. To better understand this invariant, Yang defined an equivalence relation on $k(L)$ called spectral equivalence. We combine these two stories: Up to spectral equivalence, we classify all solvable Lie algebras \mathfrak{s} with Heisenberg nilradicals $\mathfrak{h}(1), \mathfrak{h}(2)$ and an f -dimensional extension.

10:25 - 10:40 *Generalized Legendrian racks: Knot coloring invariants and algebraic classification* (Level 2)

Lýc Ta (Yale University)

We answer several open questions about generalized Legendrian racks (also called GL-racks or bi-Legendrian racks), a nonassociative algebraic structure based on the Legendrian Reidemeister moves. First, we use GL-rack coloring invariants to distinguish between Legendrian knots with identical classical invariants. In particular, we complete the classification of Legendrian 8_{13} knots. To that end, we use exhaustive search algorithms to classify GL-racks and GL-quandles of orders $n \leq 8$ up to isomorphism. Time permitting, we also discuss algebraic and categorical aspects of GL-racks. We characterize all nonisomorphic GL-structures on a given rack and classify various infinite families of GL-racks. We also compute the center of the category of GL-racks, show that the categories of racks and GL-quandles are isomorphic, and study universal-algebraic tensor products of (GL-)racks.

Abstract Algebra IB Olin 107 Chair: Margaret Robinson (Mount Holyoke College)

9:45 - 10:00 *Idempotent Matrices* (Level 1)

Josephine DiPaola (Siena College)

Karina Leibach (Siena College)

An $n \times n$ matrix A is called idempotent if and only if $A^2 = A$. In this talk, we will present a general form that can be used to determine idempotency without multiplying the matrix by itself. We will use our knowledge of linear algebra and techniques to prove why this formula works. By doing this, we will also be able to find any exceptions to the form.

10:05 - 10:20 *Locally Associated Quadratic Orders: a Reduction to Elementary Number Theory* (Level 2)

Grant Moles (Union College)

When investigating complex algebraic objects, it often helps to hearken back to simpler ideas. This talk will explore an example of this in the realm of commutative algebra. First, we will introduce the idea of a locally associated subring, the definition and investigation of which typically requires ideas from abstract algebra. Then, we will see how in a particular case of interest (an area of active research), this complicated question can be reduced to one involving only topics from elementary number theory.

10:25 - 10:40 *Finite Groups with m Elements of Order n* (Level 2)

Dana Shadoan (Marist University)

One goal in finite groups is to classify finite groups that have exactly m elements of order n . Beginning with a review of the concepts of group order, element order, cyclic subgroups, and abelian groups, this talk will present a classification of groups with exactly one element of order n and partially examine groups with exactly two elements of order n .

AI and Machine Learning I

Wold 225

Chair: Jenn Qian (Union College)

9:45 - 10:00 *Might You Wake Up and Find that Your Bank Has Collapsed?* (Level 1)

Baibhav Barwal (Union College)

Atharv Tekurkar (Union College)

Bank failures can have far-reaching consequences on customers, employees, investors, and the entire financial ecosystem. This is the problem we plan to address in this seminar by exploring the integration of machine learning modeling with the Altman Z-Score - a well-known bankruptcy prediction model - to assess the accuracy of predicting bank failures. Using a dataset of macroeconomic and microeconomic factors for 13 failed and 13 non-failed banks, we built and compared machine learning models like Random Forest, XGBoost, and Neural Networks to determine the most effective approach for predicting financial distress. In this presentation, we will dive into this interesting fusion of statistical analysis, computational techniques, and financial forecasting, offering actionable insights for proactive risk management and strategic decision-making in today's uncertain financial world.

10:05 - 10:20 *Exploring Machine Learning Methods for Survival Analysis to Predict BlackHawk UH-60 Maintenance Component Failures* (Level 1)

Eric Hong (United States Military Academy)

With the goal of increasing the U.S. Army Aviation community operational readiness across

BlackHawk UH-60 fleets, we seek to develop a predictive maintenance tool enabled by machine learning methods in a survival analysis framework. We explore the penalized regression methods Least Absolute Shrinkage and Selection Operator (LASSO) and Ridge regression to build predictive models using BlackHawk maintenance records. We aim to integrate this survival analysis blended model into the Army's Artificial Intelligence Integration Center's (AI2C) predictive maintenance tool known as GRIFFIN to provide commanders and maintainers with additional capabilities to bolster operational readiness.

Applied Math I

ISEC 222

Chair: Scott Greenhalgh (Siena College)

9:45 - 10:00 *Visualizing Neuronal Firing Patterns: A Python-Based Exploration of FitzHugh-Nagumo Model* (Level 1)

Jason Stasio (St. Lawrence University)

When stimulated by an external current, neurons fire a single electrical impulse called an action potential. However, excessive stimulation can induce repetitive firings, potentially leading to a seizure. My research focuses on developing a Python-based simulation of the FitzHugh-Nagumo model, a system of ordinary differential equations frequently used to study neuronal voltage dynamics. This simulation approximates solutions to the model across varying external current input and identifies bifurcations that mark transitions between regular and unstable neuronal activity.

By linearizing the system via the Jacobian matrix and extracting eigenvalues, the script classifies local behavior and detects shifts in dynamical behavior. The tool features interactive visualizations, including a trace-determinant plane and a phase portrait with optional nullclines and vector fields. A built-in slider enables users to explore how neuronal dynamics evolve under different current levels, providing an intuitive, hands-on approach to understanding the threshold between typical and pathological firing patterns.

10:05 - 10:20 *An Exploration of Spline Applications* (Level 1)

Lizzie Cornett (Champlain College)

Victor Diab (Champlain College)

Bezier curves allow us to interpolate a curve over a set of points. Splines are piecewise continuous functions. These tools are extremely useful in numerous fields of technology and mathematics. In video game development, for example, splines can be used for mesh generation and animation. In this talk, we will explore various applications in detail and demonstrate making a spline in 3D space using Unity.

Combinatorics and Recursion

ISEC 120

Chair: Greg Malen (Skidmore College)

9:45 - 10:00 *Fibonacci-related Recursive Sequences* (Level 1)

Kian Broderick (Ithaca College)

The Golden Ratio, an enduring fascination for artists and mathematicians since ancient times, is an irrational number approximately equal to 1.618. It is the ratio of two numbers such that the ratio between them equals the ratio of the larger number to the sum of both. Beyond its aesthetic significance, the Golden Ratio manifests in geometry, notably in the construction of Golden Rectangles and the diagonal of a pentagon. Binet's formula establishes a link between the Fibonacci numbers and the Golden Ratio, and allows the calculation of arbitrary terms

in the sequence. Using similar constructive methods to Binet, alternative ratios can produce other formulas that generate other sequences. Using the ratio of $1 + \sqrt{3}$ produces a sequence where each term is twice the sum of the two previous. This sequence exhibits several analogous properties to the Fibonacci numbers, such as having similar summation formulas and identities.

10:05 - 10:20 *Exploring a New Family of Recursively Constructed Graphs* (Level 1)

Elsa Selin Frankel (Wellesley College)

Graphs are mathematical tools, commonly used to study relationships between pairs of objects, with numerous applications in both theoretical and applied sciences. Individual objects are often represented by *vertices*, and pairs of these vertices are joined by *edges*, according to relevant relationships. You may recognize the *complete graph* on n vertices K_n , where each pair of distinct vertices is connected by a unique edge. Going further, we can glue a collection of identical complete graphs at a shared central vertex to obtain a *windmill graph*, denoted $Wd(n, k)$. Today, I'll introduce a new family of graphs, constructed from a collection of distinct windmill graphs, similarly glued at a shared central vertex. I'll additionally touch on some basic structural properties of these graphs and potential directions for the work.

10:25 - 10:40 *New Formulas for Motzkin Numbers* (Level 2)

David Vella (Skidmore College, Retired)

The Motzkin numbers M_n were introduced in 1948 as the number of ways to draw non-crossing chords on n points around a circle. They are related to Catalan numbers and have similar properties. In this talk, I will present newly discovered formulas (both recursive and explicit) for these numbers, using a theorem on the composition of generating functions. No previous exposure to Motzkin numbers is needed to understand the talk.

Graph Theory I

ISEC 150

Chair: Britny Quito (Union College)

9:45 - 10:00 *Graph Colorings and Gridlock* (Level 1)

Elaine Demetron (Smith College)

Kira Hou (Smith College)

Sharon Spaulding (Smith College)

Meredith Williams (Smith College)

The labeling of vertices of a graph with different colors has proven to be a useful tool for studying certain social coordination problems. In particular, a network of individuals who have successfully reached a consensus can be modeled as a graph in which all vertices have the same color. However, such social systems can also have local optima where every vertex has selected the same color as a plurality of their neighbors, but consensus has not been reached. In this presentation, we describe our investigation into the properties of these “gridlocked colorings.”

10:05 - 10:20 *Rainbow Matching Sequencibility of Complete Graphs* (Level 1)

Adam Serota (Skidmore College)

The matching sequencibility of a graph G is the largest integer d such that there exists an ordering of G 's edges in which every d consecutive edges form a matching. In 2006, Alspach researched the matching sequencibility number for the complete graph K_n , and in 2021, Brualdi studied a cyclic version for this parameter. In this talk, we will discuss a new version for this parameter which incorporates edge colorings. In particular, we require that every d consecutive edges in our (cyclic) ordering form a matching in which every edge's color is distinct. To determine lower bounds for the (cyclic) rainbow matching sequencibility number of K_n , we

utilize an auxiliary hypergraph and results about Hamiltonicity.

Image Analysis

ISEC 218

Chair: Francesca Morone (Union College)

9:45 - 10:00 *A Survey of Regression-Based Methods for Edge Detection and Image Segmentation*
(Level 1)

Matthew Moore (Boston University)

In this presentation, we will explore the use of regression-based techniques for edge detection and image segmentation, highlighting both linear and nonlinear regression approaches. A brief introduction to regression will be provided for those unfamiliar with the concept, followed by a demonstration of how these methods can be applied to identify edges in images. Additionally, we will discuss techniques for smoothing detected edges to facilitate image segmentation, improving the clarity and structure of the segmented regions. The talk will conclude with an overview of practical applications, showcasing the relevance of these methods in various fields such as medical imaging, object recognition, and computer vision.

10:05 - 10:20 *Optimizing DDPM: Evaluating the Effects of Hyperparameters and Training Set Quality in Generative Image Synthesis* (Level 2)

Kaiyi Liu (Boston University)

In this talk, we will introduce the application of a denoising diffusion probabilistic model for generating images from datasets containing visually similar examples. The study will examine how variations in training set size and model tuning parameters influence the quality and fidelity of the generated images. Additionally, the analysis will focus on the correlation between the similarity of training images and the final output, as well as the impact of integrating outlier images into the training dataset. Overall, the talk aims to provide critical insights into optimizing generative models for enhanced performance in machine learning-driven image synthesis.

10:25 - 10:40 *Authenticating Van Gogh: A Machine Learning Approach to Art Fraud Detection*
(Level 2)

Yiyang Xu (Boston University)

In this discussion, we will explore machine learning techniques for authenticating Vincent van Gogh's paintings by distinguishing them from forgeries and works by other artists. We will use advanced image processing techniques to extract key stylistic features and using them to train supervised learning models, including Support Vector Machines optimized through hyperparameter tuning. The presentation will emphasize the methodological framework, model training process, and classification performance, demonstrating how computer vision can be used for art authentication.

Math Throughout History

Olin 206

Chair: Sean Carney (Union College)

9:45 - 10:00 *Musically Mathematical: How Math Permeates Every Part of Music Theory*
(Level 1)

Arija Hartel (Champlain College)

Throughout history and across cultures, music has always been a part of everyday life. From elaborate theatrical productions to catchy children's tunes, humans have always been creating

music. But how do we get those sounds? What even is an octave? How do scales work? What impact did mathematicians like Pythagoras and Plato have on musical theory? This talk will explore the mathematics behind music, from pitches to rhythm, as well as the history behind the when, where, and by whom these notes were discovered.

10:05 - 10:20 *Reconstructing Early Modern Persian Trigonometry with a Base-60 “Calculator”* (Level 1)

Ziayan Omer (Union College)

Between the 14th and 15th centuries, Persian scholars, such as Ghiyāth al-Dīn Jamshīd Al-Kāshī (under the guidance of the great Timurid Sultan and astronomer/mathematician Ulugh Beg), pioneered transformative iterative mathematical approximation techniques that allowed for highly accurate approximations of $\sin(1^\circ)$. These methods were further developed in Mughal-era India in both Persian and Sanskrit texts. This talk, based on ongoing work-study research, describes how we reconstruct these mathematical techniques and implement them computationally to better understand their accuracy and efficiency.

10:25 - 10:40 *Early Algorithms For Two-Player Combinatorial Games (Like Chess)* (Level 1)

Ryan Buck (Champlain College)

For decades, the desire to teach computers how to play (and win at) complex games like chess or Go has inspired mathematicians and computer scientists alike. Today, highly sophisticated, AI-driven engines outperform human players to a degree once thought unattainable. But before these algorithmic goliaths could run, earlier machines had to crawl. These humble pioneers employed more rudimentary algorithms that fought tooth and nail for even the smallest of victories. This talk will analyze minimax, one of these simpler algorithms, and its numerous variations and improvements.

Mathematical Physics

Olin 306

Chair: Jeff Jauregui (Union College)

9:45 - 10:00 *Tensor Analysis is not dead (yet)* (Level 1)

Andrea Dziubek (SUNY Polytechnic Institute)

Tensor analysis is the language of classical continuum mechanics. Progress has been made in reformulating the equations of electromagnetism, fluids, and elasticity in terms of differential forms. However, exterior calculus will only supersede tensor analysis when this process is completed. We will compare both notations and provide some historical context. Joint with Nicholas Andrzejkiwicz, Connor Donovan, Benjamin Maiorella, Chawn Neal, and Dan Nguyen.

10:05 - 10:20 *Deriving the Radial Perturbation equations for a Neutron Star* (Level 2)

Chawn Neal (SUNY Polytechnic Institute)

The study of neutron star oscillations provides critical insight into the internal structure and stability of super compact objects under general relativity. This work explores the derivation of radial perturbation equations for neutron stars, starting from Einstein’s Field Equations (EFE) and the stress-energy tensor for a perfect fluid. By assuming a static, spherically symmetric background spacetime, the EFE reduce to the Tolman–Oppenheimer–Volkoff (TOV) equations, which describe the equilibrium structure of relativistic stars. Small, time-dependent perturbations are then introduced to both the metric and the fluid variables, preserving spherical symmetry. We linearize the EFE and enforce conservation of the perturbed stress-energy tensor. This yields a second-order differential equation governing the radial displacement of fluid elements. This solution, the radial eigenfunctions oscillation frequencies, determines the

neutron star model's stability. The resulting framework forms the basis for analyzing the dynamical response of neutron stars to radial perturbations and identifying instability thresholds in realistic stellar models.

10:25 - 10:40 *Investigating the Quantum States of a Multistep Potential Well* (Level 2)

Connor Donovan (SUNY Polytechnic Institute)

This project explores the stationary states and energy levels of a one-dimensional, multi-step, finite potential well. By solving the Time-Independent Schrodinger Equation, and applying the appropriate boundary conditions, we derive analytical expressions to solve for quantum states and their corresponding wave functions. By introducing the one dimensional, finite square well, we accumulate the necessary intuition needed to solve a multi step potential well. Numerical methods, including the Bisection and Regula-Falsi methods, are employed to compute discrete energy levels. These eigenstates are then normalized, and the corresponding wave functions are graphed. The results demonstrate how energy quantization and wave function behavior depend on the well's structure and depth, offering insights into fundamental quantum systems.

Number Theory I

ISEC 124

Chair: Vincent Ferlini (Keene State College)

9:45 - 10:00 *Arithmetic Sequences and Repdigits* (Level 1)

Karena Zhang (Mount Holyoke College)

An n -digit *repdigit* is a positive integer of the form $rr\dots r$ denoted r_n where r is a nonzero digit. A positive single digit will be called a *trivial repdigit*. The repdigit 1_n is called a *repunit*. A Berkeley Math Circle Competition (Contest 7, 23 April 2000) poses the following problem: Prove that there are infinitely many terms in the arithmetic progression 8, 21, 34, 47, ... which consist entirely of nines. The objective of this article is to generalize this problem and characterize those arithmetic sequences that have no repdigits, a finite number of repdigits, and an infinite number of repdigits. We begin with a definition that would be specific to this paper.

10:05 - 10:20 *Intro to Markov Numbers and Lagrange Spectrum* (Level 1)

Shelby Branam (Smith College)

Almanzo Gao (Smith College)

Sanjana Paul (Smith College)

Amy Pinargote (Smith College)

Yuju Shin (Smith College)

How well can we approximate real numbers with rational numbers? For example, how close to π can we get using only rational numbers? For instance, 3 is close to π , because $|\pi - 3| = 0.14\dots$ but $|\pi - \frac{22}{7}| = 0.0012\dots$ is closer to π than 3. However, $|\pi - \frac{333}{106}| = 0.000083\dots$ is even closer. The simplest answer to the question is you can get as close as you want. The trick is, how can we keep the denominator small while still getting a good approximation of π ? In this talk, we will explore this question. In the process we will use continued fractions, special types of fraction that form under the following rule: "add a fraction in the denominator of another fraction." We will also introduce the Lagrange Spectrum, and Markov Numbers, and some research questions related to these concepts.

10:25 - 10:40 *A New Definition of Convergence for Approximate p -adic Solutions to Polynomials* (Level 2)

Imaad Uzun (SUNY at Binghamton)

Hensel's lemma is the foundation of p -adic number theory. A relatively new method of proving Hensel's lemma using Newton's method was found to imply an inequality defining a bound on the rate of convergence of approximations of p -adic solutions to polynomials. The most information given by the inequality is that the number of p -adic digits of an approximation agreeing with the actual p -adic solution at least doubles with each iteration of Hensel's lemma. The inequality has been demonstrated to be an equality numerically under basic conditions, but prior to this work there existed no general definition of when equality occurs. Thus, we prove that under certain minimal conditions we have equality. When these conditions are met, the accuracy of the digits in the approximations is guaranteed to strictly double at each step. We reach this result through double induction and a careful counting argument, supplemented by an original Python program.

Statistics I

Wold 128

Chair: Phaniel Mariano (Union College)

9:45 - 10:00 *Exploring the Impacts of Social Determinants on Exercise* (Level 1)

Samuel Lagasse (United States Military Academy)

Physical exercise is an important and well-studied contributor to a healthy lifestyle. Our research goal is to determine what social and environmental factors, collectively called social determinants of health, correlate with an active lifestyle. Previous literature involving social determinants and exercise tends to focus on the impact of the physical environment such as public parks, transportation, and exercise facility access on activity. Our research uses data from the All of Us research program social determinants of health survey and accelerometry data. By clustering by social determinants and performing ANOVA tests on fitness metrics, we found social factors like community connectivity were more indicative of an active lifestyle than the individual's physical environment. In this talk we will share results of this analysis, some of which is contrary to existing literature in the field.

10:05 - 10:20 *Observing Mental Health Changes from 2007 to Present* (Level 1)

Patrick Gahan (Saint Michael's College)

This study will model trends in anxiety and depression among college students over the past 18 years using data from the Healthy Minds Network, which includes data collected from several undergraduate institutions across the United States. We will examine trends across five distinct years and apply a logistic regression model to identify predictors of these mental health disorders. Additionally, we will explore potential therapies and resources that could reduce the prevalence of anxiety and depression among students.

10:25 - 10:40 *Treatment Disparities in Substance Use Disorders: Examining the Impact of Socioeconomic Status, Location, and Age* (Level 1)

Sydney Berger (Union College)

This paper examines the effects of socioeconomic status, location, and age on the treatment of substance use disorders. Using statistical methods such as Cramer's V and cluster plotting, we analyze how these factors influence access to treatment. Our findings indicate that socioeconomic disparities and geographic differences significantly impact the availability of care, potentially leading to unequal treatment outcomes. These findings show the need for policy

interventions which would improve equitable access to substance use disorder treatment across diverse populations.

Topology

Olin 204

Chair: Nabeel Naqvi (Union College)

9:45 - 10:00 *Colorable or Knot: Colorings of Knots by Semi-Direct Product* (Level 1)

Khailah Johnson Bustamante (Smith College)

Maria Maalouf (Smith College)

Sofie Ratsimamitaka (Smith College)

A knot is an embedding of a circle in space. One goal in the field of knot theory is to find new invariants to tell knots apart. We'll explain how we can use a theorem of Hartley to determine ways to color knots diagrams by elements of certain groups using semidirect products. If time permits, we will discuss applications in four-dimensional space.

10:05 - 10:20 *Counting Open Sets that Map to Themselves* (Level 1)

Teddy Welsh (Westfield State University)

An open interval doesn't include its endpoints (or doesn't have an endpoint), and topology concerns itself with when a function maps an open set to another open set. What if both open sets are the same thing? How often does this happen? A linear function typically intersects its inverse in one point, and we can use that one point to build five examples of an open set that might map to itself. What about a cubic function? The counting gets interesting pretty quickly. Come count with us!

10:25 - 10:40 *Euler's Path: Solving an Old Problem, Starting a New Field* (Level 1)

Khanh Vu (Mount Holyoke College)

In 1736, Leonhard Euler introduced graph theory by solving the Königsberg bridge problem: Can one cross all seven bridges exactly once? Instead of focusing on locations, Euler reduced the problem to connections, laying the foundation for a new mathematical field.

This talk explores Euler's original paper, including his visualization of the problem and proof that such a path is impossible. We will also discuss the conditions needed for a valid solution and how modifying the bridges could make it possible.

Euler's work extends beyond this puzzle, influencing topology and leading to profound mathematical advancements. His insights continue to impact fields like computer science, network theory, and optimization. What began as a simple problem became the cornerstone of a discipline that now underpins modern technology and science.

Parallel Sessions – Early Afternoon

Abstract Algebra II

Olin 106

Chair: Grant Moles (Union College)

2:00 - 2:15 *Idempotents in Quandle Rings* (Level 2)

Zhaoqi Wu (College of the Holy Cross)

A quandle is a set X with a binary operation satisfying three axioms that algebraically encode the three Reidemeister moves in knot theory. Let R be an associative ring with unity, and $R[X]$ be the set of all formal finite R -linear combinations of elements of X . Then, $R[X]$ is a non-associative ring with coefficients in R . We investigate idempotents in quandle rings $R[X]$ using Gröbner bases. This is a joint work with Neranga Fernando (College of the Holy Cross).

2:20 - 2:35 *Permutation polynomials defined by functional equations over finite rings* (Level 2)

Qingjian Jiang (College of the Holy Cross)

Let R be a commutative ring with unity, and $R[X]$ be the ring of polynomials with coefficients in R . A polynomial $f(x) \in R[X]$ is called a *permutation polynomial* if it permutes the elements of the ring R . In most studies on permutation polynomials, R has been taken to be a finite field. However, there have been a few studies on permutation polynomials over finite rings \mathbb{Z}_m , where $m > 1$ is a positive integer, because of their applications in the areas of cryptography and coding theory.

Let p be a prime factor of m . We investigate the permutation behavior of the polynomial g_n defined by

$$g_n(x) = \sum_{\frac{n}{p} \leq l \leq \frac{n}{p-1}} \frac{n}{l} \binom{l}{n-l(p-1)} x^{n-l(p-1)} \in \mathbb{Z}[x]$$

over finite rings \mathbb{Z}_m . In this talk, we will explain the conditions on n and m for which the polynomial g_n is a permutation polynomial over \mathbb{Z}_m . Moreover, we will explain how the derivative of g_n plays an important role in our investigation on the permutation behaviour of the polynomial g_n .

2:40 - 2:55 *The Hilbert Series of Toric Ideals Associated to Principally Generated Borel Ideals* (Level 2)

Dennis Belotserkovskiy (Colgate University)

Toric rings are valuable objects of study in combinatorial commutative algebra and algebraic geometry. They encode the behavior of an ideal's generators in the quotient of a polynomial ring. Strongly stable ideals have several symmetries and combinatorial niceties, and their associated algebras are, therefore, relatively easier to work with. One may ask how these toric rings grow with degree. Strongly stable ideals have generators that are tightly related. Does this dependence dampen the growth enough to change the ring's dimension? For that matter, how is this growth measured?

In this talk, we find the general Hilbert series corresponding to these toric rings, which will allow us to infer their dimension. Using tools from graph theory, commutative algebra, and combinatorics, we reinterpret the computation of the Hilbert function as a combinatorial problem and produce a general formula for the Hilbert function and Hilbert series of toric rings of this type.

2:00 - 2:15 *Duplicate Screenshot Detection and Adversarial Machine Learning* (Level 1)

Gikonyo Njendu (United States Military Academy)

Misinformation remains a pervasive threat in the information dimension of warfare. An emerging trend in online misinformation is the sharing of screenshots on social media sites. Screenshots have been used to document important moments and share meaningful images, but also evade detection and propagate incorrect information. Screenshots are naturally and intentionally difficult to detect. In our work, we present a novel method based on Language Vision Models (LVMs) to obtain zero-shot screenshot classification and produce a more resilient model. We further investigate augmenting this process by combining optical character recognition to utilize the textual information frequently present in screenshots. The results of this research will aid in the development of novel tools and methodologies to assess the information space from a social-cyber perspective. The model itself will allow for misinformation propagation path backtracking to determine the source of fake adversarial misinformation attempts.

2:20 - 2:35 *New Methods for Identifying Important Structures in a Neural Network Model* (Level 2)

Olivia Kaminske (Keene State College)

Combinatorial Threshold-Linear Networks (CTLNs) are a neural network model that is used to simulate the firing rates of neurons. This model is based on a system of differential equations that compute the firing rates of each neuron in the system. In particular, we are interested in a special family of CTLNs called core motifs. Identifying these core motifs is integral to extrapolating CTLN findings to larger networks like the brain, but checking if CTLNs are core is computationally complex and difficult to scale. In an effort to more easily identify core motifs, I formed two conjectures that rule out large numbers of CTLNs as not core using simpler computations. These two conjectures use determinant sign and out-degree uniformity respectively to drastically reduce the computations required to find core motifs, allowing for greater scalability and computability.

2:40 - 2:55 *Exploring the Manifold of Neural Networks Using Diffusion Geometry* (Level 2)

Drew Steindl (Vassar College)

We apply manifold learning to the space of neural networks with different hyperparameters trained to perform the same tasks, by organizing networks on the basis of their hidden representations of the same data points. These representations are captured as data diffusion operators. Then, we introduce a distance between the hidden layer representations of the neural networks based on the Frobenius norm between these diffusion operators. We then characterize this manifold using PHATE visualizations and features of our representation. High-performing networks cluster together in this manifold. We demonstrate the utility of this approach in generating new neural networks. We use manifold scattering to encode the hidden layer representations to single points, capturing the geometry of the point cloud. Now we can interpolate between high-performing network representations. We then show that training a neural network to match this representation improves performance beyond the original networks.

2:00 - 2:15 *Some Integral Inequalities* (Level 1)

Fotios Paliogiannis (St. Francis College)

We present some important integral inequalities using standard Calculus techniques. Specifically, we prove Cauchy-Schwarz's, Minkowski's, Sobolev's and Wirtenger's inequalities.

2:20 - 2:35 *Some Applications of the Gamma Function* (Level 2)

Melie Boulianne (St. Francis College)

We will talk about two important applications of the Gamma function. The first application will be to use the Gamma function to evaluate integrals, and the second application will be to introduce fractional derivatives for the power function $f(x) = x^a$ where $a \in \mathbb{R}$ and $x > 0$.

2:40 - 2:55 *The Problem With the Riemann Integral, and How Lebesgue Solved It* (Level 2)

Cormac St. John (Vassar College)

The talk will examine integrating the characteristic function of the irrational numbers over the closed interval $[0,1]$. First, it will briefly review the Riemann integral and explain why it cannot be used to integrate such a function. Then, it will introduce the Lebesgue integral and explain how easily it can be used to integrate the characteristic function.

2:00 - 2:15 *Topology in Economics: Fixed Points & Nash Equilibria* (Level 1)

Reyna Li (Williams College)

This expository talk explores the relationship between Brouwer's fixed point theorem and the existence of Nash equilibria, visiting the Kakutani fixed-point theorem along the way. The ideal audience has some familiarity with basic point-set topology and probability, although this is not required. Prior economics knowledge will not be assumed.

2:20 - 2:35 *Implementing Topological Data Analysis in Transcriptomics Research with Mapper: A Guided Demonstration* (Level 1)

Colin Mikulski (Vassar College)

Topological data analysis (TDA) uses techniques from algebraic topology and geometry to extract information from the geometric structure of data. Although TDA has received much attention from data scientists and is used in a variety of fields from genomics to time-series analysis, few undergraduate researchers are exposed to these methods outside of mathematics. This talk will give students a guided tour of a TDA pipeline developed to analyze translational transcriptomics data. During this tour, the speaker will run through the pipeline, explain the purpose of each function call, guide the interpretation of the analysis, and take questions and input from the audience.

2:40 - 2:55 *Graphical Output Prediction Using Neural Network Structure* (Level 2)

Genevieve Steenhoek (Keene State College)

Combinatorial Threshold-Linear Networks (CTLNs) are a simplified way to represent complex neural networks and can help simulate brain activity. The CTLN model consists of a set of ordinary differential equations, and their solutions give the firing rates of each neuron. When

you plot these firing rates, you often see a pattern of neurons firing in sequence. The goal of this research is to predict this firing sequence based on the network's structure. We make these predictions by analyzing the network's graph and studying how the nodes (neurons) are related to each other.

Applied Math II

ISEC 222

Chair: Jue Wang (Union College)

2:00 - 2:15 *Investigating joint behavior of energy harvesting and energy transfer in vibro-impact systems* (Level 2)

Aine Doherty (Vassar College)

Vibro-impact systems are a novel approach to energy harvesting, harnessing ambient vibrations to generate energy. In these systems, energy is transferred from an external oscillating structure to a damped spring-mass system, and this energy is converted into electrical energy through internal collisions via a free-moving ball. It is advantageous to design these systems such that the transfer of energy and generation of energy are both high.

We use both numerical and analytical approaches to investigate a system of nonlinear differential equations and discrete dynamical maps that describe the system between impacts and at collisions. These systems are often chaotic, and exhibit a wide multitude of stable solutions with varying structures and periodicities.

Our analysis showed that energy transfer and energy harvesting do not necessarily correlate by identifying parameter regions and environmental conditions where harvesting and transfer behaved asymmetrically. We further investigated inclining the spring-mass system, and characterized the influence of gravitational forces on the behavior of the system.

2:20 - 2:35 *Parameter Identifiability in PTBP3 Binding Dynamics* (Level 1)

Alexandra Carr (Vassar College)

In this project, we analyze the parameters involved in protein dynamics using Fluorescence Recovery After Photobleaching (FRAP) intensity data, focusing on the protein PTPB3, which is involved in RNA splicing and cell proliferation. We perform a parameter identifiability analysis through both profile likelihood graphs and Fisher information to refine our model. The analysis is carried out analytically and numerically. First, by solving the underlying PDE and using limiting cases, we identify key parameters and assess their impact on the system. We also conduct numerical simulations to estimate parameters for a given dataset. In cases where identification proves unsuccessful, we explore the parameter space and identify parameter combinations that minimize the least squares error.

Combinatorics and Games

ISEC 120

Chair: Kim Plofker (Union College)

2:00 - 2:15 *Combinatorics of Nim* (Level 1)

Keaton Jilg (United States Military Academy)

The game of Nim involves two players taking turns subtracting a chosen number of tokens from a single pile until no more tokens can be removed. Assuming optimal play, the winning and losing positions form periodic and occasional preperiodic sequences. This study explores the effect of different subtraction sets on the length and structure of these sequences.

2:20 - 2:35 *Rook in the Menagerie* (Level 1)

Kai Maffucci (University of Rhode Island)

Rook is an Appalacian card game in the Bridge-Whist family of card games that utilizes a unique deck of 57 cards. Victor Mollo wrote a good book on Bridge called *Bridge in the Menagerie* with a colorful cast of characters. Amongst them are the Hideous Hog (an aggressive player), Papa Greek (a moderate player), Karapet (the Armenian, whose hardships have made him into a cautious player), and the Rueful Rabbit, who's a bit of a wildcard but sometimes stumbles into success. I wanted to analyze bidding strategies in Rook, so I wrote a simulation to bid and play the game featuring Victor Mollo's characters. My presentation will discuss my findings.

Graph Theory II

ISEC 150

Chair: Susan Beckhardt (SUNY Albany)

2:00 - 2:15 *Perfect Roman Domination and Bounds on Bondage Numbers of Graphs* (Level 1)

Sophia Child (Colgate University)

A Roman domination function is a labeling of the vertices of a graph G is with 0s, 1s, and 2s where every vertex labeled 0 is adjacent to at least one vertex labeled 2. A perfect Roman domination function (PRDF) is a labeling of the vertices of a graph with 0s, 1s, and 2s where every vertex labeled 0 is adjacent to exactly one vertex labeled 2. The perfect Roman domination number, $\gamma_{PR}(G)$, of a graph is the minimum sum of these 0s, 1s, and 2s given a valid PRDF. We found formulas for $\gamma_{PR}(G)$ of certain graph classes. We also examined the positive and negative bondage number of a graph, which are the minimum number of edges need to be removed in order to increase and decrease $\gamma_{PR}(G)$, respectively. We found formulas for positive and negative bondage for certain graph classes, along with upper bounds on the positive bondage number and the existence of a negative bondage number.

2:20 - 2:35 *Lonely Distancing Dance Graphs: An Undergraduate Research Project* (Level 1)

Matthew Glomski (Marist University)

People can crave company while simultaneously seeking to avoid too much of it. Elaborating on this goal of “lonely distancing” we construct a scenario in which k players on an $n \times n$ grid jockey to occupy space next to a single companion while maintaining at least a minimum distance from everyone else. The tension behind these conflicting needs—companionship versus isolation—induces a dynamical system on the grid we model with directed graphs. Depending on k and n , players are likely to redistribute themselves into one or more equilibrium states. In very special cases no such resolution is possible, and the players enter a cycle of perpetual redistribution in what we call a never-ending lonely distancing dance.

Mathematical Modeling I

ISEC 218

Chair: Sean Carney (Union College)

2:00 - 2:15 *From Boiling to Diffusion: A Computational and Experimental Study of Lithium Chloride in Water* (Level 1)

Ranee James (Eastern Connecticut State University)

This research investigates the effect of lithium chloride concentration on the boiling point of water under both standard and reduced pressures, as well as its diffusion behavior in water with and without evaporation effects. We experimentally demonstrate that lithium chloride elevates the boiling point of water, while reduced pressure lowers it. Additionally, as water

boils and evaporates, its loss alters the concentration and distribution of lithium chloride in solution. Computational modeling is used to simulate the diffusion and redistribution of lithium chloride as evaporation occurs. The simulations provide insight into how the solute accumulates and redistributes over time as water is lost, helping to interpret the effects observed in the experimental findings on boiling dynamics. By integrating experimental observations with computational simulations, this study offers a comprehensive analysis of how lithium chloride behaves at different concentrations under standard and reduced pressures. Understanding the thermodynamic properties of lithium chloride provides foundational insights that may support further research into the biological behavior of lithium ions in the body.

2:20 - 2:35 *Dynamics and persistence of a generalized multi-strain SIS model* (Level 1)

Scott Greenhalgh (Siena College)

Autonomous differential equation compartmental models hold broad utility in epidemiology and public health. However, such models typically cannot account explicitly for myriad factors that affect the trajectory of infectious diseases, with seasonal variations in host behavior and environmental conditions as noteworthy examples. Fortunately, using non-autonomous differential equation compartmental models can mitigate some of these deficiencies. In this presentation, I will illustrate a rare n -strain generalized Susceptible-Infectious-Susceptible (SIS) model, with a very general time-varying recovery rate, where the stability conditions are algebraic expressions. To illustrate the applicability of the model, we will apply it to recent syphilis incidence data from the United States, utilizing Akaike Information Criteria and Forecast Skill Scores to inform on the model's goodness of fit relative to complexity and the model's capacity to predict future trends. Joint with Tabitha Henriquez, Rebecca Leonard, and Michael Frutschy.

2:40 - 2:55 *Development of tools for parameterization and validation of a model of fractal growth using image analysis and embedding* (Level 1)

Farzana Chowdhury Neha (Mount Holyoke College)

Anika Nazhat (Mount Holyoke College)

Our research goal is to develop image-based metrics that measure the distance between images and can be used to parameterize models and validate them by comparing distances within and between model images and empirical images. We used FIJI to extract network elements—branches, angles, and junctions—from experimental images and compared them with model-generated structures using k-L divergence algorithms to quantify statistical differences. Additionally, a VGG-16 machine learning model is being used to calculate the similarity between experimental and simulated images, which provides a similarity index between -1 and 1, providing information on how close our model is to the actual images. This research computationally modeled the fractal growth of polyvinyl alcohol (PVOH) polymer networks formed experimentally through spin coating on PDMS substrates. Our triangular growth model in Net-Logo simulates this process of fractal formation. We demonstrated the image-based metrics techniques mentioned above by applying them to our model of polymer development.

Number Theory II

ISEC 124

Chair: Jeff Hatley (Union College)

2:00 - 2:15 *Elliptic Curve Cryptography: Data Encryption through Math* (Level 1)

William Allen (Wesleyan University)

Elliptic curves form the foundation of many modern cryptographic systems. In this talk, we will discuss one such cryptosystem, the Elliptic-Curve Diffie—Hellman algorithm. To understand

the role of elliptic curves in this algorithm, we will first explore how the set of points on elliptic curves form a group, and then how that group structure lends itself to efficiently encrypting data.

2:20 - 2:35 *Rational Points on Elliptic Curves* (Level 2)

Mengchan Geng (Mount Holyoke College)

Let E be a non-singular elliptic curve. Then the group of rational points $E(\mathbb{Q})$ is a finitely generated abelian group. We investigate the arithmetic properties of elliptic curves with a focus on the distribution of the index $i_p(E) = [E(\mathbb{Q}) : \text{red}_p(E(\mathbb{Q}))]$, which measures the discrepancy between the group of points on $E(\mathbb{Q})$ and their images under the reduction modulo p map. From a computational approach with MAGMA and elliptic curves over \mathbb{Q} from the L-functions and modular forms database, we analyze how the surjectivity of the reduction map changes as the rank increases.

2:40 - 2:55 *An Audio Analysis of Elliptic Curves* (Level 2)

Grace Newcombe (Union College)

An elliptic curve is an equation that can be expressed in the form $y^2 = x^3 + ax + b$, where a, b are certain rational constants. They can be expressed mod primes p , and the solutions to these curves in this form can be used to define a specific Fourier series (particular sums of periodic sine and cosine functions). These series can then be audialized using techniques such as additive synthesis in order to hear how they behave. The goal of this thesis is to use additive synthesis to analyze elliptic curve data.

Statistics II

Wold 128

Chair: Laura Lyman (Mount Holyoke College)

2:00 - 2:15 *A Data Analysis on What Influences the Gap between Parenting Desire and Expectation for Lesbian, Gay, and Bisexual Adults* (Level 2)

Chantal Larose (Eastern Connecticut State University)

The Generations Study surveys many aspects of life for lesbian, gay, and bisexual adults. Variables on parenting cover, among other questions, the self-reported desire and likelihood to be a parent. The presented work analyzes what factors may influence the gap between responses to these two parenting questions, including demographic and geographic information as well as reported connection to the LGBT community, social support, and stigma felt in day to day interactions. Results include cluster analysis and a classification and regression tree model for parenting expectation gap. Future work includes expanding this analysis to include the transgender community through the TransPop study.

2:20 - 2:35 *Selected Statistics on the Observed Social Dynamics of the Island of Sodor* (Level 1)

Fiona Murphy (Champlain College)

Everybody knows Thomas (of Tank Engine fame), but few pay as much attention to the other half of the series title—"and Friends". Who are these friends? How often do they appear? Who are Thomas' best friends? Who are really enemies masquerading as friends? In this talk I will explore these questions and more using statistics.

2:40 - 2:55 *Exploring the Role of First-Generation Status, Institution Type, and Income Earned on Earnings After Graduation* (Level 1)

Antoinette Martekuor Darpoh (Mount Holyoke College)

As part of the National Center for Education Statistics, IPEDS (Integrated Postsecondary Education Data System) annually collects data from US colleges, universities, and technical institutions that participate in federal student financial aid programs. This project explores how the percentage of first-generation students, the type of institution (public or private), and average family income affects (log) median earnings of students six years after graduation. In particular, I analyze the data of 3,676 schools surveyed by IPEDS. The goal of this research is to help current and prospective students, as well as their families, make informed decisions when choosing educational institutions.

Tilings and Fractals

Olin 206

Chair: Jeff Jauregui (Union College)

2:00 - 2:15 *The Art of Mathematics: Finding Beauty in Tiles, Fractals, and More* (Level 1)

Ellie Newman (Champlain College)

Whether it is in aperiodic tiles that form hypnotic patterns or in fractals such as the Mandelbrot set that dazzle with infinite levels of complexity, mathematics is a field filled with unexpected beauty and artistic inspiration. Even a page filled with numbers, symbols, and Greek letters can captivate and mesmerize an audience. Join me as we take a visual tour showcasing the aesthetics of mathematics and some of its creative applications, including software and fashion.

2:20 - 2:35 *Counting Finite Tilings: An Empirical Proof for Beekman's Theorem* (Level 1)

Sebastian Neumann (United States Military Academy)

This project focuses on counting the number of arbitrary, simple tiles in a finite, edge-to-edge tiling using an equation proposed by Mr. Richard Beekman. A proof of his equation using principles of set theory and topology will be presented, as well as outlines of the capabilities and limitations of the equation, and explorations of potential applications in a variety of fields.

Parallel Sessions – Late Afternoon

Analysis II

Olin 107

Chair: Fanhui Xu (Union College)

3:30 - 3:45 *Nonstandard Analysis* (Level 1)

Paul Schulze (Colgate University)

When Newton and Leibniz were inventing calculus, they heavily relied on the idea of the infinitesimal, a number that is infinitely small but not 0. Nonstandard Analysis is a way of making this infinitesimal-based approach to calculus rigorous, by extending the set of real numbers to the set of “hyperreals.” We learn about the hyperreals through the transfer principle, which states that any sentence of first-order logic that is true about the real numbers is also true about the hyperreals. This talk covers how we define basic calculus concepts in nonstandard terms, and how we use the transfer principle to prove basic theorems of calculus.

3:50 - 4:05 *Exploring Equivalence Between Areas Under Curves and Their Linear Regression Lines* (Level 1)

Omar Elkhadar (Bard College at Simon’s Rock)

In this paper, we explore an interesting hypothesis: for any continuous curve passing the vertical line test, the area under the curve over a specific interval equals the area under its linear regression line over the same interval. By combining calculus, statistics, and graphical insights, we demonstrate this equivalence with proofs, examples, and visualizations.

Differential Equations

ISEC 222

Chair: Sean Carney (Union College)

3:30 - 3:45 *Numbers in Nature: Parameterizing Differential Equations* (Level 1)

Jenny Krueger (St. Lawrence University)

In undergraduate Differential Equations courses, students are often introduced to the Lotka-Volterra equations as a classic model for predator-prey dynamics. While these equations effectively illustrate predator-prey population behaviors, the process of determining their parameters from real-world data is often left unexplored. In this talk, I will present a practical approach to deriving these parameters using empirical data. By demonstrating a functional method for parameter estimation, this presentation bridges the gap between theoretical modeling and applied analysis, offering insights into how mathematical models can be grounded in real-world ecological observations.

3:50 - 4:05 *A PDE for Growth of Invasive Watermilfoil in a Column* (Level 2)

Soleil Demick (Skidmore College)

Watermilfoil (*Myriophyllum*) is an invasive genus of aquatic macrophyte to North America. Starting from the floor of their inhabited body of water, milfoil grow upwards to form dense canopies on the water’s surface, which has adverse effects on both the local ecosystem and human activities. Our model investigates the growth of a small patch of milfoil within a column of water, with growth at each depth determined by the light level. Rather than directly modeling the biomass distribution, we model the distribution of plant tips within the column, from which biomass can be inferred. These tips are transported across the column according to a 1D convection-diffusion equation, with a velocity field derived from the Beer-Lambert law.

Results are given for zeros of the velocity field and steady-states, and numerical simulations are shown.

4:10 - 4:25 *Stochastic Differential Equations in Finance: Theory, Modeling, and Practical Solutions* (Level 2)

Minghui Wang (Boston University)

In this presentation, we will explore the fundamentals of stochastic differential equations (SDEs), discussing their formulation, underlying principles, and key differences from traditional differential equations. The discussion will extend to the application of SDEs in modeling and predicting the dynamic behavior of financial product prices. Furthermore, the presentation will introduce a numerical method, analogous to the Euler method for ordinary differential equations, designed to effectively approximate solutions to SDEs. Overall, the talk aims to provide a comprehensive understanding of both the theoretical and practical aspects of stochastic processes in financial contexts.

Geometry

Olin 204

Chair: Jenn Qian (Union College)

3:30 - 3:45 *The Projective Plane: Defined and Visualized* (Level 1)

Dakota Goto (Fordham University)

The projective plane is a fundamental geometric structure that extends the Euclidean plane by introducing points at infinity, providing a unified framework for understanding lines and their intersections. In this presentation, we will define the projective plane formally and develop intuition through visual representations, including an exploration of conics. Using diagrams and interactive visualizations, we aim to offer a clear and intuitive perspective on this elegant mathematical space. Out of this endeavor, we will see the idea of parallel lines meeting at a point at infinity emerges not by definition, but axiomatically.

3:50 - 4:05 *Visualizing Non-Euclidean Geometry* (Level 1)

Austin Baird (Champlain College)

What happens when you take some of the most fundamental rules governing how we define space and geometry and turn them on their heads? Unsurprisingly things get a little weird. But exactly how weird? What would these spaces look like and how could we interact with them? In this talk I will share my investigations and attempts at making strange geometries into something visual and interactable through the use of video game engines.

4:10 - 4:25 *Two-point set* (Level 2)

Cheng-Han Pan (Western New England University)

A two-point set is a subset of the plane that intersects every straight line in the plane at exactly two points. In this talk, we will introduce the method of transfinite induction and use it to construct a two-point set. The audience is expected to be familiar with mathematical induction.

3:30 - 3:45 *Curvature on Lie Groups constructed from Connected Regular Graphs* (Level 2)

John Goertemiller (United States Military Academy)

A regular directed graph can give rise to a Lie group through the definition of a bracket operator. This bracket operator gives rise to 2-step nilpotent Lie algebras are associated with Lie groups which each have their own geometric properties. The objective of this paper is to explore curvature properties of Lie groups related to directed graphs corresponding with Schreier graphs. In particular, this paper seeks to explore characteristics of a directed, connected graph that result in simplified curvature equations for an associated Lie group.

3:50 - 4:05 *A visual approach to symmetric chain decompositions of finite Young lattices* (Level 1)

Robert Donley (CUNY and Institute for Advanced Study)

The finite Young lattice $L(m, n)$ is rank-symmetric, rank-unimodal, and has the strong Sperner property. R. Stanley further conjectured that $L(m, n)$ admits a symmetric chain order. We show that the order structure on $L(m, n)$ is equivalent to a natural ordering on the lattice points of a dilated n -simplex, which in turn corresponds to a weight diagram for the root system of type A_n . Lindström's symmetric chain decompositions for $L(3, n)$ are described completely through pictures. This work is joint with Terrance Coggins, Ammara Gondal, and Arnav Krishna.

4:10 - 4:25 *Critical Groups: Exploring the Algebra of Chip-Firing Games on Graphs* (Level 2)

Marisa Zarcone (Colgate University)

The focus of this project is to study the critical groups of different families of graphs. The critical group of a graph is a finite abelian group defined using the graph's Laplacian matrix and can be described via chip-firing, a combinatorial game that involves distributing "chips" on the vertices of a graph and moving them according to specific rules. We will explore the critical groups of various families, including complete bipartite and circulant graphs. This research is motivated by a paper from Glass and Kaplan, who propose various open research problems related to the computation of critical groups of graphs.

3:30 - 3:45 *The Strong Law of Trig Functions* (Level 1)

Andrew McIntyre (Bennington College)

Richard Guy's "strong law of small numbers" is that there are not enough small numbers to meet the many demands made of them, leading to many accidental coincidences. I will argue that part of the confusion students have with trig functions is that there are not enough "simple" transcendental functions to meet the many demands of them: trig functions are, totally coincidentally, the solutions to at least three different central problems, and these three conceptually different "trigonometric functions" are in fact mathematically equal only by accident.

Everything I say will be well known to experts; this is for beginners in calculus, and for those of us who teach calculus or pre-calculus. I'll discuss some history, point you to a paper by Ne'Kiya Jackson and Calcea Johnson, and make the plea to introduce differential equations earlier in calculus.

The strong law of exponential functions and the strong law of logarithms will be left as exercises for the reader...

3:50 - 4:05 No talk – feel free to check out another session!

4:10 - 4:25 *Hands-On Mathematics* (Level 1)

Lucy Oremland (Skidmore College)

In this talk, we'll explore the process of designing custom mathematical manipulatives that enhance hands-on learning and conceptual understanding, using a 3D printer and laser cutter. Examples from Calculus, Linear Algebra, and Dynamical Systems will be shared.

Mathematical Modeling II

ISEC 218

Chair: Kim Plofker (Union College)

3:30 - 3:45 *Modeling Birth Control: Using ODEs to Understand Hormonal Dynamics* (Level 1)

Natasha Wozniak (Skidmore College)

Birth control pills have been used for decades as contraception. The contraceptive state is created by interrupting hormone dynamics in the female reproductive system. While the contraceptive effect of progesterone and estrogen is well known, the amount of progestin and estradiol used in the pills varies drastically, as do the recommendations for people who miss a pill or inconsistently use the pill. Using an Ordinary Differential Equation (ODE) model for endocrine dynamics, we modeled the hormonal effects of birth control pills using a new birth control term that represents added hormones. We varied both the quantity of progestin and estradiol, and the shape of this term from a constant concentration to an oscillating function. This let us determine the hormone concentrations needed for effective contraception. This method will be applied to look at the effects of a missed pill in various parts of the menstrual cycle.

3:50 - 4:05 *Modeling the Loss of the Menstrual Cycle in AFAB Individuals Receiving Testosterone Replacement Therapy* (Level 1)

Caro Faust (Smith College)

Many transgender (trans) men and nonbinary AFAB (assigned female at birth) individuals choose to undergo testosterone replacement therapy, TRT, where they take exogenous testosterone (T) to bring about masculinizing changes in the body. One of these changes is the loss of the menstrual cycle. T has a negative feedback effect on the hypothalamic pituitary axis (H-P axis) which dampens secretion of luteinizing hormone (LH). LH allows for secondary follicles to develop until the ovulatory phase and then prompts ovulation; therefore, as LH levels decrease, follicles are less likely to complete the ovulatory stage, thus disrupting the regular menstrual cycle. Our model explores the effects of varying dosages of T on the dynamics of the eventual loss of the menstrual cycle. Joint work with Ray Sauerwin, Jr. (Spring Hill College) and Ayezah Dar (Bowdoin College).

Number Theory IIIA

ISEC 124

Chair: Rylan Gajek-Leonard (Union College)

3:30 - 3:45 *Investigating symmetries among the gaps of numerical semigroups* (Level 1)

Caleb Shor (Western New England University)

Start with a set G of positive integers with $\gcd(G) = 1$. A numerical semigroup is the set S of all finite non-negative linear combinations of elements of G . It happens that any numerical

semigroup S contains all but finitely many positive integers. These missing integers are known as the gaps of S . In 2008, it was shown that for $G = \{a, b\}$, there is symmetry in the parity of the gaps (i.e., half are even and half are odd) precisely when both a and b are odd. In this talk, we will investigate the symmetries that exist among the gaps when working modulo m . This can lead to some unexpected places! We'll see a few. One involves counting lattice points in regions of the plane bounded by certain conics. Another is a problem about unique factorization among certain types of polynomials modulo $(x^m - 1)$.

3:50 - 4:05 *Arithmetic and statistical properties of generalized Bernoulli numbers* (Level 2)

Jane Warren (Fordham University)

The Bernoulli numbers B_n are rational numbers which have their origins in number theory but are ubiquitous in Mathematics appearing in subjects like algebraic topology and harmonic analysis. Their original importance came via Euler's celebrated results from the 1730's, which in current terminology, are about the special values of the Riemann zeta-function. The generalized Bernoulli numbers $B_{n,\chi}$ attached to a Dirichlet character χ were introduced by Leopoldt in the 50's. They appear in the special values of L-functions attached to Dirichlet characters. In this talk, we will discuss the computation on SAGE of generalized Bernoulli numbers $B_{n,p}$ in the simplest nontrivial situation of a quadratic Dirichlet character modulo an odd prime p . Then we will discuss certain arithmetic/integrality aspects of these numbers which are variations of results of Carlitz. We will also talk about comparison of certain statistical properties of $B_{n,p}$ vis-à-vis B_n . Joint work with Kaitlyn Marquez and A. Raghuram.

4:10 - 4:25 *A discussion on cryptosystems* (Level 1)

Saimon Ahmed (St. Francis College)

Sheikh Diallo (St. Francis College)

In this presentation, we will discuss the general construction of a cryptosystem. In particular, we will analyze the RSA cryptosystem, we will provide examples, and some applications.

Number Theory IIIB

ISEC 120

Chair: Tori Day (Mount Holyoke College)

3:30 - 3:45 *Exploring Properties and Patterns of Digital Roots* (Level 1)

Brianna Bownas (Ithaca College)

The digital root of a natural number is equal to the sum of the digits of the number. This concept can be applied to any base system, introducing the idea of a magic number, which is equal to one less than the base number. An application of digital roots comes in the form of Vedic Squares, multiplication tables where the digital root of the product of the row and column numbers is placed in each box. Digital roots can be used to form geometric designs. Simple shapes can be formed by connecting common digital roots within a Vedic Square. Alternatively, sequences of digital roots provide a step pattern for movement around a grid, each term corresponding to the number of units moved in a direction. The sequences can come from the rows of Vedic Squares, or from geometric sequences, like the Fibonacci sequence or those formed by other ratios.

3:50 - 4:05 *Integers, Permutations and Averages* (Level 1)

Vincent Ferlini (Keene State College)

Begin with the positive integer 629 and write down the permutations of its digits, i.e. all ways to write a 3-digit number using the digits 6, 2, and 9. In this case, they will be 629, 692, 269, 296, 926, and 962. The average of these six permutations is $3774/6 = 629$ which is one of

the permutations. The objective of this presentation is to determine all positive integers that have this property.

4:10 - 4:25 *Constructing Magic Squares: Human vs. Computer* (Level 1)

Sarah Wrzos (Ithaca College)

Magic Squares are a type of $n \times n$ matrix wherein each set of diagonals, rows, and columns sums up to the same magic sum. Magic Squares can be classified into distinct categories, each with its own construction method. These classes include $4n$, $4n + 1$, $4n + 2$, and $4n + 3$. By understanding the methods and why they work, we created a program in Java that will print a magic square for any given dimension. This work highlights the difference in how we, as people, think about patterns, versus how a computer understands only what we tell it, with no room for unambiguity. By utilizing these methods, we can now construct Magic Squares of almost any order, often in multiple ways, not only through a pen and paper through a human hand, but also through voltages and wires on a computer.

Probability

Wold 225

Chair: Phaniel Mariano (Union College)

3:30 - 3:45 *A Discretization of Bertrand's Paradox* (Level 1)

Greg Churchill (SUNY Oswego)

Dakota Williams (SUNY Oswego)

In 1889, Joseph Bertrand presented three different “answers” to a geometrical probability problem, famously known as Bertrand’s Paradox. The problem considers an equilateral triangle inscribed in a circle and a randomly chosen chord on the same circle; what is the probability that the chord will have length greater than the length of a side of the triangle? Bertrand was not interested in solving this problem per se but rather in exposing an issue with some probabilistic approaches. For this talk, we present a discretization of Bertrand’s Paradox, which makes no attempt to help resolve the paradox, but rather further shroud it in mystery.

3:50 - 4:05 *I'm Listening: Exploring Frasier Subtitle Data using Naïve Bayes* (Level 1)

Mason Lee (Champlain College)

Naïve Bayes is a statistical classification technique based on Bayes’ theorem that today is commonly applied to text-heavy datasets and can be used in tasks such as spam filtering in emails. In this talk, we discuss how Naïve Bayes works as a classification algorithm, detailing the underlying mechanics and probabilities. A demonstration of the algorithm will be provided using the transcript data from the popular 90’s sitcom Frasier, starring Kelsey Grammar, which followed the life of Frasier Crane, a psychiatrist and radio talk show host in Seattle, Washington.

4:10 - 4:25 *A Central Limit Theorem for Products of 2×2 Random Non-invertible matrices and Applications* (Level 2)

Audrey Benson (Union College)

Hunter Gould (Union College)

Grace Newcombe (Union College)

Joshua Vaidman (Union College)

The theory of products of random matrices and Lyapunov exponents have been widely studied and applied in the fields of biology, dynamical systems, economics, engineering and statistical physics. We consider the product of an i.i.d. sequence of 2×2 random non-invertible matrices with real entries. Given some mild moment assumptions we prove an explicit formula for the

Lyapunov exponent and prove a central limit theorem with an explicit formula for the variance in terms of the entries of the matrices. We also give examples where exact values for the Lyapunov exponent and variance are computed. An important example where non-invertible matrices are essential is the random Hill's equation, which has numerous physical applications, including the astrophysical orbit problem.

Statistics III

Wold 128

Chair: Kursad Tosun (Siena College)

3:30 - 3:45 *Analysis of Player-Elimination Tournaments* (Level 1)

Emily Rosaci (Smith College)

In recent years there has been a large rise of player-elimination tournaments. In this style tournament competitors are pitted against each and eliminated sequentially until a winner has been crowned. These competitions can be seen in video games, board games, endurance challenges, and reality competition series. Here we develop a flexible multinomial regression model which incorporates past results and player demographic information to predict the results of a player-elimination tournament.

3:50 - 4:05 *Updates on a Statistical Model for Player Evaluation in Basketball* (Level 2)

Kevin Braunwart (Clark University)

In team sports, Regularized Adjusted Plus-Minus (RAPM) refers to a large-scale statistical model that aims to isolate players' individual contributions to winning from those of their teammates and opponents. The pitfalls of the method are due to the huge number of samples required to stabilize the resulting linear system that needs to be solved to obtain the sought after parameter estimations. Ridge regression is one way to mitigate overly variable estimates, but a Bayesian regression allows more control. We have scraped NBA play-by-play from the past three regular seasons and applied both regularization methods. We discuss the methods and results.

4:10 - 4:25 *Representative democracy through random sampling: Should members of Congress be chosen by lottery?* (Level 1)

Zain Salim (Hampshire College)

Why do we elect our senators and representatives? Many people perceive that as the best way to ensure that Congress represents the general population. But if we look at the demographics of the US Congress, we see that it's all but representative of the general population. Women, racial minorities, and the LGBTQ community are all underrepresented in the Senate and/or the House. Meanwhile, the richest 1% and the elderly are overrepresented in Congress. So, our "representative democracy" isn't so representative. In this talk, I will use the central limit theorem to quantify the extent to which marginalized groups are underrepresented in Congress and to argue that choosing members of Congress by lottery could diminish or altogether eliminate the underrepresentation of marginalized groups in Congress. This talk will be a good introduction to random sampling and sampling distributions for students who haven't had statistics.

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