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Math 483 Reflection: Week 1

It’s only Tuesday, so I’m still starting to explore my topic. Professor Saracino recommended *Infinitesimal Calculus* by Henle and Kleinberg, which I got from the library, and *Lectures on the Hyperreals* by Goldblatt, which I’ve put in an interlibrary loan request for.

I’ve skimmed the first few chapters of Henle and Kleinberg, where they build up the idea of the hyperreals—the book only assumes a high-school level education in mathematics, and is trying to teach calculus using the hyperreals. In Mathematical Logic we constructed the hyperreals using the compactness theorem, since if we let *L\** be a language that has a symbol for every real number and every relation or function on the real numbers, along with a constant symbol *c*, we can model any finite subset of by just taking the reals and assigning *c* to an appropriate real number smaller than all the ones in the finite number of sentences from the latter set. So has a model, and in this case *c* must correspond to an infinitesimal, and we’ve invented the hyperreals.

I don’t know how standardized notation or even terminology is in mathematical logic—to be safe, is the theory of , the set of all sentences of *L\** true in . Actually, I’m kind of wondering how much prior knowledge I should assume when writing these/my thesis in general. I mean, obviously you have a PhD in math, but if the purpose of these reflections is to put to words the things I’ll eventually write in my thesis, I don’t know if I should be practicing writing things out with less assumption of prior knowledge. Also, what level of knowledge should I assume when writing my thesis? If I were doing an exploratory thesis I’d imagine I’d assume basic familiarity with the subfield I’m working on, but for an expository thesis I’m not sure where the explanation needs to “start.”

That aside—this book, the Henle and Kleinberg, defines the hyperreals as sequences of real numbers—for instance, 1 becomes the sequence “1, 1, 1, …”, while “1, 1/2, 1/3, 1/4, …” represents a positive infinitesimal. In general, a hyperreal j is “j(1), j(2), j(3), …”. j = k iff {n | j(n) = j(k)} is “quasi-big,” which is apparently something called an ultrafilter on the integers—in short, no finite set is quasi-big, if a set isn’t quasi-big then its complement is, and the intersection of two quasi-big sets is quasi-big (as a consequence of these, any superset of a quasi-big set is itself quasi-big). Then for any function f on the reals you can define f(j) = “f(j(1)), f(j(2)), …”, and for any relation R you can say R(j) iff {n | R(j(n))} is quasi-big (and analogously for many-placed relations). This is in some sense a lot more complicated, but means it’s actually viable to explain without getting into the weeds of mathematical logic. It also means I could plausibly explain the idea of the hyperreals in my thesis, even if I’m not assuming my reader knows about the compactness theorem (which… I don’t think any of my classmates do, so, that’s a bonus).

In Mathematical Logic (the class) we more or less just covered the very basics of nonstandard differential calculus, so I’m wary to start narrowing down my thesis topic until I know more about nonstandard integral calculus. A few directions I’m thinking about are areas where nonstandard analysis is particularly efficient in proving analytical theorems, looking into how different algorithms and such we’re taught in low-level calculus classes are consequences of a small set of properties of the hyperreals, or looking more into this idea of ultrafilters and the hyperreals and seeing if they have any cool applications outside of nonstandard analysis.

My goal for next time is to read chapters 5 and 6 (pp. 42-65, covering continuous functions and integral calculus) and do at least 10 exercises (perhaps not written in a human-readable format, but done nonetheless). I don’t function if I don’t have concrete deadlines, so if it’s alright with you I’d like to set a goal like this in each of my reflections.