Week #11

- What does the regression line minimise? Draw a picture
- How many regression lines are possible for a given data set?
- Give an example of restriction of range
- Explain the difference between a univariate outlier and a regression outlier, and draw a picture
- Draw an example of how an influential outlier might affect the regression line

Regression

$\bar{Y} = \Sigma(Y_i)/n$
$df_1 = 1$
$df_2 = n - df_1 - 1$
$SP = \Sigma[(X_i - \bar{X})(Y_i - \bar{Y})]$
$SS_X = \Sigma[(X_i - \bar{X})^2]$
$\beta_1 = SP/SS_X$
$\beta_0 = \bar{Y} - \beta_1 \times \bar{X}$

$\hat{Y}_i = \beta_0 + X_i \times \beta_1$
$SS_{\text{tot}} = \Sigma[(Y_i - \bar{Y})^2]$
$SS_{\text{reg}} = \Sigma[(\hat{Y} - \bar{Y})^2]$
$SS_{\text{res}} = SS_{\text{tot}} - SS_{\text{reg}}$
$MS_{\text{reg}} = SS_{\text{reg}}/df_1$
$MS_{\text{res}} = SS_{\text{res}}/df_2$
$F = MS_{\text{reg}}/MS_{\text{res}}$

Critical *F* values

		df_1		
df_2	α	1	2	3
1	0.05	161.4	199.5	215.71
	0.01	4052	4999	5404
2	0.05	18.51	19	19.16
	0.01	98.94	99	99.17
3	0.05	7.71	6.94	6.59
	0.01	34.12	30.82	29.46
4	0.05	7.71	6.94	6.59
	0.01	21.2	18	16.69

Question #1

Test the model fit at an α of 0.05.

X_i	Y_i	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
6	7	0	4	0	5.00	0.00	0.00
9	2	9	9	-9	5.23	0.23	0.05
2	3	16	4	8	4.69	-0.31	0.09
7	8	1	9	3	5.08	0.08	0.01

$SS_X = 26$
$SP = 2$
$\beta_1 = 2/26 = 0.08$
$\bar{Y} = 5$
$\bar{X} = 6$
$\beta_0 = 5 - (0.08 \times 6) = 4.54$
$\hat{Y}_i = 4.54 + (0.08 \times X_i)$
$SS_{\text{tot}} = 26$
$SS_{\text{reg}} = 0.15$

$SS_{\text{res}} = 26 - 0.15 = 25.85$
$df_1 = 1$
$df_2 = 4 - 1 - 1 = 2$
$MS_{\text{reg}} = 0.15/1 = 0.15$
$MS_{\text{res}} = 25.85/2 = 12.93$
$F = 0.15/12.93 = 0.01$
$F_{\text{crit}} = 18.51$
Fail to reject because $0.01 < 18.51$

Question #2

Test the model fit at an α of 0.01.

X_i	Y_i	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
5	7	1	4	-2
8	2	4	9	-6
9	8	9	9	9
2	3	16	4	8

Question #3

Test the model fit at an α of 0.01.

X_i	Y_i	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
4	6	1	0	-0
3	5	4	1	2
5	9	0	9	0
8	4	9	4	-6

Question #4

Test the model fit at an α of 0.05.

X_i	Y_i	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
4	4	4	1	2
6	10	0	25	0
5	6	1	1	-1
7	3	1	4	-2
8	2	4	9	-6

Question #5

Test the model fit at an α of 0.01.

X_i	Y_i	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
6	3	0	1	-0
8	2	4	4	-4
9	4	9	0	0
1	7	25	9	-15

Question #6

Test the model fit at an α of 0.01.

X_i	Y_i	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
7	1	1	25	-5
2	9	16	9	-12
6	7	0	1	0
10	5	16	1	-4
5	8	1	4	-2

Question #7

Test the model fit at an α of 0.05.

X_i	Y_i	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
9	10	4	16	8
5	2	4	16	8
7	6	0	0	0
3	5	16	1	4
10	4	9	4	-6
8	9	1	9	3

Question #8

Test the model fit at an α of 0.05.

X_i	Y_i	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
5	10	0	25	0
10	2	25	9	-15
2	4	9	1	3
7	3	4	4	-4
1	6	16	1	-4

Question #9

Test the model fit at an α of 0.05.

X_i	Y_i	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
4	3	9	9	9
10	7	9	1	3
8	8	1	4	2
6	6	1	0	-0