

## Week #1

1. What is the difference between descriptive and inferential statistics?
2. Give an example of:
  - (a) A population and a sample from that population
  - (b) A parameter and its corresponding statistic
  - (c) A descriptive research question and an inferential reframing of that question
  - (d) A research question with an independent variable, a dependent variable, and a confounding variable

For each, explain why your example is appropriate.

3. Why is a confounding variable so called? What does it prevent us from doing?

## Summing notation

$X_1$  = the first element of  $X$

$X_n$  = the last element of  $X$

$$\Sigma(X_i) = X_1 + X_2 + \dots + X_n$$

$$\Sigma(X_i + 1) = (X_1 + 1) + (X_2 + 1) + \dots + (X_n + 1)$$

$$\Sigma(X_i - Y_i) = (X_1 - Y_1) + (X_2 - Y_2) + \dots + (X_n - Y_n)$$

$$[\Sigma(X_i)]^2 = (X_1 + X_2 + \dots + X_n)^2$$

### Question #1

$$X = 3 \ 4 \ 6 \ 10 \ 3$$

$$Y = 9 \ 10 \ 7 \ 7 \ 1$$

$$\Sigma(X_i) = 26$$

$$[\Sigma(X_i)]^2 = 676$$

$$\Sigma(X_i - Y_i) = -8$$

$$\Sigma(X_i Y_i) = 182$$

$$\Sigma(X_i^2) = 170$$

$$[\Sigma(X_i - Y_i)]^2 = 64$$

$$\Sigma(X_i) \Sigma(Y_i) = 884$$

### Question #2

$$X = 2 \ 8 \ 6 \ 2 \ 10$$

$$Y = 10 \ 2 \ 9 \ 5 \ 6$$

$$\Sigma(X_i) = 28$$

$$[\Sigma(X_i)]^2 = 784$$

$$\Sigma(X_i - Y_i) = -4$$

$$\Sigma(X_i Y_i) = 160$$

$$\Sigma(X_i^2) = 208$$

$$[\Sigma(X_i - Y_i)]^2 = 16$$

$$\Sigma(X_i) \Sigma(Y_i) = 896$$

### Question #3

$$X = 2 \ 9 \ 4 \ 4 \ 7$$

$$Y = 7 \ 2 \ 3 \ 6 \ 7$$

$$\Sigma(X_i) = 26$$

$$[\Sigma(X_i)]^2 = 676$$

$$\Sigma(X_i - Y_i) = 1$$

$$\Sigma(X_i Y_i) = 117$$

$$\Sigma(X_i^2) = 166$$

$$[\Sigma(X_i - Y_i)]^2 = 1$$

$$\Sigma(X_i) \Sigma(Y_i) = 650$$

### Question #4

$$X = 6 \ 1 \ 3 \ 3 \ 9$$

$$Y = 3 \ 8 \ 10 \ 10 \ 1$$

$$\Sigma(X_i) = 22$$

$$[\Sigma(X_i)]^2 = 484$$

$$\Sigma(X_i - Y_i) = -10$$

$$\Sigma(X_i Y_i) = 95$$

$$\Sigma(X_i^2) = 136$$

$$[\Sigma(X_i - Y_i)]^2 = 100$$

$$\Sigma(X_i) \Sigma(Y_i) = 704$$

### Question #5

$$X = 3 \ 7 \ 10 \ 3 \ 2$$

$$Y = 8 \ 6 \ 9 \ 10 \ 2$$

$$\Sigma(X_i) = 25$$

$$[\Sigma(X_i)]^2 = 625$$

$$\Sigma(X_i - Y_i) = -10$$

$$\Sigma(X_i Y_i) = 190$$

$$\Sigma(X_i^2) = 171$$

$$[\Sigma(X_i - Y_i)]^2 = 100$$

$$\Sigma(X_i) \Sigma(Y_i) = 875$$

### Question #6

$$X = 7 \ 10 \ 3 \ 4 \ 9$$

$$Y = 10 \ 10 \ 8 \ 6 \ 1$$

$$\Sigma(X_i) = 33$$

$$[\Sigma(X_i)]^2 = 1089$$

$$\Sigma(X_i - Y_i) = -2$$

$$\Sigma(X_i Y_i) = 227$$

$$\Sigma(X_i^2) = 255$$

$$[\Sigma(X_i - Y_i)]^2 = 4$$

$$\Sigma(X_i) \Sigma(Y_i) = 1155$$

## Week #2

1. Give an example of:

- (a) A discrete variable
- (b) A continuous variable
- (c) A nominal variable
- (d) An ordinal variable
- (e) An interval variable
- (f) A ratio variable

For each, explain why your example is appropriate.

2. Draw a well-designed bar graph and a poorly-designed one, and explain why the first is better

3. Why might we prefer:

- (a) Relative frequency over frequency?
- (b) Cumulative relative frequency over cumulative frequency?

## Frequency table construction

For the following data sets, create a frequency table containing the frequency (freq), cumulative frequency (cfreq), relative frequency (rfreq), and cumulative relative frequency (crfreq) of each value.

freq = number equal to score

cfreq = number less than or equal to score

rfreq = frequency divided by  $n$

crfreq = cumulative frequency divided by  $n$

### Question #1

2, 2, 3, 5, 2, 5, 5, 4, 4, 1, 2, 1

score	freq	cfreq	rfreq	crfreq
1	2	2	0.17	0.17
2	4	6	0.33	0.50
3	1	7	0.08	0.58
4	2	9	0.17	0.75
5	3	12	0.25	1.00

### Question #2

1, 4, 3, 1, 5, 5, 1, 5, 3, 3, 3, 2

score	freq	cfreq	rfreq	crfreq
1	3	3	0.25	0.25
2	1	4	0.08	0.33
3	4	8	0.33	0.67
4	1	9	0.08	0.75
5	3	12	0.25	1.00

### Question #3

1, 5, 2, 2, 4, 4, 1, 2, 3, 4, 3, 3

score	freq	cfreq	rfreq	crfreq
1	2	2	0.17	0.17
2	3	5	0.25	0.42
3	3	8	0.25	0.67
4	3	11	0.25	0.92
5	1	12	0.08	1.00

### Question #4

3, 1, 2, 2, 5, 2, 4, 5, 5, 1, 4, 2

score	freq	cfreq	rfreq	crfreq
1	2	2	0.17	0.17
2	4	6	0.33	0.50
3	1	7	0.08	0.58
4	2	9	0.17	0.75
5	3	12	0.25	1.00

### Question #5

2, 4, 5, 2, 1, 4, 3, 5, 5, 1, 2, 3

score	freq	cfreq	rfreq	crfreq
1	2	2	0.17	0.17
2	3	5	0.25	0.42
3	2	7	0.17	0.58
4	2	9	0.17	0.75
5	3	12	0.25	1.00

### Question #6

4, 5, 2, 2, 5, 5, 5, 4, 3, 1, 4, 5

score	freq	cfreq	rfreq	crfreq
1	1	1	0.08	0.08
2	2	3	0.17	0.25
3	1	4	0.08	0.33
4	3	7	0.25	0.58
5	5	12	0.42	1.00

### Question #7

5, 2, 1, 1, 2, 4, 2, 5, 1, 3, 1, 2

score	freq	cfreq	rfreq	crfreq
1	4	4	0.33	0.33
2	4	8	0.33	0.67
3	1	9	0.08	0.75
4	1	10	0.08	0.83
5	2	12	0.17	1.00

### Question #8

3, 2, 4, 4, 2, 4, 2, 5, 4, 4, 3, 1

score	freq	cfreq	rfreq	crfreq
1	1	1	0.08	0.08
2	3	4	0.25	0.33
3	2	6	0.17	0.50
4	5	11	0.42	0.92
5	1	12	0.08	1.00

**Question #9**

2, 1, 2, 2, 3, 1, 2, 2, 4, 5, 1, 1

score	freq	cfreq	rfreq	crfreq
1	4	4	0.33	0.33
2	5	9	0.42	0.75
3	1	10	0.08	0.83
4	1	11	0.08	0.92
5	1	12	0.08	1.00

**Question #10**

3, 2, 3, 4, 1, 2, 2, 2, 4, 3, 4, 3

score	freq	cfreq	rfreq	crfreq
1	1	1	0.08	0.08
2	4	5	0.33	0.42
3	4	9	0.33	0.75
4	3	12	0.25	1.00

**Question #11**

2, 1, 3, 1, 1, 5, 1, 2, 5, 1, 1, 3

score	freq	cfreq	rfreq	crfreq
1	6	6	0.50	0.50
2	2	8	0.17	0.67
3	2	10	0.17	0.83
5	2	12	0.17	1.00

**Question #12**

1, 5, 5, 2, 1, 1, 1, 4, 1, 1, 2, 5

score	freq	cfreq	rfreq	crfreq
1	6	6	0.50	0.50
2	2	8	0.17	0.67
4	1	9	0.08	0.75
5	3	12	0.25	1.00

**Question #13**

4, 2, 2, 1, 5, 1, 3, 4, 5, 1, 4, 5

score	freq	cfreq	rfreq	crfreq
1	3	3	0.25	0.25
2	2	5	0.17	0.42
3	1	6	0.08	0.50
4	3	9	0.25	0.75
5	3	12	0.25	1.00

**Question #14**

2, 4, 5, 3, 5, 3, 5, 3, 3, 2, 5, 1

score	freq	cfreq	rfreq	crfreq
1	1	1	0.08	0.08
2	2	3	0.17	0.25
3	4	7	0.33	0.58
4	1	8	0.08	0.67
5	4	12	0.33	1.00

**Question #15**

4, 1, 5, 4, 2, 5, 5, 2, 4, 5, 1, 4

score	freq	cfreq	rfreq	crfreq
1	2	2	0.17	0.17
2	2	4	0.17	0.33
4	4	8	0.33	0.67
5	4	12	0.33	1.00

**Question #16**

4, 2, 3, 2, 5, 2, 1, 5, 5, 1, 2, 3

score	freq	cfreq	rfreq	crfreq
1	2	2	0.17	0.17
2	4	6	0.33	0.50
3	2	8	0.17	0.67
4	1	9	0.08	0.75
5	3	12	0.25	1.00

**Question #17**

1, 5, 3, 4, 3, 3, 2, 1, 4, 1, 3, 1

score	freq	cfreq	rfreq	crfreq
1	4	4	0.33	0.33
2	1	5	0.08	0.42
3	4	9	0.33	0.75
4	2	11	0.17	0.92
5	1	12	0.08	1.00

**Question #18**

5, 4, 5, 1, 1, 3, 2, 3, 2, 2, 4, 4

score	freq	cfreq	rfreq	crfreq
1	2	2	0.17	0.17
2	3	5	0.25	0.42
3	2	7	0.17	0.58
4	3	10	0.25	0.83
5	2	12	0.17	1.00

## Interval construction

For each scenario, find the lower limit ( $LL$ ), midpoint ( $MP$ ), and upper limit ( $UL$ ) of the first five intervals.

- $LL_1$  = the greatest multiple of the width less than or equal to the lowest score, or (if that does not exist) 0
- $UL_i = LL_i + \text{width} - 1$
- $MP_i = (LL_i + UL_i)/2$

### Question #1

The lowest value in the data set is 38, and the desired interval width is 14.

$LL$	$MP$	$UL$
28	34.5	41
42	48.5	55
56	62.5	69
70	76.5	83
84	90.5	97

### Question #2

The lowest value in the data set is 71, and the desired interval width is 10.

$LL$	$MP$	$UL$
70	74.5	79
80	84.5	89
90	94.5	99
100	104.5	109
110	114.5	119

### Question #3

The lowest value in the data set is 81, and the desired interval width is 9.

$LL$	$MP$	$UL$
81	85.0	89
90	94.0	98
99	103.0	107
108	112.0	116
117	121.0	125

### Question #4

The lowest value in the data set is 1, and the desired interval width is 30.

$LL$	$MP$	$UL$
0	14.5	29
30	44.5	59
60	74.5	89
90	104.5	119
120	134.5	149

### Question #5

The lowest value in the data set is 69, and the desired interval width is 11.

$LL$	$MP$	$UL$
66	71.0	76
77	82.0	87
88	93.0	98
99	104.0	109
110	115.0	120

### Question #6

The lowest value in the data set is 94, and the desired interval width is 31.

$LL$	$MP$	$UL$
93	108.0	123
124	139.0	154
155	170.0	185
186	201.0	216
217	232.0	247

### Question #7

The lowest value in the data set is 40, and the desired interval width is 50.

$LL$	$MP$	$UL$
0	24.5	49
50	74.5	99
100	124.5	149
150	174.5	199
200	224.5	249

**Question #8**

The lowest value in the data set is 21, and the desired interval width is 24.

<i>LL</i>	<i>MP</i>	<i>UL</i>
0	11.5	23
24	35.5	47
48	59.5	71
72	83.5	95
96	107.5	119

**Question #12**

The lowest value in the data set is 82, and the desired interval width is 4.

<i>LL</i>	<i>MP</i>	<i>UL</i>
80	81.5	83
84	85.5	87
88	89.5	91
92	93.5	95
96	97.5	99

**Question #9**

The lowest value in the data set is 3, and the desired interval width is 12.

<i>LL</i>	<i>MP</i>	<i>UL</i>
0	5.5	11
12	17.5	23
24	29.5	35
36	41.5	47
48	53.5	59

**Question #13**

The lowest value in the data set is 25, and the desired interval width is 36.

<i>LL</i>	<i>MP</i>	<i>UL</i>
0	17.5	35
36	53.5	71
72	89.5	107
108	125.5	143
144	161.5	179

**Question #10**

The lowest value in the data set is 31, and the desired interval width is 26.

<i>LL</i>	<i>MP</i>	<i>UL</i>
26	38.5	51
52	64.5	77
78	90.5	103
104	116.5	129
130	142.5	155

**Question #14**

The lowest value in the data set is 64, and the desired interval width is 13.

<i>LL</i>	<i>MP</i>	<i>UL</i>
52	58.0	64
65	71.0	77
78	84.0	90
91	97.0	103
104	110.0	116

**Question #11**

The lowest value in the data set is 1, and the desired interval width is 14.

<i>LL</i>	<i>MP</i>	<i>UL</i>
0	6.5	13
14	20.5	27
28	34.5	41
42	48.5	55
56	62.5	69

## Week #3

1. Why is the mean more sensitive to extreme scores than the median?
2. Give a scenario in which:
  - (a) The median would be a better estimate than the mean
  - (b) The mode would be a better estimate than the median/mean
  - (c) It would be crucial to report the variance along with the mean

For each, justify your example.

3. What are we squaring in a sum of squares, and why do we do it?

$$\Sigma[(X_i - \bar{X})^2]$$

4. In what sense is a variance a mean?

$$\frac{\Sigma[(X_i - \bar{X})^2]}{n - 1}$$

5. Why do we use  $n - 1$  instead of  $n$  in the denominator of sample variance?
6. When would variance be equal to zero? Explain with reference the formula
7. Is it possible to have a negative variance? Justify your answer with reference to the formula
8. Why do we use standard deviation instead of variance?
9. Why is the standard deviation so named?
10. For the following data set, calculate the mode, median, mean, and standard deviation. Then, add 5 to every number in the data set and calculate again. Then, multiply every number in the data set by 2 and calculate again. What effect does adding/multiplying every score with the same number have, and why? Explain for each statistic.  
2, 8, 1, 1, 3
11. Draw a positively skewed distribution and identify the approximate location of the mean, median, and mode. Why do they take the order they do?



## Median & IQR calculation

For each frequency table, calculate the median ( $P_{50}$ ) and interquartile range ( $P_{25}$  and  $P_{75}$ ). The equation for the 50th percentile is:

$$P_{50} = LL + W \left[ \frac{0.5(n) - cumF}{fm} \right]$$

$LL$  is (the least score with a cfreq  $\geq 0.5$ ) - 0.5,  $W$  is the interval width (1 if the data is ungrouped),  $n$  is the sample size (the greatest cfreq),  $cumF$  is the cfreq of the row below the one containing  $LL$  (0 if there is no such row), and  $fm$  is the freq of the row containing  $LL$ .

Note: the frequency table must be in *descending* order

### Question #1

score	freq	cfreq	rfreq	crfreq
62	3	30	0.10	1.00
61	10	27	0.33	0.90
60	13	17	0.43	0.57
59	4	4	0.13	0.13

$$\begin{aligned}P_{25} &= 59.5 + 1 \times [(0.25 \times 30 - 4)/13] = 59.77 \\P_{50} &= 59.5 + 1 \times [(0.5 \times 30 - 4)/13] = 60.35 \\P_{75} &= 60.5 + 1 \times [(0.75 \times 30 - 17)/10] = 61.05\end{aligned}$$

### Question #2

score	freq	cfreq	rfreq	crfreq
23	1	22	0.05	1.00
22	4	21	0.18	0.95
21	11	17	0.50	0.77
20	4	6	0.18	0.27
19	2	2	0.09	0.09

$$\begin{aligned}P_{25} &= 19.5 + 1 \times [(0.25 \times 22 - 2)/4] = 20.38 \\P_{50} &= 20.5 + 1 \times [(0.5 \times 22 - 6)/11] = 20.95 \\P_{75} &= 20.5 + 1 \times [(0.75 \times 22 - 6)/11] = 21.45\end{aligned}$$

### Question #3

score	freq	cfreq	rfreq	crfreq
69	1	24	0.04	1.00
68	4	23	0.17	0.96
67	11	19	0.46	0.79
66	8	8	0.33	0.33

$$\begin{aligned}P_{25} &= 65.5 + 1 \times [(0.25 \times 24 - 0)/8] = 66.25 \\P_{50} &= 66.5 + 1 \times [(0.5 \times 24 - 8)/11] = 66.86 \\P_{75} &= 66.5 + 1 \times [(0.75 \times 24 - 8)/11] = 67.41\end{aligned}$$

### Question #4

score	freq	cfreq	rfreq	crfreq
12	1	15	0.07	1.00
11	3	14	0.20	0.93
10	4	11	0.27	0.73
9	6	7	0.40	0.47
8	1	1	0.07	0.07

$$\begin{aligned}P_{25} &= 8.5 + 1 \times [(0.25 \times 15 - 1)/6] = 8.96 \\P_{50} &= 9.5 + 1 \times [(0.5 \times 15 - 7)/4] = 9.62 \\P_{75} &= 10.5 + 1 \times [(0.75 \times 15 - 11)/3] = 10.58\end{aligned}$$

### Question #5

score	freq	cfreq	rfreq	crfreq
8	6	21	0.29	1.00
7	8	15	0.38	0.71
6	3	7	0.14	0.33
5	4	4	0.19	0.19

$$\begin{aligned}P_{25} &= 5.5 + 1 \times [(0.25 \times 21 - 4)/3] = 5.92 \\P_{50} &= 6.5 + 1 \times [(0.5 \times 21 - 7)/8] = 6.94 \\P_{75} &= 7.5 + 1 \times [(0.75 \times 21 - 15)/6] = 7.62\end{aligned}$$

### Question #6

score	freq	cfreq	rfreq	crfreq
47	1	23	0.04	1.00
46	1	22	0.04	0.96
45	5	21	0.22	0.91
44	10	16	0.43	0.70
43	6	6	0.26	0.26

$$\begin{aligned}P_{25} &= 42.5 + 1 \times [(0.25 \times 23 - 0)/6] = 43.46 \\P_{50} &= 43.5 + 1 \times [(0.5 \times 23 - 6)/10] = 44.05 \\P_{75} &= 44.5 + 1 \times [(0.75 \times 23 - 16)/5] = 44.75\end{aligned}$$

### Question #7

score	freq	cfreq	rfreq	crfreq
13	1	14	0.07	1.00
12	2	13	0.14	0.93
11	9	11	0.64	0.79
10	1	2	0.07	0.14
8	1	1	0.07	0.07

$$\begin{aligned}P_{25} &= 10.5 + 1 \times [(0.25 \times 14 - 2)/9] = 10.67 \\P_{50} &= 10.5 + 1 \times [(0.5 \times 14 - 2)/9] = 11.06 \\P_{75} &= 10.5 + 1 \times [(0.75 \times 14 - 2)/9] = 11.44\end{aligned}$$

## Standard deviation calculation

$$SS = \sum (X_i - \bar{X})^2$$

$$df = n - 1$$

$$s^2 = \frac{SS}{df}$$

$$s = \sqrt{s^2}$$

### Question #1

4, 1, 10, 7, 3

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
4	-1	1
1	-4	16
10	5	25
7	2	4
3	-2	4

$$\bar{X} = 5$$

$$SS = 50$$

$$df = 5 - 1 = 4$$

$$s^2 = 50/4 = 12.5$$

$$s = \sqrt{12.5} = 3.54$$

### Question #2

2, 7, 5, 10, 6

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
2	-4	16
7	1	1
5	-1	1
10	4	16
6	0	0

$$\bar{X} = 6$$

$$SS = 34$$

$$df = 5 - 1 = 4$$

$$s^2 = 34/4 = 8.5$$

$$s = \sqrt{8.5} = 2.92$$

### Question #3

4, 7, 5, 3, 1

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
4	0	0
7	3	9
5	1	1
3	-1	1
1	-3	9

$$\bar{X} = 4$$

$$SS = 20$$

$$df = 5 - 1 = 4$$

$$s^2 = 20/4 = 5$$

$$s = \sqrt{5} = 2.24$$

### Question #4

4, 7, 8, 6, 5

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
4	-2	4
7	1	1
8	2	4
6	0	0
5	-1	1

$$\bar{X} = 6$$

$$SS = 10$$

$$df = 5 - 1 = 4$$

$$s^2 = 10/4 = 2.5$$

$$s = \sqrt{2.5} = 1.58$$

### Question #5

3, 4, 8, 9, 6

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
3	-3	9
4	-2	4
8	2	4
9	3	9
6	0	0

$$\bar{X} = 6$$

$$SS = 26$$

$$df = 5 - 1 = 4$$

$$s^2 = 26/4 = 6.5$$

$$s = \sqrt{6.5} = 2.55$$

### Question #6

7, 9, 1, 3, 5

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
7	2	4
9	4	16
1	-4	16
3	-2	4
5	0	0

$$\bar{X} = 5$$

$$SS = 40$$

$$df = 5 - 1 = 4$$

$$s^2 = 40/4 = 10$$

$$s = \sqrt{10} = 3.16$$

## Week #4

1. Last week we saw that

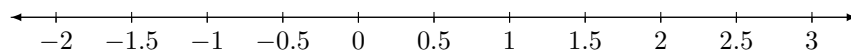
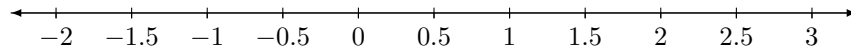
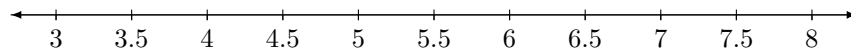
- (a) If you subtract the same number from every score, the new mean is the old mean minus that number
- (b) If you divide every score by the same number, the new standard deviation is the old standard deviation divided by that number

So if we subtract the mean from every score, and divide every score by the standard deviation, what are the new mean and standard deviation?

2. What is the advantage of standard scores over raw scores? Give an example of a scenario in which a standard score would be a lot more meaningful than a raw score
3. What does the height of the normal curve above a given score tell us about that score? What does the area under the curve for a given interval tell us about that interval?
4. On the first number line below, draw bars representing the frequency distribution of the following data:

4, 6, 4, 3, 8

Then, subtract the mean from each score and draw the new distribution on the second number line. Then, divide those by the standard deviation and draw them on the third number line. What does each step do to the location, spread, and shape of the distribution?



## Z-scores

The area between  $z_1$  and  $z_2$  is equal to the area above the lesser of the two minus the area above the greater of the two. The area below  $z = 1$  - (the area above it), the area below  $-z =$  the area above  $z$ , and the area above  $-z = 0.5 +$  (the area between  $z$  and the mean).

Calculate the area between the following pairs of  $z$  scores:

$z_1$	$z_2$	Area between $z_1$ and $z_2$
-0.56	0.21	$0.71 - 0.42 = 0.3$
-0.82	0.66	$0.79 - 0.25 = 0.54$
0.64	0.12	$0.45 - 0.26 = 0.19$
-0.3	-0.64	$0.74 - 0.62 = 0.12$
0.69	-0.58	$0.72 - 0.25 = 0.47$
0.7	-0.35	$0.64 - 0.24 = 0.39$
0.43	-0.51	$0.69 - 0.33 = 0.36$
-0.6	0.45	$0.73 - 0.33 = 0.4$
0.28	0.64	$0.39 - 0.26 = 0.13$
-0.84	0.42	$0.8 - 0.34 = 0.46$
-0.51	0.37	$0.69 - 0.36 = 0.34$
0.58	-0.87	$0.81 - 0.28 = 0.53$
-0.69	0.19	$0.75 - 0.42 = 0.33$
0.54	-0.91	$0.82 - 0.29 = 0.52$
-0.01	-0.61	$0.73 - 0.5 = 0.23$
-0.81	-0.38	$0.79 - 0.65 = 0.14$
-0.75	0.91	$0.77 - 0.18 = 0.59$
0.02	0.02	$0.49 - 0.49 = 0$
-0.62	0.87	$0.73 - 0.19 = 0.54$
-0.29	-0.66	$0.75 - 0.61 = 0.13$
0.77	0.92	$0.22 - 0.18 = 0.04$
-0.42	0.83	$0.66 - 0.2 = 0.46$
-0.41	0.63	$0.66 - 0.26 = 0.39$
0.25	0.74	$0.4 - 0.23 = 0.17$
-0.61	-0.04	$0.73 - 0.52 = 0.21$
-0.7	0.84	$0.76 - 0.2 = 0.56$
0.62	0.96	$0.27 - 0.17 = 0.1$
-0.58	0.88	$0.72 - 0.19 = 0.53$
0.82	0.05	$0.48 - 0.21 = 0.27$
-0.88	-0.17	$0.81 - 0.57 = 0.24$
0.19	0.7	$0.42 - 0.24 = 0.18$
0.74	-0.25	$0.6 - 0.23 = 0.37$
0.16	0.81	$0.44 - 0.21 = 0.23$
0.63	-0.7	$0.76 - 0.26 = 0.49$
0.2	0.49	$0.42 - 0.31 = 0.11$
-0.83	-0.69	$0.8 - 0.75 = 0.04$
-0.16	-0.29	$0.61 - 0.56 = 0.05$
-0.14	0.31	$0.56 - 0.38 = 0.18$
-0.28	-0.54	$0.71 - 0.61 = 0.1$
-0.38	0.04	$0.65 - 0.48 = 0.16$
0.86	0.13	$0.45 - 0.19 = 0.25$
-0.63	0.6	$0.74 - 0.27 = 0.46$
0.81	-0.28	$0.61 - 0.21 = 0.4$
-0.25	0.61	$0.6 - 0.27 = 0.33$
0.21	-0.88	$0.81 - 0.42 = 0.39$
0.31	-0.77	$0.78 - 0.38 = 0.4$
-0.19	0.07	$0.58 - 0.47 = 0.1$
0.92	0.51	$0.31 - 0.18 = 0.13$

$z$	Area between mean and $z$	Area above $z$
0.01	0.4960	0.0040
0.02	0.4920	0.0080
0.04	0.4840	0.0160
0.05	0.4801	0.0199
0.06	0.4761	0.0239
0.07	0.4721	0.0279
0.10	0.4602	0.0398
0.12	0.4522	0.0478
0.13	0.4483	0.0517
0.14	0.4443	0.0557
0.16	0.4364	0.0636
0.17	0.4325	0.0675
0.19	0.4247	0.0753
0.20	0.4207	0.0793
0.21	0.4168	0.0832
0.24	0.4052	0.0948
0.25	0.4013	0.0987
0.27	0.3936	0.1064
0.28	0.3897	0.1103
0.29	0.3859	0.1141
0.30	0.3821	0.1179
0.31	0.3783	0.1217
0.34	0.3669	0.1331
0.35	0.3632	0.1368
0.37	0.3557	0.1443
0.38	0.3520	0.1480
0.41	0.3409	0.1591
0.42	0.3372	0.1628
0.43	0.3336	0.1664
0.45	0.3264	0.1736
0.49	0.3121	0.1879
0.51	0.3050	0.1950
0.54	0.2946	0.2054
0.56	0.2877	0.2123
0.58	0.2810	0.2190
0.60	0.2743	0.2257
0.61	0.2709	0.2291
0.62	0.2676	0.2324
0.63	0.2643	0.2357
0.64	0.2611	0.2389
0.66	0.2546	0.2454
0.69	0.2451	0.2549
0.70	0.2420	0.2580
0.74	0.2296	0.2704
0.75	0.2266	0.2734
0.77	0.2206	0.2794
0.81	0.2090	0.2910
0.82	0.2061	0.2939
0.83	0.2033	0.2967
0.84	0.2005	0.2995
0.85	0.1977	0.3023
0.86	0.1949	0.3051
0.87	0.1922	0.3078
0.88	0.1894	0.3106
0.91	0.1814	0.3186
0.92	0.1788	0.3212
0.96	0.1685	0.3315

## Week #5

1. What is the difference between standard error and standard deviation?
2. Why is standard error so named? What does it measure?
3. What is the difference between standard error and standard error of the mean?
4. How do changes in  $n$  and  $\sigma$  affect  $\sigma_{\bar{X}}$ ? Explain with reference to the formula

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

5. What is the difference between a sampling distribution and a sampling distribution of the mean?
6. Distinguish between the population distribution, the sample distribution, and the sampling distribution, using an example
7. Distinguish between the population mean, the sample mean, and the mean of the sampling distribution of the mean, using an example
8. Imagine a bowl with five bingo chips in it, numbered 1 through 5. For this population:
  - (a) Calculate the population mean
  - (b) List all 25 possible samples of size 2

1, 1

1, 2

$\vdots$

5, 5

- (c) Calculate the mean for every sample
- (d) Count the frequency of each value of the sample mean
- (e) Calculate the mean of the sampling distribution of the mean
- (f) Draw the frequency distribution as a bar plot
- (g) Calculate the probability of drawing a sample with:
  - A mean  $\geq 4$
  - A mean  $\leq 2.5$
  - An error of at least 1.5

## Week #6

1. What are the steps in hypothesis testing? Give an example and relate each step to the concepts we've learned so far in the course (population, sample, parameter, statistic, sampling distribution of the mean, null and alternate hypotheses, critical values, etc.)
2. Draw a null distribution and illustrate the relationship between the critical values, the rejection region, and  $\alpha$
3. Draw an alternate distribution and illustrate the relationship between the critical values, the rejection region,  $\beta$ , and power
4. In what sense are  $\alpha$  and  $\beta$  *conditional* probabilities?
5. How are  $\alpha$  and  $\beta$  each affected by change in the critical values, effect size, and standard error?
6. How do we control Type I and Type II error, respectively? Why don't we make  $\alpha = 0$ ?
7.  $H_0$  and  $H_1$  must be exhaustive (one must be true) and mutually exclusive (they can't both be true). Come up with some invalid hypotheses. Here's an example to get you started:

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 > \mu_2$$

8. Why do we need hypothesis testing? Why can't we just draw a sample from the population and estimate the parameter by the sample statistic?
9. Give an example of a finding which is statistically significant but practically insignificant. When might this occur?
10. What is the goal of a confidence interval?
11. Come up with some research questions for which a nondirectional z-test would be appropriate, then reframe them so that a directional test would be appropriate. State the hypotheses formally

## Z-tests

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$z_{\text{crit}}$  = the  $z$  score with  $\alpha/2$  above it

$$z_{\text{obs}} = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}$$

$$CI_y = \bar{X} \pm (\sigma_{\bar{X}} \times z_y)$$

$z_y$  = the  $z$  score with  $(100 - y)/100$  above it

### Critical z values

$z$	Area between mean and $z$	Area above $z$
1.645	0.45	0.05
1.96	0.475	0.025
2.576	0.495	0.005

### Question #1

Researchers draw a sample of 6 with a mean of 5. The population standard deviation is known to be 2.03. Test  $H_0 : \mu = 6$  at an  $\alpha$  of 0.1, state your decision, and calculate a 90% confidence interval.

$$\sigma_{\bar{X}} = 2.03/\sqrt{6} = 0.83$$

$$z_{\text{obs}} = (5 - 6)/0.83 = -1.21$$

$$z_{\text{crit}} = \pm 1.64$$

Fail to reject because  $1.64 > -1.21 > -1.64$

$$z_{90} = 1.64$$

$$CI_{90} = 5 \pm (0.83 \times 1.64) = [3.64, 6.36]$$

### Question #2

Researchers draw a sample of 6 with a mean of 6.33. The population standard deviation is known to be 3.81. Test  $H_0 : \mu = 6$  at an  $\alpha$  of 0.05, state your decision, and calculate a 95% confidence interval.

$$\sigma_{\bar{X}} = 3.81/\sqrt{6} = 1.56$$

$$z_{\text{obs}} = (6.33 - 6)/1.56 = 0.21$$

$$z_{\text{crit}} = \pm 1.96$$

Fail to reject because  $1.96 > 0.21 > -1.96$

$$z_{95} = 1.96$$

$$CI_{95} = 6.33 \pm (1.56 \times 1.96) = [3.28, 9.38]$$

### Question #3

Researchers draw a sample of 6 with a mean of 4.83. The population standard deviation is known to be 4.23. Test  $H_0 : \mu = 4$  at an  $\alpha$  of 0.05, state your decision, and calculate a 95% confidence interval.

$$\sigma_{\bar{X}} = 4.23/\sqrt{6} = 1.73$$

$$z_{\text{obs}} = (4.83 - 4)/1.73 = 0.48$$

$$z_{\text{crit}} = \pm 1.96$$

Fail to reject because  $1.96 > 0.48 > -1.96$

$$z_{95} = 1.96$$

$$CI_{95} = 4.83 \pm (1.73 \times 1.96) = [1.45, 8.21]$$

### Question #4

Researchers draw a sample of 8 with a mean of 6.5. The population standard deviation is known to be 1.04. Test  $H_0 : \mu = 3$  at an  $\alpha$  of 0.1, state your decision, and calculate a 90% confidence interval.

$$\sigma_{\bar{X}} = 1.04/\sqrt{8} = 0.37$$

$$z_{\text{obs}} = (6.5 - 3)/0.37 = 9.52$$

$$z_{\text{crit}} = \pm 1.64$$

Reject because  $9.52 \geq 1.64$

$$z_{90} = 1.64$$

$$CI_{90} = 6.5 \pm (0.37 \times 1.64) = [5.9, 7.1]$$

### Question #5

Researchers draw a sample of 6 with a mean of 6.33. The population standard deviation is known to be 3.74. Test  $H_0 : \mu = 10$  at an  $\alpha$  of 0.1, state your decision, and calculate a 90% confidence interval.

$$\sigma_{\bar{X}} = 3.74/\sqrt{6} = 1.53$$

$$z_{\text{obs}} = (6.33 - 10)/1.53 = -2.4$$

$$z_{\text{crit}} = \pm 1.64$$

Reject because  $-2.4 \leq -1.64$

$$z_{90} = 1.64$$

$$CI_{90} = 6.33 \pm (1.53 \times 1.64) = [3.82, 8.84]$$

**Question #6**

Researchers draw a sample of 8 with a mean of 6.88. The population standard deviation is known to be 4.75. Test  $H_0 : \mu = 3$  at an  $\alpha$  of 0.01, state your decision, and calculate a 99% confidence interval.

$$\sigma_{\bar{X}} = 4.75/\sqrt{8} = 1.68$$

$$z_{\text{obs}} = (6.88 - 3)/1.68 = 2.31$$

$$z_{\text{crit}} = \pm 2.58$$

Fail to reject because  $2.58 > 2.31 > -2.58$

$$z_{99} = 2.58$$

$$CI_{99} = 6.88 \pm (1.68 \times 2.58) = [2.55, 11.21]$$

**Question #7**

Researchers draw a sample of 10 with a mean of 4.6. The population standard deviation is known to be 2.59. Test  $H_0 : \mu = 2$  at an  $\alpha$  of 0.1, state your decision, and calculate a 90% confidence interval.

$$\sigma_{\bar{X}} = 2.59/\sqrt{10} = 0.82$$

$$z_{\text{obs}} = (4.6 - 2)/0.82 = 3.17$$

$$z_{\text{crit}} = \pm 1.64$$

Reject because  $3.17 \geq 1.64$

$$z_{90} = 1.64$$

$$CI_{90} = 4.6 \pm (0.82 \times 1.64) = [3.25, 5.95]$$

**Question #8**

Researchers draw a sample of 7 with a mean of 6.71. The population standard deviation is known to be 1.83. Test  $H_0 : \mu = 8$  at an  $\alpha$  of 0.05, state your decision, and calculate a 95% confidence interval.

$$\sigma_{\bar{X}} = 1.83/\sqrt{7} = 0.69$$

$$z_{\text{obs}} = (6.71 - 8)/0.69 = -1.87$$

$$z_{\text{crit}} = \pm 1.96$$

Fail to reject because  $1.96 > -1.87 > -1.96$

$$z_{95} = 1.96$$

$$CI_{95} = 6.71 \pm (0.69 \times 1.96) = [5.35, 8.07]$$

**Question #9**

Researchers draw a sample of 6 with a mean of 4.17. The population standard deviation is known to be 1.1. Test  $H_0 : \mu = 3$  at an  $\alpha$  of 0.01, state your decision, and calculate a 99% confidence interval.

$$\sigma_{\bar{X}} = 1.1/\sqrt{6} = 0.45$$

$$z_{\text{obs}} = (4.17 - 3)/0.45 = 2.61$$

$$z_{\text{crit}} = \pm 2.58$$

Reject because  $2.61 \geq 2.58$

$$z_{99} = 2.58$$

$$CI_{99} = 4.17 \pm (0.45 \times 2.58) = [3.01, 5.33]$$

**Question #10**

Researchers draw a sample of 8 with a mean of 4.5. The population standard deviation is known to be 2.23. Test  $H_0 : \mu = 5$  at an  $\alpha$  of 0.05, state your decision, and calculate a 95% confidence interval.

$$\sigma_{\bar{X}} = 2.23/\sqrt{8} = 0.79$$

$$z_{\text{obs}} = (4.5 - 5)/0.79 = -0.63$$

$$z_{\text{crit}} = \pm 1.96$$

Fail to reject because  $1.96 > -0.63 > -1.96$

$$z_{95} = 1.96$$

$$CI_{95} = 4.5 \pm (0.79 \times 1.96) = [2.95, 6.05]$$

**Question #11**

Researchers draw a sample of 6 with a mean of 4.17. The population standard deviation is known to be 1. Test  $H_0 : \mu = 6$  at an  $\alpha$  of 0.1, state your decision, and calculate a 90% confidence interval.

$$\sigma_{\bar{X}} = 1/\sqrt{6} = 0.41$$

$$z_{\text{obs}} = (4.17 - 6)/0.41 = -4.48$$

$$z_{\text{crit}} = \pm 1.64$$

Reject because  $-4.48 \leq -1.64$

$$z_{90} = 1.64$$

$$CI_{90} = 4.17 \pm (0.41 \times 1.64) = [3.5, 4.84]$$



## Week #7

1. When should one use a  $t$  test instead of a  $z$  test?
2. What is the expected value of  $t_{\text{obs}}$  the null hypothesis?
3. Why can't we calculate  $t_{\text{obs}}$  with a sample of  $n = 1$ ?

$$t_{\text{obs}} = \frac{\bar{X} - \mu}{s_{\bar{X}}}$$

$$s_{\bar{X}} = \frac{s}{\sqrt{n}}$$

$$s = \frac{\Sigma[(X_i - \bar{X})^2]}{n - 1}$$

4. Come up with some research questions for which a nondirectional one-sample  $t$ -test would be appropriate, then reframe them so that a directional test would be appropriate. State the hypotheses formally

## One sample t-tests

$$df = n - 1$$

$$s_{\bar{X}} = \frac{s}{\sqrt{n}}$$

$$t_{\text{obs}} = \frac{\bar{X} - \mu}{s_{\bar{X}}}$$

$$CI_y = \bar{X} \pm (s_{\bar{X}} \times t_y)$$

$$t_y = \text{the critical value for } \alpha = (100 - y)/100$$

### Critical t values

df	$\alpha$			
	0.2	0.1	0.05	0.01
4	1.53	2.13	2.78	4.6
5	1.48	2.02	2.57	4.03
6	1.44	1.94	2.45	3.71

### Question #1

You draw a sample of 5 with a mean of 6.4 and a standard deviation of 3.05. Test  $H_0 : \mu = 7$  at an  $\alpha$  of 0.01, state your decision, then calculate a 99% confidence interval.

$$s_{\bar{X}} = 3.05/\sqrt{5} = 1.36$$

$$t_{\text{obs}} = (6.4 - 7)/1.36 = -0.44$$

$$t_{\text{crit}} = \pm 4.6$$

Fail to reject because  $4.6 > -0.44 > -4.6$

$$t_{99} = 4.6$$

$$CI_{99} = 6.4 \pm (1.36 \times 4.6) = [0.12, 12.68]$$

### Question #2

You draw a sample of 5 with a mean of 7.2 and a standard deviation of 3.35. Test  $H_0 : \mu = 5$  at an  $\alpha$  of 0.1, state your decision, then calculate a 90% confidence interval.

$$s_{\bar{X}} = 3.35/\sqrt{5} = 1.5$$

$$t_{\text{obs}} = (7.2 - 5)/1.5 = 1.47$$

$$t_{\text{crit}} = \pm 2.13$$

Fail to reject because  $2.13 > 1.47 > -2.13$

$$t_{90} = 2.13$$

$$CI_{90} = 7.2 \pm (1.5 \times 2.13) = [4.01, 10.39]$$

### Question #3

You draw a sample of 5 with a mean of 6.2 and a standard deviation of 2.17. Test  $H_0 : \mu = 6$  at an  $\alpha$  of 0.1, state your decision, then calculate a 90% confidence interval.

$$s_{\bar{X}} = 2.17/\sqrt{5} = 0.97$$

$$t_{\text{obs}} = (6.2 - 6)/0.97 = 0.21$$

$$t_{\text{crit}} = \pm 2.13$$

Fail to reject because  $2.13 > 0.21 > -2.13$

$$t_{90} = 2.13$$

$$CI_{90} = 6.2 \pm (0.97 \times 2.13) = [4.13, 8.27]$$

### Question #4

You draw a sample of 6 with a mean of 4.5 and a standard deviation of 3.21. Test  $H_0 : \mu = 1$  at an  $\alpha$  of 0.01, state your decision, then calculate a 99% confidence interval.

$$s_{\bar{X}} = 3.21/\sqrt{6} = 1.31$$

$$t_{\text{obs}} = (4.5 - 1)/1.31 = 2.67$$

$$t_{\text{crit}} = \pm 4.03$$

Fail to reject because  $4.03 > 2.67 > -4.03$

$$t_{99} = 4.03$$

$$CI_{99} = 4.5 \pm (1.31 \times 4.03) = [-0.78, 9.78]$$

### Question #5

You draw a sample of 5 with a mean of 6 and a standard deviation of 3.39. Test  $H_0 : \mu = 10$  at an  $\alpha$  of 0.05, state your decision, then calculate a 95% confidence interval.

$$s_{\bar{X}} = 3.39/\sqrt{5} = 1.52$$

$$t_{\text{obs}} = (6 - 10)/1.52 = -2.64$$

$$t_{\text{crit}} = \pm 2.78$$

Fail to reject because  $2.78 > -2.64 > -2.78$

$$t_{95} = 2.78$$

$$CI_{95} = 6 \pm (1.52 \times 2.78) = [1.79, 10.21]$$

### Question #6

You draw a sample of 6 with a mean of 7.67 and a standard deviation of 3.27. Test  $H_0 : \mu = 1$  at an  $\alpha$  of 0.01, state your decision, then calculate a 99% confidence interval.

$$s_{\bar{X}} = 3.27/\sqrt{6} = 1.33$$

$$t_{\text{obs}} = (7.67 - 1)/1.33 = 5$$

$$t_{\text{crit}} = \pm 4.03$$

Reject because  $5 > 4.03$

$$t_{99} = 4.03$$

$$CI_{99} = 7.67 \pm (1.33 \times 4.03) = [2.29, 13.05]$$

### Question #7

You draw a sample of 7 with a mean of 4.57 and a standard deviation of 3.26. Test  $H_0 : \mu = 2$  at an  $\alpha$  of 0.1, state your decision, then calculate a 90% confidence interval.

$$s_{\bar{X}} = 3.26/\sqrt{7} = 1.23$$

$$t_{\text{obs}} = (4.57 - 2)/1.23 = 2.09$$

$$t_{\text{crit}} = \pm 1.94$$

Reject because  $2.09 \geq 1.94$

$$t_{90} = 1.94$$

$$CI_{90} = 4.57 \pm (1.23 \times 1.94) = [2.18, 6.96]$$

### Question #8

You draw a sample of 6 with a mean of 5.5 and a standard deviation of 2.43. Test  $H_0 : \mu = 7$  at an  $\alpha$  of 0.01, state your decision, then calculate a 99% confidence interval.

$$s_{\bar{X}} = 2.43/\sqrt{6} = 0.99$$

$$t_{\text{obs}} = (5.5 - 7)/0.99 = -1.51$$

$$t_{\text{crit}} = \pm 4.03$$

Fail to reject because  $4.03 > -1.51 > -4.03$

$$t_{99} = 4.03$$

$$CI_{99} = 5.5 \pm (0.99 \times 4.03) = [1.5, 9.5]$$

### Question #9

You draw a sample of 5 with a mean of 2.8 and a standard deviation of 1.48. Test  $H_0 : \mu = 7$  at an  $\alpha$  of 0.05, state your decision, then calculate a 95% confidence interval.

$$s_{\bar{X}} = 1.48/\sqrt{5} = 0.66$$

$$t_{\text{obs}} = (2.8 - 7)/0.66 = -6.35$$

$$t_{\text{crit}} = \pm 2.78$$

Reject because  $-6.35 \leq -2.78$

$$t_{95} = 2.78$$

$$CI_{95} = 2.8 \pm (0.66 \times 2.78) = [0.96, 4.64]$$

### Question #10

You draw a sample of 6 with a mean of 3.83 and a standard deviation of 2.04. Test  $H_0 : \mu = 5$  at an  $\alpha$  of 0.1, state your decision, then calculate a 90% confidence interval.

$$s_{\bar{X}} = 2.04/\sqrt{6} = 0.83$$

$$t_{\text{obs}} = (3.83 - 5)/0.83 = -1.4$$

$$t_{\text{crit}} = \pm 2.02$$

Fail to reject because  $2.02 > -1.4 > -2.02$

$$t_{90} = 2.02$$

$$CI_{90} = 3.83 \pm (0.83 \times 2.02) = [2.15, 5.51]$$

## Week #8

1. What is  $s_{\bar{X}_1 - \bar{X}_2}$  and what does it measure?
2. What is homogeneity of variance? State it symbolically
3. When might we employ a Welch correction?
4. What is the reasoning behind using pooled variance? Why not simply take the mean of  $s_1$  and  $s_2$ ?
5. Come up with some research questions for which a nondirectional independent  $t$ -test would be appropriate, then reframe them so that a directional test would be appropriate. State the hypotheses formally

## Independent $t$ -tests

$$df_i = n_i - 1$$

$$df_{\text{tot}} = df_1 + df_2$$

$$SS_i = s_i^2 \times df_i$$

$$t_{\text{obs}} = \frac{(\bar{X}_1 - \bar{X}_2)}{s_{(\bar{X}_1 - \bar{X}_2)}}$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$$

$$s_p^2 = \frac{SS_1 + SS_2}{df_{\text{tot}}}$$

$$CI_y = (\bar{X}_1 - \bar{X}_2) \pm s_{(\bar{X}_1 - \bar{X}_2)} \times t_y$$

$$t_y = \text{is the critical value for } \alpha = (100 - y)/100$$

### Critical $t$ values

df	$\alpha$		
	0.1	0.05	0.01
10	1.81	2.23	3.17
11	1.8	2.2	3.11
12	1.78	2.18	3.05

### Question #1

Researchers draw one sample of 7 with a mean of 6.71 and a variance of 12.24, and another sample of 6 with a mean of 5.67 and a variance of 5.87. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.1, state the error, then calculate a 90% confidence interval.

$$df_1 = 7 - 1 = 6$$

$$df_2 = 6 - 1 = 5$$

$$SS_1 = 12.24 \times 6 = 73.44$$

$$SS_2 = 5.87 \times 5 = 29.35$$

$$df_{\text{tot}} = 6 + 5 = 11$$

$$s_p^2 = (73.44 + 29.35)/11 = 9.34$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{(9.34/7) + (9.34/6)} = 1.7$$

$$t_{\text{obs}}(11) = (6.71 - 5.67)/1.7 = 0.61$$

$$t_{\text{crit}} = \pm 1.8$$

Fail to reject because  $1.8 > 0.61 > -1.8$

$$t_{90} = 1.8$$

$$CI_{90} = (6.71 - 5.67) \pm (1.7 \times 1.8) = [-2.01, 4.09]$$

### Question #2

Researchers draw one sample of 7 with a mean of 6.29 and a variance of 12.24, and another sample of 7 with a mean of 5.71 and a variance of 10.57. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.1, state the error, then calculate a 90% confidence interval.

$$df_1 = 7 - 1 = 6$$

$$df_2 = 7 - 1 = 6$$

$$SS_1 = 12.24 \times 6 = 73.44$$

$$SS_2 = 10.57 \times 6 = 63.42$$

$$df_{\text{tot}} = 6 + 6 = 12$$

$$s_p^2 = (73.44 + 63.42)/12 = 11.41$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{(11.41/7) + (11.41/7)} = 1.81$$

$$t_{\text{obs}}(12) = (6.29 - 5.71)/1.81 = 0.32$$

$$t_{\text{crit}} = \pm 1.78$$

Fail to reject because  $1.78 > 0.32 > -1.78$

$$t_{90} = 1.78$$

$$CI_{90} = (6.29 - 5.71) \pm (1.81 \times 1.78) = [-2.64, 3.8]$$

### Question #3

Researchers draw one sample of 6 with a mean of 4.83 and a variance of 4.57, and another sample of 7 with a mean of 6.29 and a variance of 5.57. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.1, state the error, then calculate a 90% confidence interval.

$$df_1 = 6 - 1 = 5$$

$$df_2 = 7 - 1 = 6$$

$$SS_1 = 4.57 \times 5 = 22.85$$

$$SS_2 = 5.57 \times 6 = 33.42$$

$$df_{\text{tot}} = 5 + 6 = 11$$

$$s_p^2 = (22.85 + 33.42)/11 = 5.12$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{(5.12/6) + (5.12/7)} = 1.26$$

$$t_{\text{obs}}(11) = (4.83 - 6.29)/1.26 = -1.16$$

$$t_{\text{crit}} = \pm 1.8$$

Fail to reject because  $1.8 > -1.16 > -1.8$

$$t_{90} = 1.8$$

$$CI_{90} = (4.83 - 6.29) \pm (1.26 \times 1.8) = [-3.72, 0.8]$$

**Question #4**

Researchers draw one sample of 6 with a mean of 7.17 and a variance of 10.97, and another sample of 6 with a mean of 5.5 and a variance of 9.1. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.05, state the error, then calculate a 95% confidence interval.

$$df_1 = 6 - 1 = 5$$

$$df_2 = 6 - 1 = 5$$

$$SS_1 = 10.97 \times 5 = 54.85$$

$$SS_2 = 9.1 \times 5 = 45.5$$

$$df_{\text{tot}} = 5 + 5 = 10$$

$$s_p^2 = (54.85 + 45.5)/10 = 10.04$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{(10.04/6) + (10.04/6)} = 1.83$$

$$t_{\text{obs}}(10) = (7.17 - 5.5)/1.83 = 0.91$$

$$t_{\text{crit}} = \pm 2.23$$

Fail to reject because  $2.23 > 0.91 > -2.23$

$$t_{95} = 2.23$$

$$CI_{95} = (7.17 - 5.5) \pm (1.83 \times 2.23) = [-2.41, 5.75]$$

**Question #5**

Researchers draw one sample of 7 with a mean of 5.71 and a variance of 11.57, and another sample of 6 with a mean of 4.17 and a variance of 1.37. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.1, state the error, then calculate a 90% confidence interval.

$$df_1 = 7 - 1 = 6$$

$$df_2 = 6 - 1 = 5$$

$$SS_1 = 11.57 \times 6 = 69.42$$

$$SS_2 = 1.37 \times 5 = 6.85$$

$$df_{\text{tot}} = 6 + 5 = 11$$

$$s_p^2 = (69.42 + 6.85)/11 = 6.93$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{(6.93/7) + (6.93/6)} = 1.46$$

$$t_{\text{obs}}(11) = (5.71 - 4.17)/1.46 = 1.05$$

$$t_{\text{crit}} = \pm 1.8$$

Fail to reject because  $1.8 > 1.05 > -1.8$

$$t_{90} = 1.8$$

$$CI_{90} = (5.71 - 4.17) \pm (1.46 \times 1.8) = [-1.09, 4.17]$$

**Question #6**

Researchers draw one sample of 6 with a mean of 7.83 and a variance of 5.77, and another sample of 6 with a mean of 5.33 and a variance of 12.27. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.05, state the error, then calculate a 95% confidence interval.

$$df_1 = 6 - 1 = 5$$

$$df_2 = 6 - 1 = 5$$

$$SS_1 = 5.77 \times 5 = 28.85$$

$$SS_2 = 12.27 \times 5 = 61.35$$

$$df_{\text{tot}} = 5 + 5 = 10$$

$$s_p^2 = (28.85 + 61.35)/10 = 9.02$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{(9.02/6) + (9.02/6)} = 1.73$$

$$t_{\text{obs}}(10) = (7.83 - 5.33)/1.73 = 1.44$$

$$t_{\text{crit}} = \pm 2.23$$

Fail to reject because  $2.23 > 1.44 > -2.23$

$$t_{95} = 2.23$$

$$CI_{95} = (7.83 - 5.33) \pm (1.73 \times 2.23) = [-1.36, 6.36]$$

**Question #7**

Researchers draw one sample of 6 with a mean of 4.67 and a variance of 12.67, and another sample of 6 with a mean of 3.33 and a variance of 7.47. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.01, state the error, then calculate a 99% confidence interval.

$$df_1 = 6 - 1 = 5$$

$$df_2 = 6 - 1 = 5$$

$$SS_1 = 12.67 \times 5 = 63.35$$

$$SS_2 = 7.47 \times 5 = 37.35$$

$$df_{\text{tot}} = 5 + 5 = 10$$

$$s_p^2 = (63.35 + 37.35)/10 = 10.07$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{(10.07/6) + (10.07/6)} = 1.83$$

$$t_{\text{obs}}(10) = (4.67 - 3.33)/1.83 = 0.73$$

$$t_{\text{crit}} = \pm 3.17$$

Fail to reject because  $3.17 > 0.73 > -3.17$

$$t_{99} = 3.17$$

$$CI_{99} = (4.67 - 3.33) \pm (1.83 \times 3.17) = [-4.47, 7.15]$$

### Question #8

Researchers draw one sample of 7 with a mean of 6.71 and a variance of 5.9, and another sample of 6 with a mean of 4.17 and a variance of 12.57. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.05, state the error, then calculate a 95% confidence interval.

$$df_1 = 7 - 1 = 6$$

$$df_2 = 6 - 1 = 5$$

$$SS_1 = 5.9 \times 6 = 35.4$$

$$SS_2 = 12.57 \times 5 = 62.85$$

$$df_{\text{tot}} = 6 + 5 = 11$$

$$s_p^2 = (35.4 + 62.85)/11 = 8.93$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{(8.93/7) + (8.93/6)} = 1.66$$

$$t_{\text{obs}}(11) = (6.71 - 4.17)/1.66 = 1.53$$

$$t_{\text{crit}} = \pm 2.2$$

Fail to reject because  $2.2 > 1.53 > -2.2$

$$t_{95} = 2.2$$

$$CI_{95} = (6.71 - 4.17) \pm (1.66 \times 2.2) = [-1.12, 6.2]$$

## Week #9

1. Come up with some research questions for which a nondirectional dependent  $t$ -test would be appropriate, then reframe them so that a directional test would be appropriate. State the hypotheses formally
2. How does using a dependent design rather than an independent design affect power? Why?
3. Give an example of a research question which could be addressed by ANOVA but not a single  $t$ -test
4. What do the numerator and denominator of the  $F$  statistic represent? Why is  $F$  is equal to 1 under the null hypothesis?

$$F = \frac{MS_B}{MS_W}$$

$MS_B$  = "mean squares between"

$MS_W$  = "mean squares within"

5. How much within and between group variability is there in each of the following scenarios?

- Scenario #1
  - Group 1: [1, 1, 1, 1, 1]
  - Group 2: [1, 1, 1, 1, 1]
  - Group 3: [1, 1, 1, 1, 1]
- Scenario #2
  - Group 1: [1, 4, 3, 7, 9]
  - Group 2: [2, 3, 4, 8, 8]
  - Group 3: [1, 2, 5, 9, 7]
- Scenario #3
  - Group 1: [1, 1, 1, 1, 1]
  - Group 2: [15, 15, 15, 15, 15]
  - Group 3: [30, 30, 30, 30, 30]
- Scenario #4
  - Group 1: [1, 4, 3, 7, 9]
  - Group 2: [15, 26, 17, 29, 21]
  - Group 3: [30, 37, 43, 41, 32]

Rank them on the likelihood the null hypothesis ( $F = 1$ ) will be rejected



## Dependent *t*-tests

$$df = n - 1$$

$$\bar{D} = \Sigma(D_i)/n$$

$$s_D = \sqrt{\Sigma[(D_i - \bar{D})^2]/df}$$

$$s_{\bar{D}} = s_D/\sqrt{n}$$

$$t_{\text{obs}} = \bar{D}/s_{\bar{D}}$$

$$CI_y = \bar{D} \pm s_{\bar{D}} \times t_y$$

$$t_y \text{ is the critical value for } \alpha = (100 - y)/100$$

### Critical *t* values

df	$\alpha$		
	0.1	0.05	0.01
4	2.13	2.78	4.6
5	2.02	2.57	4.03
6	1.94	2.45	3.71

### Question #1

Test  $H_0 : \mu_{\bar{D}} = 0$  at an  $\alpha$  of 0.05, state the decision/error, then calculate a 95% confidence interval.

Pre	Post	$D_i$	$D_i - \bar{D}$	$(D_i - \bar{D})^2$
5	5	0	2	4
7	4	3	5	25
1	7	-6	-4	16
9	10	-1	1	1
3	9	-6	-4	16

$$\bar{D} = -2$$

$$\Sigma(D_i - \bar{D})^2 = 62$$

$$df = 4$$

$$s_D = \sqrt{62/4} = 3.94$$

$$s_{\bar{D}} = 3.94/\sqrt{5} = 1.76$$

$$t_{\text{obs}}(4) = -2/1.76 = -1.14$$

$$t_{\text{crit}} = 2.78$$

Fail to reject because  $2.78 > -1.14 > -2.78$

$$t_{95} = 2.78$$

$$CI_{95} = -2 \pm (1.76 \times 2.78) = [-6.89, 2.89]$$

### Question #2

Test  $H_0 : \mu_{\bar{D}} = 0$  at an  $\alpha$  of 0.01, state the decision/error, then calculate a 99% confidence interval.

Pre	Post	$D_i$	$D_i - \bar{D}$	$(D_i - \bar{D})^2$
2	2	0	0	0
8	8	0	0	0
4	7	-3	-3	9
9	4	5	5	25
7	9	-2	-2	4

$$\bar{D} = 0$$

$$\Sigma(D_i - \bar{D})^2 = 38$$

$$df = 4$$

$$s_D = \sqrt{38/4} = 3.08$$

$$s_{\bar{D}} = 3.08/\sqrt{5} = 1.38$$

$$t_{\text{obs}}(4) = 0/1.38 = 0$$

$$t_{\text{crit}} = 4.6$$

Fail to reject because  $4.6 > 0 > -4.6$

$$t_{99} = 4.6$$

$$CI_{99} = 0 \pm (1.38 \times 4.6) = [-6.35, 6.35]$$

### Question #3

Test  $H_0 : \mu_{\bar{D}} = 0$  at an  $\alpha$  of 0.05, state the decision/error, then calculate a 95% confidence interval.

Pre	Post	$D_i$	$D_i - \bar{D}$	$(D_i - \bar{D})^2$
6	2	4	3	9
5	10	-5	-6	36
9	7	2	1	1
7	6	1	0	0
8	5	3	2	4

$$\bar{D} = 1$$

$$\Sigma(D_i - \bar{D})^2 = 50$$

$$df = 4$$

$$s_D = \sqrt{50/4} = 3.54$$

$$s_{\bar{D}} = 3.54/\sqrt{5} = 1.58$$

$$t_{\text{obs}}(4) = 1/1.58 = 0.63$$

$$t_{\text{crit}} = 2.78$$

Fail to reject because  $2.78 > 0.63 > -2.78$

$$t_{95} = 2.78$$

$$CI_{95} = 1 \pm (1.58 \times 2.78) = [-3.39, 5.39]$$

**Question #4**

Test  $H_0 : \mu_{\bar{D}} = 0$  at an  $\alpha$  of 0.1, state the decision/error, then calculate a 90% confidence interval.

Pre	Post	$D_i$	$D_i - \bar{D}$	$(D_i - \bar{D})^2$
10	9	1	1	1
9	5	4	4	16
1	7	-6	-6	36
6	4	2	2	4
2	8	-6	-6	36
8	3	5	5	25

$$\bar{D} = 0$$

$$\Sigma(D_i - \bar{D})^2 = 118$$

$$df = 5$$

$$s_D = \sqrt{118/5} = 4.86$$

$$s_{\bar{D}} = 4.86/\sqrt{6} = 1.98$$

$$t_{\text{obs}}(5) = 0/1.98 = 0$$

$$t_{\text{crit}} = 2.02$$

Fail to reject because  $2.02 > 0 > -2.02$

$$t_{90} = 2.02$$

$$CI_{90} = 0 \pm (1.98 \times 2.02) = [-4, 4]$$

**Question #5**

Test  $H_0 : \mu_{\bar{D}} = 0$  at an  $\alpha$  of 0.01, state the decision/error, then calculate a 99% confidence interval.

Pre	Post	$D_i$	$D_i - \bar{D}$	$(D_i - \bar{D})^2$
7	4	3	4	16
9	8	1	2	4
3	5	-2	-1	1
1	6	-5	-4	16
5	7	-2	-1	1

$$\bar{D} = -1$$

$$\Sigma(D_i - \bar{D})^2 = 38$$

$$df = 4$$

$$s_D = \sqrt{38/4} = 3.08$$

$$s_{\bar{D}} = 3.08/\sqrt{5} = 1.38$$

$$t_{\text{obs}}(4) = -1/1.38 = -0.73$$

$$t_{\text{crit}} = 4.6$$

Fail to reject because  $4.6 > -0.73 > -4.6$

$$t_{99} = 4.6$$

$$CI_{99} = -1 \pm (1.38 \times 4.6) = [-7.35, 5.35]$$

## Week #10

1. Give an example of two variables which would have a correlation of close to 1
2. Give an example of two variables which would have a correlation of close to -1
3. Give an example of two variables which would have a correlation of close to 0
4. Draw a relationship which cannot be accurately described by Pearson's  $r$
5. Why doesn't correlation imply causation? Give a scenario in which inferring a causal relationship from correlation would be misguided
6. Why does  $df = n - 2$  when looking up critical  $r$  values? Explain using a scatterplot of the  $n = 2$  and  $n = 1$  cases

## Pearson's $r$

$$\bar{X} = \Sigma(X_i)/n$$

$$df = n - 1$$

$$SP = \Sigma[(X_i - \bar{X})(Y_i - \bar{Y})]$$

$$SS_X = \Sigma[(X_i - \bar{X})^2]$$

$$SS_Y = \Sigma[(Y_i - \bar{Y})^2]$$

$$r_{XY} = SP / \sqrt{SS_X \times SS_Y}$$

## Critical $r$ values

$\alpha$			
$(n - 2)$	0.2	0.1	0.05
2	0.8	0.9	0.95
3	0.69	0.81	0.88
4	0.61	0.73	0.81

### Question #1

Calculate  $r_{XY}$  and test  $H_0 : \rho_{XY} = 0$  at  $\alpha = 0.2$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
4	4	-1	-3	1	9	3
6	5	1	-2	1	4	-2
8	9	3	2	9	4	6
2	10	-3	3	9	9	-9

$$\bar{X} = 5$$

$$\bar{Y} = 7$$

$$SS_X = 20$$

$$SS_Y = 26$$

$$SP = -2$$

$$r_{XY} = -2 / \sqrt{20 \times 26} = -0.09$$

$$r_{\text{crit}} = \pm 0.8$$

Fail to reject because  $0.8 > -0.09 > -0.8$

### Question #2

Calculate  $r_{XY}$  and test  $H_0 : \rho_{XY} = 0$  at  $\alpha = 0.1$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
9	8	2	2	4	4	4
5	9	-2	3	4	9	-6
6	3	-1	-3	1	9	3
8	4	1	-2	1	4	-2

$$\bar{X} = 7$$

$$\bar{Y} = 6$$

$$SS_X = 10$$

$$SS_Y = 26$$

$$SP = -1$$

$$r_{XY} = -1 / \sqrt{10 \times 26} = -0.06$$

$$r_{\text{crit}} = \pm 0.9$$

Fail to reject because  $0.9 > -0.06 > -0.9$

**Question #3**

Calculate  $r_{XY}$  and test  $H_0 : \rho_{XY} = 0$  at  $\alpha = 0.2$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
8	5	1	-1	1	1	-1
6	3	-1	-3	1	9	3
10	9	3	3	9	9	9
4	7	-3	1	9	1	-3

$$\bar{X} = 7$$

$$\bar{Y} = 6$$

$$SS_X = 20$$

$$SS_Y = 20$$

$$SP = 8$$

$$r_{XY} = 8/\sqrt{20 \times 20} = 0.4$$

$$r_{\text{crit}} = \pm 0.8$$

Fail to reject because  $0.8 > 0.4 > -0.8$

**Question #4**

Calculate  $r_{XY}$  and test  $H_0 : \rho_{XY} = 0$  at  $\alpha = 0.2$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
10	3	3	-3	9	9	-9
5	8	-2	2	4	4	-4
4	6	-3	0	9	0	-0
9	4	2	-2	4	4	-4
7	9	0	3	0	9	0

$$\bar{X} = 7$$

$$\bar{Y} = 6$$

$$SS_X = 26$$

$$SS_Y = 26$$

$$SP = -17$$

$$r_{XY} = -17/\sqrt{26 \times 26} = -0.65$$

$$r_{\text{crit}} = \pm 0.69$$

Fail to reject because  $0.69 > -0.65 > -0.69$

**Question #5**

Calculate  $r_{XY}$  and test  $H_0 : \rho_{XY} = 0$  at  $\alpha = 0.1$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
7	6	2	1	4	1	2
9	3	4	-2	16	4	-8
3	2	-2	-3	4	9	6
1	9	-4	4	16	16	-16

$$\bar{X} = 5$$

$$\bar{Y} = 5$$

$$SS_X = 40$$

$$SS_Y = 30$$

$$SP = -16$$

$$r_{XY} = -16/\sqrt{40 \times 30} = -0.46$$

$$r_{\text{crit}} = \pm 0.9$$

Fail to reject because  $0.9 > -0.46 > -0.9$

**Question #6**

Calculate  $r_{XY}$  and test  $H_0 : \rho_{XY} = 0$  at  $\alpha = 0.1$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
10	6	4	0	16	0	0
3	7	-3	1	9	1	-3
4	2	-2	-4	4	16	8
6	5	0	-1	0	1	-0
7	10	1	4	1	16	4

$$\bar{X} = 6$$

$$\bar{Y} = 6$$

$$SS_X = 30$$

$$SS_Y = 34$$

$$SP = 9$$

$$r_{XY} = 9/\sqrt{30 \times 34} = 0.28$$

$$r_{\text{crit}} = \pm 0.81$$

Fail to reject because  $0.81 > 0.28 > -0.81$

**Question #7**

Calculate  $r_{XY}$  and test  $H_0 : \rho_{XY} = 0$  at  $\alpha = 0.1$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
4	10	-1	4	1	16	-4
8	2	3	-4	9	16	-12
2	9	-3	3	9	9	-9
3	3	-2	-3	4	9	6
7	5	2	-1	4	1	-2
6	7	1	1	1	1	1

$$\bar{X} = 5$$

$$\bar{Y} = 6$$

$$SS_X = 28$$

$$SS_Y = 52$$

$$SP = -20$$

$$r_{XY} = -20/\sqrt{28 \times 52} = -0.52$$

$$r_{\text{crit}} = \pm 0.73$$

Fail to reject because  $0.73 > -0.52 > -0.73$

**Question #8**

Calculate  $r_{XY}$  and test  $H_0 : \rho_{XY} = 0$  at  $\alpha = 0.2$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
5	1	0	-5	0	25	-0
8	9	3	3	9	9	9
6	7	1	1	1	1	1
2	8	-3	2	9	4	-6
4	5	-1	-1	1	1	1

$$\bar{X} = 5$$

$$\bar{Y} = 6$$

$$SS_X = 20$$

$$SS_Y = 40$$

$$SP = 5$$

$$r_{XY} = 5/\sqrt{20 \times 40} = 0.18$$

$$r_{\text{crit}} = \pm 0.69$$

Fail to reject because  $0.69 > 0.18 > -0.69$

**Question #9**

Calculate  $r_{XY}$  and test  $H_0 : \rho_{XY} = 0$  at  $\alpha = 0.2$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
1	10	-3	5	9	25	-15
2	2	-2	-3	4	9	6
9	1	5	-4	25	16	-20
4	7	0	2	0	4	0

$$\bar{X} = 4$$

$$\bar{Y} = 5$$

$$SS_X = 38$$

$$SS_Y = 54$$

$$SP = -29$$

$$r_{XY} = -29/\sqrt{38 \times 54} = -0.64$$

$$r_{\text{crit}} = \pm 0.8$$

Fail to reject because  $0.8 > -0.64 > -0.8$

**Question #10**

Calculate  $r_{XY}$  and test  $H_0 : \rho_{XY} = 0$  at  $\alpha = 0.2$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
1	3	-4	-2	16	4	8
5	5	0	0	0	0	0
10	6	5	1	25	1	5
6	1	1	-4	1	16	-4
3	10	-2	5	4	25	-10

$$\bar{X} = 5$$

$$\bar{Y} = 5$$

$$SS_X = 46$$

$$SS_Y = 46$$

$$SP = -1$$

$$r_{XY} = -1/\sqrt{46 \times 46} = -0.02$$

$$r_{\text{crit}} = \pm 0.69$$

Fail to reject because  $0.69 > -0.02 > -0.69$

## Spearman score-ranking

To rank scores when calculating Spearman's rho:

1. Order the  $n$  scores from least to greatest
2. Assign them ranks from 1 to  $n$
3. Assign tied scores the mean of their ranks from step 2

### Question #1

3, 4, 6, 9, 3

$X_i$	Sorted	Ranking 1	Ranking 2
3	3	1	1.5
4	3	2	1.5
6	4	3	3.0
9	6	4	4.0
3	9	5	5.0

### Question #2

2, 7, 6, 2, 2

$X_i$	Sorted	Ranking 1	Ranking 2
2	2	1	2.0
7	2	2	2.0
6	2	3	2.0
2	6	4	4.0
2	7	5	5.0

### Question #3

2, 8, 4, 3, 2

$X_i$	Sorted	Ranking 1	Ranking 2
2	2	1	1.5
8	2	2	1.5
4	3	3	3.0
3	4	4	4.0
2	8	5	5.0

### Question #4

6, 1, 3, 3, 6

$X_i$	Sorted	Ranking 1	Ranking 2
6	1	1	1.0
1	3	2	2.5
3	3	3	2.5
3	6	4	4.5
6	6	5	4.5

### Question #5

2, 7, 9, 3, 2

$X_i$	Sorted	Ranking 1	Ranking 2
2	2	1	1.5
7	2	2	1.5
9	3	3	3.0
3	7	4	4.0
2	9	5	5.0

### Question #6

6, 9, 3, 4, 6

$X_i$	Sorted	Ranking 1	Ranking 2
6	3	1	1.0
9	4	2	2.0
3	6	3	3.5
4	6	4	3.5
6	9	5	5.0



## Week #11

1. What does the regression line minimise? Draw a picture
2. How many regression lines are possible for a given data set?
3. Give an example of restriction of range
4. Explain the difference between a univariate outlier and a regression outlier, and draw a picture
5. Draw an example of how an influential outlier might affect the regression line
6. What is a residual and what does it tell us about model fit?

## Regression

$$\bar{Y} = \Sigma(Y_i)/n$$

$$df_1 = 1$$

$$df_2 = n - df_1 - 1$$

$$SP = \Sigma[(X_i - \bar{X})(Y_i - \bar{Y})]$$

$$SS_X = \Sigma[(X_i - \bar{X})^2]$$

$$\beta_1 = SP/SS_X$$

$$\beta_0 = \bar{Y} - \beta_1 \times \bar{X}$$

$$\hat{Y}_i = \beta_0 + X_i \times \beta_1$$

$$SS_{\text{tot}} = \Sigma[(Y_i - \bar{Y})^2]$$

$$SS_{\text{reg}} = \Sigma[(\hat{Y}_i - \bar{Y})^2]$$

$$SS_{\text{res}} = SS_{\text{tot}} - SS_{\text{reg}}$$

$$MS_{\text{reg}} = SS_{\text{reg}}/df_1$$

$$MS_{\text{res}} = SS_{\text{res}}/df_2$$

$$F = MS_{\text{reg}}/MS_{\text{res}}$$

### Critical $F$ values

		$df_1$			
$df_2$	$\alpha$	1	2	3	
1	0.05	161.4	199.5	215.71	
	0.01	4052	4999	5404	
2	0.05	18.51	19	19.16	
	0.01	98.94	99	99.17	
3	0.05	7.71	6.94	6.59	
	0.01	34.12	30.82	29.46	
4	0.05	7.71	6.94	6.59	
	0.01	21.2	18	16.69	

### Question #1

Test the model fit at an  $\alpha$  of 0.05.

$X_i$	$Y_i$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
6	7	0	4	0	5.00	0.00	0.00
9	2	9	9	-9	5.23	0.23	0.05
2	3	16	4	8	4.69	-0.31	0.09
7	8	1	9	3	5.08	0.08	0.01

$$SS_X = 26$$

$$SP = 2$$

$$\beta_1 = 2/26 = 0.08$$

$$\bar{Y} = 5$$

$$\bar{X} = 6$$

$$\beta_0 = 5 - (0.08 \times 6) = 4.54$$

$$\hat{Y}_i = 4.54 + (0.08 \times X_i)$$

$$SS_{\text{tot}} = 26$$

$$SS_{\text{reg}} = 0.15$$

$$SS_{\text{res}} = 26 - 0.15 = 25.85$$

$$df_1 = 1$$

$$df_2 = 4 - 1 - 1 = 2$$

$$MS_{\text{reg}} = 0.15/1 = 0.15$$

$$MS_{\text{res}} = 25.85/2 = 12.92$$

$$F = 0.15/12.92 = 0.01$$

$$F_{\text{crit}} = 18.51$$

Fail to reject because  $0.01 < 18.51$

### Question #2

Test the model fit at an  $\alpha$  of 0.01.

$X_i$	$Y_i$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
5	7	1	4	-2	4.70	-0.30	0.09
8	2	4	9	-6	5.60	0.60	0.36
9	8	9	9	9	5.90	0.90	0.81
2	3	16	4	8	3.80	-1.20	1.44

$$SS_X = 30$$

$$SP = 9$$

$$\beta_1 = 9/30 = 0.3$$

$$\bar{Y} = 5$$

$$\bar{X} = 6$$

$$\beta_0 = 5 - (0.3 \times 6) = 3.2$$

$$\hat{Y}_i = 3.2 + (0.3 \times X_i)$$

$$SS_{\text{tot}} = 26$$

$$SS_{\text{reg}} = 2.7$$

$$SS_{\text{res}} = 26 - 2.7 = 23.3$$

$$df_1 = 1$$

$$df_2 = 4 - 1 - 1 = 2$$

$$MS_{\text{reg}} = 2.7/1 = 2.7$$

$$MS_{\text{res}} = 23.3/2 = 11.65$$

$$F = 2.7/11.65 = 0.23$$

$$F_{\text{crit}} = 98.5$$

Fail to reject because  $0.23 < 98.5$

### Question #3

Test the model fit at an  $\alpha$  of 0.01.

$X_i$	$Y_i$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
4	6	1	0	-0	6.29	0.29	0.08
3	5	4	1	2	6.57	0.57	0.33
5	9	0	9	0	6.00	0.00	0.00
8	4	9	4	-6	5.14	-0.86	0.73

$$SS_X = 14$$

$$SP = -4$$

$$\beta_1 = -4/14 = -0.29$$

$$\bar{Y} = 6$$

$$\bar{X} = 5$$

$$\beta_0 = 6 - (-0.29 \times 5) = 7.43$$

$$\hat{Y}_i = 7.43 + (-0.29 \times X_i)$$

$$SS_{\text{tot}} = 14$$

$$SS_{\text{reg}} = 1.14$$

$$SS_{\text{res}} = 14 - 1.14 = 12.86$$

$$df_1 = 1$$

$$df_2 = 4 - 1 - 1 = 2$$

$$MS_{\text{reg}} = 1.14/1 = 1.14$$

$$MS_{\text{res}} = 12.86/2 = 6.43$$

$$F = 1.14/6.43 = 0.18$$

$$F_{\text{crit}} = 98.5$$

Fail to reject because  $0.18 < 98.5$

**Question #4**

Test the model fit at an  $\alpha$  of 0.05.

$X_i$	$Y_i$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
4	4	4	1	2	6.40	1.40	1.96
6	10	0	25	0	5.00	0.00	0.00
5	6	1	1	-1	5.70	0.70	0.49
7	3	1	4	-2	4.30	-0.70	0.49
8	2	4	9	-6	3.60	-1.40	1.96

$$SS_X = 10$$

$$SP = -7$$

$$\beta_1 = -7/10 = -0.7$$

$$\bar{Y} = 5$$

$$\bar{X} = 6$$

$$\beta_0 = 5 - (-0.7 \times 6) = 9.2$$

$$\hat{Y}_i = 9.2 + (-0.7 \times X_i)$$

$$SS_{\text{tot}} = 40$$

$$SS_{\text{reg}} = 4.9$$

$$SS_{\text{res}} = 40 - 4.9 = 35.1$$

$$df_1 = 1$$

$$df_2 = 5 - 1 - 1 = 3$$

$$MS_{\text{reg}} = 4.9/1 = 4.9$$

$$MS_{\text{res}} = 35.1/3 = 11.7$$

$$F = 4.9/11.7 = 0.42$$

$$F_{\text{crit}} = 10.13$$

Fail to reject because  $0.42 < 10.13$

**Question #5**

Test the model fit at an  $\alpha$  of 0.01.

$X_i$	$Y_i$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
6	3	0	1	-0	4.00	0.00	0.00
8	2	4	4	-4	3.00	-1.00	1.00
9	4	9	0	0	2.50	-1.50	2.25
1	7	25	9	-15	6.50	2.50	6.25

$$SS_X = 38$$

$$SP = -19$$

$$\beta_1 = -19/38 = -0.5$$

$$\bar{Y} = 4$$

$$\bar{X} = 6$$

$$\beta_0 = 4 - (-0.5 \times 6) = 7$$

$$\hat{Y}_i = 7 + (-0.5 \times X_i)$$

$$SS_{\text{tot}} = 14$$

$$SS_{\text{reg}} = 9.5$$

$$SS_{\text{res}} = 14 - 9.5 = 4.5$$

$$df_1 = 1$$

$$df_2 = 4 - 1 - 1 = 2$$

$$MS_{\text{reg}} = 9.5/1 = 9.5$$

$$MS_{\text{res}} = 4.5/2 = 2.25$$

$$F = 9.5/2.25 = 4.22$$

$$F_{\text{crit}} = 98.5$$

Fail to reject because  $4.22 < 98.5$

**Question #6**

Test the model fit at an  $\alpha$  of 0.01.

$X_i$	$Y_i$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
7	1	1	25	-5	5.32	-0.68	0.46
2	9	16	9	-12	8.71	2.71	7.32
6	7	0	1	0	6.00	0.00	0.00
10	5	16	1	-4	3.29	-2.71	7.32
5	8	1	4	-2	6.68	0.68	0.46

$$SS_X = 34$$

$$SP = -23$$

$$\beta_1 = -23/34 = -0.68$$

$$\bar{Y} = 6$$

$$\bar{X} = 6$$

$$\beta_0 = 6 - (-0.68 \times 6) = 10.06$$

$$\hat{Y}_i = 10.06 + (-0.68 \times X_i)$$

$$SS_{\text{tot}} = 40$$

$$SS_{\text{reg}} = 15.56$$

$$SS_{\text{res}} = 40 - 15.56 = 24.44$$

$$df_1 = 1$$

$$df_2 = 5 - 1 - 1 = 3$$

$$MS_{\text{reg}} = 15.56/1 = 15.56$$

$$MS_{\text{res}} = 24.44/3 = 8.15$$

$$F = 15.56/8.15 = 1.91$$

$$F_{\text{crit}} = 34.12$$

Fail to reject because  $1.91 < 34.12$

**Question #7**

Test the model fit at an  $\alpha$  of 0.05.

$X_i$	$Y_i$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
9	10	4	16	8	7.00	1.00	1.00
5	2	4	16	8	5.00	-1.00	1.00
7	6	0	0	0	6.00	0.00	0.00
3	5	16	1	4	4.00	-2.00	4.00
10	4	9	4	-6	7.50	1.50	2.25
8	9	1	9	3	6.50	0.50	0.25

$$SS_X = 34$$

$$SP = 17$$

$$\beta_1 = 17/34 = 0.5$$

$$\bar{Y} = 6$$

$$\bar{X} = 7$$

$$\beta_0 = 6 - (0.5 \times 7) = 2.5$$

$$\hat{Y}_i = 2.5 + (0.5 \times X_i)$$

$$SS_{\text{tot}} = 46$$

$$SS_{\text{reg}} = 8.5$$

$$SS_{\text{res}} = 46 - 8.5 = 37.5$$

$$df_1 = 1$$

$$df_2 = 6 - 1 - 1 = 4$$

$$MS_{\text{reg}} = 8.5/1 = 8.5$$

$$MS_{\text{res}} = 37.5/4 = 9.38$$

$$F = 8.5/9.38 = 0.91$$

$$F_{\text{crit}} = 7.71$$

Fail to reject because  $0.91 < 7.71$

**Question #8**

Test the model fit at an  $\alpha$  of 0.05.

$X_i$	$Y_i$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
5	10	0	25	0	5.00	0.00	0.00
10	2	25	9	-15	3.15	-1.85	3.43
2	4	9	1	3	6.11	1.11	1.23
7	3	4	4	-4	4.26	-0.74	0.55
1	6	16	1	-4	6.48	1.48	2.19

$$SS_X = 54$$

$$SP = -20$$

$$\beta_1 = -20/54 = -0.37$$

$$\bar{Y} = 5$$

$$\bar{X} = 5$$

$$\beta_0 = 5 - (-0.37 \times 5) = 6.85$$

$$\hat{Y}_i = 6.85 + (-0.37 \times X_i)$$

$$SS_{\text{tot}} = 40$$

$$SS_{\text{reg}} = 7.41$$

$$SS_{\text{res}} = 40 - 7.41 = 32.59$$

$$df_1 = 1$$

$$df_2 = 5 - 1 - 1 = 3$$

$$MS_{\text{reg}} = 7.41/1 = 7.41$$

$$MS_{\text{res}} = 32.59/3 = 10.86$$

$$F = 7.41/10.86 = 0.68$$

$$F_{\text{crit}} = 10.13$$

Fail to reject because  $0.68 < 10.13$

**Question #9**

Test the model fit at an  $\alpha$  of 0.05.

$X_i$	$Y_i$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
4	3	9	9	9	3.90	-2.10	4.41
10	7	9	1	3	8.10	2.10	4.41
8	8	1	4	2	6.70	0.70	0.49
6	6	1	0	-0	5.30	-0.70	0.49

$$SS_X = 20$$

$$SP = 14$$

$$\beta_1 = 14/20 = 0.7$$

$$\bar{Y} = 6$$

$$\bar{X} = 7$$

$$\beta_0 = 6 - (0.7 \times 7) = 1.1$$

$$\hat{Y}_i = 1.1 + (0.7 \times X_i)$$

$$SS_{\text{tot}} = 14$$

$$SS_{\text{reg}} = 9.8$$

$$SS_{\text{res}} = 14 - 9.8 = 4.2$$

$$df_1 = 1$$

$$df_2 = 4 - 1 - 1 = 2$$

$$MS_{\text{reg}} = 9.8/1 = 9.8$$

$$MS_{\text{res}} = 4.2/2 = 2.1$$

$$F = 9.8/2.1 = 4.67$$

$$F_{\text{crit}} = 18.51$$

Fail to reject because  $4.67 < 18.51$