

## Week #1

- Give an example of a population and a sample from that population
- Give an example of a parameter and its statistic counterpart
- Give an example of an descriptive question and then reframe it as an inferential question
- Give an example of a research question and identify the independent variable and the dependent variable

## Summing notation

$X_1$  = the first element of  $X$

$X_n$  = the last element of  $X$

$$\Sigma(X_i) = X_1 + X_2 + \dots + X_n$$

$$\Sigma(X_i + 1) = (X_1 + 1) + (X_2 + 1) + \dots + (X_n + 1)$$

$$\Sigma(X_i - Y_i) = (X_1 - Y_1) + (X_2 - Y_2) + \dots + (X_n - Y_n)$$

$$[\Sigma(X_i)]^2 = (X_1 + X_2 + \dots + X_n)^2$$

### Question #1

$$X = 3 \ 4 \ 5 \ 7 \ 2$$

$$Y = 9 \ 10 \ 6 \ 5 \ 1$$

$$\Sigma(X_i) = 21$$

$$[\Sigma(X_i)]^2 = 441$$

$$\Sigma(X_i - Y_i) = -10$$

$$\Sigma(X_i Y_i) = 134$$

$$\Sigma(X_i^2) = 103$$

$$[\Sigma(X_i - Y_i)]^2 = 100$$

$$\Sigma(X_i) \Sigma(Y_i) = 651$$

### Question #2

$$X = 2 \ 7 \ 5 \ 10 \ 6$$

$$Y = 10 \ 2 \ 7 \ 4 \ 8$$

$$\Sigma(X_i) = 30$$

$$[\Sigma(X_i)]^2 = 900$$

$$\Sigma(X_i - Y_i) = -1$$

$$\Sigma(X_i Y_i) = 157$$

$$\Sigma(X_i^2) = 214$$

$$[\Sigma(X_i - Y_i)]^2 = 1$$

$$\Sigma(X_i) \Sigma(Y_i) = 930$$

### Question #3

$$X = 2 \ 8 \ 4 \ 3 \ 9$$

$$Y = 7 \ 2 \ 3 \ 5 \ 4$$

$$\Sigma(X_i) = 26$$

$$[\Sigma(X_i)]^2 = 676$$

$$\Sigma(X_i - Y_i) = 5$$

$$\Sigma(X_i Y_i) = 93$$

$$\Sigma(X_i^2) = 174$$

$$[\Sigma(X_i - Y_i)]^2 = 25$$

$$\Sigma(X_i) \Sigma(Y_i) = 546$$

### Question #4

$$X = 6 \ 1 \ 3 \ 2 \ 5$$

$$Y = 3 \ 7 \ 8 \ 9 \ 1$$

$$\Sigma(X_i) = 17$$

$$[\Sigma(X_i)]^2 = 289$$

$$\Sigma(X_i - Y_i) = -11$$

$$\Sigma(X_i Y_i) = 72$$

$$\Sigma(X_i^2) = 75$$

$$[\Sigma(X_i - Y_i)]^2 = 121$$

$$\Sigma(X_i) \Sigma(Y_i) = 476$$

### Question #5

$$X = 3 \ 7 \ 8 \ 2 \ 1$$

$$Y = 8 \ 5 \ 7 \ 10 \ 1$$

$$\Sigma(X_i) = 21$$

$$[\Sigma(X_i)]^2 = 441$$

$$\Sigma(X_i - Y_i) = -10$$

$$\Sigma(X_i Y_i) = 136$$

$$\Sigma(X_i^2) = 127$$

$$[\Sigma(X_i - Y_i)]^2 = 100$$

$$\Sigma(X_i) \Sigma(Y_i) = 651$$

### Question #6

$$X = 7 \ 9 \ 3 \ 8 \ 5$$

$$Y = 10 \ 9 \ 7 \ 4 \ 1$$

$$\Sigma(X_i) = 32$$

$$[\Sigma(X_i)]^2 = 1024$$

$$\Sigma(X_i - Y_i) = 1$$

$$\Sigma(X_i Y_i) = 209$$

$$\Sigma(X_i^2) = 228$$

$$[\Sigma(X_i - Y_i)]^2 = 1$$

$$\Sigma(X_i) \Sigma(Y_i) = 992$$

## Week #2

- Give an example of each of the following, and explain why your example is appropriate:
  - Discrete variable
  - Continuous variable
  - Nominal variable
  - Ordinal variable
  - Interval variable
  - Ratio variable
- Draw a well-designed bar graph and a poorly-designed one, and indicate the differences

## Frequency table construction

For the following data sets, create a frequency table containing the frequency, cumulative frequency, relative frequency, and cumulative relative frequency of each value.

Freq. = number equal to score

C Freq. = number less than or equal to score

R Freq. = frequency divided by sample size

CR Freq. = cumulative frequency divided by n

### Question #1

-1, 0, -1, 2, 0, -1, 0, 1, 1, 0, 2, 0

score	freq	cfreq	rfreq	crfreq
-1	3	3	0.25	0.25
0	5	8	0.42	0.67
1	2	10	0.17	0.83
2	2	12	0.17	1.00

### Question #2

-1, 0, 2, -1, 0, 0, 1, 0, 2, 0, 0, 1

score	freq	cfreq	rfreq	crfreq
-1	2	2	0.17	0.17
0	6	8	0.50	0.67
1	2	10	0.17	0.83
2	2	12	0.17	1.00

### Question #3

-1, 0, 0, -1, 0, 0, 0, 1, -1, 1, -1, -1

score	freq	cfreq	rfreq	crfreq
-1	5	5	0.42	0.42
0	5	10	0.42	0.83
1	2	12	0.17	1.00

### Question #4

0, -1, 1, 1, 2, 1, -1, 0, 2, 2, 1, 0

score	freq	cfreq	rfreq	crfreq
-1	2	2	0.17	0.17
0	3	5	0.25	0.42
1	4	9	0.33	0.75
2	3	12	0.25	1.00

### Question #5

-1, 1, -1, 0, 2, -1, 0, -1, 0, 0, 1, -1

score	freq	cfreq	rfreq	crfreq
-1	5	5	0.42	0.42
0	4	9	0.33	0.75
1	2	11	0.17	0.92
2	1	12	0.08	1.00

### Question #6

0, -1, 1, 2, 0, 0, -1, 1, 0, -1, 2, -1

score	freq	cfreq	rfreq	crfreq
-1	4	4	0.33	0.33
0	4	8	0.33	0.67
1	2	10	0.17	0.83
2	2	12	0.17	1.00

### Question #7

2, -1, -1, 0, -1, -1, 1, 0, 0, 2, 0, 3

score	freq	cfreq	rfreq	crfreq
-1	4	4	0.33	0.33
0	4	8	0.33	0.67
1	1	9	0.08	0.75
2	2	11	0.17	0.92
3	1	12	0.08	1.00

### Question #8

0, 1, 0, -1, 1, 0, 0, -1, -3, -1, -1, 0

score	freq	cfreq	rfreq	crfreq
-3	1	1	0.08	0.08
-1	4	5	0.33	0.42
0	5	10	0.42	0.83
1	2	12	0.17	1.00

### Question #9

-1, -1, 0, 0, 0, -1, 1, 0, 0, 0, 1, 0

score	freq	cfreq	rfreq	crfreq
-1	3	3	0.25	0.25
0	7	10	0.58	0.83
1	2	12	0.17	1.00

**Question #10**

0, 0, -1, -1, 0, 0, -1, 0, -2, 0, 1, 1

score	freq	cfreq	rfreq	crfreq
-2	1	1	0.08	0.08
-1	3	4	0.25	0.33
0	6	10	0.50	0.83
1	2	12	0.17	1.00

**Question #11**

-1, 0, -2, -1, 1, -1, 1, 1, 0, -1, -1, 0

score	freq	cfreq	rfreq	crfreq
-2	1	1	0.08	0.08
-1	5	6	0.42	0.50
0	3	9	0.25	0.75
1	3	12	0.25	1.00

**Question #12**

-1, 2, -1, -1, -2, 0, 0, -1, 0, 0, -1, -1

score	freq	cfreq	rfreq	crfreq
-2	1	1	0.08	0.08
-1	6	7	0.50	0.58
0	4	11	0.33	0.92
2	1	12	0.08	1.00

**Question #13**

1, 0, 2, 0, 1, 0, 1, 0, 0, 1, -1, 0

score	freq	cfreq	rfreq	crfreq
-1	1	1	0.08	0.08
0	6	7	0.50	0.58
1	4	11	0.33	0.92
2	1	12	0.08	1.00

**Question #14**

-1, 2, 2, 1, 0, 1, 0, 1, 0, 1, 0, 0

score	freq	cfreq	rfreq	crfreq
-1	1	1	0.08	0.08
0	5	6	0.42	0.50
1	4	10	0.33	0.83
2	2	12	0.17	1.00

**Question #15**

0, 2, 0, 1, 0, -1, 0, 1, 0, -1, 1, 0

score	freq	cfreq	rfreq	crfreq
-1	2	2	0.17	0.17
0	6	8	0.50	0.67
1	3	11	0.25	0.92
2	1	12	0.08	1.00

**Question #16**

0, 0, 1, -1, 1, 0, -1, 0, 1, 1, 2, 0

score	freq	cfreq	rfreq	crfreq
-1	2	2	0.17	0.17
0	5	7	0.42	0.58
1	4	11	0.33	0.92
2	1	12	0.08	1.00

**Question #17**

-1, 0, 0, -1, 1, 0, 1, 2, 0, 0, 1, 1

score	freq	cfreq	rfreq	crfreq
-1	2	2	0.17	0.17
0	5	7	0.42	0.58
1	4	11	0.33	0.92
2	1	12	0.08	1.00

**Question #18**

1, 2, -2, 0, 0, 0, -1, 2, 0, 2, -2, 1

score	freq	cfreq	rfreq	crfreq
-2	2	2	0.17	0.17
-1	1	3	0.08	0.25
0	4	7	0.33	0.58
1	2	9	0.17	0.75
2	3	12	0.25	1.00

**Question #19**

-1, 0, 0, -1, 1, 0, 1, -1, 0, 0, 1, 1

score	freq	cfreq	rfreq	crfreq
-1	3	3	0.25	0.25
0	5	8	0.42	0.67
1	4	12	0.33	1.00

## Interval construction

For each scenario, find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #1

The lowest value in the data set is 38, and the desired interval width is 14.

LL	MP	UL
28	34.5	41
42	48.5	55
56	62.5	69
70	76.5	83
84	90.5	97

### Question #2

The lowest value in the data set is 71, and the desired interval width is 10.

LL	MP	UL
70	74.5	79
80	84.5	89
90	94.5	99
100	104.5	109
110	114.5	119

### Question #3

The lowest value in the data set is 81, and the desired interval width is 9.

LL	MP	UL
81	85.0	89
90	94.0	98
99	103.0	107
108	112.0	116
117	121.0	125

### Question #4

The lowest value in the data set is 2, and the desired interval width is 30.

LL	MP	UL
0	14.5	29
30	44.5	59
60	74.5	89
90	104.5	119
120	134.5	149

### Question #5

The lowest value in the data set is 69, and the desired interval width is 11.

LL	MP	UL
66	71.0	76
77	82.0	87
88	93.0	98
99	104.0	109
110	115.0	120

### Question #6

The lowest value in the data set is 94, and the desired interval width is 31.

LL	MP	UL
93	108.0	123
124	139.0	154
155	170.0	185
186	201.0	216
217	232.0	247

**Question #7**

The lowest value in the data set is 40, and the desired interval width is 49.

LL	MP	UL
0	24.0	48
49	73.0	97
98	122.0	146
147	171.0	195
196	220.0	244

**Question #11**

The lowest value in the data set is 1, and the desired interval width is 15.

LL	MP	UL
0	7.0	14
15	22.0	29
30	37.0	44
45	52.0	59
60	67.0	74

**Question #8**

The lowest value in the data set is 22, and the desired interval width is 24.

LL	MP	UL
0	11.5	23
24	35.5	47
48	59.5	71
72	83.5	95
96	107.5	119

**Question #12**

The lowest value in the data set is 82, and the desired interval width is 4.

LL	MP	UL
80	81.5	83
84	85.5	87
88	89.5	91
92	93.5	95
96	97.5	99

**Question #9**

The lowest value in the data set is 3, and the desired interval width is 12.

LL	MP	UL
0	5.5	11
12	17.5	23
24	29.5	35
36	41.5	47
48	53.5	59

**Question #13**

The lowest value in the data set is 25, and the desired interval width is 36.

LL	MP	UL
0	17.5	35
36	53.5	71
72	89.5	107
108	125.5	143
144	161.5	179

**Question #10**

The lowest value in the data set is 31, and the desired interval width is 26.

LL	MP	UL
26	38.5	51
52	64.5	77
78	90.5	103
104	116.5	129
130	142.5	155

**Question #14**

The lowest value in the data set is 64, and the desired interval width is 13.

LL	MP	UL
52	58.0	64
65	71.0	77
78	84.0	90
91	97.0	103
104	110.0	116

## Week #3

- Why is the mean more sensitive to extreme scores than the median?
- Why might we prefer the median over the mean?
- What are we squaring in a sum of squares?
- What is the point of the squaring in sum of squares?
- In what sense is a variance a mean?
- When would variance be equal to zero?
- Why do we use standard deviation instead of variance?
- Why do we use  $n - 1$  instead of  $n$  in the denominator of variance?
- Why is the standard deviation so named?
- For the following data set, calculate the mode, median, mean, and standard deviation. Then, add 5 to every number in the data set and calculate again. Then, multiply every number in the data set by 2 and calculate again. What effect does adding/multiplying every score with the same number have, and why? Explain for each statistic.

2, 8, 1, 1, 3



## Median calculation

For each table, calculate the median using this formula:

$$Md = LL + W \left[ \frac{0.5(n) - cumF}{fm} \right]$$

$LL$  = (score of the row with the lowest CR freq.  $\geq 0.5$ ) - 0.5

$W$  = interval width (= 1 if the data are ungrouped)

$n$  = sample size (= the highest C freq.)

$cumF$  = CR freq. of the row below the one containing  $LL$

$fm$  = frequency of the row containing  $LL$

### Question #1

score	freq	cfreq	rfreq	crfreq
14	1	1	0.08	0.08
15	9	10	0.75	0.83
16	2	12	0.17	1.00

$$\text{Median} = 14.5 + 1 \left[ \frac{0.5(12) - 1}{9} \right] = 15.06$$

### Question #2

score	freq	cfreq	rfreq	crfreq
9	2	2	0.17	0.17
10	5	7	0.42	0.58
11	3	10	0.25	0.83
12	2	12	0.17	1.00

$$\text{Median} = 9.5 + 1 \left[ \frac{0.5(12) - 2}{5} \right] = 10.3$$

### Question #3

score	freq	cfreq	rfreq	crfreq
13	1	1	0.08	0.08
14	2	3	0.17	0.25
15	5	8	0.42	0.67
16	4	12	0.33	1.00

$$\text{Median} = 14.5 + 1 \left[ \frac{0.5(12) - 3}{5} \right] = 15.1$$

### Question #4

score	freq	cfreq	rfreq	crfreq
9	2	2	0.17	0.17
10	2	4	0.17	0.33
11	6	10	0.50	0.83
12	2	12	0.17	1.00

$$\text{Median} = 10.5 + 1 \left[ \frac{0.5(12) - 4}{6} \right] = 10.83$$

### Question #5

score	freq	cfreq	rfreq	crfreq
14	5	5	0.42	0.42
15	5	10	0.42	0.83
16	2	12	0.17	1.00

$$\text{Median} = 14.5 + 1 \left[ \frac{0.5(12) - 5}{5} \right] = 14.7$$

### Question #6

score	freq	cfreq	rfreq	crfreq
12	4	4	0.33	0.33
13	5	9	0.42	0.75
14	3	12	0.25	1.00

$$\text{Median} = 12.5 + 1 \left[ \frac{0.5(12) - 4}{5} \right] = 12.9$$

### Question #7

score	freq	cfreq	rfreq	crfreq
13	1	1	0.08	0.08
14	4	5	0.33	0.42
15	4	9	0.33	0.75
16	1	10	0.08	0.83
17	2	12	0.17	1.00

$$\text{Median} = 14.5 + 1 \left[ \frac{0.5(12) - 5}{4} \right] = 14.75$$

## Standard deviation calculation

$$SS = \Sigma(X_i - \bar{X})^2$$

$$df = n - 1$$

$$s^2 = \frac{SS}{df}$$

$$s = \sqrt{s}$$

### Question #1

4, 0, 4, 9, 3

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
4	0	0
0	-4	16
4	0	0
9	5	25
3	-1	1

$$\bar{X} = 4$$

$$SS = 42$$

$$df = 5 - 1 = 4$$

$$s^2 = 42/4 = 10.5$$

$$s = \sqrt{10.5} = 3.24$$

### Question #2

10, 3, 1, 2, 9

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
10	5	25
3	-2	4
1	-4	16
2	-3	9
9	4	16

$$\bar{X} = 5$$

$$SS = 70$$

$$df = 5 - 1 = 4$$

$$s^2 = 70/4 = 17.5$$

$$s = \sqrt{17.5} = 4.18$$

### Question #3

5, 5, 5, 6, 9

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
5	-1	1
5	-1	1
5	-1	1
6	0	0
9	3	9

$$\bar{X} = 6$$

$$SS = 12$$

$$df = 5 - 1 = 4$$

$$s^2 = 12/4 = 3$$

$$s = \sqrt{3} = 1.73$$

### Question #4

6, 0, 3, 3, 8

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
6	2	4
0	-4	16
3	-1	1
3	-1	1
8	4	16

$$\bar{X} = 4$$

$$SS = 38$$

$$df = 5 - 1 = 4$$

$$s^2 = 38/4 = 9.5$$

$$s = \sqrt{9.5} = 3.08$$

### Question #5

3, 5, 3, 6, 3

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
3	-1	1
5	1	1
3	-1	1
6	2	4
3	-1	1

$$\bar{X} = 4$$

$$SS = 8$$

$$df = 5 - 1 = 4$$

$$s^2 = 8/4 = 2$$

$$s = \sqrt{2} = 1.41$$

### Question #6

6, 9, 3, 4, 8

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
6	0	0
9	3	9
3	-3	9
4	-2	4
8	2	4

$$\bar{X} = 6$$

$$SS = 26$$

$$df = 5 - 1 = 4$$

$$s^2 = 26/4 = 6.5$$

$$s = \sqrt{6.5} = 2.55$$

## Week #4

- Why do we transform scores into standard scores?
- Draw these scores on a number line:

4, 6, 4, 3, 8

Then, subtract the mean from each score and draw the result on another number line. Then, divide those by the standard deviation and draw them on yet another number line. What does each step do?

- How does the height of the normal curve correspond to the number line below it?

## Z-scores

Calculate the area between the following z scores: -0.63 and 1.12, 0.18 and -0.04, -0.84 and -0.02, 1.6 and 0.94, 0.33 and 0.82, -0.82 and 0.59, 0.49 and 0.92, 0.74 and 0.78, 0.58 and 0.07, -0.31 and -1.99, 1.51 and 0.62, 0.39 and -0.06, -0.62 and -0.16, -2.21 and -1.47

$z$	Area above $z$	Area between mean and $z$
0.02	0.4920	0.0080
0.04	0.4840	0.0160
0.06	0.4761	0.0239
0.07	0.4721	0.0279
0.16	0.4364	0.0636
0.18	0.4286	0.0714
0.31	0.3783	0.1217
0.33	0.3707	0.1293
0.39	0.3483	0.1517
0.49	0.3121	0.1879
0.58	0.2810	0.2190
0.59	0.2776	0.2224
0.62	0.2676	0.2324
0.62	0.2676	0.2324
0.63	0.2643	0.2357
0.74	0.2296	0.2704
0.78	0.2177	0.2823
0.82	0.2061	0.2939
0.82	0.2061	0.2939
0.84	0.2005	0.2995
0.92	0.1788	0.3212
0.94	0.1736	0.3264
1.12	0.1314	0.3686
1.47	0.0708	0.4292
1.51	0.0655	0.4345
1.60	0.0548	0.4452
1.99	0.0233	0.4767
2.21	0.0136	0.4864

$z_1$	$z_2$	Area between $z_1$ and $z_2$
-0.63	1.12	0.60
0.18	-0.04	0.09
-0.84	-0.02	0.29
1.60	0.94	0.12
0.33	0.82	0.16
-0.82	0.59	0.52
0.49	0.92	0.13
0.74	0.78	0.01
0.58	0.07	0.19
-0.31	-1.99	0.35
1.51	0.62	0.20
0.39	-0.06	0.18
-0.62	-0.16	0.17
-2.21	-1.47	0.06

## Week #5

- What is the difference between standard error and standard deviation?
- Why is standard error so named?
- Why can't we just draw a sample from the population and estimate the parameter by the sample statistic?
- Give an example of a sampling distribution
- What is the difference between a sampling distribution and a sampling distribution of the mean?
- What is the difference between standard error and standard error of the mean?
- Imagine a bowl with five bingo chips in it, numbered 1 through 5. For this "population":
  - List all 25 possible samples of size 2 (sampling with replacement)
  - Calculate the mean for every sample
  - Count the frequency of each value of the mean
  - Draw the frequency distribution as a bar plot
  - Calculate the probability of drawing a sample with:
    - \* A mean  $\geq 4$
    - \* A mean  $\leq 2.5$
    - \* An error of at least 1.5

## Week #6

- Draw a null distribution and illustrate the relationship between the critical values and  $\alpha$
- Draw an alternate distribution and illustrate the relationship between the critical values,  $\beta$ , and power
- What is  $\sigma_{\bar{X}}$ ? What does it measure?
- How are  $\alpha$  and  $\beta$  each affected by change in the critical values, effect size,  $\sigma_{\bar{X}}$ ? Illustrate with a drawing
- Give an example of a research question where a nondirectional test would be appropriate, then modify it so that a directional test would be appropriate
- $H_0$  and  $H_1$  must be exhaustive (one must be true) and mutually exclusive (they can't both be true). Come up with some invalid hypotheses. Here's an example to get you started:
  - $H_0: \mu_1 = \mu_2$
  - $H_1: \mu_1 > \mu_2$

## Z-tests

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$z_{\text{crit}}$  = the  $z$  score with  $\alpha/2$  above it

$$z_{\text{obs}} = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}$$

$$CI_y = \bar{X} \pm (\sigma_{\bar{X}} \times z_y)$$

$z_y$  = the  $z$  score with  $(100 - y)/100$  above it

### Critical $z$ values

$z$	Area between mean and $z$	Area above $z$
1.645	0.45	0.05
1.96	0.475	0.025
2.576	0.495	0.005

### Question #1

Researchers draw a sample of 8 with a mean of 5.81. The population variance is known to be 21.44. Test  $H_0 : \mu = 4$  at an  $\alpha$  of 0.05, state your decision, and calculate a 95% confidence interval.

$$\sigma = \sqrt{21.44} = 4.63$$

$$\sigma_{\bar{X}} = 4.63/\sqrt{8} = 1.64$$

$$z_{\text{obs}} = (5.81 - 4)/1.64 = 1.1$$

$$z_{\text{crit}} = \pm 1.96$$

Fail to reject because  $1.96 > 1.1 > -1.96$

$$z_{95} = 1.96$$

$$CI_{95} = 5.81 \pm (1.64 \times 1.96) = [2.6, 9.02]$$

### Question #2

Researchers draw a sample of 8 with a mean of 2.91. The population variance is known to be 2.79. Test  $H_0 : \mu = 8$  at an  $\alpha$  of 0.01, state your decision, and calculate a 90% confidence interval.

$$\sigma = \sqrt{2.79} = 1.67$$

$$\sigma_{\bar{X}} = 1.67/\sqrt{8} = 0.59$$

$$z_{\text{obs}} = (2.91 - 8)/0.59 = -8.63$$

$$z_{\text{crit}} = \pm 2.58$$

Reject because  $-8.63 < -2.58$

$$z_{90} = 1.64$$

$$CI_{90} = 2.91 \pm (0.59 \times 1.64) = [1.94, 3.88]$$

### Question #3

Researchers draw a sample of 7 with a mean of 3.06. The population variance is known to be 5.34. Test  $H_0 : \mu = 9$  at an  $\alpha$  of 0.05, state your decision, and calculate a 99% confidence interval.

$$\sigma = \sqrt{5.34} = 2.31$$

$$\sigma_{\bar{X}} = 2.31/\sqrt{7} = 0.87$$

$$z_{\text{obs}} = (3.06 - 9)/0.87 = -6.83$$

$$z_{\text{crit}} = \pm 1.96$$

Reject because  $-6.83 < -1.96$

$$z_{99} = 2.58$$

$$CI_{99} = 3.06 \pm (0.87 \times 2.58) = [0.82, 5.3]$$

### Question #4

Researchers draw a sample of 6 with a mean of 6.82. The population variance is known to be 4.45. Test  $H_0 : \mu = 1$  at an  $\alpha$  of 0.01, state your decision, and calculate a 95% confidence interval.

$$\sigma = \sqrt{4.45} = 2.11$$

$$\sigma_{\bar{X}} = 2.11/\sqrt{6} = 0.86$$

$$z_{\text{obs}} = (6.82 - 1)/0.86 = 6.77$$

$$z_{\text{crit}} = \pm 2.58$$

Reject because  $6.77 > 2.58$

$$z_{95} = 1.96$$

$$CI_{95} = 6.82 \pm (0.86 \times 1.96) = [5.13, 8.51]$$

### Question #5

Researchers draw a sample of 10 with a mean of 3.04. The population variance is known to be 4.58. Test  $H_0 : \mu = 7$  at an  $\alpha$  of 0.1, state your decision, and calculate a 99% confidence interval.

$$\sigma = \sqrt{4.58} = 2.14$$

$$\sigma_{\bar{X}} = 2.14/\sqrt{10} = 0.68$$

$$z_{\text{obs}} = (3.04 - 7)/0.68 = -5.82$$

$$z_{\text{crit}} = \pm 1.64$$

Reject because  $-5.82 < -1.64$

$$z_{99} = 2.58$$

$$CI_{99} = 3.04 \pm (0.68 \times 2.58) = [1.29, 4.79]$$

**Question #6**

Researchers draw a sample of 6 with a mean of 7.31. The population variance is known to be 6.35. Test  $H_0 : \mu = 10$  at an  $\alpha$  of 0.1, state your decision, and calculate a 99% confidence interval.

$$\sigma = \sqrt{6.35} = 2.52$$

$$\sigma_{\bar{X}} = 2.52/\sqrt{6} = 1.03$$

$$z_{\text{obs}} = (7.31 - 10)/1.03 = -2.61$$

$$z_{\text{crit}} = \pm 1.64$$

Reject because  $-2.61 < -1.64$

$$z_{99} = 2.58$$

$$CI_{99} = 7.31 \pm (1.03 \times 2.58) = [4.65, 9.97]$$

**Question #7**

Researchers draw a sample of 5 with a mean of 9.2. The population variance is known to be 1.64. Test  $H_0 : \mu = 4$  at an  $\alpha$  of 0.05, state your decision, and calculate a 90% confidence interval.

$$\sigma = \sqrt{1.64} = 1.28$$

$$\sigma_{\bar{X}} = 1.28/\sqrt{5} = 0.57$$

$$z_{\text{obs}} = (9.2 - 4)/0.57 = 9.12$$

$$z_{\text{crit}} = \pm 1.96$$

Reject because  $9.12 > 1.96$

$$z_{90} = 1.64$$

$$CI_{90} = 9.2 \pm (0.57 \times 1.64) = [8.27, 10.13]$$

**Question #8**

Researchers draw a sample of 9 with a mean of 4.19. The population variance is known to be 13.03. Test  $H_0 : \mu = 3$  at an  $\alpha$  of 0.05, state your decision, and calculate a 99% confidence interval.

$$\sigma = \sqrt{13.03} = 3.61$$

$$\sigma_{\bar{X}} = 3.61/\sqrt{9} = 1.2$$

$$z_{\text{obs}} = (4.19 - 3)/1.2 = 0.99$$

$$z_{\text{crit}} = \pm 1.96$$

Fail to reject because  $1.96 > 0.99 > -1.96$

$$z_{99} = 2.58$$

$$CI_{99} = 4.19 \pm (1.2 \times 2.58) = [1.09, 7.29]$$

**Question #9**

Researchers draw a sample of 6 with a mean of 3.29. The population variance is known to be 3.46. Test  $H_0 : \mu = 1$  at an  $\alpha$  of 0.05, state your decision, and calculate a 95% confidence interval.

$$\sigma = \sqrt{3.46} = 1.86$$

$$\sigma_{\bar{X}} = 1.86/\sqrt{6} = 0.76$$

$$z_{\text{obs}} = (3.29 - 1)/0.76 = 3.01$$

$$z_{\text{crit}} = \pm 1.96$$

Reject because  $3.01 > 1.96$

$$z_{95} = 1.96$$

$$CI_{95} = 3.29 \pm (0.76 \times 1.96) = [1.8, 4.78]$$

**Question #10**

Researchers draw a sample of 7 with a mean of 4.13. The population variance is known to be 14.21. Test  $H_0 : \mu = 4$  at an  $\alpha$  of 0.01, state your decision, and calculate a 95% confidence interval.

$$\sigma = \sqrt{14.21} = 3.77$$

$$\sigma_{\bar{X}} = 3.77/\sqrt{7} = 1.42$$

$$z_{\text{obs}} = (4.13 - 4)/1.42 = 0.09$$

$$z_{\text{crit}} = \pm 2.58$$

Fail to reject because  $2.58 > 0.09 > -2.58$

$$z_{95} = 1.96$$

$$CI_{95} = 4.13 \pm (1.42 \times 1.96) = [1.35, 6.91]$$

**Question #11**

Researchers draw a sample of 8 with a mean of 2.77. The population variance is known to be 1.12. Test  $H_0 : \mu = 1$  at an  $\alpha$  of 0.1, state your decision, and calculate a 99% confidence interval.

$$\sigma = \sqrt{1.12} = 1.06$$

$$\sigma_{\bar{X}} = 1.06/\sqrt{8} = 0.37$$

$$z_{\text{obs}} = (2.77 - 1)/0.37 = 4.78$$

$$z_{\text{crit}} = \pm 1.64$$

Reject because  $4.78 > 1.64$

$$z_{99} = 2.58$$

$$CI_{99} = 2.77 \pm (0.37 \times 2.58) = [1.82, 3.72]$$



## Week #7

- When should we use a  $t$  test instead of a  $z$  test?
- What is the expected value of  $t$  under the null?
- What is homogeneity of variance? State it symbolically
- Why can't we calculate a  $t$  statistic with a sample of one?
- When might we employ a Welch correction?
- Give an example of a research question for which a one-sample  $t$ -test would be appropriate, and explain why it is appropriate

## One sample $t$ -tests

$$df = n - 1$$

$$s_{\bar{X}} = \frac{s}{\sqrt{n}}$$

$$t_{\text{obs}} = \frac{\bar{X} - \mu}{s_{\bar{X}}}$$

$$CI_y = \bar{X} \pm (s_{\bar{X}} \times t_y)$$

$$t_y = \text{the critical value for } \alpha = (100 - y)/100$$

### Critical $t$ values

$df$	$\alpha$			
	0.2	0.1	0.05	0.01
4	1.53	2.13	2.78	4.6
5	1.48	2.02	2.57	4.03
6	1.44	1.94	2.45	3.71

### Question #1

Researchers draw a sample of 6 with a mean of 4.91 and a standard deviation of 2.19. Test  $H_0 : \mu = 3$  at an  $\alpha$  of 0.05, state your decision, then calculate a 90% confidence interval.

$$s_{\bar{X}} = 2.19/\sqrt{6} = 0.89$$

$$t_{\text{obs}} = (4.91 - 3)/0.89 = 2.15$$

$$t_{\text{crit}} = \pm 2.57$$

Fail to reject because  $2.15 > 2.15 > -2.57$

$$t_{90} = 2.02$$

$$CI_{90} = 4.91 \pm (0.89 \times 2.02) = [3.11, 6.71]$$

### Question #2

Researchers draw a sample of 7 with a mean of 4.13 and a standard deviation of 2.31. Test  $H_0 : \mu = 7$  at an  $\alpha$  of 0.01, state your decision, then calculate a 90% confidence interval.

$$s_{\bar{X}} = 2.31/\sqrt{7} = 0.87$$

$$t_{\text{obs}} = (4.13 - 7)/0.87 = -3.3$$

$$t_{\text{crit}} = \pm 3.71$$

Fail to reject because  $3.71 > -3.3 > -3.71$

$$t_{90} = 1.94$$

$$CI_{90} = 4.13 \pm (0.87 \times 1.94) = [2.44, 5.82]$$

### Question #3

Researchers draw a sample of 7 with a mean of 3.11 and a standard deviation of 0.88. Test  $H_0 : \mu = 10$  at an  $\alpha$  of 0.05, state your decision, then calculate a 99% confidence interval.

$$s_{\bar{X}} = 0.88/\sqrt{7} = 0.33$$

$$t_{\text{obs}} = (3.11 - 10)/0.33 = -20.88$$

$$t_{\text{crit}} = \pm 2.45$$

Reject because  $-20.88 < -2.45$

$$t_{99} = 3.71$$

$$CI_{99} = 3.11 \pm (0.33 \times 3.71) = [1.89, 4.33]$$

### Question #4

Researchers draw a sample of 5 with a mean of 5.16 and a standard deviation of 2.22. Test  $H_0 : \mu = 10$  at an  $\alpha$  of 0.05, state your decision, then calculate a 95% confidence interval.

$$s_{\bar{X}} = 2.22/\sqrt{5} = 0.99$$

$$t_{\text{obs}} = (5.16 - 10)/0.99 = -4.89$$

$$t_{\text{crit}} = \pm 2.78$$

Reject because  $-4.89 < -2.78$

$$t_{95} = 2.78$$

$$CI_{95} = 5.16 \pm (0.99 \times 2.78) = [2.41, 7.91]$$

### Question #5

Researchers draw a sample of 7 with a mean of 5.89 and a standard deviation of 2.1. Test  $H_0 : \mu = 3$  at an  $\alpha$  of 0.1, state your decision, then calculate a 95% confidence interval.

$$s_{\bar{X}} = 2.1/\sqrt{7} = 0.79$$

$$t_{\text{obs}} = (5.89 - 3)/0.79 = 3.66$$

$$t_{\text{crit}} = \pm 1.94$$

Reject because  $3.66 > 1.94$

$$t_{95} = 2.45$$

$$CI_{95} = 5.89 \pm (0.79 \times 2.45) = [3.95, 7.83]$$

**Question #6**

Researchers draw a sample of 7 with a mean of 6.86 and a standard deviation of 2.01. Test  $H_0 : \mu = 8$  at an  $\alpha$  of 0.01, state your decision, then calculate a 99% confidence interval.

$$s_{\bar{X}} = 2.01/\sqrt{7} = 0.76$$

$$t_{\text{obs}} = (6.86 - 8)/0.76 = -1.5$$

$$t_{\text{crit}} = \pm 3.71$$

Fail to reject because  $3.71 > -1.5 > -3.71$

$$t_{99} = 3.71$$

$$CI_{99} = 6.86 \pm (0.76 \times 3.71) = [4.04, 9.68]$$

**Question #10**

Researchers draw a sample of 5 with a mean of 5.53 and a standard deviation of 1.42. Test  $H_0 : \mu = 6$  at an  $\alpha$  of 0.05, state your decision, then calculate a 95% confidence interval.

$$s_{\bar{X}} = 1.42/\sqrt{5} = 0.64$$

$$t_{\text{obs}} = (5.53 - 6)/0.64 = -0.73$$

$$t_{\text{crit}} = \pm 2.78$$

Fail to reject because  $2.78 > -0.73 > -2.78$

$$t_{95} = 2.78$$

$$CI_{95} = 5.53 \pm (0.64 \times 2.78) = [3.75, 7.31]$$

**Question #7**

Researchers draw a sample of 6 with a mean of 8.21 and a standard deviation of 1.12. Test  $H_0 : \mu = 4$  at an  $\alpha$  of 0.05, state your decision, then calculate a 90% confidence interval.

$$s_{\bar{X}} = 1.12/\sqrt{6} = 0.46$$

$$t_{\text{obs}} = (8.21 - 4)/0.46 = 9.15$$

$$t_{\text{crit}} = \pm 2.57$$

Reject because  $9.15 \geq 2.57$

$$t_{90} = 2.02$$

$$CI_{90} = 8.21 \pm (0.46 \times 2.02) = [7.28, 9.14]$$

**Question #8**

Researchers draw a sample of 5 with a mean of 5.77 and a standard deviation of 2.29. Test  $H_0 : \mu = 10$  at an  $\alpha$  of 0.1, state your decision, then calculate a 90% confidence interval.

$$s_{\bar{X}} = 2.29/\sqrt{5} = 1.02$$

$$t_{\text{obs}} = (5.77 - 10)/1.02 = -4.15$$

$$t_{\text{crit}} = \pm 2.13$$

Reject because  $-4.15 \leq -2.13$

$$t_{90} = 2.13$$

$$CI_{90} = 5.77 \pm (1.02 \times 2.13) = [3.6, 7.94]$$

**Question #9**

Researchers draw a sample of 5 with a mean of 3.28 and a standard deviation of 2.37. Test  $H_0 : \mu = 10$  at an  $\alpha$  of 0.05, state your decision, then calculate a 95% confidence interval.

$$s_{\bar{X}} = 2.37/\sqrt{5} = 1.06$$

$$t_{\text{obs}} = (3.28 - 10)/1.06 = -6.34$$

$$t_{\text{crit}} = \pm 2.78$$

Reject because  $-6.34 \leq -2.78$

$$t_{95} = 2.78$$

$$CI_{95} = 3.28 \pm (1.06 \times 2.78) = [0.33, 6.23]$$

## Week #8

- Give an example of a research question for which you would use an independent  $t$ -test, and explain why it is appropriate
- What is  $s_{\bar{X}_1 - \bar{X}_2}$  and what does it measure?
- What is the sampling distribution of mean differences?
- What is the reasoning behind using pooled variance?

## Independent $t$ -tests

$$df_i = n_i - 1$$

$$df_{\text{tot}} = df_1 + df_2$$

$$SS_i = s_i^2 \times df_i$$

$$t_{\text{obs}} = \frac{(\bar{X}_1 - \bar{X}_2)}{s_{(\bar{X}_1 - \bar{X}_2)}}$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$$

$$s_p^2 = \frac{SS_1 + SS_2}{df_{\text{tot}}}$$

$$CI_y = (\bar{X}_1 - \bar{X}_2) \pm s_{(\bar{X}_1 - \bar{X}_2)} \times t_y$$

$$t_y = \text{is the critical value for } \alpha = (100 - y)/100$$

### Critical $t$ values

$df$	$\alpha$		
	0.1	0.05	0.01
10	1.81	2.23	3.17
11	1.8	2.2	3.11
12	1.78	2.18	3.05

### Question #1

Researchers draw one sample of 6 with a mean of 4.91 and a standard deviation of 2.19, and another sample of 6 with a mean of 2.79 and a standard deviation of 0.79. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.05, state the error, then calculate a 99% confidence interval.

$$s_1^2 = 2.19^2 = 4.79$$

$$s_2^2 = 0.79^2 = 0.63$$

$$df_1 = 6 - 1 = 5$$

$$df_2 = 6 - 1 = 5$$

$$SS_1 = 4.79 \times 5 = 23.95$$

$$SS_2 = 0.63 \times 5 = 3.15$$

$$df_{\text{tot}} = 5 + 5 = 10$$

$$s_p^2 = (23.95 + 3.15)/10 = 2.71$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{(2.71/6) + (2.71/6)} = 0.95$$

$$t_{\text{obs}}(10) = (4.91 - 2.79)/0.95 = 2.23$$

$$t_{\text{crit}} = \pm 2.23$$

Reject because  $2.23 \geq 2.23$

$$t_{99} = 3.17$$

$$CI_{99} = (4.91 - 2.79) \pm (0.95 \times 3.17) = [-0.89, 5.13]$$

### Question #2

Researchers draw one sample of 7 with a mean of 4.13 and a standard deviation of 2.31, and another sample of 7 with a mean of 6.61 and a standard deviation of 2.06. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.01, state the error, then calculate a 99% confidence interval.

$$s_1^2 = 2.31^2 = 5.35$$

$$s_2^2 = 2.06^2 = 4.24$$

$$df_1 = 7 - 1 = 6$$

$$df_2 = 7 - 1 = 6$$

$$SS_1 = 5.35 \times 6 = 32.1$$

$$SS_2 = 4.24 \times 6 = 25.44$$

$$df_{\text{tot}} = 6 + 6 = 12$$

$$s_p^2 = (32.1 + 25.44)/12 = 4.79$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{(4.79/7) + (4.79/7)} = 1.17$$

$$t_{\text{obs}}(12) = (4.13 - 6.61)/1.17 = -2.12$$

$$t_{\text{crit}} = \pm 3.05$$

Fail to reject because  $3.05 > -2.12 > -3.05$

$$t_{99} = 3.05$$

$$CI_{99} = (4.13 - 6.61) \pm (1.17 \times 3.05) = [-6.05, 1.09]$$

### Question #3

Researchers draw one sample of 7 with a mean of 3.11 and a standard deviation of 0.88, and another sample of 7 with a mean of 8.1 and a standard deviation of 1.24. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.1, state the error, then calculate a 99% confidence interval.

$$s_1^2 = 0.88^2 = 0.78$$

$$s_2^2 = 1.24^2 = 1.53$$

$$df_1 = 7 - 1 = 6$$

$$df_2 = 7 - 1 = 6$$

$$SS_1 = 0.78 \times 6 = 4.68$$

$$SS_2 = 1.53 \times 6 = 9.18$$

$$df_{\text{tot}} = 6 + 6 = 12$$

$$s_p^2 = (4.68 + 9.18)/12 = 1.16$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{(1.16/7) + (1.16/7)} = 0.58$$

$$t_{\text{obs}}(12) = (3.11 - 8.1)/0.58 = -8.6$$

$$t_{\text{crit}} = \pm 1.78$$

Reject because  $-8.6 \leq -1.78$

$$t_{99} = 3.05$$

$$CI_{99} = (3.11 - 8.1) \pm (0.58 \times 3.05) = [-6.76, -3.22]$$

**Question #4**

Researchers draw one sample of 6 with a mean of 5.91 and a standard deviation of 2.7, and another sample of 6 with a mean of 6.44 and a standard deviation of 1.35. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.1, state the error, then calculate a 99% confidence interval.

$$\begin{aligned}
 s_1^2 &= 2.7^2 = 7.31 \\
 s_2^2 &= 1.35^2 = 1.81 \\
 df_1 &= 6 - 1 = 5 \\
 df_2 &= 6 - 1 = 5 \\
 SS_1 &= 7.31 \times 5 = 36.55 \\
 SS_2 &= 1.81 \times 5 = 9.05 \\
 df_{\text{tot}} &= 5 + 5 = 10 \\
 s_p^2 &= (36.55 + 9.05)/10 = 4.56 \\
 s_{(\bar{X}_1 - \bar{X}_2)} &= \sqrt{(4.56/6) + (4.56/6)} = 1.23 \\
 t_{\text{obs}}(10) &= (5.91 - 6.44)/1.23 = -0.43 \\
 t_{\text{crit}} &= \pm 1.81 \\
 \text{Fail to reject because } 1.81 &> -0.43 > -1.81 \\
 t_{99} &= 3.17 \\
 CI_{99} &= (5.91 - 6.44) \pm (1.23 \times 3.17) = [-4.43, 3.37]
 \end{aligned}$$

**Question #5**

Researchers draw one sample of 7 with a mean of 5.89 and a standard deviation of 2.1, and another sample of 6 with a mean of 3 and a standard deviation of 1.01. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.01, state the error, then calculate a 95% confidence interval.

$$\begin{aligned}
 s_1^2 &= 2.1^2 = 4.4 \\
 s_2^2 &= 1.01^2 = 1.03 \\
 df_1 &= 7 - 1 = 6 \\
 df_2 &= 6 - 1 = 5 \\
 SS_1 &= 4.4 \times 6 = 26.4 \\
 SS_2 &= 1.03 \times 5 = 5.15 \\
 df_{\text{tot}} &= 6 + 5 = 11 \\
 s_p^2 &= (26.4 + 5.15)/11 = 2.87 \\
 s_{(\bar{X}_1 - \bar{X}_2)} &= \sqrt{(2.87/7) + (2.87/6)} = 0.94 \\
 t_{\text{obs}}(11) &= (5.89 - 3)/0.94 = 3.07 \\
 t_{\text{crit}} &= \pm 3.11 \\
 \text{Fail to reject because } 3.11 &> 3.07 > -3.11 \\
 t_{95} &= 2.2 \\
 CI_{95} &= (5.89 - 3) \pm (0.94 \times 2.2) = [0.82, 4.96]
 \end{aligned}$$

**Question #6**

Researchers draw one sample of 7 with a mean of 6.86 and a standard deviation of 2.01, and another sample of 7 with a mean of 6.93 and a standard deviation of 1.7. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.01, state the error, then calculate a 99% confidence interval.

$$\begin{aligned}
 s_1^2 &= 2.01^2 = 4.04 \\
 s_2^2 &= 1.7^2 = 2.89 \\
 df_1 &= 7 - 1 = 6 \\
 df_2 &= 7 - 1 = 6 \\
 SS_1 &= 4.04 \times 6 = 24.24 \\
 SS_2 &= 2.89 \times 6 = 17.34 \\
 df_{\text{tot}} &= 6 + 6 = 12 \\
 s_p^2 &= (24.24 + 17.34)/12 = 3.46 \\
 s_{(\bar{X}_1 - \bar{X}_2)} &= \sqrt{(3.46/7) + (3.46/7)} = 0.99 \\
 t_{\text{obs}}(12) &= (6.86 - 6.93)/0.99 = -0.07 \\
 t_{\text{crit}} &= \pm 3.05 \\
 \text{Fail to reject because } 3.05 &> -0.07 > -3.05 \\
 t_{99} &= 3.05 \\
 CI_{99} &= (6.86 - 6.93) \pm (0.99 \times 3.05) = [-3.09, 2.95]
 \end{aligned}$$

**Question #7**

Researchers draw one sample of 6 with a mean of 8.21 and a standard deviation of 1.12, and another sample of 6 with a mean of 3.57 and a standard deviation of 1.81. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.01, state the error, then calculate a 95% confidence interval.

$$\begin{aligned}
 s_1^2 &= 1.12^2 = 1.25 \\
 s_2^2 &= 1.81^2 = 3.26 \\
 df_1 &= 6 - 1 = 5 \\
 df_2 &= 6 - 1 = 5 \\
 SS_1 &= 1.25 \times 5 = 6.25 \\
 SS_2 &= 3.26 \times 5 = 16.3 \\
 df_{\text{tot}} &= 5 + 5 = 10 \\
 s_p^2 &= (6.25 + 16.3)/10 = 2.25 \\
 s_{(\bar{X}_1 - \bar{X}_2)} &= \sqrt{(2.25/6) + (2.25/6)} = 0.87 \\
 t_{\text{obs}}(10) &= (8.21 - 3.57)/0.87 = 5.33 \\
 t_{\text{crit}} &= \pm 3.17 \\
 \text{Reject because } 5.33 &\geq 3.17 \\
 t_{95} &= 2.23 \\
 CI_{95} &= (8.21 - 3.57) \pm (0.87 \times 2.23) = [2.7, 6.58]
 \end{aligned}$$

**Question #8**

Researchers draw one sample of 6 with a mean of 6.37 and a standard deviation of 2.51, and another sample of 6 with a mean of 3.58 and a standard deviation of 1.52. Test  $H_0 : \mu_1 = \mu_2$  at an  $\alpha$  of 0.05, state the error, then calculate a 99% confidence interval.

$$s_1^2 = 2.51^2 = 6.32$$

$$s_2^2 = 1.52^2 = 2.3$$

$$df_1 = 6 - 1 = 5$$

$$df_2 = 6 - 1 = 5$$

$$SS_1 = 6.32 \times 5 = 31.6$$

$$SS_2 = 2.3 \times 5 = 11.5$$

$$df_{\text{tot}} = 5 + 5 = 10$$

$$s_p^2 = (31.6 + 11.5)/10 = 4.31$$

$$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{(4.31/6) + (4.31/6)} = 1.2$$

$$t_{\text{obs}}(10) = (6.37 - 3.58)/1.2 = 2.33$$

$$t_{\text{crit}} = \pm 2.23$$

Reject because  $2.33 \geq 2.23$

$$t_{99} = 3.17$$

$$CI_{99} = (6.37 - 3.58) \pm (1.2 \times 3.17) = [-1.01, 6.59]$$

## Week #9

- Give an example of a research design in which you would use a dependent  $t$ -test, and explain why it is appropriate
- How does using a dependent design rather than an independent design affect power? Why?
- Give an example of a research question which could be addressed by ANOVA but not a single  $t$ -test
- Define the numerator and denominator of the  $F$  statistic, then explain why  $F$  is equal to 1 under the null hypothesis
- How much within and between group variability is there in each of the following scenarios?
  - Scenario #1
    - \* Group 1: [1, 1, 1, 1, 1]
    - \* Group 2: [1, 1, 1, 1, 1]
    - \* Group 3: [1, 1, 1, 1, 1]
  - Scenario #2
    - \* Group 1: [1, 4, 3, 7, 9]
    - \* Group 2: [2, 3, 4, 8, 8]
    - \* Group 3: [1, 2, 5, 9, 7]
  - Scenario #3
    - \* Group 1: [1, 1, 1, 1, 1]
    - \* Group 2: [15, 15, 15, 15, 15]
    - \* Group 3: [30, 30, 30, 30, 30]
  - Scenario #4
    - \* Group 1: [1, 4, 3, 7, 9]
    - \* Group 2: [15, 26, 17, 29, 21]
    - \* Group 3: [30, 37, 43, 41, 32]

Rank them on the likelihood the null hypothesis ( $F = 1$ ) will be rejected



## Dependent $t$ -tests

$$df = n - 1$$

$$\bar{D} = \Sigma(D_i)/n$$

$$s_D = \sqrt{\Sigma[(D_i - \bar{D})^2]/df}$$

$$s_{\bar{D}} = s_D/\sqrt{n}$$

$$t_{\text{obs}} = \bar{D}/s_{\bar{D}}$$

$$CI_y = \bar{D} \pm s_{\bar{D}} \times t_y$$

$t_y$  is the critical value for  $\alpha = (100 - y)/100$

### Critical $t$ values

$\alpha$			
$df$	0.1	0.05	0.01
4	2.13	2.78	4.6
5	2.02	2.57	4.03
6	1.94	2.45	3.71

### Question #1

Researchers collect the following data: Test  $H_0 : \mu_{\bar{D}} = 0$  at

Pre	Post	$D_i$	$D_i - \bar{D}$	$(D_i - \bar{D})^2$
8.00	2.00	6.00	2.83	8.01
8.00	2.00	6.00	2.83	8.01
5.00	2.00	3.00	-0.17	0.03
3.00	4.00	-1.00	-4.17	17.39
4.00	4.00	0.00	-3.17	10.05
7.00	2.00	5.00	1.83	3.35

an  $\alpha$  of 0.05, state the decision/error, then calculate a 99% confidence interval.

$$\bar{D} = 3.17$$

$$\Sigma(D_i - \bar{D})^2 = 46.84$$

$$df = 5$$

$$s_D = \sqrt{46.84/5} = 3.06$$

$$s_{\bar{D}} = 3.06/\sqrt{6} = 1.25$$

$$t_{\text{obs}}(5) = 3.17/1.25 = 2.54$$

$$t_{\text{crit}} = 2.57$$

Fail to reject because  $2.57 > 2.54 > -2.57$

$$t_{99} = 4.03$$

$$CI_{99} = 3.17 \pm (1.25 \times 4.03) = [-1.87, 8.21]$$

### Question #2

Researchers collect the following data: Test  $H_0 : \mu_{\bar{D}} = 0$  at

Pre	Post	$D_i$	$D_i - \bar{D}$	$(D_i - \bar{D})^2$
5.00	4.00	1.00	-1.80	3.24
8.00	2.00	6.00	3.20	10.24
6.00	3.00	3.00	0.20	0.04
5.00	3.00	2.00	-0.80	0.64
6.00	4.00	2.00	-0.80	0.64

an  $\alpha$  of 0.01, state the decision/error, then calculate a 99% confidence interval.

$$\bar{D} = 2.8$$

$$\Sigma(D_i - \bar{D})^2 = 14.8$$

$$df = 4$$

$$s_D = \sqrt{14.8/4} = 1.92$$

$$s_{\bar{D}} = 1.92/\sqrt{5} = 0.86$$

$$t_{\text{obs}}(4) = 2.8/0.86 = 3.26$$

$$t_{\text{crit}} = 4.6$$

Fail to reject because  $4.6 > 3.26 > -4.6$

$$t_{99} = 4.6$$

$$CI_{99} = 2.8 \pm (0.86 \times 4.6) = [-1.16, 6.76]$$

### Question #3

Researchers collect the following data: Test  $H_0 : \mu_{\bar{D}} = 0$  at

Pre	Post	$D_i$	$D_i - \bar{D}$	$(D_i - \bar{D})^2$
8.00	9.00	-1.00	-4.00	16.00
10.00	7.00	3.00	0.00	0.00
8.00	4.00	4.00	1.00	1.00
10.00	2.00	8.00	5.00	25.00
9.00	8.00	1.00	-2.00	4.00

an  $\alpha$  of 0.01, state the decision/error, then calculate a 95% confidence interval.

$$\bar{D} = 3$$

$$\Sigma(D_i - \bar{D})^2 = 46$$

$$df = 4$$

$$s_D = \sqrt{46/4} = 3.39$$

$$s_{\bar{D}} = 3.39/\sqrt{5} = 1.52$$

$$t_{\text{obs}}(4) = 3/1.52 = 1.97$$

$$t_{\text{crit}} = 4.6$$

Fail to reject because  $4.6 > 1.97 > -4.6$

$$t_{95} = 2.78$$

$$CI_{95} = 3 \pm (1.52 \times 2.78) = [-1.23, 7.23]$$

**Question #4**

Researchers collect the following data: Test  $H_0 : \mu_{\bar{D}} = 0$  at

Pre	Post	$D_i$	$D_i - \bar{D}$	$(D_i - \bar{D})^2$
4.00	9.00	-5.00	-0.67	0.45
5.00	8.00	-3.00	1.33	1.77
1.00	10.00	-9.00	-4.67	21.81
3.00	10.00	-7.00	-2.67	7.13
7.00	4.00	3.00	7.33	53.73
3.00	8.00	-5.00	-0.67	0.45

an  $\alpha$  of 0.1, state the decision/error, then calculate a 99% confidence interval.

$$\bar{D} = -4.33$$

$$\Sigma(D_i - \bar{D})^2 = 85.34$$

$$df = 5$$

$$s_D = \sqrt{85.34/5} = 4.13$$

$$s_{\bar{D}} = 4.13/\sqrt{6} = 1.69$$

$$t_{\text{obs}}(5) = -4.33/1.69 = -2.56$$

$$t_{\text{crit}} = 2.02$$

Reject because  $-2.56 < -2.02$

$$t_{99} = 4.03$$

$$CI_{99} = -4.33 \pm (1.69 \times 4.03) = [-11.14, 2.48]$$

**Question #5**

Researchers collect the following data: Test  $H_0 : \mu_{\bar{D}} = 0$  at

Pre	Post	$D_i$	$D_i - \bar{D}$	$(D_i - \bar{D})^2$
4.00	5.00	-1.00	0.40	0.16
7.00	8.00	-1.00	0.40	0.16
8.00	8.00	0.00	1.40	1.96
2.00	7.00	-5.00	-3.60	12.96
5.00	5.00	0.00	1.40	1.96

an  $\alpha$  of 0.05, state the decision/error, then calculate a 95% confidence interval.

$$\bar{D} = -1.4$$

$$\Sigma(D_i - \bar{D})^2 = 17.2$$

$$df = 4$$

$$s_D = \sqrt{17.2/4} = 2.07$$

$$s_{\bar{D}} = 2.07/\sqrt{5} = 0.93$$

$$t_{\text{obs}}(4) = -1.4/0.93 = -1.51$$

$$t_{\text{crit}} = 2.78$$

Fail to reject because  $2.78 > -1.51 > -2.78$

$$t_{95} = 2.78$$

$$CI_{95} = -1.4 \pm (0.93 \times 2.78) = [-3.99, 1.19]$$

## Week #10

- Give an example of two variables which would have a correlation of close to 1
- Give an example of two variables which would have a correlation of close to -1
- Give an example of two variables which would have a correlation of close to 0
- Draw a relationship which cannot be accurately described by Pearson's  $r$
- Why doesn't correlation imply causation? Does causation imply correlation?
- Why does  $df = n - 2$ ?
- Rank each set of scores as though you were calculating Spearman's rho:

4, 0, 4, 9, 3  
3, 5, 3, 6, 3

## Pearson's $r$

$$\bar{X} = \Sigma(X_i)/n$$

$$df = n - 1$$

$$SP = \Sigma[(X_i - \bar{X})(Y_i - \bar{Y})]$$

$$SS_X = \Sigma[(X_i - \bar{X})^2]$$

$$SS_Y = \Sigma[(Y_i - \bar{Y})^2]$$

$$r_{XY} = SP / \sqrt{SS_X \times SS_Y}$$

## Critical $r$ values

$\alpha$			
$(n - 2)$	0.2	0.1	0.05
2	0.8	0.9	0.95
3	0.69	0.81	0.88
4	0.61	0.73	0.81

## Question #1

Calculate  $r_{XY}$  and test  $H_0: \rho_{XY} = 0$  at  $\alpha = 0.2$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
4	9	-0	2	0	6	-1
5	8	0	2	0	2	1
7	5	2	-2	6	2	-4
2	4	-2	-2	6	6	6

$$\bar{X} = 4.5$$

$$\bar{Y} = 6.5$$

$$SS_X = 13$$

$$SS_Y = 17$$

$$SP = 2$$

$$r_{XY} = 2 / \sqrt{13 \times 17} = 0.13$$

$$r_{\text{crit}} = \pm 0.8$$

Fail to reject because  $0.8 > 0.13 > -0.8$

## Question #2

Calculate  $r_{XY}$  and test  $H_0: \rho_{XY} = 0$  at  $\alpha = 0.1$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
7	9	2	4	4	16	8
5	2	0	-3	0	9	-0
2	6	-3	1	9	1	-3
6	3	1	-2	1	4	-2

$$\bar{X} = 5$$

$$\bar{Y} = 5$$

$$SS_X = 14$$

$$SS_Y = 30$$

$$SP = 3$$

$$r_{XY} = 3 / \sqrt{14 \times 30} = 0.15$$

$$r_{\text{crit}} = \pm 0.9$$

Fail to reject because  $0.9 > 0.15 > -0.9$

**Question #3**

Calculate  $r_{XY}$  and test  $H_0: \rho_{XY} = 0$  at  $\alpha = 0.1$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
8	6	2	2	4	6	5
4	1	-2	-2	4	6	5
3	3	-3	-0	9	0	2
9	4	3	0	9	0	2

$$\bar{X} = 6$$

$$\bar{Y} = 3.5$$

$$SS_X = 26$$

$$SS_Y = 13$$

$$SP = 13$$

$$r_{XY} = 13/\sqrt{26 \times 13} = 0.71$$

$$r_{\text{crit}} = \pm 0.9$$

Fail to reject because  $0.9 > 0.71 > -0.9$

**Question #4**

Calculate  $r_{XY}$  and test  $H_0: \rho_{XY} = 0$  at  $\alpha = 0.2$ . Error in ifelse(robs > rcrit, ">=", "<=") : object 'robs' not found

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
1	7	-3	1	7	1	-3
3	8	-1	2	0	5	-1
2	9	-2	3	3	10	-5
5	1	1	-5	2	23	-7
7	4	3	-2	12	3	-6

**Question #5**

Calculate  $r_{XY}$  and test  $H_0: \rho_{XY} = 0$  at  $\alpha = 0.2$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
7	7	2	0	6	0	1
8	5	4	-2	12	3	-6
2	6	-2	-1	6	1	2
1	9	-4	2	12	5	-8

$$\bar{X} = 4.5$$

$$\bar{Y} = 6.75$$

$$SS_X = 37$$

$$SS_Y = 8.75$$

$$SP = -11.5$$

$$r_{XY} = -11.5/\sqrt{37 \times 8.75} = -0.64$$

$$r_{\text{crit}} = \pm 0.8$$

Fail to reject because  $0.8 > -0.64 > -0.8$

**Question #6**

Calculate  $r_{XY}$  and test  $H_0: \rho_{XY} = 0$  at  $\alpha = 0.1$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
9	9	3	3	8	10	9
3	7	-3	1	10	1	-4
8	4	2	-2	3	3	-3
5	1	-1	-5	1	23	6
6	8	-0	2	0	5	-0

$$\bar{X} = 6.2$$

$$\bar{Y} = 5.8$$

$$SS_X = 22.8$$

$$SS_Y = 42.8$$

$$SP = 7.2$$

$$r_{XY} = 7.2 / \sqrt{22.8 \times 42.8} = 0.23$$

$$r_{\text{crit}} = \pm 0.81$$

Fail to reject because  $0.81 > 0.23 > -0.81$

**Question #7**

Calculate  $r_{XY}$  and test  $H_0: \rho_{XY} = 0$  at  $\alpha = 0.2$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
4	9	-1	3	1	9	-3
1	2	-4	-4	16	16	16
8	4	3	-2	9	4	-6
2	8	-3	2	9	4	-6
9	6	4	0	16	0	0
6	7	1	1	1	1	1

$$\bar{X} = 5$$

$$\bar{Y} = 6$$

$$SS_X = 52$$

$$SS_Y = 34$$

$$SP = 2$$

$$r_{XY} = 2 / \sqrt{52 \times 34} = 0.05$$

$$r_{\text{crit}} = \pm 0.61$$

Fail to reject because  $0.61 > 0.05 > -0.61$

**Question #8**

Calculate  $r_{XY}$  and test  $H_0: \rho_{XY} = 0$  at  $\alpha = 0.2$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
2	3	-3	-3	12	9	10
7	8	2	2	3	4	3
5	6	-0	0	0	0	-0
9	4	4	-2	13	4	-7
4	9	-1	3	2	9	-4

$$\bar{X} = 5.4$$

$$\bar{Y} = 6$$

$$SS_X = 29.2$$

$$SS_Y = 26$$

$$SP = 2$$

$$r_{XY} = 2 / \sqrt{29.2 \times 26} = 0.07$$

$$r_{\text{crit}} = \pm 0.69$$

Fail to reject because  $0.69 > 0.07 > -0.69$

**Question #9**

Calculate  $r_{XY}$  and test  $H_0: \rho_{XY} = 0$  at  $\alpha = 0.1$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
1	2	-2	-2	6	2	4
2	4	-2	0	2	0	-1
8	3	4	-0	20	0	-2
3	5	-0	2	0	2	-1

$$\bar{X} = 3.5$$

$$\bar{Y} = 3.5$$

$$SS_X = 29$$

$$SS_Y = 5$$

$$SP = 0$$

$$r_{XY} = 0/\sqrt{29 \times 5} = 0$$

$$r_{\text{crit}} = \pm 0.9$$

Fail to reject because  $0.9 > 0 > -0.9$

**Question #10**

Calculate  $r_{XY}$  and test  $H_0: \rho_{XY} = 0$  at  $\alpha = 0.1$ .

$X_i$	$Y_i$	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
3	3	0	-3	0	8	-0
4	9	1	3	1	10	3
5	5	2	-1	4	1	-2
1	8	-2	2	4	5	-4
2	4	-1	-2	1	3	2

$$\bar{X} = 3$$

$$\bar{Y} = 5.8$$

$$SS_X = 10$$

$$SS_Y = 26.8$$

$$SP = -1$$

$$r_{XY} = -1/\sqrt{10 \times 26.8} = -0.06$$

$$r_{\text{crit}} = \pm 0.81$$

Fail to reject because  $0.81 > -0.06 > -0.81$

## Week #11

- What does the regression line minimise? Draw a picture
- How many regression lines are possible for a given data set?
- Give an example of restriction of range
- Explain the difference between a univariate outlier and a regression outlier, and draw a picture
- Draw an example of how an influential outlier might affect the regression line



## Regression

$$\bar{Y} = \Sigma(Y_i)/n$$

$$df_1 = 1$$

$$df_2 = n - df_1 - 1$$

$$SP = \Sigma[(X_i - \bar{X})(Y_i - \bar{Y})]$$

$$SS_X = \Sigma[(X_i - \bar{X})^2]$$

$$\beta_1 = SP/SS_X$$

$$\beta_0 = \bar{Y} - \beta_1 \times \bar{X}$$

$$\hat{Y}_i = \beta_0 + X_i \times \beta_1$$

$$SS_{\text{tot}} = \Sigma[(Y_i - \bar{Y})^2]$$

$$SS_{\text{reg}} = \Sigma[(\hat{Y}_i - \bar{Y})^2]$$

$$SS_{\text{res}} = SS_{\text{tot}} - SS_{\text{reg}}$$

$$MS_{\text{reg}} = SS_{\text{reg}}/df_1$$

$$MS_{\text{res}} = SS_{\text{res}}/df_2$$

$$F = MS_{\text{reg}}/MS_{\text{res}}$$

## Critical $F$ values

		$df_1$			
$df_2$	$\alpha$	1	2	3	
1	0.05	161.4	199.5	215.71	
	0.01	4052	4999	5404	
2	0.05	18.51	19	19.16	
	0.01	98.94	99	99.17	
3	0.05	7.71	6.94	6.59	
	0.01	34.12	30.82	29.46	
4	0.05	7.71	6.94	6.59	
	0.01	21.2	18	16.69	

## Question #1

Test the model fit at an  $\alpha$  of 0.05.

$X_i$	$(X_i - \bar{X})^2$	$Y_i$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
4.00	0.25	9.00	6.25	-1.25	6.43	-0.07	0.00
5.00	0.25	8.00	2.25	0.75	6.58	0.08	0.01
7.00	6.25	5.00	2.25	-3.75	6.88	0.38	0.14
2.00	6.25	4.00	6.25	6.25	6.13	-0.37	0.14

$$SS_X = 13$$

$$SP = 2$$

$$\beta_1 = 2/13 = 0.15$$

$$\bar{Y} = 6.5$$

$$\bar{X} = 4.5$$

$$\beta_0 = 6.5 - (0.15 \times 4.5) = 5.83$$

$$\hat{Y}_i = 5.83 + (0.15 \times X_i)$$

$$SS_{\text{tot}} = 17$$

$$SS_{\text{reg}} = 0.29$$

$$SS_{\text{res}} = 17 - 0.29 = 16.71$$

$$df_1 = 1$$

$$df_2 = 4 - 1 - 1 = 2$$

$$MS_{\text{reg}} = 0.29/1 = 0.29$$

$$MS_{\text{res}} = 16.71/2 = 8.36$$

$$F = 0.29/8.36 = 0.03$$

$$F_{\text{crit}} = 18.51$$

Fail to reject because  $0.03 < 18.51$

**Question #2**

Test the model fit at an  $\alpha$  of 0.01.

$X_i$	$(X_i - \bar{X})^2$	$Y_i$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
7.00	4.00	9.00	16.00	8.00	5.42	0.42	0.18
5.00	0.00	2.00	9.00	0.00	5.00	0.00	0.00
2.00	9.00	6.00	1.00	-3.00	4.37	-0.63	0.40
6.00	1.00	3.00	4.00	-2.00	5.21	0.21	0.04

$$SS_X = 14$$

$$SP = 3$$

$$\beta_1 = 3/14 = 0.21$$

$$\bar{Y} = 5$$

$$\bar{X} = 5$$

$$\beta_0 = 5 - (0.21 \times 5) = 3.95$$

$$\hat{Y}_i = 3.95 + (0.21 \times X_i)$$

$$SS_{\text{tot}} = 30$$

$$SS_{\text{reg}} = 0.62$$

$$SS_{\text{res}} = 30 - 0.62 = 29.38$$

$$df_1 = 1$$

$$df_2 = 4 - 1 - 1 = 2$$

$$MS_{\text{reg}} = 0.62/1 = 0.62$$

$$MS_{\text{res}} = 29.38/2 = 14.69$$

$$F = 0.62/14.69 = 0.04$$

$$F_{\text{crit}} = 98.5$$

Fail to reject because  $0.04 < 98.5$

**Question #3**

Test the model fit at an  $\alpha$  of 0.01.

$X_i$	$(X_i - \bar{X})^2$	$Y_i$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
8.00	4.00	6.00	6.25	5.00	4.50	1.00	1.00
4.00	4.00	1.00	6.25	5.00	2.50	-1.00	1.00
3.00	9.00	3.00	0.25	1.50	2.00	-1.50	2.25
9.00	9.00	4.00	0.25	1.50	5.00	1.50	2.25

$$SS_X = 26$$

$$SP = 13$$

$$\beta_1 = 13/26 = 0.5$$

$$\bar{Y} = 3.5$$

$$\bar{X} = 6$$

$$\beta_0 = 3.5 - (0.5 \times 6) = 0.5$$

$$\hat{Y}_i = 0.5 + (0.5 \times X_i)$$

$$SS_{\text{tot}} = 13$$

$$SS_{\text{reg}} = 6.5$$

$$SS_{\text{res}} = 13 - 6.5 = 6.5$$

$$df_1 = 1$$

$$df_2 = 4 - 1 - 1 = 2$$

$$MS_{\text{reg}} = 6.5/1 = 6.5$$

$$MS_{\text{res}} = 6.5/2 = 3.25$$

$$F = 6.5/3.25 = 2$$

$$F_{\text{crit}} = 98.5$$

Fail to reject because  $2 < 98.5$

**Question #4**

Test the model fit at an  $\alpha$  of 0.01.

$X_i$	$(X_i - \bar{X})^2$	$Y_i$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
1.00	11.56	6.00	1.00	-3.40	4.69	-0.31	0.10
6.00	2.56	3.00	4.00	-3.20	5.14	0.14	0.02
5.00	0.36	5.00	0.00	0.00	5.05	0.05	0.00
2.00	5.76	4.00	1.00	2.40	4.78	-0.22	0.05
8.00	12.96	7.00	4.00	7.20	5.32	0.32	0.10

$$SS_X = 33.2$$

$$SP = 3$$

$$\beta_1 = 3/33.2 = 0.09$$

$$\bar{Y} = 5$$

$$\bar{X} = 4.4$$

$$\beta_0 = 5 - (0.09 \times 4.4) = 4.6$$

$$\hat{Y}_i = 4.6 + (0.09 \times X_i)$$

$$SS_{\text{tot}} = 10$$

$$SS_{\text{reg}} = 0.27$$

$$SS_{\text{res}} = 10 - 0.27 = 9.73$$

$$df_1 = 1$$

$$df_2 = 5 - 1 - 1 = 3$$

$$MS_{\text{reg}} = 0.27/1 = 0.27$$

$$MS_{\text{res}} = 9.73/3 = 3.24$$

$$F = 0.27/3.24 = 0.08$$

$$F_{\text{crit}} = 34.12$$

Fail to reject because  $0.08 < 34.12$

**Question #5**

Test the model fit at an  $\alpha$  of 0.05.

$X_i$	$(X_i - \bar{X})^2$	$Y_i$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
7.00	6.25	7.00	0.06	0.62	5.97	-0.78	0.61
8.00	12.25	5.00	3.06	-6.12	5.66	-1.09	1.19
2.00	6.25	6.00	0.56	1.88	7.52	0.77	0.59
1.00	12.25	9.00	5.06	-7.88	7.83	1.08	1.17

$$SS_X = 37$$

$$SP = -11.5$$

$$\beta_1 = -11.5/37 = -0.31$$

$$\bar{Y} = 6.75$$

$$\bar{X} = 4.5$$

$$\beta_0 = 6.75 - (-0.31 \times 4.5) = 8.14$$

$$\hat{Y}_i = 8.14 + (-0.31 \times X_i)$$

$$SS_{\text{tot}} = 8.75$$

$$SS_{\text{reg}} = 3.56$$

$$SS_{\text{res}} = 8.75 - 3.56 = 5.19$$

$$df_1 = 1$$

$$df_2 = 4 - 1 - 1 = 2$$

$$MS_{\text{reg}} = 3.56/1 = 3.56$$

$$MS_{\text{res}} = 5.19/2 = 2.6$$

$$F = 3.56/2.6 = 1.37$$

$$F_{\text{crit}} = 18.51$$

Fail to reject because  $1.37 < 18.51$

**Question #6**

Test the model fit at an  $\alpha$  of 0.01.

$X_i$	$(X_i - \bar{X})^2$	$Y_i$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
9.00	7.84	9.00	10.24	8.96	6.70	0.90	0.81
3.00	10.24	7.00	1.44	-3.84	4.78	-1.02	1.04
8.00	3.24	4.00	3.24	-3.24	6.38	0.58	0.34
5.00	1.44	1.00	23.04	5.76	5.42	-0.38	0.14
6.00	0.04	8.00	4.84	-0.44	5.74	-0.06	0.00

$$SS_X = 22.8$$

$$SP = 7.2$$

$$\beta_1 = 7.2/22.8 = 0.32$$

$$\bar{Y} = 5.8$$

$$\bar{X} = 6.2$$

$$\beta_0 = 5.8 - (0.32 \times 6.2) = 3.82$$

$$\hat{Y}_i = 3.82 + (0.32 \times X_i)$$

$$SS_{\text{tot}} = 42.8$$

$$SS_{\text{reg}} = 2.33$$

$$SS_{\text{res}} = 42.8 - 2.33 = 40.47$$

$$df_1 = 1$$

$$df_2 = 5 - 1 - 1 = 3$$

$$MS_{\text{reg}} = 2.33/1 = 2.33$$

$$MS_{\text{res}} = 40.47/3 = 13.49$$

$$F = 2.33/13.49 = 0.17$$

$$F_{\text{crit}} = 34.12$$

Fail to reject because  $0.17 < 34.12$

**Question #7**

Test the model fit at an  $\alpha$  of 0.05.

$X_i$	$(X_i - \bar{X})^2$	$Y_i$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
4.00	1.00	9.00	9.00	-3.00	5.96	-0.04	0.00
1.00	16.00	2.00	16.00	16.00	5.84	-0.16	0.03
8.00	9.00	4.00	4.00	-6.00	6.12	0.12	0.01
2.00	9.00	8.00	4.00	-6.00	5.88	-0.12	0.01
9.00	16.00	6.00	0.00	0.00	6.16	0.16	0.03
6.00	1.00	7.00	1.00	1.00	6.04	0.04	0.00

$$SS_X = 52$$

$$SP = 2$$

$$\beta_1 = 2/52 = 0.04$$

$$\bar{Y} = 6$$

$$\bar{X} = 5$$

$$\beta_0 = 6 - (0.04 \times 5) = 5.8$$

$$\hat{Y}_i = 5.8 + (0.04 \times X_i)$$

$$SS_{\text{tot}} = 34$$

$$SS_{\text{reg}} = 0.08$$

$$SS_{\text{res}} = 34 - 0.08 = 33.92$$

$$df_1 = 1$$

$$df_2 = 6 - 1 - 1 = 4$$

$$MS_{\text{reg}} = 0.08/1 = 0.08$$

$$MS_{\text{res}} = 33.92/4 = 8.48$$

$$F = 0.08/8.48 = 0.01$$

$$F_{\text{crit}} = 7.71$$

Fail to reject because  $0.01 < 7.71$

**Question #8**

Test the model fit at an  $\alpha$  of 0.05.

$X_i$	$(X_i - \bar{X})^2$	$Y_i$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
2.00	11.56	3.00	9.00	10.20	5.76	-0.24	0.06
7.00	2.56	8.00	4.00	3.20	6.11	0.11	0.01
5.00	0.16	6.00	0.00	0.00	5.97	-0.03	0.00
9.00	12.96	4.00	4.00	-7.20	6.25	0.25	0.06
4.00	1.96	9.00	9.00	-4.20	5.90	-0.10	0.01

$$SS_X = 29.2$$

$$SP = 2$$

$$\beta_1 = 2/29.2 = 0.07$$

$$\bar{Y} = 6$$

$$\bar{X} = 5.4$$

$$\beta_0 = 6 - (0.07 \times 5.4) = 5.62$$

$$\hat{Y}_i = 5.62 + (0.07 \times X_i)$$

$$SS_{\text{tot}} = 26$$

$$SS_{\text{reg}} = 0.14$$

$$SS_{\text{res}} = 26 - 0.14 = 25.86$$

$$df_1 = 1$$

$$df_2 = 5 - 1 - 1 = 3$$

$$MS_{\text{reg}} = 0.14/1 = 0.14$$

$$MS_{\text{res}} = 25.86/3 = 8.62$$

$$F = 0.14/8.62 = 0.02$$

$$F_{\text{crit}} = 10.13$$

Fail to reject because  $0.02 < 10.13$

**Question #9**

Test the model fit at an  $\alpha$  of 0.01.

$X_i$	$(X_i - \bar{X})^2$	$Y_i$	$(Y_i - \bar{Y})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\hat{Y}_i$	$(\hat{Y}_i - \bar{Y})$	$(\hat{Y}_i - \bar{Y})^2$
1.00	6.25	2.00	2.25	3.75	3.50	0.00	0.00
2.00	2.25	4.00	0.25	-0.75	3.50	0.00	0.00
8.00	20.25	3.00	0.25	-2.25	3.50	0.00	0.00
3.00	0.25	5.00	2.25	-0.75	3.50	0.00	0.00

$$SS_X = 29$$

$$SP = 0$$

$$\beta_1 = 0/29 = 0$$

$$\bar{Y} = 3.5$$

$$\bar{X} = 3.5$$

$$\beta_0 = 3.5 - (0 \times 3.5) = 3.5$$

$$\hat{Y}_i = 3.5 + (0 \times X_i)$$

$$SS_{\text{tot}} = 5$$

$$SS_{\text{reg}} = 0$$

$$SS_{\text{res}} = 5 - 0 = 5$$

$$df_1 = 1$$

$$df_2 = 4 - 1 - 1 = 2$$

$$MS_{\text{reg}} = 0/1 = 0$$

$$MS_{\text{res}} = 5/2 = 2.5$$

$$F = 0/2.5 = 0$$

$$F_{\text{crit}} = 98.5$$

Fail to reject because  $0 < 98.5$