

## Summing notation

$X_1$  = the first element of  $X$

$X_i$  = an arbitrary element of  $X$

$X_n$  = the last element of  $X$

$\Sigma(X_i) = X_1 + X_2 + \dots + X_n$

$\Sigma(X_i + 1) = (X_1 + 1) + (X_2 + 1) + \dots + (X_n + 1)$

$\Sigma(X_i - Y_i) = (X_1 - Y_1) + (X_2 - Y_2) + \dots + (X_n - Y_n)$

$[\Sigma(X_i)]^2 = (X_1 + X_2 + \dots + X_n)^2$

### Question #1

$X = 3\ 4\ 5\ 7\ 2$

$Y = 9\ 10\ 6\ 5\ 1$

$$\Sigma(X_i) = 21$$

$$[\Sigma(X_i)]^2 = 441$$

$$\Sigma(X_i - Y_i) = -10$$

$$\Sigma(X_i Y_i) = 134$$

$$\Sigma(X_i^2) = 103$$

$$[\Sigma(X_i - Y_i)]^2 = 100$$

$$[\Sigma(X_i)][\Sigma(Y_i)] = 651$$

### Question #4

$X = 6\ 1\ 3\ 2\ 5$

$Y = 3\ 7\ 8\ 9\ 1$

$$\Sigma(X_i) =$$

$$[\Sigma(X_i)]^2 =$$

$$\Sigma(X_i - Y_i) =$$

$$\Sigma(X_i Y_i) =$$

$$\Sigma(X_i^2) =$$

$$[\Sigma(X_i - Y_i)]^2 =$$

$$[\Sigma(X_i)][\Sigma(Y_i)] =$$

### Question #2

$X = 2\ 7\ 5\ 10\ 6$

$Y = 10\ 2\ 7\ 4\ 8$

$$\Sigma(X_i) =$$

$$[\Sigma(X_i)]^2 =$$

$$\Sigma(X_i - Y_i) =$$

$$\Sigma(X_i Y_i) =$$

$$\Sigma(X_i^2) =$$

$$[\Sigma(X_i - Y_i)]^2 =$$

$$[\Sigma(X_i)][\Sigma(Y_i)] =$$

### Question #5

$X = 3\ 7\ 8\ 2\ 1$

$Y = 8\ 5\ 7\ 10\ 1$

$$\Sigma(X_i) =$$

$$[\Sigma(X_i)]^2 =$$

$$\Sigma(X_i - Y_i) =$$

$$\Sigma(X_i Y_i) =$$

$$\Sigma(X_i^2) =$$

$$[\Sigma(X_i - Y_i)]^2 =$$

$$[\Sigma(X_i)][\Sigma(Y_i)] =$$

### Question #3

$X = 2\ 8\ 4\ 3\ 9$

$Y = 7\ 2\ 3\ 5\ 4$

$$\Sigma(X_i) =$$

$$[\Sigma(X_i)]^2 =$$

$$\Sigma(X_i - Y_i) =$$

$$\Sigma(X_i Y_i) =$$

$$\Sigma(X_i^2) =$$

$$[\Sigma(X_i - Y_i)]^2 =$$

$$[\Sigma(X_i)][\Sigma(Y_i)] =$$

### Question #6

$X = 7\ 9\ 3\ 8\ 5$

$Y = 10\ 9\ 7\ 4\ 1$

$$\Sigma(X_i) =$$

$$[\Sigma(X_i)]^2 =$$

$$\Sigma(X_i - Y_i) =$$

$$\Sigma(X_i Y_i) =$$

$$\Sigma(X_i^2) =$$

$$[\Sigma(X_i - Y_i)]^2 =$$

$$[\Sigma(X_i)][\Sigma(Y_i)] =$$

## Frequency table construction

For the following data sets, create a frequency table containing the frequency, cumulative frequency, relative frequency, and cumulative relative frequency of each value.

Freq. = number equal to score

C Freq. = number less than or equal to score

R Freq. = frequency divided by sample size

CR Freq. = cumulative frequency divided by n

### Question #1

-1, 0, -1, 2, 0, -1, 0, 1, 1, 0, 2, 0

score	freq	cfreq	rfreq	crfreq
-1	3	3	0.25	0.25
0	5	8	0.42	0.67
1	2	10	0.17	0.83
2	2	12	0.17	1.00

### Question #2

-1, 0, 2, -1, 0, 0, 1, 0, 2, 0, 0, 1

### Question #3

-1, 0, 0, -1, 0, 0, 0, 1, -1, 1, -1, -1

### Question #4

0, -1, 1, 1, 2, 1, -1, 0, 2, 2, 1, 0

### Question #5

-1, 1, -1, 0, 2, -1, 0, -1, 0, 0, 1, -1

### Question #6

0, -1, 1, 2, 0, 0, -1, 1, 0, -1, 2, -1

### Question #7

2, -1, -1, 0, -1, -1, 1, 0, 0, 2, 0, 3

### Question #8

0, 1, 0, -1, 1, 0, 0, -1, -3, -1, -1, 0

### Question #9

-1, -1, 0, 0, 0, -1, 1, 0, 0, 0, 1, 0

### Question #10

0, 0, -1, -1, 0, 0, -1, 0, -2, 0, 1, 1

### Question #11

-1, 0, -2, -1, 1, -1, 1, 1, 0, -1, -1, 0

### Question #12

-1, 2, -1, -1, -2, 0, 0, -1, 0, 0, -1, -1

### Question #13

1, 0, 2, 0, 1, 0, 1, 0, 0, 1, -1, 0

### Question #14

-1, 2, 2, 1, 0, 1, 0, 1, 0, 1, 0, 0

### Question #15

0, 2, 0, 1, 0, -1, 0, 1, 0, -1, 1, 0

### Question #16

0, 0, 1, -1, 1, 0, -1, 0, 1, 1, 2, 0

### Question #17

-1, 0, 0, -1, 1, 0, 1, 2, 0, 0, 1, 1

### Question #18

1, 2, -2, 0, 0, 0, -1, 2, 0, 2, -2, 1

### Question #19

-1, 0, 0, -1, 1, 0, 1, -1, 0, 0, 1, 1

## Interval construction

### Question #1

The lowest value in the data set is 38, and the desired interval width is 14. Find the lower limit, midpoint, and upper limit of the first five intervals.

LL	MP	UL
28	34.5	41
42	48.5	55
56	62.5	69
70	76.5	83
84	90.5	97

### Question #2

The lowest value in the data set is 71, and the desired interval width is 10. Find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #3

The lowest value in the data set is 81, and the desired interval width is 9. Find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #4

The lowest value in the data set is 2, and the desired interval width is 30. Find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #5

The lowest value in the data set is 69, and the desired interval width is 11. Find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #6

The lowest value in the data set is 94, and the desired interval width is 31. Find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #7

The lowest value in the data set is 40, and the desired interval width is 49. Find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #8

The lowest value in the data set is 22, and the desired interval width is 24. Find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #9

The lowest value in the data set is 3, and the desired interval width is 12. Find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #10

The lowest value in the data set is 31, and the desired interval width is 26. Find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #11

The lowest value in the data set is 1, and the desired interval width is 15. Find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #12

The lowest value in the data set is 82, and the desired interval width is 4. Find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #13

The lowest value in the data set is 25, and the desired interval width is 36. Find the lower limit, midpoint, and upper limit of the first five intervals.

### Question #14

The lowest value in the data set is 64, and the desired interval width is 13. Find the lower limit, midpoint, and upper limit of the first five intervals.

Median

For each table, calculate the median using this formula:

$$Md = LL + W \left[ \frac{0.5(n) - cumF}{fm} \right]$$

$LL$  = (score of the row with the lowest CR freq.  $\geq 0.5$ ) - 0.5  
 $W$  = interval width (= 1 if the data are ungrouped)  
 $n$  = sample size (= the highest C freq.)  
 $cumF$  = CR freq. of the row below the one containing  $LL$   
 $fm$  = frequency of the row containing  $LL$

Question #1

score	freq	cfreq	rfreq	crfreq
14	1	1	0.08	0.08
15	9	10	0.75	0.83
16	2	12	0.17	1.00

Median = 14.5 + 1  $\left[ \frac{0.5(12)-1}{9} \right]$  = 15.06

Question #2

score	freq	cfreq	rfreq	crfreq
9	2	2	0.17	0.17
10	5	7	0.42	0.58
11	3	10	0.25	0.83
12	2	12	0.17	1.00

Question #3

score	freq	cfreq	rfreq	crfreq
13	1	1	0.08	0.08
14	2	3	0.17	0.25
15	5	8	0.42	0.67
16	4	12	0.33	1.00

Question #4

score	freq	cfreq	rfreq	crfreq
9	2	2	0.17	0.17
10	2	4	0.17	0.33
11	6	10	0.50	0.83
12	2	12	0.17	1.00

Question #5

score	freq	cfreq	rfreq	crfreq
14	5	5	0.42	0.42
15	5	10	0.42	0.83
16	2	12	0.17	1.00

Question #6

score	freq	cfreq	rfreq	crfreq
12	4	4	0.33	0.33
13	5	9	0.42	0.75
14	3	12	0.25	1.00

Question #7

score	freq	cfreq	rfreq	crfreq
13	1	1	0.08	0.08
14	4	5	0.33	0.42
15	4	9	0.33	0.75
16	1	10	0.08	0.83
17	2	12	0.17	1.00

Standard deviation

$$SS = \Sigma(X_i - \bar{X})^2$$
$$df = n - 1$$
$$s^2 = \frac{SS}{df}$$
$$s = \sqrt{s}$$

Question #1

4, 0, 4, 9, 3

$X_i$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
4	0	0
0	-4	16
4	0	0
9	5	25
3	-1	1

$$\bar{X} = 4$$
$$SS = 42$$
$$df = 5 - 1 = 4$$

$$s^2 = 42/4 = 10.5$$
$$s = \sqrt{10.5} = 3.24$$

Question #2

10, 3, 1, 2, 9

Question #3

5, 5, 5, 6, 9

Question #4

6, 0, 3, 3, 8

Question #5

3, 5, 3, 6, 3

Question #6

6, 9, 3, 4, 8

## Z-scores

Calculate the area between the following z scores: -0.63 and 1.12, 0.18 and -0.04, -0.84 and -0.02, 1.6 and 0.94, 0.33 and 0.82, -0.82 and 0.59, 0.49 and 0.92, 0.74 and 0.78, 0.58 and 0.07, -0.31 and -1.99, 1.51 and 0.62, 0.39 and -0.06, -0.62 and -0.16, -2.21 and -1.47

$z$	Area above $z$	Area between mean and $z$
0.02	0.4920	0.0080
0.04	0.4840	0.0160
0.06	0.4761	0.0239
0.07	0.4721	0.0279
0.16	0.4364	0.0636
0.18	0.4286	0.0714
0.31	0.3783	0.1217
0.33	0.3707	0.1293
0.39	0.3483	0.1517
0.49	0.3121	0.1879
0.58	0.2810	0.2190
0.59	0.2776	0.2224
0.62	0.2676	0.2324
0.62	0.2676	0.2324
0.63	0.2643	0.2357
0.74	0.2296	0.2704
0.78	0.2177	0.2823
0.82	0.2061	0.2939
0.82	0.2061	0.2939
0.84	0.2005	0.2995
0.92	0.1788	0.3212
0.94	0.1736	0.3264
1.12	0.1314	0.3686
1.47	0.0708	0.4292
1.51	0.0655	0.4345
1.60	0.0548	0.4452
1.99	0.0233	0.4767
2.21	0.0136	0.4864

## Z-tests

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$z_{\text{crit}}$  = the  $z$  score with  $\alpha/2$  above it

$$z_{\text{obs}} = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}$$

$$CI_y = \bar{X} \pm (\sigma_{\bar{X}} \times z_y)$$

$z_y$  = the  $z$  score with  $(100 - y)/100$  above it

## Critical $z$ values

$z$	Area between mean and $z$	Area above $z$
1.645	0.45	0.05
1.96	0.475	0.025
2.576	0.495	0.005

### Question #1

Researchers draw a sample of 8 with a mean of 5.81. The population variance is known to be 21.44. Test  $H_0: \mu = 4$  at an  $\alpha$  of 0.05 state the error/decision, and calculate a 95% confidence interval. The population mean is actually 3.

$$z_{\text{crit}} = 1.96$$

$$\sigma = \sqrt{21.44} = 4.63$$

$$\sigma_{\bar{X}} = \frac{4.63}{\sqrt{8}} = 1.64$$

$$z_{\text{obs}} = \frac{5.81 - 4}{1.64} = 1.1$$

They make a false acceptance (Type II error)

$$CI_{95} = 5.81 \pm (1.64 \times 1.96) = [5.71, 5.91]$$

### Question #2

Researchers draw a sample of 8 with a mean of 2.91. The population variance is known to be 2.79. Test  $H_0: \mu = 8$  at an  $\alpha$  of 0.01 state the error/decision, and calculate a 90% confidence interval. The population mean is actually 2.

### Question #3

Researchers draw a sample of 7 with a mean of 3.06. The population variance is known to be 5.34. Test  $H_0: \mu = 9$  at an  $\alpha$  of 0.05 state the error/decision, and calculate a 99% confidence interval. The population mean is actually 2.

### Question #4

Researchers draw a sample of 6 with a mean of 6.82. The population variance is known to be 4.45. Test  $H_0: \mu = 1$  at an  $\alpha$  of 0.01 state the error/decision, and calculate a 95% confidence interval. The population mean is actually 6.

### Question #5

Researchers draw a sample of 10 with a mean of 3.04. The population variance is known to be 4.58. Test  $H_0: \mu = 7$  at an  $\alpha$  of 0.1 state the error/decision, and calculate a 99% confidence interval. The population mean is actually 3.

### Question #6

Researchers draw a sample of 6 with a mean of 7.31. The population variance is known to be 6.35. Test  $H_0: \mu = 10$  at an  $\alpha$  of 0.1 state the error/decision, and calculate a 99% confidence interval. The population mean is actually 7.

### Question #7

Researchers draw a sample of 5 with a mean of 9.2. The population variance is known to be 1.64. Test  $H_0: \mu = 4$  at an  $\alpha$  of 0.05 state the error/decision, and calculate a 90% confidence interval. The population mean is actually 10.

### Question #8

Researchers draw a sample of 9 with a mean of 4.19. The population variance is known to be 13.03. Test  $H_0: \mu = 3$  at an  $\alpha$  of 0.05 state the error/decision, and calculate a 99% confidence interval. The population mean is actually 5.

### Question #9

Researchers draw a sample of 6 with a mean of 3.29. The population variance is known to be 3.46. Test  $H_0: \mu = 1$  at an  $\alpha$  of 0.05 state the error/decision, and calculate a 95% confidence interval. The population mean is actually 3.

### Question #10

Researchers draw a sample of 7 with a mean of 4.13. The population variance is known to be 14.21. Test  $H_0: \mu = 4$  at an  $\alpha$  of 0.01 state the error/decision, and calculate a 95% confidence interval. The population mean is actually 6.

### Question #11

Researchers draw a sample of 8 with a mean of 2.77. The population variance is known to be 1.12. Test  $H_0: \mu = 1$  at an  $\alpha$  of 0.1 state the error/decision, and calculate a 99% confidence interval. The population mean is actually 3.