CSM 16A **Designing Information Systems and Devices**

Week 3 Worksheet

Term: **Spring 2020** *Name:*

Problem 1: Conceptual Checks

For each of the following statements, determine if they are **TRUE** or **FALSE**. If they are **FALSE**, try to come up with a counterexample; if they are **TRUE**, give a brief explanation.

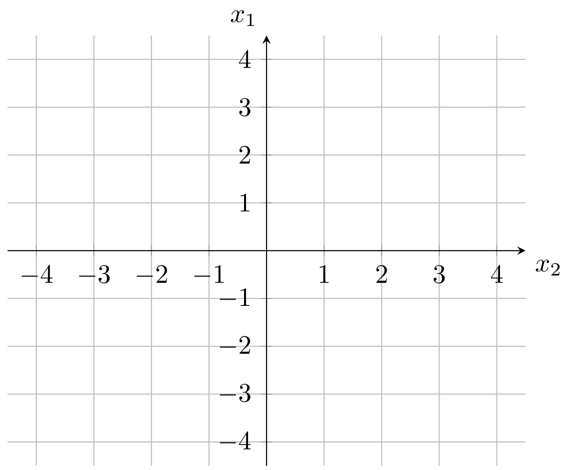
1. If the augmented matrix of the linear system represented by $A\vec{x} = \vec{b}$ has a pivot in the last column, then the matrix vector equation $A\vec{x} = \vec{b}$ has no solution.

2. If A is a 3×3 matrix such that the matrix vector equation $A\vec{x} = \vec{0}$ has only the trivial solution ($\vec{x} = \vec{0}$), then the matrix vector equation $A\vec{x} = \vec{b}$ is consistent for every vector $\vec{b}$ in $\mathbb{R}^3$

3. If the matrix vector equation $A\vec{x} = \vec{0}$ is true only when $\vec{x} = \vec{0}$, then the matrix A has an inverse (A is invertible).

4. A matrix A is called *symmetric* if it is equal to its transpose: $A = A^T$. If A is an invertible and *symmetric* matrix, A^{-1} must also be *symmetric*.

Problem 2: Range Intuition



$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

1. Draw the space on the figure above that is represented by the span of all the column vectors in A. Also draw the space covered by the span of all the row vectors in A. What dimension are these spaces?

2. Consider some arbitrary vector $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$. Write out the product $\mathbf{A}\vec{v}$ in terms of v_1, v_2, and the columns of $\mathbf{A}$.

3. We have talked about how matrices like $\mathbf{A}$ have no inverse. Give a geometric explanation for why this is the case.

4. Consider all points $\vec{y}$ such that $\mathbf{A}\vec{y} = 0$ Draw the space that the $\vec{y}$'s will make up. What do you notice geometrically? What is the dimension of this space?

Problem 3: More on Linear Transformation

1. Consider a matrix $\mathbf{S}$ that transforms a vector $\vec{x} = \begin{bmatrix} a \\ b \end{bmatrix}$ to $\vec{y} = \begin{bmatrix} a+b \\ a-b \end{bmatrix}$. Note that a, b can take on any values in $\mathbb{R}$. In other words, $\mathbf{S}\vec{x} = \vec{y}$. Is this transformation linear?

2. What is the matrix $\mathbf{S}$? Is the matrix invertible? Is the transformation invertible?

3. Consider a matrix $\mathbf{S}$ that transforms a vector $\vec{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ to $\vec{y} = \begin{bmatrix} a-b-c \\ a-b-c \\ a-b+c \end{bmatrix}$. Note that a, b, c can take on any values in $\mathbb{R}$. In other words, $\mathbf{S}\vec{x} = \vec{y}$. Is this transformation linear?

4. Write out the matrix **S** from part 3. Is it invertible? Combining with what you saw in the previous part, what can you say about the relationship between whether a matrix is invertible and whether the matrix transformation is a linear transformation?

Problem 4: Subsets v.s. Subspaces

Learning Goal: Prereqs: What are vector spaces and subspaces?

Description: Explains how to read set notation, tries to make students really realize that the notation means a set of vectors, and that a subspace is also a set of vectors. And what a subspace intuitively means.

1. Consider the set $W = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, a_1, a_2, a_3 \in \mathbb{R} : a_1 + 2a_2 - 3a_3 = 0 \right\}$. Is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ an element of the set W?

2. Write any 3 elements from this set.

3. Is the set W a subspace?

4. How can we now quickly find more elements of this set?

5. Consider the set $X = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, a_1, a_2, a_3 \in \mathbb{R} : a_1 * a_2 * a_3 = 0 \right\}$. Is X? is subspace?

Problem 5: Null Spaces and Transformations

Assume that the vector $\vec{x} = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix}$. For each of the following matrices $\mathbf{A} \in \mathbb{R}^{n \times m}$, answer the following:

- Compute the matrix product $\mathbf{A}\vec{x}$. Explain in words how the matrix transforms the vector.
- Suppose you know that A transforms $\vec{x}$ to give $\vec{y}$. Given $\vec{y}$, can you find what the original vector $\vec{x}$ was?
- Is the matrix $\mathbf{A}$ invertible? How do you know? If it is invertible, find the inverse.
- Verify that (dimension of nullspace) + (dimension of column space) = # of columns.

(a). $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \vec{y} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

(b). $\mathbf{A} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}, y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c). $\mathbf{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$, $\vec{y} = \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}$

(d). $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $\vec{y} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$