

# Week 5 Worksheet Metas

Term: Spring 2020

Name:

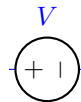
## Problem 1: Introduction to Circuit Components

**Meta:** Introduction to basic circuit components. Mentors: do a mini lecture on what is charge, what is voltage, and what is current. See lecture notes for definitions.

In this problem, we will introduce the fundamental circuit components.

1. What is a voltage source?

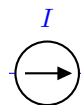
**Solution:** Firstly, a voltage source is represented in this manner:



A voltage source **guarantees** that the potential at its positive end will be  $V$  more than the potential at its negative end, no matter what.

2. What is a current source?

**Solution:** A current source is represented in this manner:



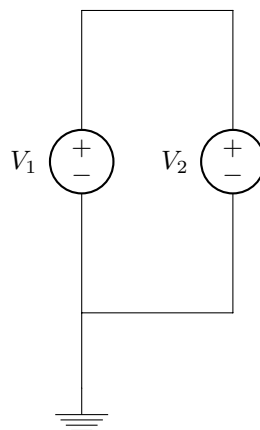
A current source **guarantees** that the current passing through the unit in the direction of the arrow will be its designated value.

3. What is voltage? What is a voltage drop?

**Solution:** For our discussion, it suffices to think of voltage as a kind of driver for current. Current is the movement of charges. A voltage difference forces current to move from the point (node) that has higher voltage, to the point that has lower voltage.

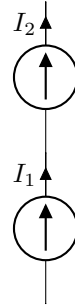
Voltage drop is the voltage lost (decline of nodal voltage) across a circuit component.

4. Consider the figure below. If  $V_1 \neq V_2$ , what will happen to the circuit?



**Solution:** Let us designate the potential at the positive end of  $V_1$  to be  $V_1^+$ , the potential at the negative end of  $V_1$  to be  $V_1^-$ , the potential at the positive end of  $V_2$  to be  $V_2^+$ , and the potential at the negative end of  $V_2$  to be  $V_2^-$ .  $V_1^-$  and  $V_2^-$  are equal to 0 because of the ground. Then, the potential across  $V_1$  is  $V_1^+$ , and the potential across  $V_2$  is  $V_2^+$ . Since  $V_1^+$  and  $V_2^+$  are connected by a wire, they must be the same voltage; we know that a wire does not affect a circuit's behavior, so the voltage must stay constant across it. This means that  $V_1^+ = V_2^+$ . However, we know that the voltage potential  $V_1^+ - V_1^-$  is not equal to  $V_2^+ - V_1^-$  as given in the question. Hence, we see that we cannot have two voltage sources connected in this configuration.

5. What happens in this case if  $I_1 \neq I_2$ ?



**Solution:** The current source at the bottom guarantees that through that wire there will be  $I_1$  current going through, and the current source at the top guaranteed that  $I_2$  current goes through that wire. This is a contradiction, and is not theoretically possible in a circuit.

Also, look at the point in between the two current sources.  $I_1$  enters on one end, and  $I_2$  leaves on the other end. This is impossible.

6. What is a resistor?

**Solution:** A resistor is represented in this manner:



A resistor is a circuit unit designed to ‘resist’ the flow of current. Following convention, there is a “voltage drop” across a resistor from the positive end to the negative end. The voltage drop across a resistor is  $V_R = I_R R$ , where  $V_R$  is the voltage drop,  $I_R$  is the current through the resistor and  $R$  is the resistance of the resistor.

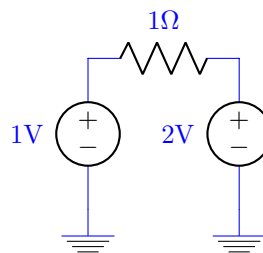
7. What is power?

**Solution:** Power is the rate at which work is done, where work is in terms of electrical energy.

For circuits, the power *consumed* or *dissipated* by a device is  $P = IV$ , where the current and voltage abide by passive sign convention.

**Common Misconceptions:**

- Active components do not necessarily dissipate negative power! Consider the following circuit:

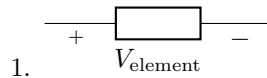


When calculating the power dissipated by the LHS voltage source, we see the current flows counterclockwise about the circuit. With passive sign convention, we calculate the power  $P_{V_1} = 1V \times 1A$ , which is positive! The left-side voltage source is dissipating power.

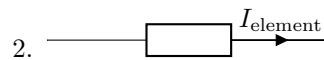
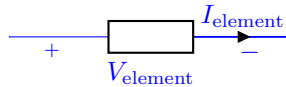
**Problem 2: Passive Sign Convention**

For the following components, label all the missing  $V_{\text{element}}$ ,  $I_{\text{element}}$ , and  $+/-$  signs. *Hint: The value of the voltage and current sources shouldn't affect passive sign convention—remember that voltage and current can be negative!*

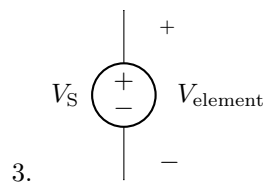
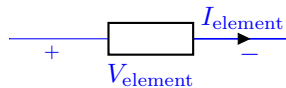
**Meta:** For parts (a) and (b), make sure to clarify to your students that the box figure can represent any arbitrary circuit element (a resistor, a voltage source, etc.).



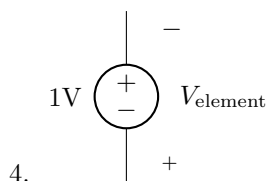
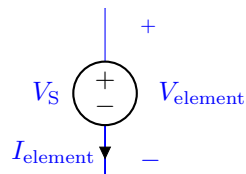
**Solution:**



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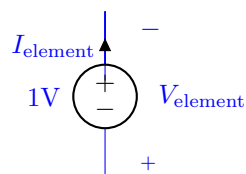


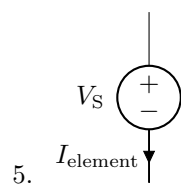
**Solution:**



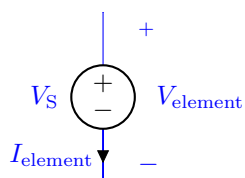
**Meta:** If students are confused by this answer, draw the source on the board, then a box around it. Shade in the box so the voltage source is obscured, and now the problem is identical to having a black box voltage source. This particular style of question has featured on several exams, and it's good to have lots of practice with this when calculating power.

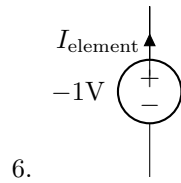
**Solution:**



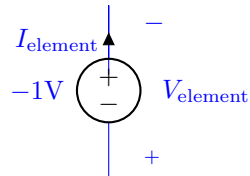


**Solution:**

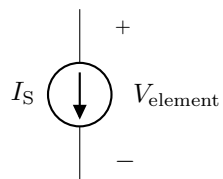




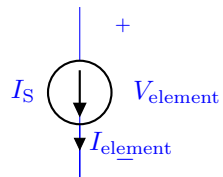
**Solution:**



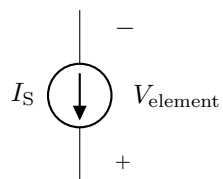
7. (PRACTICE)



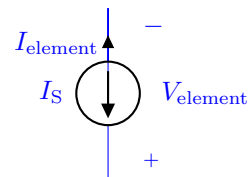
**Solution:**



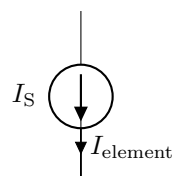
8. (PRACTICE)



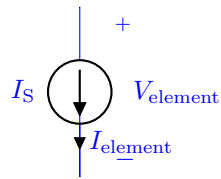
**Solution:**



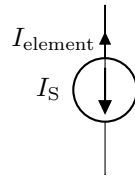
9. (PRACTICE)



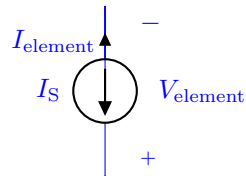
**Solution:**



10. (PRACTICE)



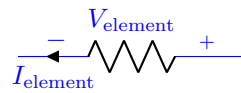
**Solution:**



11. (PRACTICE)



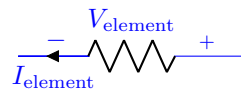
**Solution:**



12. (PRACTICE)

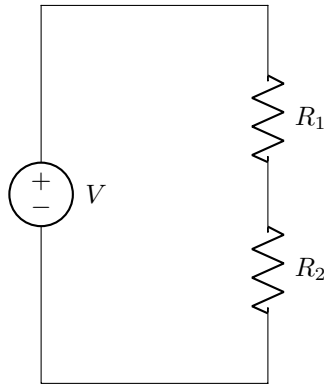


**Solution:**



**Problem 3: Voltage Divider Properties**

Let's take a systematic look at the voltages across a resistor, and see how other components in the circuit can affect it. Consider the following circuit:



1. Calculate the voltage drop across  $R_1$  and  $R_2$  using series resistance calculations.

**Solution:** The current out of the voltage source is given by Ohm's law:

$$i = \frac{V}{R_{eq}}$$

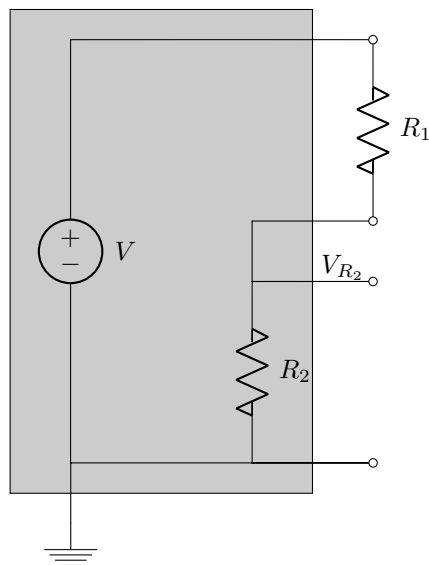
$$i = \frac{V}{R_1 + R_2}$$

Again, by Ohm's law, we have that

$$V_{R_1} = iR_1 = V \frac{R_1}{R_1 + R_2}$$

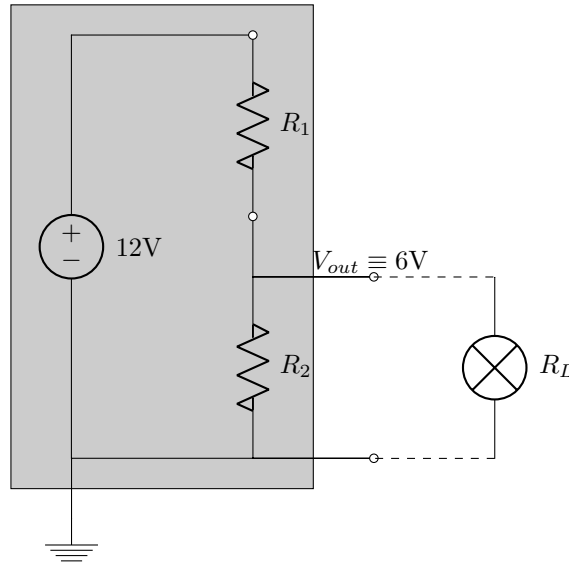
$$V_{R_2} = iR_2 = V \frac{R_2}{R_1 + R_2}$$

2. Suppose we want to manipulate the voltage across  $R_2$ , but it's locked in a box with the voltage source, as denoted below. Can we use  $R_1$  to manipulate  $V_{R_2}$ ? What range of voltages can we achieve?



**Solution:** Any voltage in the range  $(0, V]$ ! Notice from the equations above that  $V_{R_2} = V \frac{R_2}{R_{Total}}$ . If we increase  $R_1$  indefinitely, holding  $R_2$  constant, we can make the fraction arbitrarily small. Intuitively, since the same current flows through both  $R_1$  and  $R_2$ , they have to split the total voltage of the power source, and larger resistances correspond to larger voltage drops (by Ohm's Law). If we decrease  $R_1$  to 0,  $V_{R_2} = V$ , so the voltage can be at most whatever is supplied by the power source. That the voltage source limits the achievable voltage in the circuit is a concept we will see again when we cover clipping in op-amps.

3. Now let's try using our new variable voltage source to power a light bulb with resistance  $R_L$ , where the threshold voltage for lighting the bulb is 6V. Find  $R_1$  and  $R_2$  so that the voltage across  $R_2$  is this threshold voltage; that is,  $V_{R_2} \equiv V_{out} = 6V$ . Assume we have a 12V voltage source.



**Solution:** We want to split the voltage in half (from 12 to 6). Based on the voltage divider formula above, that means  $R_1 = R_2 \equiv R$ . Note that, under this condition, the voltage is evenly split regardless of what the actual resistance values are! While the current depends on actual resistance values, the voltage only depends on the ratio of resistances.

4. Now that we found an  $R_1$  and  $R_2$  that seem to divide our voltage source appropriately, let's try to connect the bulb to the ends of  $R_2$ . Remember, the bulb has a resistance  $R_L$ . Calculate the voltage across  $R_1$ ,  $R_2$  and the light bulb when it is connected. Will the light bulb turn on?

**Solution:** Let's reapply the voltage divider formula, but now notice that the "second" resistor is  $R_2 \parallel R_L$ ! When we change the resistance in a voltage divider circuit, the voltage across that resistance changes as well. We find that  $V_{R_1} = V \frac{R}{R + R \parallel R_L} = 12V \frac{R}{R + \frac{R R_L}{R + R_L}} = 12V \frac{R + R_L}{R + 2R_L}$ . The rest of the potential drop must be across the  $R_2$ - $R_L$  system, so we have  $V_{R_2} = V_{bulb} = 12V \left(1 - \frac{R + R_L}{R + 2R_L}\right) = 12V \frac{R_L}{R + 2R_L} \leq 12V \frac{R_L}{2R_L} = 6V$ .

So the voltage *decreased*, and the light bulb doesn't turn on.

The takeaway from this is that, while it may seem like the voltage divider can make voltage sources of arbitrary voltages, the act of connecting a component actually changes the output of the voltage divider. We will later learn about a way to stop a light bulb (or other devices that use up power) from "affecting" the circuit that supplies power by placing a "buffer" between the two.