CSM 16A — Designing Information Systems and Devices

Week 5 Worksheet Metas

Term: **Spring 2020** — *Name:*

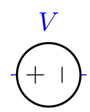
Problem 1: Introduction to Circuit Components

Meta: Introduction to basic circuit components. Mentors: do a mini lecture on what is charge, what is voltage, and what is current. See lecture notes for definitions.

In this problem, we will introduce the fundamental circuit components.

1. What is a voltage source?

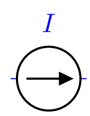
   Solution: Firstly, a voltage source is represented in this manner:

   A voltage source **guarantees** that the potential at its positive end will be V more than the potential at its negative end, no matter what.

2. What is a current source?

   Solution: A current source is represented in this manner:

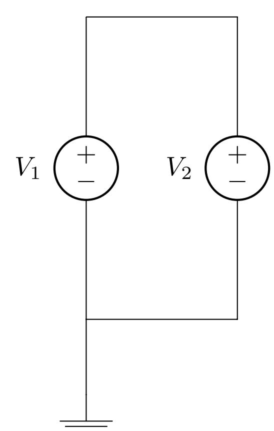
   A current source **guarantees** that the current passing through the unit in the direction of the arrow will be its designated value.

3. What is voltage? What is a voltage drop?

   Solution: For our discussion, it suffices to think of voltage as a kind of driver for current. Current is the movement of charges. A voltage difference forces current to move from the point (node) that has higher voltage, to the point that has lower voltage.

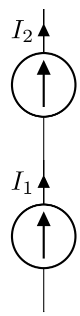
   Voltage drop is the voltage lost (decline of nodal voltage) across a circuit component.

4. Consider the figure below. If $V_1 \neq V_2$, what will happen to the circuit?

Solution: Let us designate the potential at the positive end of V_1 to be V_1^+, the potential at the negative end of V_1 to be V_1^-, the potential at the positive end of V_2 to be V_2^+, and the potential at the negative end of V_2 to be V_2^-. V_1^- and V_2^- are equal to 0 because of the ground. Then, the potential across V_1 is V_1^+, and the potential across V_2 is V_2^+. Since V_1^+ and V_2^+ are connected by a wire, they must be the same voltage; we know that a wire does not affect a circuit's behavior, so the voltage must stay constant across it. This means that $V_1^+ = V_2^+$. However, we know that the voltage potential $V_1^+ - V_1^-$ is not equal to $V_2^+ - V_1^-$ as given in the question. Hence, we see that we cannot have two voltage sources connected in this configuration.

5. What happens in this case if $I_1 \neq I_2$?



Solution: The current source at the bottom guarantees that through that wire there will be I_1 current going through, and the current source at the top guaranteed that I_2 current goes through that wire. This is a contradiction, and is not theoretically possible in a circuit.

Also, look at the point in between the two current sources. I_1 enters on one end, and I_2 leaves on the other end. This is impossible.

6. What is a resistor?

Solution: A resistor is represented in this manner:

A resistor is a circuit unit designed to 'resist' the flow of current. Following convention, there is a "voltage drop" across a resistor from the positive end to the negative end. The voltage drop across a resistor is $V_R = I_R R$, where V_R is the voltage drop, I_R is the current through the resistor and R is the resistance of the resistor.

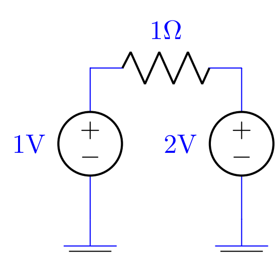
7. What is power?

Solution: Power is the rate at which work is done, where work is in terms of electrical energy.

For circuits, the power *consumed* or *dissipated* by a device is $P = IV$, where the current and voltage abide by passive sign convention.

Common Misconceptions:

- Active components do not necessarily dissipate negative power! Consider the following circuit:



When calculating the power dissipated by the LHS voltage source, we see the current flows counterclockwise about the circuit. With passive sign convention, we calculate the power $P_{V_1} = 1\text{V} \times 1\text{A}$, which is positive! The left-side voltage source is dissipating power.

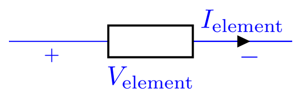
Problem 2: Passive Sign Convention

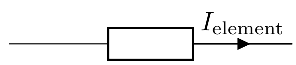
For the following components, label all the missing V_{element}, I_{element}, and +/- signs. *Hint: The value of the voltage and current sources shouldn't affect passive sign convention—remember that voltage and current can be negative!*

Meta: For parts (a) and (b), make sure to clarify to your students that the box figure can represent any arbitrary circuit element (a resistor, a voltage source, etc.).

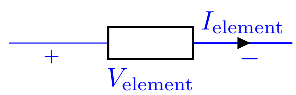
1. 

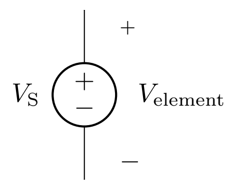
Solution:



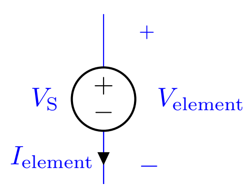
2. 

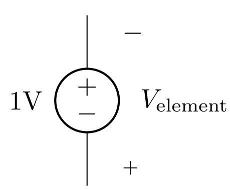
Solution:



3. 

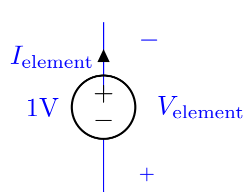
Solution:

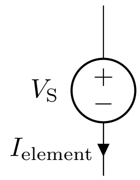


4. 

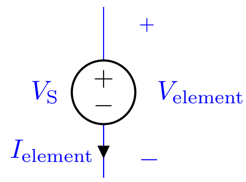
Meta: If students are confused by this answer, draw the source on the board, then a box around it. Shade in the box so the voltage source is obscured, and now the problem is identical to having a black box voltage source. This particular style of question has featured on several exams, and it's good to have lots of practice with this when calculating power.

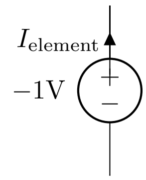
Solution:



5. 

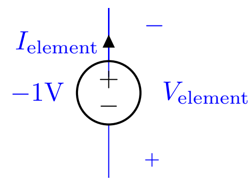
Solution:



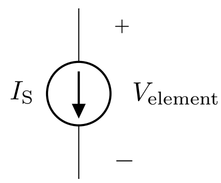


6.

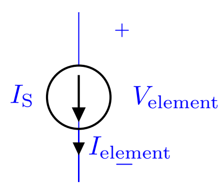
Solution:



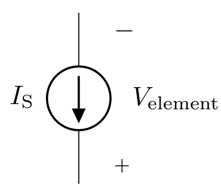
7. **(PRACTICE)**



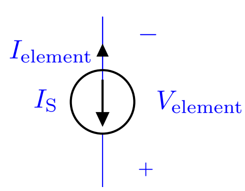
Solution:



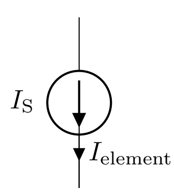
8. **(PRACTICE)**



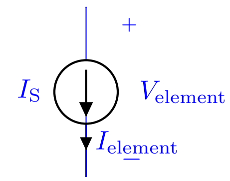
Solution:



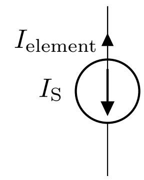
9. **(PRACTICE)**



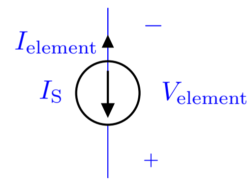
Solution:



10. **(PRACTICE)**

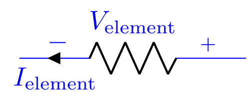


Solution:

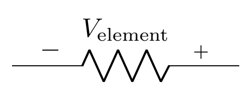


11. **(PRACTICE)**

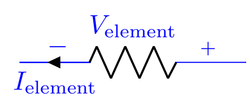
Solution:



12. **(PRACTICE)**

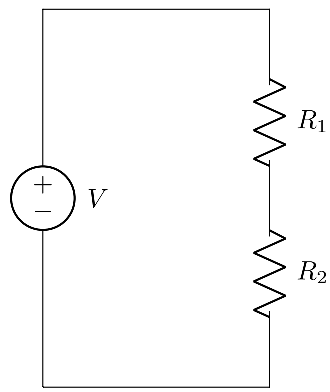


Solution:



Problem 3: Voltage Divider Properties

Let's take a systematic look at the voltages across a resistor, and see how other components in the circuit can affect it. Consider the following circuit:



1. Calculate the voltage drop across R_1 and R_2 using series resistance calculations.

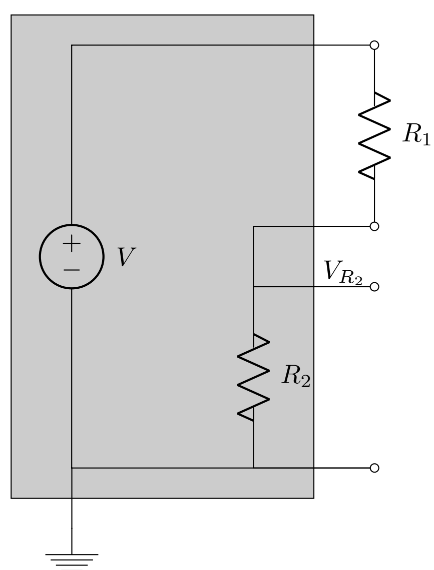
   Solution: The current out of the voltage source is given by Ohm's law:

$$i = \frac{V}{R_{eq}}$$
$$i = \frac{V}{R_1 + R_2}$$

   Again, by Ohm's law, we have that

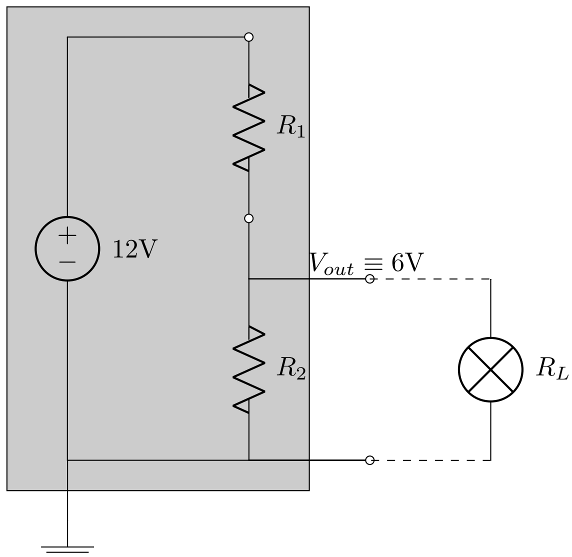
$$V_{R_1} = iR_1 = V\frac{R_1}{R_1 + R_2}$$
$$V_{R_2} = iR_2 = V\frac{R_2}{R_1 + R_2}$$

2. Suppose we want to manipulate the voltage across R_2, but it's locked in a box with the voltage source, as denoted below. Can we use R_1 to manipulate V_{R_2}? What range of voltages can we achieve?



Solution: Any voltage in the range $(0, V]$! Notice from the equations above that $V_{R_2} = V\frac{R_2}{R_{Total}}$. If we increase R_1 indefinitely, holding R_2 constant, we can make the fraction arbitrarily small. Intuitively, since the same current flows through both R_1 and R_2, they have to split the total voltage of the power source, and larger resistances correspond to larger voltage drops (by Ohm's Law). If we decrease R_1 to 0, $V_{R_2} = V$, so the voltage can be at most whatever is supplied by the power source. That the voltage source limits the achievable voltage in the circuit is a concept we will see again when we cover clipping in op-amps.

3. Now let's try using our new variable voltage source to power a light bulb with resistance R_L, where the threshold voltage for lighting the bulb is 6V. Find R_1 and R_2 so that the voltage across R_2 is this threshold voltage; that is, $V_{R_2} \equiv V_{out} = 6\text{V}$. Assume we have a 12V voltage source.



Solution: We want to split the voltage in half (from 12 to 6). Based on the voltage divider formula above, that means $R_1 = R_2 \equiv R$. Note that, under this condition, the voltage is evenly split regardless of what the actual resistance values are! While the current depends on actual resistance values, the voltage only depends on the ratio of resistances.

4. Now that we found an R_1 and R_2 that seem to divide our voltage source appropriately, let's try to connect the bulb to the ends of R_2. Remember, the bulb has a resistance R_L. Calculate the voltage across R_1, R_2 and the light bulb when it is connected. Will the light bulb turn on?

Solution: Let's reapply the voltage divider formula, but now notice that the "second" resistor is $R_2 \| V_L$!. When we change the resistance in a voltage divider circuit, the voltage across that resistance changes as well. We find that $V_{R_1} = V\frac{R}{R+R\|R_L} = 12\text{V}\frac{R}{R+\frac{RR_L}{R+R_L}} = 12\text{V}\frac{R+R_L}{R+2R_L}$. The rest of the potential drop must be across the R_2-R_L system, so we have $V_{R_2} = V_{\text{bulb}} = 12\text{V}\left(1 - \frac{R+R_L}{R+2R_L}\right) = 12\text{V}\frac{R_L}{R+2R_L} \leq 12\text{V}\frac{R_L}{2R_L} = 6\text{V}$.

So the voltage *decreased*, and the light bulb doesn't turn on.

The takeaway from this is that, while it may seem like the voltage divider can make voltage sources of arbitrary voltages, the act of connecting a component actually changes the output of the voltage divider. We will later learn about a way to stop a light bulb (or other devices that use up power) from "affecting" the circuit that supplies power by placing a "buffer" between the two.