CSM 16A **Designing Information Systems and Devices**

Week 7 Worksheet

Term: **Spring 2020** *Name:*

Problem 1: Linear Algebra Review

1. Suppose λ is an eigenvalue for the matrix A. Consider the λ-eigenspace of A: the set of all vectors v satisfying the equation $\mathbf{A}\vec{v} = \lambda\vec{v}$. Show that this eigenspace is a subspace by directly checking the three conditions needed to be a subspace.

2. Solve for the eigenvalue-eigenvector pairs for the following 2 by 2 matrix:

$$\mathbf{A} = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

3. Projection of a vector $\vec{u}$ onto $\vec{v}$ is given by:

$$\frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2}\vec{v}$$

Prove that projection onto a vector $\vec{v}$ is a linear transformation.

Problem 2: Introduction to Inner Products

Learning Goal:

Description: goes over definition, properties, and simple applications of inner products

Prereqs: basic linear algebra, i.e. what vectors are

1. What is an inner product?

2. What is the dot product between two vectors $\vec{x}, \vec{y} \in \mathbb{R}^n$?

   In the next four parts, we prove that the dot product is an inner product. Do note that the dot product is simply a type of inner product, and other inner products are also possible.

3. Prove that the dot product satisfies symmetry, i.e. that $\vec{x}\vec{y} = \vec{y}\vec{x}$

4. Prove that the dot product satisfies homogeneity, i.e. that $c\vec{x}\vec{y} = c\vec{x}\vec{y}$: $c \in \mathbb{R}$

5. Prove that the dot product satisfies additivity, i.e. that $\vec{x} + \vec{y}\vec{z} = \vec{x}\vec{z} + \vec{y}\vec{z}$

6. Prove that the dot product satisfies positive-definiteness, i.e., that $\vec{x}\vec{x} \geq 0$, and is equal to 0 iff $\vec{x} = \vec{0}$

   We will now consider ways to use dot products to do neat things. For each of the following, assume that you're given a $\vec{x}$, and that you get a pick $\vec{y}$ of your choosing. Describe a $\vec{y}$, such that when you compute $\vec{x}\vec{y}$, you get:

7. The sum of every element in $\vec{x}$

8. The sum of certain elements in $\vec{x}$

9. The mean of all the items in $\vec{x}$ (for $\vec{x}$ in $\mathbb{R}^n$)

10. The sum of the elements of $\vec{x}$ squared

    We will conclude by making some observations based on that last case.

11. Consider that last case, where we summed the squares of the elements of a vector. Try doing that for a few 2-dimensional vectors (vectors of length 2). What do you notice about the resulting answer? What about for vectors of length 3, or for vectors of any length n?