

## Worksheet #13 Metas

Term: Fall 2019

Name:

## Problem 1: And You Thought You Could Ignore Circuits Until Dead Week

**Learning Goal:** Understand when Least Squares is helpful for estimating values, and how to translate a word problem with given data points into a Least Squares set up.

1. Write Ohm's Law for a resistor.

**Solution:** For the resistor

$$V_R = I_R R$$

2. You're given the following test setup and told to find  $R_{Th}$  between the two terminals of the mystery box. What is  $R_{Th}$  of the mystery box between the two terminals in terms of  $V_S$  and  $I_{\text{measure}}$ ?

**Solution:**

$$R_{Th} = \frac{V_S}{I_{\text{measure}}}$$

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3. You think you've figured out how to find  $R_{Th}$ ! You've taken the following measurements:

| Measurement # | $I_{\text{measure}}$ | $V_S$  |
|---------------|----------------------|--------|
| 1             | 1A                   | 1.25kV |
| 2             | 2A                   | 1kV    |
| 3             | 3A                   | 4kV    |
| 4             | 4A                   | 3.5kV  |

Using the information above, formulate a least squares problem whose answer provides an estimate of  $R_{Th}$ .

**Meta:** Since voltage and current are directly proportional, as seen in Ohm's Law, there is no additional input. Therefore, the corresponding matrix  $A$  is a 4 x 1 vector.

**Solution:** According to Ohm's Law,  $V = IR$ . We are estimating the resistance so  $R_{Th}$  corresponds to  $\vec{x}$  in the  $A\vec{x} = \vec{b}$  Least Squares equation. Additionally,  $I$  corresponds to  $A$  and  $V$  corresponds to  $\vec{b}$ .

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 1.25 \\ 1 \\ 4 \\ 3.5 \end{bmatrix}$$

By running Least Squares,  $R_{Th} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \vec{b} = 975 \text{ kohms}$