

Worksheet #13 Metas

Term: Fall 2019

Name:

Problem 1: And You Thought You Could Ignore Circuits Until Dead Week

Learning Goal: Understand when Least Squares is helpful for estimating values, and how to translate a word problem with given data points into a Least Squares set up.

1. Write Ohm's Law for a resistor.

Solution: For the resistor

$$V_R = I_R R$$

2. You're given the following test setup and told to find R_{Th} between the two terminals of the mystery box. What is R_{Th} of the mystery box between the two terminals in terms of V_S and I_{measure} ?

Solution:

$$R_{Th} = \frac{V_S}{I_{\text{measure}}}$$

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3. You think you've figured out how to find R_{Th} ! You've taken the following measurements:

Measurement #	I_{measure}	V_S
1	1A	1.25kV
2	2A	1kV
3	3A	4kV
4	4A	3.5kV

Using the information above, formulate a least squares problem whose answer provides an estimate of R_{Th} .

Meta: Since voltage and current are directly proportional, as seen in Ohm's Law, there is no additional input. Therefore, the corresponding matrix A is a 4 x 1 vector.

Solution: According to Ohm's Law, $V = IR$. We are estimating the resistance so R_{Th} corresponds to \vec{x} in the $A\vec{x} = \vec{b}$ Least Squares equation. Additionally, I corresponds to A and V corresponds to \vec{b} .

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 1.25 \\ 1 \\ 4 \\ 3.5 \end{bmatrix}$$

By running Least Squares, $R_{Th} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \vec{b} = 975 \text{ kohms}$