Robustness of Measurement

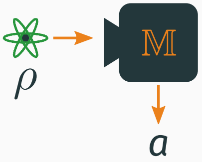
Paul Skrzypczyk
Joint work with Noah Linden

September 21, 2018

Outline

1. Robustness of Measurement & Its Properties
2. Operational Significance I: State discrimination
3. Operational Significance II: Single-Shot Accessible Information
4. Summary

Robustness of Measurement & Its Properties

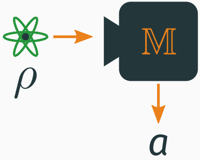


$$\mathbb{M} = \{M_a\}_a$$

$$a = 1, \ldots, o$$

$$p(a|\rho) = \mathrm{tr}[M_a \rho]$$

Informativeness of a measurement



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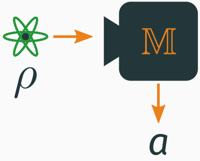

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- Completely uninformative measurement $\mathbb{C} = \{C_a\}_a$

$$C_a = q(a)\mathbb{I}$$

where $\mathbf{q} = \{q(a)\}_a$ probability vector

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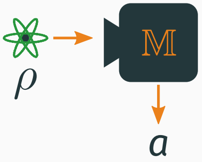

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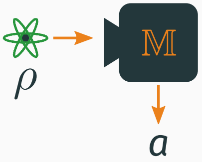
- For all ρ, $\quad p(a|\rho) = q(a)$.

Informativeness of a measurement



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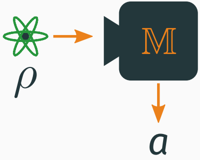
- For all ρ, $\quad p(a|\rho) = q(a)$.
- How far is a measurement from being completely uninformative?

Robustness of Measurement



- Minimal amount of 'noise' that needs to be added to make a measurement completely uninformative

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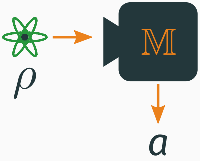

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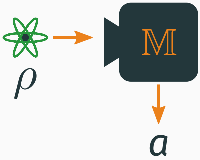
Robustness of Measurement

$$\begin{aligned} R(\mathbb{M}) = \min_{r, \mathbb{N}, \mathbf{q}} \quad & r \\ \text{s.t.} \quad & \frac{M_a + rN_a}{1+r} = q(a)\mathbb{I} \quad \forall a, \\ & N_a \geq 0 \quad \forall a, \\ & \sum_a N_a = \mathbb{I}. \end{aligned}$$



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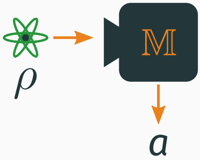

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Properties

1. Faithfulness: $R(\mathbb{M}) = 0$ if and only if $M_a = q(a)\mathbb{I}$ for all a.

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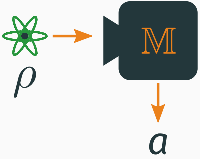

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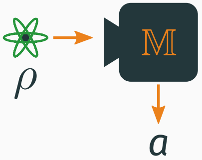
2. Convexity: $R(p\mathbb{M}_1 + (1-p)\mathbb{M}_2) \leq pR(\mathbb{M}_1) + (1-p)R(\mathbb{M}_2)$

Properties



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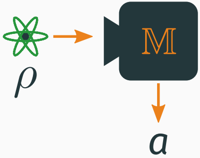
1. Faithfulness: $R(\mathbb{M}) = 0$ if and only if $M_a = q(a)\mathbb{I}$ for all a.
2. Convexity: $R(p\mathbb{M}_1 + (1-p)\mathbb{M}_2) \leq pR(\mathbb{M}_1) + (1-p)R(\mathbb{M}_2)$
3. Non-increasing under measurement simulation: $R(\mathbb{M}') \leq R(\mathbb{M})$ where $\mathbb{M}' = \{M'_b\}_b$,

$$M'_b = \sum_a p(b|a) M_a,$$

$p(b|a) \geq 0$, $\sum_b p(b|a) = 1$ for all a.



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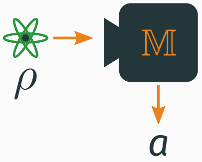
SDP formulation

$$R(\mathbb{M}) = \min_{\tilde{q}} \sum_a \tilde{q}(a) - 1$$

$$\text{s.t.} \quad \tilde{q}(a)\mathbb{I} \geq M_a \quad \forall a,$$

Explicit form



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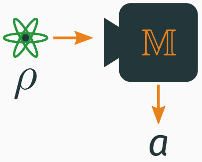
Explicit form

$$R(\mathbb{M}) = \sum_a \|M_a\|_\infty - 1$$

- Function only of the largest eigenvalue of each POVM element

Bounds



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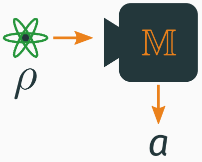

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- Universal solution

$$N_a = \frac{\mathrm{tr}[M_a]\mathbb{I} - M_a}{d-1}, \quad q(a) = \frac{1}{d}\mathrm{tr}[M_a]$$

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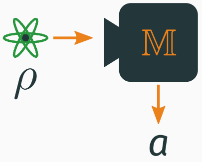

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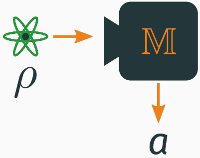
Bound

$$R(\mathbb{M}) \leq \min(o, d) - 1$$
$$\leq d - 1$$

Examples: Maximally robust measurements



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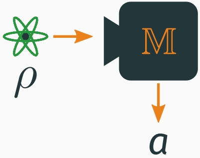

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- Ideal projective measurements $M_a = |\psi_a\rangle\langle\psi_a|$
  $\|M_a\|_\infty = 1$ for all $a = 1, \ldots, d$

$$R(\mathbb{M}) = d - 1$$

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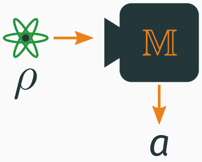

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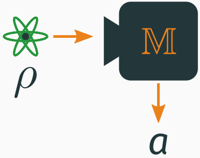
- rank-1 measurements $M_a = \alpha_a |\phi_a\rangle \langle\phi_a|$

$$\sum_a \alpha_a = d$$

$$R(\mathbb{M}) = d - 1$$

Dual semidefinite program formulation



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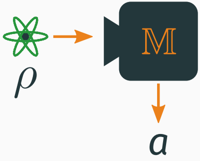
Dual semidefinite program formulation



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$$\text{s.t.} \quad \rho_a \geq 0 \quad \forall a,$$

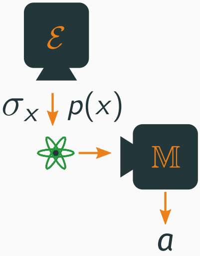
$$\mathrm{tr}[\rho_a] = 1 \quad \forall a.$$

- By inspection ρ_a^* any pure state in max-eigenvalue eigenspace of M_a

$$\mathrm{tr}[M_a \rho_a^*] = \|M_a\|_\infty$$

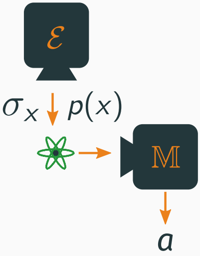
Operational Significance I: State discrimination

State discrimination



$$\mathcal{E} = \{p(x), \sigma_x\}$$

- Consider situation where state σ_x produced with probability $p(x)$



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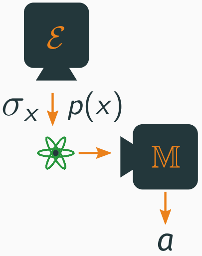

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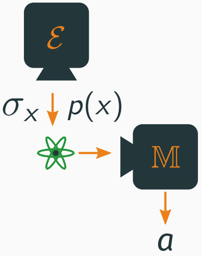
- $$P^{Q}_{guess}(\mathcal{E}, \mathbb{M}) = \max_{\mathbb{M}'} \quad \sum_{x,g} p(x) \mathrm{tr}[\sigma_x M'_g] \delta_{g,x}$$
  $$\text{s.t.} \quad M'_g = \sum_a p(g|a) M_a$$



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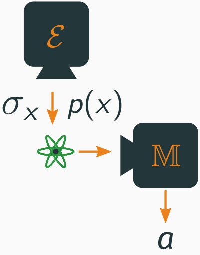
- Compare to completely uninformative measurement

$$P^{C}_{guess}(\mathcal{E}) = \max_x p(x)$$

Robustness of Measurement as advantage in state discrimination



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Operational Significance

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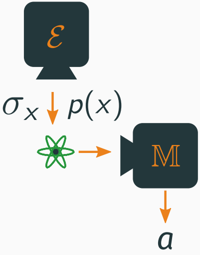

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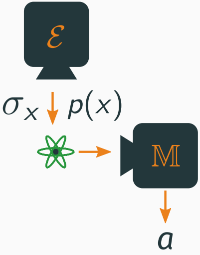
Operational Significance

$$\frac{p^{Q}_{guess}(\mathcal{E}, \mathbb{M})}{p^{C}_{guess}(\mathcal{E})}$$



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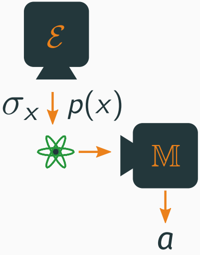

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Operational Significance

$$\max_{\mathcal{E}} \frac{p^{Q}_{guess}(\mathcal{E}, \mathbb{M})}{p^{C}_{guess}(\mathcal{E})} = 1 + R(\mathbb{M})$$

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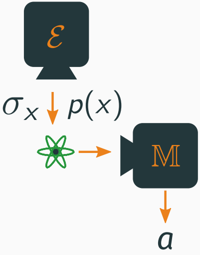

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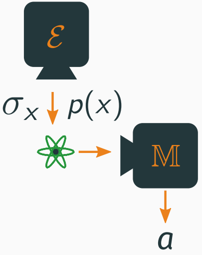
Robustness of Measurement as advantage in state discrimination



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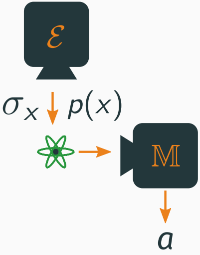
- Proof idea: Upper bound from primal SDP, lower bound from dual SDP



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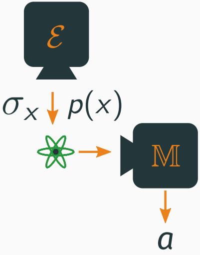
- $\mathcal{E}^* = \{\frac{1}{o}, \rho_x^*\}$

Examples: Maximally robust measurements



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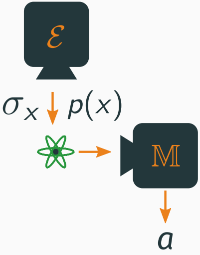

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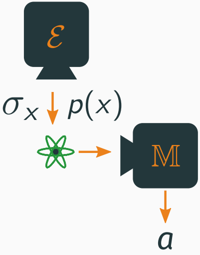

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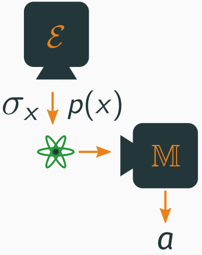
$$p_{guess}^{Q}(\mathcal{E}^*, \mathbb{M}) = 1, \quad p_{guess}^{C}(\mathcal{E}^*) = \tfrac{1}{d}$$



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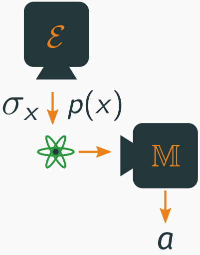

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$$p_{guess}^{Q}(\mathcal{E}^*, \mathbb{M}) = 1, \quad p_{guess}^{C}(\mathcal{E}^*) = \frac{1}{d}$$

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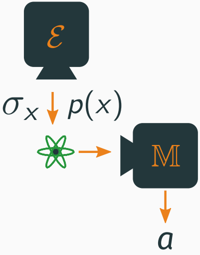

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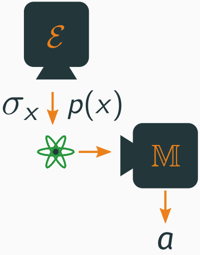
Examples: Maximally robust measurements



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Operational Significance II: Single-Shot Accessible Information

- Measurements can be associated to quantum-to-classical channels

$$\Lambda_{\mathbb{M}}(\rho) = \sum_a \mathrm{tr}[M_a \rho] \left|a\right\rangle \left\langle a\right|$$

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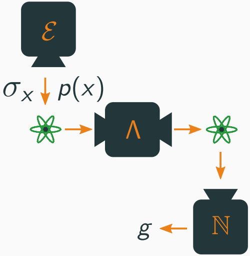
- Does $R(\mathbb{M})$ have operational significance from this viewpoint also?



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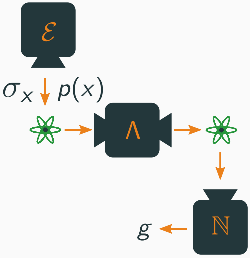
$$I^{acc}(\Lambda(\cdot)) = \max_{\mathcal{E},\mathbb{N}} I(X : G)$$



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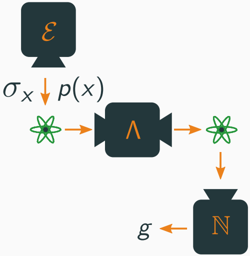

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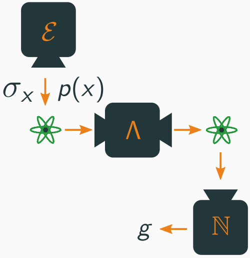

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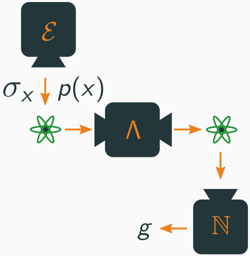

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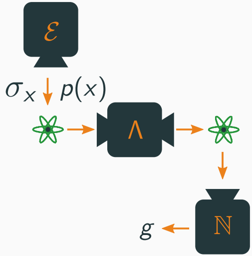
Single-Shot Accessible Information of a Channel



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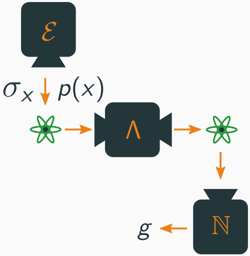

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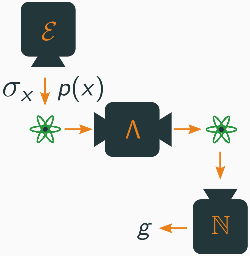

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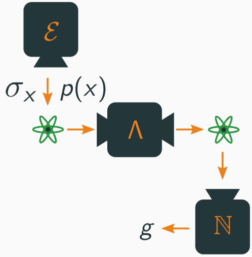

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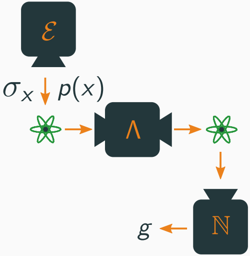

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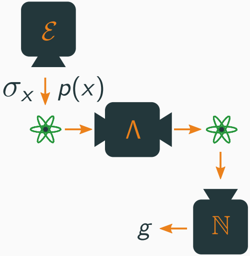
$$I_{min}^{acc}(\Lambda_{\mathbb{M}}(\cdot)) = \log(1 + R(\mathbb{M}))$$



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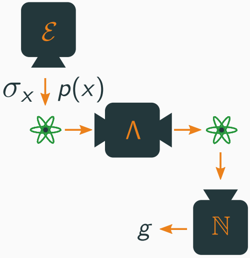
Single-Shot Accessible Information of a $Q \to C$ channel



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- Operational significance in terms of state discrimination and single-shot information theory
- Open questions
    - Look at instruments instead of POVMs
    - Revisit information-disturbance trade-off
    - Extend the connection between robustness, discrimination problems and single-shot information theory to other contexts

Thank you