# Quantum teleportation from the perspective of nonlocality

Paul Skrzypczyk
Joint work with Ivan Šupić and Daniel Cavalcanti

June 20, 2017

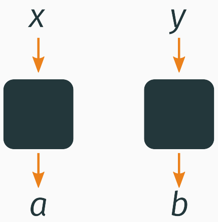
University of Bristol
ICFO Institut de Ciències Fotòniques
The Royal Society

# Outline

1. Family tree of Nonlocality
2. Which states demonstrate non-classical teleportation?
3. Testing for teleportation with convex optimisation
4. Witnesses of non-classical teleportation
5. Quantifying entanglement with teleportation
6. Conclusions

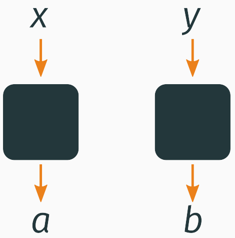
# Family tree of Nonlocality

# Family tree of nonlocality




Bell nonlocality

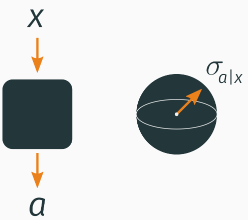
# Bell nonlocality



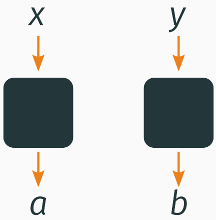

$$P(a, b|x, y) = \mathrm{tr}[(M_{a|x} \otimes M_{b|y})\rho^{AB}]$$

- About the correlations between the measurements of Alice and Bob
- Alice and Bob receive classical inputs
- Playground for device-independent quantum information

# Family tree of nonlocality

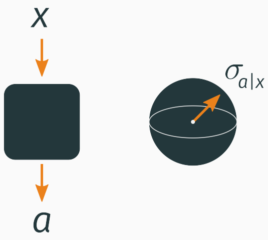



EPR Steering




Bell nonlocality

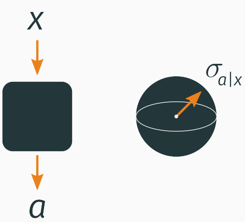
# EPR Steering



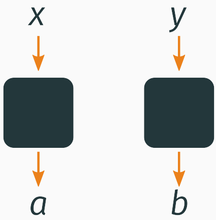

$$\sigma_{a|x} = \mathrm{tr}_A[(M_{a|x} \otimes \mathbb{I})\rho^{AB}]$$

- About the correlations between the measurements of Alice and the states prepared for Bob
- Assume Alice can do tomography or perform known measurements
- Playground for one-sided device-independent quantum information

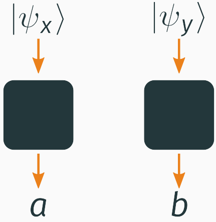
# Family tree of nonlocality




EPR Steering

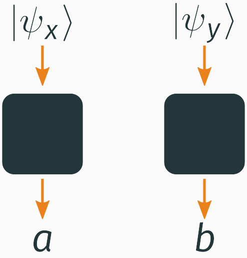



Bell nonlocality




Buscemi nonlocality

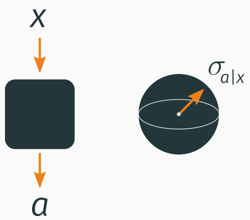
# Buscemi nonlocality



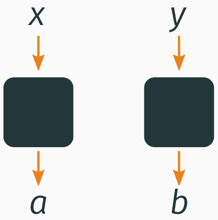

$$P(a,b|\psi_x,\psi_y) = \mathrm{tr}[(M_a^{A'A} \otimes M_b^{BB'}) \times (\psi_x^{A'} \otimes \rho^{AB} \otimes \psi_y^{B'})]$$

- About the correlations between the measurement of Alice and Bob
- Inputs encoded in unknown quantum states $\psi_x$ and $\psi_y$.
- Alice and Bob perform joint measurements on input state and on shared state
- Playground for measurement-device-independent quantum information

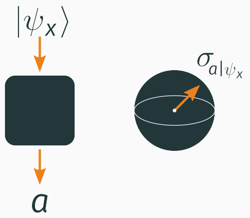
# Family tree of nonlocality



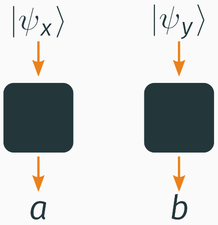

EPR Steering




Bell nonlocality

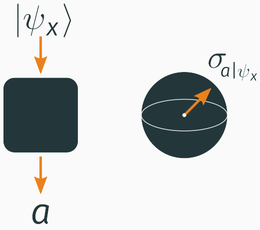



Teleportation




Buscemi nonlocality

# Teleportation

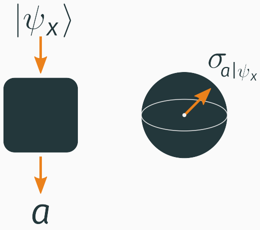



$$\sigma_{a|\psi_x} = \mathrm{tr}_{A'A}[(M_a^{A'A} \otimes \mathbb{I}) \times (\psi_x^{A'} \otimes \rho^{AB})]$$

- About the correlations between the measurements of Alice and the states prepared for Bob
- Alice performs joint measurement on unknown input state $\psi_x$
- Nonlocal part of teleportation – affect Alice's measurement has on Bob's system
- Do not consider the local part – classical communication + local operations (corrections) on Bob's side

# Which states demonstrate non-classical teleportation?

# Which states are useful for teleportation?



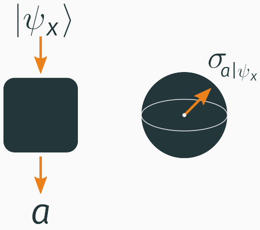

$$\sigma_{a|\psi_x} = \mathrm{tr}_{A'A}[(M_a^{A'A} \otimes \mathbb{I}) \times (\psi_x^{A'} \otimes \rho^{AB})]$$

- State is useful for teleportation if average fidelity of teleportation is better than the classical fidelity of teleportation
- Average fidelity of teleportation

$$\overline{F}_{\mathrm{tel}} = \frac{1}{|X|} \sum_{a,x} \langle \psi_x | U_a \sigma^B_{a|\psi_x} U_a^\dagger \, |\psi_x\rangle$$

- Classical fidelity of teleportation best fidelity that can be obtained without sharing entanglement

# Which states are useful for teleportation?




$$\sigma_{a|\psi_x} = \mathrm{tr}_{A'A}[(M_a^{A'A} \otimes \mathbb{I}) \times (\psi_x^{A'} \otimes \rho^{AB})]$$

- HHH '99

$$\bar{F}_{\mathrm{tel}} = \frac{d\mathrm{SF}(\rho) + 1}{d + 1}$$

  SF($\rho$) singlet fraction

- HHH '99

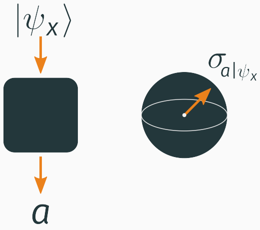
$$\bar{F}_{\mathrm{cl}} = \frac{2}{d + 1}$$

- Useful for teleportation iff

$$\mathrm{SF}(\rho) > \frac{1}{d}$$

- Bound entangled states useless

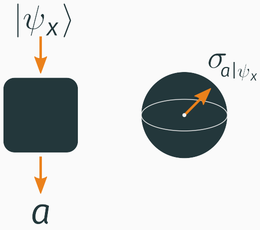
# Using the full data in a teleportation experiment




$$\sigma_{a|\psi_x} = \mathrm{tr}_{A'A}[(M_a^{A'A} \otimes \mathbb{I}) \times (\psi_x^{A'} \otimes \rho^{AB})]$$

- Average fidelity of teleportation is only a single figure of merit with which to judge an experiment
- In principle can use full observable data available in analysis.
- Related question: When can the observable data in a teleportation experiment be explained classically?
- Do there exists states for which SF $< 1/d$ but nevertheless $\sigma_{a|\psi_x}$ cannot be explained classically?

# All entangled states demonstrate non-classical teleportation



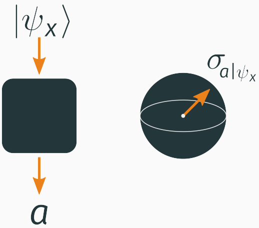

$$\sigma_{a|\psi_x} = \mathrm{tr}_{A'A}[(M_a^{A'A} \otimes \mathbb{I}) \times (\psi_x^{A'} \otimes \rho^{AB})]$$

- All entangled states produce data in a teleportation experiment that cannot be reproduced classically
- Use $\psi_x$ which are tomographically complete i.e. allow for process tomography
- Alice performs partial (or full) Bell state measurement

$$M_0^{A'A} = |\Phi^+\rangle\langle\Phi^+|$$
$$M_1^{A'A} = \mathbb{I} - |\Phi^+\rangle\langle\Phi^+|$$

# Testing for teleportation with convex optimisation

# Membership problem for classical teleportation




- Teleportation operators

$$\sigma_{a|\psi_x} = \mathrm{tr}_{A'A}[(M_a^{A'A} \otimes \mathbb{I})(\psi_x^{A'} \otimes \rho^{AB})]$$
$$= \mathrm{tr}_{A'}[\tilde{M}_a^{A'B}(\psi_x^{A'} \otimes \mathbb{I})]$$

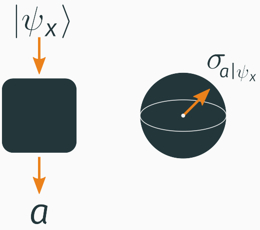
-

$$\tilde{M}_a^{A'B} = \mathrm{tr}_A[(M_a^{A'A} \otimes \mathbb{I})(\mathbb{I}^{A'} \otimes \rho^{AB})]$$

- No-signalling condition

$$\sum_a \tilde{M}_a^{A'B} = \mathbb{I}^{A'} \otimes \rho^B$$

# Membership problem for classical teleportation



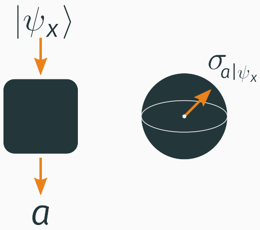

- If $\rho^{AB}$ is separable $\rho^{AB} = \sum_\lambda p_\lambda \rho_\lambda^A \otimes \rho_\lambda^B$

$$\tilde{M}_a^{A'B} = \mathrm{tr}_A[(M_a^{A'A} \otimes \mathbb{I})(\mathbb{I}^{A'} \otimes \sum_\lambda p_\lambda \rho_\lambda^A \otimes \rho_\lambda^B)]$$

$$= \sum_\lambda p_\lambda \tilde{M}_{a|\lambda}^{A'} \otimes \rho_\lambda^B$$

- $$\tilde{M}_{a|\lambda}^{A'} = \mathrm{tr}_A[M_a^{A'A}(\mathbb{I}^{A'} \otimes \rho_\lambda^A)]$$
- $\tilde{M}_a^{A'B}$ seperable operator

# Membership problem for classical teleportation



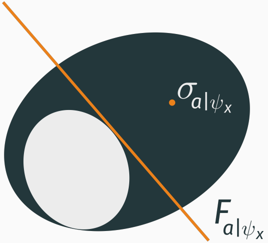

- Test for classical teleportation for observed data $\sigma_{a|\psi_x}$

$$
\begin{aligned}
\text{given} \quad & \sigma_{a|\psi_x} \\
\text{find} \quad & \tilde{M}_a^{A'B} \\
\text{s.t.} \quad & \mathrm{tr}_{A'}[\tilde{M}_a^{A'B}(\psi_x^{A'} \otimes \mathbb{I})] = \sigma_{a|\psi_x}, \\
& \tilde{M}_a^{A'B} \text{ separable}, \\
& \sum_a \tilde{M}_a^{A'B} = \mathbb{I} \otimes \rho^B
\end{aligned}
$$

- If feasible solution exists, provides classical teleportation operators which reproduce data
- If problem is infeasible, certifies that the data is non-classical.

# Witnesses of non-classical teleportation

# Membership problem for classical teleportation

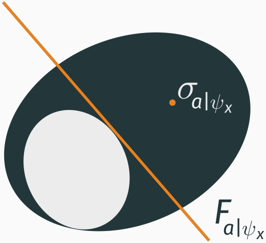



- Dual formulation of membership problem provides teleportation witness
- Operators $F_{a|\psi_x}$ such that

$$\beta = \sum_{a,x} \mathrm{tr}[F_{a|\psi_x}\sigma_{a|\psi_x}] > \beta_{\mathrm{cl}}$$

  certify the infeasibility of the membership problem
- $\beta_{\mathrm{cl}}$ classical bound of witness

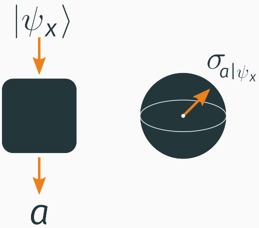
# Comparison with average fidelity of teleportation




- Average fidelity of teleportation special instance of teleportation witness
- $$F_{a|\psi_x} = \frac{1}{|X|} U_a^\dagger |\psi_x\rangle \langle\psi_x| U_a$$
- Bound entangled states cannot violate average fidelity witness, but violate other witnesses
- General construction of teleportation witness from entanglement witness, similar to MDI nonlocality witnesses

# Quantifying entanglement with teleportation

# Quantifying entanglement with teleportation




- Teleportation witnesses can be made quantitative if additional structure is imposed

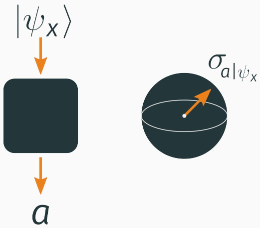
$$W_a = -\sum_x \psi_x \otimes F_{a|\psi_x} \text{ ent witness}$$

$$1 + \frac{1}{od}\mathrm{tr}\sum_{a,x} F_{a|\psi_x} \geq 0$$

- $$E_R(\rho^{AB}) \geq \sum_{a,x} \mathrm{tr}[F_{a|\psi_x}\sigma_{a|\psi_x}]$$

$E_R(\rho^{AB})$ Robustness of entanglement

# Comparison with average fidelity of teleportation




- Average fidelity of teleportation

$$E_R(\rho^{AB}) \geq \frac{\overline{F}_{\mathrm{tel}} - \overline{F}_{\mathrm{cl}}}{\overline{F}_{\mathrm{cl}} - 1/d}$$

- Ideal teleportation, $\overline{F}_{\mathrm{tel}} = 1$

$$E_R(\rho^{AB}) \geq d$$

# Conclusions

# Conclusions

- Teleportation can be placed in a family tree of nonlocality, as a one-sided measurement-device-independent paradigm
- Every entangled state can produce non-classical teleportation data
- Tools of convex optimisation can be used to analyse teleportation, leading to teleportation witnesses and to teleportation quantifiers

Thank you