Nonlocality

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Introduction

- (One of the most) fascinating predictions of quantum mechanics
  - Correlations that can arise between measurement outcomes on entangled particles defy any 'reasonable' (aka 'local') explanation
- More general than quantum theory → phrase phenomenon abstractly
- Experiments have confirmed that nature is fundamentally nonlocal

Structure

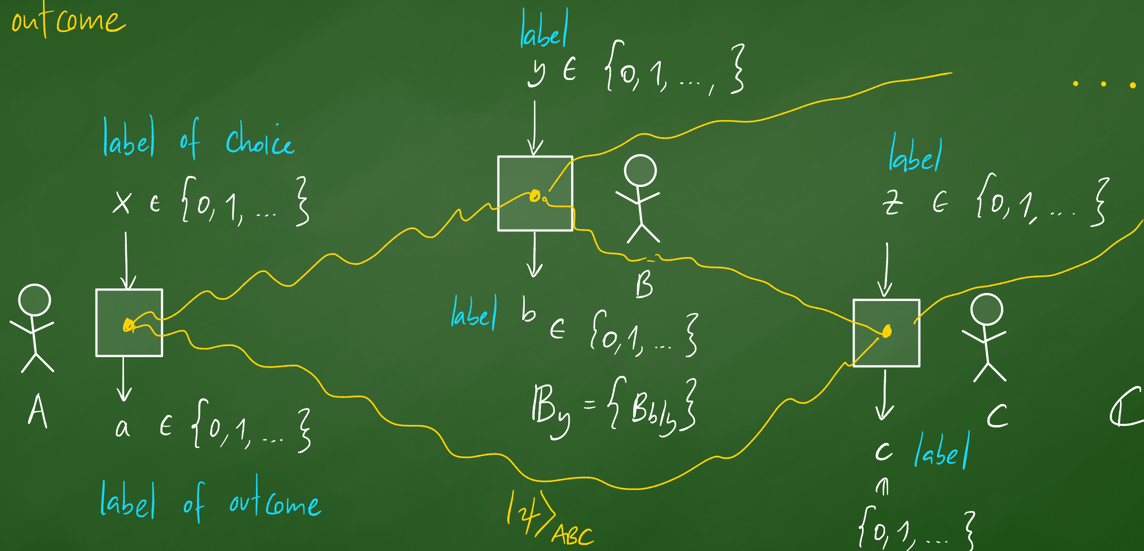
1. Bell scenarios & local correlations
2. Nonlocal games, quantum nonlocality & 'loopholes'
3. Nonlocality beyond quantum mechanics & non-signalling polytope.

Part I : Bell scenarios

& local correlations

Bell Scenarios

- Multiple parties / agents spatially separated holding quantum particle
- In each round / run, each party chooses a measurement to perform & obtains outcome



measurements

$A_x = \{A_{a|x}\}$

$A_{a|x} \geq 0$

$\sum_a A_{a|x} = \mathbb{1}$

$\mathbb{C}_z = \{C_{c|z}\}$

- Collect statistics $P(a, b, c, \dots \mid x, y, z, \dots)$ by repeating experiments many times.

Question what correlations are possible to produce?

Caveats

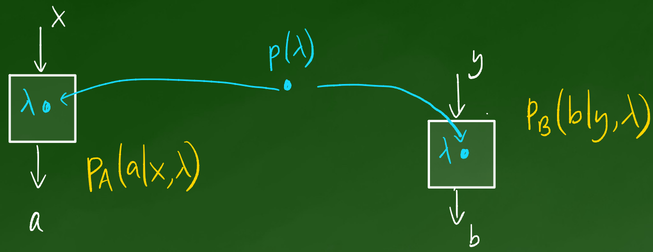
- very particular scenario, but surprisingly rich phenomena
- Conclusions only on labels & NOT on specific properties of measurements/states.

→ Often referred to as device independent scenario

- reason why Bell nonlocality can be viewed as a powerful resource in QIS. e.g. DI – crypto
  – randomness generation
  – certification ...

'Local' Correlations & Local Hidden Variable (LHV) models (2 parties from now on)

- In order to see that entanglement leads to interesting correlations, first need to identify what is 'uninteresting'
- Forget about QM, & study correlations in classical setting involving only Random Variables.



- Each party might hold a 'hidden (random) variable' λ distributed as $p_\Lambda(\lambda)$

- Given λ & measurement label, probabilistically produce result according to some response function

Statistics $\quad P(a,b\,|\,x,y) \;=\; \underbrace{\sum_\lambda}_{\text{average over hidden variable}} p_\Lambda(\lambda)\, P_A(a|x,\lambda)\, P_B(b|y,\lambda) \qquad (*)$

(could take measure/integrate)

- $(*)$ is called a local (hidden variable) model (LHV)

- Captures the correlations obtainable classically. (e.g. you could model this on a computer) / play tonight at dinner by rolling dice / flipping coins.

Two other ways of arriving at local correlations

1. Measure separable quantum state: $\rho_{AB} = \sum_\mu p(\mu)\, \rho_\mu^A \otimes \rho_\mu^B$

Born rule: $p(a,b|x,y) = \mathrm{tr}\left[(A_{a|x} \otimes B_{b|y})\rho_{AB}\right]$

$$= \mathrm{tr}\left[(A_{a|x} \otimes B_{b|y}) \sum_\mu p(\mu)\, \rho_\mu^A \otimes \rho_\mu^B\right]$$

$$= \sum_\mu p(\mu)\, \underbrace{\mathrm{tr}\left[A_{a|x}\, \rho_\mu^A\right]}_{\substack{\text{local response} \\ P_A(a|x,\mu)}}\, \underbrace{\mathrm{tr}\left[B_{b|y}\, \rho_\mu^B\right]}_{P_B(b|y,\mu)}$$

$$= \sum_\mu p(\mu)\, P_A(a|x,\mu)\, P_B(b|y,\mu)$$

local form ✓

$\Rightarrow$ correlations that arise from measurements on separable states are always local.

2. Make Compatible measurements

For POVMs operational notion of compatibility is joint measurability:

(i) Perform single parent measurement $G = \{G_\lambda\}$

(ii) Prob. post-process parent result to give child result using $p(a|x,\lambda)$

mathematically: $\{A_x\}$ jointly measurable if $A_{a|x} = \sum_\lambda p(a|x,\lambda) G_\lambda$

In Bell scenario:

$$p(a,b|x,y) = \mathrm{tr}\left[\left(A_{a|x} \otimes B_{b|y}\right)\sigma_{AB}\right]$$

$$= \sum_\lambda p(a|x,\lambda)\, \underbrace{\mathrm{tr}\left[\left(G_\lambda \otimes B_{b|y}\right)\sigma_{AB}\right]}_{p(b,\lambda|y) \,=\, p(b|y,\lambda)\,p(\lambda|\cancel{y})}$$

$$= \sum_\lambda p(\lambda)\, p(a|x,\lambda)\, p(b|y,\lambda) \quad \text{local}$$

→ if either party uses compatible measurements → local correlations again.

Structure of local set of correlations

- Because $p_A(\lambda)$, $p_A(a|x,\lambda)$ & $p_B(b|y,\lambda)$ all arbitrary, local correlations appear 'complex'
- In fact, have a simple form in terms of deterministic response functions
  (interesting conceptually & useful in calculations!)

- Basic idea: can push all randomness into $p_\Lambda(\lambda)$

Let $\lambda = (a_0, a_1, \ldots, b_0, b_1, \ldots)$ list of ficticious measurement results.

w.l.o.g assume $p_A(a|x,\lambda) = \delta_{a,a_x} = \begin{cases} 1 & \text{if } a = a_x \\ 0 & \text{otherwise} \end{cases}$

(i.e. deterministically set $a = a_x$ when input is x)

$p_B(b|y,\lambda) = \delta_{b,b_y}$

$$\rightarrow \quad P(a,b|x,y) = \sum_{\substack{a_0,a_1,\ldots \\ b_0,b_1,\ldots}} \underbrace{p(a_0,a_1,\ldots,b_0,b_1,\ldots)}_{\text{average / mixture}} \underbrace{\delta_{a,a_x}\,\delta_{b,b_y}}_{D_{\underline{a}\,\underline{b}}(a,b|x,y)}$$

deterministic correlations

i.e. All local correlations are mixtures of deterministic behaviours.

→ A performs m_A meas, O_A outcomes each   B performs m_B meas, O_B outcomes each

→ $O_A^{m_A}$ deterministic correlations for A

$O_B^{m_B}$ — " — for B

& $O_A^{m_A} O_B^{m_B}$ correlations $D_{\underline{a}\underline{b}}(a,b|x,y)$

[later: local polytope].

<u>Summary</u>: - Bell scenario

- Seen def$^{\underline{n}}$ of local correlations + properties.

<u>key Question</u>: How to see that local correlations are limited?

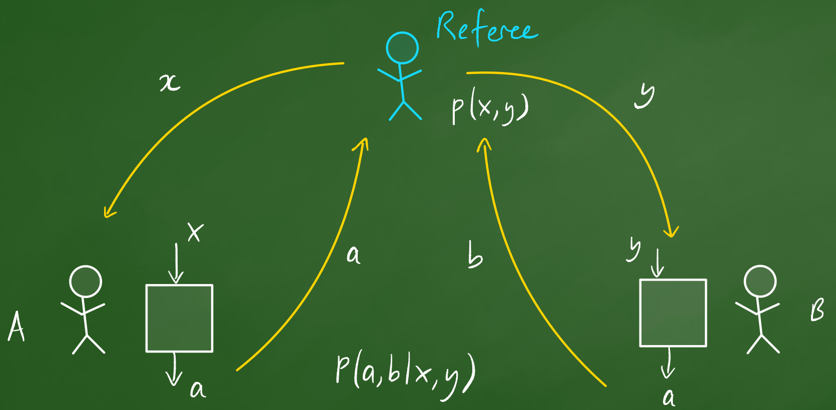
Part II: Nonlocal games, quantum nonlocality & 'loopholes'

Nonlocal Games & Bell Inequalities

- Limitations of local correlations can be witnessed by use of nonlocal games (or Bell inequalities)
  - Co-operative game / task played by separated, non-communicating parties
  - Can analyse best strategy of players given access to classical or quantum resources

Quantum nonlocality / Bells Theorem:

Quantum strategies outperform classical strategies.



win game if $V(a,b,x,y) = 0$ (predicate)

lose game if $V(a,b,x,y) \neq 0$

$$P_{suc} = \sum_{abxy} p(x,y)\, p(a,b|x,y)\, \delta_{V(a,b,x,y),0}$$

Example: CHSH game

most important game. workhorse of field!

$p(x,y) = \frac{1}{4}$

$x, y, a, b \in \{0,1\}$

win if $a \oplus b = xy$

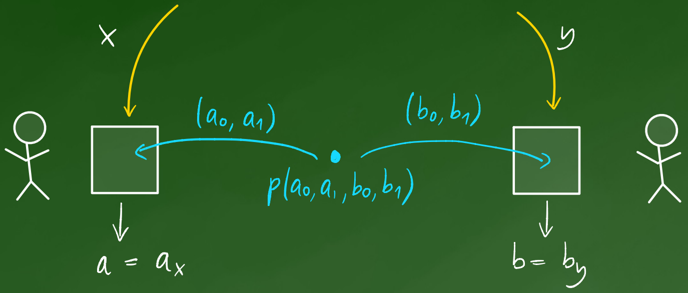
lose if $a \oplus b \neq xy$

Simplest possible non-trivial game

Reminder: $0 \oplus 0 = 1 \oplus 1 = 0$

$0 \oplus 1 = 1 \oplus 0 = 1$

- Using a classical strategy (LHV) $\quad P_{suc}^{local} \leq \frac{3}{4}$



(x,y)	a	b	$a \oplus b$	xy	win ?
$(0,0)$	a_0	b_0	$a_0 \oplus b_0$	0	$a_0 \oplus b_0 = 0$
$(0,1)$	a_0	b_1	$a_0 \oplus b_1$	0	$a_0 \oplus b_1 = 0$
$(1,0)$	a_1	b_0	$a_1 \oplus b_0$	0	$a_1 \oplus b_0 = 0$
$(1,1)$	a_1	b_1	$a_1 \oplus b_1$	1	$a_1 \oplus b_1 = 1$

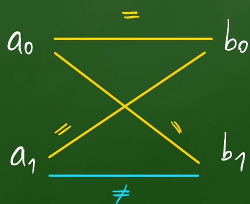
- No solution to all 4 equations simultaneously.

i) Contradiction:

$$\underbrace{(a_0 \oplus b_0) \oplus (a_0 \oplus b_1) \oplus (a_1 \oplus b_0) \oplus (a_1 \oplus b_1)}_{0} = \underbrace{0 \oplus 0 \oplus 0 \oplus 1}_{1}$$

$0 = 1$ can't be satisfied.

ii) 'frustration'



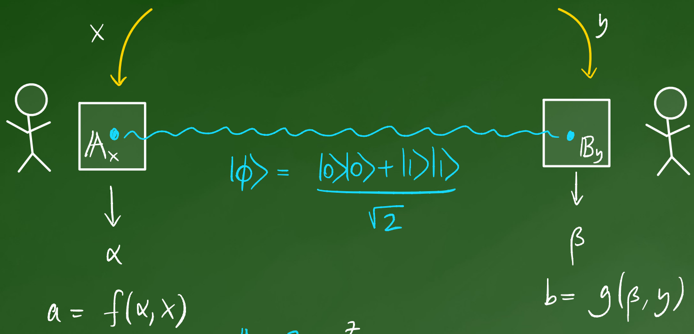
3 relations are consistent

4^{th} always contradicts.

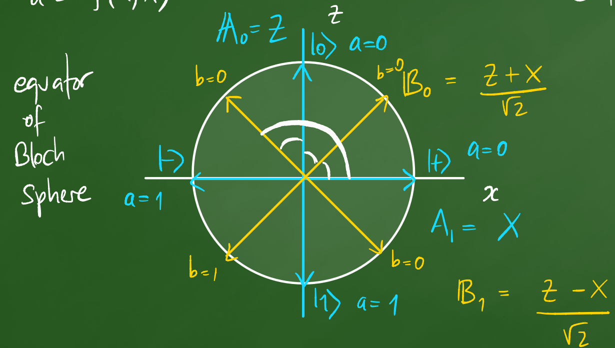
- This analysis is for deterministic strategies. General strategy mix / average over det. strats

→ this clearly can't help you win better ⇒ CHSH game has $p_{suc}^{LHV} \leq \frac{3}{4}$.

Quantum strategy for CHSH game



- pre-share entangled state between players
- upon recieving x, y → choose which measurement to make.
- Process meas. outcomes into answers.



$$S(\theta) = \cos\theta\, Z + \sin\theta\, X$$

$$\langle\phi|\, S(\theta) \otimes S(\phi)\, |\phi\rangle = \cos(\theta - \phi)$$

$$P(\alpha = \beta \mid \theta, \phi) = \tfrac{1}{2}\left(1 + \cos(\theta - \phi)\right)$$

$$p_{suc}^{Q} = \tfrac{1}{2}\left(1 + \tfrac{1}{\sqrt{2}}\right) \approx 0.85$$

→ Quantum strategy outperforms best classical strategy by ≈ 10% !

Correlations arising from measurements on entangled states are stronger than those that can arise from LHV models!

– Alice & Bob coordinate much better given quantum resources compared to classical.

Experiments & 'loopholes'

- Long history of experimental demonstrations of quantum nonlocality
- Since prediction of nonlocality is so remarkable ⟶ demanded remarkable evidence.
- Imperfections in experimental realisation of theoretical setup open up 'loopholes'
- two major loopholes: 1. 'detection' loophole: photonic experiments ⟶ photons go missing

  ↳ make 'fair sampling' assumption ignore rounds with no results

  * Smart / malicious LHV model can use this to fake $P_{suc}^{LHV} > \frac{3}{4}$.

e.g.

a_0	a_1	b_0	b_1
0	no click	0	0

50% rounds dicarded $(x=1)$

in non-discarded rounds ⟶ win 100%

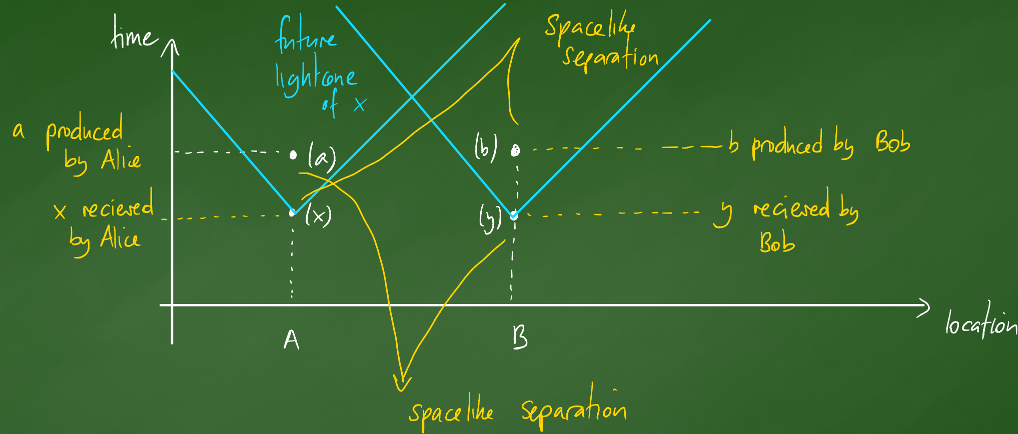
- mix such strategies (to match experimental obs)

Resolution:
- set no-click = 0 $\quad P_{suc}^{LHV} \leq \frac{3}{4}$
- set no-click = 2 $\quad$ nonlocal game with 3 outcomes

2. 'locality' loophole : hidden communication can easily win all time

↳ e.g. if Bob knows x, $a = \lambda$ $b = \lambda \oplus xy$ $p(\lambda) = \frac{1}{2}$

Resolution: rule out communication based upon relativity



- In practice: requires large distances & fast measurements

Only in 2015 did 3 landmark experiments close both detection & locality loopholes in same experiment. 'Conclusive' (even if very skeptical!) demonstration of nonlocality.

Part III : Nonlocality beyond quantum mechanics & non-signalling polytope

Nonlocality beyond quantum mechanics

- $p_{suc}^{QM} = \frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right)$ provably best possible win probability for CHSH game in QM.

  question: why can't we win the game all time?

- Crucial property of quantum correlations: non-signalling

  – Impossible for one party to signal to another.

$$P(a,b|x,y) = \mathrm{tr}\left[\rho_{AB}\left(A_{a|x} \otimes B_{b|y}\right)\right]$$

$$\to \sum_a P(a,b|x,y) = \mathrm{tr}\Big[\rho_{AB}\,\underbrace{\Big(\sum_a A_{a|x}\Big)}_{\mathbb{1}\text{ to be valid POVM for all } x} \otimes B_{b|y}\Big]$$

$$= P(b|y) \quad \text{independent of } x.$$

Similarly: $\sum_b P(a,b|x,y) = P(a|x)$ independent of y.

These conditions are called NON-SIGNALLING conditions.

– Must be satisfied by ANY REASONABLE THEORY

Question: what correlations $P(a,b|x,y)$ are consistent with no-signalling?

Conditions: $P(a,b|x,y) \geq 0 \quad \forall\ a,b,x,y$ — probs. non-negative

$$\sum_{a,b} P(a,b|x,y) = 1 \quad \forall\ x,y$$ probs. normalised — $m_A m_B$

$$\sum_{a} P(a,b|x,y) = P(b|y)$$ no-signalling $A \rightarrow B$ — $o_B(m_A - 1)m_B$

$$\sum_{b} P(a,b|x,y) = P(a|x)$$ no-signalling $B \rightarrow A$ — $o_A(m_B - 1)m_A$

Can collect $p(a,b|x,y)$ together into a vector $\underline{p} \in \mathbb{R}^d$ $\quad d = o_A o_B m_A m_B \quad$ CHSH: 16

- Linear equality constraints constrain $\underline{p}$ to lie in lower dimensional hypersurface
  $\left[d' = m_A(o_A - 1) + m_B(o_B - 1) + (o_A - 1)(o_B - 1)m_A m_B\right]$
  CHSH: 8

- Linear inequality constraints constrain $\underline{p}$ to lie in non-negative orthant

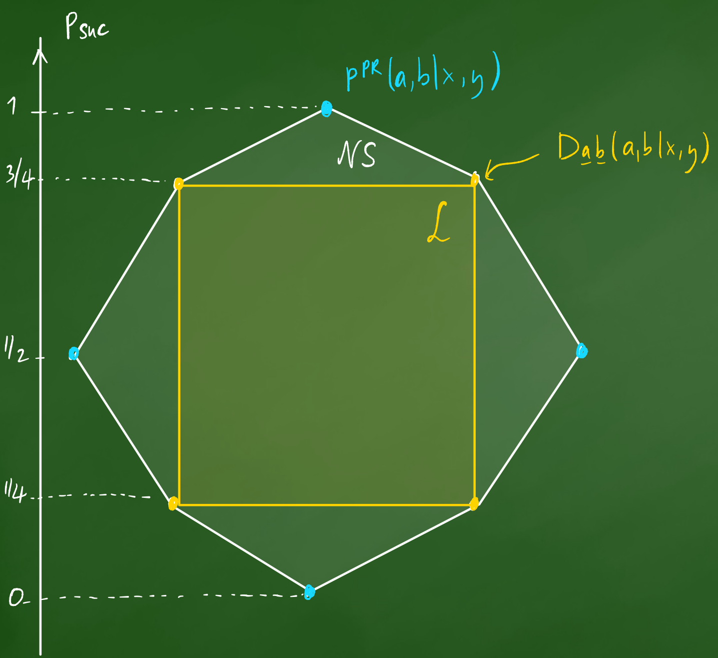
→ Geometrically set of non-signalling correlations lie in a convex polytope

↑ generalisation of polygon

– called Non-Signalling polytope

- finite number of vertices / extreme points
- all faces are flat

CHSH 'cartoon':



<u>24 vertices</u>

– 16 local deterministic strategies $D_{\underline{ab}}(a,b|x,y)$

→ these vertices define a polytope too – local polytope

Recall: local correlations are mixtures of det. strats.

– this is geometrical perspective / understanding

– 8 nonlocal 'Popescu-Rohrlich boxes' (PR boxes)

'maximally' nonlocal & win CHSH game perfectly + Symmetries

$$p^{PR}(a,b|x,y) = \begin{cases} \frac{1}{2} & \text{if } a \oplus b = xy \\ 0 & \text{otherwise} \end{cases}$$ ← winning condition

e.g. $P(0,0|0,0) = \frac{1}{2}$, $P(0,1|0,1) = 0$ etc

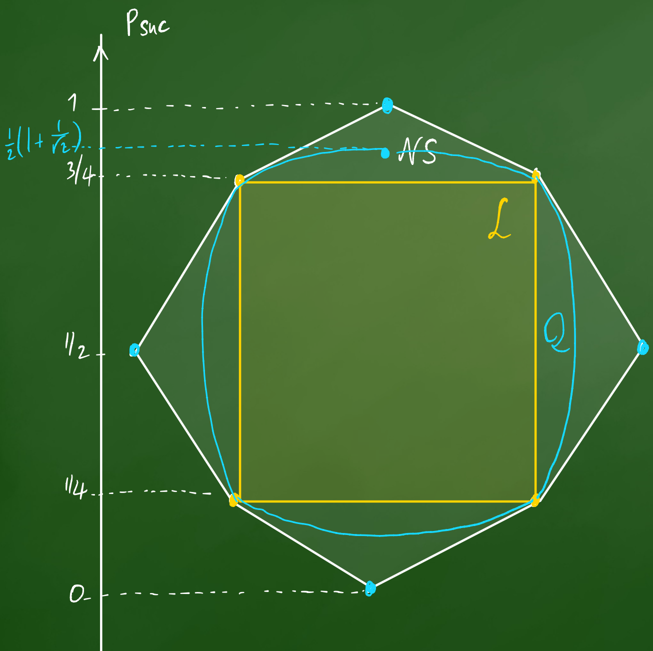
Symmetries of CHSH: Change winning condition: $a \oplus b = \underset{\text{flip } x}{(x \oplus \alpha)}\underset{\text{flip } y}{(y \oplus \beta)} \oplus \underset{\text{flips winning condition}}{\gamma}$

- 8 unique symmetries $\longrightarrow$ 1 × PR box perfectly winning each strat.

- PR box correlations cannot arise in QM.

↳ Question: what can arise?

Answer: Complicated!



Intuitive explanation: $P(a,b|x,y)$ can arise from measuring arbitrary dimensional quantum system (even CV system, like x, p)

- Q is union of correlations that can arise from every single quantum state!

- No closed-form expression for set Q

In fact … (uncomputable?)

Approximating Quantum Correlations

- In many device-independent applications want to restrict to quantum nonlocality
  - Often good enough to have bounds: " best case cannot be better than ... "
    " worst case cannot be worse than ... "

→ For this we need outer approximation to set Q.

* Fortunately we have a sequence of approximations $Q^{(1)} \supseteq Q^{(2)} \dots \supseteq Q$
  each of which is "simple" = feasible set of semidefinite program (of increasing size)

  Called Navascues-Pironio-Acin (NPA) hierarchy.

Principles for quantum Correlations

- Realisation that nonlocality beyond QM that is still consistent with principle of no-signalling lead to important question: Is there a physical or information theoretic reason why nonlocality should be limited?

lead to search to principles satisfied by quantum nonlocality but violated by post-QM NL

Candidates:
- communication complexity should be non-trivial
- macroscopic world should be local
- 'Information causality'
- 'local orthogonality'

none known to single out quantum NL

(ask me for me info^{n} if interested!)