All quantum measurements are asymptotically equivalent

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Introduction

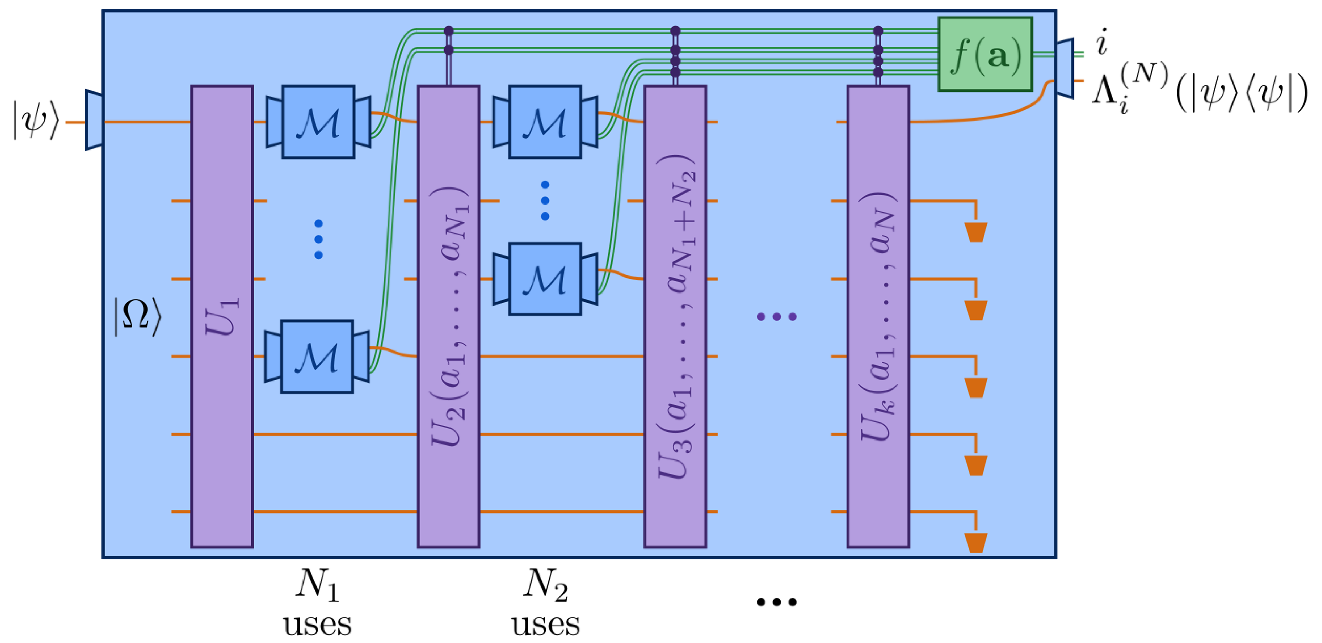
- Classical information is obtained about the quantum state of a system through measurement.
- In practice, never possible to perfectly perform a target measurement.
- Different architectures have fundamental/engineering constraints making certain target measurements demanding.

Main question

How well can a target measurement be approximated when making use of only a fixed available measuring device?

General set-up

- General set-up we consider:



- N uses of **instrument** $\mathcal{M}$ in k rounds.
- Arbitrary ancillary system $|\Omega\rangle$.
- Arbitrary classically-controlled global unitaries.
- **Goal**: Optimally approximate target instrument i.e. recover both measurement statistics and post-measurement state for arbitrary input state.

It is possible to perform arbitrarily well **any** measurement given the ability to perform any* fixed measurement enough times.

* non-trivial

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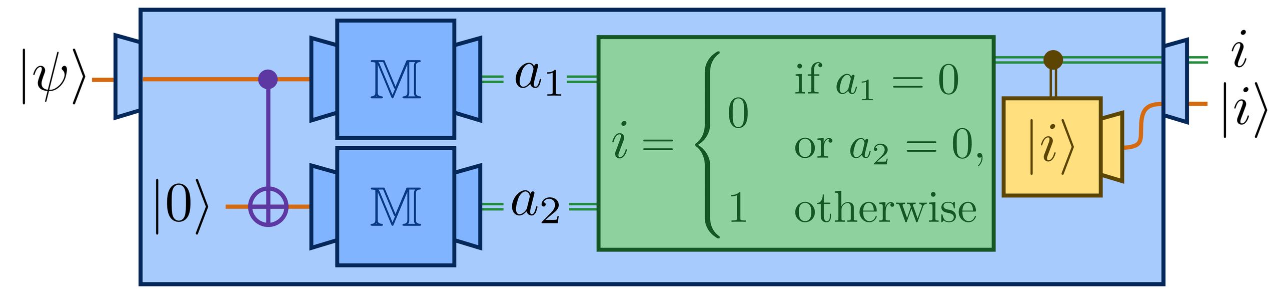
Illustrative example: 'classical cloning' protocol

- Before measuring POVM $\mathbb{M}$, first 'clone' comp. basis $|i\rangle \mapsto |i\rangle\cdots|i\rangle$ (i.e. CNOTS).
- Measure system and each clone using $\mathbb{M}$, performing N measurements in total.
- Process string of outcomes $\boldsymbol{a} = a_1\cdots a_N$ into single final outcome $i = i(\boldsymbol{a})$, and prepare state $|i\rangle$, e.g.



Sub-routine 1: von Neumann measurement reproduction

- Given ability to perform POVM $\mathbb{M}$ N times, it is possible to reproduce a von Neumann measurement arbitrarily well.
- von Neumann measurement equivalent to perfectly discriminating a basis.
- Protocol uses generalised classical-cloning.
- Error in reproduction drops exponentially in N.

Sub-routine 2: von Neumann to general measurement reproduction

- Given ability to perform von Neumann measurement, it is possible to perform a completely general measurement.
- Consider $\mathcal{T} = \{\Gamma_i\}_i$, where $\Gamma_i(\cdot) = \sum_j K_j^i(\cdot)K_j^{i\dagger}$.
- Prepare ancillary particles in state $|0\rangle|0\rangle$, and apply $V|\psi\rangle|0\rangle|0\rangle = \sum_{i,j} K_i^j|\psi\rangle|i\rangle|j\rangle$.
- Measuring final particle with von Neumann measurement reproduces target measurement.

Main results

- In combination, sub-routines 1 and 2 imply that any target measurement can be approximated arbitrarily well, given the ability to perform any (non-trivial) fixed measurement.
- Available and target measurement can be **completely different**, e.g. 'trine' measurement and von Neumann.
- Using **block-coding** techniques, protocols can further be made **finite rate**.