9^{th} Maths - Chapter 7

This is Problem-5 from Exercise 7.1

1. Line l is the bisector of an angle $\angle A$ and B is a point on line l. $BP = BQ$ are perpendiculars from B to the arms of $\angle A$.
   (a) $\triangle APB \cong \triangle AQB$
   (b) $BP = BQ$ or B is equidistant from the arms of $\angle A$

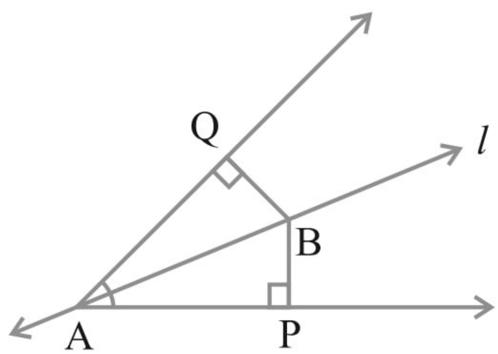

Figure 1: l bisecting $\angle A$

Construction:
The input parameters for the construction are shown in Table

Symbol	Value	Description
θ	$30°$	$\angle BAQ = \angle BAP$
a	7	Length of AB
c	6	Length of AQ
$\mathbf{e}_1$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	Basis vector

Table 1: values and descriptions.

Let $\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\mathbf{B} = a\mathbf{e_1}$, $\mathbf{Q} = \begin{pmatrix} c\cos\theta \\ c\sin\theta \end{pmatrix}$, and $\mathbf{P} = \begin{pmatrix} c\cos\theta \\ -c\sin\theta \end{pmatrix}$.

Solution:

Given:

$$\angle BAQ = \angle BAP \quad \text{(line `l' bisects } \angle A) \tag{1}$$

$$\angle AQB = \angle APB \quad \left(\text{Both angles are } 90^{\circ}\right) \tag{2}$$

To Prove:

1. $\triangle APB \cong \triangle AQB$
2. $BP = BQ$ or B is equidistant from the arms of $\angle A$

Proof:

In $\triangle APB$ and $\triangle AQB$

$$\angle BAQ = \angle BAP \quad \text{(line `l' bisects } \angle A) \tag{3}$$

$$\angle AQB = \angle APB \quad \left(\text{Both angles are } 90^{\circ}\right) \tag{4}$$

$$AB = AB \quad \text{(common side for both triangles)} \tag{5}$$

Therefore, by Angle-Angle-Side (A-A-S) congruence rule, $\triangle APB \cong \triangle AQB$.
Now we have to prove $BP = BQ$

$$\|\mathbf{B} - \mathbf{P}\| = \sqrt{(\mathbf{B} - \mathbf{P})^\top (\mathbf{B} - \mathbf{P})} \tag{6}$$

$$(\mathbf{B} - \mathbf{P}) = \begin{pmatrix} 7 \\ 0 \end{pmatrix} - \begin{pmatrix} 6\cos\theta \\ -6\sin\theta \end{pmatrix} \implies \begin{pmatrix} 7 - 6\cos\theta \\ -6\sin\theta \end{pmatrix} \implies \begin{pmatrix} 7 - 5.19 \\ -3 \end{pmatrix} \implies \begin{pmatrix} 1.81 \\ -3 \end{pmatrix} \tag{7}$$

$$(\mathbf{B} - \mathbf{P})^\top = \begin{pmatrix} 7 - 6\cos\theta & -6\sin\theta \end{pmatrix} \implies \begin{pmatrix} 1.81 & -3 \end{pmatrix} \tag{8}$$

$$(\mathbf{B} - \mathbf{P})^\top (\mathbf{B} - \mathbf{P}) = \begin{pmatrix} 1.81 & -3 \end{pmatrix} \begin{pmatrix} 1.81 \\ -3 \end{pmatrix} \implies 12.27 \tag{9}$$

$$\|\mathbf{B} - \mathbf{P}\| = \sqrt{12.27} = 3.5 \tag{10}$$

$$Similarly Now, \|\mathbf{B} - \mathbf{Q}\| = \sqrt{(\mathbf{B} - \mathbf{Q})^\top (\mathbf{B} - \mathbf{Q})} \tag{11}$$

$$(\mathbf{B} - \mathbf{Q}) = \begin{pmatrix} 7 \\ 0 \end{pmatrix} - \begin{pmatrix} 6\cos\theta \\ 6\sin\theta \end{pmatrix} \implies \begin{pmatrix} 7 - 6\cos\theta \\ 6\sin\theta \end{pmatrix} \implies \begin{pmatrix} 7 - 5.19 \\ 3 \end{pmatrix} \implies \begin{pmatrix} 1.81 \\ 3 \end{pmatrix} \tag{12}$$

$$(\mathbf{B} - \mathbf{Q})^\top = \begin{pmatrix} 7 - 6\cos\theta & 6\sin\theta \end{pmatrix} \implies \begin{pmatrix} 1.81 & 3 \end{pmatrix} \tag{13}$$

$$(\mathbf{B} - \mathbf{Q})^\top ((\mathbf{B} - \mathbf{Q}) = \begin{pmatrix} 1.81 & 3 \end{pmatrix} \begin{pmatrix} 1.81 \\ 3 \end{pmatrix} \implies 12.27 \tag{14}$$

$$\|\mathbf{B} - \mathbf{Q}\| = \sqrt{12.27} = 3.5 \tag{15}$$

$$\|\mathbf{B} - \mathbf{P}\| = \|\mathbf{B} - \mathbf{Q}\| \tag{16}$$

Therefore, $BP = BQ$ or B is equidistant from the arms of $\angle A$ is proved

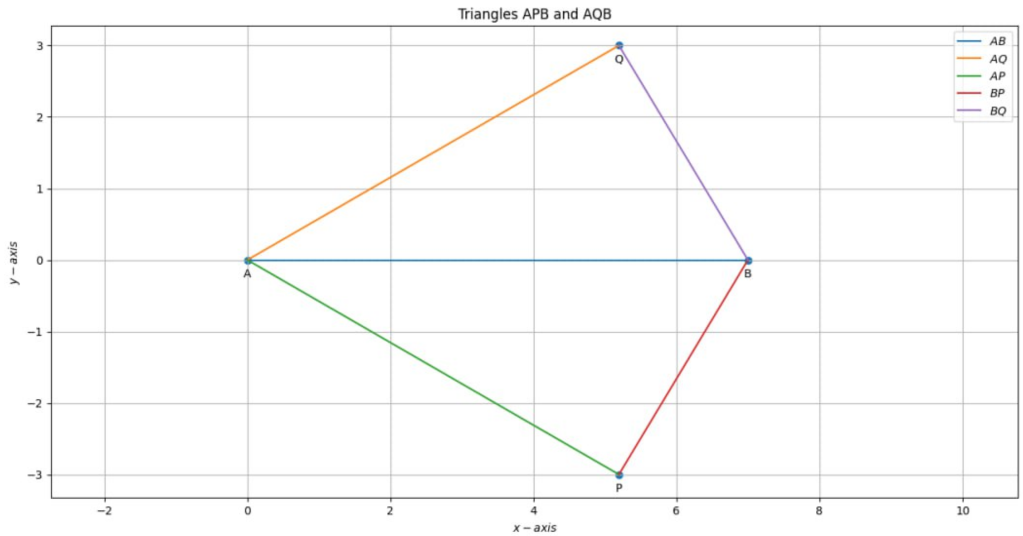

Figure 2: python graph plot