

Heat Advection Tests

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1 One-dimensional heat advection via a single-phase fluid

Consider the case of a single-phase fluid in 1 dimension, $0 \leq x \leq 1$, with the porepressure fixed at the boundaries:

$$P(x=0, t) = 1 \quad \text{and} \quad P(x=1, t) = 0 . \quad (1.1)$$

With zero gravity, and high fluid bulk modulus, the Darcy equation implies that the solution is $P(x, t) = 1 - x$, with

$$v = k/\mu \quad (1.2)$$

being the constant¹ “Darcy velocity” from $x = 0$ to $x = 1$. Here k is the porous medium’s permeability, and μ is the fluid dynamic viscosity.

Suppose that the fluid internal energy is given by CT , where C is the specific heat capacity and T is its temperature. Assuming that $P/\rho \ll CT$, then the fluid’s enthalpy is also CT . In this case, the energy equation reads

$$((1 - \phi)\rho_R C_R + \phi \rho C) \frac{\partial T}{\partial t} + C \rho v \frac{\partial T}{\partial x} = 0 . \quad (1.3)$$

This is the wave equation with velocity

$$v_T = \frac{C \rho v}{(1 - \phi)\rho_R C_R + \phi \rho C} . \quad (1.4)$$

Recall that the “Darcy velocity” is $v = k/\mu$.

Let the initial condition for T be $T(x, t=0) = 200$. Apply the boundary conditions

$$T(x=0, t) = 300 \quad \text{and} \quad T(x=1, t) = 200 . \quad (1.5)$$

At $t = 0$ this creates a front at $x = 0$. Choose the parameters $C = 2$, $C_R = 1$, $\rho = 1000$, $\rho_R = 125$, $\phi = 0.2$, $k = 1.1$, $\mu = 4.4$ (all in consistent units), so that $v_T = 1$ is the front’s velocity.

The sharp front is *not* maintained by MOOSE. This is due to numerical diffusion, which is particularly strong in the upwinding scheme implemented in the PorousFlow module. Nevertheless, MOOSE advects the smooth front with the correct velocity, as shown in Figure 1.1.

¹To get the velocity of the individual fluid particles, this should be divided by the porosity.

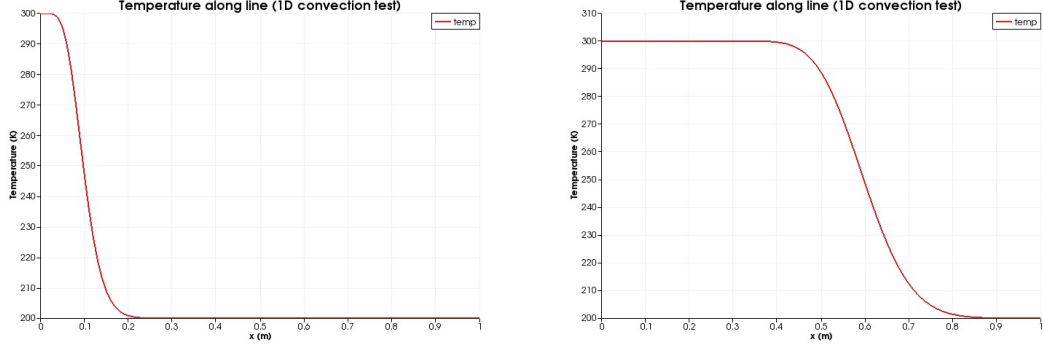


Figure 1.1: Results of heat advection via a fluid in 1D. The fluid flows with constant Darcy velocity of 0.25 m.s^{-1} to the right, and this advects a temperature front at velocity 1 m.s^{-1} to the right. This pictures above that the numerical implementation of porous flow within MOOSE diffuses sharp fronts, but advects them at the correct velocity (notice the centre of the front is at the correct position in each picture). Left: temperature $t = 0.1 \text{ s}$. Right: temperature at $t = 0.6 \text{ s}$.