RANDOMIZED RESPONSE: A SURVEY TECHNIQUE FOR ELIMINATING EVASIVE ANSWER BIAS

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For various reasons individuals in a sample survey may prefer not to confide to the interviewer the correct answers to certain questions. In such cases the individuals may elect not to reply at all or to reply with incorrect answers. The resulting evasive answer bias is ordinarily difficult to assess. In this paper it is argued that such bias is potentially removable through allowing the interviewee to maintain privacy through the device of randomizing his response. A randomized response method for estimating a population proportion is presented as an example. Unbiased maximum likelihood estimates are obtained and their mean square errors are compared with the mean square errors of conventional estimates under various assumptions about the underlying population.

1. INTRODUCTION

For reasons of modesty, fear of being thought bigoted, or merely a reluctance to confide secrets to strangers, many individuals attempt to evade certain questions put to them by interviewers. In survey vernacular, these people become the "non-cooperative" group [5, pp. 235–72], either refusing outright to be surveyed, or consenting to be surveyed but purposely providing wrong answers to the questions. In the one case there is the problem of refusal bias [1, pp. 355–61], [2, pp. 33–6], [5, pp. 261–9]; in the other case there is the problem of response bias [3, p. 89], [4, pp. 280–325].

The questions that people tend to evade are the questions which demand answers that are too revealing. Innocuous questions ordinarily receive good response, but questions requiring personal or controversial assertions excite resistance. When resistance is encountered, the usual modification of the survey method is simply an added effort on the part of the interviewer to gain the confidence of the interviewee. There is, however, a natural reticence of the general individual to confide certain things to anyone—let alone a stranger—and there is also a natural reluctance to have confidential statements on a paper containing his name and address. For some questions at least, probably only limited gains are possible through trying to persuade the interviewee that he surrenders little by confiding to the interviewer.

This paper suggests an alternate method for increasing cooperation. The method is built on the premise that cooperation should be naturally better if the questions allow answers which reveal less even to the interviewer. Essentially the method involves the device that—for certain questions not already innocuous—the interviewee responds with answers that furnish information only on a probability basis. As an example, one application might involve the interviewee's only making a true statement with a given probability less than 1. In this case, even the interviewer would know only the probability that the given answer was true. Inasmuch as this type of answer is less revealing than an answer required to be truthful with probability 1, it is suggested that this type of approach may encourage greater cooperation for certain survey problems. As another more detailed application of the randomized response method, the following section outlines a particular model for estimating a population proportion. The resulting estimates are then compared with conventional estimates under various assumptions about the cooperation of those interviewed.

2. A RANDOM RESPONSE MODEL FOR PROPORTIONS

Suppose that every person in a population belongs to either Group A or Group B and it is required to estimate by survey the proportion belonging to Group A. A simple random sample of n people is drawn with replacement from the population and provisions made for each person to be interviewed. Before the interviews, each interviewer is furnished with an identical spinner with a face marked so that the spinner points to the letter A with probability p and to the letter B with probability $(1-p)$. Then, in each interview, the interviewee is asked to spin the spinner unobserved by the interviewer and report only whether or not the spinner points to the letter representing the group to which the interviewee belongs. That is, the interviewee is required only to say yes or no according to whether or not the spinner points to the correct group; he does not report the group to which the spinner points. Under the assumption that these yes and no reports are made truthfully, maximum likelihood estimates of the true population proportion are straightforward.

Let

$\pi =$ the true probability of A in the population,
$p =$ the probability that the spinner points to A, and

$$X_i = \begin{cases} 1 \text{ if the } i\text{th sample element says yes} \\ 0 \text{ if the } i\text{th sample element says no.} \end{cases}$$

Then

$$P(X_i = 1) = \pi p + (1 - \pi)(1 - p),$$
$$P(X_i = 0) = (1 - \pi)p + \pi(1 - p),$$

and arranging the indexing of the sample so that the first n_1 report "yes" while the second $(n-n_1)$ report "no," the likelihood of the sample is

$$L = [\pi p + (1 - \pi)(1 - p)]^{n_1}[(1 - \pi)p + \pi(1 - p)]^{n-n_1}. \tag{1}$$

The log of the likelihood is

$$\log L = n_1 \log [\pi p + (1 - \pi)(1 - p)] + (n - n_1) \log [(1 - \pi)p + \pi(1 - p)], \tag{2}$$

and necessary conditions on π for a maximum are

$$\frac{(n - n_1)(2p - 1)}{(1 - \pi)p + \pi(1 - p)} = \frac{n_1(2p - 1)}{\pi p + (1 - \pi)(1 - p)}$$

or

$$\pi p + (1 - \pi)(1 - p) = \frac{n_1}{n} \cdot \tag{3}$$

Then, supposing $p \neq 1/2$, the maximum likelihood estimate of π is

$$\hat{\pi} = \frac{p-1}{2p-1} + \frac{n_1}{(2p-1)n}. \tag{4}$$

The expected value of the estimate is

$$\begin{aligned} E\hat{\pi} &= \frac{1}{2p-1}\left[p - 1 + (1/n)\sum^{n} EX_i\right] \\ &= \frac{1}{2p-1}[p - 1 + \pi p + (1-\pi)(1-p)] \\ &= \pi, \end{aligned} \tag{5}$$

and the variance of $\hat{\pi}$ is

$$\begin{aligned} \operatorname{Var} \hat{\pi} &= \frac{n \operatorname{Var} X_i}{(2p-1)^2 n^2} \\ &= \frac{[\pi p + (1-\pi)(1-p)][(1-\pi)p + \pi(1-p)]}{(2p-1)^2 n} \\ &= \frac{1/4 + (2p^2 - 2p + 1/2)(-2\pi^2 + 2\pi - 1/2)}{(2p-1)^2 n} \\ &= \frac{1}{n}\left[\frac{1}{16(p-1/2)^2} - (\pi - 1/2)^2\right]. \end{aligned} \tag{6}$$

Expression (5) shows $\hat{\pi}$ is an unbiased estimate of the true population proportion π.[1] Moreover, since $\hat{\pi}$ is a maximum likelihood estimate and any useful n's are apt to be large, $\hat{\pi}$ may be assumed normally distributed about π with the variance indicated in expression (6). Thus all the usual confidence intervals are easily established. Expression (6) also sets out the separate dependence of the variance of $\hat{\pi}$ upon the choice of p. In fact, identifying

$$\frac{\frac{1}{4} - (\pi - 1/2)^2}{n} = \frac{\pi(1-\pi)}{n}$$

as the variance due to sampling and writing expression (6) as

$$\operatorname{Var} \hat{\pi} = \frac{\frac{1}{4} - (\pi - 1/2)^2}{n} + \frac{\frac{1}{16(p-1/2)^2} - \frac{1}{4}}{n}, \tag{7}$$

it is clear that the variance of $\hat{\pi}$ can be expressed as the sum of the variance due to sampling plus the variance due to the random device.

Two practical questions concern the estimation method implied by $\hat{\pi}$. First, how likely are people to cooperate and tell the truth when asked to respond in

[1] The possibility of $\hat{\pi}$ taking values outside the 0–1 range cannot be ruled out, but this possibility is remote in large samples.

the manner described? Second, how large a sample is required to obtain various degrees of precision by this estimate as compared to the conventional estimate?

The first question is primarily an empirical question, but the rationale for expecting better cooperation is clear. The individual being interviewed is asked for less. The matter of how much less is summarized by the parameter p. Note first from expression (1) that if $p=1/2$, the likelihood function does not even depend on π. Thus, for a $p=1/2$, the interviewee would be furnishing no information at all. Then note that if $p=1$, the entire procedure would reduce to the conventional procedure of requiring the individual to state unreservedly whether or not he belonged to Group A. For p's between 1/2 and 1 (or between 1/2 and 0) the person interviewed provides useful but not absolute information as to exactly which group he is in. In this context the p can be thought of as describing the nature of the cooperation between the interviewer and the interviewee. As p goes from 1 to 1/2 the burden of cooperating passes from the interviewee to the interviewer. It therefore seems reasonable to expect that for some questions at least, p's less than 1 should induce greater cooperation on the part of the person interviewed.

The question of the sample size required for a given level of precision also depends on the parameter p. If a p close to 1 (or close to 0) is adequate to insure cooperation, then a smaller sample size is required than if a p close to 1/2 is required to insure cooperation. Values of p close to 1/2 convey less information from each interview, thus they also imply either a larger variance of the estimate or a larger sample size. Substituting values of p in expression (6) sets out the precise relation. As an example, supposing a $\pi=.5$ and a p halfway between the zero and full information points, i.e., a p of .75, the variance shown by (6) is $1/n$. This would imply that the sample size should be about 400 in order to secure a standard deviation of .05. By way of comparison, the conventional estimation method (equivalent to a $p=1$) would imply that a sample of only about 100 would be sufficient for a standard deviation of .05—provided that all the interviewees told the truth for the regular method.

The more pertinent comparisons are between the randomized estimates and regular estimates under the assumption that the regular estimates are handicapped by less than 100 per cent truthfulness. Suppose that in a regular survey all consent to be surveyed, but members of Group A tell the truth only with probability T_a and members of Group B tell the truth only with probability T_b. Then, if $Y_i=1$ or 0 according as the ith member of the sample reports he is or is not in Group A, the conventional estimate of the true population proportion π is

$$\hat{\pi} = \frac{\sum^{n} Y_i}{n}. \tag{8}$$

The expected value, response bias [3, p. 89], and variance of this regular estimate are given by

$$E\hat{\pi} = \pi T_a + [(1-\pi)(1-T_b)], \tag{9}$$

$$\text{Bias } \hat{\pi} = E(\hat{\pi} - \pi)$$
$$= \pi[T_a + T_b - 2] + [1 - T_b], \text{ and} \tag{10}$$
$$\text{Var } \hat{\pi} = \frac{[\pi T_a + (1-\pi)(1-T_b)][1 - \pi T_a - (1-\pi)(1-T_b)]}{n}. \tag{11}$$

Tables 1 and 2 then compare the mean square errors (the variance plus the square of the bias) of the randomized and regular methods of estimation under the assumption that the interviewed individuals tell the truth in the randomized method but only tell the truth in the non-random method with probabilities given by T_a and T_b. The left-hand two columns of each table indicate various

TABLE 1. COMPARISON OF RANDOMIZED AND REGULAR ESTIMATES FOR TRUE PROBABILITY OF A=.6 AND n=1000

Regular Estimates			Mean Square Error Randomized / Mean Square Error Regular			
Probability of Truth T_a	T_b	Bias	p=.6	p=.7	p=.8	p=.9
.95	1.00	−.03	5.45	1.36	.60	.33
.90	1.00	−.06	1.62	.40	.18	.10
.70	1.00	−.18	.19	.05	.02	.01
.50	1.00	−.30	.07	.02	.01	.00
1.00	.95	.02	9.82	2.44	1.08	.60
1.00	.90	.04	3.41	.85	.37	.21
1.00	.70	.12	.43	.11	.05	.03
1.00	.50	.20	.16	.04	.02	.01
.95	.95	−.01	18.25	4.54	2.00	1.11
.90	.90	−.02	9.70	2.41	1.06	.59
.70	.70	−.06	1.62	.40	.18	.10
.50	.50	−.10	.61	.15	.07	.04

TABLE 2. COMPARISON OF RANDOMIZED AND REGULAR ESTIMATES FOR TRUE PROBABILITY OF A=.5 AND n=1000

Regular Estimates			Mean Square Error Randomized / Mean Square Error Regular			
Probability of Truth T_a	T_b	Bias	p=.6	p=.7	p=.8	p=.9
.95	1.00	−.03	7.15	1.79	.79	.45
.90	1.00	−.05	2.28	.57	.25	.14
.70	1.00	−.15	.28	.07	.03	.02
.50	1.00	−.25	.10	.03	.01	.01
.95	.95	.00	25.00	6.25	2.78	1.56
.90	.90	.00	25.00	6.25	2.78	1.56
.70	.70	.00	25.00	6.25	2.78	1.56
.50	.50	.00	25.00	6.25	2.78	1.56

paired values for T_a and T_b. The third column shows the bias of the non-random method, and the remaining columns exhibit the ratios of the mean square errors of the randomized estimates to the mean square errors of the regular estimates for various values of p. Tables 1 and 2 are respectively appropriate for the cases where the true probability of A is .6 and .5. The sample size is set at 1000 in each case.

3. CONCLUSIONS

Both tables are constructed under the assumption that the p in each case is low enough to induce full cooperation in the randomized approach. Thus the advantages of the randomized method, shown by those ratios in the tables that are less than 1, are in the nature of potential advantages that depend upon the cooperation actually achieved by the randomized method. Nevertheless, there is the clear suggestion that the randomized method is apt to out-perform the regular method in a variety of situations. Table 1 with $T_a=1$ and $T_b=.9$, for example, exhibits the situation in which members of the minority population B resent directly confiding to their interviewer their minority status to the point where ten per cent of them say A instead of B. The bias created is $+.04$, and the ratio of mean square errors varies from 3.41 to .21, depending on the value of p. The possible improvement through randomization in this case is evident. An even greater improvement is possible if it is the larger population that hesitates to identify itself openly. This latter case is exemplified by the row in which $T_a=.9$ and $T_b=1$.

More generally it is to be observed that—except for the cases where the bias of the regular estimate is 0 or negligible—there appear to be sizable potential gains through the randomized response. It should also be kept in mind that the potential advantages of randomizing are even larger for larger samples. For example, a sample size of 2000 would imply that the entry in Table 1, column 4, row 2, would change from 1.62 to .84. Thus the randomized method is to be preferred in this instance even if a p as low as .6 is required to assure cooperation.

The question is still open as to what methods of randomized response will prove the most useful. Even with regard to estimating proportions, the method set out in Section 2 is only one of many possibilities. It is interesting to note in this connection that a mathematically equivalent model to the one of Section 2 is furnished by simply requiring each interviewee to make a statement that is true with probability p as to which of the two groups he is in. Thus in this model, the interviewee, again out of sight of the interviewer, spins a spinner which points to "true" with given probability p and to "false" with probability $(1-p)$. Then the interviewee makes a statement that is true or false according to the way the spinner pointed. Psychologically this would appear to be quite a different model from that of Section 2, but the statistical properties of the two models are equivalent.[2] The maximum likelihood estimate for the latter scheme has the same form and the same variance as the estimate of Section 2. There is

[2] As before, a p of $\frac{1}{2}$ furnishes no information, a p of 1 furnishes full information, and other p's furnish information depending on how far they are from $\frac{1}{2}$. It is a feature of the dichotomous nature of the population that telling the truth .2 of the time is equivalent to telling the truth .8 of the time.

thus a question as to which of these or other equivalent randomized models is to be preferred from the standpoint of increasing cooperation.

Finally, it should be noted that it is easy to extend the randomized response technique to estimate distributions other than that appropriate to a simple dichotomous variable. As one example, the technique could be applied to estimate a five-class income distribution through the obvious device of estimating the proportion in each class separately by the method of Section 2. In this case each interviewee might be simply asked to make five separate randomized responses concerning whether or not he was in each of the five separate classes. Just as with the proportion problem, it is clear that other randomized response methods may be imagined for this more general estimation problem. And just as with the proportion problem, the question of which specific technique will prove superior is a matter for empirical investigation.

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