Shallow Intro to Deep Learning'19

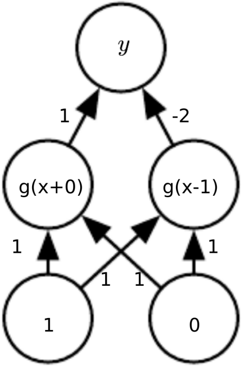
Feed Forward, Chapters 6, 6.1, 4.3, 5.9

Delft University of Technology

Lecturer: Jan van Gemert

What is the outcome of this Feed Forward net?

Book: chapter 6, 6.1



$$g(z) = \max\{0, z\}$$

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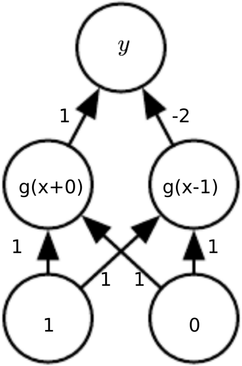
Outcome: 1



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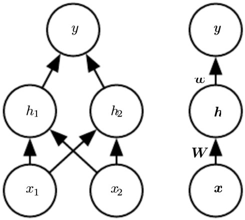
Deep Feed forward Networks (Multi-layered perceptron)

- Approximate some function f^*
- Eg: classifier $y = f^*(x)$
- Learn parameters θ for mapping $y = f(x; \theta)$



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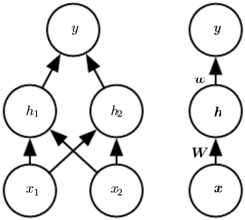

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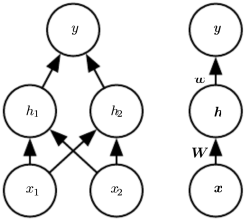

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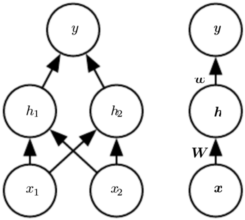

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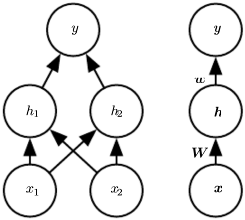

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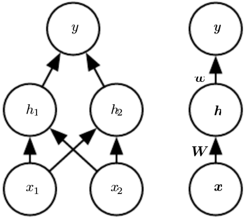

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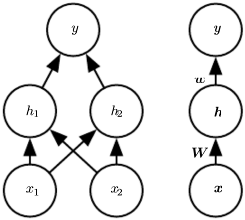

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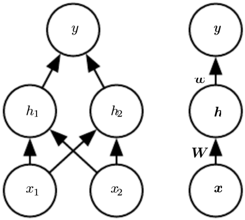

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Goal: drive $f(x)$ to match $f^*(x)$

Training a network

Chapter 4.3

Training set:

Data		Label
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1	1	0
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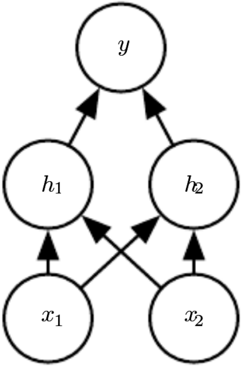
While not converged:

1. Present a training sample
2. Compare the results
3. Update the weights



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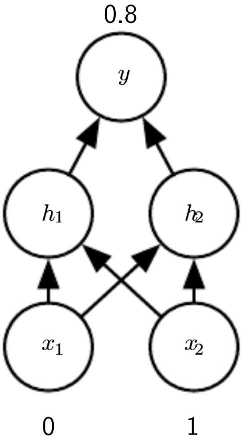

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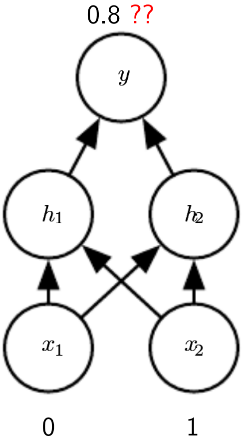

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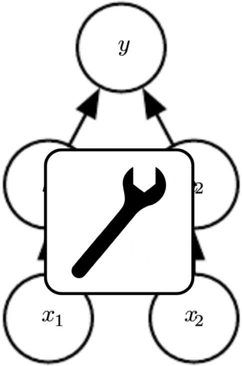

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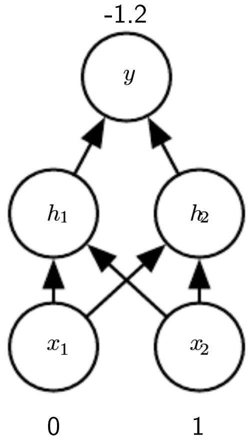

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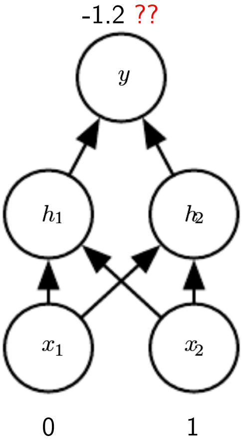

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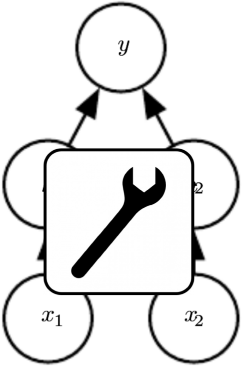

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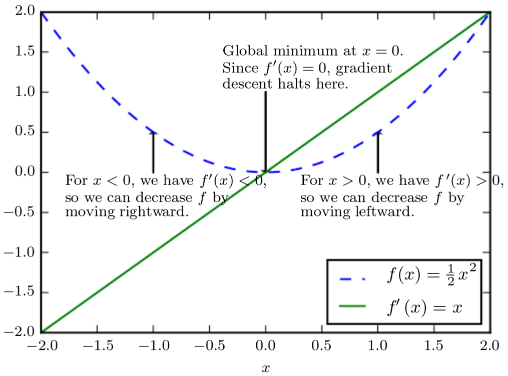

Stochastic Gradient Descent

Chapter 5.9

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This is the procedure we've already seen.

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Chapter 4.3

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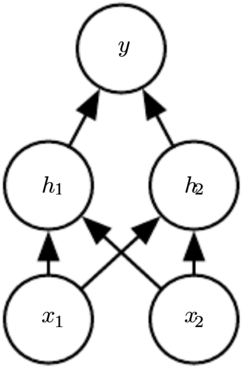

Questions?

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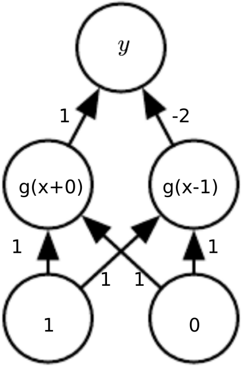
- Q: What is the equation of the output unit?



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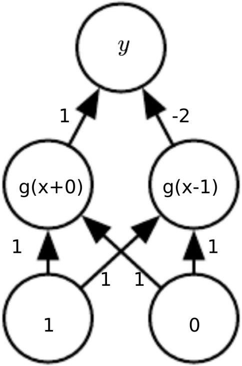

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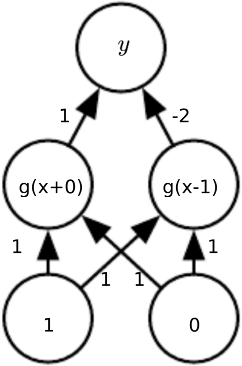

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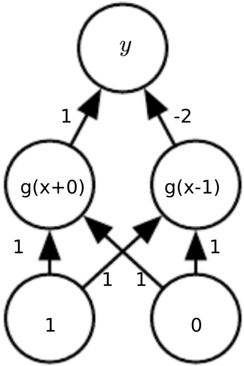

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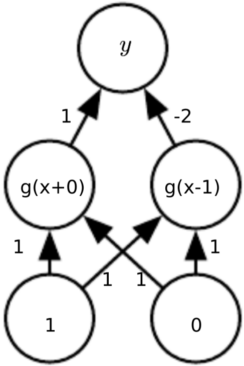

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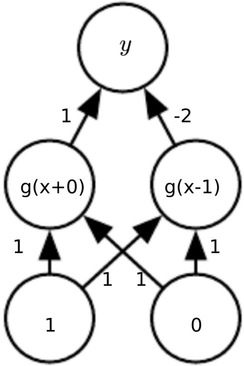

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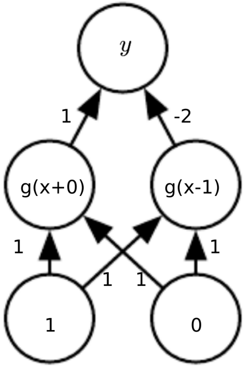

Three ways to make the input non-linear:

1. Generic kernel function
2. Designing feature extractors
3. Learn it..

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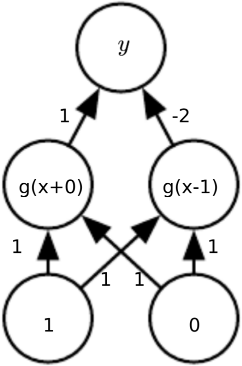

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3. Learn it.. $\quad y = f(x, \theta, w, b) = \phi(x, \theta)^\top w + b$

Deep Feed forward Networks (Multi-layered perceptron)

- Q: What is the equation of the output unit?
  A: A linear model, $f(x, w, b) = x^\top w + b$
- Linear is good, but possibly too rigid
- Extend to non-linear by applying a non-linear transformation $\phi(x)$ on the input
- The non-linear equation of f is
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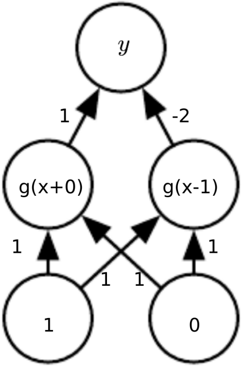

Three ways to make the input non-linear:

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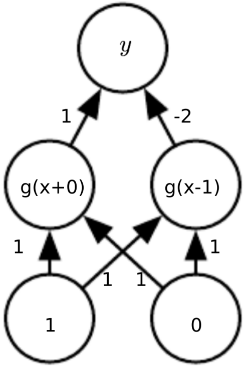

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Step 3 allows to add knowledge to restrict 1 but more flexible than 2.
It learns the representation (features)

Example: XOR

- Input: $\mathbb{X} = \{[0,0]^\top, [0,1]^\top, [1,0]^\top, [1,1]^\top\}$
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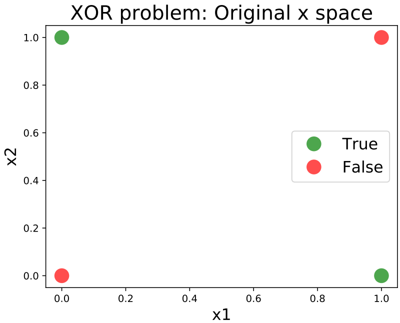

Can it learn this?

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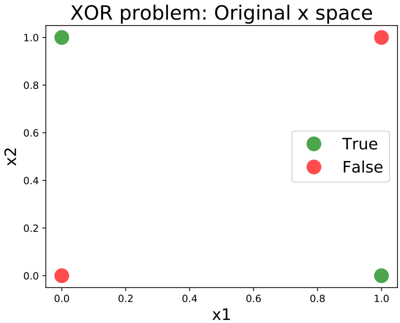

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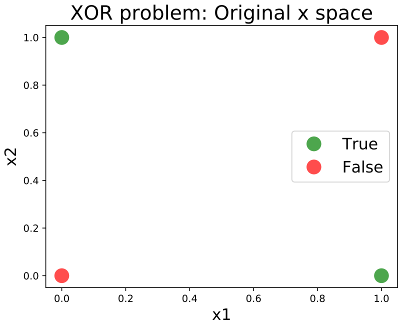

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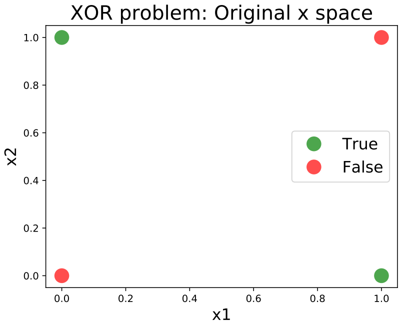

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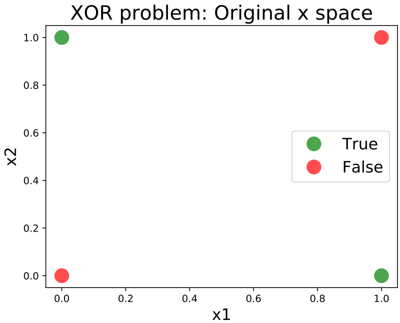

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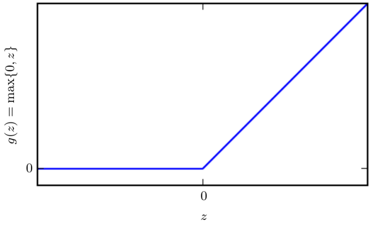

Solution to XOR

Full model $f(x, W, c, w, b) = w^\top \max\{0, W^\top x + c\} + b$

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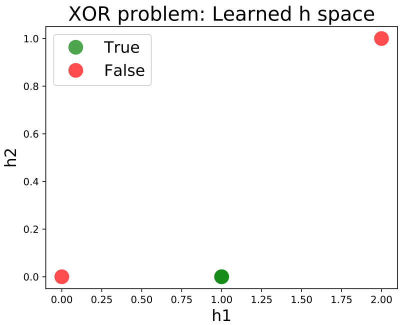

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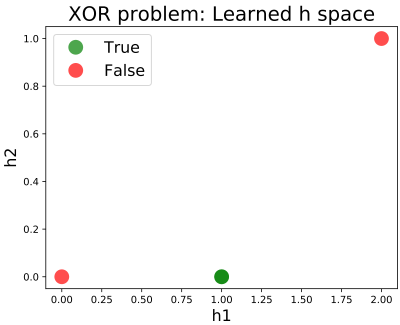

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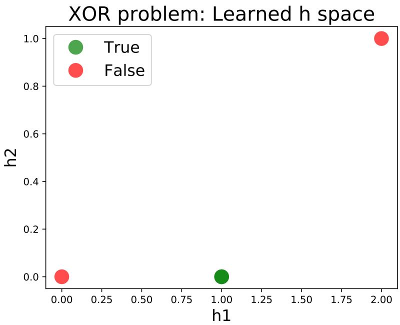

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Questions?

Summary

- Feed forward networks
- How to train them using SGD
- How representations are learned (XOR example)