

EXERCISE #3 - Distributions - ANSWER KEY

1. What is a distribution?

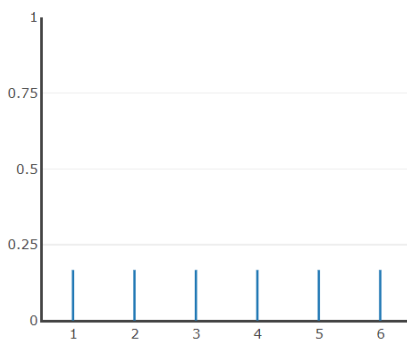
A distribution describes all of the probable outcomes of a variable.

2. What is the difference between a discrete and a continuous distribution?

In a discrete distribution, the sum of all the individual probabilities must equal 1.

In a continuous distribution, the area under the probability curve equals 1.

3. Fill in the graph below to show the probability mass function of the outcomes of a fair die. What is the name of this type of distribution where the probability of each event is the same?



Discrete Uniform Distribution

4. In the above graph, does changing the number of trials change the appearance of the graph? Why or why not?

No. The graph shows the expected outcome as the number of trials approaches infinity.

BINOMIAL DISTRIBUTIONS

5. If you flip a fair coin five times, what is the probability that heads come up twice?

$$\begin{aligned}
 P(x: n, p) &= \left(\frac{n!}{x!(n-x)!} \right) (p)^x (1-p)^{(n-x)} \quad x = \underline{2} \quad n = \underline{5} \quad p = \underline{1/2} \\
 &= \left(\frac{5!}{2!(5-2)!} \right) (1/2)^2 (1 - 1/2)^{(5-2)} \\
 &= \left(\frac{5 \cdot 4}{2} \right) \left(\frac{1^2}{2^2} \right) \left(\frac{1^3}{2^3} \right) = \frac{5}{16} = \mathbf{0.3125}
 \end{aligned}$$

6. What is the difference between a Binomial Distribution and a Poisson Distribution?

Where a Binomial Distribution considers the number of successes out of n trials, the Poisson Distribution considers the number of success *per unit* of some measure of time or distance.

7. On average, five hurricanes form in the Atlantic Ocean each year. Given this, what is the probability that 7 hurricanes form next year?

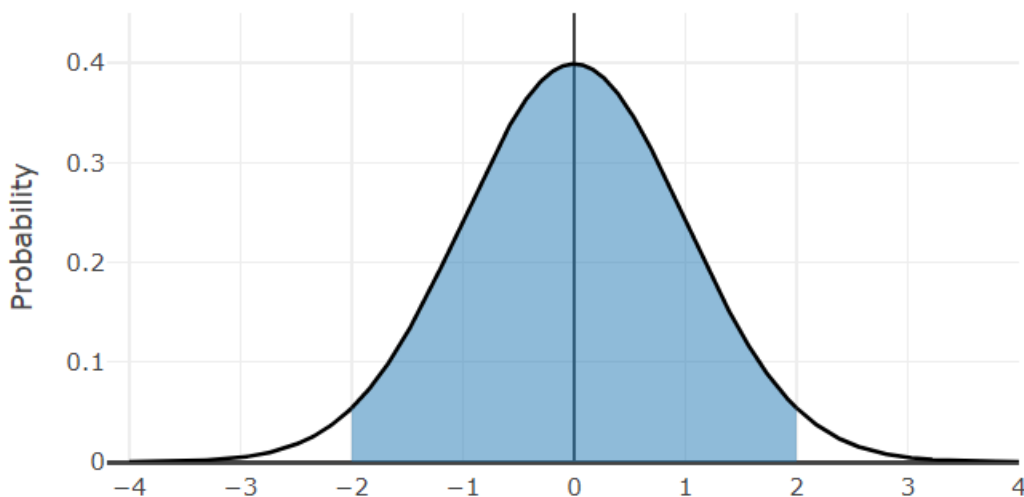
$$\begin{aligned} P(x) &= \frac{\lambda^x e^{-\lambda}}{x!} & e &= 2.71828 & x &= \underline{7} & \lambda &= \underline{5} \\ &= \frac{5^7 \cdot 2.71828^{-5}}{7!} \\ &= \frac{78125 \cdot \left(\frac{1}{148.41}\right)}{5040} = \mathbf{0.1044} \end{aligned}$$

NORMAL DISTRIBUTIONS

8. In a Normal Distribution, what is meant by the "68-95-99.7 Rule"?

This refers to the fact that in any normal distribution, 68.27%, 95.45% and 99.73% of the values lie within one, two and three standard deviations of the population mean.

9. The graph below shows a Standard Normal Distribution curve. Shade in the area that represents 95.45% of values (that is, the area inside of 2 standard deviations).



10. The IQ Test is designed to have a mean score of 100 with a standard deviation of 15 points. A score above 140 is considered to be genius level. What is the calculated z-score for an IQ of 140?

$$z = \frac{x - \mu}{\sigma} = \frac{140 - 100}{15} = 2.667$$

11. Into what percentile does this put people who have an IQ of 140?

A z-score of 2.67 corresponds to an area under the curve of 0.9962

This means that a genius would have outscored 99.62% of the rest of the population.

<i>z</i>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981