ECE 606, Fall 2019, Assignment 3

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1. We observed that the algorithm is recursive, then, when the input is $<1, n>$ we can write the running-time of ADD as following.

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + \Theta(1) + \Theta(1) & \text{otherwise} \end{cases}$$

If $n = 1$, then, the algorithm will return at the Line 2. Thus, the running-time should be $\Theta(1)$. If $n > 1$, then, for the Line 3, it takes $\Theta(1)$ to compute $\lfloor \frac{p+q}{2} \rfloor$. For the Line 4, it takes $T(\lceil n/2 \rceil)$ to compute ADD(p, m) and $T(\lfloor n/2 \rfloor)$ to compute ADD$(m + 1, q)$, and $\Theta(1)$ to compute the sum of them.

Now, if $n > 1$, suppose $n = 2^k$, we can rewrite the equation above as following:

$$\begin{aligned} T(n) &= 2T(\frac{n}{2}) + \Theta(2) \\ &= 2(2T(\frac{n}{2^2}) + 2) + 2 \\ &= 2^3(T(\frac{n}{2^3})) + 2^3 + 2 \\ &= \dots \\ &= 2^{k-1} + 2^k - 2 \\ &= \frac{3}{2} * 2^k - 2 \\ &= \Theta(2^k) \\ &= \Theta(n) \end{aligned}$$

Therefore, when the input is $<1, n>$, the running-time will be $\Theta(n)$ or said $\Theta(2^k)$, where $k = \lg n$.

2. Each $A[i]$ has a size of m bits. Then, for each loop Line (2) and Line (7) both have a running-time of $\Theta(m)$. Now, for the worst-case, Line (5) will execute j times, then have a running time of $j * \Theta(m)$.

Therefore, for the whole function, the worst-case running-time should be $\sum_{i=1}^{n} 2\Theta(m) + i\Theta(m)$.

Finally, we can give a characterization of the worst-case running-time of this function as $\Theta(m)$

3. Here we denote X_k as $A[i]$, which k represent Line (3) has executed k times. When the Line (3) is executed first time, the probability that $A[i]$ is the median m is $Pr\{X_1 = m\} = \frac{1}{n}$. Then, the probability that Line (3) will execute again is $Pr\{X_1 \neq m\} = 1 - Pr\{X_1 = m\}$. Hence, for the second time the probability that $A[i]$ is the median is $Pr\{X_2 = m\} = Pr\{X_2 = m | X_1 \neq m\} * Pr\{X_1 \neq m\} = \frac{1}{n-1} * (1 - \frac{1}{n}) = \frac{1}{n}$.

Therefore, $Pr\{X_k = m\} = \frac{1}{n}$.
$E(X) = \sum_{i=1}^{n} i * \frac{1}{n} = \frac{1+n}{2}$
Line (3) is expected to run $(n + 1)/2$ times when the input is $A[1, \dots, n]$.

4. a3p4.py