CSC 148: Introduction to Computer Science

Week 8

Recursive structures

Trees

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Recursion, natural or otherwise

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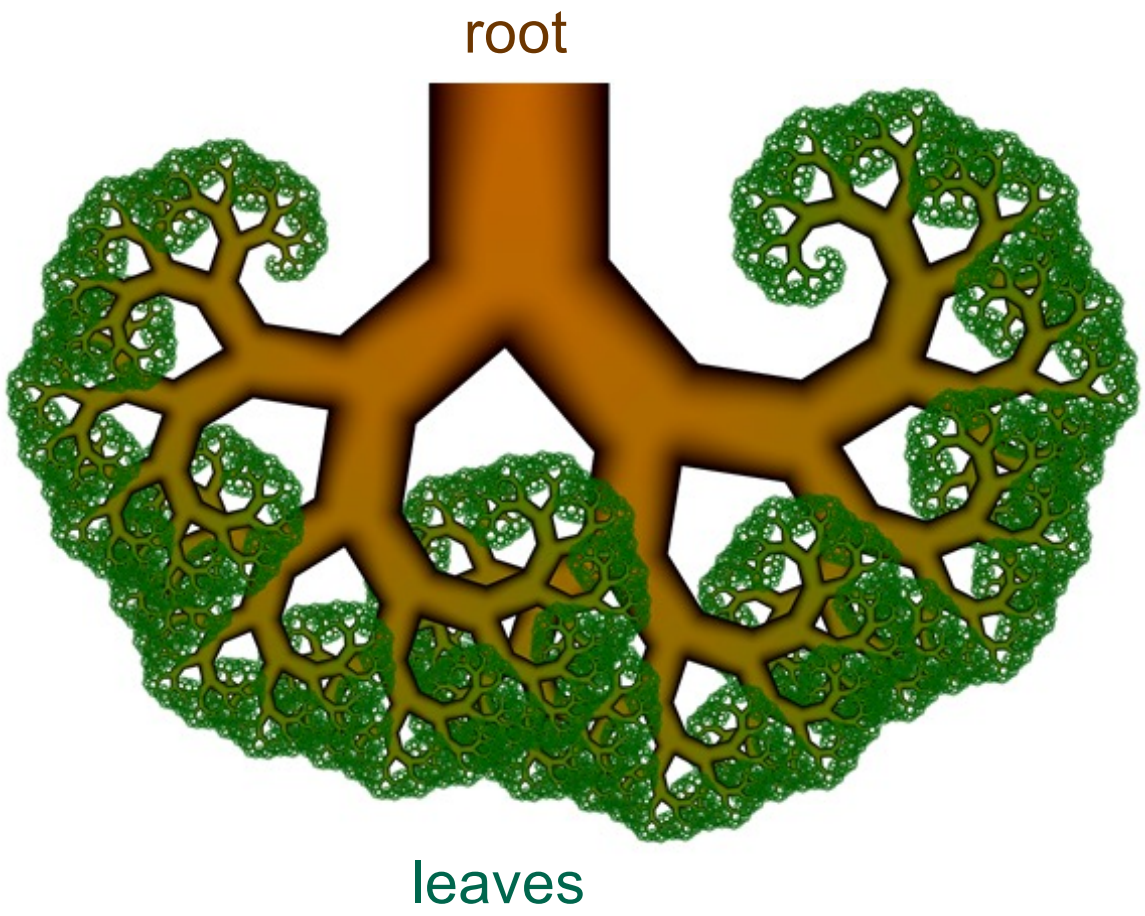
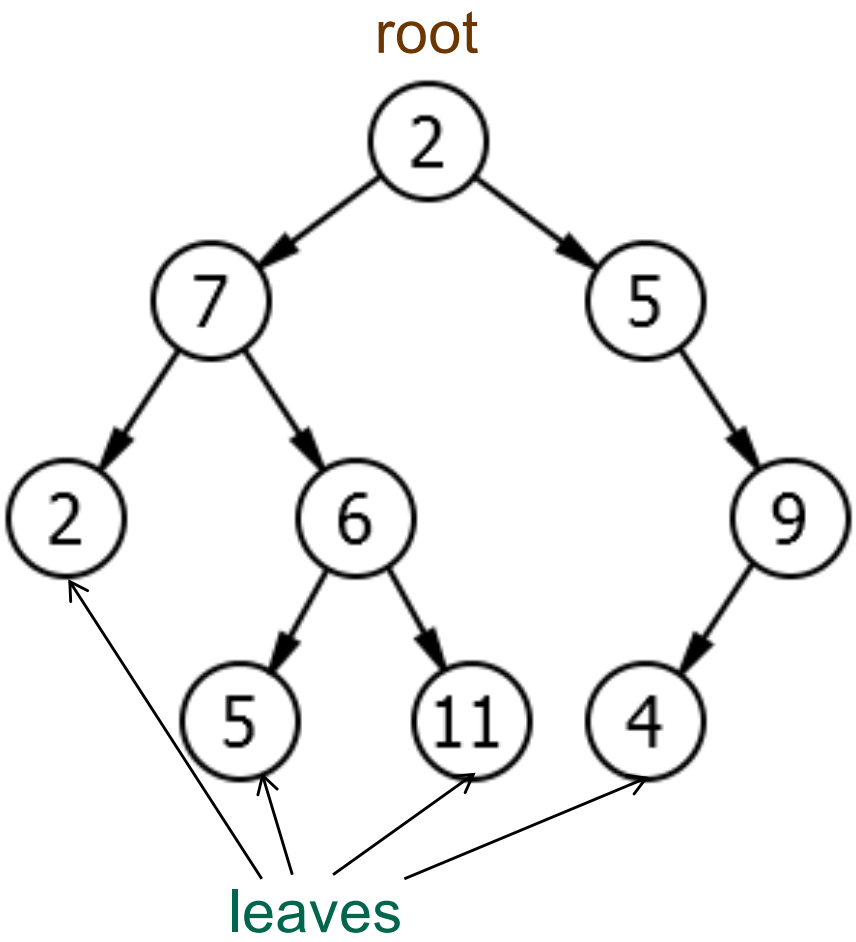




Tree terminology

- A collection of values (aka nodes), with directed edges between them
  - … and no cycles (more detail soon)

- A tree is either empty, or non-empty

- Every non-empty tree has a root, connected to 0 or more subtrees

More Tree Terminology

- The parent of a tree value (node) is the value (node) immediately above and connected to it

- The children of a tree value (node) are the values (nodes) directly connected underneath it.

- The descendants of a value (node) are its children, its children's children, etc.
  - Can be defined recursively: its children + descendants of its children

- The ancestors of a value (node) are its parent, its parent's parent, etc.
  - Can be defined recursively: its parent + ancestors of its parent

Even More Tree Terminology

- A path is a sequence of values (nodes) $n_1, n_2, \ldots, n_k$, where there is an edge between each pair of n_i - n_{i+1}, $i<k$

- The length of a path is the number of edges in it

- There is a unique path from the root of the tree to each node in that tree. In the case of the root itself this is just n_1, if the root is node n_1

- There are no cycles (no paths that form loops)

Yet More Tree Terminology

- leaf: a value (node) with no children (with no subtrees)
- internal node: a value (node) with one or more children

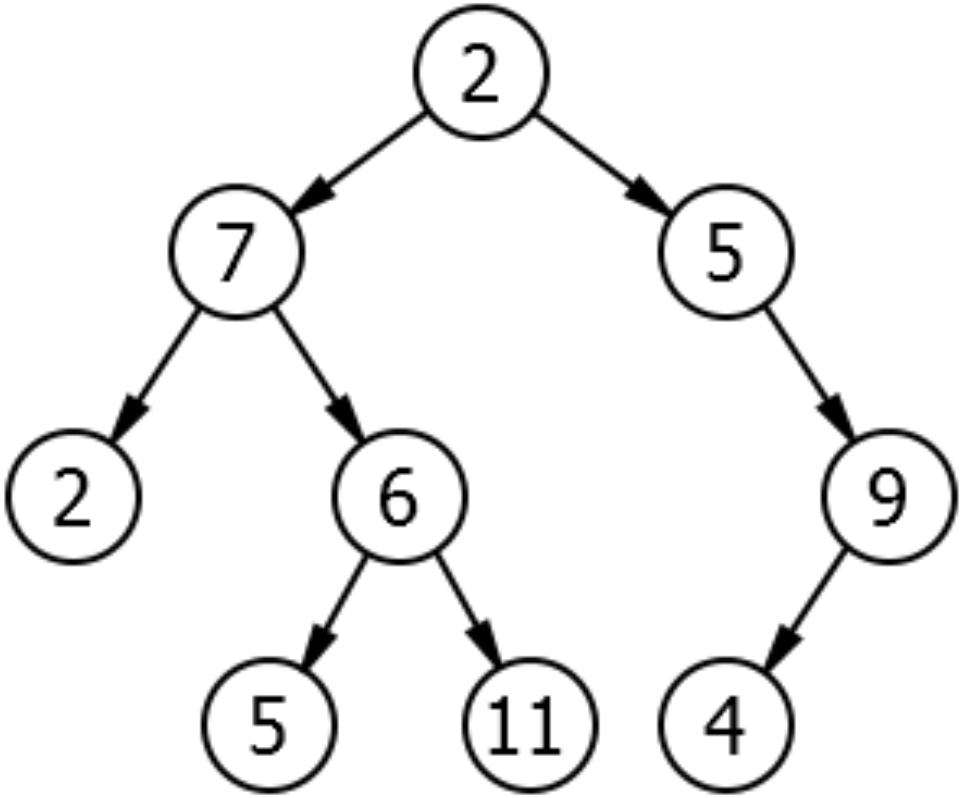
- height of a tree: longest path from the root to one of the leaves
  - Count the number of values on the path (not the edges!)
- depth of a value (node): length of the path from the root to that value
  - The root itself has depth 0

- arity, branching factor: maximum number of children for any node

- Practice these terms!

Examples

- root of this tree?
- parent of value/node 6?
- name a child of node 7?
- name the leaves?
- internal nodes?
- example of a subtree?
- example path?
- height of the tree?
- depth of values/nodes: 9, 5, 7?
- arity, branching factor?



Tree Attributes and Recursive Template

```
class Tree:
    _root: Optional[Any]
    _subtrees: list[Tree]
```

```
def method(self) -> ...:
    if self.is_empty():
        ...
    else:
        for subtree in self._subtrees:
            ... subtree.method() ...
```

Remember: Practice is Crucial!

- Worksheet: practice with trees ...