STA260 Summer 2025 - Tutorial 4

July 2025

Overview:

1. Bias
2. MSE

Summary

Bias and MSE

Given a point estimator $\hat{\theta}$ for the parameter θ, the bias is calculated as $\mathrm{B}(\hat{\theta}) = \mathrm{E}[\hat{\theta}] - \theta$.
We say a point estimator is unbiased if $\mathrm{E}[\hat{\theta}] = \theta$.
The mean square error (MSE) of $\hat{\theta}$ is $\mathrm{MSE}(\hat{\theta}) = \mathrm{E}\big((\hat{\theta} - \theta)^2\big) = \mathrm{Var}(\hat{\theta}) + \mathrm{B}(\hat{\theta})^2$.

Question 1

Let $X_1, X_2, \ldots X_n$ denote a random sample from a distribution with the probability density function

μ is the smallest val possible

$$f(x) = \begin{cases} e^{-(x-\mu)} & \text{if } x \geq \mu \\ 0 & \text{otherwise} \end{cases}$$

Let $\hat{\mu} = \text{Min}(X_1, X_2, \ldots, X_n)$. Calculate the bias of $\hat{\mu}$.

$$B(\hat{\theta}) = E(\hat{\theta}) - \theta \quad \Rightarrow B[\hat{\mu}] = E[\hat{\mu}] - \mu$$

$$f_{\hat{\mu}}(x) = n f_X(\mu) \left(1 - F_X(\mu)\right)^{n-1}$$

$$= n e^{-(x-\mu)} \left(1 - \cancel{1} - e^{-(x-\mu)}\right)^{n-1}$$

$$F_X(x) = \int_{\mu}^{x} e^{-(t-\mu)} \, dt = -e^{-(t-\mu)} \Big|_{\mu}^{x}$$

$$= -e^{-(x-\mu)} + 1$$

$$= 1 - e^{-(x-\mu)}$$

$$= n e^{-(x-\mu)} \left(e^{-(x-\mu)}\right)^{n-1}$$

$$= n e^{-(x-\mu)^n}$$

Question 2

Let $X_1, X_2, \ldots X_n$ denote a random sample from a distribution with the probability density function

$$f(x) = \begin{cases} e^{-(x-\mu)} & \text{if } x \geq \mu \\ 0 & \text{otherwise} \end{cases}$$

Calculate the mean square error of $\hat{\mu}$.

$$MSE(\hat{u}) = Var(\hat{u}) + B(\hat{u})^2$$

Question 3

Let $Y_1, Y_2, \ldots Y_n$ denote a random sample from a Uniform$(\theta, \theta + 1)$ distribution. Consider the following estimators:

$$\hat{\theta}_1 = \bar{Y} - \frac{1}{2} \qquad \hat{\theta}_2 = Y_{(n)} - \frac{n}{n+1}$$

Which estimators are unbiased?