STA260 Summer 2025 - Tutorial 2

July 2025

Overview:

Some Relevant Distribution Relationships Assume Y_i are i.i.d, $\mathrm{i} = 1, 2, \ldots, \mathrm{n}$, and $Y_i \sim \mathrm{Normal}(\mu, \sigma^2)$

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i, \quad Z = \frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{Normal}(0,1), \quad \sum_{i=1}^{n}\left(\frac{Y_i-\mu}{\sigma}\right)^2 \sim \chi^2_{(n)}$$

$$\chi^2_{(n)} = \mathrm{Gamma}(n/2, 2),$$

$$\frac{(n-1)S^2}{\sigma^2} = \sum_{i=1}^{n}\frac{\left(Y_i-\bar{Y}\right)^2}{\sigma^2} \sim \chi^2_{(n-1)}, \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(Y_i-\bar{Y}\right)^2$$

$$\mathrm{Exp}(\beta) = \mathrm{Gamma}(\alpha=1,\beta), \quad (\mathrm{Normal}(0,1))^2 = \chi^2_{(1)} = \mathrm{Gamma}(1/2, 2)$$

Questions:

Question 1

1. If U has a χ^2 distribution with v degrees of freedom, find $E(U)$ and $\mathrm{Var}(U)$.
2. Let $Y_1, \ldots, Y_n$ be a random sample from the normal distribution with mean μ and variance σ^2. Find $E\left(S^2\right)$ and $\mathrm{Var}\left(S^2\right)$, where S is defined as:

$$S^2 = \frac{\sum_{i=1}^{n}\left(Y_1-\bar{Y}\right)^2}{n-1}$$

Question 2

Let $\bar{Y}$ and S^2 be the mean and the variance of a random sample of size 25 from $N\left(\mu = 3, \sigma^2 = 100\right)$. Find $P\left((0 < \bar{Y} < 6) \cap \left(55.2 < S^2 < 145.6\right)\right)$.
Hint: recall the following facts:
1. $\bar{Y}$ and S^2 are independent.
2. $\bar{Y} \sim N\left(\mu, \sigma^2/n\right)$.
3. $(n-1)S^2/\sigma^2 \sim \chi^2(n-1)$