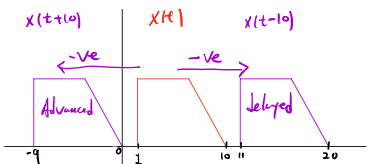
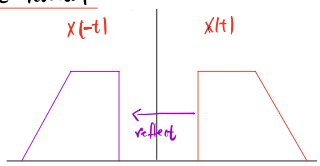
Time shift

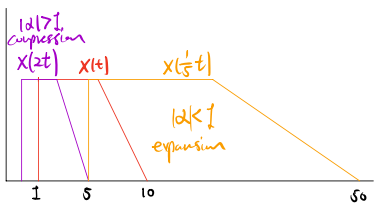


shape of the signal remains same only shift.

Time reversal



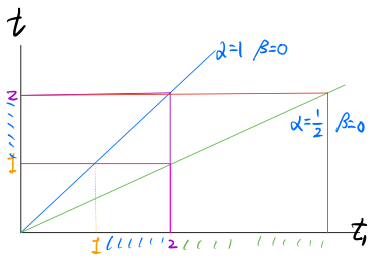
Time scaling



scaling & shifting

$$y(t) = y(t=\alpha t_1 + \beta) = y\left(t_1 = \frac{t-\beta}{\alpha}\right)$$ $t-\beta$ first then $/\alpha$

$$= y\left(t_1 = \frac{t}{\alpha} - \frac{\beta}{\alpha}\right)$$ t/α first then $-\beta/\alpha$



Periodic signal

A signal is Periodic iff
$y(t) = y(t + mT_0)$ m is a integer.
$x[n] = x[n + mN_0]$

fundamental period (smallest)

Even & odd signal

even: $x(t) = x(-t)$ $x[n] = x[-n]$
odd: $x(t) = -x(-t)$ $x[n] = -x[-n]$

if $x(t)$ is odd then $x(0) = 0$

odd and even parts of a function

$$e(t) = \frac{1}{2}[x(t) + x(-t)] = e(-t)$$
$$o(t) = \frac{1}{2}[x(t) - x(-t)] = -o(-t)$$

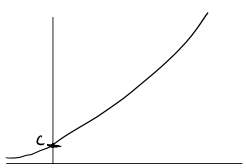
$$x(t) = e(t) + o(t)$$

Continuous Complex exponential & Sinusoidal signal

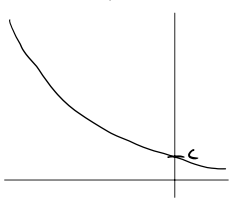
$$x(t) = C\exp(at)$$

if $C, a \in \mathbb{R}$: $x(t) = C\exp(at)$ ← only exponential

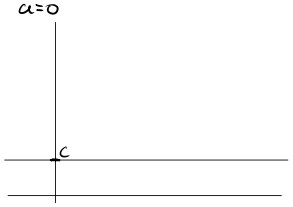
$a > 0$



$a < 0$

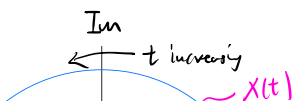


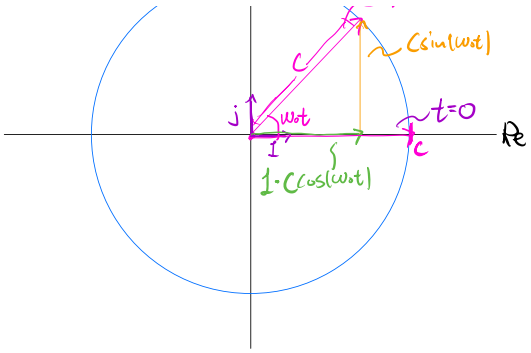
$a = 0$



if c real and a imaginary:

$x(t) = C\exp(j\omega t)$ ← Periodic with $T_0 = \frac{2\pi}{|\omega_0|}$





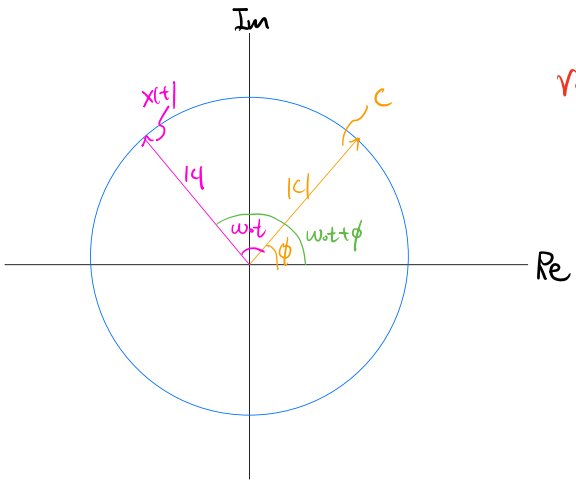
ω_0 - # many full circle the vector spin in 1 1s

T_0 - how many seconds does it take for the vector to spin 1 full circle.

if C complex and a imaginary:

$$x(t) = C e^{j\omega t} = |c| e^{j\phi} e^{j\omega t} = |c| e^{j(\omega t + \phi)}$$

$$T_0 = \frac{2\pi}{|\omega|}$$



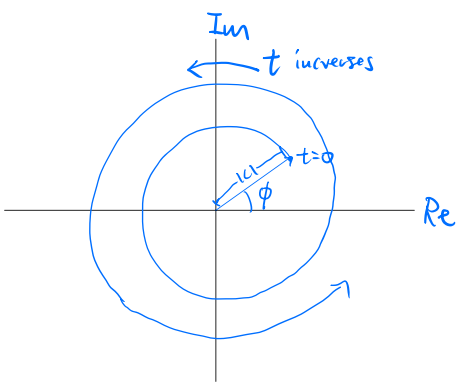
rotates $x(t)$ by an fixed angle ϕ

if C is complex and a is complex:

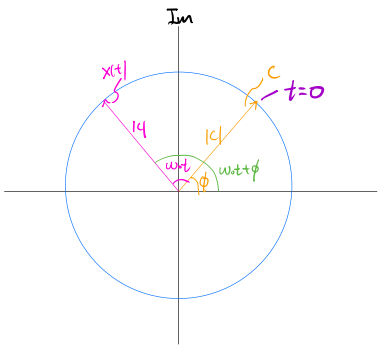
$$C = |c| e^{j\phi} \qquad a = r + j\omega_0$$

$$x(t) = C e^{at} = |c| e^{rt} e^{j(\omega t + \phi)}$$

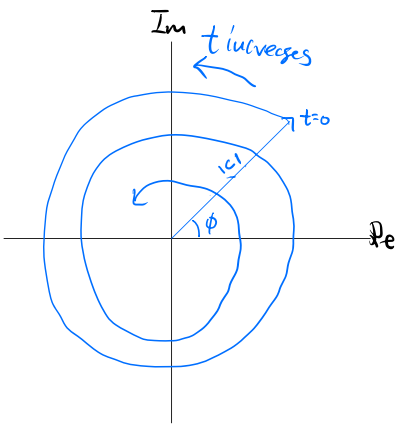
$r>0$



$r=0$



$r<0$



discrete complex exponential and sinusoidal signal

$X[n] = C\alpha^n$

C and α are real numbers:

- α +ve $\Rightarrow$ $X[n]$ has same sign for all n
- α -ve $\Rightarrow$ $X[n]$ has alternating signs.
- $\alpha = 1$ $\Rightarrow$ $x[n] = 1$ $\forall n$
- $|\alpha| > 1$ $\Rightarrow$ growing / diverging to infinity
- $|\alpha| < 1$ $\Rightarrow$ decaying / converge to 0
- $\alpha = -1$ $\Rightarrow$ alternates b/w +C and -C