Signal Analysis & Communication

ECE355H1 F

Ch 1.4:

Unit Step & Unit Impulse Functions (Contd.)

Lec. 1, Wk 3

19-09-2022

In continuation to the last lecture...

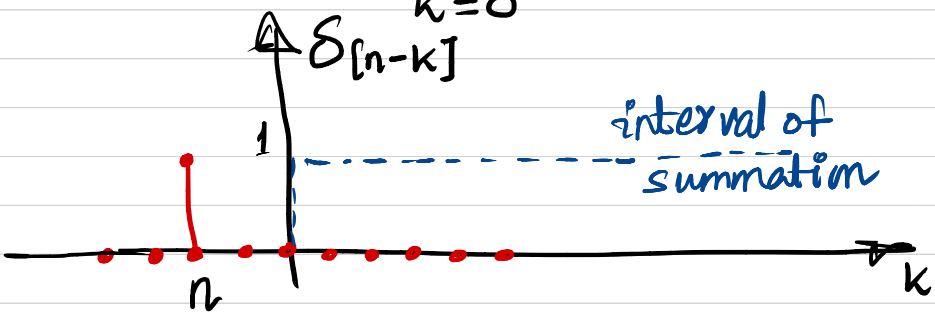
Assume $k = n - m$

$$u[n] = \sum_{k=\infty}^{0} \delta[n-k]$$

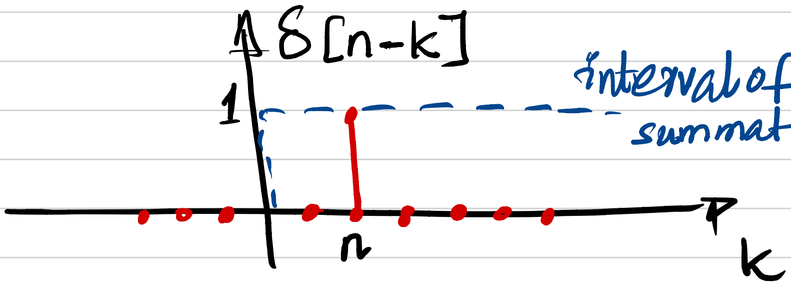
Remember $\delta[n-k] = \begin{cases} 1 & \text{at } k=n \\ 0 & \text{o/w} \end{cases}$

Equivalently,

$$u[n] = \sum_{k=0}^{\infty} \delta(n-k)$$



$n < 0$



$n \geq 0$

* Summation is '0' for $n<0$ & '1' for $n>0$.

Unit step: superposition of delayed impulses. [We will use later]

Unit Impulse seq. can be used to sample the value of the sig. at $n=0$.

$$x[n]\delta[n] = x[0]\delta[n]$$

Generally,

$$x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$$

this will be '0' at $n=n_0$

Sampling Property of $\delta[n]$

CT Unit Step & Unit Impulse Functions

CT Unit Step

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

$u(t)$ is discontinuous at $t=0$

Analogous to the relationship b/w $u[n]$ & $\delta[n]$

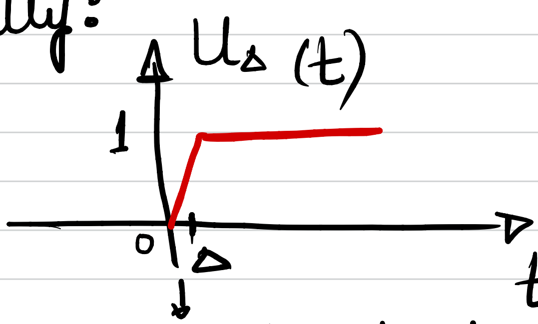
$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

Similarly,

$$\delta(t) = \frac{d}{dt} u(t)$$

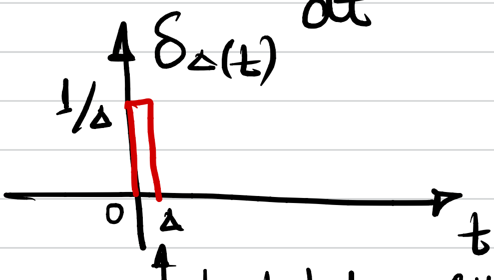
But since $u(t)$ is discontinuous at $t=0$, it is not differentiable.

More formally:



so short duration that it doesn't matter practically

$$\delta_\Delta(t) = \frac{du_\Delta(t)}{dt}$$



$\Delta \to 0$

short pulse with unit area

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$

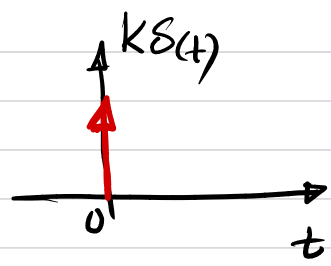
area = 1

$$\boxed{\int_{-\infty}^{+\infty} \delta(t)\, dt = 1}$$

def^n of CT $\delta(t)$

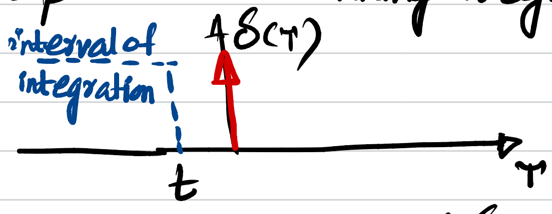
Scaled Impulse $k\delta(t) \rightarrow$ area k

$$\int_{-\infty}^{t} k\delta(\tau)\,d\tau = k\,u(t)$$



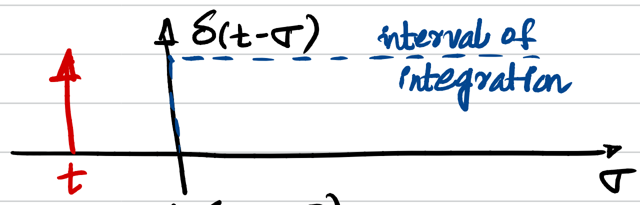
As with DT, simple graphical interpretation of running integral.

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$



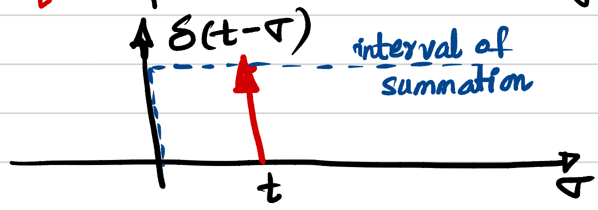
With $\sigma = t - \tau$

$$u(t) = \int_{\infty}^{0} \delta(t-\sigma)\,(-d\sigma)$$



Equivalently,

$$u(t) = \int_{0}^{\infty} \delta(t-\sigma)\,d\sigma$$



The integral is '0' for $t<0$ & '1' for $t>0$.

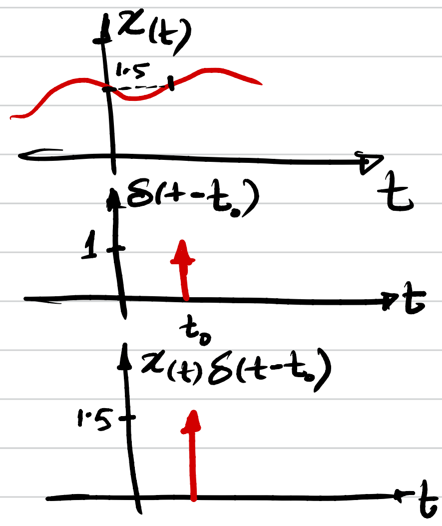
Sampling Property of Unit Impulse

$$x_1(t) = x(t)\,\delta_\Delta(t)$$

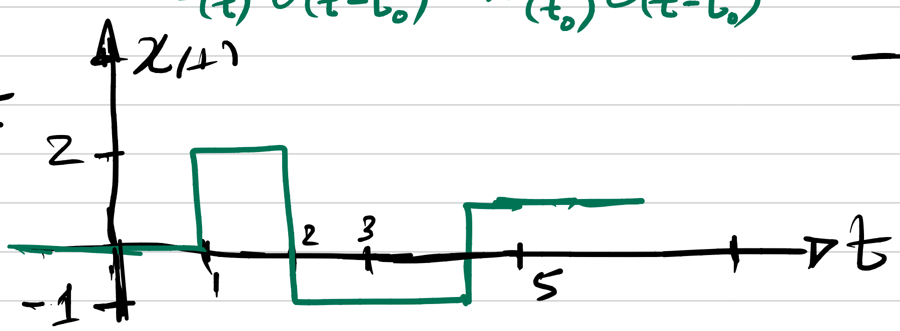
$$x(t)\,\delta_\Delta(t) \simeq x(0)\,\delta_\Delta(t)$$

$$x(t)\,\delta(t) = x(0)\,\delta(t)$$

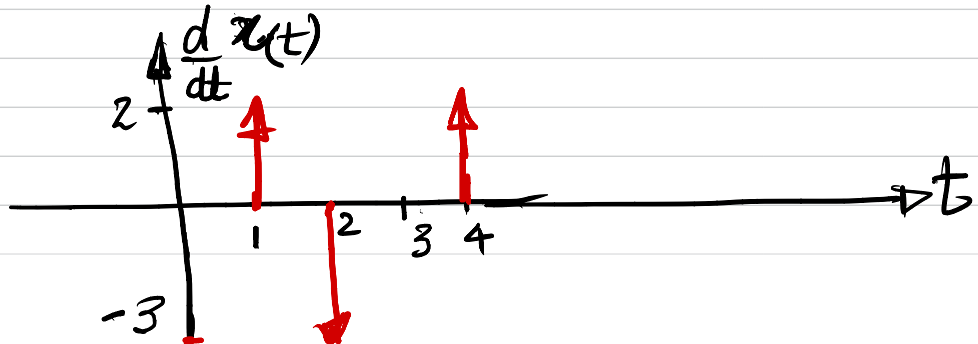
$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$



Example



* Expressing the sig. mathematically



$$\frac{d}{dt}x(t) = 2\delta(t-1) - 3\delta(t-2) + 2\delta(t-4)$$

$$x(t) = \int_{-\infty}^{t} 2\delta(\tau-1)\,d\tau - \int_{-\infty}^{t} 3\delta(\tau-2)\,d\tau + \int_{-\infty}^{t} 2\delta(\tau-4)\,d\tau$$

$$= 2u(t-1) - 3u(t-2) + 2u(t-4)$$

Check: $x(3) = 2-3+0 = -1$ ✓

NOTE

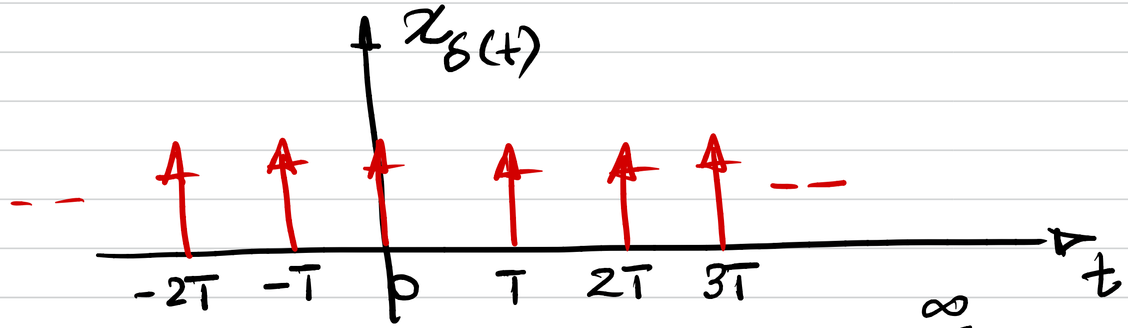
$$\int_{-\infty}^{+\infty} x(t)\,\delta(t-t_0)\,dt$$

$$= \int_{-\infty}^{+\infty} x(t_0)\,\delta(t-t_0)\,dt$$

$$= x(t_0)$$

APP

Sampling of a CT Signal via Impulse Train $\quad x_{\delta}(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$



$$x(t)\,x_{\delta}(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$$