Signal Analysis & Communication

ECE355H1F

Lec. 1, Wk 2

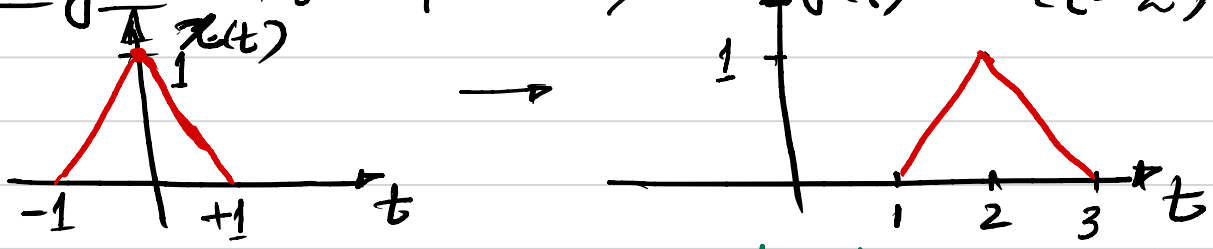
12-09-2022

SIGNAL TRANSFORMATION (indep. var.) [Ch. 1-2]

① Time Shift: $x(t) \rightarrow y(t) = x(t - t_0)$

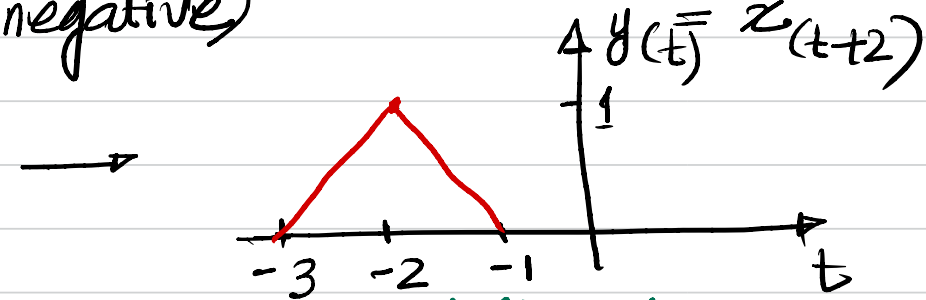
$y(t)$ - identical in shape
- but shifted relative to $x(t)$

a. Delayed (t_0 is positive)



Delayed → shifted right

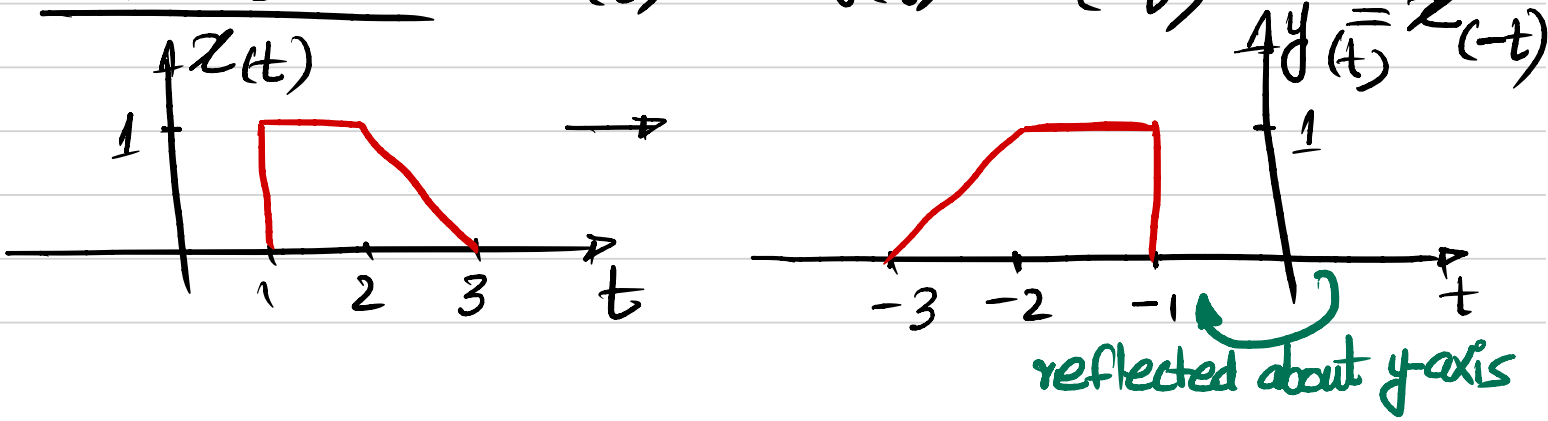
b. Advanced (t_0 is negative)



shifted left ← Advanced

Application: Radar, Sonar
Transmission Delay

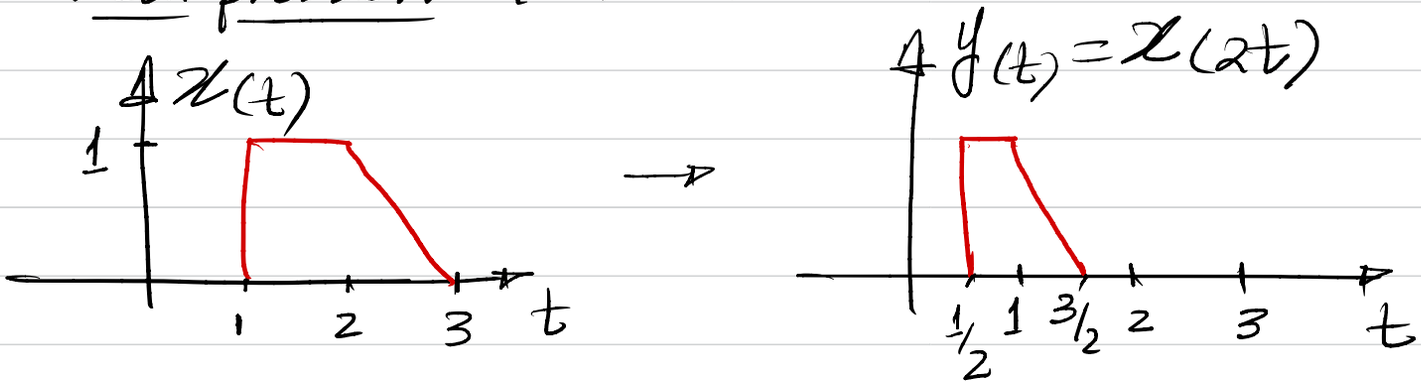
② Time Reversal $x(t) \rightarrow y(t) = x(-t)$



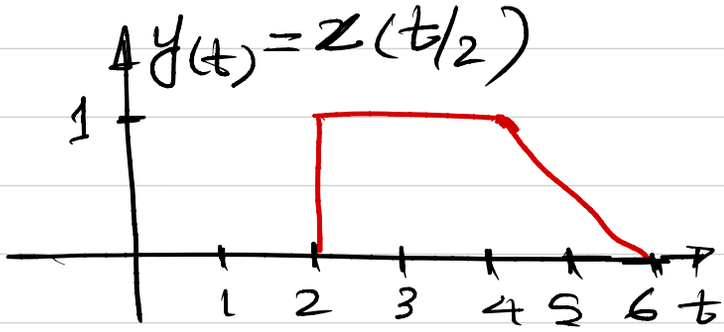
reflected about y-axis

③ Time Scaling $x(t) \rightarrow y(t) = x(\alpha t)$

a. Compression $|\alpha| > 1$



b. Expansion $|\alpha| < 1$



c. $\alpha < 0$

Signal reversed – compressed or expanded

✓ Application: Recording at twice / half the speed of original signal

*OFTEN – Scaling & shifting both

$y(t) = x(\alpha t + \beta)$

2 ways

1. step 1 Shift $v(t) = x(t + \beta)$
   step 2 scale $y(t) = x(\alpha t + \beta)$

2. step1 scale $w(t) = x(\alpha t)$
step2 shift $y(t) = x(\alpha t + \frac{\beta}{\alpha})$

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* If in step2, you do $y(t) = x(\alpha t + \beta)$ X
↑ as it is already scaled, the shifting should take care of that scaling.

Similar signal transformation in DT signals.

The transformation is useful in analyzing the properties of a system.

Periodic Signals:

$$x(t) = x(t+T)$$

For example: - ideal LC circuit (without R ← no loss)
- ideal mech. system (without friction ← no loss)

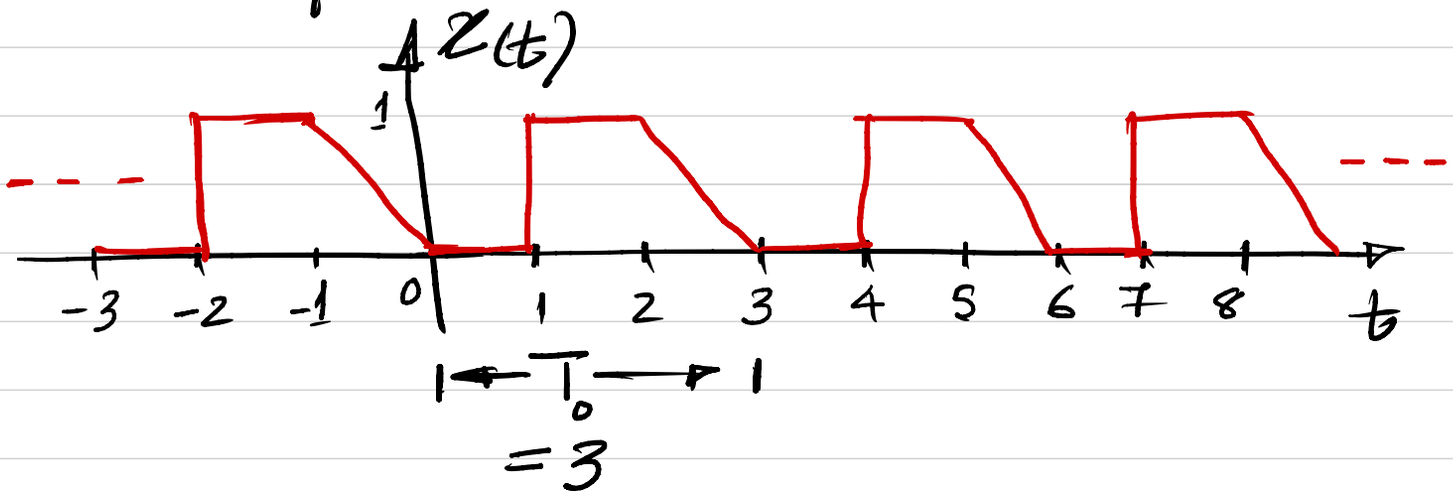
} Energy is conserved

Also, if $x(t)$ is periodic

$$x(t) = x(t+mT)$$
↑ integer

The signal is periodic with $2T, 3T, \ldots$

T_0 is called the fundamental period
→ smallest positive value of T



* If $x_{(t)}$ is a constant signal, the fundamental period is undefined.
& $x_{(t)}$ is periodic with any choice of T.

* If $x_{(t)}$ is not periodic → Aperiodic

For DT Signal: $x[n] = x[n+N]$ (periodic)

Fundamental period N_0

If $x_{[n]}$ not periodic → Aperiodic

Even & Odd Signals:

a) Even Signals

CT: $x(-t) = x(t)$

DT: $x[-n] = x[n]$

b) Odd Signals

CT: $x_{(-t)} = -x_{(t)}$

DT: $x[-n] = -x[n]$

* Odd signals must be zero at $t=0$ since $x_{(0)} = -x_{(0)}$ & $x[0] = -x[0]$.

Any signal can be broken into two

$$Ev[x_{(t)}] = \frac{1}{2}[x_{(t)} + x_{(-t)}]$$ Even part

$$Od[x_{(t)}] = \frac{1}{2}[x_{(t)} - x_{(-t)}]$$ Odd part