Signal Analysis & Communication

ECE355H1 F

ch 1.5 :

CT & DT Systems

ch 1.6 :

Basic System Properties

Lec 2, klk3

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Continuous Time & Discrete Time Systems (ch. 1-5)

System A process that transforms I/P sig. into O/P sig.

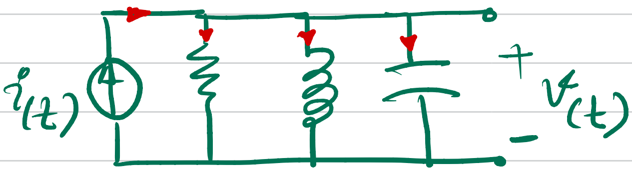
CT: $x(t) \rightarrow \boxed{\text{Sys.}} \rightarrow y(t)$

DT: $x[n] \rightarrow \boxed{\text{Sys.}} \rightarrow y[n]$

The sys. is characterized by their I/P-O/P relationship.

Examples

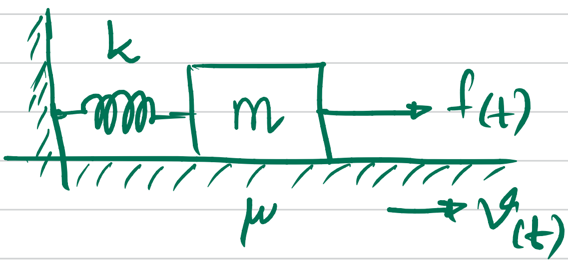
① RLC Sys.



$$i(t) = \frac{v(t)}{R} + \frac{1}{L}\int_0^t v(\tau)\,d\tau + C\frac{dv(t)}{dt}$$

$i(t)$ - I/P
$v(t)$ - O/P

② Spring-mass Sys.



$f(t)$: force
m: mass
k: spring const.
μ: friction coeff.
$v(t)$: velocity

$f(t) \rightarrow \boxed{\text{Spring-mass Sys.}} \rightarrow v(t)$

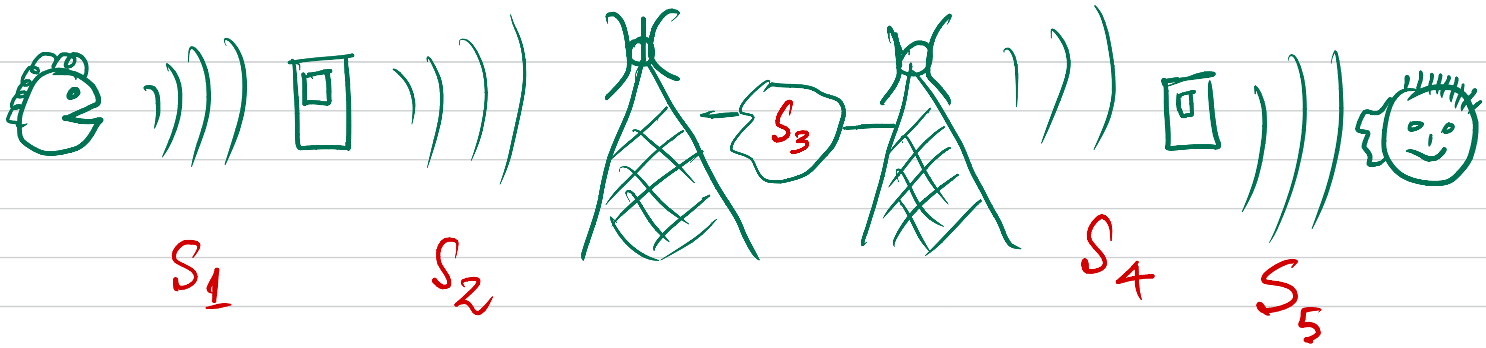
$$f(t) = \mu\, v(t) + k\int_{-\infty}^{t} v(\tau)\,d\tau + m\frac{dv(t)}{dt}$$

Interconnection of Sys.

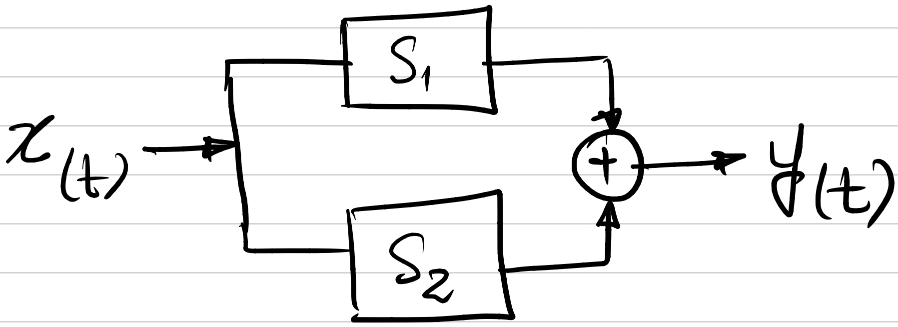
① Series

$x(t) \rightarrow \boxed{S_1} \rightarrow \boxed{S_2} \rightarrow y(t)$

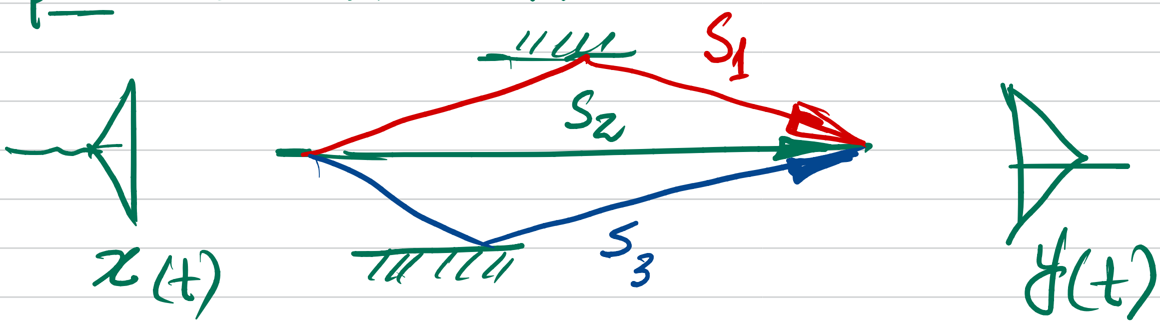
Example: Wireless Sys.



② Parallel

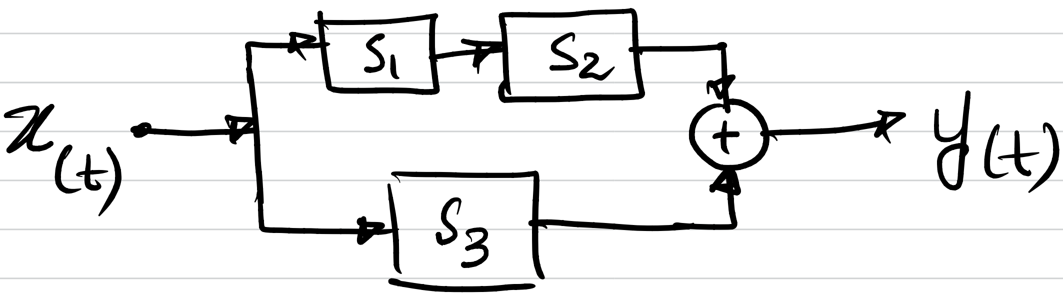


Example Wireless channel

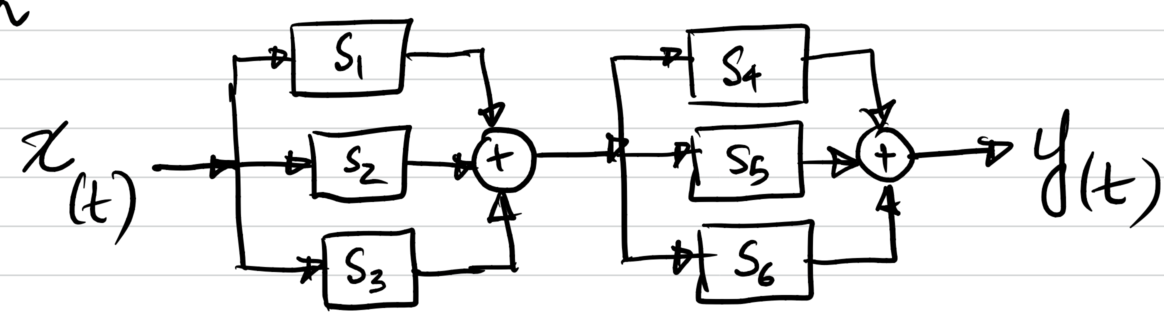


③ Series-Parallel

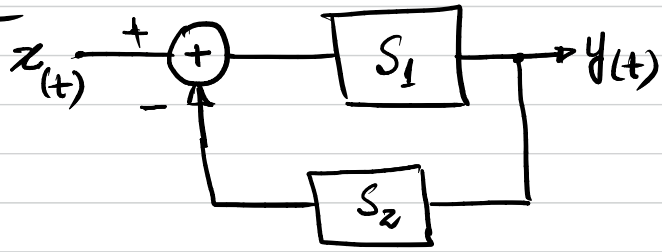
- Combination of the above two



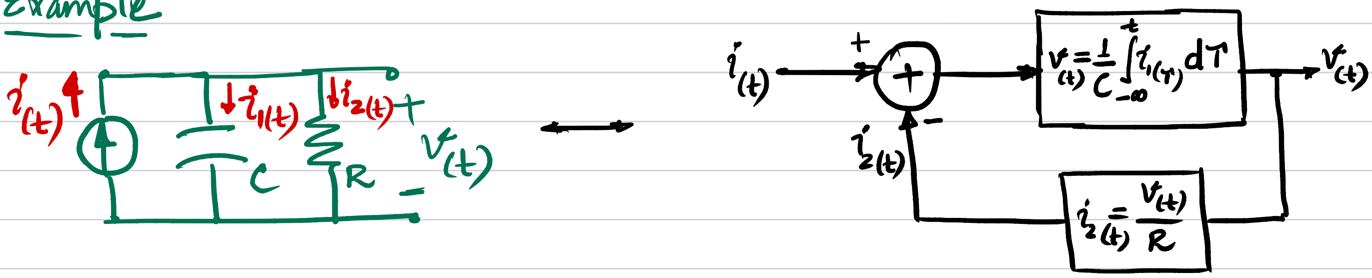
OR



④ Feedback



Example



$$i_1(t) = i(t) - i_2(t)$$

Basic System Properties (ch. 1·6)

Provides: - Important physical interpretation
- Relatively simple mathematical description

① System with & without Memory:

A. Memoryless

- If the output of the sys. for each value of independent variable at a given time is dependent only on the input at the same time.

Eg. ① $y[n] = (2x[n] - x^2[n])^2$

② Resistor: $y(t) = Rx(t)$

③ identity: $y(t) = x(t)$ (CT)

$y[n] = x[n]$ (DT)

B. With Memory

- Presence of a mechanism in the sys. that retains or stores info. about i/p values at non-current times

Eg. ① $y[n] = \sum_{k=-\infty}^{n} x[k]$

② $y[n] = x[n-1]$

③ Capacitor: $v(t) = \frac{1}{C} \int_{-\infty}^{t} i(\tau) d\tau$

④ Computers: storage registers (memory)

* Some cases where current o/p is dependent on future i/p & o/p values, eg $y[n] = x[n+1]$ ← We'll see later

(2) Invertibility

- If distinct I/Ps leads to distinct O/Ps - for all times.

Eg. (1) $y(t) = 2x(t)$ — Invertible

Inverse $w(t) = \frac{1}{2} y(t) = x(t)$

(2) $y(t) = x^2(t)$ — Non-invertible (No distinct values)

$w(t) = \pm\sqrt{y(t)}$

$x(t)$ could be $+\sqrt{y(t)}$ or $-\sqrt{y(t)}$

For $+\sqrt{y(t)}$ & $-\sqrt{y(t)}$, the O/P is same, $y(t)$

(3) $y(t) = 0$ — Non-invertible

For any I/P, O/P is '0'

(4) $y[n] = \sum_{k=-\infty}^{n} x[k]$ — ✓ Invertible

(Remember to look at one given time)

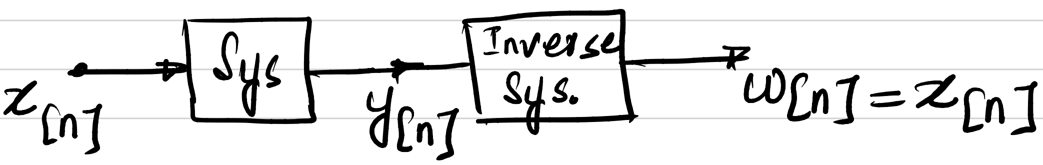
$= \sum_{k=-\infty}^{n-1} x[k] + x[n]$

$= y[n-1] + x[n]$

* Should be valid for all times for the considered system!

Inverse $x[n] = y[n] - y[n-1]$

- Application: Encoding in digital Communication Sys. (Compression, Error Control, Encryption)



(3) Causality

DT: A sys. is causal if $y[n_0]$ for any n_0 does not depend on values of $x[n]$ for $\underbrace{n > n_0}_{\text{future}}$.

(i.e., Non-anticipative of future)

Eg.
(1) In dig. image processing, moving average to smoothout

$$y[n] = \frac{1}{2M+1} \sum_{k=-M}^{M} x[n+k]$$

(indep. var.: spatial coordinates)
(Not time)
Non-Causal

(2)
$$y[n] = \frac{1}{2M+1} \sum_{k=-2N}^{0} x[n+k]$$
Causal

CT: A sys. is causal if for any time t_0, its O/P $y(t)$ for $t<t_0$ doesnot depend on I/P $x(t)$ for $t \geq t_0$.

Equivalently, if two I/Ps are identical for $t<t_0$, the corresponding O/Ps are also equal for $t<t_0$.

Eg.
(1) $y(t) = x(t+1)$ Non-Causal

(2) $y(t) = x^2(t)$ Causal

(3) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$ Causal

(4) Stability

- A sys. is Bounded-I/P-Bounded-O/P (BIBO) stable if bounded I/P signals i.e., $|x(t)| < \infty, \forall t$, lead to bounded O/P signal, i.e., $|y(t)| < \infty, \forall t$.
- Small I/Ps lead to O/P that do not diverge!
- Stability of physical sys. results from the presence of mechanisms that dissipate energy.

Eg.
(1) RLC Sys. (R dissipates energy) Stable

(2) Spring-mass Sys. Stable

(3) $y[n] = \sum_{k=-\infty}^{n} x[k]$ Not stable

* (if $x[k]=1$ $\forall k$)

④ $y[n] = \frac{1}{2M+1} \sum_{k=-M}^{M} x[n-k]$ **Stable**

↑ Bounded

⑤ <u>Time Invariance</u>

- A sys. is time invariant if a finite shift in I/P sig. results in an <u>identical time shift</u> in the O/P sig.

- if

$$x(t) \xrightarrow{S} y(t)$$

then

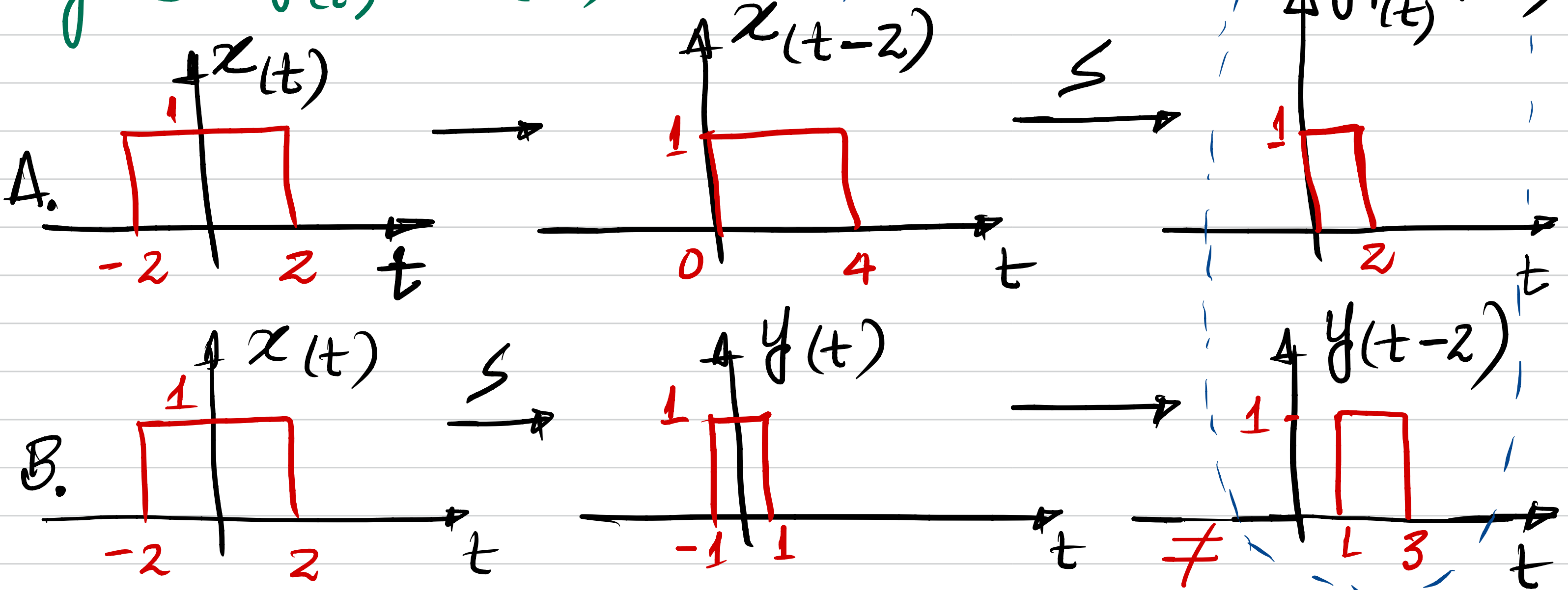
$$x(t-t_0) \xrightarrow{S} y(t-t_0) \quad , \forall\, t, t_0$$

A. $x(t) \to \boxed{\text{Delay}} \xrightarrow{x(t-t_0)} \boxed{S} \to y(t-t_0)$

$\updownarrow$ = time invariant

B. $x(t) \to \boxed{S} \xrightarrow{y(t)} \boxed{\text{Delay}} \to y(t-t_0)$

Eg. ① $y(t) = x(2t)$ $\quad z_1(t) =$



② $y(t) = t\,x(t)$ TV

for I/P $x(t-t_0) \to$ O/P $t\,x(t-t_0)$

$\neq y(t-t_0) = (t-t_0)\,x(t-t_0)$

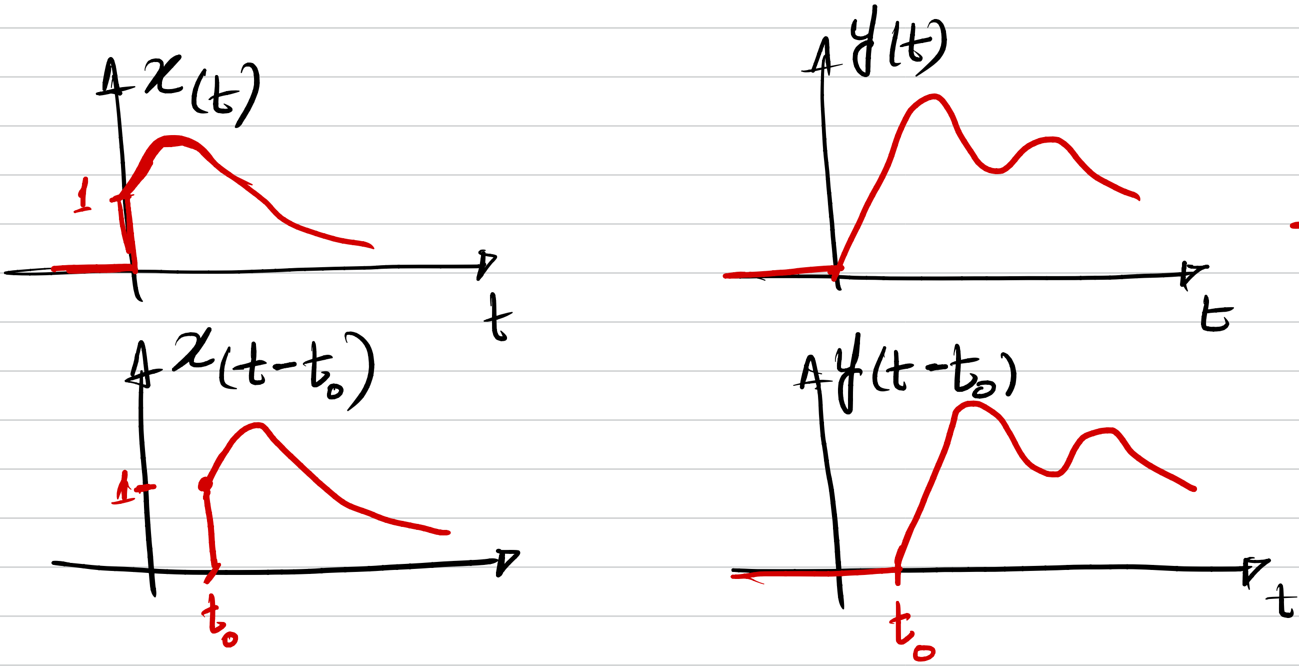
$x(t-t_0)$

③ $y(t) = \sin[x(t)]$

$y(t-t_0) = \sin[x(t-t_0)]$

Time invariant (TIV)

④



TIV

⑤ for RLC Circuits — differential Eqn.

I. If coeff. of terms - constant TIV

II. If coeff. of terms - time varying TV

⑥ Linearity

- A sys. is linear if the following conditions are satisfied:

- For any arbitrary I/Ps $x_1(t)$ & $x_2(t)$

$$x_1(t) \xrightarrow{S} y_1(t)$$

$$x_2(t) \xrightarrow{S} y_2(t)$$

i. $x_1(t) + x_2(t) \xrightarrow{S} y_1(t) + y_2(t)$ "additivity"

ii. $a x_1(t) \xrightarrow{S} a y_1(t)$ "homogeneity" "scaling"

$\forall a \in \mathbb{C}$

"Superposition"

NOTE: A compact way to write this

$$a x_1(t) + b x_2(t) \xrightarrow{S} a y_1(t) + b y_2(t)$$

$\forall a, b \in \mathbb{C}$

Also equivalent:

$$\sum_k a_k x_k(t) \xrightarrow{S} \sum_k a_k y_k(t)$$

Eq ① $y(t) = \sin(t)\, x(t)$

Let $y_1(t) = \sin(t)\, x_1(t)$

$y_2(t) = \sin(t)\, x_2(t)$

Suppose $x(t) = x_1(t) + x_2(t)$

then $y(t) = \sin(t)\,[x_1(t) + x_2(t)] = y_1(t) + y_2(t)$

Suppose $x(t) = a\, x_1(t)$

then $y(t) = \sin(t)\, a x_1(t) = a\, y_1(t)$

Linear

② $y[n] = n\, x[n]$ Linear

③ $y(t) = \frac{1}{2T} \int_{t-T}^{t+T} x(\tau)\, d\tau$ Linear

④ $y(t) = x^2(t)$ (Sys.-S)

$x_1(t) + x_2(t) \longrightarrow (x_1(t) + x_2(t))^2$

$y_1(t) + y_2(t) = x_1^2(t) + x_2^2(t)$

$\neq$ Not linear

⑤ $y(t) = 2x(t) + 3$ (Sys-S)

$a x_1(t) \xrightarrow{S} 2a x_1(t) + 3$

$a y_1(t) = a(2x_1(t) + 3)$

$= 2a x_1(t) + 3a$

$\neq$ Not linear

NOTE 1: A linear sys. commutes with scaling & summation.

$y(t) = \sin(t)\, x(t)$

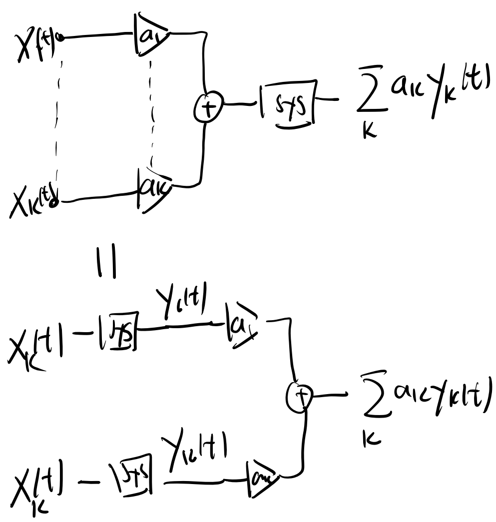
$y_1(t) = \sin(t)\, x_1(t)$

$y_2(t) = \sin(t)\, x_2(t)$

$x_3(t) = x_1(t) + x_2(t)$

$$y_3(t) = \sin(t)\,(x_1(t) + x_2(t)) = \sin(t)\, x_1(t) + \sin(t)\, x_2(t) = y_1(t) + y_2(t)$$

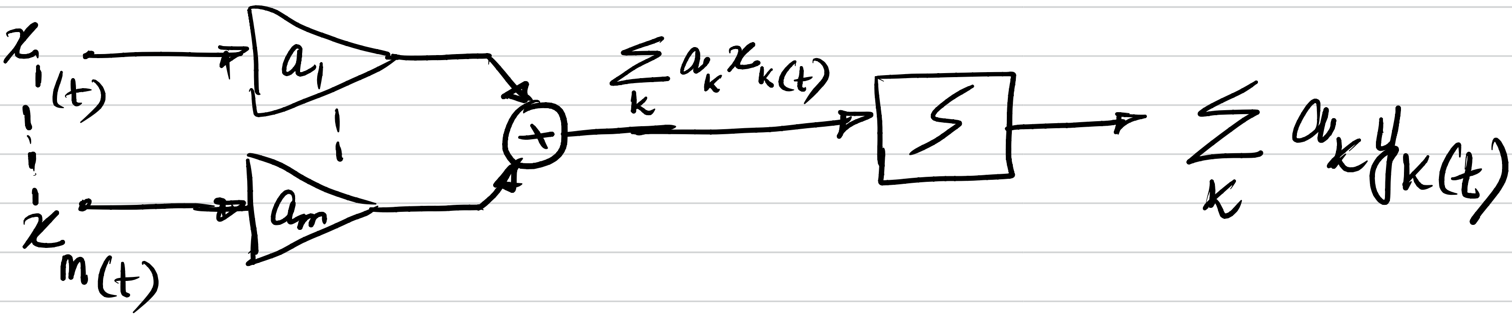
linear if

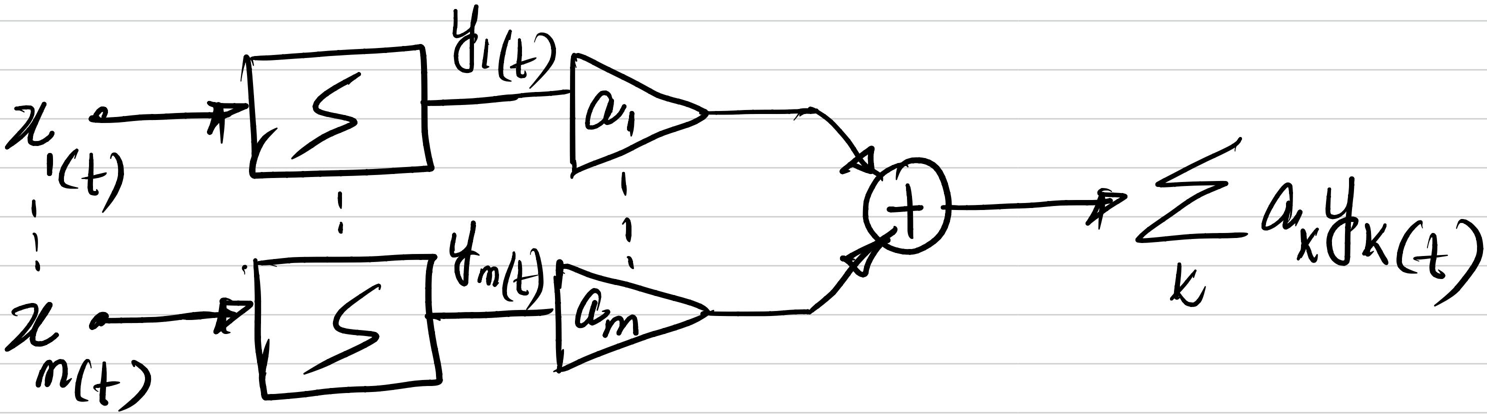


① $$t(ax_1(t) + bx_2(t)) = a\,t x_1(t) + b\,t x_2(t) = a y_1(t) + b y_2(t)$$

② $$(ax_1(t) + bx_2(t))^2 = a^2 x_1^2(t) + ab\,x_1(t)x_2(t) + b^2 x_2(t)$$

$$\neq a y_1^2(t) + b y_2^2(t)$$





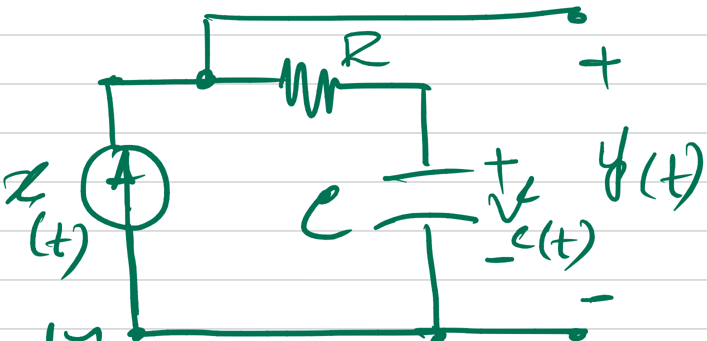
NOTE 2: For a linear sys., we always have

$$x(t)=0,\ \forall t \xrightarrow{S} y(t)=0,\ \forall t$$

Eg. ① $y(t)=2x(t)+3$

due to this when $x(t)=0$, $y(t)\neq 0$

② RC Circuit



$$y(t)=Rx(t)+\frac{1}{C}\int_{-\infty}^{t} x(\tau)\,d\tau$$

$$y(t)=Rx(t)+\underbrace{v_c(0)}+\frac{1}{C}\int_{0}^{t} x(\tau)\,d\tau$$

If it is '0' linear sys.

o/w Non-linear sys.

Formally called 'Initial rest condition'. [for any time t_0, any I/p $x(t)$ such that $x(t)=0$ for $t<t_0$, the corresponding o/p $y(t)=0$ for $t<t_0$]