Project presentation

Programming Concepts in Scientific Computing

EPFL, Master class

November 15, 2023

Rules

1. Project realized in groups of **two** students
2. Delivery on a GIT repository (Sources and report): Deadline **Friday 17th December 2023, 14h**
3. CMake build system
4. One central executable **that reads input**
5. Inline documentation of your code (Doxygen)
6. Test suite
7. Make a report delivered as an extended README:
   - how to compile the program
   - typical program execution (the flow) and usage
   - List of features & tests
   - TODOs and perspectives
8. Make one oral presentation **per student**
   - the structure of the program
   - list of features
   - limitations and problems

Rules

What is important in the evaluation:

- The code
  1. must be compiling
  2. different options are inputs (**no need to recompile to change behavior**)
  3. should be clean (coding convention)
  4. should have inline comments (and Doxygen)
  5. must pass tests
  6. The git log entries/comments must be understandable
- The report should describe:
  1. the implementation in a consise way
  2. the validating tests
  3. the limitations and problems

Project 1: Eigenvalue problems

Implementation of numerical methods for eigenvalue computation.

For a matrix A, finding all scalars λ such that

$$\mathbf{Ax} = \lambda \mathbf{x}$$

- Power and Inverse power method.
- Implementation of Power and Inverse power methods with shift
- Implementation of the QR method

Project 2: Ordinary Differential Equations

- This project focuses on ODE, with generic non-linear function:

$$y'(t,x) = f(t,x)$$

Description:

- Implementation of explicit methods, such as Forward Euler and the multistep Adams Bashforth (up to 4 steps) for both projects.
- Implementation of the implicit Backward Euler method.
- Implementation of explicit Runge-Kutta methods and/or Backward Differentiation Formulas (BDF schemes) and/or multistep Adams-Moulton.

Project 3: Non-linear systems

Implementation of numerical methods for the solution of nonlinear equations.

For a function f, find x such that:

$$f(x) = 0$$

- Mandatory: consider a scalar nonlinear problem and implement the bisection, aitken, chord, newton and fixed point methods.
- Extension to systems of nonlinear equations solved by the Newton and/or modified Newton method.
- Sources: here for the Newton's, chord and bisection method, and here for the fixed point method. Here for the Aitken acceleration to be applied to these methods.

Project 4: Data approximation

This project deals with interpolation and data fitting.

For a function f, parametrized by parameters $[a_1, a_2, \ldots]$:

- Interpolation: Assuming $f[a_1, \ldots](x_i)$ are known evaluations of f, computes $f[a_1, \ldots](x)$ everywhere.
- Data fitting: Provided points (x_i, y_i) find the parameters $[a_1, a_2, \ldots]$ so that $f[a_1, \ldots](x_i) \simeq y_i$ (in some weak sense).

- Implement the Lagrange polynomial approximation and Barycentric interpolation for the solution of interpolation problems.
- For the data fitting, the least squares method has to be implemented.
- Input data by reading a file
- Fourier approximation of periodic data or Cubic spline interpolation.
- Sources: see Chapter 3 of the book Scientific Computing with MATLAB and Octave (Quarteroni, Saleri) for the description of the methods. It can be downloaded here.

Project 5: Numerical Integration

Implementation of methods for the numerical computation of integrals in one or two dimensions.

For a function $f : \mathbb{R}^n \to \mathbb{C}^m$, with $0 < n \leq 3$, and m arbitrary, this project aims at computing:

$$\int_{\Omega} f(x_1, \ldots, x_n) dx^1 \ldots dx^n$$

- At first, a simple geometrical domain can be considered (square, rectangle) and it will consist in generating grids which can be structured.
- The numerical integration has to be carried out by the implementation of the following methods: Midpoint/Trapezoidal/Cavalieri-Simpson.
- Extension to more complex shaped domain, assembled by union.
- Sources: see Section 4.2 of the book Scientific Computing with MATLAB and Octave (Quarteroni, Saleri) for the description of the methods. It can be downloaded here.

Project 6: Image/sound processing

This project deals with the treatment of images or sound

- Computation of intensity histograms (discrete probability density of "pixel" intensity)
- Implementation of the discrete Fourier transform with the Fast Fourier Transform algorithm for 1D or 2D. (Find the algorithm at here)
- Contour extraction of an image or noise removal
- Filtering image/sound (By using the Fourier transform)

Project 7: Monte Carlo

This project deals with the statistical study of non-linear operators

For a (vectorial) function f, some statistical information is expected, such as the statistical moments:

$$\langle f^m \rangle \simeq \frac{1}{N} \sum_{i}^{N} f^m(x_i)$$

with the evaluations x_i taken from a random variable.

- Implement random number generators following a probability distribution for the normal and the uniform distributions. Use inverse transform sampling for the normal distribution.
- Computing numerically the expectation value of a user defined function, based on a random input variable.
- Same task for the extraction of the statistical moments
- Verification of the central limit theorem