$$\begin{pmatrix} 0 & 0 & B_R \\ 0 & K_{cc} & K_{cR} \\ B_R^T & K_{Rc} & K_{RR} \end{pmatrix} \begin{pmatrix} \lambda \\ u_c \\ u_R \end{pmatrix} = \begin{pmatrix} d \\ f_c \\ f_R \end{pmatrix} \begin{array}{l} \\ \leftarrow \text{Corners} \\ \\ \end{array}$$

$$u_c = \begin{pmatrix} u_P \\ u_D \end{pmatrix} \begin{array}{l} \leftarrow \text{Primal} \\ \leftarrow \text{Dirichlet} \end{array} \qquad K_{cc} = \begin{pmatrix} K_{PP} & K_{PD} \\ K_{DP} & K_{DD} \end{pmatrix} \qquad f_c = \begin{pmatrix} f_P \\ f_D \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & B_R \\ 0 & K_{PP} & K_{PD} & K_{PR} \\ 0 & K_{DP} & K_{DD} & K_{DR} \\ B_R^T & K_{RP} & K_{RD} & K_{RR} \end{pmatrix} \begin{pmatrix} \lambda \\ u_P \\ u_D \\ u_R \end{pmatrix} = \begin{pmatrix} d \\ f_P \\ f_D \\ f_R \end{pmatrix} \begin{array}{l} \\ \\ \leftarrow \text{Already} \\ \quad \text{Known} \end{array}$$

$$\begin{pmatrix} 0 & 0 & B_R \\ 0 & K_{PP} & K_{PR} \\ B_R^T & K_{RP} & K_{RR} \end{pmatrix} \begin{pmatrix} \lambda \\ u_P \\ u_R \end{pmatrix} = \begin{pmatrix} d \\ \tilde{f}_P \\ \tilde{f}_R \end{pmatrix} \qquad \left. \begin{array}{l} \tilde{f}_P = f_P - K_{PD} u_D \\ \tilde{f}_R = f_R - K_{RD} u_D \end{array} \right\} \text{Contain BC}$$

$$B_R u_R = d \quad (1)$$

$$K_{PP} u_P + K_{PR} u_R = \tilde{f}_P \quad (2) \quad \rightarrow \quad K_{PP} u_P = \tilde{f}_P - K_{PR} u_R$$

$$B_R^T \lambda + K_{RP} u_P + K_{RR} u_R = \tilde{f}_R \quad (3) \quad \rightarrow \quad u_R = K_{RR}^{-1} \left(\tilde{f}_R - B_R^T \lambda - K_{RP} u_P \right)$$

(3) → (2)

$$K_{PP} u_P = \tilde{f}_P - K_{PR} K_{RR}^{-1} \tilde{f}_R + K_{PR} K_{RR}^{-1} B_R^T \lambda + K_{PR} K_{RR}^{-1} K_{RP} u_P$$

$$u_P = S_{PP}^{-1} \left(\tilde{f}_P - K_{PR} K_{RR}^{-1} \tilde{f}_R + K_{PR} K_{RR}^{-1} B_R^T \lambda \right) \quad (4)$$

$$\rightarrow \quad S_{PP} = K_{PP} - K_{PR} K_{RR}^{-1} K_{RP} \quad (5)$$

(4) → (3)

!!

$$u_R = K_{RR}^{-1} \tilde{f}_R - K_{RR}^{-1} B_R^T \lambda - K_{RR}^{-1} K_{RP} S_{PP}^{-1} \tilde{f}_P + K_{RR}^{-1} K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1} \tilde{f}_R$$

!! $$- K_{RR}^{-1} K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1} B_R^T \lambda$$

$$u_R = K_{RR}^{-1}\left(I_R + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}\right) \tilde{f}_R - K_{RR}^{-1} K_{RP} S_{PP}^{-1} \tilde{f}_P \qquad (6)$$

$$\quad - K_{RR}^{-1}\left(I_R + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}\right) B_R^T \lambda$$

(6) → (1)

!!

$$B_R K_{RR}^{-1}\left(\left(I_R + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}\right) \tilde{f}_R - K_{RP} S_{PP}^{-1} \tilde{f}_P\right)$$

$$\quad - B_R K_{RR}^{-1}\left(I_R + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}\right) B_R^T \lambda = d$$

$$F\lambda = \bar{d}$$

$$F = - B_R K_{RR}^{-1}\left(I_R + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}\right) B_R^T$$

$$\bar{d} = d - B_R K_{RR}^{-1}\left(\left(I_R + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}\right) \tilde{f}_R - K_{RP} S_{PP}^{-1} \tilde{f}_P\right)$$