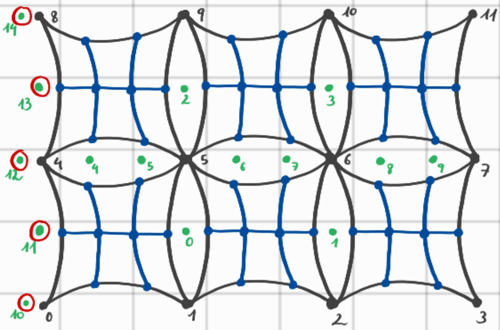
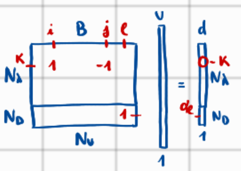
FETI DP SERIAL IMPLEMENTATION

- P: PRINCIPAL (N_p)
- R: REMAINING (N_R)
- λ: INTERFACE (N_λ)
- D: DIRICHLET (N_D)



(1) $\begin{pmatrix} 0 & B \\ B^T & K \end{pmatrix}\begin{pmatrix} \lambda \\ u \end{pmatrix} = \begin{pmatrix} d \\ f \end{pmatrix}$ (1.1) $Bu = d$



(1.2) $B^T\lambda + Ku = f$

$u = \begin{pmatrix} u_P \\ u_R \end{pmatrix}$

$(N_\lambda + N_D) \times N_p \quad N_p \times 1 \quad N_\lambda + N_D$

(2) $\begin{pmatrix} 0 & B_P & B_R \\ B_P^T & K_{PP} & K_{PR} \\ B_R^T & K_{RP} & K_{RR} \end{pmatrix}\begin{pmatrix} \lambda \\ u_P \\ u_R \end{pmatrix} = \begin{pmatrix} d \\ f_P \\ f_R \end{pmatrix}$

(3) $B_P u_P + B_R u_R = d$

(4) $B_P^T\lambda + K_{PP}u_P + K_{PR}u_R = f_P$

(5) $B_R^T\lambda + K_{RP}u_P + K_{RR}u_R = f_R$

(6) $K_{PP}u_P = f_P - K_{PR}u_R - B_P^T\lambda$

(7) $u_R = K_{RR}^{-1}\left[f_R - K_{RP}u_P - B_R^T\lambda\right]$

(8) $K_{PP}u_P = f_P - K_{PR}K_{RR}^{-1}f_R + K_{PR}K_{RR}^{-1}K_{RP}u_P + K_{PR}K_{RR}^{-1}B_R^T\lambda - B_P^T\lambda$ (7) → (6)

$u_P = [\underbrace{K_{PP} - K_{PR}K_{RR}^{-1}K_{RP}}_{S_{PP}}]^{-1}\left(\underbrace{f_P - K_{PR}K_{RR}^{-1}f_R}_{\hat{f}_P} + K_{PR}K_{RR}^{-1}B_R^T\lambda - B_P^T\lambda\right)$

(9) $u_P = S_{PP}^{-1}\hat{f}_P + S_{PP}^{-1}K_{PR}K_{RR}^{-1}B_R^T\lambda - S_{PP}^{-1}B_P^T\lambda$

(10) $u_R = K_{RR}^{-1}f_R - K_{RR}^{-1}B_R^T\lambda - K_{RR}^{-1}K_{RP}S_{PP}^{-1}\hat{f}_P$
$- K_{RR}^{-1}K_{RP}S_{PP}^{-1}K_{PR}K_{RR}^{-1}B_R^T\lambda + K_{RR}^{-1}K_{RP}S_{PP}^{-1}B_P^T\lambda$ (9) → (7)

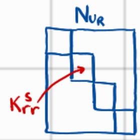
(11) $d = B_P\left(S_{PP}^{-1}\hat{f}_P + S_{PP}^{-1}K_{PR}K_{RR}^{-1}B_R^T\lambda - S_{PP}^{-1}B_P^T\lambda\right) + B_R\left(K_{RR}^{-1}f_R - K_{RR}^{-1}\lambda\right.$
$\left. - K_{RR}^{-1}K_{RP}S_{PP}^{-1}\hat{f}_P - K_{RR}^{-1}K_{RP}S_{PP}^{-1}K_{PR}K_{RR}^{-1}B_R^T\lambda + K_{RR}^{-1}K_{RP}S_{PP}^{-1}B_P^T\lambda\right)$

(9)(10) → (3)

(12) $\overbrace{-d + B_P S_{PP}^{-1}\hat{f}_P + B_R K_{RR}^{-1}f_R - B_R K_{RR}^{-1}K_{RP}S_{PP}^{-1}\hat{f}_P}^{\bar{d}} =$
$\underbrace{\left(-B_P S_{PP}^{-1}K_{PR}K_{RR}^{-1}B_R^T + B_P S_{PP}^{-1}B_P^T + B_R K_{RR}^{-1}K_{RP}S_{PP}^{-1}K_{PR}K_{RR}^{-1}B_R^T - B_R K_{RR}^{-1}K_{RP}S_{PP}^{-1}B_P^T + B_R K_{RR}^{-1}\right)}_{F}\lambda$

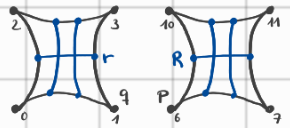
(13) $F\lambda = \bar{d}$

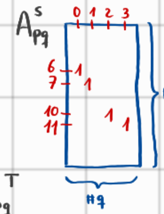
K_{RR}



K_{PP}

LOCAL → GLOBAL



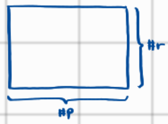


$K_{PP} = \sum_s A_{Pq}^s K_{qq}^s A_{Pq}^{sT}$

$u^s = \begin{pmatrix} u_q^s \\ u_r^s \end{pmatrix}$ $K^s = \begin{pmatrix} K_{qq}^s & K_{qr}^s \\ K_{rq}^s & K_{rr}^s \end{pmatrix}$ $A_{Pq}^s K_{qq}^s A_{Pq}^{sT}$

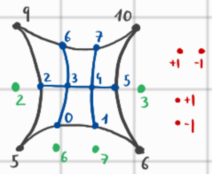
K_{PR}

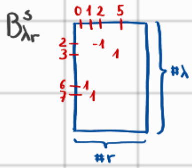
$K_{rP}^s = K_{rq}^s A_{Pq}^{sT}$



$K_{RP} = \sum_s A_{Rr}^s \overbrace{K_{rq}^s A_{Pq}^{sT}}^{K_{rP}^s}$

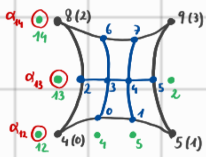
B_λ





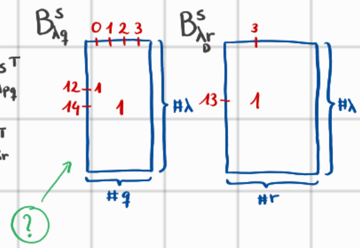
$B_\lambda = \sum_s B_{\lambda r}^s A_{Rr}^{sT}$

B

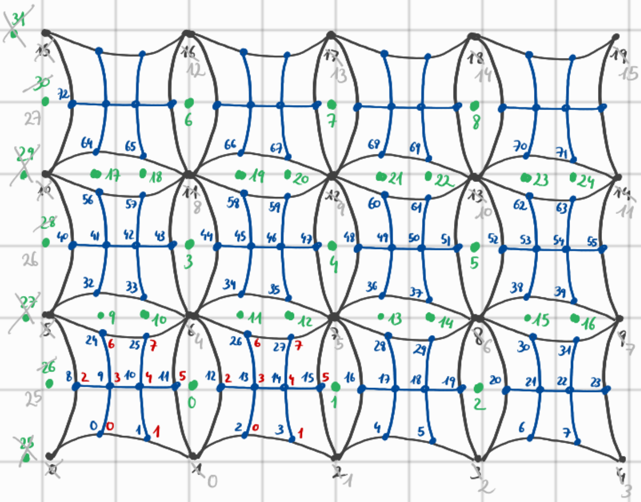


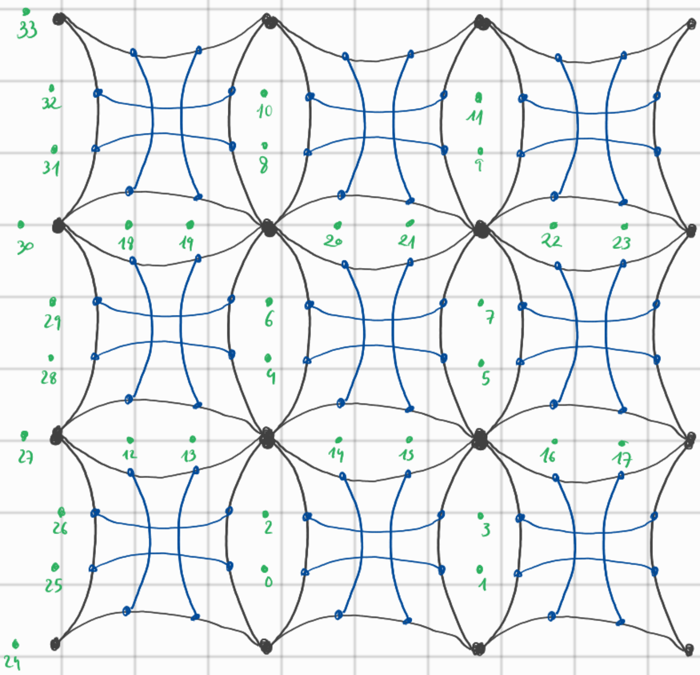
$B_{PD} = \sum_s B_{\lambda q}^s A_{Pq}^{sT}$

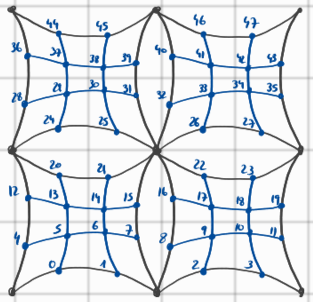
$B_{RD} = \sum_s B_{\lambda r}^s A_{Rr}^{sT}$



$B_R = \sum_s B_{\lambda Dr}^s A_{Rr}^{sT} + B_{\lambda r}^s A_{Rr}^{sT}$
$= \sum_s (B_{\lambda Dr}^s + B_{\lambda r}^s) A_{Rr}^{sT}$







$$\begin{pmatrix} 0 & 0 & B_R \\ 0 & K_{cc} & K_{cR} \\ B_R^T & K_{Rc} & K_{RR} \end{pmatrix} \begin{pmatrix} \lambda \\ u_c \\ u_R \end{pmatrix} = \begin{pmatrix} d \\ f_c \\ f_R \end{pmatrix} \begin{matrix} \\ \leftarrow \text{Corners} \\ \\ \end{matrix}$$

$$u_c = \begin{pmatrix} u_P \\ u_D \end{pmatrix} \begin{matrix} \leftarrow \text{Primal} \\ \leftarrow \text{Dirichlet} \end{matrix} \qquad K_{cc} = \begin{pmatrix} K_{PP} & K_{PD} \\ K_{DP} & K_{DD} \end{pmatrix} \qquad f_c = \begin{pmatrix} f_P \\ f_D \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & B_R \\ 0 & K_{PP} & K_{PD} & K_{PR} \\ 0 & K_{DP} & K_{DD} & K_{DR} \\ B_R^T & K_{RP} & K_{RD} & K_{RR} \end{pmatrix} \begin{pmatrix} \lambda \\ u_P \\ u_D \\ u_R \end{pmatrix} = \begin{pmatrix} d \\ f_P \\ f_D \\ f_R \end{pmatrix} \begin{matrix} \\ \\ \leftarrow \text{Already Known} \\ \\ \end{matrix}$$

$$\begin{pmatrix} 0 & 0 & B_R \\ 0 & K_{PP} & K_{PR} \\ B_R^T & K_{RP} & K_{RR} \end{pmatrix} \begin{pmatrix} \lambda \\ u_P \\ u_R \end{pmatrix} = \begin{pmatrix} d \\ \tilde{f}_P \\ \tilde{f}_R \end{pmatrix} \qquad \left.\begin{matrix} \tilde{f}_P = f_P - K_{PD} u_D \\ \tilde{f}_R = f_R - K_{RD} u_D \end{matrix}\right\} \text{Contain BC}$$

$$B_R u_R = d \quad (1)$$

$$K_{PP} u_P + K_{PR} u_R = \tilde{f}_P \quad (2) \quad \rightarrow \quad K_{PP} u_P = \tilde{f}_P - K_{PR} u_R$$

$$B_R^T \lambda + K_{RP} u_P + K_{RR} u_R = \tilde{f}_R \quad (3) \quad \rightarrow \quad u_R = K_{RR}^{-1} \left(\tilde{f}_R - B_R^T \lambda - K_{RP} u_P \right)$$

(3) → (2)

$$K_{PP} u_P = \tilde{f}_P - K_{PR} K_{RR}^{-1} \tilde{f}_R + K_{PR} K_{RR}^{-1} B_R^T \lambda + K_{PR} K_{RR}^{-1} K_{RP} u_P$$

$$u_P = S_{PP}^{-1} \left(\tilde{f}_P - K_{PR} K_{RR}^{-1} \tilde{f}_R + K_{PR} K_{RR}^{-1} B_R^T \lambda \right) \quad (4)$$

$$\rightarrow \quad S_{PP} = K_{PP} - K_{PR} K_{RR}^{-1} K_{RP} \quad (5)$$

(4) → (3)

!!

$$u_R = K_{RR}^{-1} \tilde{f}_R - K_{RR}^{-1} B_R^T \lambda - K_{RR}^{-1} K_{RP} S_{PP}^{-1} \tilde{f}_P + K_{RR}^{-1} K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1} \tilde{f}_R$$

!! $$- K_{RR}^{-1} K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1} B_R^T \lambda$$

$$u_R = K_{RR}^{-1}\left(I_R + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}\right)\tilde{f}_R - K_{RR}^{-1} K_{RP} S_{PP}^{-1} \tilde{f}_P$$
$$- K_{RR}^{-1}\left(I_R + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}\right) B_R^T \lambda \qquad (6)$$

(6) → (1)

!!

$$B_R K_{RR}^{-1}\left(\left(I_R + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}\right)\tilde{f}_R - K_{RP} S_{PP}^{-1} \tilde{f}_P\right)$$
$$- B_R K_{RR}^{-1}\left(I_R + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}\right) B_R^T \lambda = d$$

$$F\lambda = \bar{d}$$

$$F = - B_R K_{RR}^{-1}\left(I_R + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}\right) B_R^T$$
$$\bar{d} = d - B_R K_{RR}^{-1}\left(\left(I_R + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}\right)\tilde{f}_R - K_{RP} S_{PP}^{-1} \tilde{f}_P\right)$$