FETI-DP

$$\begin{pmatrix} 0 & B_P & B_R \\ B_P^T & K_{PP} & K_{PR} \\ B_R^T & K_{RP} & K_{RR} \end{pmatrix} \begin{pmatrix} \lambda \\ u_P \\ u_R \end{pmatrix} = \begin{pmatrix} d \\ f_P \\ f_R \end{pmatrix} , \quad K_{PR} = K_{RP}^T$$

$$B_P u_P + B_R u_R = d \qquad (1)$$

$$B_P^T \lambda + K_{PP} u_P + K_{PR} u_R = f_P \qquad (2)$$

$$B_R^T \lambda + K_{RP} u_P + K_{RR} u_R = f_R \qquad (3)$$

From (2): $K_{PP} u_P = f_P - K_{PR} u_R - B_P^T \lambda$ (4)

From (3): $u_R = K_{RR}^{-1} (f_R - K_{RP} u_P) - K_{RR}^{-1} B_R^T \lambda$ (5)

We plug (5) in (4):

$$K_{PP} u_P = f_P - K_{PR} K_{RR}^{-1} f_R + K_{PR} K_{RR}^{-1} K_{RP} u_P + K_{PR} K_{RR}^{-1} B_R^T \lambda - B_P^T \lambda$$

$$\underbrace{(K_{PP} - K_{PR} K_{RR}^{-1} K_{RP})}_{\substack{S_{PP} \\ \text{(Schur complement)}}} u_P = \underbrace{f_P - K_{PR} K_{RR}^{-1} f_R}_{\hat{f}_P} + (K_{PR} K_{RR}^{-1} B_R^T - B_P^T) \lambda$$

$$u_P = S_{PP}^{-1}\hat{f}_P + S_{PP}^{-1}\left(K_{PR}K_{RR}^{-1}B_R^T - B_P^T\right)\lambda \qquad (6)$$

In (6), u_P only depends on λ. We want the same for u_R. So, we plug (6) in (5):

$$\begin{aligned} u_R &= K_{RR}^{-1} f_R - K_{RR}^{-1} K_{RP} S_{PP}^{-1} \hat{f}_P \\ &\quad - K_{RR}^{-1} K_{RP} S_{PP}^{-1} \left(K_{PR} K_{RR}^{-1} B_R^T - B_P^T \right)\lambda - K_{RR}^{-1} B_R^T \lambda \\ &= K_{RR}^{-1}\left[f_R - K_{RP} S_{PP}^{-1}\left(f_P - K_{PR} K_{RR}^{-1} f_R \right)\right] \\ &\quad - K_{RR}^{-1}\left[K_{RP} S_{PP}^{-1}\left(K_{PR} K_{RR}^{-1} B_R^T - B_P^T \right) + B_R^T \right]\lambda \end{aligned}$$

$$\begin{aligned} u_R &= K_{RR}^{-1}\left[\left(I + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1} \right) f_R - K_{RP} S_{PP}^{-1} f_P \right] \\ &\quad - K_{RR}^{-1}\left[K_{RP} S_{PP}^{-1}\left(K_{PR} K_{RR}^{-1} B_R^T - B_P^T \right) + B_R^T \right]\lambda \qquad (7) \end{aligned}$$

Finally, we introduce (6) and (7) in (1)

$$B_P S_{PP}^{-1} \hat{f}_P + B_P S_{PP}^{-1} (K_{PR} K_{RR}^{-1} B_R^T - B_P^T) \lambda$$
$$+ B_R K_{RR}^{-1} \left[(I + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}) f_R - K_{RP} S_{PP}^{-1} f_P \right]$$
$$- B_R K_{RR}^{-1} \left[K_{RP} S_{PP}^{-1} (K_{PR} K_{RR}^{-1} B_R^T - B_P^T) + B_R^T \right] \lambda = d$$

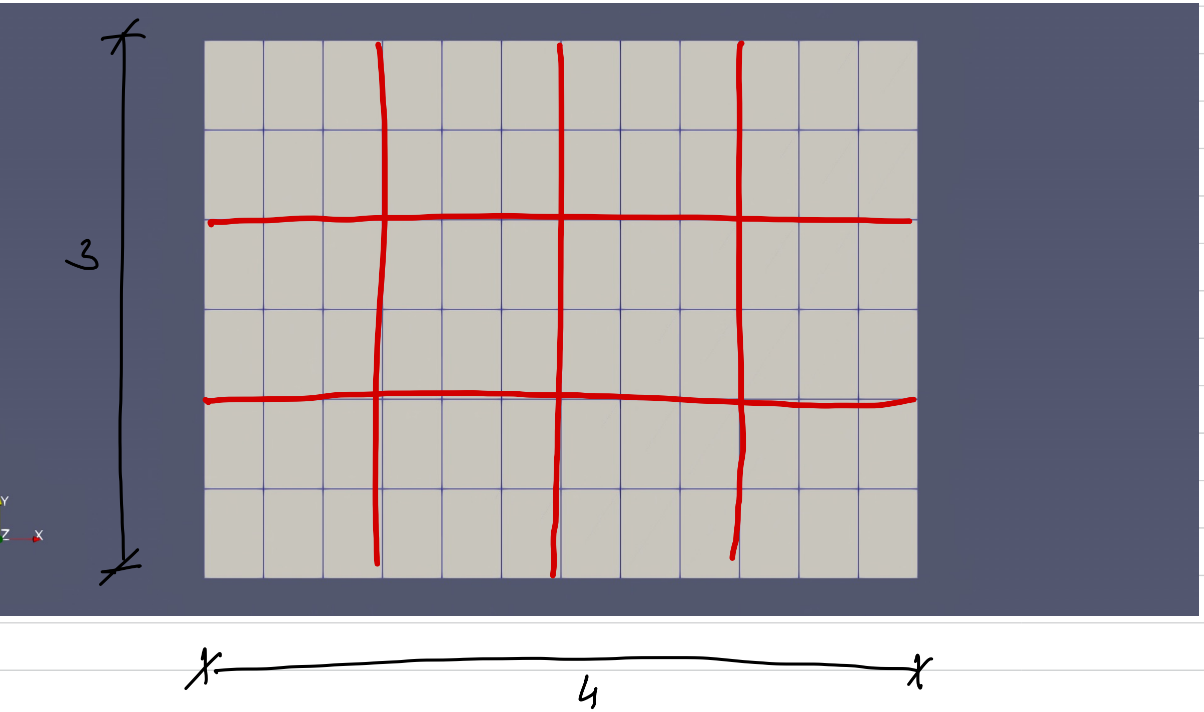
$$\Big[B_R K_{RR}^{-1} \left[K_{RP} S_{PP}^{-1} (K_{PR} K_{RR}^{-1} B_R^T - B_P^T) + B_R^T \right]$$
$$- B_P S_{PP}^{-1} (K_{PR} K_{RR}^{-1} B_R^T - B_P^T) \Big] \lambda$$
$$= -d + B_P S_{PP}^{-1} \hat{f}_P$$
$$+ B_R K_{RR}^{-1} \left[(I + K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1}) f_R - K_{RP} S_{PP}^{-1} f_P \right]$$

That we write as $F \lambda = \bar{d}$, where

$$F = B_R K_{RR}^{-1} B_R^T + B_R K_{RR}^{-1} K_{RP} S_{PP}^{-1} K_{PR} K_{RR}^{-1} B_R^T$$
$$+ B_P S_{PP}^{-1} B_P^T - B_P S_{PP}^{-1} K_{PR} K_{RR}^{-1} B_R^T - B_R K_{RR}^{-1} K_{RP} S_{PP}^{-1} B_P^T$$

$$\bar{d} = -d + (B_P - B_R K_{RR}^{-1} K_{RP}) S_{PP}^{-1} \hat{f}_P + B_R K_{RR}^{-1} f_R$$

TEST PROBLEM



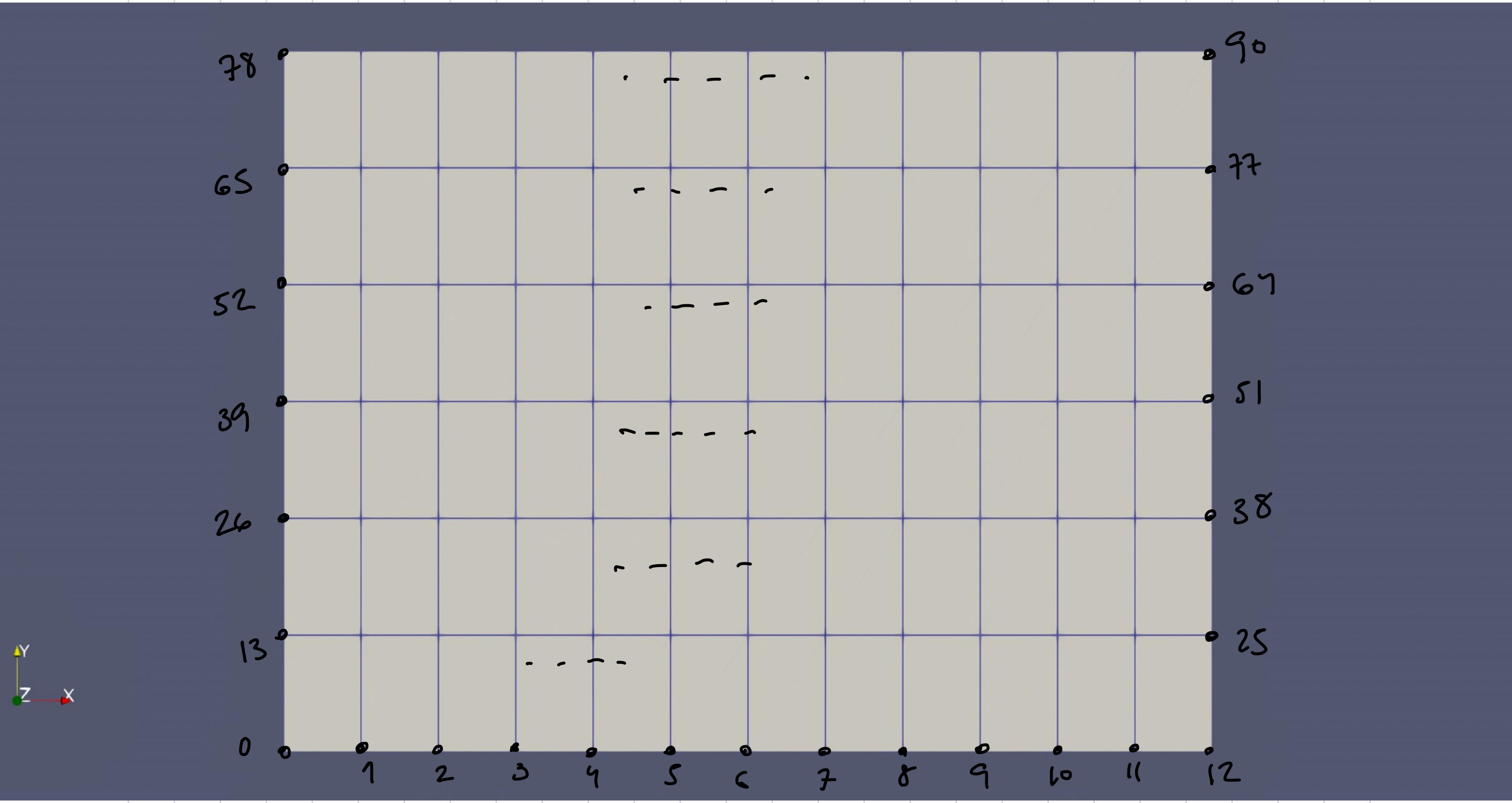
$$\Delta u + f = 0 \text{ in } \Omega$$

$$u = u_D \qquad \text{on left face}$$

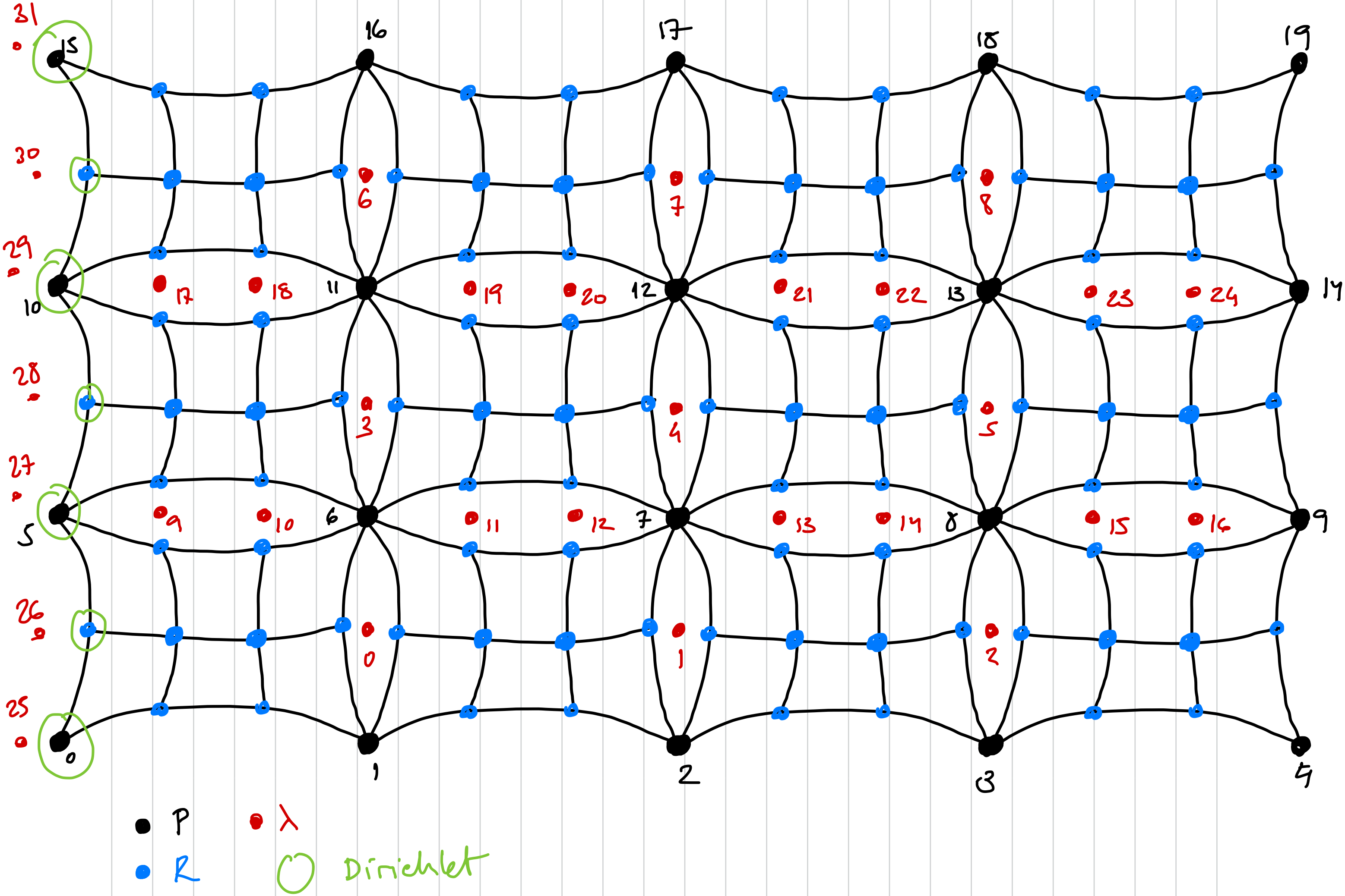
where $f = \frac{1}{2}$

$$u_D(x,y) = 1 + \frac{1}{3} y$$ linear temperature from 1 (bottom) to 2 (top)

GLOBAL NUMBERING : From 0 to 90.

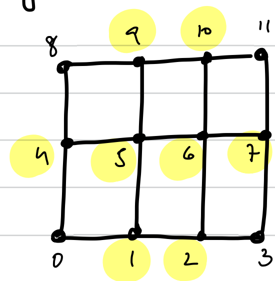


Degrees of freedom numbering help for FETI-DP



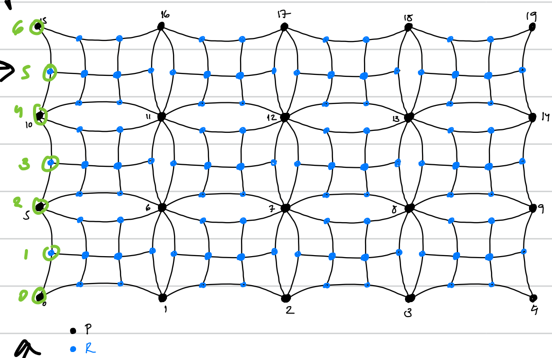
List of files provided

- fr.dat : right-hand side vector F restricted to only the remainder dofs o a single subdomain, i.e., to degrees of freedom



- local K.dat : stiffness matrix of subdomain referred to the 12 dofs in

- fP.dat : right-hand side vector F restricted to primal dofs of the full domain, i.e., to the black ones →



- dr.dat : Dirichlet values associated to the o

- solution.dat : Problem's solution associated to the 91 nodes