

$$\begin{aligned}
 &1 \quad 13 \bmod 3 \\
 &a = dq + r. \\
 &a = 13 \\
 &d = 3 \\
 &13 = 3q + r \\
 &q = 4, r = 1. \\
 &\therefore 13 \bmod 3 = 1 \neq.
 \end{aligned}$$

$$\begin{aligned}
 &-22 \bmod 23. \\
 &a = dq + r \\
 &a = -22 \\
 &q = 23. \\
 &-22 = 23q + r. \\
 &\cancel{q=0, r=-22 \neq} \\
 &\text{or} \\
 &\cancel{q=-1, r=1 \neq} \\
 &q = -1, r = 1 \neq.
 \end{aligned}$$

$$2 \quad a \equiv -15 \bmod 27.$$

Since -15 is between 0 and -26,
 $a = -15.$

$$3. \quad 5 \bmod 17 \quad 80, 103.$$

$$\begin{aligned}
 &5 + 17 = 22 \\
 &+ 17 = 39 \\
 &+ 17 = 56 \\
 &+ 17 = 73 \\
 &+ 17 = 90 \\
 &+ 17 = 107
 \end{aligned}$$

$\therefore 80$ not congruent
 103 not congruent.

$$4. -133 \bmod 23$$

$$23 \times (-6) = -138 \neq -133$$

$$-138 \neq -133$$

$$-133$$

$$x = 5.$$

$$26 \bmod 23$$

$$23 \times 1 + x = 26$$

$$x = 26 - 23$$

$$= 3.$$

$$\therefore -133 \bmod 23 + 26 \bmod 23 = 5 + 3 = 8.$$

$$5 (89^3 \bmod 79)^4 \bmod 26.$$

$$89 \bmod 79 = 10$$

$$(10^3 \bmod 79)^4 \bmod 26$$

$$\neq$$

$$100$$

$$100 \bmod 79 = 21$$

$$\therefore$$

$$100 \bmod 79 = 21 \cdot 210$$

$$(210)^4 \bmod 26 = (210 \bmod 79)^4 \bmod 26$$

$$210 \bmod 79 = 52$$

$$52^4 \bmod 26$$

$$52 \bmod 26 = 0.$$

$$\therefore 5 \cdot 0^4 \bmod 26 = 0.$$

$$6a) 97 = 1 \times 97$$

$$\therefore \text{prime}$$

$$b) 111 = 3 \times 37$$

$$\therefore \text{not prime}$$

$$c) 143 = 11 \times 13$$

$$\therefore \text{not prime.}$$

$$7a) 7 \times 11 \times 13 = 1001$$

$$b) 11 \times 101 = 1111$$

8. a) $11 = 11$
 $15 = 3 \times 5$
 $19 = 19$
 $11, 15 = 1$
 $11, 19 = 1$
 $15, 19 = 1$
 \therefore relatively prime

b) $12 = 3 \times 4$
 $17 = 17$
 $31 = 31$
 $37 = 37$
 \therefore relatively prime

9. a) $3^5 \times 5^3 \times 7^3$
 $= 10418625$

b) 1

c) $41 \times 43 \times 53 = 93439$

d) 1

10. $1000 = 2^3 \times 5^3$

$625 = 5^4$

$\text{gcd} = 5^3$

$= 125$

$\text{lcm} = 2^3 \times 5^4$

$= 5000$

$\text{gcd. lcm} = 125 \cdot 5000$

$= 625000$

$\therefore 625000 = 625 \times 1000$

11 a) $\text{gcd}(111, 201) = \text{gcd}(111, 90) = \text{gcd}(90, 21) = \text{gcd}(21, 6)$
 $= \text{gcd}(6, 3) = \text{gcd}(3, 0) = 3$

b) $\text{gcd}(1001, 1331) = \text{gcd}(1001, 330) = \text{gcd}(330, 11) = \text{gcd}(11, 0) = 11$

No.

Date

$$c) \gcd(1000, 6040) = \gcd(1000, 40) = \gcd(40, 0) = 40$$

$$12 \quad 117, 213$$

$$\gcd(117, 213) = \gcd(117, 96) = \gcd(96, 21) = \gcd(21, 12) = \gcd(12, 9) = \gcd(9, 3) = \gcd(3, 0) = 3$$

$$3 = 12 - (1 \times 9)$$

$$= (1 \times 12) + (-1 \times 9)$$

$$= 12 + (-1 \times (21 - 1 \times 12))$$

$$= 2 \times 12 + (-21)$$

$$= 2(96) + (-9(21))$$

$$= 11 \times 96 + (-9(117))$$

$$= (11 \times 213) + (-20 \times 117)$$

$$b) \gcd = 2$$

$$1128 \times 3454 + -835 \times 444$$