IMPLEMENTATION AND ANALYSIS OF A CODED OFDM COMMUNICATION SYSTEM OVER A WIRELESS FADING CHANNEL

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In previous works on channel modeling, the Rayleigh channel was simulated from the Clarkes spectrum. With this foundation an orthogonal frequency Division Multiplexing (OFDM) system will be constructed. OFDM is used to carry data over multi-carrier frequencies. The main advantage of OFDM is the simplified equalization process to counter the severe channel conditions. The challenge is to design an OFDM system that is inter-symbol interference (ISI) free and inter-carrier interference (ICI) free by tuning the system to meet the desired requirements. By analyzing and comparing the BER results with theory the simulated uncoded system will be compared to an equivalent uncoded AWGN channel. Improvements to the Rayleigh OFDM system are explored and implemented. Time diversity is also explored in combination with a simple repetition coding scheme. It was determined that the Rayleigh channel performed poorly in comparison with the AWGN channel, but as a higher order diversity was used the BER characteristics approached the AWGN performance.

***Index Terms*—OFDM, Fading channel, ISI, ICI, Simulation, Time Diversity, Repetition Coding, BER analysis, Cyclic prefix, QPSK, Yolo, Interleaver, Frequency-selective channels, MRC, Path loss**

I. INTRODUCTION

OFDM is a state of the art technique commonly used in today's communication systems. With the evolving connection-based infrastructure, higher gross rates are in demand. However, when increasing the data rates, the frequency spectrum will consequently expand, causing neighboring symbols to overlap, this is known as inter-symbol interference (ISI). A popular technique used to negate this effect is OFDM. For OFDM to be efficient, subcarriers should be orthogonal, but some inter-carrier interferences (ICI) could cause a lose of orthogonality. Among other functions the cyclic prefix is designed to help avoid interference between the neighboring carriers. This report focuses on the performance of an OFDM system, operating over a wireless time-varying, frequency-selective, Rayleigh fading channel. Different parameters, such as number of sub-carriers and length of cyclic prefix are designed in order to avoid both ISI and ICI. This scheme divides the spectrum into orthogonal sub-carriers, each occupying a fraction of the spectrum compared to that of the equivalent single carrier. Each OFDM symbol now passes through a channel that is slower varying than before OFDM was applied (narrowband).

Furthermore, both uncoded and coded transmission is simulated and their performance curves in terms of BER against SNR are compared. By using a simple repetition coding scheme along with an interleaver, diversity is achieved, by means of sending the same signal over different channels. The methods of achieving diversity are; position (multiple antennas), frequency (different subcarriers) and time (different OFDM frames). The latter two are investigated in this report and the optimal alternative is presented in the section *Method and Results*, where different diversity orders (M = 1,2,3) are compared.

II. METHOD AND RESULTS

This section starts off by explaining the general method for creating an OFDM system, simulation and detailed design of the system (uncoded) follows. After this the coded system is introduced and the parameters (see Appendix A for complete list) that are set in order to meet the specified requirements are motivated.

A. General Design of an OFDM system

The methodology of the uncoded OFDM communication system can be explained by inspecting Figure 1;

1) A sequence of random bits is generated.

2) The bits are transformed into QPSK-symbols according to
$$s[p] = \sqrt{\frac{E_s}{2}}(b[2p-1] + jb[2p]) \tag{1}$$
The energy per symbol is calculated according to $E_s = P_t T_s$, resulting in $E_s = 0.1\mu J$ per QPSK symbol.

3) The symbols are put into a matrix of size $N_{symb} \times N$, where N_{symb} is the total number of OFDM carriers and N is the amount of symbols for each carrier.

4) An IFFT of N points is applied to each row of the matrix. N is chosen such that it is a power of 2, making the algorithm significantly faster. This is the first condition that is added to the number of subcarriers, N.

5) For each sub-carrier a cyclic prefix is prepended, which acts as a guard interval and leads the MATLAB linear convolution to perform a circular convolution. In order to calculate the factor $f_D T_s$, the doppler shift needs to be calculated as follows,

$$f_D = \frac{v \cdot cos(\theta)}{\lambda} \tag{2}$$

where $v = 15$ m/s , $f = 2MHz$, $cos(\theta) = 1$, which results in $f_D = 100Hz$.
In order to ensure that the channel is constant for one whole OFDM symbol transmission, the following criterias must be met:

$$\begin{cases} (N + N_{cp}) f_D T_s \ll 1 \\ N_{cp} \geq \lceil \tau_L \rceil \end{cases} \tag{3}$$

With $f_D T_s = 0.1ms$ N is limited to being much below 10 000. As previously stated N must be a power of 2, with both of these constraints N was chosen to be 128. It is worth noting that a higher number of subcarriers could've been chosen and as it will become evident, the cyclic prefix will have a bound to be above 4, making a choice of N being 512 or 1024 more appropriate as the cyclic prefix will take up a smaller percentage of the overall OFDM frame. Still all the simulations were done with N equal to 128. Furthermore, according to equation 3, N_{cp} must be greater than the delay spread of the channel, i.e, greater than $\tau_L = 4$ samples. Since the cyclic prefix takes samples from the frame, the resulting limit is $\tau_{max} \leq N_{cp} \leq N$, which in our case equals $4 \leq N_{cp} \leq 128$. With respect to these two conditions N_{cp} was set to 10. This is assumed to be a safe limit to prevent ICI.

6) The digital to analog converter block is not used for this project. Including it would not have changed the simulation results, as well it is not required as we are not transmitting over an analog channel since the simulation is running on a computer.

7) The properties of the wireless fading channel are hidden in a protected matlab file, it was troublesome to decrypt. Complex noise is added after the channel and the given path-loss of 101 [dB] is applied to the signal. Since the output of the channel is in linear scale, the path-loss is added by dividing the output with $\sqrt{10^{10.1}}$, the square-root is needed in order to convert the path-loss from power-domain to amplitude. To compute the variance of the white-noise, N_0 needs to be divided by $2 \cdot T_s$, according to equation (25) in the PM. The factor of two emerges from the fact that the noise is added in both the in-phase and quadrature dimensions.

8) The output of the channel is taken one OFDM symbol at a time until all N_{symb} OFDM frames are received. The cyclic prefix of each individual frame is stripped and discarded. Frames are then concatenated into an array of N_{symb} OFDM symbols.

9) For each sub-carrier a FFT of N points is applied. The data is back into the frequency domain.

10) In order to compensate for the phase rotation caused by the channel, an estimation matrix **C** is produced, which is then multiplied with the received vector **r**. However, for this project the channel is assumed to be perfectly estimated, and returned by the channel. If condition 3 is satisfied all the entries in the rows of the returned **h** matrix are identical. Therefore, the received signal is multiplied with the Hermitian of **C** as follow

$$\begin{aligned} (C^{(m)})^* r^{(m)} = (C^{(m)})^* C^{(m)} s^{(m)} + (C^{(m)})^* n^{(m)} = \\ = diag(|C^{(m)}[0,0]|^2, |C^{(m)}[1,0]|^2, .. \\ .., |C^{(m)}[N-1,0]|^2) s^{(m)} + (C^{(m)})^* n^{(m)} \end{aligned} \tag{4}$$

Consequently, the noise phase is taking into account. The signal $s^{(m)}$ is multiplied by the magnitude of the **C** matrix (Hermitian multiplication) which will not influence the sign, it will only push the respective signals deeper into the their planes, and since decisions are made solely on the sign they will only be made more certain. The errors introduced come from the noise term of equation 4.

11) As previously stated decisions are made on the signs of the real and imaginary components outputting the resulting bits.

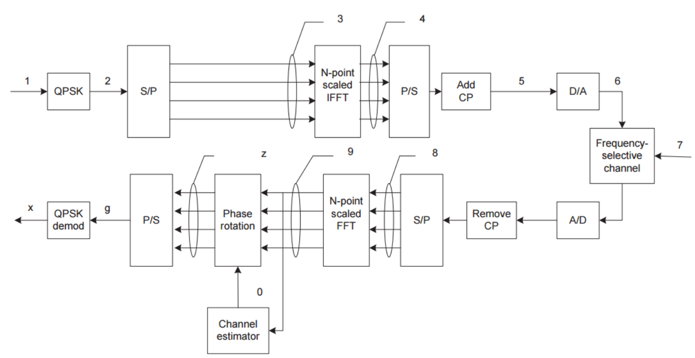

Fig. 1. *Showing the general outline of the OFDM system*

B. Uncoded OFDM System

The goal of this section is to design an uncoded OFDM system to satisfy the following requirements: BER of $5 \cdot 10^{-3}$, at data rate 1.85 Mbps, $N_0/2 = 2.07 \cdot 10^{-14} \mu W/Hz$, along with the general constraints mentioned above. As aforementioned $N = 128, N_{cp} = 10$, and $E_s = \bar{P}_t T_s = 10^{-7}$J. The effective data rate is reduced by the length of the cyclic prefix according to,

$$R_{required} = R \cdot \frac{N}{N + N_{cp}} \tag{5}$$

From this the data rate can be determined to be $R = 1.99Mbps$. The average SNR per bit $\bar{\gamma}_b$, can be calculated using equation (6) with $M = 1$ and the resulting value for BER, $\bar{P}_b = 5 \cdot 10^{-3}$ is $16.92[dB]$. Using this value it can be determined that a rate of 1.85 Mbps can be achieved, with the appropriate BER.

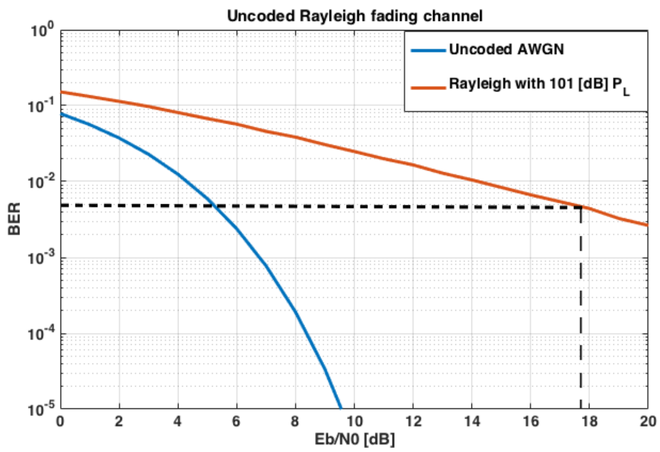

Fig. 2. *The uncoded AWGN curve together with uncoded Rayleigh curve. The SNR value for a BER of* $5 \cdot 10^{-3}$ *is pointed out*

The designed OFDM system is simulated through a sweep of SNR values and the BER behavior is observed. From Figure (3) it can seen the target BER is achieved at around $17.5[dB]$ which is close to what was predicted in theory. A comparison is done against the uncoded AWGN channel, and it can be observed that an enormous gain of around $12[dB]$ occurs between the uncoded Rayleigh and AWGN channels. This gap is bridged in the next section where diversity gain and coding are used.

With the choices mentioned above, the following parameters are obtained;

Delay spread: $T_m = \tau_{max} \cdot T_s = 4\mu s$
Coherence Bandwidth: $B_c = \frac{1}{T_m} = 250kHz$
Coherence Time: $T_c \geq (N + N_{cp})T_s$

C. Coded OFDM System

In order to simulate diversity, a repetition coding scheme as well as an interleaver is added to the previous communication system. The two repetition (REP) coding schemes used are (2,1) and (3,1). The overview of the system schematics is illustrated in Figure (4). As can be seen in the figure, the first step is to encode the bits. However, in this system implementation, the QPSK-modulator was added before the encoder (this does not affect the BER analysis in any way).

The interleaver has multiple options where it can separate identical frames any distance apart. The hypothesis here was that if a fade occurred at some instance it would disappear at some later time, and it was expected that the channel would've totally changed if sufficient time elapsed. It was found that separating the frames far apart made no difference in the simulated channel, as the mechanics of the channel are not documented it was assumed that there was no memory and therefore the separation in time made no difference.

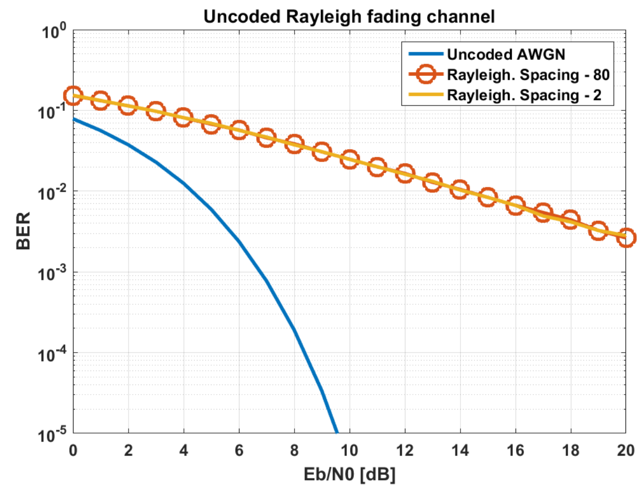

Fig. 3. *Showing how the spacing does not affect the BER curve*

In order to achieve frequency diversity the same signal needs to be sent on different carrier frequencies, which is basically what OFDM is doing [1]. But, the sub-carriers have to be separated by at least the coherence time T$_c$. Frequency diversity requires more transmit power as the signal has to be sent over different sub-carriers. But, the transmit power was set to 0.1W so it could not be changed in order to implement frequency diversity.

Moreover, to achieve time diversity, which does not require an increased transmit power, a coding and interleaver can be used. Indeed, it is what Figure (4) suggests and it is also the second reason why time diversity has been preferred from frequency diversity.

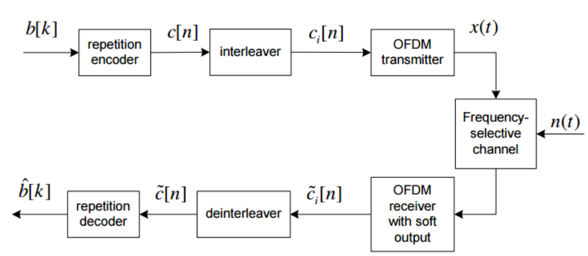

Fig. 4. *Showing the system schematics for the coded OFDM*

D. Comparison of the OFDM systems

As seen in Figure (5), our simulation results corresponds nicely to the theoretical curves, which are generated using Equation (6). This equation is solely for Rayleigh fading distribution together with Maximum-Ratio Combining. When increasing the order of diversity, the slope of the curve gets steeper, providing better BER values for a given SNR.

$$\bar{P}_b = \int_0^{\infty} Q(\sqrt{2\gamma}) p_{\gamma\Sigma} \gamma \mathrm{d}\gamma =$$
$$= \frac{1-\Gamma}{2}^M \sum_{m=0}^{M-1} \binom{M-1+m}{m} (\frac{1+\Gamma}{2})^M \tag{6}$$

Where $\Gamma = \sqrt{\bar{\gamma}/(1+\bar{\gamma})}$

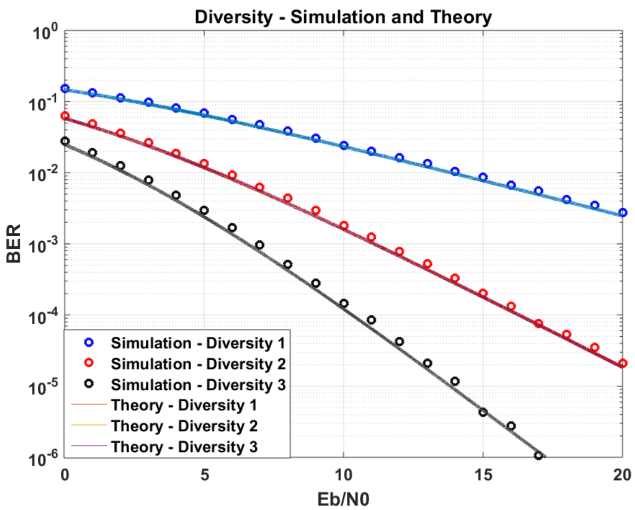

Fig. 5. *Showing the theoretical curves for diversity 1,2 and 3 together with the simulated data points*

III. Conclusion

A complete OFDM system has been successfully constructed. This system has been evaluated for coded and uncoded schemes. The performance of the coded system accomplished lower BER as expected. The simulation results closely matched the theoretical calculations of the diversity schemes. Time diversity was used as the only diversity scheme, improving the BER as much as 3 orders of magnitude in the case of uncoded versus diversity of 3. The reason for using time diversity, instead of frequency, was because there was a constraint on the power of the transmitter, which was fixed. An interleaver and an encoder naturally leads to the implementation of time diveristy.

For the uncoded scheme the desired BER was achieved at SNR of approximately 17 [dB], as was shown in figure (3). The gap between the AWGN and Rayleigh channels was observed in the same figure, which emphasized the need for a method to bring the BER closer towards the BER of the AWGN.

Simulations were time consuming in the cases of high diversity. That was because the BER was theoretically able to reach very low values, and in simulation BER of less than 10^{-7} are extremely time consuming due to the large number of bits that are required to generate these errors.

IV. Contribution

The team effort was great, in which, the participants sequentially solved problems together.

V. References

[1] A. Goldsmith *”Wireless Communications,”* Cambridge university Press, chap 7, pp. 205-206, 2005

VI. Appendix

A. Table of parameters

```
fc    = 2e9;                % Carrier frequency
fs    = 1e6;                % Sample frequency
Ts    = 1./fs;              % Symbol period (size of a QPSK symbol)
fd    = 15/(3e8/fc);        % Doppler frequency (with v = 15 m/s)
fdTs  = fd*Ts;
Pt    = 0.1;                % Transmit power
Pr    = 7.94e-12;           % Receivered power after channel
m     = 2;                  % Bits per symbol (QPSK)
N     = 128;                % Number of subcarriers (arbitrary
                            % chosen)
                            % Must be power of 2 (due to FFT
                            % speed up)
Nsym = 200000;              % Number of OFDM symbols to transmit
Nbits = 2 * N * Nsym;       % Total Number of bits
tau   = [0; 4];             % Channel delays
Ncp = ceil(max(tau))+6;     % Length of the c.prefix ( > length of
                            % channel delay spread
                            % (4 microseconds)
E = Ts*Pt;                  % Transmitted energy per second joule
Tb = 5.405e-7;              % Tb effective. Tb = 1/Rb.
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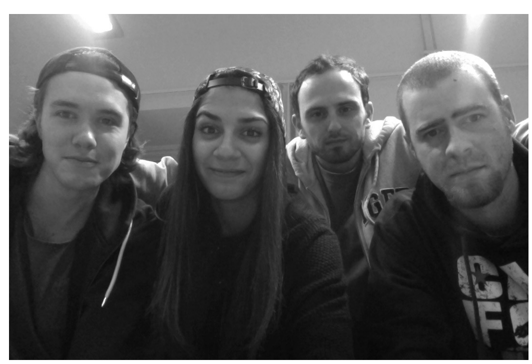

Fig. 6. The members of group 1. From left to right: Patrick Bateman, Jeanne Of Arc, Aqua man, Captain America