Design and Simulation of a Communication System over a Wireless Rayleigh Fading Channel

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Rayleigh fading was primarily used to model tropospheric and ionospheric communication channels, which covers communication over the horizon spanning wide distances. As mobile communication systems expanded, Rayleigh fading turned out to be a suitable model of urban environments. This model is the most appropriate when modeling multi-path signal propagation, where clear line of sight communication is limited. This project focuses on how to model a Rayleigh fading channel using two different methods by scrutinizing the impact of the modeling parameters. By acquiring an understanding of the channel, a foundation is built for further development of an OFDM communications system. As it turned out, both methods shared similar properties, but differed in terms of performance. Thus, the Spectrum Method is preferred.

***Index Terms*—Rayleigh, Fading channels, Filter method, Spectrum method, Simulation, Clarke's Doppler Spectrum**

I. Introduction

THE Rayleigh fading channel is one of the many important models used when simulating wireless communications. It is commonly applied when considering channels where obstacles are blocking the direct line of sight (LOS). Henceforth, Rayleigh channel simulation is essential when designing wireless communication networks in large metropolitan cities. This project investigates two different methods of simulating the Rayleigh fading channel; one of which is done in the time-domain using filtering functions, while the other one is computed in the frequency domain. These two methods are compared to each other in terms of their autocorrelation, PDF, CDF and PSD. Furthermore, a time and frequency-varying channel is simulated while investigating the effects of changing different parameters such as symbol period (T_s) and number of taps.

II. Methods and Milestones

A. Filter Design

One way of simulating the channel is by using the so called Filter Method. The following steps outline the procedure of the method.

1) f_D is calculated according to:

$$f_D = \frac{v \cdot cos(\theta)}{\lambda} \tag{1}$$

where $v = 8.33$ m/s , $f = 2GHz$, $cos(\theta) = 1$, which results in $f_D = \frac{500}{9} = 55.5Hz$. The imposed condition that must be satisfied in order to avoid aliasing is

$$1/T_s > 2f_D \tag{2}$$

therefore, a sampling frequency, $f_s = 1/T_s = 10kHz$ is selected. This will satisfy the above condition.

2) A rectangular window was chosen to retain as much out of the main lobe as possible, as the sidelobe attenuation offered by the different windows is of no interest. The duration of the window corresponds to where $g(t)$ is to be truncated. A reasonable length was used which was much smaller than the simulation number, N_s.
3) This technique is based on equation

$$S_c(f) = |G(f)|^2 S_x(f) = |G(f)|^2 \tag{3}$$

Where the PSD of white noise is unity, and g(t) can be found from the inverse Fourier transform of the square root of (3). The constant K is chosen to normalize the generated $\hat{g}$.

4) A zero mean complex Gaussian is formed using the MATLAB function `randn`.
5) After the convolution of $\hat{g}$ with x the result has both its sides truncated according to the $N_h - 1$, where N_h is the selected filter length, which produces a steady-state response.

Difficulties: It was difficult to understand which filter window was best suitable for the task. From different filter windows it was determined that the rectangular window produced the best results.

B. Spectrum Design

1) T_s is chosen according to the Filter Design, subpoint 1).
2) Clarke's Doppler spectrum is chosen as the basis of this approach, taken from equation

$$S_c(f) = \begin{cases} \frac{1}{\pi f_D} \frac{1}{\sqrt{1-(f/f_D)^2}} & , |f| \leq f_D \\ 0 & , Otherwise \end{cases} \tag{4}$$

G(f) is duplicated and the copy is centered around f_s.

3) The resulting spectrum was sampled at kf_s/N_s.

4) Finally the variance of a white noise process was chosen such that the resulting $c(nT_s)$ would have unit variance. This is derived with the help of Parseval's Theorem.

$$
\begin{aligned}
Var[X] &= E[X^2] - (E[X])^2 = E[X^2] \\
Parseval's\ theorem: \sum_{n=0}^{N-1} |y(n)|^2 &= \frac{1}{N}\sum_{k=0}^{N-1}|y(k)|^2 \\
\frac{1}{N}\sum_{n=0}^{N-1}|c(n)|^2 &= \frac{1}{N^2}\sum_{k=0}^{N-1}|X(k)\tilde{G}(k)|^2 \\
1 = \frac{1}{N^2}\sum_{k=0}^{N-1}|c(k)|^2 &\leq \frac{1}{N^2}\sum_{k=0}^{N-1}|X(k)|^2|\tilde{G}(k)|^2 \\
\frac{N_s^2}{\sum_{k=0}^{N-1}|\tilde{G}(k)|^2} &\leq \sum_{k=0}^{N-1}|X(k)|^2 = \sigma^2 \\
\Rightarrow \sigma &\leq \frac{N_s}{\sqrt[2]{\sum_{k=0}^{N-1}|\tilde{G}(k)|^2}}
\end{aligned}
\tag{5}
$$

By using this upper-bound σ was set so that the variance of c(n) was within 10^{-2} of unity.

Difficulties: It was complicated to form the resulting spectrum specified by step 2), in which the frequency axis needed to be properly transformed. It was also troublesome to calculate the resulting variance of $X(k)$. Equation (5) was used to calculate the appropriate variance of $X(k)$.

III. Results and Discussion

A. Filter and Spectrum method

After creating the channel as seen in *figure 1*, the channel gain is low pass filtered. It is now varying slower, which was expected from the low pass characteristic of the Doppler Spectrum, which was used in the filter generation.

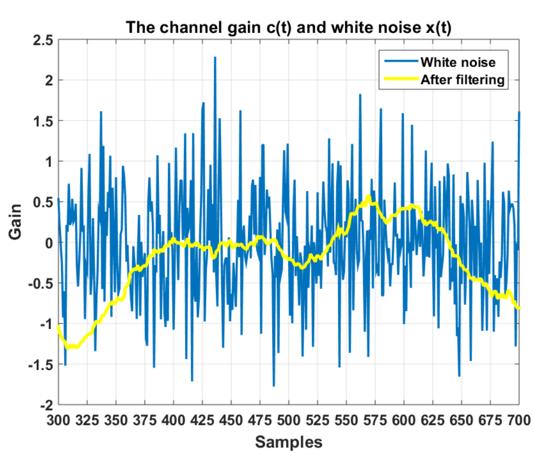


Fig. 1. *Showing white noise vs. filtered white noise using the channel model*

Looking at the envelope of both simulations we see an uncanny resemblance. As seen in *Figures (2,3)* the variance appears to be the same, while the spectrum method has a lower average magnitude.

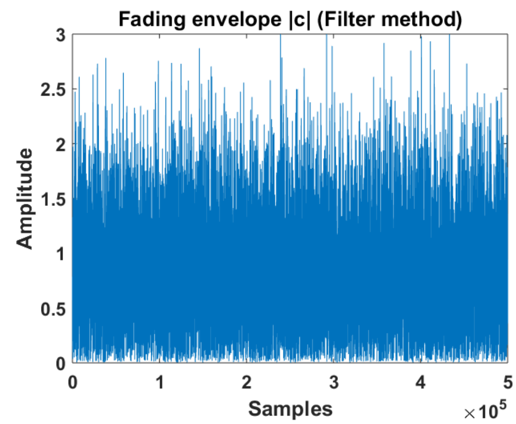


Fig. 2. *Envelope of the channel using the filter method*

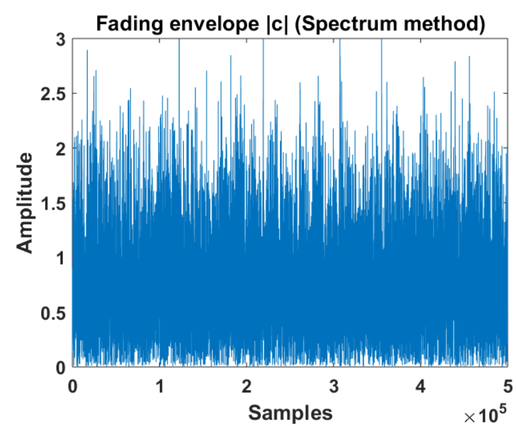


Fig. 3. *Envelope of the channel using the spectrum method*

Furthermore, by inspecting the Doppler spectrum functions for the two methods in *Figures (4,5)*, it is noticed that they share similar characteristics and are close to the theoretical bath-tub shape. The spectrum is band-limited to f_D and $fs - f_D$ with the majority of the power being in the impulses at the respective frequencies.

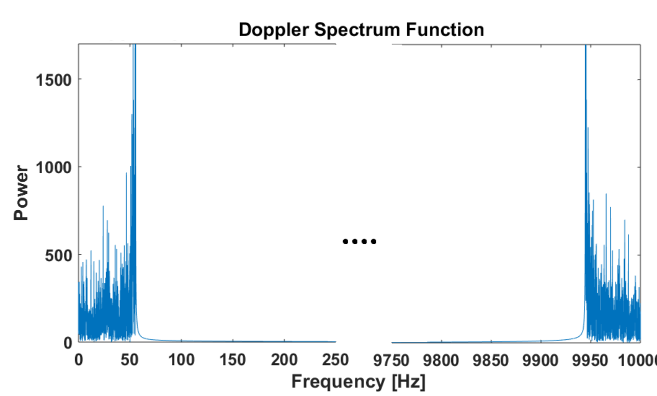


Fig. 4. *This is the PSD for the filter method*

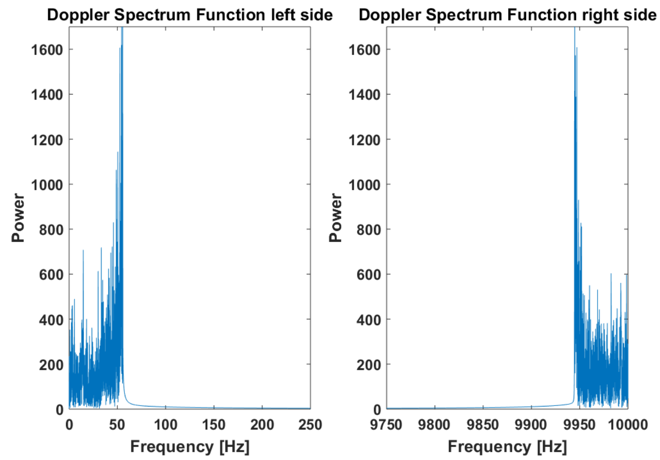


Fig. 5. *This is the PSD for the spectrum method*

According to Equation 6, the received signal is a function depending on the channel gain, transmitted symbol and additive noise. For this experiment, noise is neglected and we assume s(t) to be a constant signal. By varying the Doppler frequency, the receiver is being simulated at different speeds under small scale fading (multi-path Rayleigh uniform scattering). The resulting graph (Figure. 6) shows the signal power envelope in dB scale. Notice that the faster moving object has a lot more variation in the power level compared to the slower one. Furthermore, notice the deep fades (e.g. at time 240 ms the signal drops significantly).

$$r(t) = c(t)s(t) + n(t) \tag{6}$$

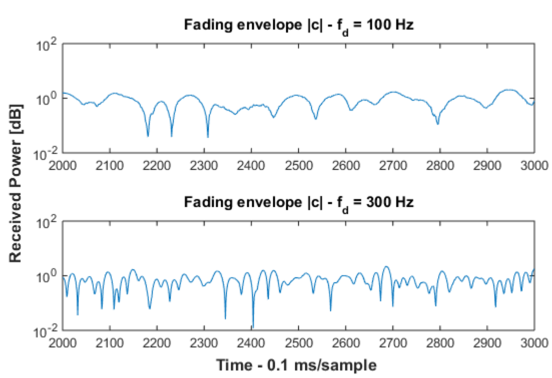


Fig. 6. *Showing the effect of different doppler frequencies. The total time interval correspond to 1 s*

In Figure. (7), the Auto Correlation Function (ACF) for the filter method, the spectrum method, as well as for the theoretical one are shown. The theoretical ACF is calculated by applying the Bessel-function according to:

$$A_c(\Delta t) = J_0(2\pi f_D \Delta t) \tag{7}$$

Furthermore, according to Equation (8), it is noticed that at delay zero the value is one, which corresponds to the total power of the signal, which also is correct across all three cases. As seen in Figure. (7), the two methods have very similar ACFs, however, the spectrum method seems to correspond a bit more to the theoretical curve. The observant reader will note how the channel decorrelates after a certain time delay and then again recorrelates.

$$A_c(0) = \int_{-\infty}^{\infty} S_c(\rho)\mathrm{d}\rho \tag{8}$$

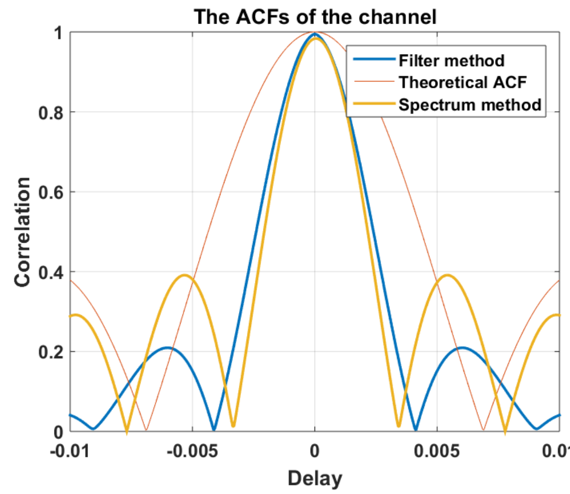


Fig. 7. *Showing the two ACFs for the respective methods vs the theoretical one*

Regardless of the method used to model the channel, both the PDF and CDF, respectively Figure. (8) and Figure. (9), are following a Rayleigh distribution. These results are not surprising as both the filter and spectrum methods are based on the Doppler spectrum function, which is in turn is originally derived from Rayleigh fading according to, see Equation (4).

Notice that the theoretical PDF and CDF have a variance equal to one, for which the corresponding slope is different from the empirical ones obtained by these methods. By setting the theoretical variance to 0.75 it will be very similar to our results (observe: this is not illustrated in the figure).

B. Time and frequency varying channel

Here we are approximating a time and frequency varying Rayleigh fading channel using the filter method in Figure (4) of the PM. It is based on Equation (9), where we assume that the input is a complex sinusoidal. We extract the channel from this channel approximation.

$$r(nT_s) = \sum_{l=1}^{L} c_l(nT_s)s(nT_s - \tau_l T_s) \tag{9}$$

The results can be seen in Figure. (10), where the number of taps (L), as well as the factors $T_s \cdot f_D$ are being varied. As can be seen the plots with L=1 generates frequency flat channels for all variations. This is easily explained by looking at Equation 9, in reality this results in a frequency flat channel, which is equivalent to having only one delay

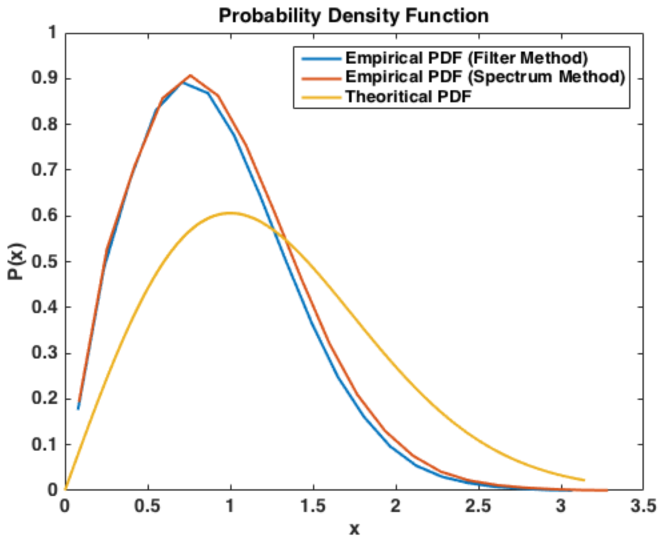


Fig. 8. *PDF comparisons*

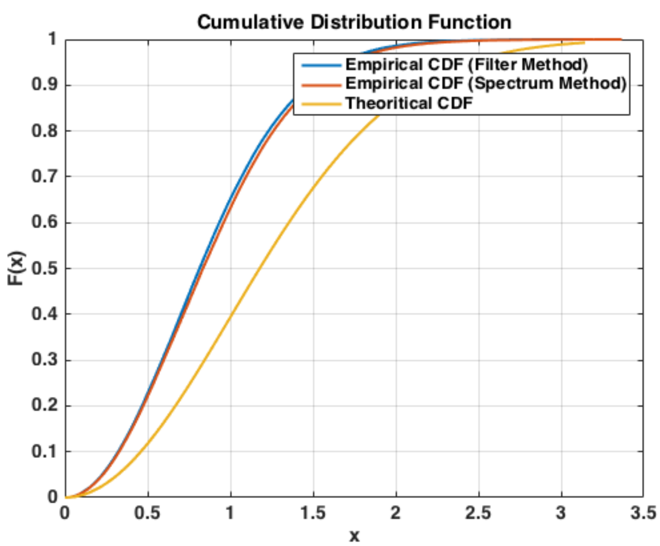


Fig. 9. *CDF comparisons*

coming in. In the next two cases, when L = 2 and L = 3, the channel becomes frequency selective. When increasing L, the channel gets sensitive for low values of $T_s \cdot f_D$.

The Doppler spread is defined as the frequency over which the PSD function is greater than zero, which relates closely to the Doppler frequency. Furthermore, the Doppler spread equals the inverse of the coherence time. Therefore, increasing f_D will make the coherence time go down, hence making the channel less stable. This makes sense in reality, where the channel for a fast moving object will change more rapidly.

For low values of $T_s \cdot f_D$ we always have a quickly changing channel, as seen in the plots (L=1), where it becomes highly time selective. In reality channels are a mix of frequency selective and time selective, as seen in the middle plots.

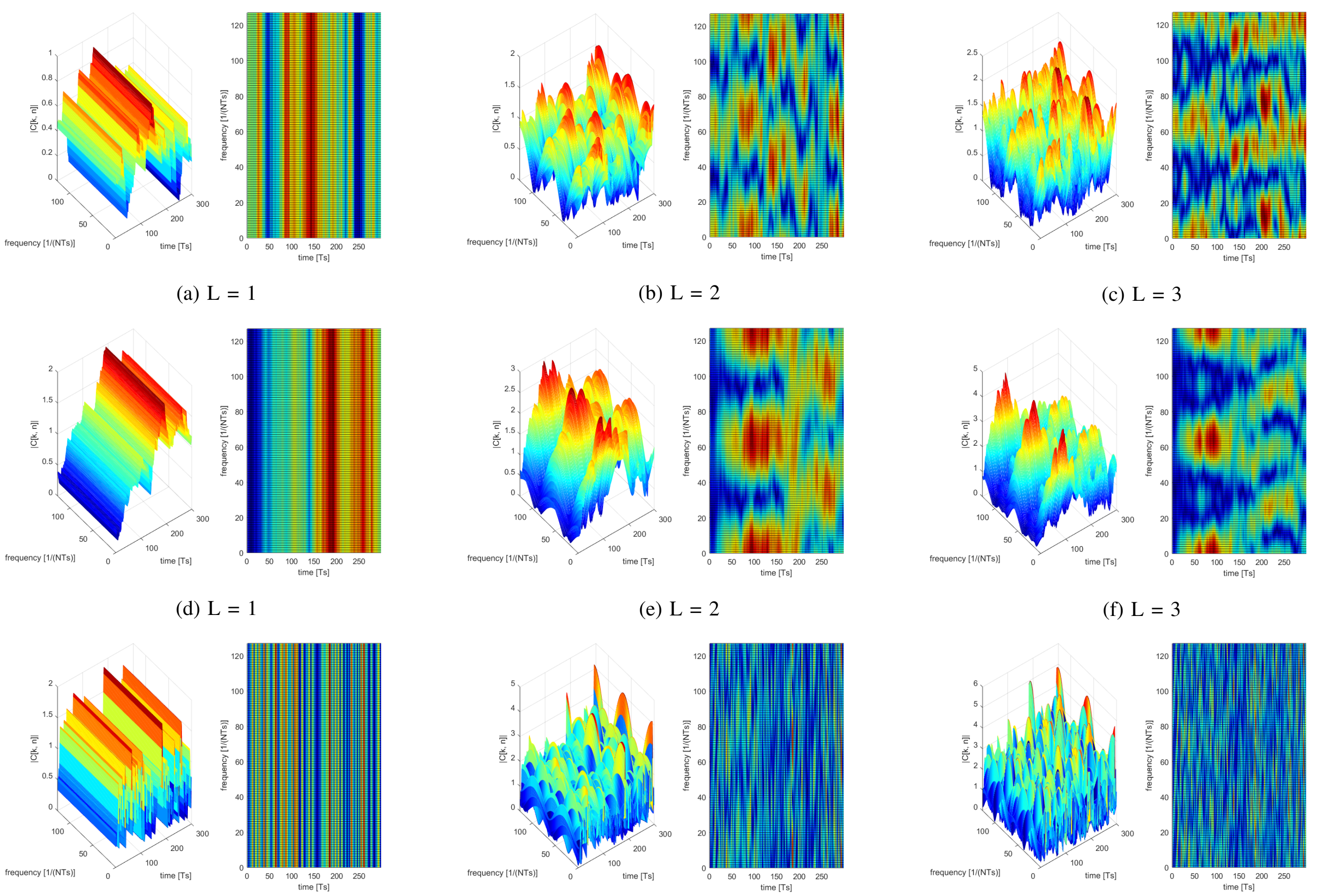
IV. Conclusion

By simulating the Rayleigh fading channel using the two different methods, it can be concluded that they both end up with quite similar results, except the Auto Correlation Function, where the spectrum method gets the best result. It is also important to take into account the complexity of each method. The filter method is significantly more computational demanding compared to the spectrum method.

In regards to the time and frequency varying channels, it is concluded that in order to generate frequency selective channels, L must be greater than 1. In addition, the Doppler frequency plays a key role, as it will ultimately effect the coherence time of the channel.

V. Contribution

It was a great team effort, in which all participants worked together on the same problems sequentially.



(a) L = 1 (b) L = 2 (c) L = 3

(d) L = 1 (e) L = 2 (f) L = 3

(g) L = 1) (h) L = 2 (i) L = 3

Fig. 10. Variation of taps and $T_s \cdot f_D$