Chebyshev transformer

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1 Introduction

The spectrum of the reflection coefficient Γ for a quarter length transformer, in the region where $\theta - \frac{\pi}{2} \ll 1$ is given by

$$|\Gamma| = \frac{|Z_L - Z_0|}{2\sqrt{Z_0 Z_L}} |\cos\theta|, \tag{1}$$

where $\theta = \beta l = \frac{2\pi l}{c} f$. In a **multisection transformer** N sections of quarter transformer are used in series. In this configuration the spectrum of the total reflection coefficient is

$$\Gamma = \Gamma_0 + \Gamma_1 e^{-2i\theta} + \Gamma_2 e^{-4i\theta} + ... + \Gamma_N e^{-2iN\theta}, \tag{2}$$

A **Chebyshev trasformer** is obtained by equating Γ to a **Chebyshev polynomial** $T_n(x)$ in terms of

$$x = \frac{\cos\theta}{\cos\theta_m}, \tag{3}$$

where θ_m is the maximum electrical length shift from $\frac{\pi}{2}$ for the quarter transformer, and it is given by

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} \tag{4}$$

In order to chose the number of section and the value of each reflection coefficient we can calculate

$$\theta_m = \left(2 - \frac{\Delta f}{f_0}\right)\frac{\pi}{4} = 0.225\pi \tag{5}$$

and therefore the number of sections from

$$N = \frac{\cosh^{-1}\left(\frac{1}{\Gamma_m}\frac{1-\frac{Z_0}{Z_L}}{1+\frac{Z_0}{Z_L}}\right)}{\cosh^{-1}(\sec\theta_m)} = \frac{\cosh^{-1}\left(\frac{1}{0.05}\frac{1-3}{1+3}\right)}{\cosh^{-1}(1.315)} = 3.866 \tag{6}$$

Therefore we need 4 sections. In this case the bandwidth is given by

The reflection coefficients for the each quarter wavelength transformer are given by

Considering the wave impedance for TE modes in a rectangular waveguide

$$Z_{TE} = \frac{k\eta}{\beta}; \tag{7}$$

supposing that the system impedance is given by a waveguide filled with air ($\epsilon_r = 1$), that the propagating mode is TE_{10} and considering the dimensions of the waveguide $a = 80$mm and $b = 10$mm, we can calculate for the central frequency $f_0 = 6$GHz

$$\begin{cases} k_c = \dfrac{\pi}{a} = 39.27\text{m}^{-1} \\ k = \dfrac{\omega}{\sqrt{\mu\epsilon}} = \dfrac{2\pi f_0}{c} = 125.7\text{m}^{-1} \\ \beta = \sqrt{k^2 - k_c^2} = 119.4\text{m}^{-1} \\ Z_0 = \dfrac{k\eta}{\beta} = 395\Omega \end{cases} \tag{8}$$

This imply that the impedance of the load is

$$Z_L = \frac{Z_0}{3} = 132\Omega. \tag{9}$$

Using the impedance ratios found before we can calculate the value of the impedance for each element of the transformer:

$$\begin{cases} Z_1 = 2.589 Z_L = 340\Omega \\ Z_2 = 2.017 Z_L = 265\Omega \\ Z_3 = 1.487 Z_L = 196\Omega \\ Z_4 = 1.158 Z_L = 153\Omega \end{cases} \tag{10}$$

Considering that

$$Z_{TE} = \frac{k\eta}{\beta} = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{\omega^2\mu\epsilon - k_c^2}}\sqrt{\frac{\mu}{\epsilon}} \tag{11}$$

and inverting this formula

$$\epsilon_r = \frac{c^2}{\omega^2}k_c^2 + \frac{\eta_0^2}{Z_{TE}^2} = 0.0976 + \left(\frac{377\Omega}{Z_{TE}}\right)^2 \tag{12}$$

Therefore the dielectric constant for the various sections are

$$\begin{cases} \epsilon_{\text{system}} = 1 \\ \epsilon_1 = 1.327 \\ \epsilon_2 = 2.123 \\ \epsilon_3 = 3.803 \\ \epsilon_4 = 6.201 \\ \epsilon_L = 8.313 \end{cases} \tag{13}$$

In order to calculate the length in each section we need to calculate the value of one forth of wavelength

$$\frac{\lambda}{4} = \frac{\pi}{2\sqrt{\frac{\omega^2}{c^2}\epsilon_r - k_c^2}}. \tag{14}$$

In our case the lengths of the sessions are

$$\begin{cases} L_1 = 11.27\text{mm} \\ L_2 = 8.78\text{mm} \\ L_3 = 6.49\text{mm} \\ L_4 = 5.06\text{mm} \end{cases} \tag{15}$$

2 Simulation Results

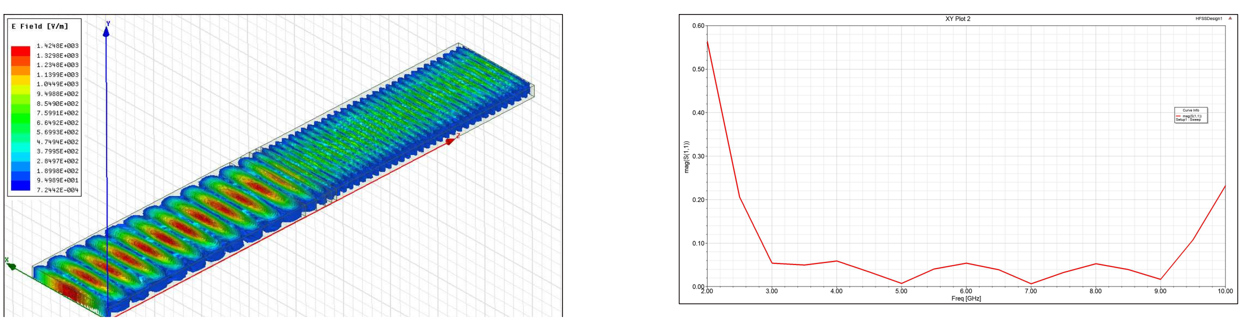


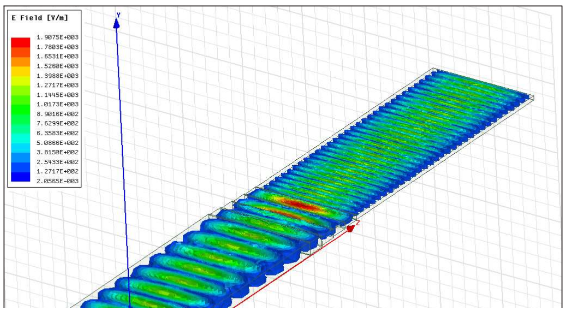
Figure 1: Mode TE_{10}

3 Height transformer

The values of the heights that we calculated are

$$\begin{cases} b_0 = 10\text{mm} \\ b_1 = 8.63\text{mm} \\ b_2 = 6.73\text{mm} \\ b_3 = 4.96\text{mm} \\ b_4 = 3.86\text{mm} \\ b_{load} = 3.33\text{mm} \end{cases} \tag{16}$$

4 Simulation results



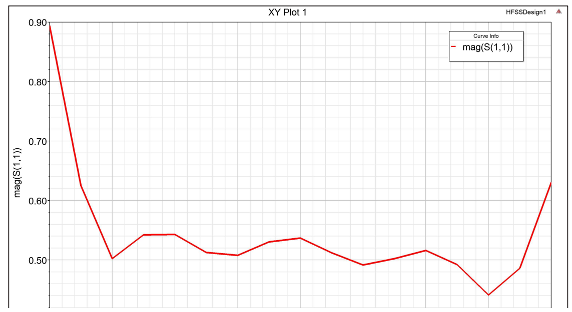


Figure 2: Mode TE_{10}