Stochastic Decoding: Summary and Details

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1 Article Summary

As we have studied over the course of this period, low-density parity check codes (LDPC) codes are often used in modern communication standards and are, for an appropriately chosen block length, capacity approaching. In addition decoding an LDPC code of a longer size using the global optimal maximum aposteriori (MAP) or the bit-wise MAP requires exponential time and is therefore in efficient in practice.

Any LDPC code can be represented by a factor graph (bipartite), which is composed of check nodes (CN), variable nodes (VN) and interconnecting edges. The sum-product algorithm (SPA) using iterations of passing probability distribution messages between the CNs and the VNs will converge to the bit-wise MAP in cycle free graphs and be a very accurate approximation on graphs with cycles (all practical LDPC codes).

Constructing the SP algorithm into a hardware chip requires many interconnected wires which stem from the nearly random LDPC code matrix construction. These wires make the integrated circuit (IC) for the SPA implementation complicated to design, in addition it also leads to effects known as parasitic capacitance, high power consumption and large dye area. In literature, there are several solutions proposed but this particular paper presents a solution called a stochastic decoder and it builds on previous works. As well it presents mathematical bounds for a particular stochastic decoder design (which is favourable towards the mathematical derivations) yet it shares all the benefits of the traditional stochastic decoders.

Based on the concepts of stochastic computing developed in the 1960s; the algorithm approximates the SPA algorithm further by representing the SP messages (binary random variables (RVs) with distribution $\alpha_i = \Pr(x_i = 1|y_i), i = 1, 2, ..., n$) by encoding them in Bernoulli sequences with corresponding marginal probabilities. These sequences are then used in derivations of new check and variable node equations, which in turn result in significantly simpler hardware implementations. Due to the random construction of the stochastic decoder there are a few questions that are natural to ask. For example the convergence

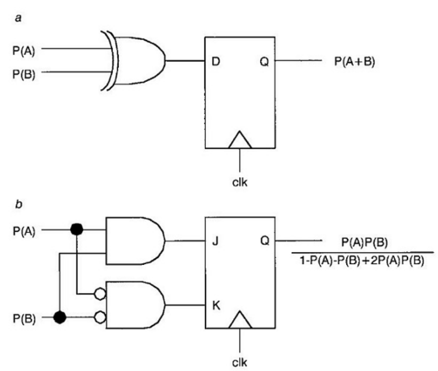

Figure 1: Stochastic a: Check Node b: Variable Node

to the approximated SPA algorithm, average behaviour, is the average performance typical or is there a measure of cencentration around the average. These are addressed in the paper mathematically on a modified stochastic algorithm called Markov based stochastic decoding (MbSD).

2 Article Contributions

As previously stated common difficulties of the SPA are the large number of interleaved wires making the production of decoder ICs a challenge. The parasitic capacitance - capacitance due to close proximity of wires; especially in high frequency circuits leads to unwanted non-ideal behaviour.

After derivation of the SPA algorithm, the MbSD algorithm is presented by using probability distributions in the form of Bernoulli random variables instead of the traditional LLRs. The complete hardware implementation requires XOR gates for the CN, JK flip-flops and AND gates, and a random number generator (RNG).

Figure 1 shows the check and variable nodes.

For the mathematical bounds on the algorithm, an assumption is first made on the LDPC code of interest is well behaved. The messages on the directed edges reach the true marginals after $\lim t \to +\infty$ iterations. This leads to a condition of finite stopping time:

$$T = inf\{t | \max_{1 \leq i \leq n} |\mu_i^t - \mu_i^*| \leq \delta\} \tag{1}$$

For δ being a arbitrarily small but fixed parameter. μ^* are the true marginal. μ^t are the marginals generated by the MbSD.And n is the length of the received

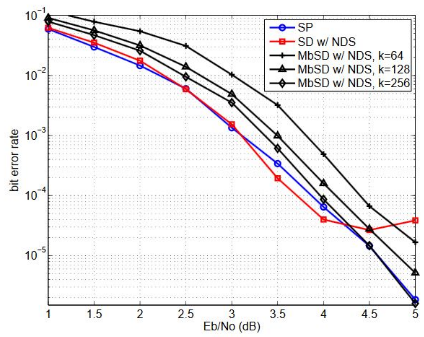

Figure 2: BER for different approaches. Bernoulli sequence with lengths 64,128,256 (times 2 for the algorithm). SP is the SPA algorithm. SD is another stochastic decoder, for comparison. NDS is an approach called noise dependent scaling, from which stochastic decoders benefit greatly.

codeword.

The proven results from the paper are for general factor graphs, concerning marginals generated by the MbSD algorithm on an LDPC that satisfies the aforementioned assumption. Then for sufficiently large $k = k(\delta, T, \vartheta)$ where ϑ is the graph.

(a) The expected stochastic marginals become arbitrarily close to the SP marginals:

$$|\mathbb{E}[n_i^t] \hat{-} \mu_i^t| \leq \delta$$

for all i = 1,2,3,...,n and t = 0,1,2,3,...,T.

(b) As k increases the variance of the generated marginals will tend to their means:

$$\max_{1 \leq i \leq n} \max_{0 \leq t \leq T} var(\hat{n_i^t}) = \mathcal{O}(\frac{1}{k})$$

From these results it can be shown that the stochastic decoder converges to that of SPA, as k increases (k is the design parameter of the length of the Bernoulli sequences). There is a finite stopping time T, under the assumption. It can be proved that after T iterations the estimated marginals approach the true marginals μ^*.

$$\max_{0 \leq i \leq n} |\mathbb{E}[\hat{n_i^T}] - \mu_i^*| \leq \max_{0 \leq i \leq n} |\mathbb{E}[\hat{n_i^T}] - \mu_i^T| + \max_{0 \leq i \leq n} |\mu_i^T - \mu_i^*| \leq 2\delta$$

Where the first δ comes from (a) and the second comes from equation (1).

Figure 2 shows the BER curves for several algorithms.