In [1]:
```python
import pandas as pd
import numpy as np
from tqdm.notebook import tqdm
import matplotlib.pyplot as plt
from PIL import Image
from urllib.request import urlopen
from sklearn.decomposition import PCA
```

In [2]:
```python
data = np.genfromtxt("data.txt",delimiter=",")
x_array = data[:,0]
y_array = data[:,1]
```

In [3]:
```python
#initiating values
β0 = 1;β1 = 1;n=len(data);
```

In [4]:
```python
#Question 1.3 : for β0 = 1 and β1 = 1
y_pred = β0 + β1*x_array
errors = y_array-y_pred
squared_errors = errors**2
sum_of_errors = sum(squared_errors)

loss_value = sum_of_errors/n

print("Loss value for β0 = 1 and β1 = 1 is : ",loss_value)
```

```
Loss value for β0 = 1 and β1 = 1 is :  30068.440558989987
```

In [5]:
```python
#Question 1.4:

iterations = 10
learning_rate = 0.001

for iteration in tqdm(range(iterations)):
    y_pred = β0 + β1*x_array
    δβ1 = -2*sum(x_array*(y_array-y_pred))/n
    δβ0 = -2*sum(y_array-y_pred)/n
    β0 = β0-learning_rate*δβ0 #changing β0 using learning rate and δβ0
    β1 = β1-learning_rate*δβ1 #changing β1 using learning rate and δβ1

print("After {} iterations, β0 = {} and β1 = {}".format(iterations,β0,β1))
```

```
After 10 iterations, β0 = 0.6855626678299438 and β1 = 14.233847598602836
```

Problem 1 answers

1) Loss Function is : Averaged Sum of Squared Errors = Mean Squared Error

$$= \frac{1}{n}\sum_{i=0}^{n}(y_{actual} - y_{predicted})^2 = \frac{1}{n}\sum_{i=0}^{n}(y_i - (\beta_0 + \beta_1 x_i))^2$$

2) For Gradient function, we partially differentiate the loss function wrt both β_0 and β_1.

$$\delta\beta_1 = \frac{1}{n}\sum_{i=0}^{n}2(y_i - (\beta_0 + \beta_1 x_i))(-x_i)$$

$$\delta\beta_1 = \frac{-2}{n}\sum_{i=0}^{n}x_i(y_i - \hat{y}_i)$$

Similarly,

$$\delta\beta_0 = \frac{-2}{n}\sum_{i=0}^{n}(y_i - \hat{y}_i)$$

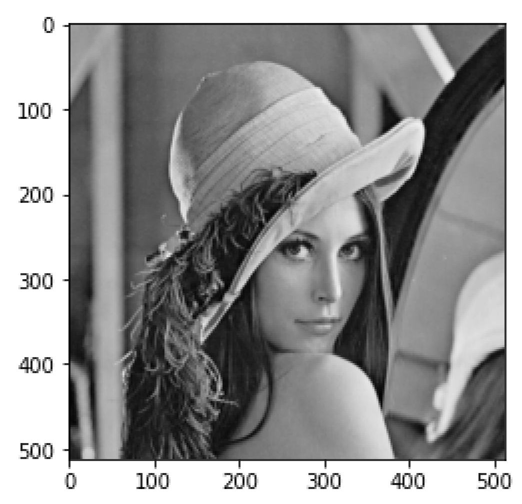
3) Loss value for β_0 = 1 and β_1 = 1 : 30068.440558989987

4) Starting with $\beta_0^{(1)}$=1 and $\beta_1^{(1)}$=1, after 10 iterations, we get $\beta_0^{(11)}$=0.6855626678299438 and $\beta_1^{(11)}$=14.233847598602836

Problem 2

In [8]:
```python
image_data = Image.open(urlopen("https://raw.githubusercontent.com/
                                 changyaochen/MECE4520/master/lectures/lecture_1/leena
X = np.array(image_data)

plt.figure()
plt.imshow(X, cmap="gray")
# plt.imshow(image_data, cmap="gray")   # this also works
plt.show()
```



In [9]:
```python
#Question 2.1:
print("The element at index (128,128) of matrix X is :",str(X[128][128]))
```

```
The element at index (128,128) of matrix X is : 173
```

In [10]:
```python
#Question 2.2:
X_scaled = []
for i in tqdm(range(X.shape[1])):
    col = X[:,i] #one column at a time
    new_col = (col-col.mean())/col.std() #standardizing
    X_scaled.append(new_col) #appending to a new list

X_scaled = np.asarray(X_scaled).transpose()
#consverting list ndarray and transposing to bring it back to original position

print("The element at index (128,128) of standardized matrix X is :",
      str(X_scaled[128][128]))
```

```
The element at index (128,128) of standardized matrix X is : 1.2794041765058435
```

In [11]:
```python
#Question 2.3:

pca = PCA()
pca_X = pca.fit_transform(X_scaled)
print("The first element of scaled X matrix after PCA is : ", pca.components_[0][0])
```

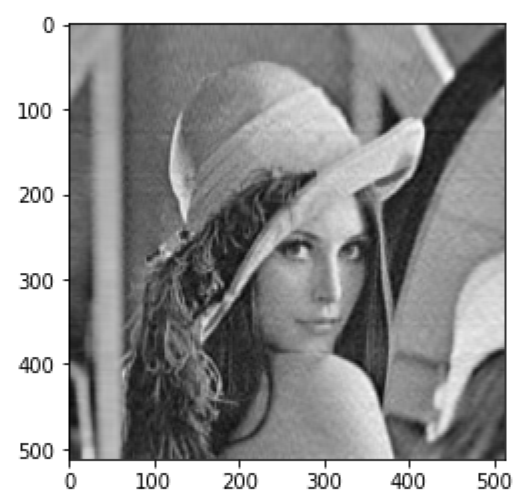
```
The first element of scaled X matrix after PCA is :  0.05842033740661615
```

In [12]:
```python
#Question 2.4:

pca_50 = PCA(n_components=50)
pca_50_X = pca_50.fit_transform(X_scaled)
```

In [13]:
```python
covariance_matrix = X - np.mean(X , axis = 1)
eigen_values, eigen_vectors = np.linalg.eigh(np.cov(covariance_matrix))
p = np.size(eigen_vectors, axis =1)
idx = np.argsort(eigen_values)
idx = idx[::-1]
eigen_vectors = eigen_vectors[:,idx]
eigen_values = eigen_values[idx]
principal_components = 50
if principal_components <p or principal_components >0:
    eigen_vectors = eigen_vectors[:, range(principal_components)]
    score = np.dot(eigen_vectors.T, covariance_matrix)
    reconstruction = np.dot(eigen_vectors, score) + np.mean(X, axis = 1).T
    reconstructed_matrix = np.uint8(np.absolute(reconstruction))
```

In [14]:
```python
plt.figure()
plt.imshow(reconstructed_matrix, cmap="gray")
plt.show()
```



In [15]:
```python
k_sum = 0
n_sum = 0
for i in range(len(reconstructed_matrix)):
    k_sum += reconstructed_matrix[i][i]
    n_sum += X[i][i]

print("The Reconstruction Error if 50 Principal components are used : ",
      str(1-k_sum/n_sum))
```

```
The Reconstruction Error if 50 Principal components are used :  0.008731721488750455
```

Problem 2 answers

1) The element at index (128,128) of matrix X is : 173

2) The element at index (128,128) of standardized matrix X is : 1.2794041765058435

3) The first element of scaled X matrix after PCA is : 0.05842033740661615

4) The Reconstruction Error if 50 Principal components are used : 0.008731721488750455

Problem 3 answers

1) Total pool size is n. For bootstrapping, items are drawn with replacement. This suggests that there are n-1 items in the population that are not the j^{th} observation. So, the probability that the first item is not j is:

$$\left(1 - \frac{1}{n}\right)$$

2) Since we have n chances to draw something other than j, the probability that none of them are the j^{th} sample is :

$$\left(1 - \frac{1}{n}\right)^n$$

3) When n=10000:

$$\prod_{i=1}^{10000} P_{Si\neq j} = \left(1 - \frac{1}{n}\right)^n = \left(1 - \frac{1}{10000}\right)^{10000} = 0.3678$$