Event-triggered stabilization of linear systems under channel blackouts

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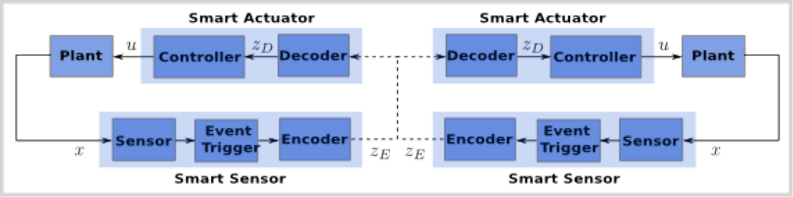
Networked control systems



Shared communication resource

- Time-varying communication rates
- Channel may not be available during some intervals (blackouts)
- Time-triggered strategies would be very conservative
- Event-triggered controllers typically assume *on-demand* availability of channel[1]

[1] An important exception: Anta, Tabuada (2009)

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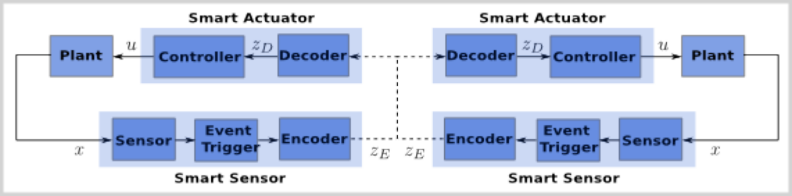

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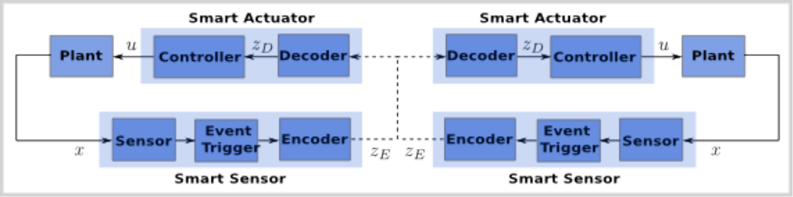

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Key to online state based transmission policy: data capacity

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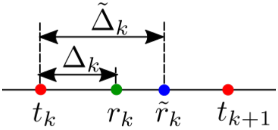

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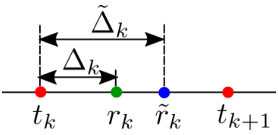

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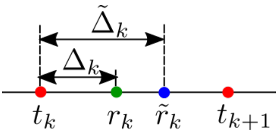

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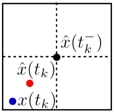

Dynamic controller jump: $\hat{x}(\tilde{r}_k) \triangleq q_k(x(t_k), \hat{x}(t_k^-))$

Encoding error: $x_e \triangleq x - \hat{x}$

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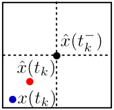
- Both encoder and decoder compute recursively:

$$d_e(t) \triangleq \|e^{A(t-t_k)}\|_\infty \delta_k, \; t \in [\tilde{r}_k, \tilde{r}_{k+1}), \; k \in \mathbb{Z}_{\geq 0}$$

$$\delta_{k+1} = \frac{1}{2^{p_{k+1}}} d_e(t_{k+1}).$$



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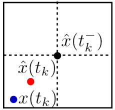

- Then, $\|x_e(t)\|_\infty \le d_e(t)$, for all $t \ge t_0$

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Suppose $\bar{A} = A + BK$ is Hurwitz $\iff$ $P\bar{A} + \bar{A}^T P = -Q$

Lyapunov function: $x \mapsto V(x) = x^T P x$

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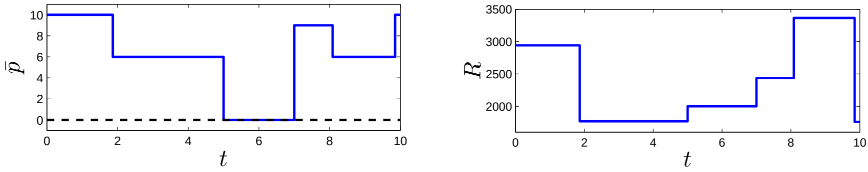
Design objective:

- Design event-triggered communication policy that is applicable to channels with time-varying rates and blackouts
- Recursively determine $\{t_k\}$, $\{p_k\}$ and $\{\tilde{r}_k\}$
- Ensure a uniform positive lower bound for $\{t_k - t_{k-1}\}_{k \in \mathbb{Z}_{>0}}$

Time-slotted channel model



$$R(t) = R_j, \quad \forall t \in (\theta_j, \theta_{j+1}], \quad \textbf{min comm. rate: } \frac{p_k}{\Delta(t_k, p_k)} \geq R(t_k)$$

$$\bar{p}(t) = \bar{\pi}_j, \quad \forall t \in (\theta_j, \theta_{j+1}], \quad \textbf{max packet size: } p_k \leq \bar{p}(t_k)$$

- j^{th} time-slot is of length $T_j = \theta_{j+1} - \theta_j$
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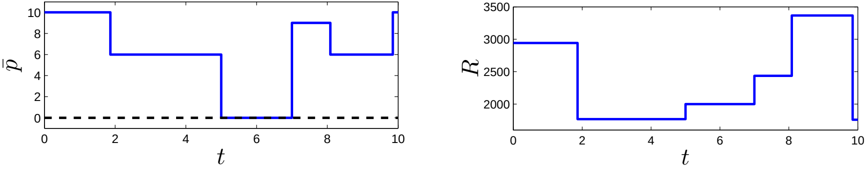

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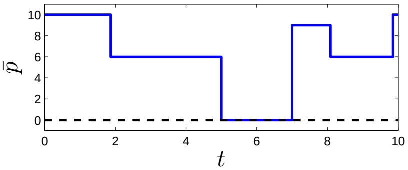

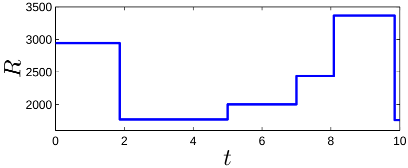

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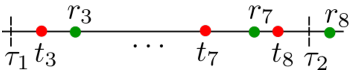
Need to quantify *data capacity*

Data capacity

max # of bits that can be *communicated* during the time interval $[\tau_1, \tau_2]$, overall all possible $\{t_k\}$ and $\{p_k\}$

$$\mathcal{D}(\tau_1,\tau_2) \triangleq \max_{\substack{\{t_k\},\{p_k\} \\ \text{s.t. } \ldots}} n \sum_{k=\underline{k}_{\tau_1}}^{\overline{k}_{\tau_2}} p_k$$



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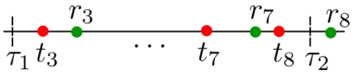

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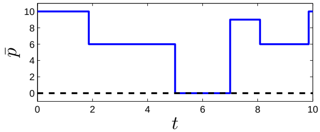
Equivalent to optimal allocation of *discrete* # bits to be transmitted in each time slot

Data capacity as allocation problem

Max # bits that may be transmitted in slot j

$$n\phi_j \leq \begin{cases} nR_jT_j + n\bar{\pi}_j, & \text{if } \bar{\pi}_j > 0 \\ 0, & \text{if } \bar{\pi}_j = 0 \end{cases}$$



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Available time in slot j is affected by prior transmissions

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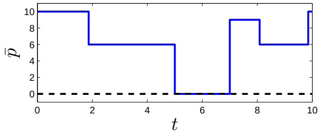

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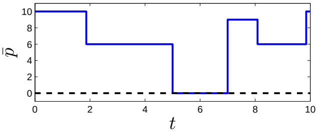

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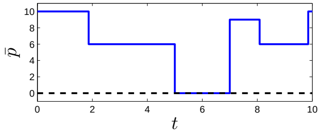

$$\mathcal{D}(\theta_{j_0}, \theta_{j_f}) = \max_{\substack{\phi_j \in \mathbb{Z}_{\geq 0} \\ \text{s.t. } \ldots}} n \sum_{j=j_0}^{j_f-1} \phi_j.$$

A suboptimal solution for "slowly varying channels"

Proposition

Assume $\frac{\bar{\pi}_j}{R_j} < T_{j+1}, \forall j \in \mathcal{N}_{j_0}^{j_f}$ (*any bits transmitted in slot j are received before the end of slot $j+1$*).

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Let

$$\phi^N \triangleq \lfloor \phi^r \rfloor \triangleq (\lfloor \phi^r_{j_0} \rfloor, \dots, \lfloor \phi^r_{j_f-1} \rfloor), \quad \mathcal{D}_s(\theta_{j_0}, \theta_{j_f}) \triangleq n \sum_{j=j_0}^{j_f-1} \phi^N_j.$$

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Then

- ϕ^N *is a sub-optimal solution*
- $\mathcal{D}(\theta_{j_0}, \theta_{j_f}) - \mathcal{D}_s(\theta_{j_0}, \theta_{j_f}) \leq n(j_f - 1 - j_0)$.

Real time computation of data capacity

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Significance: Sufficient to solve the data capacity problem for intervals $[\theta_{j_0}, \theta_{j_f}]$ of interest.

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Lemma

If $h_{\text{pf}}(t) \leq 1$ *and* $h_{\text{ch}}(t) \leq 1$ *then* $h_{\text{pf}}(s) \leq 1$, $\forall s \in [t, t + T']$.

Elements of the event-trigger

Recall performance objective: ensure $h_{\mathrm{pf}}(t) \triangleq \frac{V(x(t))}{V_d(t)} \leq 1$, for all $t \geq t_0$

Channel trigger function: $h_{\mathrm{ch}}(t) \triangleq \frac{\epsilon(t)}{\rho_T(h_{\mathrm{pf}}(t))}, \quad \epsilon(t) \triangleq \frac{d_e(t)}{c\sqrt{V_d(t)}}$

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Idea for triggering:

- Make sure $h_{\mathrm{pf}}(t) \leq 1, \forall t \in [t_k, \tilde{r}_k]$

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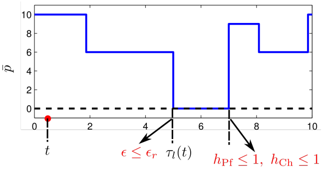
$$\tilde{\mathcal{L}}_1(t) \triangleq \bar{h}_{\text{pf}}\left(\mathcal{T}(t), h_{\text{pf}}(t), \epsilon(t)\right)$$

$$\tilde{\mathcal{L}}_2(t) \triangleq \bar{h}_{\text{ch}}\left(\mathcal{T}(t), h_{\text{pf}}(t), \epsilon(t), \psi^{\tau_l}(t)\right)$$

$$\mathcal{T}(t) \triangleq \begin{cases} T_M(\psi^{\tau_l}(t)), & \text{if } \psi^{\tau_l}(t) \geq 1 \\ \frac{2}{R(t)}, & \text{if } \psi^{\tau_l}(t) = 0. \end{cases}$$

Role of data capacity in control



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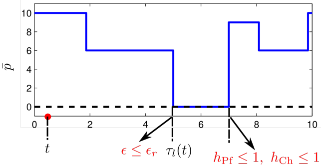

$$\tilde{\mathcal{L}}_3(t) \triangleq n \log_2 \left(\frac{e^{\bar{\mu}(\tau_l(t)-t)}\epsilon(t)}{\epsilon_r(t)} \right) - \sigma_1 \hat{\mathcal{D}}_s(t, \tau_l(t))$$

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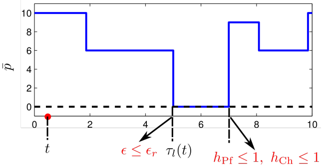

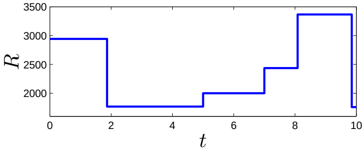

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Transmission policy should be in tune with the optimal allocation

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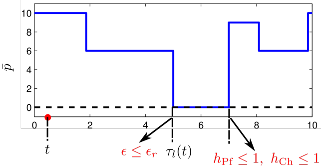

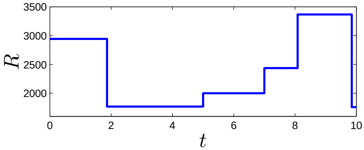

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$\Phi^{\tau_l}(t) \triangleq \left[\lfloor \mathcal{P}_j - R_j(t-\theta_j) \rfloor \right]_+ ,\ t \in (\theta_j, \theta_{j+1}]$ (optim. alloc. in $(t, \theta_{j+1}]$)

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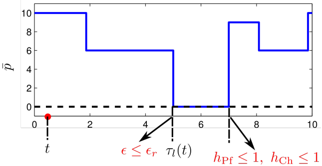

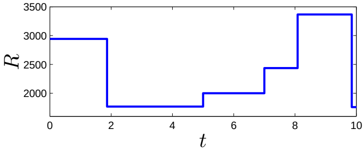

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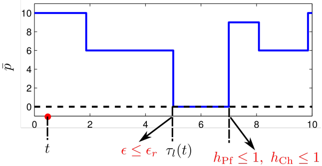

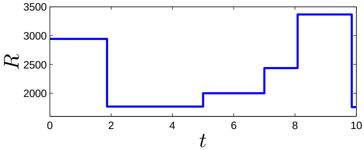

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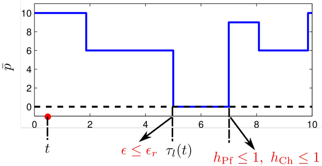
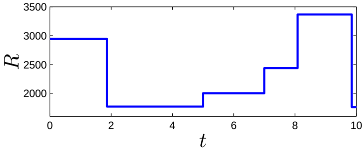
If data capacity was "sufficient" at t_k and p_k respects artificial bound

Role of data capacity in control





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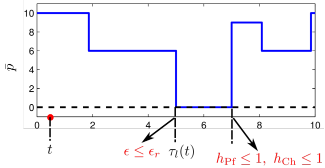

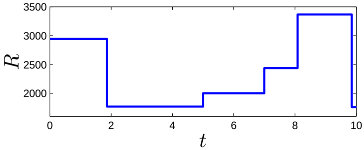

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But $\psi^{\tau_l}(t)$ can be 0 when $\bar{p}(t) > 0$ (artificial blackouts)

Control policy in the presence of blackouts

$$t_{k+1} = \min \Big\{ t \geq \tilde{r}_k : \psi^{\tau_l}(t) \geq 1 \wedge \Big(\max\{\tilde{\mathcal{L}}_1(t), \tilde{\mathcal{L}}_1(t^+), \tilde{\mathcal{L}}_2(t), \tilde{\mathcal{L}}_2(t^+)\} \geq 1 \vee \max\{\tilde{\mathcal{L}}_3(t), \tilde{\mathcal{L}}_3(t^+)\} \geq 0 \Big) \Big\},$$

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Control policy in the presence of blackouts

Theorem

If

- $R(t) \geq \frac{(p+2)}{T_M(p)}, \ \forall p \in \{1, \ldots, p^{Max}\}, \ \forall t$
- $\tilde{\mathcal{L}}_1(t_0) \leq 1$, $\tilde{\mathcal{L}}_2(t_0) \leq 1$ *and* $\tilde{\mathcal{L}}_3(t_0) \leq 0$ *(initial feasibility)*
- *Conditions on blackout lengths*

Control policy in the presence of blackouts

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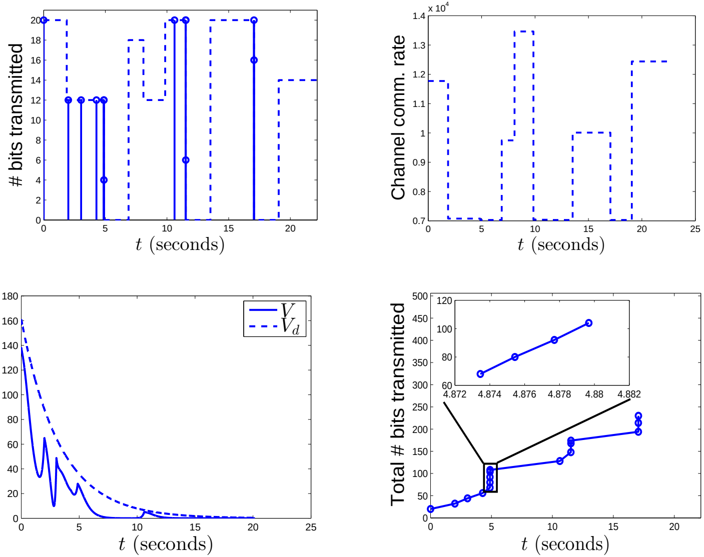
Then

- $\{t_k\}$, $\{p_k\}$, $\{\tilde{r}_k\}$ *well defined*
- *inter-transmission times have uniform positive lower bound*
- $V(x(t)) \leq V_d(t_0)e^{-\beta(t-t_0)}$ *for* $t \geq t_0$ *(origin is exponentially stable)*

Simulation results: 2D linear system



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Future work:

- Address conservatism in the design
- Stochastic model of channel evolution
- Impact of the available information pattern at the encoder