

Decentralized Event-Triggering for Control of LTI Systems

Pavankumar Tallapragada and Nikhil Chopra

Abstract—This paper considers Linear Time Invariant (LTI) control systems with full state feedback, a central controller and distributed sensors not co-located with the central controller. We present a methodology for designing decentralized asynchronous event-triggers, which utilize only locally available information, for determining the time instants of transmission from the sensors to the central controller. The proposed design guarantees a positive lower bound for the inter-transmission times of each sensor, while ensuring global asymptotic stability of the origin of the system. Additionally, the proposed decentralized asynchronous event-triggers are shown to preserve scale invariance of inter-transmission times. The proposed method is illustrated through simulations of a linear example.

I. INTRODUCTION

State based aperiodic event-triggering is receiving increased attention (a representative list of the recent literature includes [1]–[7]) as an alternative to the traditional time-triggering (example: periodic triggering) in sampled data control systems. In event based control systems, a state or data dependent event-triggering condition implicitly determines time instances at which control is updated or when a sensor transmits data to a controller. Such updates or transmissions are in general aperiodic and depend on the system state. Such a paradigm is particularly appealing in control systems with limited computational and/or communication resources.

Much of the literature on event-triggered control utilizes the full state information in the triggering conditions. However, in two very important classes of problems full state information is not available to the event-triggers. These are systems with decentralized sensing and/or dynamic output feedback control. In the latter case, full state information is not available even when the sensors and the controller are centralized (*co-located*). In systems with decentralized sensing, each individual sensor has to base its decision to transmit data to a central controller only on locally available information. These two classes of problems are receiving attention in the community only recently - [8]–[12] (decentralized sensing) and [13]–[18] (output feedback control). This paper is an important addition to the limited literature on decentralized event-triggering in control systems with distributed sensors.

The basic **contribution** of this paper is a methodology for designing implicitly verified decentralized event-triggers for control of Linear Time Invariant (LTI) systems. The system

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architecture we consider is one with full state feedback but with the sensors distributed and not co-located with a central controller. The proposed design methodology provides event-triggers that determine when each sensor transmits data to the central controller. The event-triggers are designed to utilize only locally available information, making the transmissions from the sensors asynchronous. The proposed design guarantees global asymptotic stability of the origin of the system and a positive lower bound for the inter-transmission times of each sensor individually. In addition, scale invariance of inter-transmission times is preserved.

In the literature, distributed event-triggered control was studied in [11], [12] with the assumption that the subsystems are weakly coupled, which allowed the design of event-triggers depending on only local information. Our proposed design method requires much less restrictive assumptions. In [8]–[10], each sensor checks a local condition (based on threshold crossing) that triggers asynchronous transmission of data by sensors to a central controller. However, this design guarantees only semi-global practical stability (even for linear systems) if the sensors do not listen to the central controller. Compared to this work, our proposed design holds for Linear Time Invariant (LTI) systems and guarantees global asymptotic stability without the sensors having to listen to the central controller. A similarity between the current paper and [8]–[10] is that both are partially motivated by the need to eliminate or drastically reduce the listening effort of the sensors to save energy.

In the dynamic output feedback control literature, [13]–[15] consider asynchronous and decentralized event-triggering for Linear Time Invariant (LTI) systems. Again, the method in [13] can guarantee only semi-global practical stability. In [14], [15], we have proposed a method that guarantees global asymptotic stability and positive minimum inter-transmission times. In principle, the method in [14], [15] can be utilized to design decentralized asynchronous event-triggers for the special case with full state feedback. Thus, apart from clearly spelling out the design for the case with full state feedback, the additional contribution of this paper with respect to [14], [15] is an improved presentation of the design method that is much more intuitive and the result on the preservation of the scale invariance of inter-transmission times.

The rest of the paper is organized as follows. Section II describes and formally sets up the problem under consideration. In Section III, the design of asynchronous decentralized event-triggers for Linear Time Invariant (LTI) systems is presented. The proposed design methodology is illustrated through simulations in Section IV and finally Section V

provides some concluding remarks.

II. PROBLEM SETUP

Consider a Linear Time Invariant (LTI) system and the feedback control law

$$\dot{x} = Ax + Bu, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m \quad (1)$$

$$u = K(x + x_e) \quad (2)$$

where A , B and K are matrices of appropriate dimensions while x_e is the error in the measurement of x . In general, the measurement error can be due to many factors such as sensor noise and quantization. However, we consider measurement error that is purely a result of “sampling” of the sensor data x . Before going into the precise definition of this measurement error, we first describe the broader problem. Let us assume that for each $i \in \{1, 2, \dots, n\}$, $x_i \in \mathbb{R}$ is sensed by the i^{th} sensor. Thus, it is useful to explicitly express (1)-(2) as a collection of n scalar differential equations

$$\dot{x}_i = r_i(A)x + r_i(BK)(x + x_e) \quad (3)$$

where $x = [x_1, x_2, \dots, x_n]^T$ and the notation $r_i(H)$ denotes the i^{th} row of the matrix H .

In this paper we are concerned with a distributed sensing scenario where each component, x_i , of the state vector x is sensed at a different location. Although the i^{th} sensor senses x_i continuously in time, it transmits this data to a central controller only intermittently. In other words, the controller is a sampled-data controller that uses intermittently transmitted/sampled sensor data. In particular, we are interested in designing an asynchronous decentralized sensing mechanism based on local event-triggering that renders the origin of the closed loop system asymptotically stable.

To precisely describe the sampled-data nature of the problem, we now introduce the following notation. Let $\{t_j^{x_i}\}$ be the increasing sequence of time instants at which x_i is sampled and transmitted to the controller. The resulting piecewise constant sampled signal is denoted by $x_{i,s}$, that is,

$$x_{i,s} \triangleq x(t_j^{x_i}), \quad \forall t \in [t_j^{x_i}, t_{j+1}^{x_i}), \quad \forall j \in \{0, 1, 2, \dots\} \quad (4)$$

As mentioned previously, the sampled data, $x_{i,s}$, may also be viewed as resulting from an error in the the measurement of the continuous-time signal, x_i . This measurement error is denoted by

$$x_{i,e} \triangleq x_{i,s} - x_i = x_i(t_j^{x_i}) - x_i, \quad \forall t \in [t_j^{x_i}, t_{j+1}^{x_i})$$

Finally, we define the sampled-data vector and the measurement error vector as

$$x_s \triangleq [x_{1,s}, x_{2,s}, \dots, x_{n,s}]^T, \quad x_e \triangleq [x_{1,e}, x_{2,e}, \dots, x_{n,e}]^T$$

Note that, in general, the components of the vector x_s are asynchronously sampled components of the plant state x . The components of x_e are also defined accordingly.

In time-triggered implementations, the time instants $t_j^{x_i}$ are pre-determined and are commonly a multiple of a fixed sampling period. However, in event-triggered implementations

the time instants $t_j^{x_i}$ are determined implicitly by a state/data based triggering condition at run-time. Consequently, an event-triggering condition may result in the inter-sample times $t_{j+1}^{x_i} - t_j^{x_i}$ to be arbitrarily close to zero or it may even result in the limit of the sequence $\{t_j^{x_i}\}$ to be a finite number (*Zeno* behavior). Thus for practical utility, an event-trigger has to ensure that these scenarios do not occur.

Thus, the problem under consideration may be stated more precisely as follows. For the n sensors, design event-triggers that depend only on local information and implicitly define the non-identical sequences $\{t_j^{x_i}\}$ such that (i) the origin of the closed loop system is rendered globally asymptotically stable and (ii) inter-sample (inter-transmission) times $t_{j+1}^{x_i} - t_j^{x_i}$ are lower bounded by a positive constant.

Finally, a point regarding the notation in the paper is that the notation $|.|$ denotes the Euclidean norm of a vector. In the next section, event-triggering for the decentralized sensing architecture is developed.

III. DECENTRALIZED ASYNCHRONOUS EVENT-TRIGGERING

In this section, our aim is to constructively show that for LTI systems decentralized asynchronous event-triggering can be used to globally asymptotically stabilize $x = 0$ (the trivial solution or the origin) while also guaranteeing positive minimum inter-sample times. The proposed design relies on the emulation of continuous-time feedback control. Thus, it is assumed that the matrix $(A + BK)$ is Hurwitz, which is equivalent to the following statement.

(A1) Suppose that for any given symmetric positive definite matrix Q , there exists a symmetric positive definite matrix P such that $P(A+BK)+(A+BK)^TP=-Q$.

The proposed design of decentralized asynchronous event-triggering progresses in stages. In the first stage, centralized event-triggers for asynchronous sampling of the sensors are proposed.

A. Centralized Asynchronous Event-Triggering

The following lemma describes a centralized asynchronous sensing mechanism for linear systems that guarantees global asymptotic stability of the origin.

Lemma 1: Consider the closed loop system (1)-(2) and assume (A1) holds. Let Q be any symmetric positive definite matrix and let Q_m be the smallest eigenvalue of Q . For each $i \in \{1, 2, \dots, n\}$, let

$$\theta_i \in (0, 1) \quad \text{s.t.} \quad \theta = \sum_{i=1}^n \theta_i \leq 1 \quad (5)$$

$$w_i = \frac{\sigma \theta_i Q_m}{|c_i(2PBK)|} \quad (6)$$

where $0 < \sigma < 1$ is a constant and $c_i(2PBK)$ is the i^{th} column of the matrix $(2PBK)$. Suppose the sampling instants are such that for each $i \in \{1, \dots, n\}$, $|x_{i,e}| \leq w_i |x|$ for all time $t \geq 0$. Then, the origin is globally asymptotically stable.

Proof: Consider the candidate Lyapunov function $V(x) = x^T Px$ where P satisfies (A1). The derivative of

the function V along the flow of the closed loop system satisfies

$$\begin{aligned}\dot{V} &= x^T [P(A + BK) + (A + BK)^T P]x + 2x^T PBKx_e \\ &\leq -(1 - \sigma)x^T Qx + |x| \left[|2PBKx_e| - \sigma Q_m |x| \right] \\ &\leq -(1 - \sigma)x^T Qx + |x| \left[\sum_{i=1}^n |c_i(2PBK)x_{i,e}| - \sigma Q_m |x| \right] \\ &\leq -(1 - \sigma)x^T Qx + |x| \left[\sum_{i=1}^n |c_i(2PBK)| |x_{i,e}| - \sigma Q_m |x| \right]\end{aligned}$$

The sensor update instants have been assumed to be such that $|x_{i,e}|/|x| \leq w_i = \frac{\sigma\theta_i Q_m}{|c_i(2PBK)|}$ for each i and for all time $t \geq 0$. Thus,

$$\dot{V} \leq -(1 - \sigma)x^T Qx$$

which implies that the origin is globally asymptotically stable. \blacksquare

The lemma does not mention a specific choice of event-triggers but rather a family of them - all those that ensure the conditions $|x_{i,e}| \leq w_i|x|$ are satisfied. Thus, any decentralized event-triggers in this family automatically guarantee asymptotic stability with the desired region of attraction. To enforce the conditions $|x_{i,e}| \leq w_i|x|$ strictly, event-triggers at each sensor would need to know $|x|$, which is possible only if we have centralized information. One obvious way to decentralize these conditions is to enforce $|x_{i,e}| \leq w_i|x_i|$. However, such event-triggers cannot guarantee any positive lower bound for the inter-transmission times, which is not acceptable. So, we take an alternative approach, in which the next step is to derive lower bounds for the inter-transmission times when the conditions in Lemma 1 are enforced strictly.

Before analyzing the lower bounds for the inter-transmission times that emerge from the event-triggers in Lemma 1, we introduce some notation. Let the function τ be defined as

$$\tau(w, a_0, a_1, a_2) = \{t \geq 0 : \phi(t, 0) = w\} \quad (7)$$

where a_0, a_1, a_2 are non-negative constants and $\phi(t, \phi_0)$ is the solution of

$$\dot{\phi} = a_0 + a_1\phi + a_2\phi^2, \quad \phi(0, \phi_0) = \phi_0$$

Lemma 2: Consider the closed loop system (1)-(2). For each $i \in \{1, \dots, n\}$, let θ_i, w_i be defined as in (5)-(6) and

let $W_i = \sqrt{\left(\sum_{j=1}^n w_j^2 \right)} - w_i^2$. Suppose the sampling instants are such that $|x_{i,e}|/|x| \leq w_i$ for each $i \in \{1, \dots, n\}$ for all time $t \geq t_0$. Then, for all $t \geq t_0$, the time required for $|x_{i,e}|/|x|$ to evolve from 0 to w_i is lower bounded by $T_i > 0$, where

$$T_i = \tau(w_i, a_0, a_1, a_2) \quad (8)$$

where the function τ is given by (7) and

$$\begin{aligned}a_0 &= |r_i(A + BK)| + |r_i(BK)|W_i, \\ a_1 &= |A + BK| + |r_i(BK)| + |BK|W_i, \quad a_2 = |BK|\end{aligned}$$

Proof: Letting $\nu_i \triangleq |x_{i,e}|/|x|$, for $i \in \{1, \dots, n\}$, an upper bound for the time derivative of ν_i can be found by direct calculation.

$$\begin{aligned}\frac{d\nu_i}{dt} &= \frac{-(x_{i,e}^T x_{i,e})^{-1/2} x_{i,e}^T \dot{x}_{i,e}}{|x|} - \frac{x^T \dot{x} |x_{i,e}|}{|x|^3} \\ &\leq \frac{|x_{i,e}| |\dot{x}_{i,e}|}{|x_{i,e}| |x|} + \frac{|x| |\dot{x}| |x_{i,e}|}{|x|^3} \\ &\leq \frac{|r_i(A + BK)| |x| + |r_i(BK)| |x_e|}{|x|} \\ &\quad + \frac{(|A + BK| |x| + |BK| |x_e|) |x_{i,e}|}{|x|^2}\end{aligned}$$

where for $x_{i,e} = 0$ the relation holds for all directional derivatives while the notation $r_i(H)$ denotes the i^{th} row of the matrix H . Next, notice that

$$\frac{|x_e|}{|x|} = \sqrt{\sum_{j=1}^{j=n} \nu_j^2} \leq \sqrt{\left(\sum_{j=1}^{j=n} w_j^2 \right) - w_i^2 + \nu_i^2} \leq W_i + \nu_i$$

where the condition that $\nu_i \leq w_i$, the definition of W_i and the triangle inequality property have been utilized. Thus,

$$\begin{aligned}\frac{d\nu_i}{dt} &\leq |r_i(A + BK)| + |A + BK| \nu_i \\ &\quad + (|r_i(BK)| + |BK| \nu_i)(W_i + \nu_i) \\ &= a_0 + a_1 \nu_i + a_2 \nu_i^2\end{aligned}$$

The claim of the Lemma now directly follows. \blacksquare

Now, by combining Lemmas 1 and 2, we get the following result for the centralized asynchronous event-triggering.

Theorem 1: Consider the closed loop system (1)-(2) and assume (A1) holds. Let Q be any symmetric positive definite matrix and let Q_m be the smallest eigenvalue of Q . For each $i \in \{1, 2, \dots, n\}$, let θ_i and w_i be defined as in (5)-(6). Also suppose the i^{th} sensor transmits its measurement to the controller whenever $|x_{i,e}|/|x| \geq w_i$. Then, the origin is globally asymptotically stable and the inter-transmission times have a positive lower bound.

Proof: The result follows from Lemmas 1 and 2. \blacksquare

B. Decentralized Asynchronous Event-Triggering

Now, we turn to the main subject of this paper. In the decentralized sensing case, unlike in the centralized sensing case, no single sensor knows the exact value of $|x|$ from the locally sensed data. As mentioned earlier, the centralized asynchronous event-triggers that enforce $|x_{i,e}| \leq w_i|x|$ may be decentralized by making them enforce the more conservative conditions $|x_{i,e}| \leq w_i|x_i|$ and still satisfy the assumptions of Lemma 1. However, such a choice cannot guarantee a positive minimum inter-sample time. At this stage, it might seem that Lemma 2 cannot be used to design an implicitly verified event-triggering in the decentralized

sensing case. However, the lemma can be interpreted in an alternative way, which would aid in our design goal.

Rather than providing a minimum inter-sampling time for an event-triggering mechanism, Lemma 2 can be interpreted as providing a minimum time threshold only after which it is necessary to check a data based event-triggering condition. For example, the event-triggers in Theorem 1,

$$t_{j+1}^{x_i} = \min\{t \geq t_j^{x_i} : |x_{i,e}| \geq w_i|x|\}, \quad i \in \{1, \dots, n\} \quad (9)$$

can be equivalently expressed as

$$t_{j+1}^{x_i} = \min\{t \geq t_j^{x_i} + T_i : |x_{i,e}| \geq w_i|x|\} \quad (10)$$

where T_i are the known positive lower bounds for inter-sample times provided by Lemma 2 in (8). In the latter interpretation, a lower bound for inter-sample times is explicitly enforced, only after which, the state based condition is checked. Now, in order to let the event-triggers depend only on locally sensed data, one can let the sampling times, for $i \in \{1, \dots, n\}$, be determined as

$$t_{j+1}^{x_i} = \min\{t \geq t_j^{x_i} + T_i : |x_{i,e}| \geq w_i|x_i|\} \quad (11)$$

where T_i are given by (8). This allows us to implement decentralized asynchronous event-triggering. The following theorem is the core result of this paper. It prescribes the constants T_i and w_i in the event triggers, (11), that guarantee global asymptotic stability of the origin while also explicitly enforcing positive minimum inter-transmission times.

Theorem 2: Consider the closed loop system (1)-(2) and assume (A1) holds. Let Q be any symmetric positive definite matrix and let Q_m be the smallest eigenvalue of Q . For each $i \in \{1, 2, \dots, n\}$, let θ_i , w_i and T_i be defined as in (5), (6) and (8), respectively. Suppose the sensors asynchronously transmit the measured data at time instants determined by (11). Then, the origin is globally asymptotically stable and the inter-transmission times are explicitly enforced to have a positive lower threshold.

Proof: The statement about the positive lower threshold for inter-transmission times is obvious from (11) and only asymptotic stability remains to be proven. This can be done by showing that the event-triggers (11) are included in the family of event-triggers considered in Lemma 1. From the equivalence of (9) and (10), it is clearly true that $|x_{i,e}|/|x| \leq w_i$ for $t \in [t_j^{x_i}, t_j^{x_i} + T_i]$, for each $i \in \{1, 2, \dots, n\}$ and each j . Next, for $t \in [t_j^{x_i} + T_i, t_{j+1}^{x_i}]$, (11) enforces $|x_{i,e}| \leq w_i|x_i|$, which implies $|x_{i,e}| \leq w_i|x|$ since $|x_i| \leq |x|$. Therefore, the event-triggers in (11) are included in the family of event-triggers considered in Lemma 1. Hence, $x \equiv 0$ (the origin) is globally asymptotically stable. ■

Remark 1: Although at first sight our approach of explicitly enforcing a lower bound on inter-transmission times may seem similar to that of [19], there are important differences. Unlike in [19], the combination of time and event triggering, as in (11), is used *at the sensors rather than at the central controller*. Further, in our approach, the controller utilizes the asynchronously transmitted sensor data rather than synchronous measurements from the sensors (which are requested by the central controller in [19]).

The decentralized asynchronous event-triggers, (11), ensure a type of scale invariance for LTI systems. Scaling laws of inter-execution times for centralized *synchronous* event-triggering have been studied in [20]. In particular, Theorem 4.3 of [20], in the special case of linear systems, guarantees scale invariance of the inter-execution times determined by a centralized event-trigger $|x_e| = W|x|$. The centralized and decentralized asynchronous event-triggers developed in this paper are under-approximations of this kind of central event-triggering. In the following, we show that the scale invariance is preserved in the asynchronous event-triggers. As an aside, we would like to point out that the decentralized event-triggers proposed in [8]–[10] are not scale invariant for LTI systems. In order to precisely state the notion of scale invariance and to state the result the following notation is useful. Let $x(t)$ and $z(t)$ be two solutions to the system: (1)-(2) along with the event-triggers (11).

Theorem 3: Consider the closed loop system (1)-(2) and assume (A1) holds. Let Q be any symmetric positive definite matrix and let Q_m be the smallest eigenvalue of Q . For each $i \in \{1, 2, \dots, n\}$, let θ_i , w_i and T_i be defined as in (5), (6) and (8), respectively. Suppose the sensors asynchronously transmit the measured data at time instants determined by (11). Assuming b is any scalar constant, let $[z(0)^T, z_s(0)^T]^T = b[x(0)^T, x_s(0)^T]^T \in \mathbb{R}^n \times \mathbb{R}^n$ be two initial conditions for the system. Further let $t_0^{z_i} = t_0^{x_i} < 0$ for each $i \in \{1, \dots, n\}$. Then, $[z(t)^T, z_s(t)^T]^T = b[x(t)^T, x_s(t)^T]^T$ for all $t \geq 0$ and $t_j^{x_i} = t_j^{z_i}$ for each i and j .

Proof: First of all, let us introduce two strictly increasing sequences of time, $\{t_j^{z_s}\}$ and $\{t_j^{x_s}\}$, at which one or more components of z_s and x_s are updated, respectively. Further, without loss of generality, assume $t_0^{z_s} = t_0^{x_s}$. The proof proceeds by mathematical induction. Let us suppose that $t_j^{z_s} = t_j^{x_s} = t_j$ for each $j \in \{0, \dots, k\}$ and that $[z(t)^T, z_s(t)^T]^T = b[x(t)^T, x_s(t)^T]^T$ for all $t \in [0, t_k]$. Then, letting $\underline{t}_{k+1} = \min\{t_{k+1}^{z_s}, t_{k+1}^{x_s}\}$ the solution, z , in the time interval $[t_k, \underline{t}_{k+1}]$ satisfies

$$\begin{aligned} z(t) &= e^{A(t-t_k)} z(t_k) + \int_{t_k}^t e^{A(t-\sigma)} B K z_s(\tau) d\sigma \\ &= b e^{A(t-t_k)} x(t_k) + b \int_{t_k}^t e^{A(t-\sigma)} B K x_s(\tau) d\sigma \end{aligned}$$

Hence,

$$z(t) = b x(t), \quad \forall t \in [t_k, \underline{t}_{k+1}] \quad (12)$$

Further, in the time interval $[t_k, \underline{t}_{k+1}]$

$$z_{i,e}(t) = z_i(t_k) - z_i(t) = b(x_i(t_k) - x_i(t)) = b x_{i,e}(t) \quad (13)$$

Similarly, for all $t \in [t_k, \underline{t}_{k+1}]$,

$$\frac{|z_{i,e}(t)|}{|z(t)|} = \frac{|x_{i,e}(t)|}{|x(t)|} \quad (14)$$

Without loss of generality, assume $z_{i,s}$ is updated at \underline{t}_{k+1} . Then, clearly, at least T_i amount of time has elapsed

since $z_{i,s}$ was last updated. Next, by the assumption that $t_0^{z_i} = t_0^{x_i} < 0$ and the induction statement, it is clear that at least T_i amount of time has elapsed since $x_{i,s}$ also was last updated. Further, it also means that $|z_{i,s}(t_k) - z_i(t_{k+1})| \geq w_i |z_i(t_{k+1})|$. Then, (12)-(13) imply that $|x_{i,s}(t_k) - x_i(t_{k+1})| \geq w_i |x_i(t_{k+1})|$, meaning $t_{k+1} = t_{k+1}^{z_s} = t_{k+1}^{x_s} = t_{k+1}$. Arguments analogous to the preceding also hold for multiple $z_{i,s}$ updated at t_{k+1} instead of one or even $x_{i,s}$ instead of $z_{i,s}$. Since the induction statement is true for $k = 0$, we conclude that the statement of theorem is true. ■

Remark 2: From the proof of Theorem 3, (14) specifically, it is clear that the *centralized asynchronous* event-triggers of Theorem 1 also guarantee scale invariance.

Remark 3: Scale invariance, as described in Theorem 3, means that the inter-transmission times over an arbitrary length of time is independent of the scale (or the magnitude) of the initial condition of the system. Similarly for any given scalar, $0 < \delta < 1$, the time and the number of transmissions it takes for $|x(t)|$ to reduce to $\delta|x(0)|$ is independent of $|x(0)|$. So, the advantage is that the ‘average’ network usage remains the same over large portions of the state space.

IV. SIMULATION RESULTS

In this section, the proposed decentralized asynchronous event-triggered sensing mechanism is illustrated for a linearized model of a batch reactor, [21]. The plant and the controller are given by (1)-(2) with

$$A = \begin{bmatrix} 1.38 & -0.20 & 6.71 & -5.67 \\ -0.58 & -4.29 & 0 & 0.67 \\ 1.06 & 4.27 & -6.65 & 5.89 \\ 0.04 & 4.27 & 1.34 & -2.10 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 5.67 & 0 \\ 1.13 & -3.14 \\ 1.13 & 0 \end{bmatrix}$$

$$K = -\begin{bmatrix} 0.1006 & -0.2469 & -0.0952 & -0.2447 \\ 1.4099 & -0.1966 & 0.0139 & 0.0823 \end{bmatrix}$$

which places the eigenvalues of the matrix $(A + BK)$ at around $\{-2.98 + 1.19i, -2.98 - 1.19i, -3.89, -3.62\}$. The matrix Q was chosen as the identity matrix. The system matrices and Q have been chosen to be the same as in [8]. Lastly, the controller parameters were chosen as $[\theta_1, \theta_2, \theta_3, \theta_4] = [0.6, 0.17, 0.08, 0.15]$ and $\sigma = 0.95$. For the simulations presented here, the initial condition of the plant was selected as $x(0) = [4, 7, -4, 3]^T$ and the initial sampled data that the controller used was $x_s(0) = [4.1, 7.2, -4.5, 2]^T$. The zeroth sampling instant was chosen as $t_0^{x_i} = -T_i$ for sensor i . This is to ensure sampling at $t = 0$ if the local triggering condition was satisfied. Finally the simulation time was chosen as $T_{sim} = 10s$.

Figures 1a and 1b show the evolution of the Lyapunov function and its derivative along the flow of the closed loop system, respectively. Figure 1c shows the time evolution of the inter-transmission times for each sensor. The frequency distribution of the inter-transmission times is another useful

metric to understand the closed loop event-triggered system. Thus, given a time interval of interest $[0, T_{INT}]$ consider

$$\mathcal{N}^{x_i}(\mathcal{T}, T_{INT}) = \left\{ j \in \mathbb{N}_0 : t_{j+1}^{x_i} \in [0, T_{INT}] \text{ and } (t_{j+1}^{x_i} - t_j^{x_i}) \in [0, \mathcal{T}] \right\}$$

where $\mathbb{N}_0 = \{0, 1, 2, \dots\}$ is the set of natural numbers. Hence, the cumulative distribution of the inter-transmission times during $[0, T_{INT}]$ is given as

$$\mathcal{D}^{x_i}(\mathcal{T}, T_{INT}) = \frac{\#\mathcal{N}^{x_i}(\mathcal{T}, T_{INT})}{\#\mathcal{N}^{x_i}(T_{INT}, T_{INT})} \quad (15)$$

where $\#$ denotes the cardinality of a set. Figure 1d shows the cumulative frequency distribution of the inter-transmission times, $\mathcal{D}^{x_i}(\mathcal{T}, T_{sim})$, for each sensor. The cumulative frequency distribution of the inter-transmission times is a measure of the performance of the event-triggers. A distribution that rises sharply to 100% indicates that event-trigger is not much better than a time-trigger. Thus, slower the rise of the cumulative distribution curves, greater is the justification for using the event-trigger instead of a time-trigger.

The minimum thresholds for the inter-transmission times T_i for the example can be computed as in Lemma 2 and have been obtained as $[T_1, T_2, T_3, T_4] = [11, 15.4, 12.6, 19.9]\text{ms}$, which are also the minimum inter-transmission times in the simulations presented here. These numbers are a few orders of magnitude higher and an order higher than the guaranteed minimum inter-transmission times and the observed minimum inter-transmission times in [8], [9]. The average inter-transmission times obtained in the presented simulations were $[\bar{T}_1, \bar{T}_2, \bar{T}_3, \bar{T}_4] = [24.9, 27.7, 34.5, 34.2]\text{ms}$, which are about an order of magnitude lower than those reported in [8], [9]. A possible explanation for this phenomenon is that in [8], [9], the average inter-transmission times depends quite critically on the evolution of the threshold η . Although the controller gain matrix K and the matrix Q have been chosen to be the same, by inspection of the plots in [8], [9], it appears that the rate of decay of the Lyapunov function V is roughly about half of that in our simulations. However, we would like to point out that our average inter-transmission times are of the same order as in [10] by the same authors. In any case, for LTI systems, our proposed method does not require communication from the controller to sensors to achieve global asymptotic stability. Lastly, as a measure of the usefulness of the event-triggering mechanism compared to a purely time-triggered mechanism, T_i/\bar{T}_i was computed for each i and were obtained as $[T_1/\bar{T}_1, T_2/\bar{T}_2, T_3/\bar{T}_3, T_4/\bar{T}_1] = [0.44, 0.55, 0.36, 0.58]$. The lower these numbers are, the better it is.

V. CONCLUSIONS

In this paper, we have developed a method for designing decentralized event-triggers for control of Linear Time Invariant (LTI) systems. The architecture of the systems considered in this paper included full state feedback, a central controller and distributed sensors not co-located with the central controller. The aim was to develop event-triggers

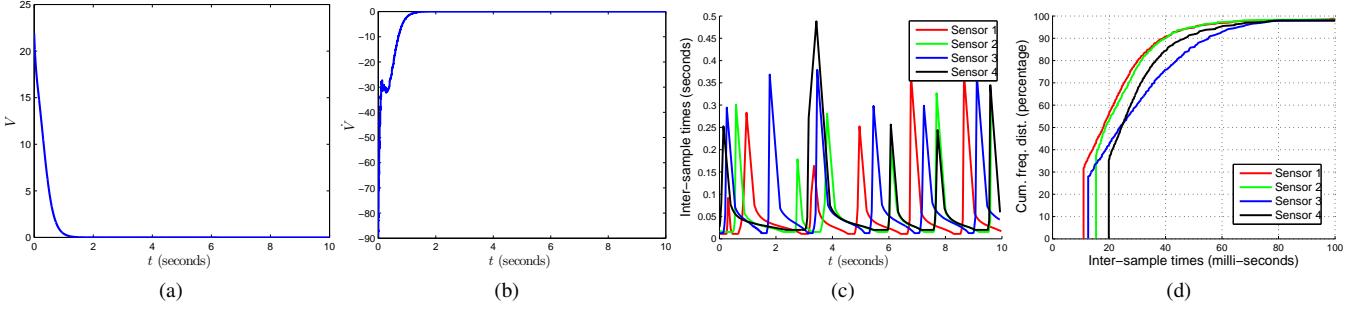


Fig. 1: Batch reactor example: evolution of the (a) Lyapunov function, (b) time derivative of Lyapunov function, along the flow of the closed loop system. (c) Sensor inter-transmission times (d) cumulative frequency distribution of the sensor inter-transmission times, $\mathcal{D}^{x_i}(\mathcal{T}, T_{sim})$, where $T_{sim} = 10s$ is the simulation time.

for determining the time instants of transmission from the sensors to the central controller. The proposed design ensures that the event-triggers at each sensor depend only on locally available information, thus allowing for asynchronous transmissions from the sensors to the central controller. Further, for LTI systems, the design succeeds in completely eliminating the need for the sensors to listen to other sensors and/or the controller. The proposed design was shown to guarantee a positive lower bound for inter-transmission times of each sensor. The origin of the closed loop system is also guaranteed to be globally asymptotically stable. Finally, the proposed design method was illustrated through simulations of a linear system.

In the system architecture considered in this paper, although the control input to the plant is updated intermittently, it is not exactly event-triggered. In fact, in all the results the inter-transmission times of each sensor individually have been shown to have a positive lower bound. And the time interval between receptions of the central controller from two different sensors can be arbitrarily close to zero. Since the control input to the plant is updated each time the controller receives some information, no positive lower bound can be guaranteed for the inter-update times of the controller. However, it is not very difficult to incorporate event-triggering (with guaranteed positive minimum inter-update times) or explicit thresholds on inter-update times of the control by choosing smaller σ values in the event-triggers for the sensors. Future work will include results with event-triggered actuation in addition to event-triggered communication on the sensing side. Finally, although time delays have not been considered explicitly, they may be handled as in most event-triggered control literature (see [1] for example).

REFERENCES

- [1] P. Tabuada, “Event-triggered real-time scheduling of stabilizing control tasks,” *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1680–1685, 2007.
- [2] K. Åström, “Event based control,” in *Analysis and Design of Nonlinear Control Systems: In Honor of Alberto Isidori*, A. Astolfi and L. Marconi, Eds. Springer Berlin Heidelberg, 2008, pp. 127–147.
- [3] W. Heemels, J. Sandee, and P. Van Den Bosch, “Analysis of event-driven controllers for linear systems,” *International Journal of Control*, vol. 81, no. 4, pp. 571–590, 2008.
- [4] M. Velasco, P. Martí, and E. Bini, “Control-driven tasks: Modeling and analysis,” in *Real-Time Systems Symposium*, 2008, pp. 280–290.
- [5] X. Wang and M. Lemmon, “Self-triggering under state-independent disturbances,” *IEEE Transactions on Automatic Control*, vol. 55, no. 6, pp. 1494–1500, 2010.
- [6] J. Lunze and D. Lehmann, “A state-feedback approach to event-based control,” *Automatica*, vol. 46, no. 1, pp. 211–215, 2010.
- [7] M. D. Lemmon, “Event-triggered feedback in control, estimation, and optimization,” in *Networked Control Systems*, ser. Lecture Notes in Control and Information Sciences, A. Bemporad, M. Heemels, and M. Johansson, Eds. Springer Berlin / Heidelberg, 2011, vol. 406, pp. 293–358.
- [8] M. Mazo Jr and M. Cao, “Decentralized event-triggered control with asynchronous updates,” in *IEEE Conference on Decision and Control and European Control Conference*, 2011, pp. 2547–2552.
- [9] ———, “Decentralized event-triggered control with one bit communications,” in *IFAC Conference on Analysis and Design of Hybrid Systems*, 2012, pp. 52–57.
- [10] M. Mazo Jr and M. Cao, “Asynchronous decentralized event-triggered control,” *arXiv preprint arXiv:1206.6648v1 [math.OC]*, 2012.
- [11] X. Wang and M. Lemmon, “Event-triggering in distributed networked systems with data dropouts and delays,” *Hybrid systems: Computation and control*, pp. 366–380, 2009.
- [12] ———, “Event triggering in distributed networked control systems,” *IEEE Transactions on Automatic Control*, vol. 56, no. 3, pp. 586–601, 2011.
- [13] M. Donkers and W. Heemels, “Output-based event-triggered control with guaranteed \mathcal{L}_∞ -gain and improved event-triggering,” in *IEEE Conference on Decision and Control*, 2010, pp. 3246–3251.
- [14] P. Tallapragada and N. Chopra, “Event-triggered dynamic output feedback control for LTI systems,” in *IEEE Conference on Decision and Control*, 2012, pp. 6597–6602.
- [15] ———, “Event-triggered decentralized dynamic output feedback control for LTI systems,” in *Estimation and Control of Networked Systems*, vol. 3, no. 1, 2012, pp. 31–36.
- [16] D. Lehmann and J. Lunze, “Event-based output-feedback control,” in *Mediterranean Conference on Control & Automation*, 2011, pp. 982–987.
- [17] L. Li and M. Lemmon, “Weakly coupled event triggered output feedback control in wireless networked control systems,” in *Annual Allerton Conference on Communication, Control, and Computing*, 2011, pp. 572–579.
- [18] J. Almeida, C. Silvestre, and A. M. Pascoal, “Observer based self-triggered control of linear plants with unknown disturbances,” in *American Control Conference*, 2012, pp. 5688–5693.
- [19] M. Mazo Jr. and P. Tabuada, “Decentralized event-triggered control over Wireless Sensor/Actuator Networks,” *IEEE Transactions on Automatic Control*, vol. 56, no. 10, pp. 2456–2461, 2011.
- [20] A. Anta and P. Tabuada, “To sample or not to sample: Self-triggered control for nonlinear systems,” *IEEE Transactions on Automatic Control*, vol. 55, no. 9, pp. 2030–2042, 2010.
- [21] G. Walsh and H. Ye, “Scheduling of networked control systems,” *IEEE Control Systems Magazine*, vol. 21, no. 1, pp. 57–65, 2001.