Coordinated intersection traffic management

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Motivation

- Emerging technologies such as networked and automated vehicles enable us to radically redesign traffic systems of the future, with the potential to hugely improve safety, traveling ease, travel time, and energy consumption.
- A particularly useful application of these technologies is in the coordination of traffic at and near intersections.
- Traffic coordination can be achieved by controlling the vehicles much before they arrive at the intersection.
- Such a paradigm offers the possibility of significantly reduced stop times and increased fuel efficiency.

Problem summary

- *Assumptions:* (i) Single lane in each direction, (ii) all vehicles are identical with length L, (iii) no turning at the intersection, (iv) no sources or sinks for vehicles along the branches.

- *Vehicle dynamics:*

$$\dot{x}_j^v(t) = v_j^v(t), \quad \text{position}$$
$$\dot{v}_j^v(t) = u_j^v(t), \quad \text{velocity}$$

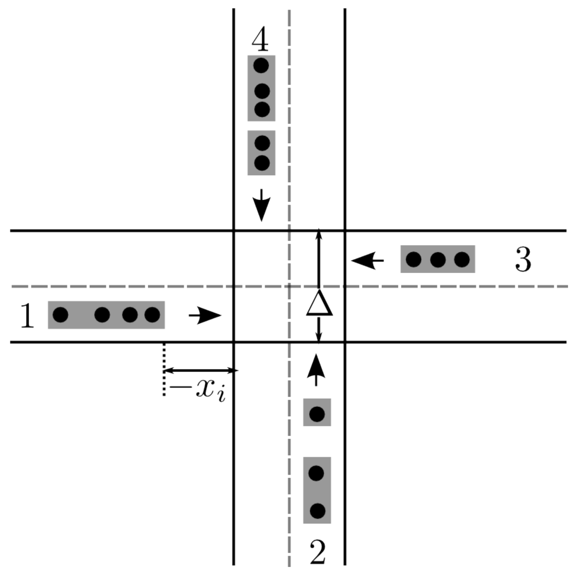
  - Bounded control: $u_j^v(t) \in [u_m, u_M]$, with $u_m \leq 0$ and $u_M \geq 0$.
  - For each vehicle j, $v_j^v(t)$ must be constrained to the interval $[0, v^M]$ for all time t that the vehicle is in the system.

- *Objective:* Design a traffic coordination mechanism for networked and automated vehicles that minimizes a combination of cumulative travel time and fuel cost.
- *Challenges:* Problem is combinatorial. Solving it at the level of individual cars is computationally expensive and not scalable.

Contributions

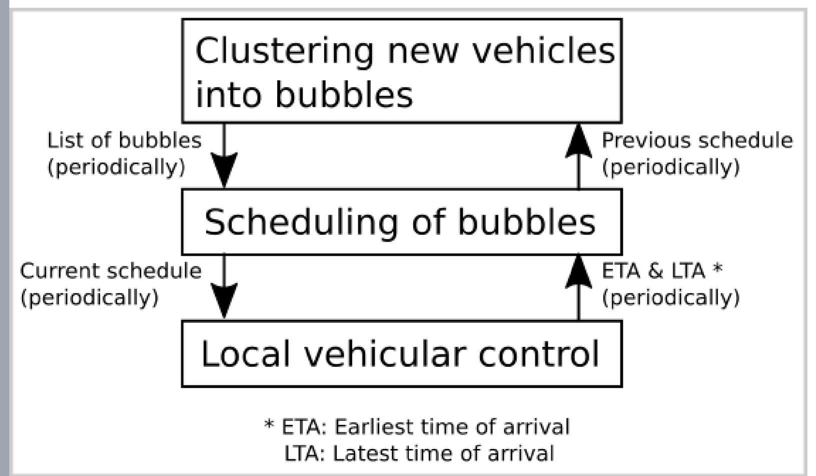
- We propose a *provably safe hierarchical* intersection management system
- Extensive simulations are not required for scheduling and safety-verification
- Applicable to a wide range of traffic conditions

A scalable solution



- Black dots are individual vehicles
- Vehicles are clustered into *bubbles* represented by the grey boxes
- x_i is the position of the lead vehicle in the bubble
- Δ is the length of the intersection
- The numbers $\{1, 2, 3, 4\}$ are labels for the incoming branches

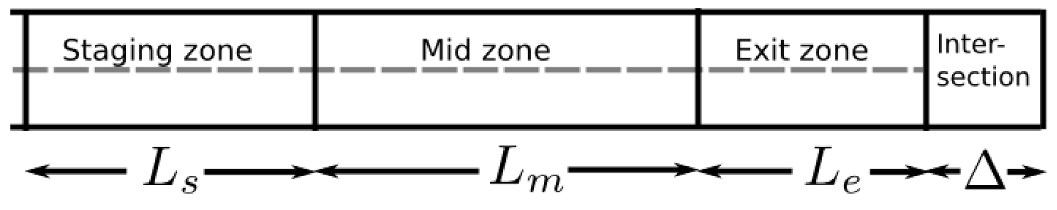
Overview of hierarchical solution



State of bubble i:
$\xi_i = (x_i, v_i, m_i, \mathcal{I}_i) \in \mathbb{R}^3 \times \{1, 2, 3, 4\}$,
x_i: position of the lead vehicle in the bubble
v_i: velocity of the lead vehicle in the bubble
m_i: number of vehicles in the bubble
$\mathcal{I}_i$: branch label that the bubble is on

Other important variables:
τ_i: scheduled *approach time* at the beginning of the intersection for the lead vehicle in bubble i
τ_i^{occ}: *occupancy time* - time for which bubble i occupies the intersection
$\bar{\tau}_i^{\text{occ}}$: guaranteed upper-bound on τ_i^{occ}

Dynamic vehicle clustering



Division of an incoming branch into zones.

- Executed every $T_{cs} < \frac{L_s}{v^M}$ units of time
- New vehicles on branch i are clustered based on their position using k-means algorithm.
- List of bubbles to be scheduled, $\mathcal{L}$ consists of newly created bubbles and possibly some of the previously created bubbles, subject to $|\mathcal{L}| \leq \bar{N}$.
- Update τ^{min}, the earliest time the intersection is available.

Constraints and cost function

Constraints:

$\tau_i \in [\max\{\tau^{\text{min}}, \tau_i^m\}, \tau_i^M]$, interval determined by initial conditions
$\tau_j \geq \tau_i + \bar{\tau}_i^{\text{occ}}$, if bubbles i and j on same branch and j follows i
$\tau_i \geq \tau_j + \bar{\tau}_j^{\text{occ}}$ OR $\tau_j \geq \tau_i + \bar{\tau}_i^{\text{occ}}$, if $\mathcal{I}_i \neq \mathcal{I}_j$,

Cost function:

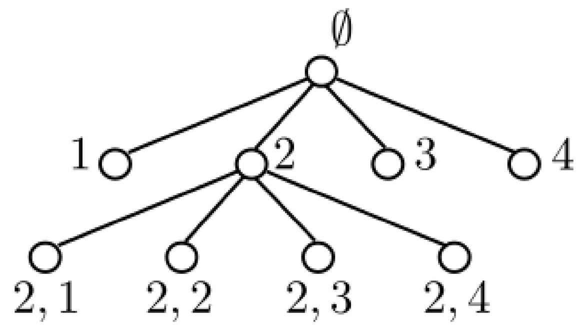
$$\mathcal{C} \triangleq \sum_{i=1}^{N} m_i(\tau_i - t_0 + F_i(\bar{v}_i)) = \sum_{i=1}^{N} m_i\Big(\frac{-x_i}{\bar{v}_i} + F_i(\bar{v}_i)\Big) \triangleq \sum_{i=1}^{N} \phi_i(\bar{v}_i),$$

$\bar{v}_i$: average velocity of the lead vehicle in bubble i for $t \in [t_0, t_0 + \tau_i]$.

Assumption: For each i, $F_i : [\bar{v}_i^m, \bar{v}_i^M] \mapsto \mathbb{R}_{>0}$ is a monotonically decreasing function.

Scheduling of bubbles

- Algorithm to find optimal approach times τ_i and optimal cost given an order P.
- Algorithm to find a lower-bound on optimal cost given part of the order.
- Schedule optimization using *branch and bound*.



Tree of possible bubble passage orders.

Lemma: Safe-following distance

For a vehicle j following $j-1$,
$\mathcal{D}(v_{j-1}^v(t), v_j^v(t)) = L + \max\Big\{0, \frac{-1}{2u_m}\Big((v_j^v(t))^2 - (v_{j-1}^v(t))^2\Big)\Big\}$ is a *safe-following distance* (a control action exists to avoid collision).

Local vehicular control

Consists of two parts

- an *uncoupled optimal feedback controller* for reaching the intersection at a nominal deadline with a nominal speed: g_{uc}
- a *controller for safe following*: $g_{sf} \triangleq \min\{g_{uc}, [g_{us}]_{u_m}^{u_M}\}$,

$$g_{us} \triangleq \left(\frac{v_{j-1}^v}{v_j^v}\left(1 + \sigma_j \frac{u_{j-1}^v}{-u_m}\right) - 1\right)\left(\frac{-u_m}{\sigma_j}\right), \quad \sigma_j(t) \triangleq \frac{x_{j-1}^v(t) - x_j^v(t)}{\mathcal{D}(v_{j-1}^v(t), v_j^v(t))}.$$

Control law:
$$u_j^v(t) = \begin{cases} g_{uc}, & \text{if } (v_{j-1}^v, v_j^v, \sigma_j) \notin \mathcal{C}_s \\ g_{sf}, & \text{if } (v_{j-1}^v, v_j^v, \sigma_j) \in \mathcal{C}_s, \end{cases}$$

where $\mathcal{C}_s$ is the *coupling set* given by

$$\mathcal{C}_s \triangleq \{(v_{j-1}^v, v_j^v, \sigma_j) : v_j^v \geq v_{j-1}^v \ \wedge \sigma_j \in [1, \sigma_0]\}.$$

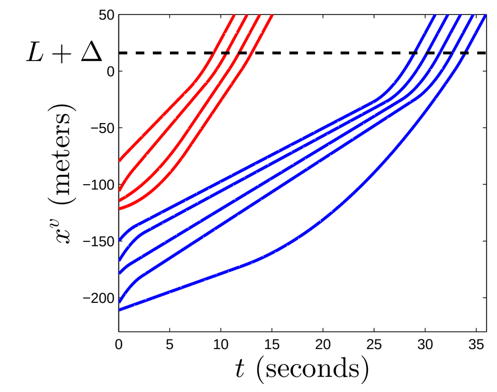
Idea: When there is sufficient gap in front, use g_{uc} otherwise use g_{sf}.

Theorem: Provably safe traffic coordination

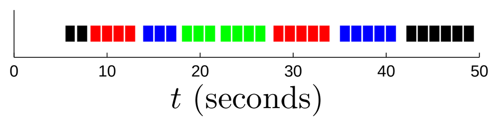
The local vehicular control law results in a provably safe traffic coordination and the prescribed schedule is satisfied.

Simulations

Static example: 32 vehicles on 4 branches, clustered into 8 bubbles



Evolution of the vehicle positions on one of the branches. The color indicates the bubble the vehicles belong to.



Intersection occupancy times for each vehicle. The color indicates the branch that the vehicle originates from.

- The local vehicular control ensures tight platooning of the vehicles of a bubble at the time of intersection crossing.
- This ensures the intersection, is efficiently used.

Dynamic arrival: Also performed simulations with dynamic Poisson arrivals with safety constraints.

Future Work

- Characterization of performance and comparison with traditional intersection management.
- Coordination with limited/uncertain data.
- Coordinated management for networks of intersections.

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