PYTHON PROGRAMMING ON MATRICES

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FWC22011 IITH Future Wireless Communication (FWC) Matrix:Lines

1 Problem

In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ. Show that:
(i) $\Delta APD \cong \Delta CQB$
(ii) AP = CQ
(iii)$\Delta AQB \cong \Delta CPD$
(iv) AQ = CP
(v) APCQ is a parallelogram

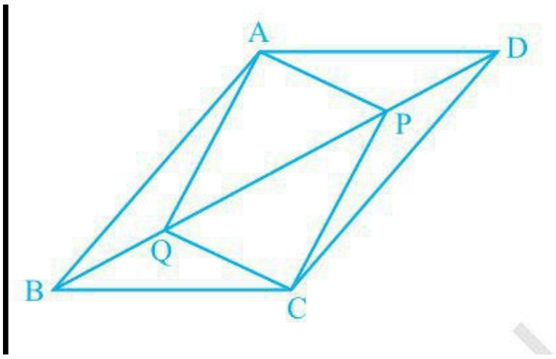


Figure 1: Given Figure

2 Solution

The input parameters for this construction are

Symbol	Value
r	5
k	3
b	4
θ	$\frac{pi}{3}$

$$\vec{A} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}$$

$$\vec{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\vec{C} = \begin{pmatrix} b \\ 0 \end{pmatrix}$$

$$\vec{D} = \vec{A} + \vec{C} - \vec{B}$$

$$\vec{P} = \frac{\vec{B} + K \times \vec{D}}{1 + K}$$

$$\vec{Q} = \frac{K \times \vec{B} + \vec{D}}{1 + K}$$

Direction vectors:

$$\vec{m_1} = \vec{A} - \vec{B}$$
$$\vec{m_2} = \vec{C} - \vec{B}$$
$$\vec{m_3} = \vec{D} - \vec{C}$$
$$\vec{m_4} = \vec{D} - \vec{A}$$
$$\vec{m_5} = \vec{Q} - \vec{B}$$
$$\vec{m_6} = \vec{D} - \vec{P}$$

$$\vec{n_1} = \vec{A} - \vec{P}$$
$$\vec{n_2} = \vec{C} - \vec{Q}$$
$$\vec{n_3} = \vec{A} - \vec{Q}$$
$$\vec{n_4} = \vec{C} - \vec{P}$$

(i)To prove:$\Delta APD \cong \Delta CQB$
Distance between D and P is $\left\|\vec{D-P}\right\|$
Distance between B and Q is $\left\|\vec{B-Q}\right\|$
if $\left\|\vec{D-P}\right\| = \left\|\vec{B-Q}\right\|$
then DP = BQ..........(1)

Distance between A and D is $\left\|\vec{A-D}\right\|$
Distance between B and C is $\left\|\vec{B-C}\right\|$
if $\left\|\vec{A-D}\right\| = \left\|\vec{B-C}\right\|$
then AD = BC..........(2)

$\angle CBQ = \arccos \frac{\vec{m5}^T \vec{m2}}{\|\vec{m5}\|\|\vec{m2}\|}$
$\angle ADP = \arccos \frac{\vec{m4}^T \vec{m6}}{\|\vec{m4}\|\|\vec{m6}\|}$
if $\angle CBQ = \angle ADP$ ....(3)
From (1),(2) and (3) $\Delta APD \cong \Delta CQB$

ii.TO prove:AP=CQ
Distance between A and P is $\left\|\vec{A-P}\right\|$
Distance between C and Q is $\left\|\vec{C-Q}\right\|$
if $\left\|\vec{A-P}\right\| = \left\|\vec{C-Q}\right\|$
then AP = CQ..........(4)

(iii)To prove:$\Delta AQB \cong \Delta CPD$
Distance between D and P is $\left\|\vec{D-P}\right\|$
Distance between B and Q is $\left\|\vec{B-Q}\right\|$
if $\left\|\vec{D-P}\right\| = \left\|\vec{B-Q}\right\|$

then DP = BQ..........(5)

Distance between A and B is $\left\|\vec{A-B}\right\|$

Distance between C and D is $\left\|\vec{C-D}\right\|$

if $\left\|\vec{A-B}\right\| = \left\|\vec{C-D}\right\|$
then AB = CD..........(6)
$\angle ABQ = \arccos \frac{\vec{m5}^T \vec{m1}}{\|\vec{m5}\|\|\vec{m1}\|}$
$\angle CDP = \arccos \frac{\vec{m6}^T \vec{m3}}{\|\vec{m6}\|\|\vec{m3}\|}$
if $\angle ABQ = \angle CDP$ ....(7)
Then, from (5),(6) and (7) $\Delta AQB \cong \Delta CPD$

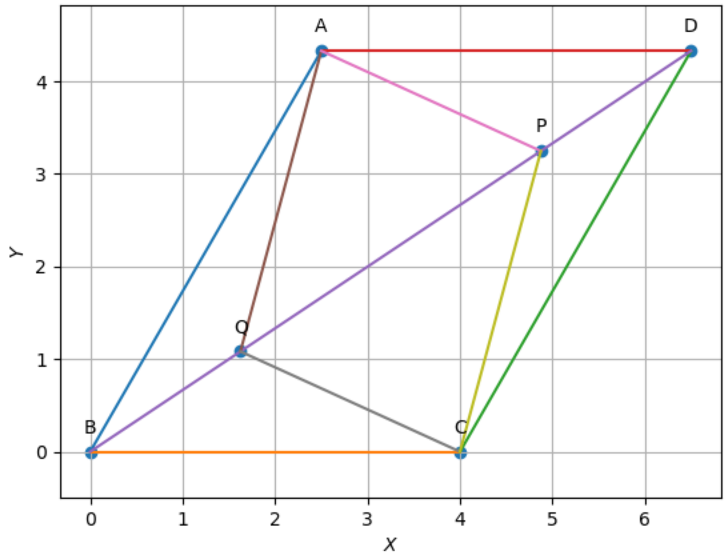
iV.TO prove:AQ=CP
Distance between A and Q is $\left\|\vec{A-Q}\right\|$
Distance between C and P is $\left\|\vec{C-P}\right\|$
if $\left\|\vec{A-Q}\right\| = \left\|\vec{C-P}\right\|$
then AQ = CP..........(8)

V.APCQ is a parallelogram
From (4) and (8) ;AP=CQ and AQ=CP Therefore, each pair Opposite sides are equal in quadrilateral APCQ
$\angle AQC = \arccos \frac{\vec{n2}^T \vec{n3}}{\|\vec{n2}\|\|\vec{n3}\|}$
$\angle APC = \arccos \frac{\vec{n1}^T \vec{n4}}{\|\vec{n1}\|\|\vec{n4}\|}$
$\angle QAP = \arccos \frac{\vec{n1}^T \vec{n3}}{\|\vec{n1}\|\|\vec{n3}\|}$
$\angle QCP = \arccos \frac{\vec{n2}^T \vec{n4}}{\|\vec{n2}\|\|\vec{n4}\|}$

If $\angle AQC = \angle APC$ and
$\angle QAP = \angle QCP$

Then, each pair of Opposite angles are equal in quadrilateral APCQ
Theorem:
If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram. (0r) If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.
∴ Quadrilateral APCQ is a parallelogram.

3 Construction



4 Execution

*Verify the above proofs in the following code.

https://github.com/pavan170850/Fwciith2022/blob/main/Matrix_Lines/codes/para.py