Limits, Continuity & Differentiability

G V V Sharma

1) Let $f(x)$=$\begin{cases} (x-1)^2 \sin \frac{1}{(x-1)} - |x|, & \text{if } x \neq 1. \\ -1, & \text{if x=1.} \end{cases}$

   be real-valued function. Then find the set of points where $f(x)$ is not differentiable ?
2) Let
$$f(x) = \begin{cases} \frac{(x^3+x^2-16x+20)}{(x-2)^2}, & \text{if } x \neq 2 \\ k, & \text{if x=2} \end{cases}$$
   If $f(x)$ is continuous for all x, then find k ?
3) A discontinuous function $y = f(x)$ satisfying $x^2 + y^2 = 4$ is given by $f(x) =$ ....
4) $\lim_{x \to 1} (1-x) \tan \frac{\pi x}{2}$= ....
5) If f(x)=$\begin{cases} \sin x, & x \neq n\pi, n = 0, \pm 1, \pm 2, \pm 3, ... \\ 2, & \text{otherwise} \end{cases}$

   and g(x)=$\begin{cases} x^2+1, & x \neq 0, 2 \\ 4, & x = 0 \\ 5, & x = 2 \end{cases}$ then

   $\lim_{x \to 0} g\left[f(x)\right]$ is ....
6) $\lim_{x \to -\infty} \left[\frac{x^4 \sin\left(\frac{1}{x}\right) + x^2}{(1+|x|^3)} \right]$ = ...
7) If $f(9) = 9, f'(9) = 4$, then $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ equals ....
8) ABC is an isosceles triangle inscribed in a circle of radius r. If AB = AC and h is the altitude from A to BC then the triangle ABC has perimeter P = $\left(2\left(\sqrt{2hr-h^2}\right) + \sqrt{2hr}\right)$ and area A= .... also $\lim_{h \to 0} \frac{A}{P^3}$= ....
9) $\lim_{x \to \infty} \left(\frac{x+6}{x+1}\right)^{x+4}$ = ...
10) Let $f(x) = x|x|$. The set of points where $f(x)$ is twice differentiable is ....
11) Let $f(x) = [x] \sin\left(\frac{\pi}{[x+1]}\right)$, where

    $[\bullet]$ denotes the greatest integer function. The domain of f is .... and the points of discontinuity of f in the domain are ....
12) $\lim_{x \to 0} \left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$ = ...
13) Let $f(x)$ be a continuous function defined for $1 \leq x \leq 3$. If $f(x)$ takes rational values for all x and $f(2)$ = 10, then $f(1.5)$ = ...
14) If $\lim_{x \to a} [f(x)g(x)]$ exists then both $\lim_{x \to a} f(x)$

    and $\lim_{x \to a} g(x)$ exist. (True / False)
15) If $f(x) = \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$, then $\lim_{x \to \infty} f(x)$ is
    - (a) 0
    - (b) ∞
    - (c) 1
    - (d) none of these
16) For a real number y, let $[y]$ denotes the greatest integer less than or equal to y : Then

    the function $f(x)$= $\frac{\tan(\pi[x-\pi])}{1+[x]^2}$ is
    - (a) discontinuous at some x
    - (b) continuous at all x, but the derivative $f'(x)$ does not exist for some x
    - (c) $f'(x)$ exists for all x, but the second derivative $f''(x)$ does not exist for some x
    - (d) $f'(x)$ exists for all x
17) There exists a function $f(x)$, satisfying $f(0) = 1$, $f'(0) = -1$, $f(x) > 0$ for all x, and

(a) $f''(x) > 0$ for all x

(b) $-1 < f''(x) < 0$ for all x

(c) $-2 \leq f''(x) \leq -1$ for all x

(d) $f''(x) < -2$ for all x

18) If G(x)= $-\sqrt{25 - x^2}$ then $\lim\limits_{x\to 1} \dfrac{G(x) - G(1)}{x - 1}$ has the value

(a) $\dfrac{1}{24}$ (b) $\dfrac{1}{5}$

(c) $-\sqrt{24}$ (d) none of these

19) If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2$, then the value of $\lim\limits_{x\to a} \dfrac{g(x)f(a) - g(a)f(x)}{x - a}$ is

(a) -5 (b) $\dfrac{1}{5}$

(c) 5 (d) none of these

20) The function

$$f(x) = \frac{ln(1 + ax) - ln(1 - bx)}{x}$$

is not defined at $x = 0$. The value which should be assigned to f at $x = 0$ so that it is continuous at $x = 0$, is

(a) $a - b$ (b) $a + b$

(c) $lna - lnb$ (d) none of these

21) $\lim\limits_{n\to\infty} \left\{ \dfrac{1}{1 - n^2} + \dfrac{2}{1 - n^2} + ... + \dfrac{n}{1 - n^2} \right\}$ is equal to

(a) 0 (b) $-\dfrac{1}{2}$

(c) $\dfrac{1}{2}$ (d) none of these

22) If $f(x) = \begin{cases} =\dfrac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$

Where [x] denotes the greatest integer less than or equal to x, then $\lim\limits_{x\to 0} f(x)$ equals

(a) 1 (b) 0

(c) -1 (d) none of these

23) Let $f : R \to R$ be differentiable function and $f(1) = 4$. Then the value of $\lim\limits_{x\to 1} \int\limits_{4}^{f(x)} \dfrac{2t}{x - 1} dt$ is

(a) $8f'(1)$ (b) $4f'(1)$

(c) $2f'(1)$ (d) $f'(1)$

24) Let [•] denote the greatest integer function and $f(x) = [\tan^2 x]$, then

(a) $\lim\limits_{x\to 0} f(x)$ does not exist

(b) $f(x)$ is continuous at $x = 0$

(c) $f(x)$ is not differentiable at $x = 0$

(d) $f'(0) = 1$

25) The function $f(x) = [x] \cos\left(\dfrac{2x - 1}{2}\right)\pi$, [•] denotes the greatest integer function, is discontinuous at

(a) All x

(b) All integer points

(c) No x

(d) x which is not an integer

26) $\lim\limits_{n\to\infty} \dfrac{1}{n} \sum\limits_{r=1}^{2n} \dfrac{r}{\sqrt{n^2 + r^2}}$ equals

(a) $1 + \sqrt{5}$ (b) $-1 + \sqrt{5}$

(c) $-1 + \sqrt{2}$ (d) $1 + \sqrt{2}$

27) The function $f(x) = [x^2] - [x^2]$ (where $[y]$ is the greatest integer less than or equal to y), is discontinuous at

(a) all integers

(b) all integers except 0 and 1

(c) all integers except 0
(d) all integers except 1

28) The function $f(x) = (x^2-1)\,|x^2-3x+2| + \cos(|x|)$ is NOT differentiable at

(a) -1 (b) 0 (c) 1 (d) 2

29) $\lim_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ is

(a) 2 (b) -2 (c) 1/2 (d) -1/2

30) For x ∈ R, $\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ =

(a) e (b) e^{-1} (c) e^{-5} (d) e^5

31) $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ equals

(a) $-\pi$ (b) π (c) $\pi/2$ (d) 1

32) The left-hand derivative of $f(x)$=$[x]sin(\pi x)$ at $x = k$, k and integer, is

(a) $(-1)^k(k-1)\pi$ (b) $(-1)^{k-1}(k-1)\pi$

(c) $(-1)^k k\pi$ (d) $(-1)^{k-1}k\pi$

33) Let $f : R \to R$ be a function defined by $f(x) = max\{x, x^3\}$. The set fo all points where $f(x)$ is NOT differentiable is

(a) {-1,1} (b) {-1,0}

(c) {0,1} (d) {-1,0,1}

34) Which of the following functions is differentiable at x = 0 ?

(a) $\cos(|x|) + |x|$ (b) $\cos(|x|) - |x|$

(c) $\sin(|x|) + |x|$ (d) $\sin(|x|) - |x|$

35) The domain of the derivative of the function $f(x) = \begin{cases} \tan^{-1} x & \text{if } |x| \le 1 \\ \frac{1}{2}(|x|-1) & |x| > 1 \end{cases}$ is

(a) $R - \{0\}$ (b) $R - \{-1\}$

(c) $R - \{1\}$ (d) $R - \{-1, 1\}$

36) The integer for which

$\lim_{x\to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite

non-zero number is

(a) 1 (b) 2 (c) 3 (d) 4

37) Let $f : R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ eqauls

(a) 1 (b) $e^{1/2}$ (c) e^2 (d) e^3

38) If $\lim_{x\to 0} \frac{((a-n)nx - \tan x)\sin nx}{x^2} = 0$, where n is nonzero real number, then a is equal to

(a) 0 (b) $\frac{n+1}{n}$

(c) n (d) $n + \frac{1}{n}$

39) $\lim_{h\to 0} \frac{f(2h+2+h^2) - f(2)}{f(h-h^2+1) - f(1)}$, given that $f'(2) = 6$ and $f'(1) = 4$

(a) does not exist (b) is equal to -3/2
(c) is equal to 3/2 (d) is equal to 3

40) If (x) is differentiable and strictly increasing function, then the value of

$\lim_{x\to 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$ is

(a) 1 (b) 0 (c) -1 (d) 2

41) The function given by $y = ||x| - 1|$ is differentiable for all real numbers except the points

(a) {0,1,-1} (b) ±1
(c) 1 (d) -1

42) If $f(x)$ is continuous and differentiable function and $f(1/n) = 0 \forall n \ge 1$ and $n \in$ I, then

(a) $f(x) = 0, x \in (0,1]$
(b) $f(0) = 0, f'(0) = 0$
(c) $f(0) = 0 = f'(0), x \in (0,1]$
(d) $f(0) = 0$ and $f'(0)$ need not to be zero

43) The value of $\lim x \to 0 \left((\sin x)^{1/x} + (1+x)^{\sin x}\right)$, where $x > 0$ is

(a) 0 (b) -1 (c) 1 (d) 2

44) Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and

$$\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1 \text{ for each } x > 0.$$

Then $f(x)$ is

(a) $\frac{1}{3x} + \frac{2x^2}{3}$ (b) $\frac{-1}{3x} + \frac{4x^2}{3}$
(c) $\frac{-1}{x} + \frac{2}{x^2}$ (d) $\frac{1}{x}$

45) $\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec x^2} f(t)dt}{x^2 - \frac{\pi^2}{16}}$ equals

(a) $\frac{8}{\pi} f(2)$ (b) $\frac{2}{\pi} f(2)$
(c) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$ (d) $4f(2)$

46) Let $g(x) = \frac{(x-1)^n}{\log \cos^m (x-1)}$; $0 < x < 2$, m and n are integers, $m \neq 0, n > 0$, let p be the left hand derivative of $|x - 1|$ at x = 1. If $\lim_{x \to 1} g(x) = p$, then

(a) $n = 1, m = 1$ (b) $n = 1, m = 1$
(c) $n = 2, m = 2$ (d) $n > 2, m = n$

47) If $\lim_{x \to 0} [1 + xln(1 + b^2)]^{1/x} = 2bsin^2\theta, b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is

(a) $\pm\frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $\pm\frac{\pi}{6}$ (d) $\pm\frac{\pi}{2}$

48) If $\lim_{x \to \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b\right) = 4$, then

(a) $a = 1, b = 4$ (b) $a = 1, b = -4$
(c) $a = 2, b = -3$ (d) $a = 2, b = 3$

49) Let $f(x) = \begin{cases} x^2 \left|\cos \frac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in R$ then f is

(a) differentiable both at x = 0 and at x = 2
(b) differentiable at x= 0 but not differentiable at x = 2
(c) not differentiable at x = 0 but differentiable at x = 2
(d) differentiable neither at x = 0 nor at x = 2

50) Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $\left(\sqrt[3]{1+a} - 1\right) x^2 + \left(\sqrt{1+a} - 1\right) x + \left(\sqrt[6]{1+a} - 1\right) = 0$ where a>-1. Then $\lim_{a \to 0^+} \alpha(a)$ and $\lim_{x \to 0^+} \beta(a)$ are

(a) $-\frac{5}{2}$ and 1 (b) $-\frac{1}{2}$ and -1
(c) $-\frac{7}{2}$ and 2 (d) $-\frac{9}{2}$ and 3

51) If x+$|y|$ = 2y, then y as a function of x is

(a) defined for all real x
(b) continuous at x = 0
(c) differentiable for all x
(d) such that $\frac{dy}{dx} = \frac{1}{3}$ for $x < 0$

52) If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then

(a) f(x) is continuous but not differentiable at x = 0
(b) f(x) is differentiable at x = 0
(c) f(x) is not differentiable at x = 0
(d) none of these

53) The function $f(x) = 1 + |\sin x|$ is

(a) continuous nowhere
(b) continuous everywhere
(c) differentiable nowhere
(d) not differentiable at $x = 0$
(e) not differentiable at infinite number of points

54) Let [x] denote the greatest integer less than or equal to x. If $f(x)=[x\sin\pi x]$, then $f(x)$ is
   (a) continuous at $x=0$
   (b) continuous in (-1,0)
   (c) differentiable at $x=1$
   (d) differentiable in (-1,1)
   (e) none of these

55) The set of all points where the function $f(x)=\dfrac{x}{(1+|x|)}$ is differentiable, is
   (a) $(-\infty,\infty)$ (b) $[0,\infty)$
   (c) $(-\infty,0)\cup(0,\infty)$ (d) $(0,\infty)$
   (e) None

56) The function $f(x)=\begin{cases}|x-3|, & x\ge 1\\ \dfrac{x^2}{4}-\dfrac{3x}{2}+\dfrac{13}{4}, & x<1\end{cases}$ is
   (a) continuous at $x=1$
   (b) differentiable at $x=1$
   (c) continuous at $x=3$
   (d) differentiable at $x=3$

57) If $f(x)=\dfrac{1}{2}x-1$, then on the interval $[0,\pi]$
   (a) $\tan[f(x)]$ and $1/f(x)$ are both continuous
   (b) $\tan[f(x)]$ and $1/f(x)$ are both discontinuous
   (c) $\tan[f(x)]$ and $f^{-1}(x)$ are both continuous
   (d) $\tan[f(x)]$ is continuous but $1/f(x)$ is not

58) The value of $\lim\limits_{x\to 0}\dfrac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$
   (a) 1 (b) -1
   (c) 0 (d) none of these

59) The following functions are continuous on $(0,\pi)$
   (a) $\tan x$
   (b) $\int\limits_0^x t\sin\dfrac{1}{t}dt$
   (c) $\begin{cases}1, & 0<x\le\dfrac{3\pi}{4}\\ 2\sin\dfrac{2}{9}x, & \dfrac{3\pi}{4}<x<\pi\end{cases}$
   (d) $\begin{cases}x\sin x, & 0<x\le\dfrac{\pi}{2}\\ \dfrac{\pi}{2}\sin(\pi+x), & \dfrac{\pi}{2}<x<\pi\end{cases}$

60) Let $f(x)=\begin{cases}0, & x<0\\ x^2, & x\ge 0\end{cases}$ then for all x
   (a) f' is differentiable
   (b) f is differentiable
   (c) f' is continuous
   (d) f is continuous

61) Let $g(x)=xf(x)$, where
   $f(x)=\begin{cases}x\sin\dfrac{1}{x}, & x\ne 0\\ 0, & x=0\end{cases}$. At $x=0$
   (a) g is differentiable but g' is not contionuous
   (b) g is differentiable while f is not
   (c) both f and g are differentiable
   (d) g is differentiable and g' is continuous

62) The function $f(x)$=max$\{(1-x),(1+x),2\}$, $x\in(-\infty,\infty)$ is
   (a) continuous at all points
   (b) diffrentiable at all points
   (c) diffrentiable at all points except at $x=1$ and $x=-1$
   (d) continuous at all points except at $x=1$ and $x=-1$, where it is discontinuous

63) Let $h(x)$=min$\{x,x^2\}$, for every real number of x, then
   (a) h is continuous for all x
   (b) h is differentiable for all x
   (c) $h'(x)=1$, for all $x>1$
   (d) h is differentiable at two values of x

64) $\lim\limits_{x\to 1}\dfrac{\sqrt{1-\cos 2(x-1)}}{x-1}$
   (a) exists and it equals $\sqrt{2}$
   (b) exists and it equals $-\sqrt{2}$
   (c) does not exist because $x-1\to 0$
   (d) does not exist because the left hand limit is not equal to the right hand limit

65) If $f(x)$=min$\{1, x^2, x^3\}$, then
(a) $f(x)$ is continuous $\forall x \in R$
(b) $f(x)$ is continuous and differentiable everywhere
(c) $f(x)$ is not differentiable at two points
(d) $f(x)$ is not differentiable at one point

66) Let $L = \lim_{x\to 0} \dfrac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0$. If l is finite, then
(a) $a = 2$ (b) $a = 1$
(c) $L = \dfrac{1}{64}$ (d) $L = \dfrac{1}{32}$

67) Let $f : R \to R$ be a function such that $f(x+y) = f(x) + f(y), \forall x, y \in R$. If $f(x)$ is differentiable at $x = 0$, then
(a) $f(x)$ is differentiable only in a finite interval containing zero
(b) $f(x)$ is continuous $\forall x \in R$
(c) $f(x)$ is constant $\forall x \in R$
(d) $f(x)$ is differentiable except at finitely many points

68) If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ lnx, & x > 1 \end{cases}$, then
(a) $f(x)$ is continuous at $x = -\dfrac{\pi}{2}$
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f(x)$ is differentiable at $x = 1$
(d) $f(x)$ is differentiable at $x = -\dfrac{3}{2}$

69) For every integer n, let a_n and b_n be real numbers. Let function $f : IR \to IR$ be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & x \in [2n, 2n+1] \\ b_n + \cos \pi x, & x \in (2n-1, 2n) \end{cases}$$

for all integers n. If f is continuous, then which of the flowing hold(s) for all n ?
(a) $a_{n-1} - b_{n-1} = 0$ (b) $a_n - b_n = 1$
(c) $a_n - b_{n+1} = 1$ (d) $a_{n-1} - b_n = -1$

70) For $a \in R$(the set of all real numbers), a≠-1,

$$\lim_{x\to\infty} \frac{(1^a + 2^a + .. + n^a)}{(n+1)^{a-1}[(na+1) + (na+2) + .. + (na+n)]}$$

$=\dfrac{1}{60}$. Then $a =$
(a) 5 (b) 7 (c) $\dfrac{-15}{2}$ (d) $\dfrac{-17}{2}$

71) Let $f : [a, b] \to [1, \infty)$ be a continuous function and let $g : R \to R$ be defined as

$$g(x) = \begin{cases} 0, & \text{if } x < a \\ \int_a^x f(t)dt, & \text{if } a \leq x \leq b \\ \int_a^b f(t)dt, & \text{if } x > b \end{cases}; \text{ then}$$

(a) $g(x)$ is continuous but not differentiable at a
(b) $g(x)$ is differentiable on R
(c) $g(x)$ is continuous but not differentiable at b
(d) $g(x)$ is continuous and differentiable at either (a) or (b) but not both

72) For every pair of continuous functions $f, g : [0, 1] \to R$ such that max$\{f(x) : x \in [0, 1]\}$=max$\{g(x) : x \in [0, 1]\}$, the correct statement(s) is(are)
(a) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
(b) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
(c) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$
(d) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$

73) Let $g : R \to R$ be a differentiable function with $g(0) = 0, g'(0) = 0$ and $g'(1) \neq 0$. Let

$$f(x) = \begin{cases} \frac{x}{|x|} g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and $h(x) = e^{|x|}$ for all $x \in R$. Let $(foh)(x)$ denote $f(h(x))$ and $(hof)(x)$ denote $h(f(x))$. Then which of the following is(are) true ?
(a) f is differentiable at $x = 0$
(b) h is differentiable at $x = 0$
(c) foh is differentiable at $x = 0$
(d) hof is differentiable at $x = 0$

74) Let $a, b \in \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = a\cos(|x^3 - x|) + b|x|\sin(|x^3 + x|)$. Then f is
   (a) differentiable at $x = 0$ if $a = 0$ and $b = 1$
   (b) differentiable at $x = 1$ if $a = 1$ and $b = 0$
   (c) NOT differentiable at $x = 0$ if $a = 1$ and $b = 0$
   (d) NOT differentiable at $x = 1$ if $a = 1$ and $b = 1$

75) Let $f : \left[-\frac{1}{2}, 2\right] \to \mathbb{R}$ and $g : \left[-\frac{1}{2}, 2\right] \to \mathbb{R}$ be functions defined by $f(x) = [x^2 - 3]$ and $g(x) = |x|f(x) + |4x - 7|f(x)$, where [y] denotes the greatest integer less than or equal to y for y $\in R$. Then
   (a) f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$
   (b) f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$
   (c) g is NOT differentiable excatly at four points in $\left(-\frac{1}{2}, 2\right)$
   (d) g is NOT differentiable exactly at five points in $\left(-\frac{1}{2}, 2\right)$

76) Let [x] be the greatest integer less than or equal to x. Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x+[x]))$ is discontinuous ?

   (a) $x = -1$ (b) $x = 0$
   (c) $x = 1$ (d) $x = 2$

77) Let $f(x) = \frac{1 - x(1 + |1 - x|)}{|1 - x|}\cos\left(\frac{1}{1 - x}\right)$ for $x \neq 1$. Then
   (a) $\lim_{x\to 1^-} f(x) = 0$
   (b) $\lim_{x\to 1^-} f(x)$ does not exist
   (c) $\lim_{x\to 1^+} f(x) = 0$
   (d) $\lim_{x\to 1^+} f(x)$ does not exist

78) Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two non-constant differentiable functions. If $f'(x) = \left(e^{(f(x)-g(x))}\right) g'(x)$ for all $x \in \mathbb{R}$, and $f(1) = g(2) = 1$, then which of the following statement(s) is(are) TRUE ?

   (a) $f(2) < 1 - log_e 2$ (b) $f(2) > 1 - log_e 2$

   (c) $g(1) > 1 - log_e 2$ (d) $g(1) < 1 - log_e 2$

79) Let $f : R \to R$ given by
$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0 \\ x^2 - x + 1, & 0 \leq x < 1 \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \leq x < 3 \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \geq 3 \end{cases}$$
then which of the following options is/are correct ?
   (a) f' has a local maximum at $x = 1$
   (b) f is increasing on $(-\infty, 0)$
   (c) f' is NOT differentiable at $x = 1$
   (d) f is onto

80) Let $f : R \to R$ be a function. We say that f has
PROPERTY 1 if $\lim_{h\to 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$ exists and is finite, and
PROPERTY 2 if $\lim_{h\to 0} \frac{f(h) - f(0)}{h^2}$ exists and is finite

   (a) $f(x) = x^{2/3}$ has **PROPERTY 1**
   (b) $f(x) = \sin x$ has **PROPERTY 2**
   (c) $f(x) = |x|$ has **PROPERTY 1**
   (d) $f(x) = x|x|$ has **PROPERTY 2**

81) Evaluate $\lim_{x\to a} \frac{\sqrt{a + 2x} - \sqrt{3x}}{\sqrt{3a + x} - 2\sqrt{x}}$, $(a \neq 0)$

82) $f(x)$ is the integral of $\frac{2\sin x - \sin 2x}{x^3}, x \neq 0$, find $\lim_{x\to 0} f'(x)$.

83) Evaluate $\lim_{h\to 0} \frac{(a + h)^2\sin(a + h) - a^2\sin a}{h}$

84) Let $f(x + y) = f(x) + f(y)$ for all x and y. If the function $f(x)$ is continuous at $x = 0$, then show that $f(x)$ is continuous at all x.

85) Use the formula $\lim_{x\to 0}\frac{a^x-1}{x}=lna$ to find $\lim_{x\to 0}\frac{2^x-1}{(1+x)^{1/2}-1}$

86) Let $f(x)=\begin{cases}1+x, & 0\le x\le 2\\ 3-x, & 2\le x\le 3\end{cases}$

Determine the form of $g(x)=fIf(x)$ and hence find the points of discontinuity of g, if any.

87) Let $f(x)=\begin{cases}\frac{x^2}{2}, & 0\le x<1\\ 2x^2-3x+\frac{3}{2}, & 1\le x\le 2\end{cases}$

Discuss the continuity of f, f' and f'' on [0,2].

88) Let $f(x)=x^3-x^2+x+1$ and $g(x)=\begin{cases}\max\{f(t); 0\le t\le x\}, & 0\le x\le 1\\ 3-x, & 0\le x\le 2\end{cases}$

Discuss the continuity and differentiability f the function $g(x)$ in the interval (0,2).

89) Let $f(x)$ be defined in the interval [-2,2] such that $f(x)=\begin{cases}-1, & -2\le x\le 0\\ x-1, & 0<x\le 2\end{cases}$

and $g(x)=f(|x|)+|f(x)|$. Test the differentiability of $g(x)$ in (-2,2).

90) Let $f(x)$ be a continuous and $g(x)$ be a discontinuous function. Prove that $f(x)+g(x)$ is a discontinuous function.

91) Let $f(x)$ be a function satisfying the condition $f(-x)=f(x)$ for all real x. If $f'(0)$ exists, find its value.

92) Find the values of a and b so that the function

$$f(x)=\begin{cases}x+a\sqrt{2}\sin x, & 0\le x\le \pi/4\\ 2x\cot x+b, & \pi/4\le x\le \pi/2\\ a\cos 2x-b\sin x, & \pi/2<x\le \pi\end{cases}$$

is continuous for $0\le x\le \pi$.

93) Draw a graph of the function $y=[x]+|1-x|, -1\le x\le 3$. Determine the points, if any, where this function is not differentiable.

94) Let $f(x)=\begin{cases}\frac{1-\cos 4x}{x^2}, & x<0\\ a, & x=0\\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & x>0\end{cases}$

Determine the value of a, if possible, so that the function is continuous at $x=0$.

95) A function $f: R\to R$ satisfies the equation $f(x+y)=f(x)f(y)$ for all x, y in R and $f(x)\ne 0$ for any x in R. Let the function be differentiable at $x=0$ and $f'(0)=2$. Show that $f'(x)=2f(x)$ for all x in R. hence, determine $f(x)$.

96) Find $\lim_{x\to 0}\{\tan(\pi/4+x)\}^{1/x}$.

97) Let

$$f(x)=\begin{cases}\{1+|\sin x|\}^{a/|\sin x|}; & \frac{\pi}{6}<x<0\\ b: & x=0\\ e^{\tan 2x/\tan 3x}; & 0<x<\frac{\pi}{6}\end{cases}$$

Determine a and b such that $f(x)$ is continuous at $x=0$.

98) Let $f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}$ for all real x and y. If $f'(0)$ exists and equal -1 and $f(0)=1$, find $f(2)$.

99) Determine the values of x for which the following function fails to be continuous or differentiable :

$$f(x)=\begin{cases}1-x, & x<1\\ (1-x)(2-x), & 1\le x\le 2\\ 3-x, & x>2\end{cases}$$

Justify your answer.

100) Let $f(x), x\ge 0$, be non-negative continuous function, and let $F(x)=\int_0^x f(t)dt, x\ge 0$. If for some $c>0, f(x)\le cF(x)$ for all $x\ge 0$, then show that $f(x)=0$ for all $x\ge 0$.

101) Let $\alpha \in R$. Prove that a function $f : R \to R$ is differentiable at α if and only if there is a function $g : R \to R$ which is continuous at α and satisfies $f(x) - f(\alpha) = g(x)(x - \alpha)$ for all $x \in R$.

102) Let $f(x) = \begin{cases} x+1, & \text{if } x < 0 \\ |x-1|, & \text{if } x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x+1, & \text{if } x < 0 \\ (x-1)^2 + b, & \text{if } x \geq 0 \end{cases}$ where a and b are non-negative numbers. Determine the composite function gof. If $(gof)(x)$ is continuous for all real x, determine the values of a and b. Further, for these values of a and b, is gof differentiable at $x = 0$? Justify your answer.

103) If a function $f : [-2a, 2a] \to R$ is an odd function such that $f(x) = f(2a - x)$ for $x \in [a, 2a]$ and the left hand derivative at $x = a$ is 0 then find the left hand derivative at $x = -a$.

104) $f'(0) = \lim_{n\to\infty} nf\left(\frac{1}{n}\right)$ and $f(0) = 0$. Using this find

$$\lim_{n\to\infty}\left((n+1)\frac{2}{\pi}\cos^{-1}\left(\frac{1}{n}\right) - n\right),$$

$$\left|cos^{-1}\frac{1}{n}\right| < \frac{\pi}{2}$$

105) if $|c| \leq \frac{1}{2}$ and $f(x)$ is a differentiable function at $x = 0$ given by

$$f(x) = \begin{cases} b\sin^{-1}\left(\frac{c+x}{2}\right), & -\frac{1}{2} < x < 0 \\ \frac{1}{2}, & x = 0 \\ \frac{e^{ax/2} - 1}{x}, & 0 < x < \frac{1}{2} \end{cases}$$

Find the value of 'a' and prove that $64b^2 = 4 - c^2$.

106) If $f(x - y) = f(x)\dot{}g(y) - f(y)\dot{}g(x)$ and $g(x - y) = g(x)\dot{}g(y) - f(x)\dot{}f(y)$ for all $x, y \in R$. If right hand derivative at $x = 0$ exists for $f(x)$. Find derivative of $g(x)$ at $x = 0$.

107) Let $f : [1, \infty) \to [2, \infty)$ be a differentiable functions such that $f(1) = 2$. If $6\int_1^x f(t)dt = 3xf(x) - x^3$ for all $x \geq 14$, then the value of $f(2)$ is ?

108) The largest value of non-negative integer a for which

$$\lim_{x\to1}\left\{\frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

is

109) Let $f : R \to R$ and $g : R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : R \to R$ by

$$h(x) = \begin{cases} max\{f(x).g(x)\}, & \text{if } x \leq 0 \\ min\{f(x).g(x)\}, & \text{if } x > 0 \end{cases}$$

The number of points at which $h(x)$ is not differentiable is

110) Let m and n be two positive integers greater than 1. If $\lim_{\alpha\to0}\left(\frac{e^{\cos(\alpha^n)} - e}{\alpha^m}\right) = -\left(\frac{e}{2}\right)$ then the value of $\frac{m}{n}$ is

111) Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim_{x\to0}\frac{x^2\sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals

112) $\lim_{x\to0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is

(a) 1 (b) -1
(c) zero (d) does not exist

113) $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^x$

(a) e^4 (b) e^2 (c) e^3 (d) 1

114) Let $f(x) = 4$ and $f'(x) = 4$. Then

$$\lim_{x\to2}\frac{xf(2) - 2f(x)}{x-2}$$ is given by

(a) 2 (b) -2 (c) -4 (d) 3

115) $\lim_{n\to\infty}\frac{1^p + 2^p + 3^p + ... + n^p}{n^{p+1}}$

(a) $\frac{1}{p+1}$ (b) $\frac{1}{p}-\frac{1}{p-1}$
(c) $\frac{1}{1-p}$ (d) $\frac{1}{p+2}$

116) $\lim\limits_{x\to 0}\frac{\log x^n-[x]}{[x]}, n\in N$, ([x] denotes greatest integer less than or equal to x)

(a) has value -1 (b) has value 0
(c) has value 1 (d) does not exist

117) If $f(1)=1, f'(1)=2$, then $\lim\limits_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is

(a) 2 (b) 4 (c) 1 (d) 1/2

118) f is defined in [-5,5] as
$f(x)=\begin{cases} x, & \text{if } x \text{ is rational} \\ -x, & \text{if } x \text{ is irrational}\end{cases}$. Then
(a) $f(x)$ is continuous at every x, except $x=0$
(b) $f(x)$ is discontinuous at every x, except $x=0$
(c) $f(x)$ is continuous everywhere
(d) $f(x)$ is discontinuous everywhere

119) $f(x)$ and $g(x)$ are two differentiable functions on [0,2] such that $f''(x)-g''(x)=0, f'(1)=2g'(1)=4f(2)=3g(2)=9$ then $f(x)-g(x)$ at x = 3/2 is

(a) 0 (b) 2 (c) 10 (d) 5

120) If $(x+y)=f(x).f(y)\forall x,y$ and $f(5)=2$,

$f'(0)=3$, then $f'(5)$ is

(a) 0 (b) 1 (c) 6 (d) 2

121) $\lim\limits_{n\to\infty}\frac{1+2^4+3^4+...+n^4}{n^5}-\lim\limits_{n\to\infty}\frac{1+2^3+3^3+..+n^3}{n^5}$

(a) $\frac{1}{5}$ (b) $\frac{1}{30}$ (c) Zero (d) $\frac{1}{4}$

122) If $\lim\limits_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$, the value of k is

(a) $-\frac{2}{3}$ (b) 0 (c) $-\frac{1}{3}$ (d) $\frac{2}{3}$

123) The value of $\lim\limits_{x\to 0}\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}$ is

(a) 0 (b) 3 (c) 2 (d) 1

124) Let $f(a)=g(a)=k$ and their nth derivatives $f^n(a), g^n(a)$ exist and are not equal for some n. Further if

$\lim\limits_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+f(a)}{g(x)-f(x)}$=4

then the value of k is

(a) 0 (b) 4 (c) 2 (d) 1

125) $\lim\limits_{x\to\frac{\pi}{2}}\frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x]^3}$ is

(a) ∞ (b) $\frac{1}{8}$ (c) 0 (d) $\frac{1}{32}$

126) If $f(x)=\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x\neq 0 \\ 0, & x=0\end{cases}$ then $f(x)$ is
(a) discontinuous everywhere
(b) continuous as well as differentiable for all x
(c) continuous for all x but not differentiable at $x=0$
(d) neither differentiable nor continuous at $x=0$

127) If $\lim\limits_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, then the values of a and b, are

(a) $a=1$ and $b=2$ (b) $a=1, b\in R$
(c) $a\in R, b=2$ (d) $a\in R, b\in R$

128) Let $f(x)=\frac{1-\tan x}{4x-\pi}, x\neq\frac{\pi}{4}, x\in\left[0,\frac{\pi}{2}\right]$.
If $f(x)$ is continuous in $\left[0,\frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

(a) -1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

129) $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}...+\frac{1}{n}\sec^2 1\right]$ equals

(a) $\frac{1}{2}\sec 1$ (b) $\frac{1}{2}cosec1$

(c) $\tan 1$ (d) $\frac{1}{2}\tan 1$

130) Let α and β be the distinct roots of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(a) $\frac{\alpha^2}{2}(\alpha-\beta)^2$ (b) 0

(c) $\frac{-\alpha^2}{2}(\alpha-\beta)^2$ (d) $\frac{1}{2}(\alpha-\beta)^2$

131) Suppose $f(x)$ is differentiable at $x=1$ and $\lim_{h\to 0}\frac{1}{h}f(1+h)=5$, then $f'(1)$ equals

(a) 3 (b) 4 (c) 5 (d) 6

132) Let f be differentiable for all x. If $f(1)=-2$ and $f'(x)\geq 2$ for $x\in[1,6]$, then

(a) $f(6)\geq 8$ (b) $f(6)<8$
(c) $f(6)<5$ (d) $f(6)=5$

133) If f is a real valued differentiable function satisfying $|f(x)-f(y)|\leq(x-y)^2, x,y\in R$ and $f(0)=0$, then $f(1)$ equals

(a) -1 (b) 0 (c) 2 (d) 1

134) Let $f : R\to R$ be a function defined by$f(x)$ =min$\{x+1,|x|+1\}$, Then which of the following is true ?

(a) $f(x)$ is differentiable everywhere
(b) $f(x)$ is not differentiable at $x=0$
(c) $f(x)\geq 1$ for all $x\in R$
(d) $f(x)$ is not differentiable at $x=1$

135) The function $f : R/\{0\}\to R$ is given by $f(x)=\frac{1}{x}-\frac{2}{e^{2x}-1}$ can be made continuous at $x=0$ by defininig $f(0)$ as

(a) 0 (b) 1 (c) 2 (d) -1

136) Let $f(x)=\begin{cases}(x-1)\sin\frac{1}{x-1}, & \text{if } x\neq 1\\ 0, & \text{if } x=1\end{cases}$

Then which of the following is true ?

(a) f is neither differentiable at $x=0$ nor at $x=1$
(b) f is differentiable at $x=0$ and at $x=1$
(c) f is differentiable at $x=0$ but not at $x=1$
(d) f is differentiable at $x=1$ but not at $x=0$

137) Let $f : R\to R$ be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$. then $\lim_{x\to\infty}\frac{f(2x)}{f(x)}=$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) 1

138) $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

(a) equals $\sqrt{2}$ (b) equals $-\sqrt{2}$
(c) equals $\frac{1}{\sqrt{2}}$ (d) does not exist

139) The values of p and q for which the function

$$f(x)=\begin{cases}\frac{\sin(p+1)x+\sin x}{x}, & x<0\\ q, & x=0\\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0\end{cases}$$

is continuous for all x in R, are

(a) $p=\frac{5}{2}, q=\frac{1}{2}$ (b) $p=\frac{1}{2}, q=\frac{3}{2}$

(c) $p=-\frac{3}{2}, q=\frac{1}{2}$ (d) $p=\frac{1}{2}, q=-\frac{3}{2}$

140) Let $f : R\to[0,\infty)$ be such that $\lim_{x\to 5}f(x)$ exists and $\lim_{x\to 5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}=0$. Then $\lim_{x\to 5}f(x)$ equals

(a) 0 (b) 1 (c) 2 (d) 3

141) If $f : R \to R$ is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is
(a) continuous for every real x
(b) discontinuous only at $x = 0$
(c) discontinuous only at non-zero integral values of x
(d) continuous only at $x = 0$

142) Consider the function, $f(x) = |x-2| + |x-5|, x \in R$.
Statement-1:$f'(4) = 0$
Statement-2:f is continuous in [2,5], differentiable in (2,5) and $f(2) = f(5)$
(a) Statement-1 is false, Statement-2 is true
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is false

143) $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to
(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

144) $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to
(a) $-\pi$ (b) π (c) $\frac{\pi}{2}$ (d) 1

145) If the function,

$$g(x) = \begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx+2, & 3 < x \le 5 \end{cases}$$ is

differentiable, then the value of k + m is

(a) $\frac{10}{3}$ (b) 4 (c) 2 (d) $\frac{16}{5}$

146) For $x \in R, f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then
(a) $g'(0) = -\cos(\log 2)$
(b) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$
(c) g is not differentiable at $x = 0$
(d) $g'(0) = \cos(\log 2)$

147) $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)....3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to
(a) $\frac{9}{e^2}$ (b) $3\log 3 - 2$
(c) $\frac{18}{e^4}$ (d) $\frac{27}{e^2}$

148) Let $p = \lim_{x\to 0^+}(1+\tan^2\sqrt{x})^{\frac{1}{2x}}$ then $\log p$ is equal to
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) 2 (d) 1

149) $\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi-2x)^3}$ equals
(a) $\frac{1}{4}$ (b) $\frac{1}{24}$ (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

150) For each $t \in R$, let [t] be the greatest integer less than or equal to t/. Then

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$$

(a) is equal to 15 (b) is equal to 120
(c) does not exist(in R) (d) is equal to 0

151) Let $S = t \in R : f(x) = |x-\pi|(e^{|x|}-1)\sin|x|$ is not differentiable at t. Then the set S is equal to

(a) $\{0\}$ (b) $\{\pi\}$
(c) $\{0,\pi\}$ (d) ϕ(an empty set)

152) $\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$
(a) exists and equals $\frac{1}{4\sqrt{2}}$
(b) exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$
(c) exists and equals $\frac{1}{2\sqrt{2}}$

(d) does not exist

153) Let $f : R \to R$ be a function defined as

$$f(x) = \begin{cases} 5, & \text{if } x \leq 1 \\ a + bx, & \text{if } 1 < x < 3 \\ b + 5x, & \text{if } 3 \leq x < 5 \\ 30, & \text{if } x \geq 5 \end{cases}$$

Then f is

(a) continuous if a = 5 and b = 5
(b) continuous if a = -5 and b = 10
(c) continuous if a = 0 and b = 5
(d) not continuous for any values of a and b

154) if the function f is defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by

$$f(x) = \begin{cases} \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \neq \dfrac{\pi}{4} \\ k, & x = \dfrac{\pi}{4} \end{cases}$$

is continuous, then k is equal to

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{\sqrt{2}}$

155) Let $f(x) = 15 - |x - 10|; x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is

(a) {5,10,15} (b) {10,15}
(c) {5,10,15,20} (d) {10}

156) In this questions there are entries in columns I and II. Each entry in column I is related to exactly one entry in column II.

Column I	**Column II**
(A) $\sin(\pi[x])$	(p) differentiable everywhere
(B) $\sin(\pi(x - [x]))$	(q) nowhere differentiable
	(r) not differentiable at 1 and -1

157) In the following [x] denotes the greatest integer less than or equal to x.Match the functions in **Column I** with the properties in **Column II**

Column I	**Column II**
(A) $x\|x\|$	(p) continuous in (-1,1)
(B) $\sqrt{\|x\|}$	(q) differentiable in (-1,1)
(C) $x + [x]$	(r) strictly increasing in (-1,1)
(D) $\|x - 1\| + \|x + 1\|$	(s) not differentiable at least at one point (-1,1)

158) Let $f_1 : R \to R, f_2 : [0, \infty) \to R, f_3 : R \to R$ and $f_4 : R \to [0, \infty)$ be defined by $f_1(x) = \begin{cases} |x|, & \text{if } x < 0 \\ e^x, & \text{if } x \geq 0 \end{cases}$; $f_2(x) = x^2$;

$f_3(x) = \begin{cases} \sin x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0; \end{cases}$ and

$$f_4(x) = \begin{cases} f_2(f_1(x)), & \text{if } x < 0 \\ f_2(f_1(x)) - 1, & \text{if } x \geq 0; \end{cases}$$

List-I	**List-II**
(P) f_4 is	1. onto but not one-one
(Q) f_3 is	2. Neither continuous nor one-one
(R) $f_2 o f_1$ is	3. differentiable but not one-one
(S) f_2 is	4. continuous and one-one

(a) P $\rightarrow$ 3; Q $\rightarrow$ 1; R $\rightarrow$ 4; S $\rightarrow$ 2
(b) P $\rightarrow$ 1; Q $\rightarrow$ 3; R $\rightarrow$ 4; S $\rightarrow$ 2
(c) P $\rightarrow$ 3; Q $\rightarrow$ 1; R $\rightarrow$ 2; S $\rightarrow$ 4
(d) P $\rightarrow$ 1; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 4

159) let $f_1 : \mathbb{R} \rightarrow \mathbb{R}, f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R}$
$f_3 : \left(-1, e^{\frac{\pi}{2}-2}\right)$ and $f_4 : \mathbb{R} \rightarrow \mathbb{R}$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right)$

(ii) $f_2(x) = \begin{cases} \frac{|\sin x|}{tan^{-1}x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$, where the inverse trignometric function

$\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iii) $f_3(x) = [\sin(\log_e(x+2))]$, where for $t \in \mathbb{R}, [t]$ denotes the greatest integer less than or equal to t

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

List-I	**List-II**
(P) The function f_1 is	1. **NOT** continuous at $x = 0$
(Q) The function f_2 is	2. Continuous at $x = 0$ and **NOT** differentiable at $x = 0$
(R) The function f_3 is	3. differentiable at $x = 0$ and its derivative is **NOT** continuous at $x = 0$
(S) The function f_4 is	4. differentiable at $x = 0$ and its derivative is continuous at $x = 0$

(a) P $\rightarrow$ 2; Q $\rightarrow$ 3; R $\rightarrow$ 1; S $\rightarrow$ 4
(b) P $\rightarrow$ 4; Q $\rightarrow$ 1; R $\rightarrow$ 2; S $\rightarrow$ 3
(c) P $\rightarrow$ 4; Q $\rightarrow$ 2; R $\rightarrow$ 1; S $\rightarrow$ 3
(d) P $\rightarrow$ 2; Q $\rightarrow$ 1; R $\rightarrow$ 4; S $\rightarrow$ 3