## **SVD-LLM:**

# Truncation-aware Singular Value Decomposition for Large Language Model Compression

Xin Wang<sup>1</sup> Yu Zheng<sup>2</sup> Zhongwei Wan<sup>1</sup> Mi Zhang<sup>1\*</sup>

The Ohio State University <sup>2</sup>Michigan State University https://github.com/AIoT-MLSys-Lab/SVD-LLM

#### **Abstract**

The advancements in Large Language Models (LLMs) have been hindered by their substantial sizes, which necessitate LLM compression methods for practical deployment. Singular Value Decomposition (SVD) offers a promising solution for LLM compression. However, state-of-the-art SVD-based LLM compression methods have two key limitations: truncating smaller singular values may lead to higher compression loss, and the lack of update on the compressed weight after SVD truncation. In this work, we propose SVD-LLM, a new SVD-based LLM compression method that addresses the limitations of existing methods. SVD-LLM incorporates a truncation-aware data whitening strategy to ensure a direct mapping between singular values and compression loss. Moreover, SVD-LLM adopts a layerwise closed-form model parameter update strategy to compensate for accuracy degradation under high compression ratios. We evaluate SVD-LLM on a total of 10 datasets and eight models from three different LLM families at four different scales. Our results demonstrate the superiority of SVD-LLM over state-of-the-arts, especially at high model compression ratios.

#### 1 Introduction

Large Language Models (LLMs) have demonstrated remarkable capabilities in a wide range of tasks such as natural language understanding and generation [31, 9]. Despite such capabilities, the democratization of LLMs is primarily restricted by their substantial resource demands [25, 26, 32]. One of the most effective techniques to reduce the resource demands of LLMs is model compression [33]. Compression techniques based on quantization [6, 15, 27], parameter pruning [16, 5], and knowledge distillation [10, 11] specifically designed for LLMs have been intensively studied. Regardless of their success, these techniques have their own constraints, such as hardware dependency and the need for expensive retraining. Compared to those techniques, compression techniques based on low-rank approximation, such as Singular Value Decomposition (SVD) are not limited by those constraints. Moreover, the KV cache of LLMs compressed via SVD at runtime can also be reduced.

Despite these advantages, the potential of SVD for LLM compression has not been thoroughly explored. A few SVD-based LLM compression methods such as FWSVD [12] and ASVD [28] have recently been proposed. However, these methods exhibit severe performance degradation when the model compression ratio<sup>2</sup> is high. Such limitation can be attributed to two fundamental issues involved in their approaches: **①** Imprecise Data Preprocessing: although the data preprocessing strategy proposed by ASVD reduces the negative impact of activation outliers, it does not establish a direct relationship between singular values and the model compression loss. As a consequence,

<sup>\*</sup>Corresponding author. Email: mizhang.1@osu.edu

<sup>&</sup>lt;sup>2</sup>The compression ratio refers to the percentage of parameter reduction achieved through compression.

truncating smaller singular values in SVD could lead to significant compression loss. **②** Lack of Model Parameter Update after SVD Truncation: as the model compression ratio increases, the number of singular values that need to be truncated in SVD increases as well. To compensate for the accuracy degradation caused by truncating a large number of singular values, it is required to update the remaining parameters in the compressed model. Unfortunately, existing SVD-based LLM compression methods do not take such update into account, and thus fail to compensate for the accuracy degradation under high model compression ratios.

In this paper, we propose a new SVD-based LLM compression method named SVD-LLM that effectively addresses the two fundamental issues of the existing methods. SVD-LLM differs from existing SVD-based LLM compression methods in two key aspects: **①** Truncation-Aware Data Whitening: supported by the theoretical proof, SVD-LLM incorporates a truncation-aware data whitening technique that ensures a *direct mapping* between singular values and model compression loss. In doing so, the proposed truncation-aware data whitening technique is able to identify which singular values should be truncated to incur minimal model compression loss. **②** Layer-Wise Closed-Form Model Parameter Update: to compensate for accuracy degradation under high compression ratios, SVD-LLM incorporates a layer-wise closed-form model parameter update strategy to progressively update the compressed weights layer by layer.

We compare SVD-LLM with three SVD-based methods for LLM compression, including vanilla SVD as well as state-of-the-art SVD-based LLM compression methods FWSVD and ASVD. To demonstrate the generability of SVD-LLM, we conduct our evaluation on a total of 10 datasets and eight models from three different LLM families (LLaMA, OPT, and Mistral) at four different scales (7B, 13B, 30B, and 65B). We highlight six of our findings: (1) SVD-LLM consistently outperforms vanilla SVD, FWSVD, and ASVD across all 10 datasets, three different LLM families, and four different scales, especially under higher compression ratios from 30% to 60%. (2) SVD-LLM also exhibits superiority over state-of-the-arts in terms of compression speed. Specifically, when compressing LLaMA-7B under 20% compression ratio on an A100 GPU, ASVD takes about 5.5 hours whereas SVD-LLM completes the compression process in 15 minutes. (3) Our ablation studies show that with one of the two key components of SVD-LLM alone, it still outperforms state-of-the-art SVD-based compression methods under different compression ratios. (4) SVD-LLM is able to benefit other LLM compression methods. Our evaluation results show that SVD-LLM is able to further enhance the compression performance of well-recognized quantization (GPTO [6]) and parameter pruning-based (LLM-Pruner [16]) LLM compression methods. (5) SVD-LLM is able to not only compress LLMs but also enhance inference efficiency on real hardware including both GPU and CPU. (6) Lastly, SVD-LLM brings additional benefit beyond compressing the sizes of LLMs, and is also able to reduce the footprint of KV cache during inference at runtime (shown in Appendix A.5).

### 2 Related Work

**Large Language Model Compression:** LLMs in general contain billion-scale parameters. Applying conventional model compression methods for LLMs is not feasible as they necessitate resourceintensive retraining. Given that, post-training methods that avoid retraining in the compression process have been developed. In general, these methods can be grouped into four categories: unstructured pruning, structured pruning, quantization, and low-rank approximation. Specifically, unstructured pruning methods set the individual weights of an LLM to zero without changing its shape. A notable contribution is SparseGPT [5] which prunes the least important weight elements with the inversion of the Hessian matrix. However, the irregular sparsification of unstructured pruning is difficult to achieve the desired speedup or memory saving and can only demonstrate its best efficiency on certain hardware architecture such as NVIDIA Ampere GPU. Unlike unstructured pruning, structured pruning methods directly remove entire channels or other structured components from LLMs, making them easier to implement on hardware. For example, LLM-Pruner [16] utilizes a small amount of data to obtain the weight, parameter, and group importance of the coupled structure for pruning with LoRA to recover precision. However, due to the great modification of the weight matrix in LLM, it suffers from a great accuracy degradation, especially under high compression ratios. Quantization methods, on the other hand, achieve model compression by reducing the precision of weight matrices of an LLM. For example, GPTO [6] uses layer-wise quantization and updates the weights with inverse Hessian information. However, quantization has the drawback of only providing a limited range of compression options, typically ranging from 3 to 8 bits. This limited range could prevent full utilization of the available memory budget.

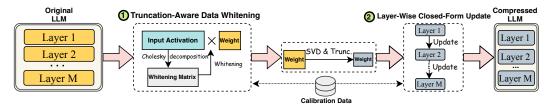


Figure 1: Overview of SVD-LLM.

**SVD for LLM Compression:** Singular Value Decomposition (SVD) is a widely used technique to reduce matrix size by approximating a matrix with two smaller low-ranking matrices [8]. In the context of LLM compression, only a few SVD-based LLM compression methods have been proposed. Specifically, vanilla SVD only focuses on the compression of the original weight matrix without considering the importance of the parameters, potentially giving a larger compression error. To address this problem, [12] propose FWSVD, which introduces Fisher information to weigh the importance of parameters. However, FWSVD requires a complex gradient calculation that demands substantial resources for LLM compression. Another problem of vanilla SVD is the distribution of activation can affect the compression error. To address this issue, [28] propose ASVD, which scales the weight matrix by a diagonal matrix that represents the impact of input channels on the weights. However, both FWSVD and ASVD do not establish a direct relationship between singular values and compression loss. As a result, truncating the smaller singular values may lead to higher compression loss. Moreover, as the compression ratio increases, it is necessary to update the compressed weights due to truncating a great number of singular values. However, existing methods have no design for this update and thus incur severe accuracy degradation under high compression ratios.

#### 3 SVD-LLM

Figure 1 provides an overview of SVD-LLM. At a high level, SVD-LLM is a SVD-based post-training LLM compression method. Specifically, following the standard procedure of post-training LLM compression methods [5, 28, 27], SVD-LLM uses a random set of sentences as calibration data to generate activation for truncation-aware data whitening and layer-wise closed-form update for model compression. SVD-LLM whitens the activation through Cholesky decomposition, and performs SVD to truncate the weight matrices to compress the LLM. Under high model compression ratios, SVD-LLM performs a layer-wise closed-form update to progressively update the remaining weights layer by layer after compression. In the following, we describe both truncation-aware data whitening and layer-wise closed-form update in detail. The pseudocode is provided in Appendix A.2.

#### 3.1 Truncation-Aware Data Whitening

**Motivation:** Due to high variance of the input activation, simply applying vanilla SVD for LLM compression leads to severe accuracy degradation [28]. To address this issue, ASVD [28] formulates LLM compression as an optimization problem with the following optimization objective:

$$O = \min(||WX - W'X||_F) \tag{1}$$

where W is the weight matrix of the original LLM, X is the activation of W given an input, W' is the compressed weight matrix, and  $L = ||WX - W'X||_F$  is the compression loss in the form of Frobenius loss.

Specifically, ASVD extracts a diagonal matrix  $S_0$  from X where each element in the diagonal is the absolute mean value of each channel. It then uses  $S_0$  to normalize X and converts WX into  $(WS_0)(S_0^{-1}X)$ . Subsequently, SVD is performed on  $WS_0$  to obtain the decomposed matrices  $U_0$ ,  $\Sigma_0$ , and  $V_0$ . Lastly, ASVD truncates the smallest singular values in  $\Sigma_0$  to obtain the compressed weight matrix  $W_0' = U_0 \times \operatorname{Trunc.}(\Sigma_0) \times V_0 \times S_0^{-1}$ .

Although normalizing the activation improves the performance, ASVD does not establish a direct relationship between singular values and compression loss (a detailed proof is included in Appendix A.1). To better illustrate this point, we show two concrete examples in Figure 2(a). In the first example  $\bullet$  where only one singular value is truncated, truncating the smallest singular value 0.1 results in a higher compression loss (loss = 1.1) than truncating the second smallest singular value 0.9 (loss = 0.7). In the second example  $\bullet$  where multiple singular values are truncated, truncating the smallest

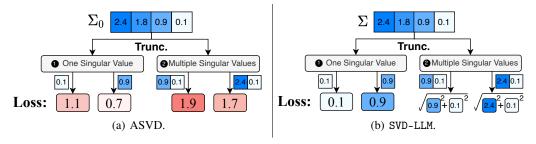


Figure 2: Comparison of data whitening methods between ASVD and SVD-LLM.

two singular values 0.9 and 0.1 also leads to a higher loss (loss = 1.9) than truncating 2.4 and 0.1 (loss = 1.7). Hence, truncating the smallest singular values does not lead to minimal loss.

Key Design: The key idea of SVD-LLM is to incorporate a truncation-aware data whitening technique that ensures a direct mapping between singular values and compression loss. To achieve this, SVD-LLM enforces the whitened activation  $S^{-1}X$  to be orthogonal such that each channel is independent of each other, i.e.,  $(S^{-1}X)(S^{-1}X)^T = S^{-1}XX^T(S^{-1})^T = I$ , where S is derived through Cholesky decomposition [19]. Then we perform SVD on WS to obtain the decomposed matrices  $U, \Sigma, V$ , where  $U = [u_1, u_2, u_3, ..., u_r]$ ,  $\Sigma = \operatorname{diag}(\sigma_1, \sigma_2, \sigma_3, \cdots, \sigma_r)$ , and  $V = [v_1, v_2, v_3, ..., v_r]$ . Lastly, the smallest singular values in  $\Sigma$  are truncated to obtain the compressed weight matrix  $W' = U \times \operatorname{Trunc}(\Sigma) \times V^T \times S^{-1}$ .

Figure 2(b) illustrates the effect of the proposed truncation-aware data whitening technique. In the first example **①** where only one singular value is truncated, the compression loss is equal to the truncated singular value. In the second example **②**, the compression loss of truncating multiple singular values is equal to the square root of the sum of their squares. As such, under the proposed truncation-aware data whitening technique, truncating the smallest singular values leads to minimal compression loss.

Below, we provide a theoretical proof on why the proposed truncation-aware data whitening technique ensures a direct mapping between singular values and compression loss in the case of one singular value (Theorem 3.2) and multiple singular values (Corollary 3.3).

**Lemma 3.1.** The Frobenius norm of matrix A with dimension  $m \times n$  can be deduced into the square root of the trace of its gram matrix, which is:

$$||A||_F \triangleq \left(\sum_{j=1}^n \sum_{i=1}^m |a_{ij}|^2\right)^{\frac{1}{2}} = \left[\operatorname{trace}\left(A^T A\right)\right]^{\frac{1}{2}}$$
 (2)

Using Lemma 3.1, we obtain the compression loss  $L_i$  when truncating the  $i^{th}$  singular value of  $S^{-1}X$  to reduce its rank for compression:

$$L_{i} = ||(W - W')X||_{F} = ||\sigma_{i}u_{i}v_{i}^{T}S^{-1}X||_{F} = \sigma_{i} \operatorname{trace}(u_{i}v_{i}^{T}S^{-1}XX^{T}(S^{-1})^{T}v_{i}u_{i}^{T})^{\frac{1}{2}}$$
(3)

Since both  $U = [u_1, u_2, u_3, ..., u_r]$  and  $V = [v_1, v_2, v_3, ..., v_r]$  are orthogonal matrices, we have:

$$v_i^T v_i = u_i^T u_i = I; v_i^T v_j = u_i^T u_j = 0, \forall i \neq j; trace(v_i v_i^T) = trace(u_i u_i^T) = 1$$
 (4)

**Theorem 3.2.** If S is the Cholesky decomposition of  $XX^T$ , the compression loss  $L_i$  equals to  $\sigma_i$ . Proof. Since the whitening matrix S is the Cholesky decomposition of  $XX^T$ , we have  $SS^T = XX^T$ . We can further infer Equation (3) to obtain:

$$L_{i} = ||\sigma_{i}u_{i}v_{i}^{T}S^{-1}X||_{F} = \sigma_{i} \operatorname{trace}(u_{i}v_{i}^{T}S^{-1}XX^{T}(S^{-1})^{T}v_{i}u_{i}^{T})^{\frac{1}{2}} = \sigma_{i} \operatorname{trace}(u_{i}v_{i}^{T}v_{i}u_{i}^{T})^{\frac{1}{2}} = \sigma_{i}$$
(5)
Therefore,  $L_{i}$  of truncating  $\sigma_{i}$  equals to the singular value  $\sigma_{i}$  itself.

**Corollary 3.3.** If S is the Cholesky decomposition of  $XX^T$ , truncating the smallest singular values leads to the lowest loss L compared to truncating others.

*Proof.* If we truncate  $\sigma_{m+1}, \sigma_{m+2}, \sigma_{m+3}, ..., \sigma_r$  in  $\Sigma$  for compression, the square of the loss L is:

$$L^{2} = \left\| \sum_{i=m+1}^{r} \sigma_{i} u_{i} v_{i}^{T} S^{-1} X \right\|_{F}^{2} = \sum_{j=m+1}^{r} \sum_{i=m+1}^{r} \sigma_{i} \sigma_{j} \operatorname{trace}(u_{i} v_{i}^{T} S^{-1} X X^{T} (S^{-1})^{T} v_{j} u_{j}^{T})$$

$$= \sum_{i=m+1}^{r} \sigma_{i}^{2} \operatorname{trace}(u_{i} v_{i}^{T} S^{-1} X X^{T} (S^{-1})^{T} v_{i} u_{i}^{T}) = \sum_{i=m+1}^{r} (L_{i})^{2} = \sum_{i=1}^{k} (\sigma_{i})^{2}$$

$$(6)$$

The squared loss  $L^2$  is equal to the sum of the squared singular values. Therefore, truncating the smallest singular values  $\sigma_i$  achieves the lowest compression loss.

#### 3.2 Layer-Wise Closed-Form Update

**Motivation:** Given the same calibration data as input, the compressed weight matrix W' generates a new activation X' that is different from X generated by the original weight matrix W. As the compression ratio increases, W needs to truncate a larger number of singular values to obtain W', thus X' deviates further from X. Therefore, it becomes necessary to design a strategy for updating W' to minimize  $||WX' - W'X'||_F$ . However, existing SVD-based LLM compression methods have no design of parameter update after compression, leading to less competitive performance at high compression ratios.

Key Design: The key idea of SVD-LLM is to incorporate a layer-wise closed-form strategy to update W' to minimize  $||WX'-W'X'||_F$ . Figure 3 illustrates the overall process. To perform the update in layer i, SVD-LLM uses the new activation  $X'_{i-1}$  from the previous layer i-1 that has been updated. To perform the update without destroying the low-rank structure of  $W'_i$ , SVD-LLM only updates the matrix  $U_i$  to its closed-form solution  $U'_i$  while keeping  $Trunc.(\Sigma)_i$  and  $V_i$  fixed as:

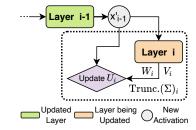


Figure 3: Layer-Wise Closed-Form Update.

$$U_{i}' = \arg\min_{U_{i}'} ||W_{i}X_{i-1}' - W_{i}'X_{i-1}'||_{F}$$

$$= W_{i}X_{i-1}'D^{T}(DD^{T})^{-1}, D = \text{Trunc.}(\Sigma)_{i}V_{i}^{T}S_{i}^{-1}X_{i-1}'$$
(7)

#### 4 Experiments

**Baselines.** We compare SVD-LLM against three baselines including vanilla SVD as well as state-of-the-art SVD-based LLM compression methods FWSVD [12] and ASVD [28].

Models and Datasets. To demonstrate the generability of our method, we evaluate the performance of SVD-LLM and the baselines on eight models from three different LLM families at four different scales (LLaMA-7B, 13B, 30B, 65B [23], LLaMA2-7B [24], OPT-6.7B [30], Vicuna-7B [3] and Mistral-7B [14]) and 10 datasets including three language modeling datasets (WikiText-2 [18], PTB [17] and C4 [21]) and seven common sense reasoning datasets (OpenbookQA [20], WinoGrande [22], HellaSwag [29], PIQA [2], MathQA [1], ARC-e, and ARC-c [4]) in zero-shot setting with the LM-Evaluation-Harness framework [7].

**Implementation Details.** To ensure a fair comparison, we followed ASVD [28] to randomly select 256 samples from WikiText-2 as the calibration data. Since layer-wise closed-form update is intended to mitigate the accuracy drop under higher compression ratios, we only apply it when the compression ratios are at 40% and above. All of our experiments are conducted on NVIDIA A100 GPUs.

#### 4.1 Overall Performance

We evaluate the overall performance of SVD-LLM from four aspects: (1) performance under different compression ratios, (2) performance on different LLMs, (3) performance on LLMs with larger scales, and (4) performance with LoRA fine-tuning. Some generated contents by the compressed LLMs are listed in Appendix A.3 and Appendix A.4 to provide a more straightforward comparison.

Performance under Different Compression Ratios. First, we evaluate the performance of LLaMA-

Table 1: Zero-shot performance of LLaMA-7B compressed by SVD-LLM and baselines under 20% to 60% compression ratio on three language modeling datasets (measured by perplexity  $(\downarrow)$ ) and seven common sense reasoning datasets (measured by both individual and average accuracy  $(\uparrow)$ ). The best performance is marked in bold. The relative performance gain compared to the best-performing baseline is marked in green color inside bracket.

RATIO	Метнор	WikiText-2↓	PTB↓	C4↓	Openb.	ARC_e	WinoG.	HellaS.	ARC_c	PIQA	MathQA	Average↑
0%	Original	5.68	8.35	7.34	0.28	0.67	0.67	0.56	0.38	0.78	0.27	0.52
	SVD	20061	20306	18800	0.14	0.27	0.51	0.26	0.21	0.53	0.21	0.31
20%	FWSVD	1727	2152	1511	0.15	0.31	0.50	0.26	0.23	0.56	0.21	0.32
20%	ASVD	11.14	16.55	15.93	0.25	0.53	0.64	0.41	0.27	0.68	0.24	0.43
	SVD-LLM	<b>7.94</b> (\129%)	<b>16.22</b> (\dagger*2%)	<b>15.84</b> (\1%)	0.22	0.58	0.63	0.43	0.29	0.69	0.24	0.44 (†2%)
	SVD	13103	17210	20871	0.13	0.26	0.51	0.26	0.21	0.54	0.22	0.30
30%	FWSVD	20127	11058	7240	0.17	0.26	0.49	0.26	0.22	0.51	0.19	0.30
30%	ASVD	51	70	41	0.18	0.43	0.53	0.37	0.25	0.65	0.21	0.38
	SVD-LLM	<b>9.56</b> (\$1%)	<b>26.39</b> (\ddot 62%)	<b>25.11</b> (\139%)	0.20	0.48	0.59	0.37	0.26	0.65	0.22	0.40 (†5%)
	SVD	52489	59977	47774	0.15	0.26	0.52	0.26	0.22	0.53	0.20	0.30
40%	FWSVD	18156	20990	12847	0.16	0.26	0.51	0.26	0.22	0.53	0.21	0.30
40%	ASVD	1407	3292	1109	0.13	0.28	0.48	0.26	0.22	0.55	0.19	0.30
	SVD-LLM	13.11 (↓99%)	<b>63.75</b> (↓98%)	<b>49.83</b> (↓96%)	0.19	0.42	0.58	0.33	0.25	0.60	0.21	<b>0.37</b> (†23%)
	SVD	131715	87227	79815	0.16	0.26	0.50	0.26	0.23	0.52	0.19	0.30
50%	FWSVD	24391	28321	23104	0.12	0.26	0.50	0.26	0.23	0.53	0.20	0.30
30%	ASVD	15358	47690	27925	0.12	0.26	0.51	0.26	0.22	0.52	0.19	0.30
	SVD-LLM	23.97 (↓99%)	150.58 (↓99%)	118.57 (↓99%)	0.16	0.33	0.54	0.29	0.23	0.56	0.21	<b>0.33</b> (†10%)
	SVD	105474	79905	106976	0.16	0.26	0.50	0.26	0.22	0.52	0.21	0.30
60%	FWSVD	32194	43931	29292	0.15	0.26	0.49	0.26	0.22	0.53	0.18	0.30
00%	ASVD	57057	45218	43036	0.12	0.26	0.49	0.26	0.21	0.51	0.18	0.29
	SVD-LLM	42.30 (↓99%)	312.28 (↓99%)	<b>246.89</b> (↓99%)	0.14	0.28	0.50	0.27	0.22	0.55	0.21	<b>0.31</b> (†7%)

Table 2: Perplexity (↓) of four different LLMs – OPT-6.7B, LLaMA 2-7B, Mistral-7B, and Vicuna-7B – under 20% compression ratio on WikiText-2. The relative performance gain compared to the best-performing baseline is marked in green color inside bracket.

Метнор	OPT-6.7B	LLaMA 2-7B	Mistral-7B	Vicuna-7B
SVD	66275	18192	159627	18644
FWSVD	14559	2360	6357	2758
ASVD	82	10.10	13.72	16.23
SVD-LLM	<b>16.04</b> (↓80%)	<b>8.50</b> (\16%)	10.21 (\126%)	<b>6.78</b> (\$\\$58\%)

Table 3: Perplexity (↓) of LLaMA-7B, 13B, 30B, 65B under 20% compression ratio on WikiText-2. Some results from baselines are not available due to out of memory (OOM) error during model compression. The relative performance gain compared to the best-performing baseline is marked in green color inside bracket.

Метнор	LLaMA-7B	LLaMA-13B	LLaMA-30B	LLaMA-65B
SVD	20061	946.31	54.11	11.27
FWSVD	1630	OOM	OOM	OOM
ASVD	11.14	6.74	22.71	OOM
SVD-LLM	7.94 (\129%)	6.61 (\12%)	5.63 (\175%)	6.58 (\.42%)

7B compressed by SVD-LLM and the baselines under compression ratios ranging from 20% to 60% on all 10 datasets. Table 1 summarizes the results. On the three language modeling datasets, SVD-LLM consistently outperforms vanilla SVD, FWSVD and ASVD across all the compression ratios. More importantly, SVD-LLM exhibits significant advantages over the baselines under higher compression ratios. Specifically, under 30% compression ratio, compared to the best-performing baseline (ASVD), SVD-LLM reduces the perplexity on WikiText-2, PTB, and C4 by 81%, 62%, and 39%, respectively; when the compression ratio reaches 40% and above, SVD-LLM reduces the perplexity by more than 96%. These results indicate that SVD-LLM is more effective in compressing LLMs for more resource-constrained devices such as smartphones and IoT devices. On the seven common sense reasoning datasets, SVD-LLM performs better than the best-performing baseline on most of the datasets and consistently achieves at least 2% higher average accuracy across all the compression ratios.

**Performance on Different LLMs.** To examine the generability of SVD-LLM across different LLMs, we compare the performance between SVD-LLM and the baselines on four different models from three different LLM families – OPT-6.7B (OPT family), LLaMA 2-7B (LLaMA family), Vicuna-7B (LLaMA family), and Mistral-7B (Mistral family) – under 20% compression ratio on WikiText-2. As shown in Table 2, SVD-LLM consistently outperforms vanilla SVD, FWSVD, and ASVD on all four LLMs, and exhibits more stable performance across different LLMs, especially compared to vanilla SVD and FWSVD.

**Performance on LLMs with Larger Scales.** To examine the generability of SVD-LLM on LLMs across different scales, we compare the performance between SVD-LLM and the baselines on LLaMA series at four different scales – 7B, 13B, 30B, and 65B – under 20% compression ratio on WikiText-2. As shown in Table 3, SVD-LLM consistently outperforms vanilla SVD, FWSVD, and ASVD across all four model sizes. Moreover, due to memory-intensive operations for estimating the importance

Table 4: Perplexity of LLaMA-7B compressed by SVD-LLM and ASVD (w/ and w/o LoRA) under 20% to 80% compression ratio on WikiText-2. The relative performance gain of applying LoRA after compression over compression without LoRA is marked in green color.

Метнор	20%	30%	40%	50%	60%	70%	80%
ASVD	11.14	51	1407	15358	57057	67166	43406
ASVD + LoRA	8.37 (\dold 25%)	10.16 (↓80%)	14.86 (↓99%)	21.83 (↓99%)	44.81 (↓99%)	99 (↓99%)	271 (↓99%)
SVD-LLM	7.94	9.56	13.11	23.97	42.30	185	819
SVD-LLM + LoRA	<b>7.78</b> (\pm2%)	<b>9.14</b> (↓4%)	<b>10.65</b> (\19%)	<b>13.26</b> (↓45%)	<b>17.93</b> (↓63%)	<b>28.45</b> (↓85%)	<b>33.19</b> (↓96%)

of model weights, both FWSVD and ASVD demand excessive memory resources, causing out of memory (OOM) error when compressing LLMs at larger scales on an A100 GPU. In contrast, SVD-LLM does not involve such estimation operations and thus avoids OOM.

Performance with LoRA Fine-Tuning. LoRA [13] has become one of the most widely used fine-tuning techniques for LLMs. It has been applied with pruning-based LLM compression methods such as LLM-Pruner [16] to mitigate accuracy drop after pruning. LoRA can also be combined with SVD-based LLM compression methods by modifying the forward pass of a linear layer as:

$$Y = W_u W_v X \tag{8}$$

where  $W_u = W_u + B_u A_u$ ,  $W_v = W_v + B_v A_v$ , and  $A_u$ ,  $B_u$ ,  $A_v$ , and  $B_v$  are low-rank weights fine-tuned using LoRA.

To examine the performance of SVD-LLM in combination with LoRA, we follow the same configuration used in LLM-Pruner [16] to fine-tune LLaMA-7B compressed by SVD-LLM and ASVD under the compression ratios from 20% to 80% with LoRA. The results are shown in Table 4. We have three observations. (1) As the compression ratio increases, LoRA plays a more significant role in improving the performance of SVD-LLM. (2) Compared to ASVD + LoRA, SVD-LLM + LoRA consistently achieves better perplexity across all the compression ratios. (3) Finally, even without LoRA, SVD-LLM is able to achieve perplexity comparable to ASVD with LoRA (i.e., ASVD + LoRA) under the compression ratios from 20% and 60%.

#### 4.2 Compression Speed Evaluation

Besides compression performance, we also evaluate the compression speed of SVD-LLM and the baselines. Specifically, we measured the GPU hours used for SVD-LLM and ASVD when compressing LLaMA-7B under 20% compression ratio on an A100 GPU. The results are shown in Table 5. As shown, ASVD takes about 5.5 hours whereas SVD-LLM completes the compression process in 15 minutes, which is 32 times faster. When breaking down the time, most of the time consumed by ASVD is dedicated to searching for the specific compression ratio for each weight matrix based on its calculated importance score. In contrast, SVD-LLM maintains a consistent compression ratio across all weight matrices and thus gets rid of the time-consuming search process.

Table 5: Compression time of SVD-LLM and Table 6: Performance of LLaMA-7B compressed color inside bracket.

SVD\_IIM

	010 000			10.10	
Truncation-Aware Data Whitening	Layer-Wise Closed-Form Update	Total	Normalize	Search	Total
10min	5min	15min (↓95%)	5min	5.5h	5.5h
	data (x2048 tokens)		10 42 d for Rando	om Sam	
(a) Chang	ge of Number	(b) C	Change	or Se	ea -

Figure 4: Perplexity of LLaMA-7B under 20% compression ratio using calibration data with different number or sampled with different seeds from WikiText-2.

ASVD on LLaMA-7B under 20% compression by SVD-LLM under 20% compression ratio using ratio. The relative speedup is marked in green calibration data randomly sampled from WikiText-2 (by default in our paper) and C4. The performance on WikiText-2, PTB, and C4 is reported by perplexity (\$\dagger\$), while the performance on OpenbookQA and HellaSwag are reported by accuracy (†). The relative performance drop (gain) for data sampled from C4 compared to that sampled from WikiText-2 is marked in red (green) color inside bracket.

WikiText-2	PTB	C4	Openb.	HellaS.		
Calibration data sampled from WikiText-2 (seed=3)						
7.94	16.22	15.84	0.22	0.43		
-	Calibration data sampled from C4 (seed=3)					
8.62(†9%)	14.63(\dagger*9%)	13.77(\13%)	0.23(↑5%)	0.43		

Table 7: Perplexity (\$\psi\$) of compressed LLaMA-7B on WikiText-2. SVD-LLM (\$\Walls\*) denotes the version of SVD-LLM with truncation-aware data whitening only; SVD-LLM (\$\Walls\*) denotes the version of SVD-LLM with layer-wise closed-form update only; SVD-LLM (\$\Walls\*+U)\$ denotes the version of SVD-LLM with both truncation-aware data whitening and layer-wise closed-form update. The relative performance gain compared to ASVD is marked in green color.

Метнор	20%	30%	40%	50%	60%
ASVD	11.14	51	1407	15358	57057
SVD-LLM (W) SVD-LLM (U) SVD-LLM (W+U)	7.94 (\\\\)29%) 9.54 (\\\\)14%) 8.25 (\\\\\)26%)	<b>9.56</b> (\$\\$1\%) 12.98 (\$\\$75\%) 9.95 (\$\\$0\%)	13.73 (\$\psi 99\%) 24.16 (\$\psi 99\%) <b>13.11</b> (\$\psi 99\%)	26.11 (\$\dagge 99\%) 72.13 (\$\dagge 99\%) <b>23.97</b> (\$\dagge 99\%)	66.62 (\$\dagge 99\%) 204 (\$\dagge 99\%) <b>42.30</b> (\$\dagge 99\%)
SVD-LLM	7.94 (\129%)	<b>9.56</b> (↓81%)	13.11 (↓99%)	23.97 (↓99%)	42.30 (\$99%)

#### 4.3 Ablation Study

Modular Sensitivity Study. We conduct ablation studies to evaluate the separate contributions of the two key components (i.e., truncation-aware data whitening and layer-wise closed-form update) of SVD-LLM. Let SVD-LLM (W) denote the version of SVD-LLM with truncation-aware data whitening only; SVD-LLM (U) denote the version of SVD-LLM with layer-wise closed-form update only; and SVD-LLM (W+U) denote the version of SVD-LLM with both truncation-aware data whitening and layer-wise closed-form update. The results are shown in Table 7. We have three observations. (1) Both SVD-LLM (W) and SVD-LLM (U) consistently outperform ASVD across all the compression ratios. Notably, when the compression ratio is at and above 40%, both variants reduce the perplexity by more than 99% compared to ASVD. (2) Under 20% and 30% compression ratios, SVD-LLM (W) achieves the lowest perplexity compared to SVD-LLM (U) and SVD-LLM (W+U). (3) Under 40%, 50% and 60% compression ratios, SVD-LLM (W+U) achieves the lowest perplexity compared to SVD-LLM (W) and SVD-LLM (U), highlighting the importance of combining both truncation-aware data whitening and layer-wise closed-form update when compression ratio increases.

**Impact of Calibration Data.** Next, we examine the impact of calibration data used for both truncation-aware data whitening and layer-wise closed-form update on the compression performance. Figure 4 and Table 6 summarize the performance of compressed LLaMA-7B when changing three key characteristics of the calibration data: (1) the number of the calibration data, (2) the seed used to randomly sample the calibration data, and (3) the data set from which the calibration data is sampled. As shown, the changes on calibration data incur no more than 13% to the final performance, demonstrating that the sensitivity of SVD-LLM on calibration data is limited.

#### 4.4 Benefits to other LLM Compression Methods

SVD-LLM is orthogonal to other LLM compression methods including quantization and parameter pruning. In this experiment, we combine SVD-LLM with quantization and parameter pruning-based LLM compression methods that are widely recognized by the community to examine how SVD-LLM could further enhance their performance.

**Integrate** SVD-LLM **with Quantization.** We select GPTQ [6] as the quantization method. Specifically, we compress LLaMA-7B by GPTQ-4bit combined with SVD-LLM, and compare the compressed model against LLaMA-7B compressed by GPTQ-3bit. As shown in Table 8, combining GPTQ-4bit with SVD-LLM achieves a perplexity that is 18% lower than GPTQ-3bit even with a smaller memory footprint (2.1 GB vs. 2.8 GB). This result demonstrates that compared to directly quantizing using smaller number of bits, GPTQ achieves better compression performance with the help of SVD-LLM.

**Integrate** SVD-LLM with Parameter Pruning. We select LLM-Pruner [16] as the parameter pruning method. Specifically, we compress LLaMA-7B by LLM-Pruner under 30% compression ratio combined with SVD-LLM, and compare the compressed model against LLaMA-7B compressed by LLM-Pruner under 40% compression ratio. As shown in Table 9, LLM-Pruner achieves better compression performance when used in conjunction with SVD-LLM. In particular, with the same memory footprint of 8.8 GB, combining LLM-Pruner under 30% compression ratio with SVD-LLM achieves a perplexity that is 13% lower than LLM-Pruner under 40% compression ratio.

Table 8: Perplexity (↓) of LLaMA-7B compressed Table 9: Perplexity (↓) of LLaMA-7B comby GPTQ w/ and w/o SVD-LLM on WikiText-2. The relative performance gain of combined compression compared to GPTQ-3bit is marked in green color inside bracket.

METRIC	GPTQ-4bit	GPTQ-3bit	SVD-LLM + GPTQ-4bit
Memory	3.9 GB	2.8 GB	2.1 GB
Perplexity	6.21	16.28	13.29 (↓18%)

pressed by LLM-Pruner w/ and w/o SVD-LLM on WikiText-2. The relative performance gain of combined compression compared to LLM-Pruner under 40% compression ratio is marked in green color.

METRIC	LLM-Pruner-30%	LLM-Pruner-40%	LLM-Pruner-30% + SVD-LLM
Memory	9.8 GB	8.8 GB	8.8 GB
Perplexity	9.88	12.21	10.58 (↓13%)

#### **Enhancing Inference Efficiency on Real Hardware**

SVD-LLM is able to not only compress LLMs but also enhance inference efficiency on real hardware. To demonstrate this, we measure the numbers of tokens that the original LLaMA-7B and its compressed version by SVD-LLM generate per second with different batch sizes and sequence lengths on both GPU and CPU. The results are illustrated in Figure 5. As shown, SVD-LLM consistently ensures an enhancement in the generation speed across all the compression ratios. More importantly, the enhancement becomes more significant as the batch size increases and the sequence length decreases, resulting in a maximum speedup of 1.2x on GPU and 1.1x on CPU under the 20% compression ratio, 1.7x on GPU and 1.5x on CPU under the 40% compression ratio, 2.1 x on GPU and 1.64x on CPU under the 60% compression ratio, and 3.1x on GPU and 2.3x on CPU under the 80% compression ratio. These results highlight the effectiveness of SVD-LLM in enhancing the inference efficiency of LLMs on real hardware.

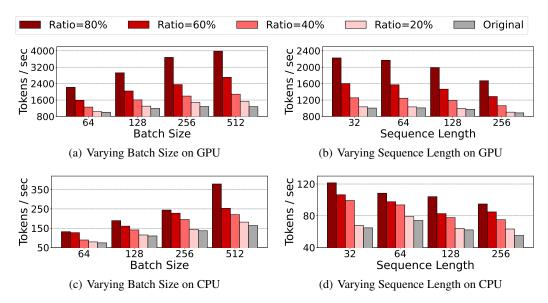


Figure 5: Throughput (Tokens/sec) of original LLaMA-7B and its compressed version by SVD-LLM under 20%, 40%, 60% and 80% compression ratio on single A100 GPU (Figure (a),(b)) and single AMD EPYC 7643 CPU (Figure (c),(d)). Figure (a),(c) is the comparison with different batch size while sequence length = 32, Figure (b), (d) is the comparison with different sequence length while batch size = 64.

#### 5 Conclusion

In this paper, we presented SVD-LLM, a SVD-based post-training LLM compression method. SVD-LLM proposes a novel truncation-aware data whitening strategy to guide which singular values to be truncated with minimal compression loss. It also introduces a layer-wise closed-form model parameter update scheme to compensate for accuracy degradation under high compression ratios. We have demonstrated the effectiveness of SVD-LLM on 10 datasets and seven models from three LLM families at four scales and have shown its superiority over state-of-the-arts. We also show its effectiveness in further enhancing the performance of other LLM compression methods.

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#### A Appendix.

#### A.1 The compression error of ASVD

The previous state-of-the-art method ASVD introduced a diagonal scaling matrix  $S_0$  that modifies the weight matrix to reflect the varying significance of different input channels. The linear layer is formulated as  $Y = (WS_0)S_0^{-1}X$ . The compression is made by keeping the largest m singular value of  $WS_0$ :

$$WS_0 \approx \sum_{i=1}^m \sigma_i' u_i' {v'}_i^T$$

The resulting activation is expressed as:

$$Y \approx \sum_{i=1}^{m} \sigma'_{i} u'_{i} {v'}_{i}^{T} S_{0}^{-1} X$$
.

The compression error  $L = ||(WS_0 - \sum_{i=1}^m \sigma_i' u_i' {v'}_i^T) S_0^{-1} X||_F$  is demonstrated below:

$$\begin{split} L^2 = &||(WS_0 - \sum_{i=1}^m \sigma_i' u_i' v_i'^T) S_0^{-1} X||_F^2 \\ = &\left| \left| \sum_{i=m+1}^r \sigma_i' u_i' v_i'^T S_0^{-1} X \right| \right|_F^2 \\ = &\sum_{j=m+1}^r \sum_{i=m+1}^r \sigma_i' \sigma_j' \ trace(u_i' v_i'^T X X^T v_j' u_j'^T) \\ = &\sum_{j=m+1}^r \sum_{i=m+1}^r \sigma_i' \sigma_j' \ trace(u_j'^T u_i' v_i'^T S_0^{-1} X X^T (S_0^{-1})^T v_j') \\ = &\sum_{i=m+1}^r \sigma_i'^2 \ trace(v_i'^T S_0^{-1} X X^T (S_0^{-1})^T v_i') \\ = &\sum_{i=m+1}^r \sigma_i'^2 \ ||v_i'^T S_0^{-1} X||_F^2 \ , \end{split}$$

which is still a complex function that involves the activation X, the diagonal matrix  $S_0$ , the singular vector  $v_i'$  and the singular value  $\sigma_i'$ . As a result, compression error is not directly related to the singular value, and the conventional SVD compression by truncating the smallest singular values may lead to suboptimal compression error.

#### A.2 Pseudocode of SVD-LLM

Algorithm 1 shows the pseudocode of SVD-LLM. Before compression, SVD-LLM randomly collects a small amount of sentences as the calibration data C, it then runs the truncation-aware data whitening process as shown in Algorithm 2 to obtain the set of whitening matrix  $Set_S$  for the weight to compress. After that, it runs the SVD and truncation with  $Set_S$  on each weight matrix in the LLM. Instead of directly finishing the whole compression, it stores the decomposed matrices and further utilizes these matrices to run the layer-wise closed-form update as shown in Algorithm 3.

#### A.3 Contents Generated from the Model Compressed by SVD-LLM and ASVD

We compare some examples of sentences generated by LLaMA-7B compressed with SVD-LLM and ASVD in Table 10. As shown, the sentences generated by the model compressed by SVD-LLM

#### Algorithm 1 Pseudocode of SVD-LLM

```
1: Input: M: Original LLM
 2: Output: M': Compressed LLM by SVD-LLM
    procedure SVD-LLM(M)
 4:
         Randomly collect several sentences as the calibration data C
         Set_S \leftarrow TRUNCATION-AWARE DATA WHITENING(M, C)
 5:
 6:
         Set_{SVD} \leftarrow \emptyset
                                    ▶ Initialize the set of decomposed matrices for the weight to compress
 7:
         Set_W \leftarrow M
                                                                    \triangleright Obtain the set of weights in M to compress
         for W in Set_W do
 8:
 9:
                                                           \triangleright Extract the whitening matrix of current weight W
              S \leftarrow \operatorname{Set}_S(W)
              U, \Sigma, V \leftarrow \overrightarrow{SVD}(WS)
10:
                                                                     \triangleright Apply singular value decomposition on W
              \Sigma_1 \leftarrow \text{Trunc.}(\Sigma)
11:
                                                                      \triangleright Truncate the smallest singular values in \Sigma
12:
              \operatorname{Set}_{SVD} \leftarrow (U, \Sigma_1, V) \cap \operatorname{Set}_{SVD} \triangleright \operatorname{Store} the decomposed matrices of the weight in the
     set
13:
         end for
         M' \leftarrow \text{LAYER-WISE CLOSED-FORM UPDATE}(M, C, \text{Set}_S, \text{Set}_{SVD})
14:
         return M'
15:
16: end procedure
```

#### Algorithm 2 Pseudocode of Truncation-Aware Data Whitening

```
1: Input: M: Original LLM
 2: Input: C: Calibration Data
 3: Output: Set<sub>S</sub>: Set of whitening matrices for the weight to compress in M
 4: procedure Truncation-Aware Data Whitening(M, C)
 5:
        Set_S \leftarrow \emptyset
                                                                   ▶ Initialize the set of whitening matrices
 6:
        Set_W \leftarrow M
                                                             \triangleright Obtain the set of weights in M to compress
 7:
        for W in \operatorname{Set}_W do
 8:
             X \leftarrow M(W, D)
                                                    \triangleright Obtain the input activation of the weight matrix W
             S \leftarrow \text{Cholesky\_Decomposition}(XX^T)
                                                                \triangleright Apply cholesky decomposition on XX^T
 9:
10:
             Set_S \leftarrow S \cup Set_S
                                                             > Store the whitening weight matrix in the set
11:
        end for
12:
        return Sets
13: end procedure
```

exhibit better fluency, relevance, and informativeness compared to that compressed by ASVD. More importantly, when the compression ratio is increased to 40%, the previous state-of-the-art method ASVD completely loses its generation ability. In contrast, even when the compression ratio is up to 60%, SVD-LLM is still capable of generating complete sentences.

#### A.4 Contents Generated from the Fine-Tuned LLMs with LoRA after Compression

Since LoRA can also enhance the accuracy of the compressed LLM. In this section, wcompare some examples of sentences generated by the fine-tuned LLaMA-7B with LoRA after its compression by SVD-LLM and ASVD in Table 11. As shown, with the assistant of LoRA, SVD-LLM can even generate high quality answer under the 80% compression ratio, which indicates that user can reduce 80% of the parameters from the LLM but still maintain a good generation using SVD-LLM in some cases. On the contrary, even with the help of LoRA, ASVD still losses its generation ability under 50% compression ratio.

#### A.5 Benefits on Reducing Footprint of KV Cache at Runtime

SVD-LLM is able to not only compress LLMs but also compress the runtime KV cache *at the same time*. Specifically, instead of keeping the original intermediate state matrix m=WX with shape  $M\times L$  inside the KV cache, after decomposing and compressing W into  $W_uW_v$ , SVD-LLM only needs to store  $m'=W_vX$  with shape  $r\times L$ . Therefore, the size of the KV cache can be compressed to  $\frac{r}{M}$  of the original. Moreover, since  $W_u$  is already stored as the weight matrix in the decomposed

#### Algorithm 3 Pseudocode of Layer-Wise Closed-Form Update

```
1: Input: M: Original LLM
 2: Input: C: Calibration Data
 3: Input: Set<sub>S</sub>: Set of whitening matrices for the weight to compress in M
 4: Input: Set SVD: Set of decomposed matrices for the weight to compress in M
 5: Output: M': Compressed LLM by SVD-LLM
 6: procedure LAYER-WISE CLOSED-FORM UPDATE(M, C, \operatorname{Set}_S, \operatorname{Set}_{SVD})
         M' \leftarrow M
                                                                                                 \triangleright Initialize M' with M
 7:
 8:
         Set_L \leftarrow M'
                                                           \triangleright Obtain the set of encoder and decoder layers in M'
         X' \leftarrow M'(C)
 9:
                                                            \triangleright Obtain the input activation of the first layer in M'
         for L in Set_L do
10:
11:
              Set_W \leftarrow L
                                                                      \triangleright Obtain the set of weights in L to compress
              for W in Set_W do
12:
13:
                   S \leftarrow \operatorname{Set}_S(W)
                                                            \triangleright Extract the whitening matrix of current weight W
14:
                   U, \Sigma_1, V \leftarrow \underline{\operatorname{Set}_{SVD}(W)} \quad \triangleright \operatorname{Obtain} the decomposed matrices of W from \underline{\operatorname{Set}_{SVD}}
                   U' = WX'X'^{T}(S^{-1})^{T}V\Sigma_{1}(\Sigma_{1}V^{T}S^{-1}X'X'^{T}(S^{-1})^{T}V\Sigma_{1})^{-1}
                                                                                                           15:
                   W_u \leftarrow U'(\Sigma_1)^{1/2}, W_v \leftarrow S^{-1}V(\Sigma_1)^{1/2}
                                                                                      ▷ Obtain two low-rank matrices
16:
                   L(W) \leftarrow L(W_u, W_v)
17:
                                                                                \triangleright Replace W with W_u and W_v in L
18:
              end for
               X' \leftarrow L(X') \triangleright \text{Use the compressed layer to calculate the new input activation } X' \text{ for the}
19:
     next layer
         end for
20:
21:
         return M'
22: end procedure
```

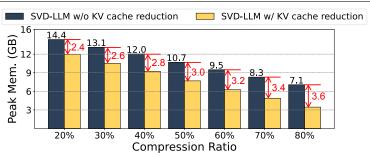


Figure 6: Peak memory to generate 128 tokens with batch size of 32 using LLaMA-7B compressed by SVD-LLM under different compression ratios w/ and w/o KV-cache compression. The difference between the blue and yellow bars marked in red indicates the reduced footprint of the KV cache.

LLM, the original intermediate state matrix can still be recovered by  $m=W_um'$  without accuracy drop. Therefore, SVD-LLM provides a unified solution that combines model compression and KV cache compression into a single process. This is different from existing quantization or parameter pruning-based LLM compression methods that need to be combined with other techniques for compressing both weights and KV cache..

In our last experiment, we evaluate this benefit on KV cache compression brought by SVD-LLM. This is a new avenue since KV cache compression has not been evaluated in previous LLM compression studies. Specifically, we measure the peak memory footprint during inference when generating 128 tokens with batch size of 32 using LLaMA-7B compressed by SVD-LLM under different compression ratios w/ and w/o considering KV cache compression. The results are illustrated in Figure 6 where the difference between the blue and yellow bars marked in red represents the reduced footprint of the KV cache. As shown, SVD-LLM is able to effectively reduce the footprint of KV cache. Therefore, the peak memory during inference at runtime across all the compression ratios.

Table 10: An example of contents generated by the compressed LLaMA-7B by SVD-LLM and ASVD under different compression ratios. The input is marked in bold and the normal texts are the generated sentences.

RATIO	SVD-LLM	ASVD		
Original	What is the universe? The universe is a single unified, indivinfinite in extent, does not actually include any other space be which there would be absolutely nothing, not even infinity			
20%	What is the universe? The universe is the system of planets located by our closest planets. We have nine planets that have nine planets; these planets are made by the solar system	What is the universe? The universe is everything that exists, that ever has existed, or that may exist in the future. The visible universe includes all the objects and events seen by the naked eye and visible objects		
30%	What is the universe? The universe is all the bodies of visible matter found in the solar systems, including the Solar System. The Sun is the most important part of the solar system because Earth is in the	What is the universe? If, ultimately, this universe is the "final form of physical theory, the basic laws will have a great future. AI/Safa R-B-Safar is. This conference provides a great opportunity to get in person contact with		
40%	What is the universe? The universe has numerous stars, with some forming the central core. Within the core of a giant star, there are smaller stars known as the main sequence. These main sequence stars, in turn, contain	What is the universe? Yes the people standing for. There You to T M Ch is f I is Cl id id In id In la id In In tub id id id did id id id id id id id id id		
50%	What is the universe? The universe remained stable for around 8 million years. Before the 'Big Collapse,' it was perceived as the encompassing reality, consisting of celestial bodies like planets and stars. During this period, the universe itself was akin to a planet, with stars forming within it	What is the universe? Great Is Seighting with with has consultéoreferrer or CCirgretr jourfiVi i rif en i and du20 with the an in has a la — n i a i has a n i is n 9. R has-p ii fori see in 'R re i I get and t		
60%	What is the universe? The universe contains a sun, planets, and moons. The sun is a solar star, and it has its own orbit. Planets orbit around the sun, and some of these planets have moons. Moons orbit their respective planets	What is the universe? Theo the or and shh i I sheh SDc2D32232222222222222222222222222222222		

Table 11: An example of contents generated by the fine-tuned LLaMA-7B with LoRA after being compressed by SVD-LLM and ASVD under different compression ratios. The input is marked in bold and the normal texts are the generated sentences.

RATIO	SVD-LLM	ASVD
Original	What is the responsibility of the AI assistant? The responsibility of the AI assistant? The responsibility of the AI assistant has no problem with taking a complete the task and it needs to learn from his past experience	a decision and executing it. An AI assistant is responsible to
20%	What is the responsibility of the AI assistant? The AI assistant's role and responsibilities are often ambiguous and can vary depending on the context and the type of assistant being used. Some AI assistants may have preset responses to common queries, but their behavior can also depend on the user's interaction and expectations	What is the responsibility of the AI assistant? 1. To act in the same way as the programmer intended. 2. To make a value assessment of the user's intentions and offer an appropriate response. 3. To help the user carry out specific tasks or follow a strict script. 4. To make the system as independent of the user as possible
30%	What is the responsibility of the AI assistant? 1. Keep a record of user activities and interactions. 2. Provide accurate and useful information to user. 3. Answer user's questions or provide information on topics requested. 4. Execute tasks and operations in the user's line of work or personal life. 5. Keep user's schedule, appointments and other relevant information organized	What is the responsibility of the AI assistant? 1. To provide intelligent assistance that saves time and reduces the need for human effort 2. To help make better decisions, provide valuable insights, and warn about potential risks 3. To always respond in a way that is logical and reasonable 4. None of the above Is the role of the AI assistant always about making human life better or helping people to save time
40%	What is the responsibility of the AI assistant? 1. The AI assistant is responsible for performing tasks that can only be done by AI technology such as scheduling appointments, setting reminders and making suggestions, while also providing quick and accurate responses. 2. The AI assistant is also responsible for learning and updating its knowledge base through continuous improvement, thus making it more efficient and effective. 3. The AI assistant is responsible for handling and managing user-generated and customer-oriented input	What is the responsibility of the AI assistant? 1. Define the definition of the job specification. 2. Define the role of the AI assistant for the job. 3. Define the skills necessary in order to carryout the tasks assigned to the job to do the work of the same capacity in the market. 4. Determine the evaluation of the job competence of the AI assistant to ensure their professional integrity. 5. What type of AI assistant jobs are available? 6. What industries require technology assistance work to complete the tasks that it own personnel cannot perform
50%	What is the responsibility of the AI assistant? Answer: The AI assistant is responsible for performing tasks assigned to it by the user. For example, if a user asks the assistant for "How can you help in finding a new friend?" or "What's the weather forecast at this time?" the AI assistant will find the right answer for them. Additionally, the AI assistant could also provide useful information or assistance in other areas, such as scheduling appointments, creating a reminder, or helping users navigate a website or app	What is the responsibility of the AI assistant? It helps to guide, provide and teach people to learn tasks.ightarrow and IBM J o d m and D I o d are also used on the ightarrow is considered to be a type of electronic assistant designed to act similarly to. Interest in b and David is increasing from the fact that the most common name a refers to the number
60%	What is the responsibility of the AI assistant? In a nutshell, the responsibility of the AI assistant could vary depending on the task, but generally, the focus would be on automatic tasks, without the need for human intervention. Some common tasks could include setting reminders, scheduling appointments, and making routine household or productivity tasks. The AI assistant also serves as a backup or a relief system, taking on responsibilities when the user is not available	What is the responsibility of the AI assistant? 2.3. ?? Brush for a discussion I wonder is it worth doing is important.2- It isn't useful just reducing labor costs; it helps employees feel a sense of connected to your attention which leads to better workplace values among staffers and leads to long relationships among org
70%	What is the responsibility of the AI assistant? As an AI assistant, my primary goal is to provide you with the most efficient and accurate services you need, whether it's scheduling to visit a restaurant, sending notifications when you are on vacation or even responding to an email message or phone call	What is the responsibility of the AI assistant? Contact your TechatUser with any questionsgZyguta 1. Where are most common appointment setting
80%	What is the responsibility of the AI assistant? Our Design is based on our understanding of the world, and we are actively learning, adapting and adapting, so we're always evolving new ideas, which we see to be most unique and relevant in our community	What is the responsibility of the AI assistant? ygua AIeltemperaturen/2, (64mbz/.3/.1/, 7.kbld.org.0/2/ In these puthebout les bnvols n merginels