University of Padua – Department of Industrial Engineering
Bachelor’s Degree in Aerospace Engineering

Final project presentation «Development of a numerical method for airfoil optimization»

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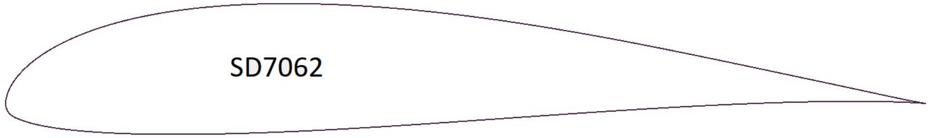
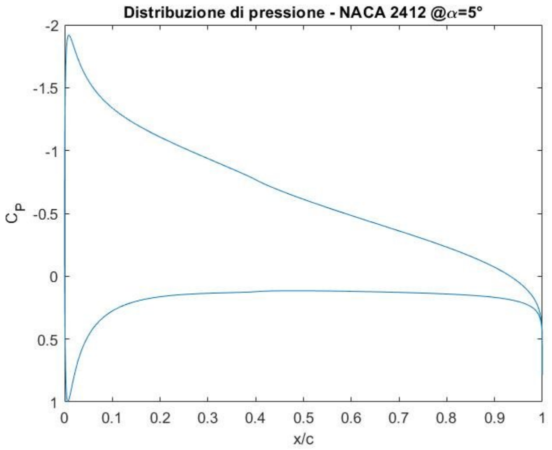
- The goal of the LiftUp team is to design and build a remote-controlled aeromodel that can compete in the «Air Cargo Challenge» (ACC) international competition. The goal is to transport as much payload as possible at the highest possible speed;
- 2019 Regulations: the aircraft must be able to take off in 60 meters, complete 10 sections of 100 meters each, and then land on a grass field;
- The rules require the use of a preselected propulsion system. No active control and/or stabilization software is allowed;

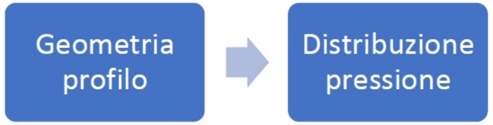
  → Aerodynamics play a dominant role in determining the score.
- Instead of selecting an airfoil profile from literature, an algorithm was developed to generate and optimize it numerically;



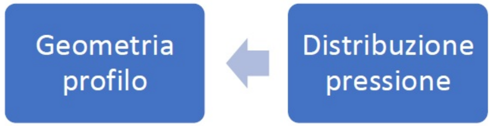
- The traditional approach is to parameterize the airfoil geometry (e.g. using Bézier curves) and then optimize the parameters to find the best shape;
- In this work it is proposed to parameterize the pressure distribution $C_P(x)$ instead of the airfoil geometry, which will be seen to offer several advantages;
- The algorithm development is divided into three stages:
  1. Panel method
  2. Inverse design method
  3. C_P parametrization and optimization (EMFID method)
- In practice, this approach is just another way to parametrize an airfoil geometry;



Metodo a pannelli:



Inverse design:

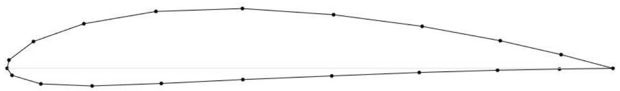


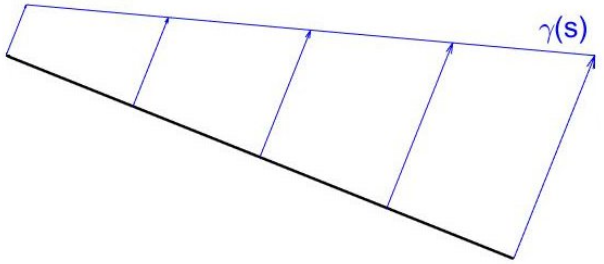
PANEL METHOD

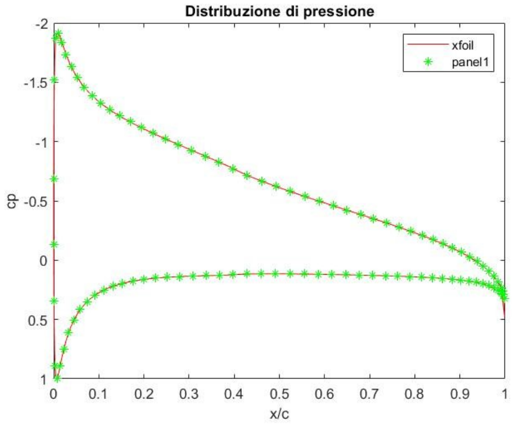
- As a first step, a low-fidelity inviscid panel method was implemented in Matlab to quickly compute C_P (~30ms on an average desktop computer);
  - → It does not depend on running external software;
- The pressure distribution C_P in the inviscid case is similar to that in the viscous case, but the computational cost is significantly lower;
- panel1.m uses a linear distribution of vortex density $\gamma(s)$ on each panel. Impermeability conditions are calculated at the midpoint, while the Kutta condition is applied at the trailing edge node;
- Once the distribution $\gamma(s)$ is found, the pressure distribution C_P can be calculated using the local velocity jump formula at no additional cost:

$$V_t = \gamma, \qquad C_P = 1 - \frac{\gamma^2}{V_\infty^2}$$

- panel1.m also returns the values of γ at the nodes, thereby avoiding the need for interpolation (which is useful for inverse design);



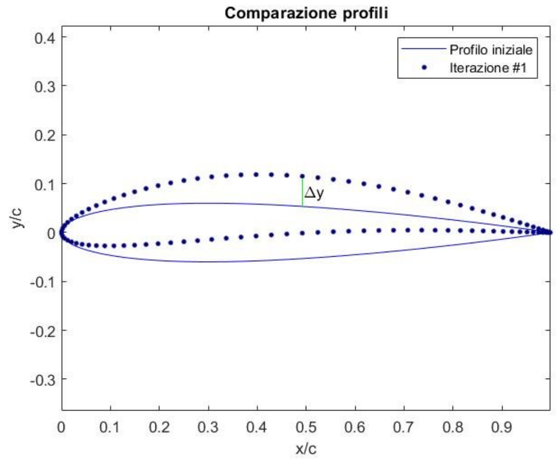


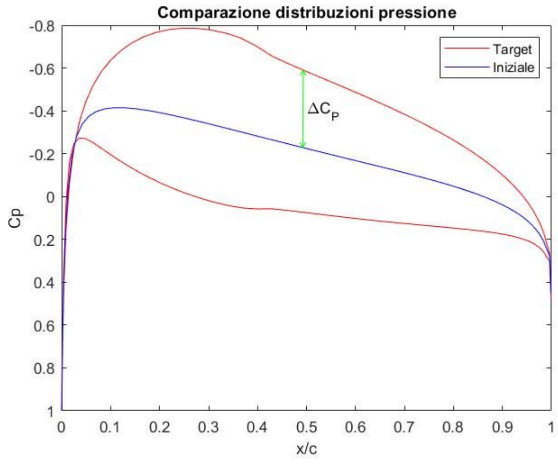


- The Modified Garabedian-McFadden Method (MGM) is a design approach that generates a profile capable of producing a specified target pressure distribution;
- This is an 'elastic membrane' method that uses an analysis code (e.g. panel1) in an iterative manner, modifying the airfoil shape at each step;
- The underlying idea of this method is that the pressure distribution around a profile is influenced by the position of its points $y(s)$, their local slope $\frac{dy}{ds}$, and their curvature $\frac{d^2y}{ds^2}$:

$$\Delta C_P = f\left(\Delta y, \frac{d(\Delta y)}{ds}, \frac{d^2(\Delta y)}{ds^2}\right)$$

- The algorithm begins with an initial profile of arbitrary shape, and then iteratively adjusts the vertical position of each point by a calculated Δy, until the target C_P is achieved;
- Depending on how the displacements Δy are calculated, it is possible to construct different elastic membrane methods;





- In the MGM method, the vertical displacements Δy are the solution of the following ODE:

$$A\,\Delta y + B\frac{d\,(\Delta y)}{ds} + C\frac{d^2\,(\Delta y)}{ds^2} = \Delta C_P$$

Where A, B, and C are arbitrary constants that prevent the profile from changing too much.

The boundary conditions are set as follows: $\Delta y_{LE} = 0,\ \Delta y_{TE} = 0.$

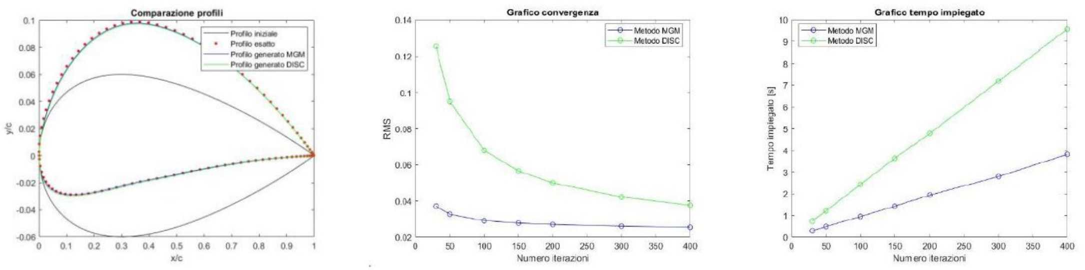
- It is worth noting that the equation has no physical meaning, yet it proves effective in practice. When $\Delta C_P\,(s) = 0$, the equation returns $\Delta y\,(s) = 0$, meaning that the profile remains unchanged;
- The constants A, B, and C influence the speed and convergence of the method. There is no way to determine the optimal values, which strongly depend on the problem. In the literature, the values $A = 1.4,\ B = 0,\ C = 0.6$ are often used, but in this work, more conservative values were chosen: $A = 14,\ B = 0,\ C = 6$;
- The ODE in the MGM method is discretized using a first-order finite difference scheme with uneven spacing. The solution is computed separately for the upper and lower surfaces, which requires displacements in opposite directions. This results in two tridiagonal linear systems, which can be solved using the Thomas algorithm;

INVERSE DESIGN – DISC METHOD

- In the DISC method, two separate equations are defined for the upper and lower surfaces, coupled through the boundary conditions at the leading edge:
  - Upper: $A\,\Delta y^t + B\frac{d(\Delta y^t)}{ds} - C\frac{d^2(\Delta y^t)}{ds^2} = \Delta C_P^t$ $\quad \Delta y_{TE}^t = 0 \quad \Delta y_{LE}^t = \Delta y_{LE}^b$
  - Lower: $-A\,\Delta y^b + B\frac{d(\Delta y^b)}{ds} + C\frac{d^2(\Delta y^b)}{ds^2} = \Delta C_P^b$ $\quad \Delta y_{TE}^b = 0 \quad \frac{d(\Delta y_{LE}^t)}{ds} = \frac{d(\Delta y_{LE}^b)}{ds}$
- By expanding the function $\Delta C_P(s) = C_{P\_target} - C_{P\_actual}$ in a Fourier series, it is possible to numerically extract the Fourier coefficients a_j and b_j. These coefficients can then be used to calculate the coefficients of the series expansion of the function $\Delta y(s)$ through simple algebraic relations. This ultimately leads to the formulation of a 4x4 linear system, which can be solved very quickly;
- The computational cost is dominated by the series expansion of $\Delta C_P(s)$. This expansion is truncated at the index $j_{max} = 3n$. Despite this, the operation remains computationally intensive, resulting in the DISC method being slower than the MGM technique.
- In general, the DISC method is more reliable than the MGM method, as it allows the profile to change its angle of attack during the iterations;

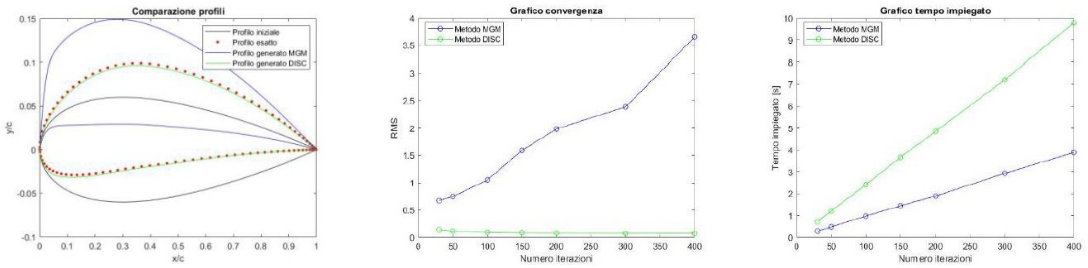
INVERSE DESIGN - COMPARISON

2.5.1 Caso 1 – Profilo NACA-4412 ad $\alpha = 0°$



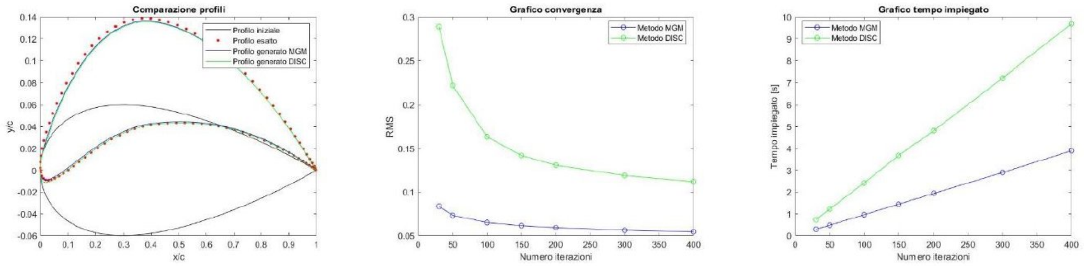
(2.6) Risultati di "mgm.m" (in blu) e "disc.m" (in verde) al variare del numero di iterazioni.

2.5.2 Caso 2 – Profilo NACA-4412 ad $\alpha = 5°$



(2.7) Risultati di "mgm.m" (in blu) e "disc.m" (in verde) al variare del numero di iterazioni.

2.5.3 Caso 3 – Profilo NACA-9410 ad $\alpha = 0°$

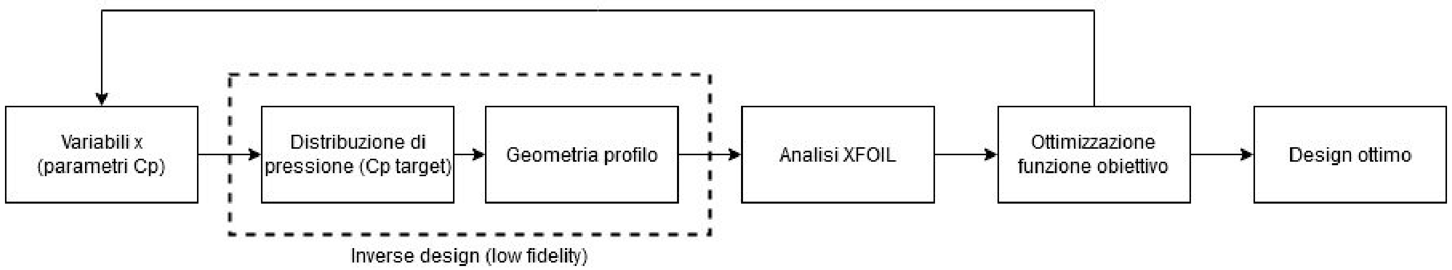


(2.8) Risultati di "mgm.m" (in blu) e "disc.m" (in verde) al variare del numero di iterazioni.

The MGM method offers low computational cost and performs best for zero AoA cases, but it suffers significantly when deviating from this condition.

The DISC method is slower and less accurate, but it is more reliable. According to the literature, it appears to perform better when used with high-fidelity solvers.
It can also be extended to the full 3D case.

EMFID METHOD



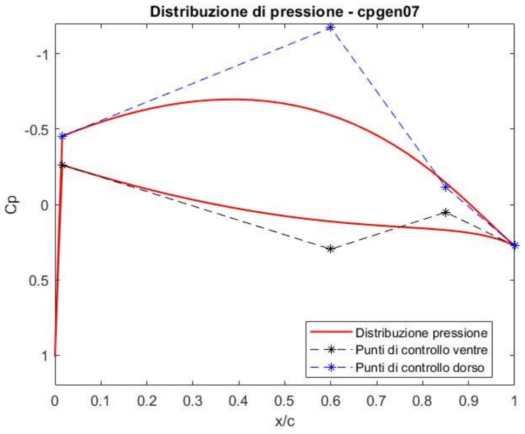
- The EMFID (*Embedded Multi-Fidelity Inverse Design*) method involves parameterizing the pressure distribution C_P instead of the traditional profile geometry;

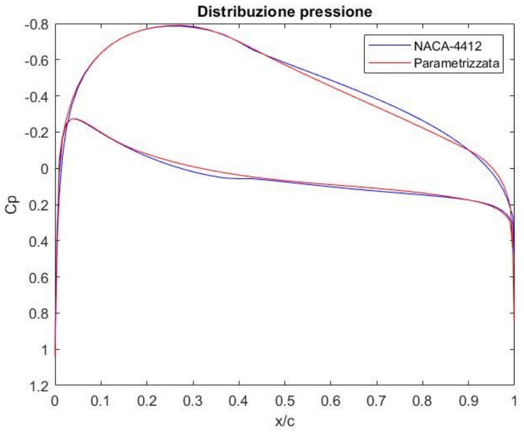
1. The process begins by setting the values of 10 parameters, which are then used to generate a target pressure distribution C_P;
2. The MGM method is applied, which designs the airfoil that generates the given C_P;
3. The resulting profile is analyzed using XFOIL to determine its aerodynamic polars;
4. Finally, the objective function is calculated (e.g. drag);

- The optimization process is carried out using an optimizer, such as a genetic algorithm, to determine the optimal parameters that minimize the objective function;

PARAMETRIZATION

- A 7-variable parameterization (cpgen07), has been found in the literature as a suitable representation for subsonic distributions;
- It uses a Bézier curve generated from 4 control points that can move vertically (see figure on the right). The main problem is that the parameters do not have a clear geometric meaning and it is not straightforward to set limits on the search space. Also, the areas near the LE and TE are not well reproduced;
- To address the limitations of cpgen07, a 10-variable parameterization based on the Hicks-Henne «bump» function is proposed. This has proven to be particularly well-suited to the optimization problems in this work;
- Advantages of the «*flow feature parametrization*»:
  - Lower dimensionality (7-10 vs >=14);
  - Easy to impose constraints on the internal area (=C_L);
  - Local C_P changes affect the entire profile geometry, making convergence easier for the optimizer and avoiding noisy geometries;





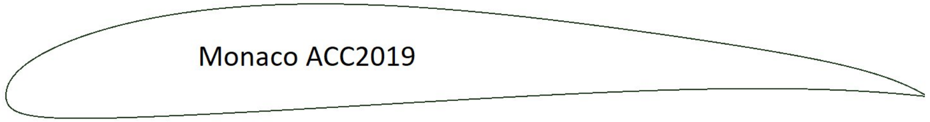
OBJECTIVE FUNCTION AND OPTIMIZATION

- During the race, the wing operates essentially in two different conditions (cruise and turn), respectively for 40% and 60% of the time. The objective function is therefore set as follows:

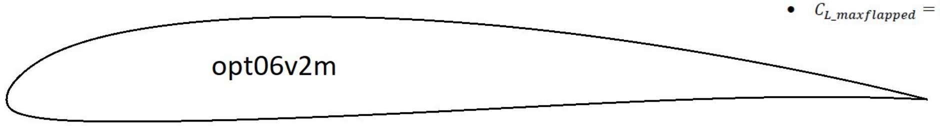
$$f_{opt} = 0.4051\ C_{D\_cruise} + 0.5949\ C_{D\_turn} + P_1 + P_2$$

- Aerodynamic analysis of the airfoil in both conditions (using XFOIL) is required to calculate f_{opt}: cruise $C_{L\_cruise} = 0.37,\ Re = 430k$ and turn $C_{L\_turn} = 0.84,\ Re = 430k$;
- Penalty functions must then be added to satisfy various requirements:
  - $P_1 = 20\max\left[0,\ 1.90 - C_{L\_max\,flapped}\right]^2$ requirement for maximum lift at takeoff;
  - $P_2 = 20\max\left[0,\ 11 - T_{hk}\right]^2$ minimum thickness requirement;
- The following optimization algorithms were used: genetic algorithm (GA), particle swarm optimization (PSO) and pattern search (PS);

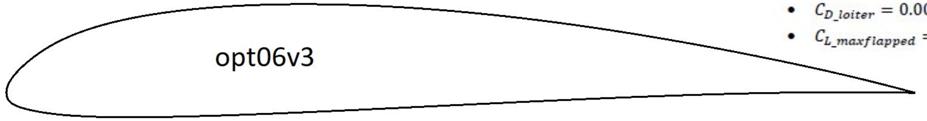
RESULTS

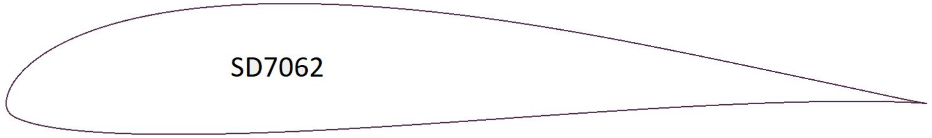


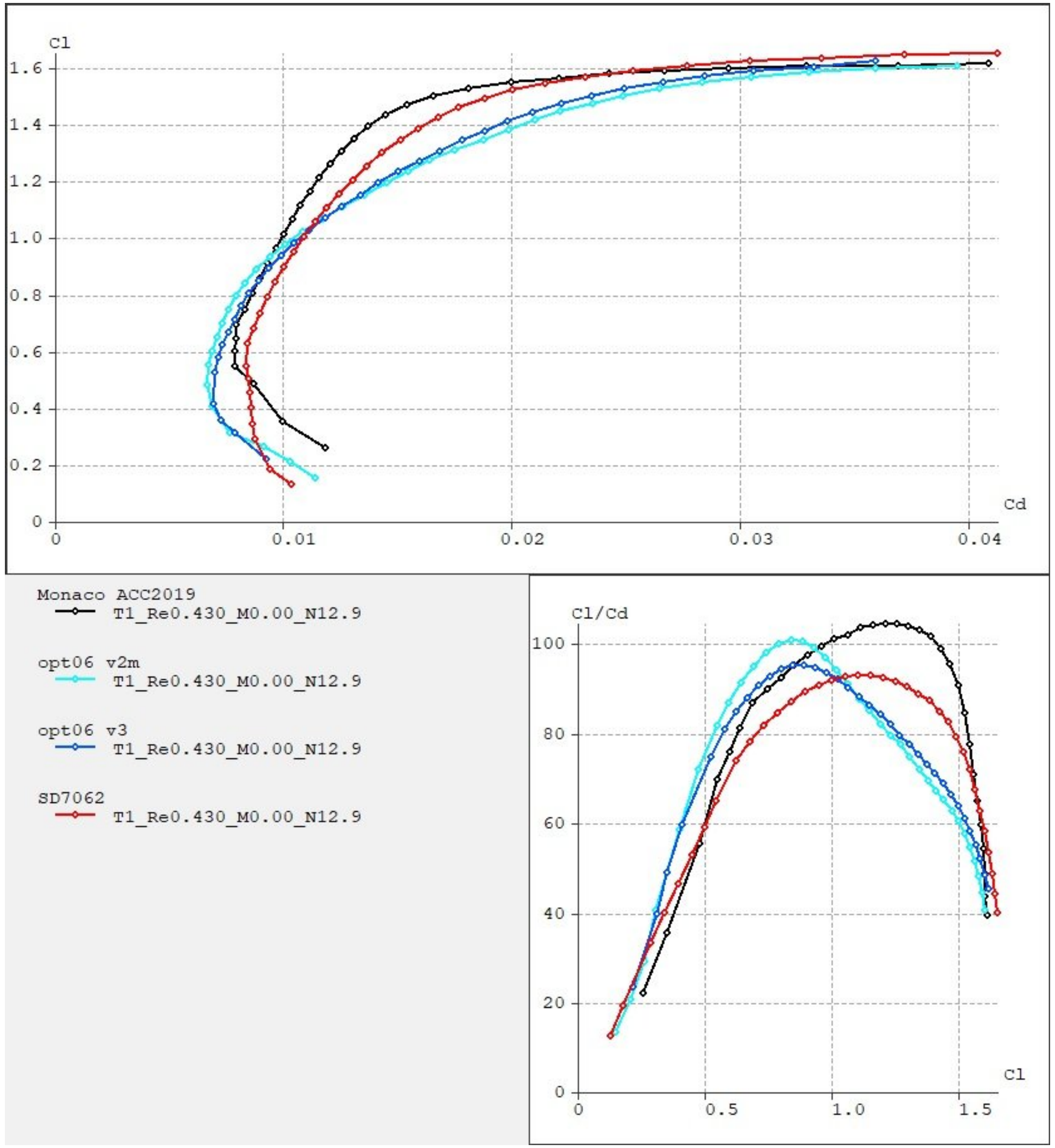
- Spessore max: 11.0%
- Max camber: 3.6%
- $C_{D\_cruise} = 0.00710$ $@C_L = 0.370,\ Re = 430k$
- $C_{D\_loiter} = 0.00835$ $@C_L = 0.840,\ Re = 430k$
- $C_{L\_maxflapped} = 1.85$



- Spessore max: 12.0%
- Max camber: 3.9%
- $C_{D\_cruise} = 0.00711$ $@C_L = 0.370,\ Re = 430k$
- $C_{D\_loiter} = 0.00885$ $@C_L = 0.840,\ Re = 430k$
- $C_{L\_maxflapped} = 1.85$







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