AE 647 – Assignment 1
Particle Distribution Function – 15 marks

Deadline: 28 August 2016

From the discussion in class

We discussed the particle distribution functions in 3-D Configuration Space $(x, y, z) \equiv$ 6-D Phase Space (x, y, z, v_x, v_y, v_z). For uniform velocity, in the reference frame of the gas, we had the following expressions:

In terms of Velocity

$$f(\vec{v}) = n \left(\frac{m}{2\pi kT} \right)^{3/2} \exp\left\{ -\frac{m\vec{v} \cdot \vec{v}}{2kT} \right\}$$

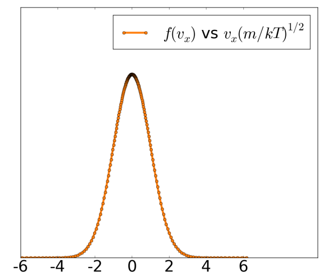

Figure 1: Particle distribution function in terms of velocity

In terms of Speed

$$f(v) = n \left(\frac{m}{2\pi kT} \right)^{3/2} 4\pi v^2 \exp\left\{ -\frac{mv^2}{2kT} \right\}$$

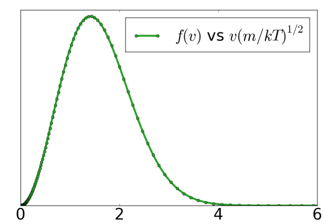

Figure 2: Particle distribution function in terms of speed

In terms of Particle Energy, $\varepsilon = mv^2/2$,

$$f(\varepsilon) = n \frac{\pi}{(\pi kT)^{3/2}} \sqrt{\varepsilon} \exp\left\{ -\frac{\varepsilon}{kT} \right\}$$

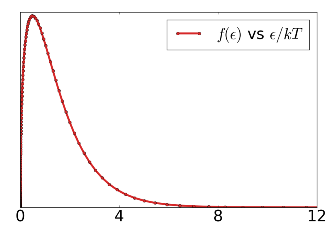

Figure 3: Particle distribution function in terms of particle energy

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Repeat the assignment described in the next page for the following cases:

- A 1-D gas => 1-D in configuration space (x) and an equivalent velocity dimensions
  1-D in configuration space $(x) \leftrightarrow$ 2-D in phase space (x, v_x)
- A 2-D gas => 2-D in configuration space (x,y) and equivalent velocity dimensions (v_x, v_y)
  2-D in configuration space $(x, y) \leftrightarrow$ 2-D in phase space (x, y, v_x, v_y)
- The plots should be readable, i.e., when included in a report, the plot test should be of the same size as the report text size.

Assignment

1. Write the expressions for the equilibrium distribution function and the mean and rms values for the respective variables for $1-D$ and $2-D$ gases.

2. Define the pdfs for a gas whose molecular weight, density and temperature can be given as input parameters.

3. Plot the distributions in terms of velocity, speed and energy

4. Highlight the mean and rms values of the particles in the respective plots.

5. Compute the moments (density, velocity, pressure, internal energy, entropy) in each of the cases.

6. Perform a 'grid independence' study for the moment computations. Can you comment on how the error in moment computation decays with the grid size and grid distribution?

7. Consider any three pdfs that you know from your experience with statistics. Define these pdfs such that they have the same moments as that of the gas being considered. Compare the entropy for these pdfs with the Maxwellian. Verify that the Maxwellian distribution function has a higher entropy than any other distribution function.

8. Consider a *quasi-neutral* Hydrogen plasma at *thermal equilibrium*. Plot the pdfs for ions and electrons in the same figure. Is there a way to make the qualitative details stand out, without compromising accuracy?