Assignment 2

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1 Question 1:

The following schemes were used to solve the motion of particle in presence of the magnetic field. The value of charge was taken to be 5C wih a mass of 10kg. The value of magnetic field was taken to be 1T in $\cap k$ direction. An initial velocity of 10^4 m/s were given to the charge which was initially kept at origin. For the first question we used a time step of 0.01s and the final time was taken to be 200s

1.1 Explicit Euler method

Following is the plot of the position of the charge over many cycles

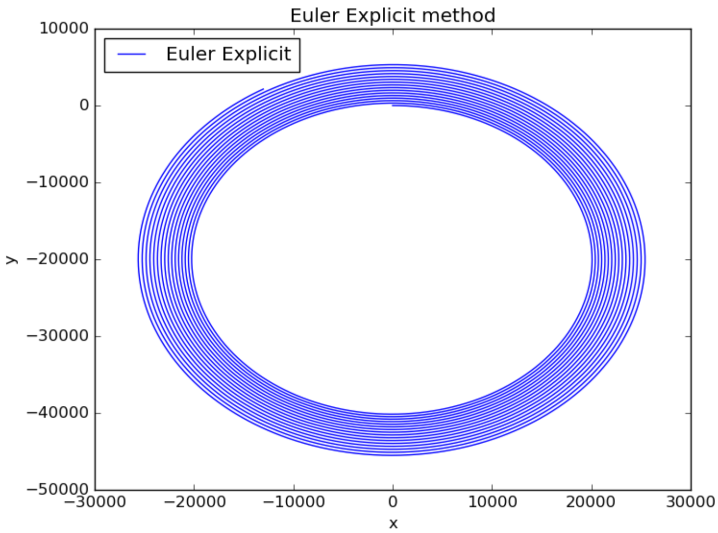

Figure 1: Graph of the position of charged particle over many cycles

We see that the particle spirals outwards.

The evolution of the energy of the particle is shown in the figure below

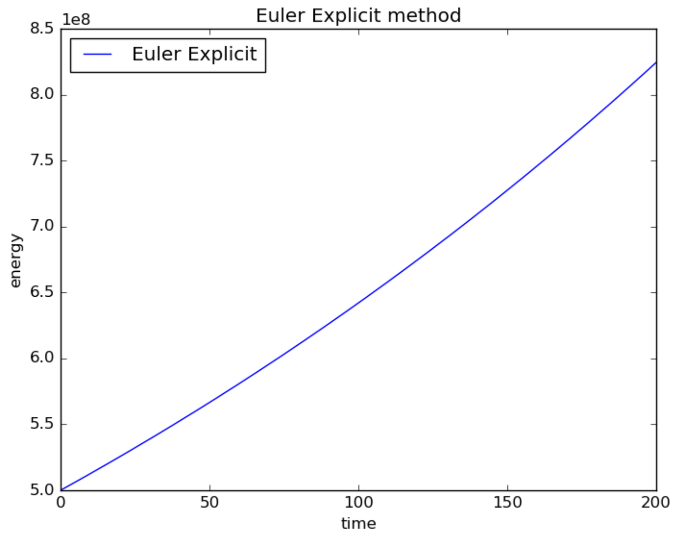

Figure 2: Evolution of energy with time

Thus the energy of the particle increases quadratically with time

1.1.1 Position error:

$$\Delta(x) = O(\Delta t) \tag{1}$$

Following plot shows the variation of error in position with time:

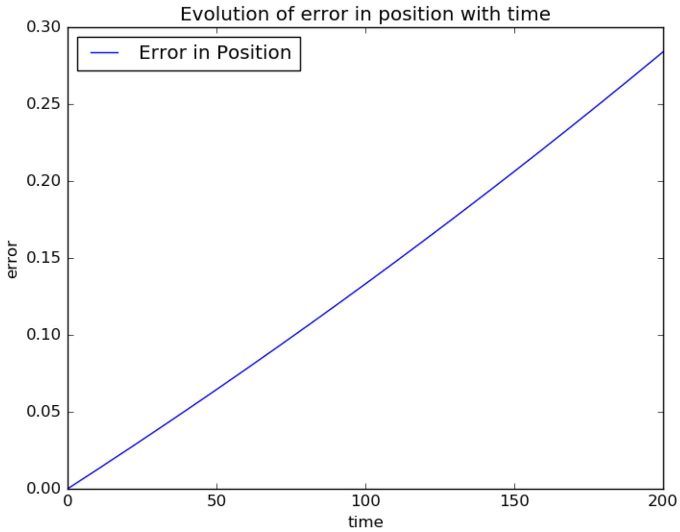

Figure 3: Evolution of position error

1.1.2 Dissipation:

$$\Delta(E) = O(\delta t) \tag{2}$$

1.2 Semi-implicit Euler:

Following is the plot of the position of the charge over many cycles

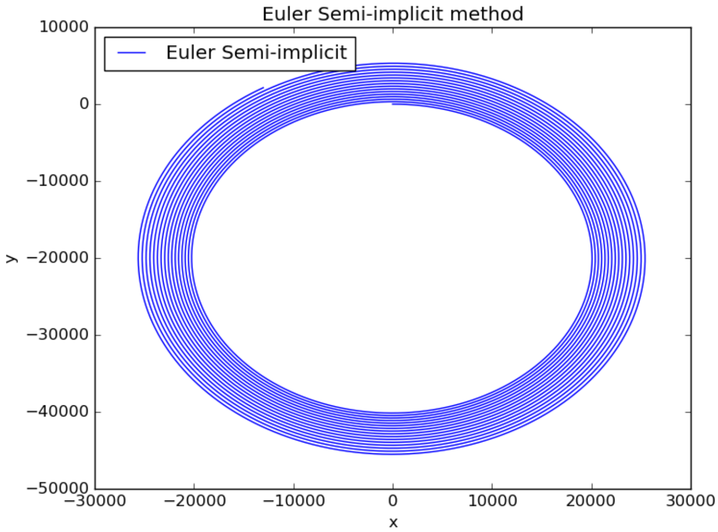

Figure 4: Graph of the position of charged particle over many cycles

We see that the particle spirals outwards.

The evolution of the energy of the particle is shown in the figure below

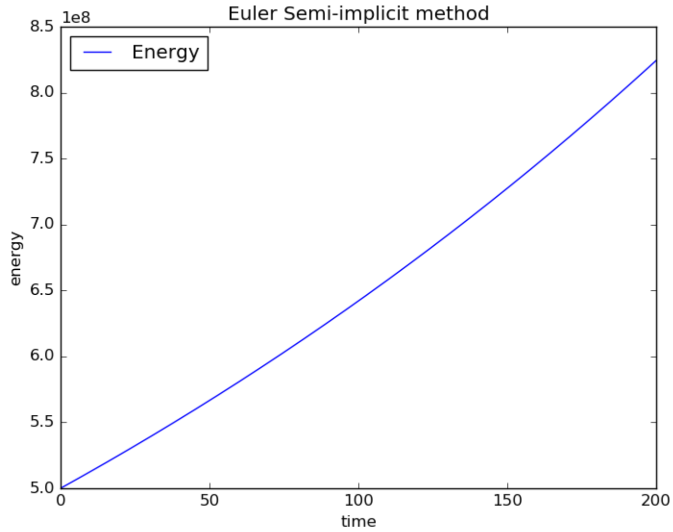

Figure 5: Evolution of energy with time

Thus the energy of the particle increases quadratically with time

1.2.1 Position error:

$$\Delta(x) = O(\Delta t) \tag{3}$$

Following plot shows the variation of error in position with time:

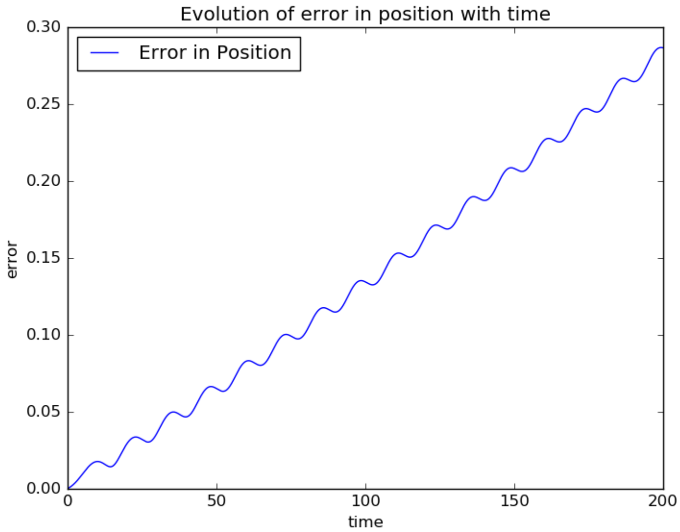

Figure 6: Evolution of position error

1.2.2 Dissipation:

$$\Delta(E) = O(\delta t) \tag{4}$$

1.3 Runge-Kutta method:

Following is the plot of the position of the charge over many cycles

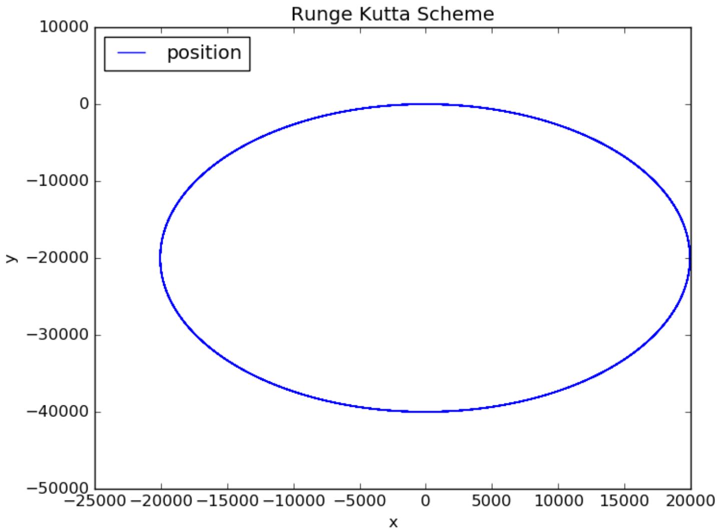

Figure 7: Graph of the position of charged particle over many cycles

We see that the particle spirals outwards but very slowly.

The evolution of the energy of the particle is shown in the figure below

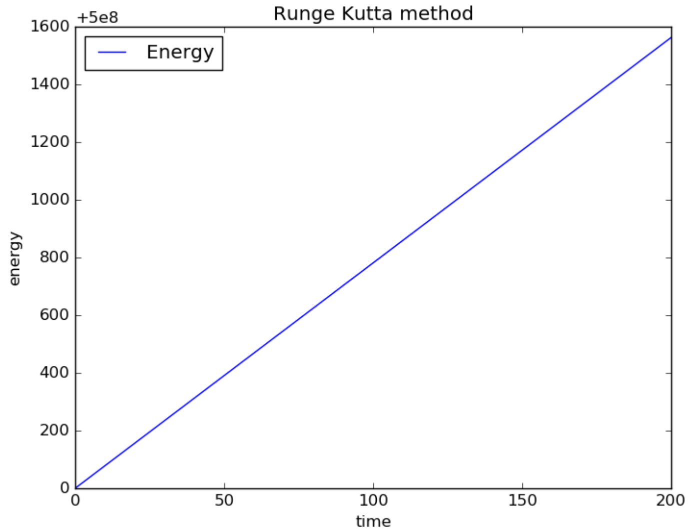

Figure 8: Evolution of energy with time

Thus the energy of the particle increases quadratically with time

1.3.1 Position error:

$$\Delta(x) = O(\Delta t^2) \tag{5}$$

Following plot shows the variation of error in position with time:

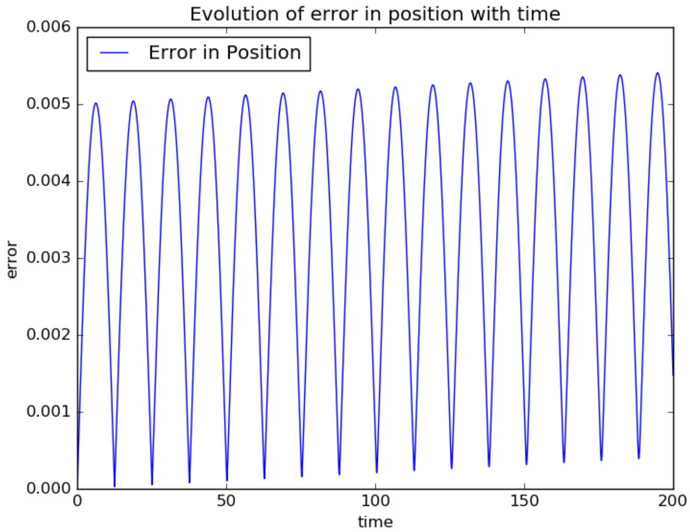

Figure 9: Evolution of position error expressed as phase

1.3.2 Dissipation:

$$\Delta(E) = O(\delta t) \tag{6}$$

1.4 Boris pusher:

Following is the plot of the position of the charge over many cycles

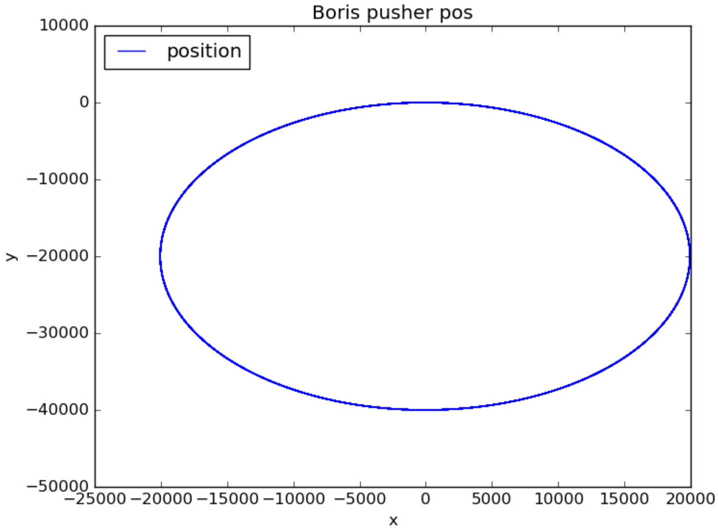

Figure 10: Graph of the position of charged particle over many cycles

We see that the particle spirals outwards.

The evolution of the energy of the particle is shown in the figure below

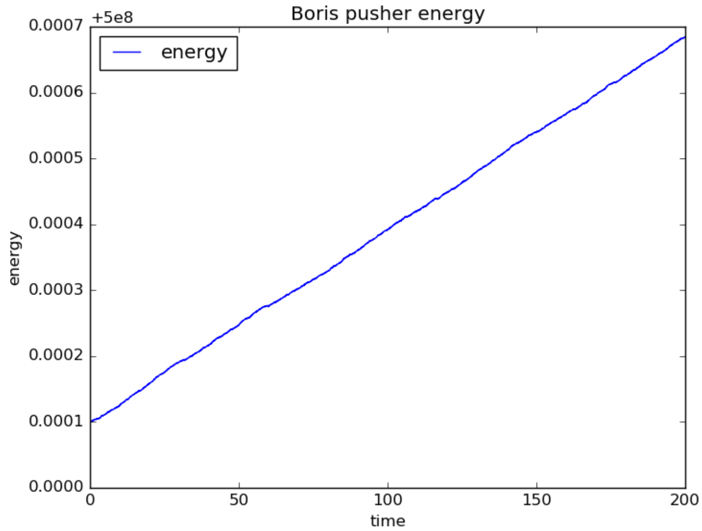

Figure 11: Evolution of energy with time

Thus the energy of the particle increases quadratically with time

1.4.1 Position error:

$$\Delta(x) = O(\Delta t) \tag{7}$$

Following plot shows the variation of error in position with time:

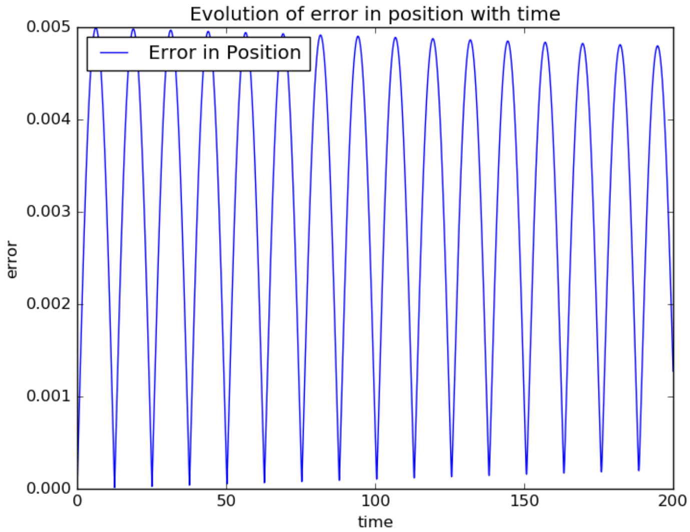

Figure 12: Evolution of position error expressed as phase

1.4.2 Dissipation:

$$\Delta(E) = O(\delta t) \tag{8}$$

2 Question 2:

The variation in the position error in various schemes are plotted below

The variation in energy error in various schemes are plotted below

3 Question 3:

The evolution of position of the charge particle is shown below

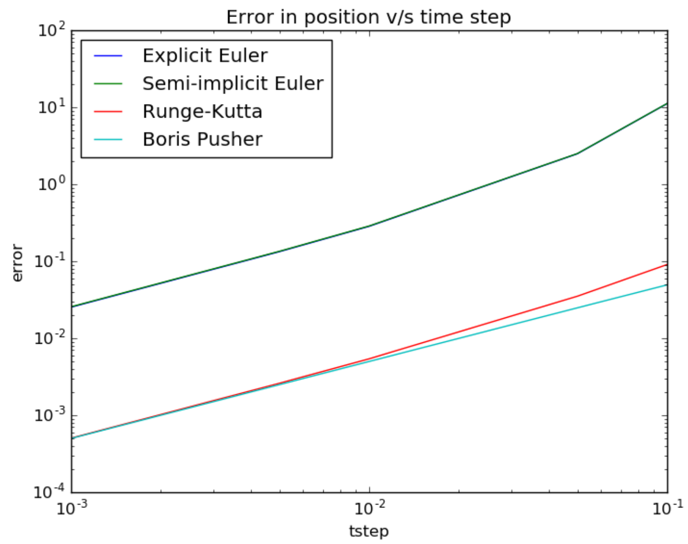

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Figure 13: Variation in the position error

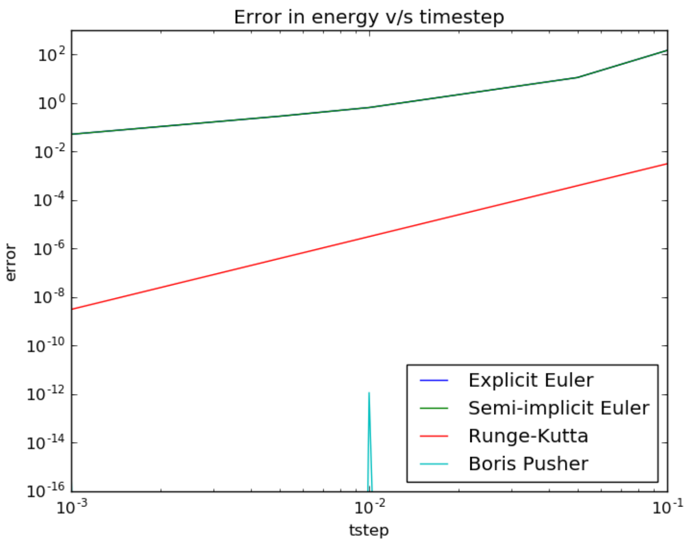

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Figure 14: Variation in the position error

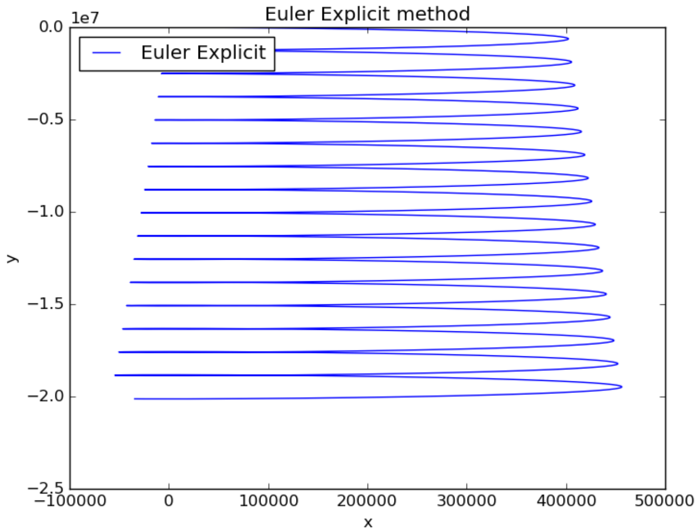

Figure 15: Euler method

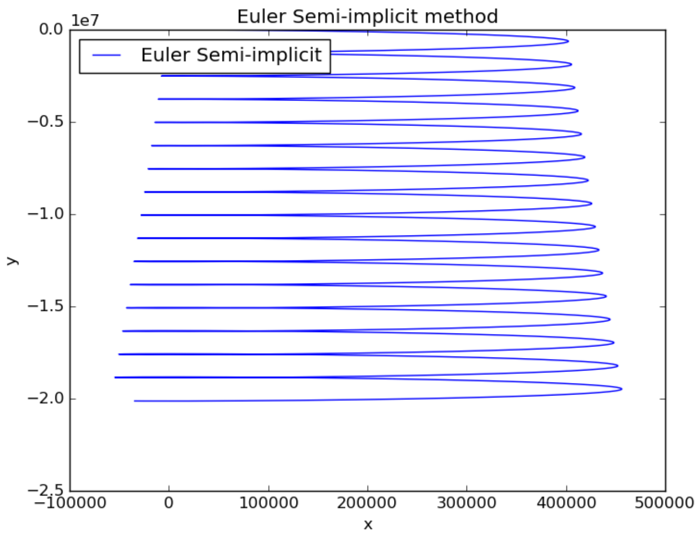

Figure 16: Semi-implicit euler

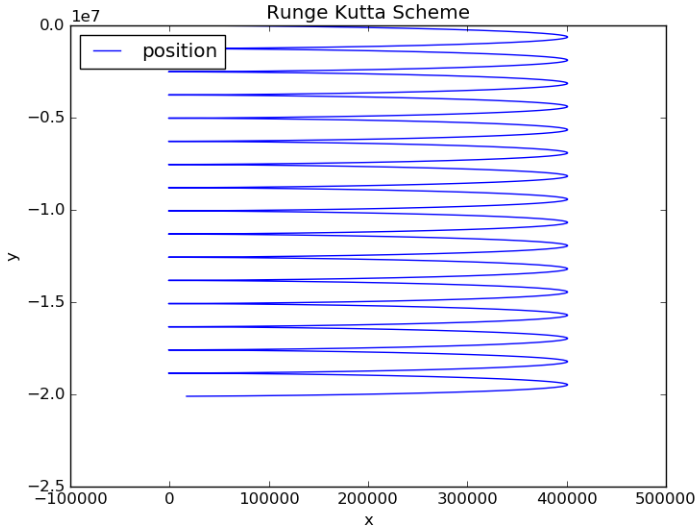

Figure 17: RK2 method

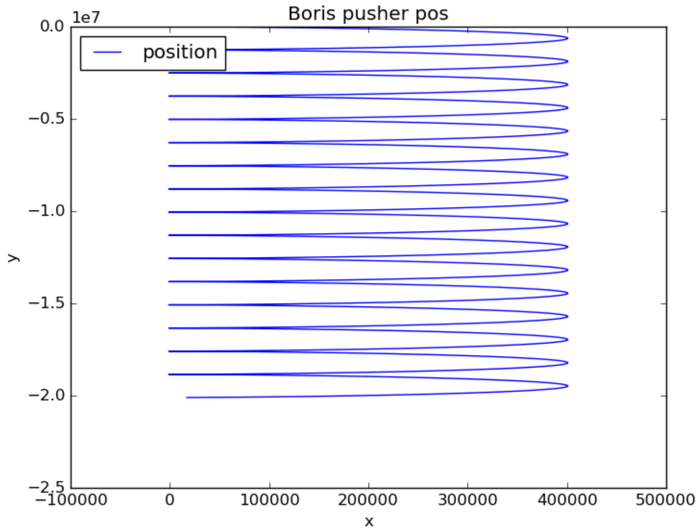

Figure 18: Boris pusher method

Observed y positon in Boris pusher method after 200s = $-2.01 * 10^7$ m/s Observed x position in Boris pusher method after 200s = $1.71 * 10^4$ m/s Since y position due to the gyrating motion will be of the order of 10^4 and observed is $O(10^7)$, we can say that the observed y position is due to the drift velocity i.e Drift velocity = $-2.01 * 10^7/200 = -1.005 * 10^5$ in $\cap j$

Calculated drift velocity = -10^5 Thus the observed drift velocity is within 1% of the calculated drift velocity

4 Question 4:

The motion of the particle under gradB is shown below

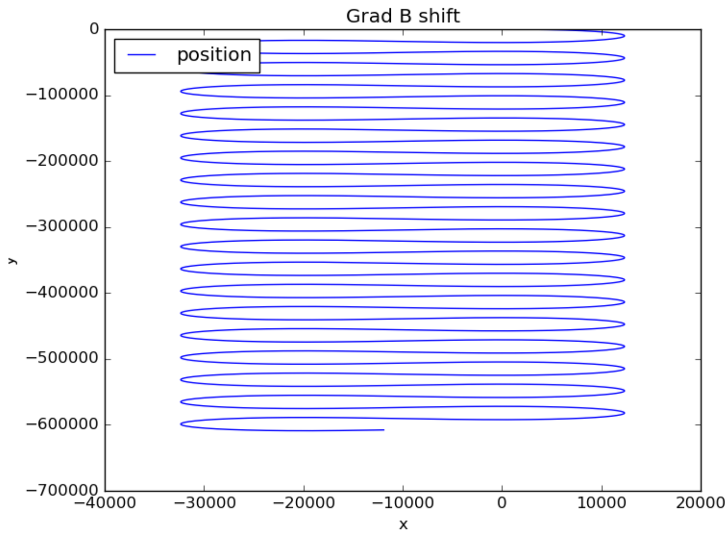

Figure 19: GradB shift using Boris pusher method