AE647 Assignment 4 : Kinetic Theory

Krishna Dutt 163010019

1^{st} Year MTech, Dept. of Aerospace Engineering IIT bombay

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Part I

BGK Model :

Two cold beams are incident on a uniform warm plasma from opposite directions. The warm plasma has a temperature of 1ev and density of $10^{10}m^{-1}$.The cold beams have same density and temperature $(1/100)^{th}$ of the warm plasma.The beams are incident at a velocity 50 times the thermal velocity of the warm plasma.

Applying BGK Collision Model to single species equation for ions, we get

$$\frac{\partial f}{\partial t} + \bar{V}.\nabla f + \frac{\bar{F}}{m}.\nabla_v f = \frac{f_M(\bar{V}) - f(\bar{V})}{\tau_C} \tag{1}$$

Where

1. f is the Particle Distribution Function
2. $\bar{F}$ is the external force
3. f_M is the Equilibrium Boltzmann Distribution
4. $\tau_C = 1/\nu_C$ where ν_C is the collisional frequency
5. m is the mass of the particle

In the absense of external fields and a uniform plasma, the above equation reduces to

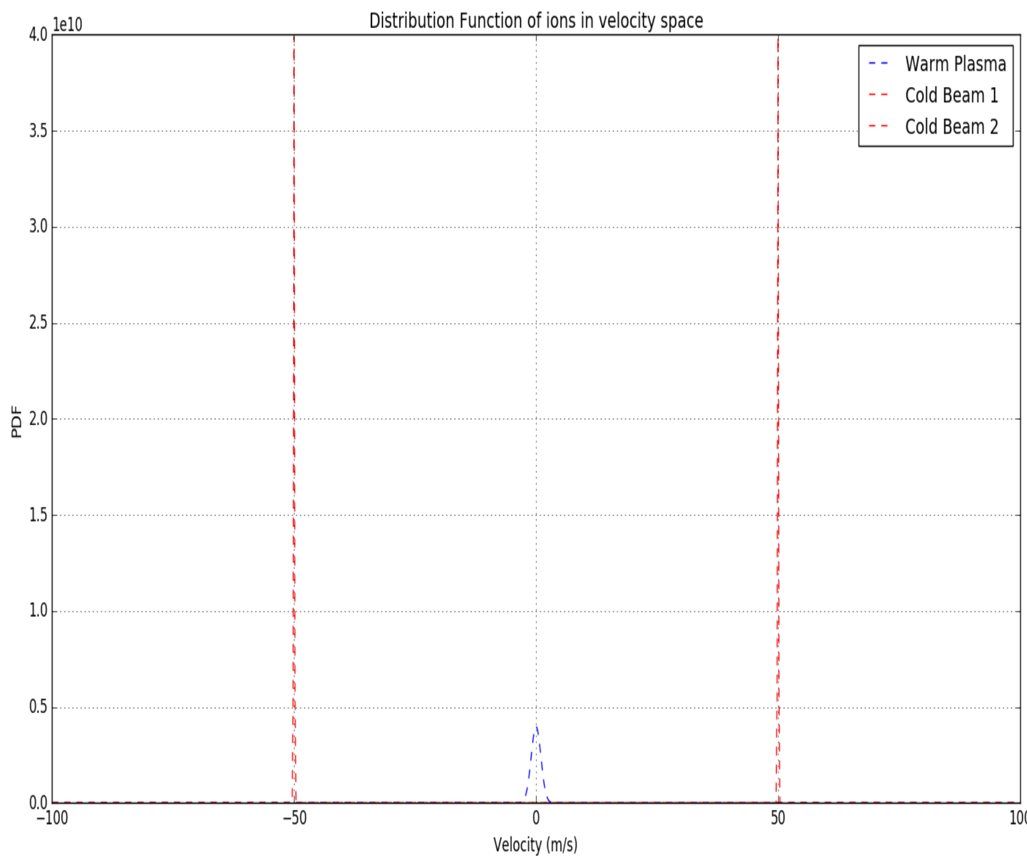
$$\frac{\partial f}{\partial t} = \frac{f_M(\bar{V}) - f(\bar{V})}{\tau_C} \tag{2}$$

At steady state the PDF becomes equal to the Equilibrium Particle ditribution Function.So, for the given problem to be steady in time, $\frac{\partial}{\partial t} << \nu_C$ or $t >> 1/\nu_C$, where $\nu_C = \frac{Ne^4}{\epsilon_0(mKT)^{3/2}}$ whose value for the given problem comes out to be $4.37 * 10^{-16}s^{-1}$.

0.1 Initial PDF for Ions :

The attached figure shows the initial PDF for ions in velocity space for the plasma (which includes the warm plasma ad the two cold beams.)

Figure 1: PDF of Ions in 1D Velocity space



0.2 Equilibrium Distribution Function :

The number density and Temperature of the plasma at equilibrium was computed and were found to be :

1. Number Density N = $2.999 * 10^{10} m^{-1}$
2. Temperature KT = 0.3401ev

Thus , the Equilibrium Boltzmann distribution Function can be written as :

$$f_M(\bar{V}) = N(\frac{m}{2\pi KT})^{1/2} \exp(-\frac{m(\bar{V})^2}{2KT}) \tag{3}$$

Where

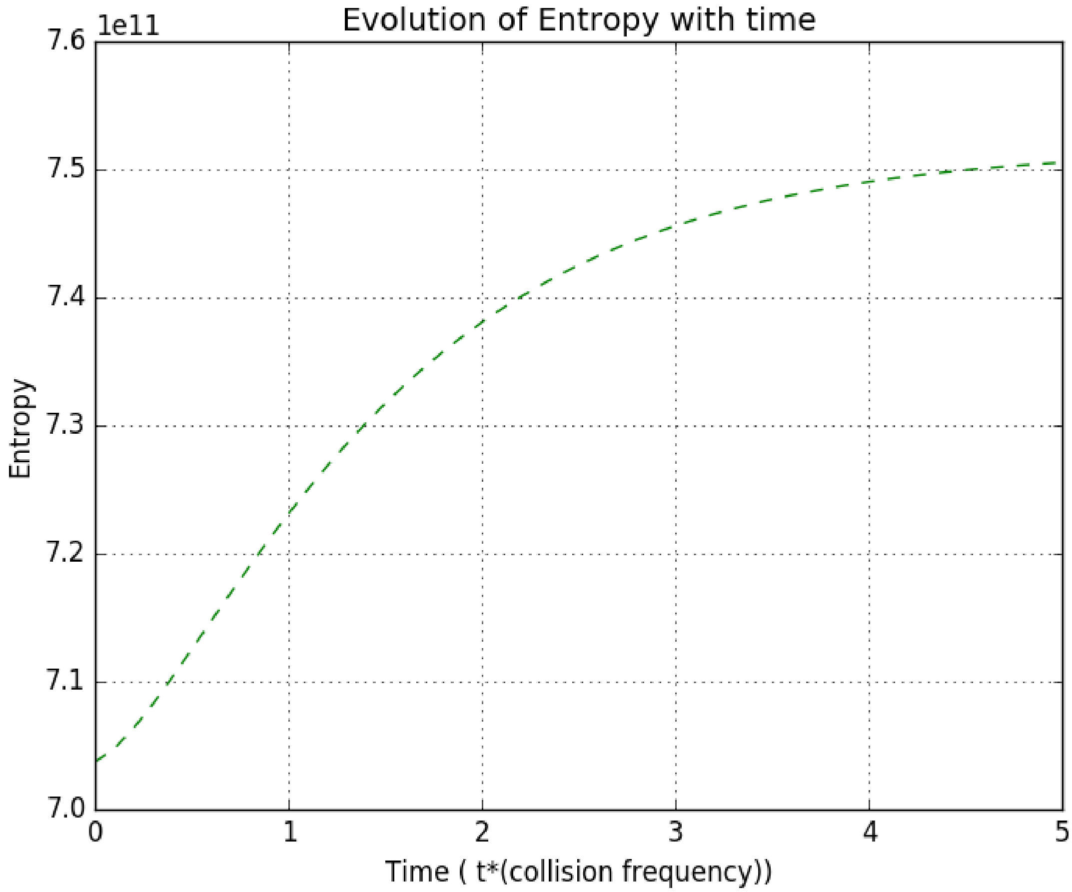
1. N and KT take the computed value
2. m is taken to be $1.6 * 10^{-19}$ Kg
3. The net drift velocity is zero to conserve momentum.

0.3 Simulation using BGK :

The above equation contiaining BGK model was simulated to find the Equilibrium PArticle distribution Function. The simulation took an approximate time of $t = 5.0/\nu_C$ to reach within 1% of the Equilibrium Boltzmann Distribution,where $\nu_C = 4.37 * 10^{-16} s^{-1}$

The evolution of entropy with time was also calculated. It was observed that the entropy increases with time and attains a maximum at Equilibrium Distribution.

Figure 2: Evolution of Entropy with time



Part II

Conclusion

0.4 Summary :

Using the BGK model, the equilibrium PDF for the given problem was arrived at and the corresponding time to attain equilibrium was also computed.

The evolution of entropy with time was also measured and was found to increase with time attaining its maximum value at equilibrium.