AE 706: Assignment 3

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March 3, 2017

Contents

1 Introduction:

In this assignement we implement numerical schemes to solve the linear advection equation and analyse their stability

2 Question 1:

In this question we solve the given intial condition and boundary conditions using FTBS, FTFS and FTCS schemes with CFL values 0.8, 1.0 and 1.2.

2.1 FTBS

2.1.1 CFL = 0.8

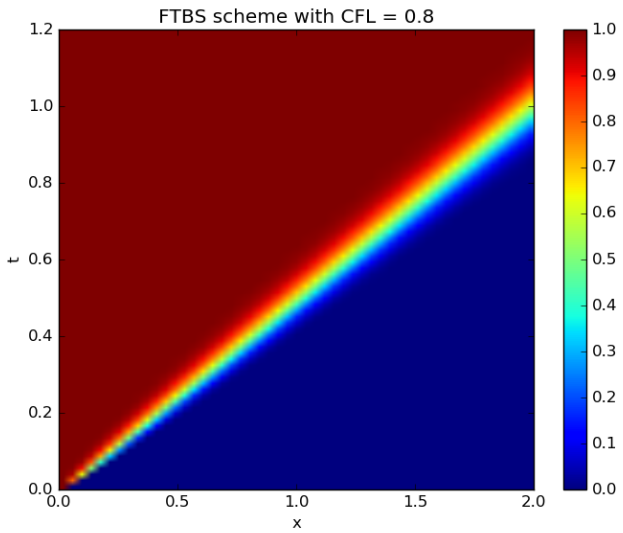

Figure 1: Color represents the magnitude of u at the given x and t

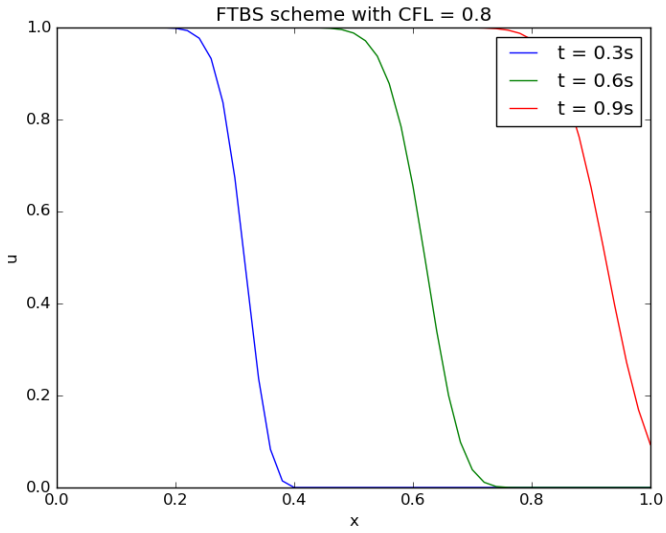

Figure 2: Plot of u v/s x for various t

We see that that the step function starts smoothening. This is because the higher frequency components dampen faster in in comparison to lower frequencies.

2.1.2 CFL = 1.0

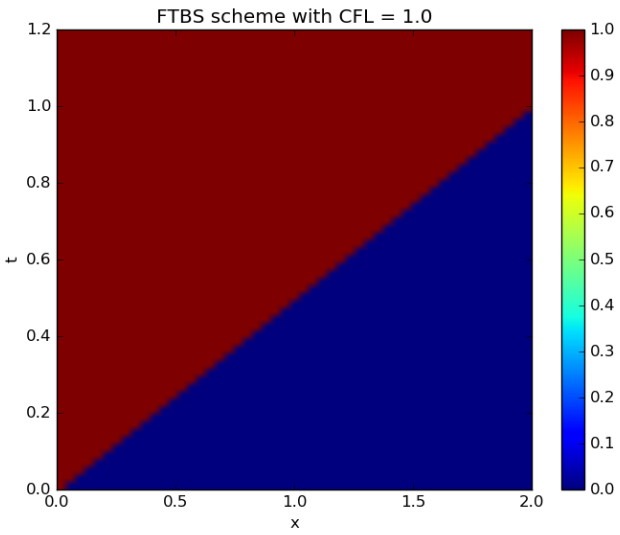

Figure 3: Color represents the magnitude of u at the given x and t

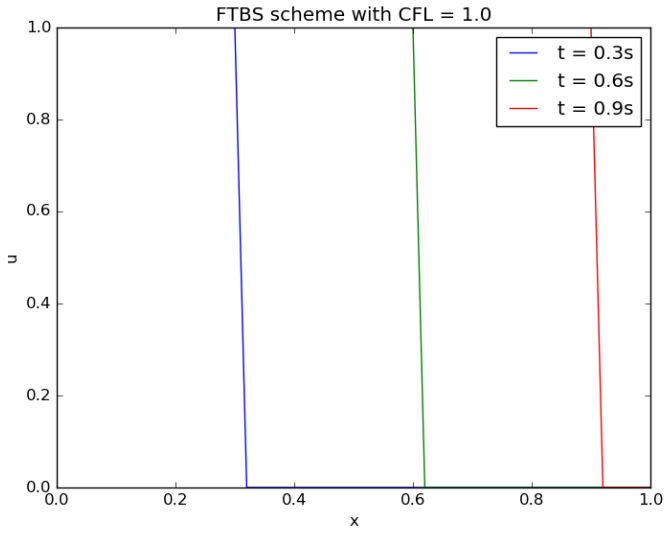

Figure 4: Plot of u v/s x for various t

We see that the function propogates without any change in its shape. Though the shape should technically be a step function the figure shown is that of a ramp. This can be improved by decreasing the grid size.

2.1.3 CFL = 1.2

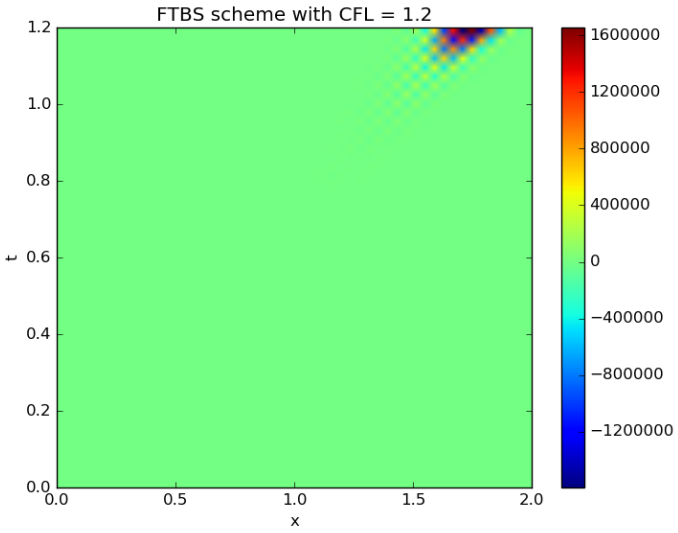

Figure 5: Color represents the magnitude of u at the given x and t

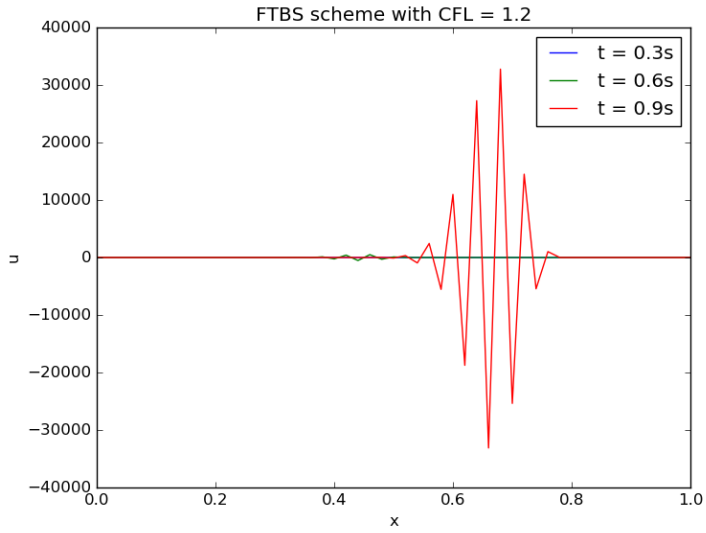

Figure 6: Plot of u v/s x for various t

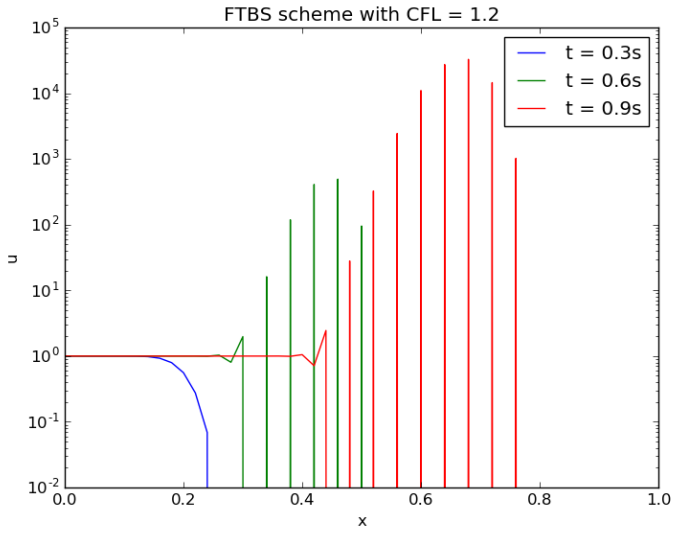

Figure 7: Plot of u v/s x for various t in log scale

We see that the solution diverges to very high values with time. i,e solution isn't stable

2.2 FTFS

2.2.1 CFL = 0.8

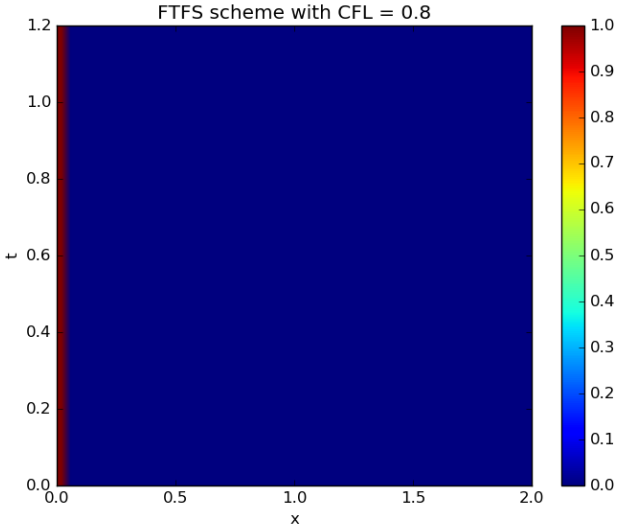

Figure 8: Color represents the magnitude of u at the given x and t

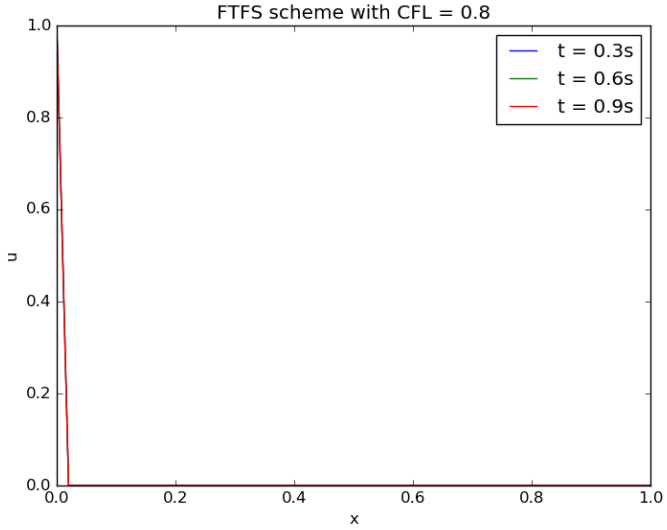

Figure 9: Plot of u v/s x for various t

2.2.2 CFL = 1.0

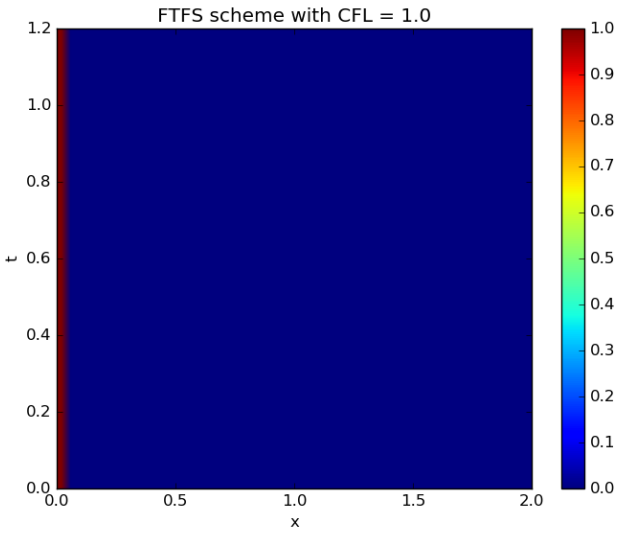

Figure 10: Color represents the magnitude of u at the given x and t

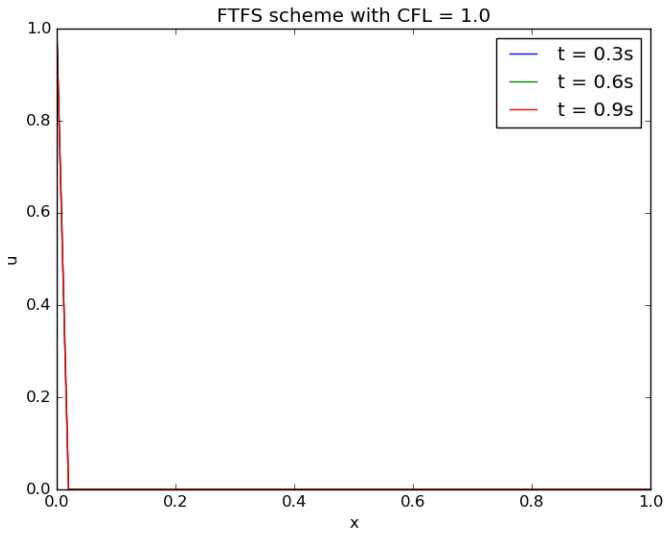

Figure 11: Plot of u v/s x for various t

2.2.3 CFL = 1.2

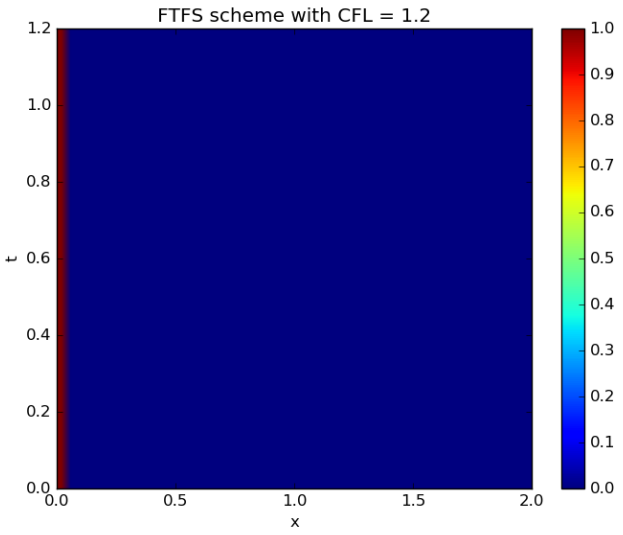

Figure 12: Color represents the magnitude of u at the given x and t

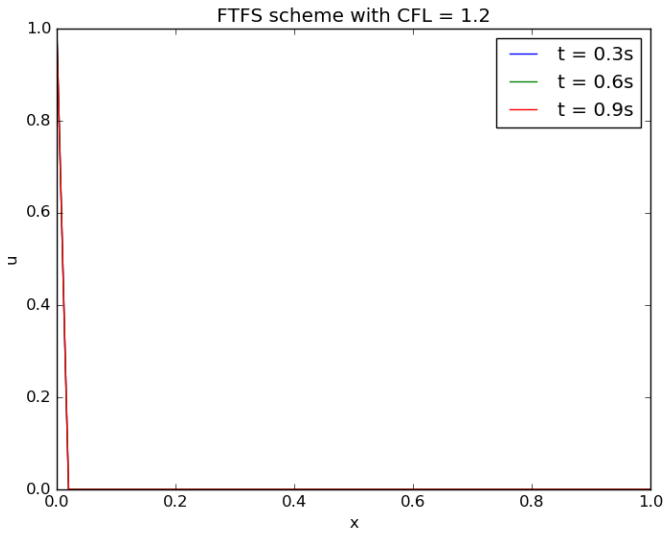

Figure 13: Plot of u v/s x for various t

We see that the solution doesn't propogate at all under FTFS. This is because the initial condition is set to xero at all points

2.3 FTCS

2.3.1 CFL = 0.8

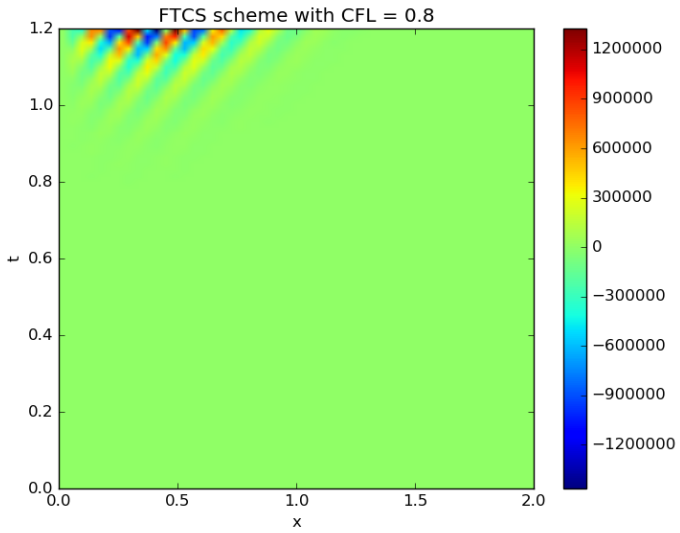

Figure 14: Color represents the magnitude of u at the given x and t

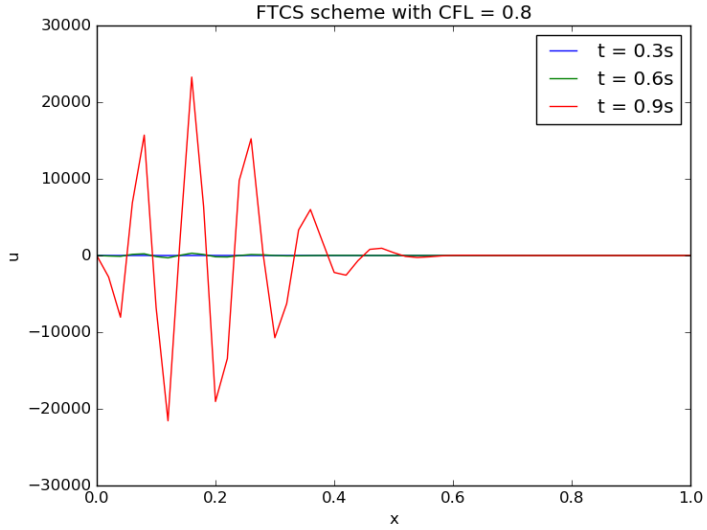

Figure 15: Plot of u v/s x for various t

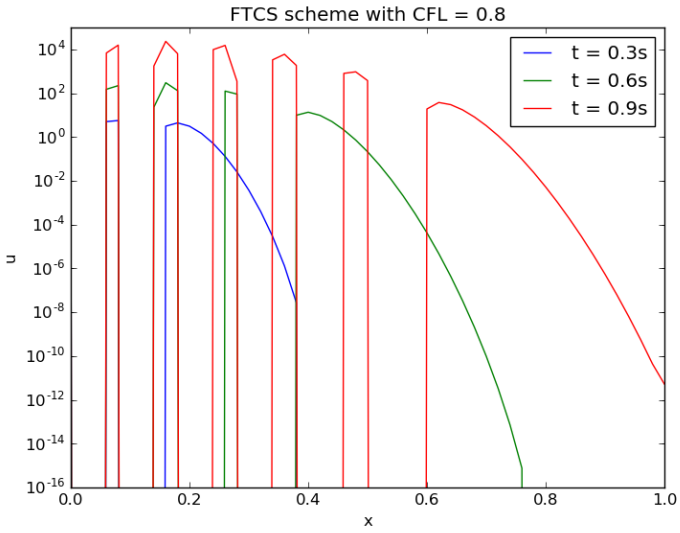

Figure 16: Plot of u v/s x for various t in log scale

2.3.2 CFL = 1.0

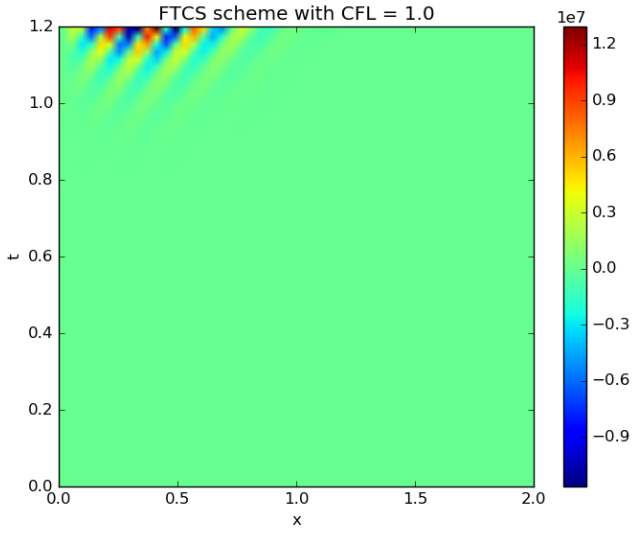

Figure 17: Color represents the magnitude of u at the given x and t

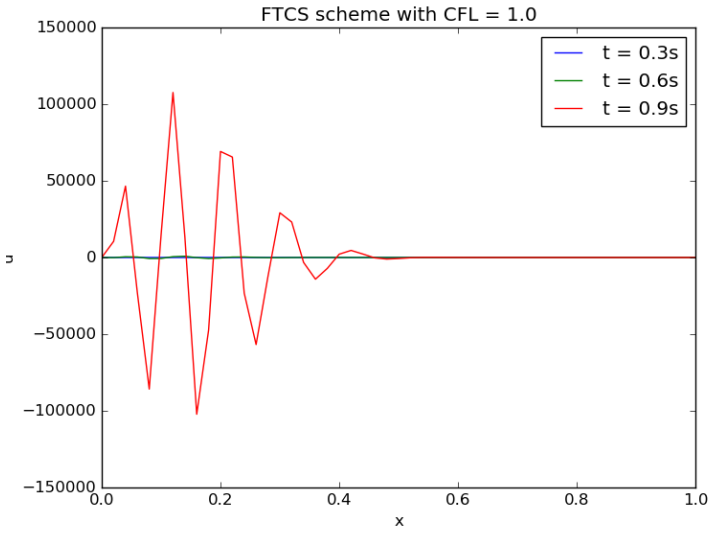

Figure 18: Plot of u v/s x for various t

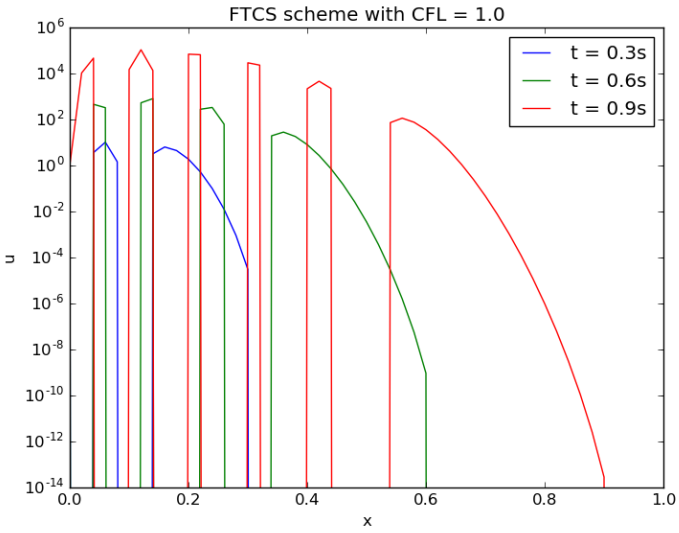

Figure 19: Plot of u v/s x for various t in log scale

2.3.3 CFL = 1.2

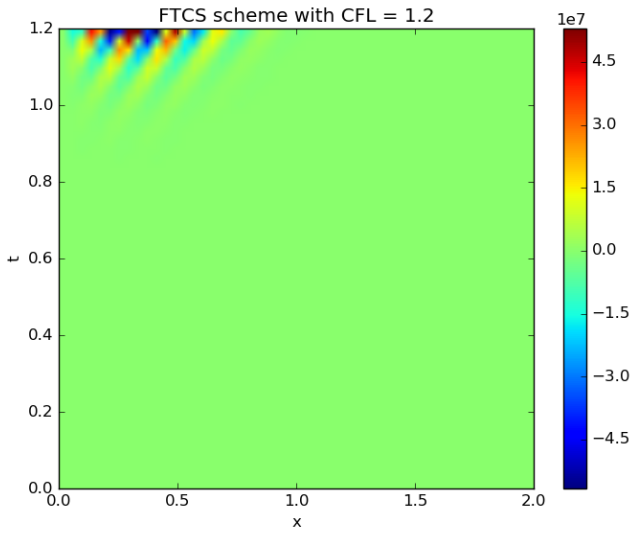

Figure 20: Color represents the magnitude of u at the given x and t

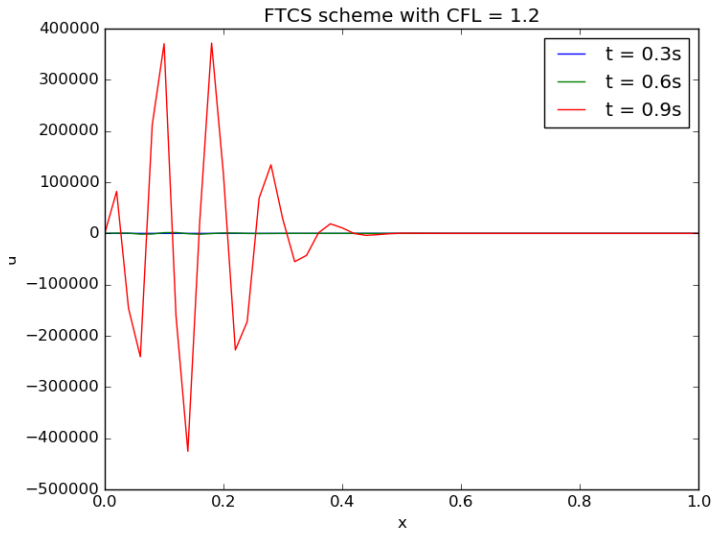

Figure 21: Plot of u v/s x for various t

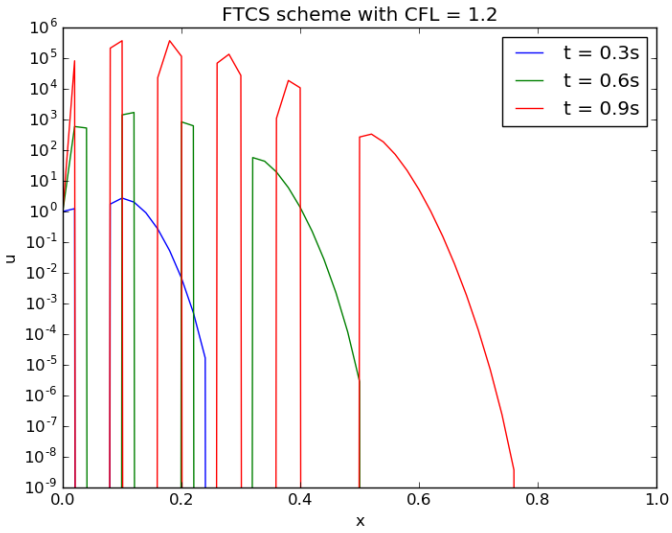

Figure 22: Plot of u v/s x for various t in log scale

We see that the solution diverges for all CFL values.

3 Question 2:

In this question we solve the linear wave equation when the initial condition is a sine wave and/or a combination of sine waves

3.1 Single frequency

3.1.1 FTBS

CFL = 0.8

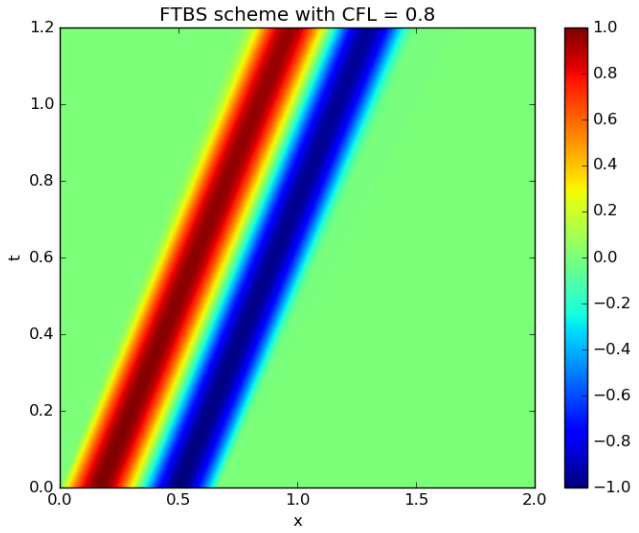

Figure 23: Color represents the magnitude of u at the given x and t

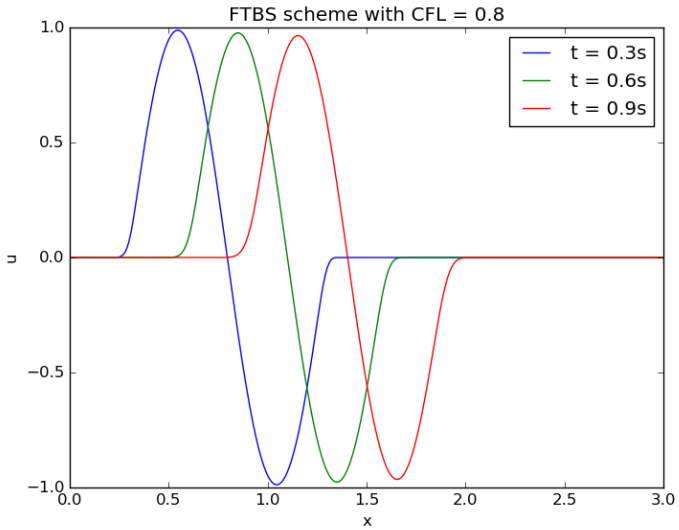

Figure 24: Plot of u v/s x for various t

We see that that the function starts smoothening and also dampens with time.

3.1.2 CFL = 1.0

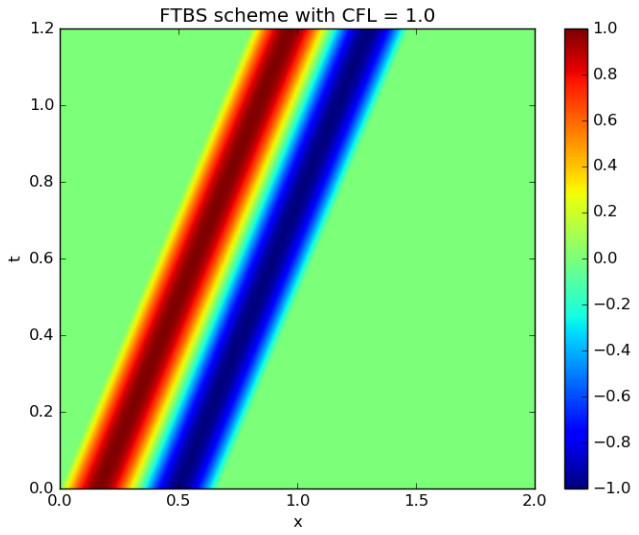

Figure 25: Color represents the magnitude of u at the given x and t

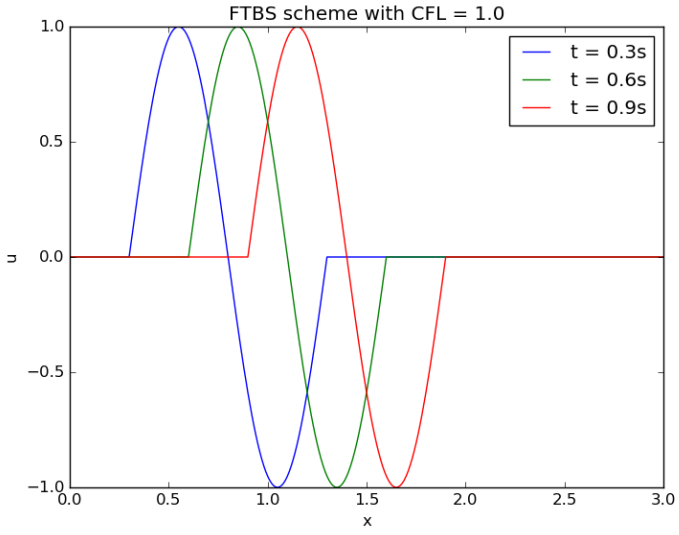

Figure 26: Plot of u v/s x for various t

The sine wave propogates without any damping.

3.1.3 CFL = 1.2

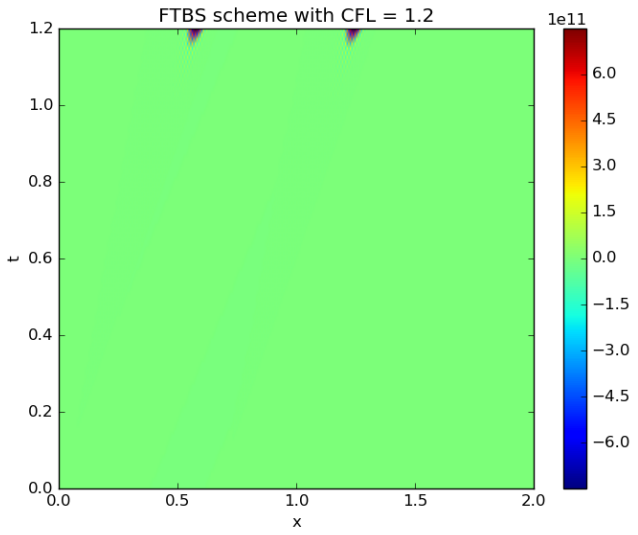

Figure 27: Color represents the magnitude of u at the given x and t

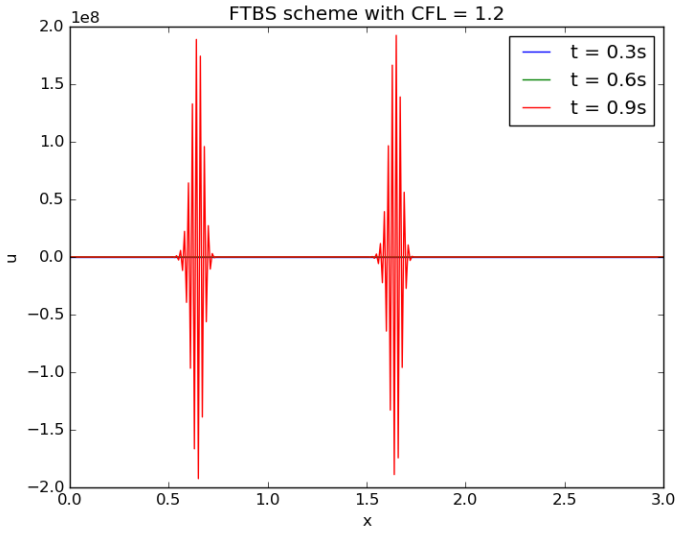

Figure 28: Plot of u v/s x for various t

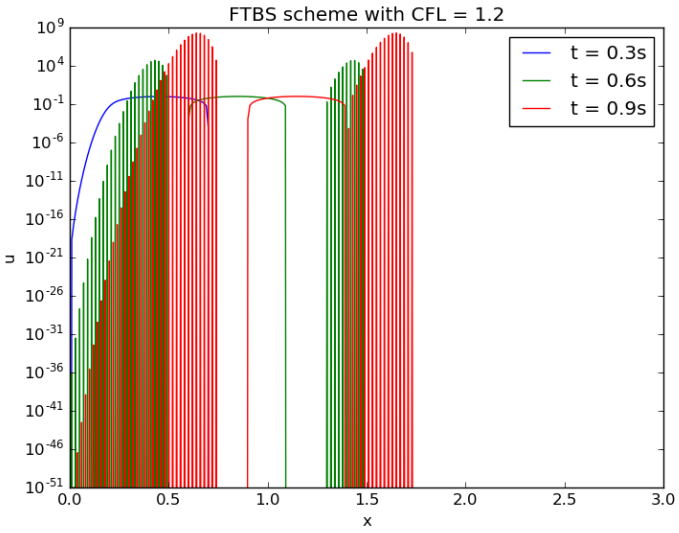

Figure 29: Plot of u v/s x for various t in log scale

We see that the solution diverges to very high values with time. i,e solution isn't stable

3.2 FTFS

3.2.1 CFL = 0.8

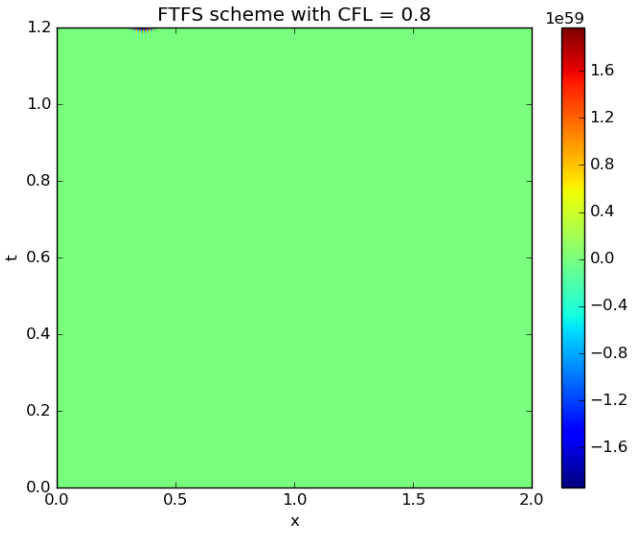

Figure 30: Color represents the magnitude of u at the given x and t

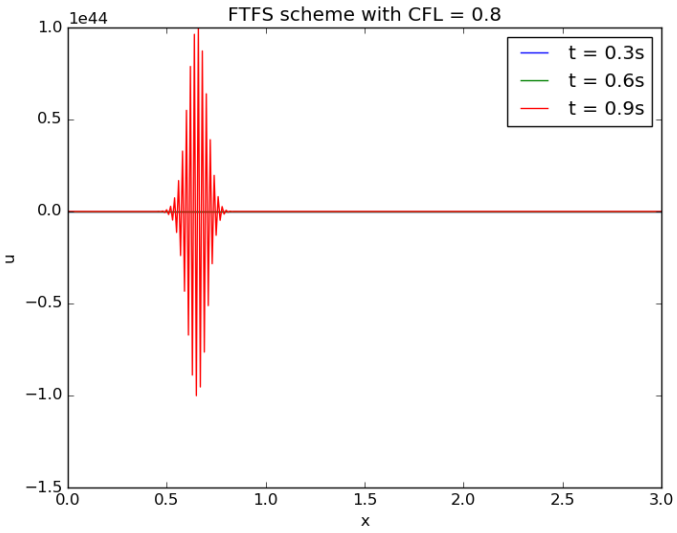

Figure 31: Plot of u v/s x for various t

3.2.2 CFL = 1.0

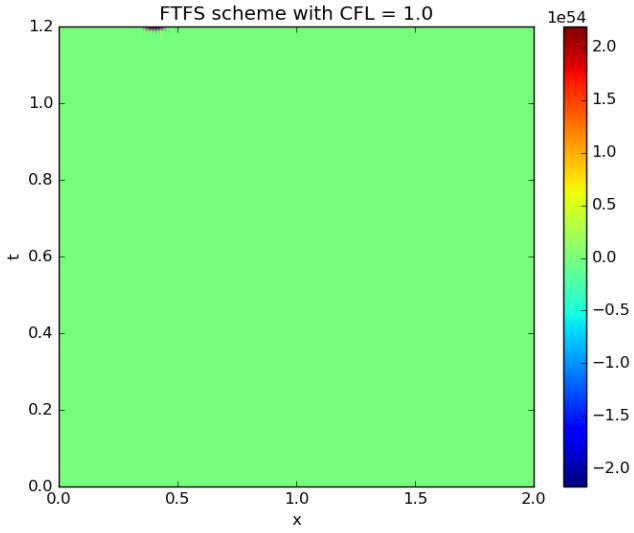

Figure 32: Color represents the magnitude of u at the given x and t

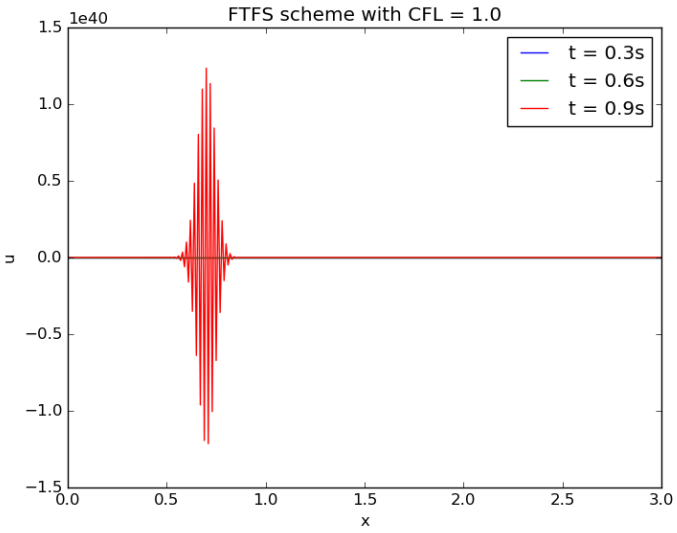

Figure 33: Plot of u v/s x for various t

3.2.3 CFL = 1.2

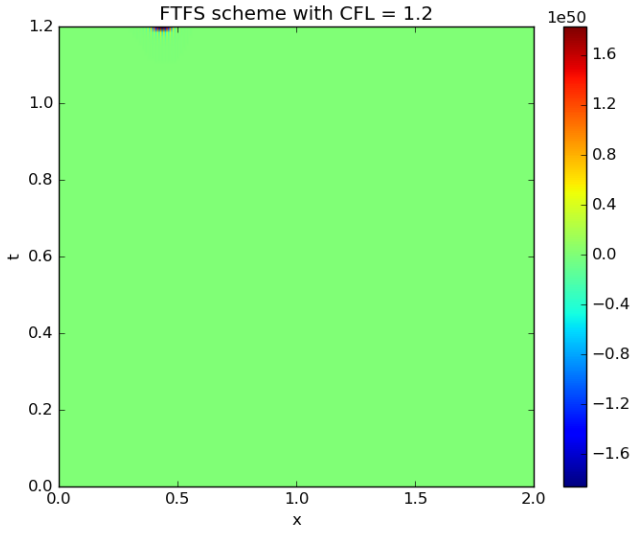

Figure 34: Color represents the magnitude of u at the given x and t

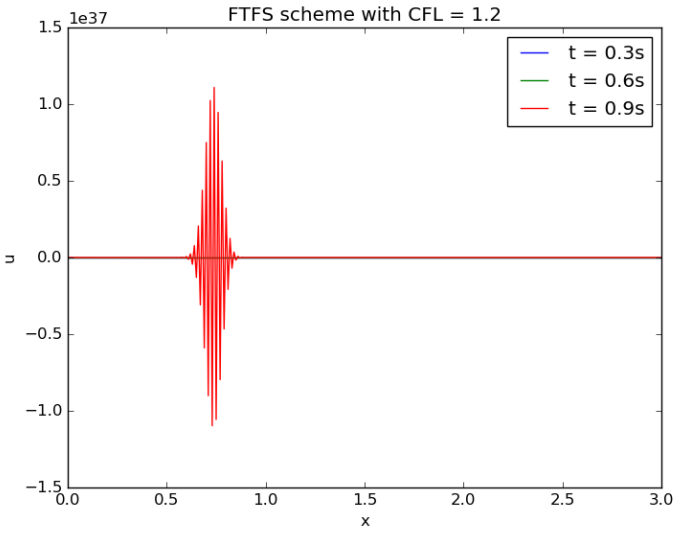

Figure 35: Plot of u v/s x for various t

We see that the solution diverges i.e the scheme is unstable

3.3 FTCS

3.3.1 CFL = 0.8

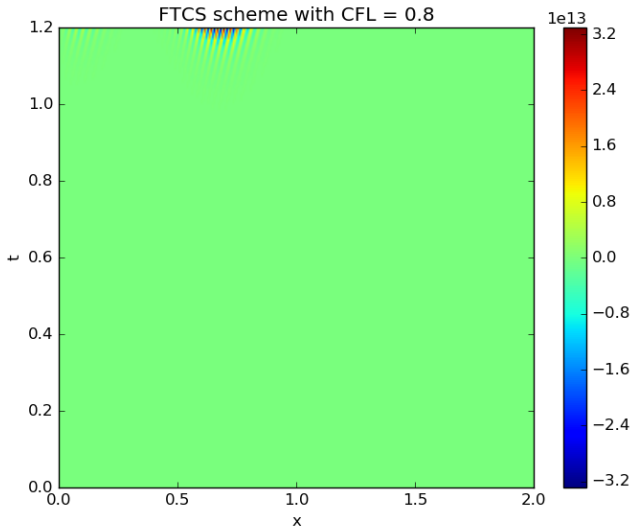

Figure 36: Color represents the magnitude of u at the given x and t

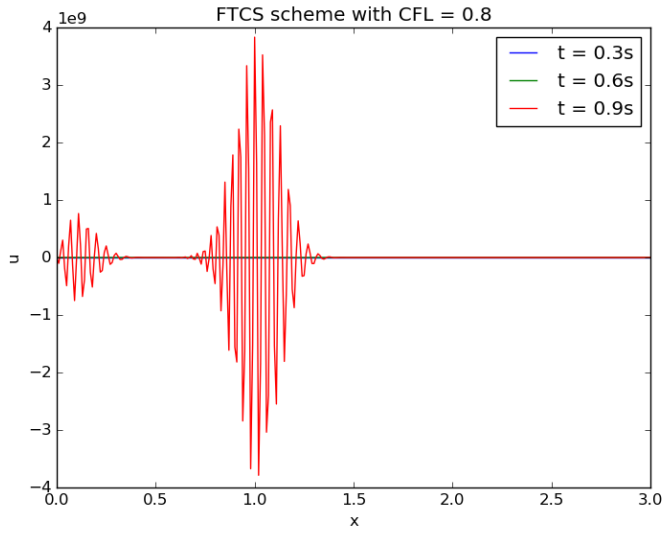

Figure 37: Plot of u v/s x for various t

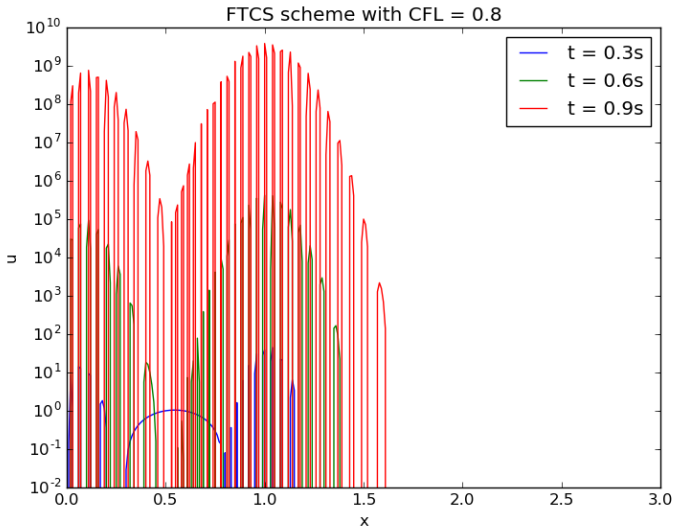

Figure 38: Plot of u v/s x for various t in log scale

3.3.2 CFL = 1.0

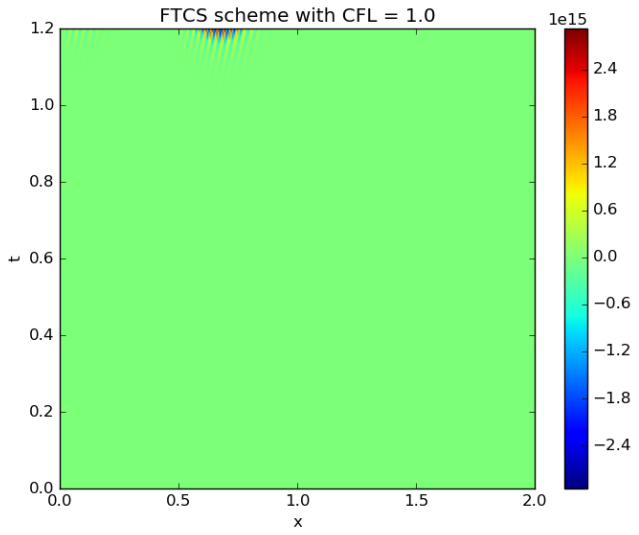

Figure 39: Color represents the magnitude of u at the given x and t

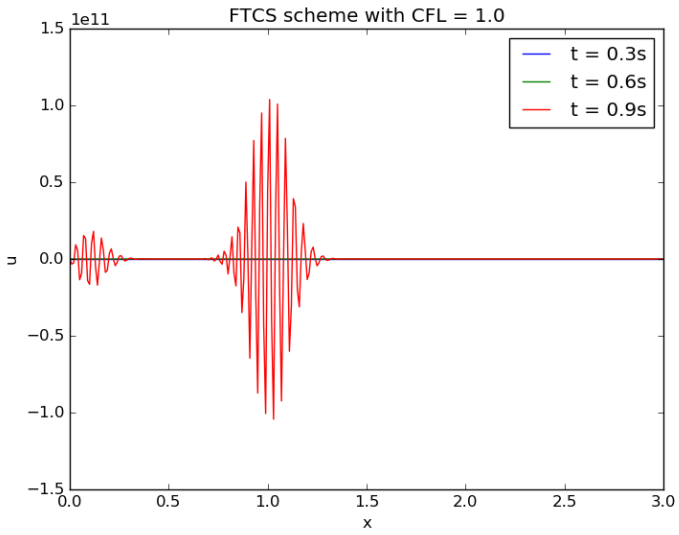

Figure 40: Plot of u v/s x for various t

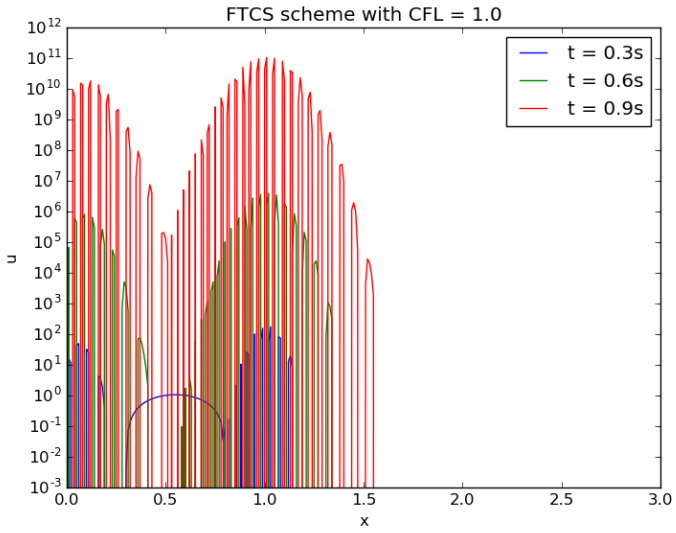

Figure 41: Plot of u v/s x for various t in log scale

3.3.3 CFL = 1.2

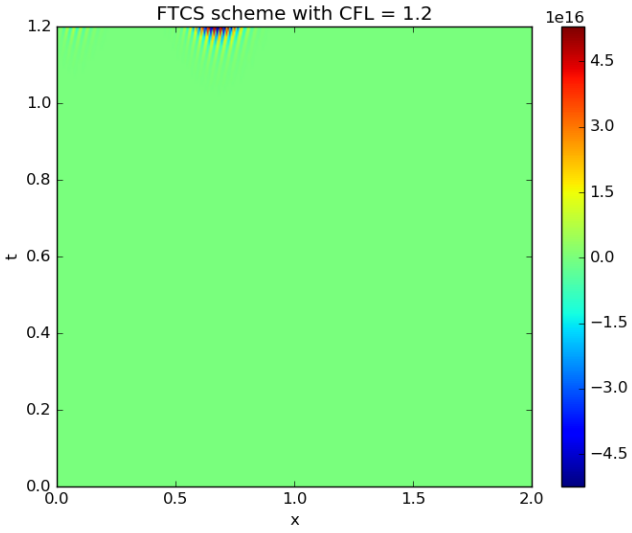

Figure 42: Color represents the magnitude of u at the given x and t

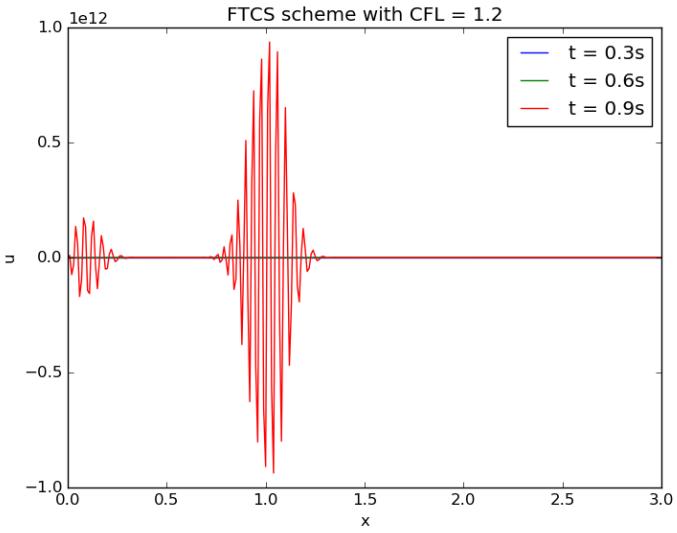

Figure 43: Plot of u v/s x for various t

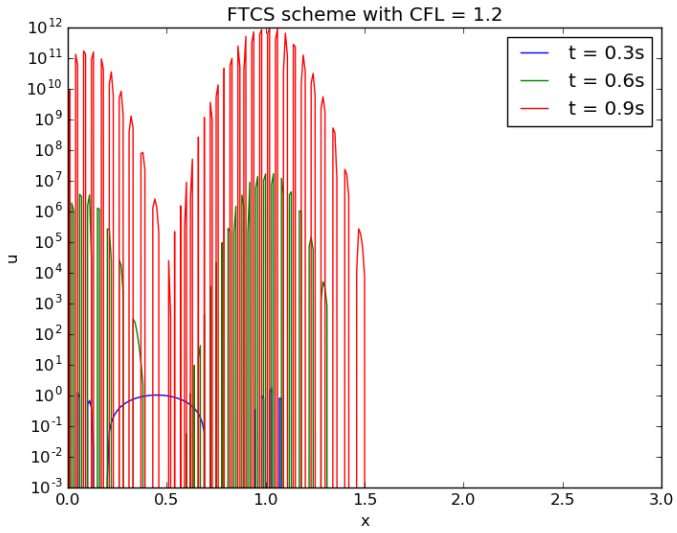

Figure 44: Plot of u v/s x for various t in log scale

We see that the solution doesn't converge.

3.4 Multiple frequencies

3.4.1 FTBS

CFL = 0.8

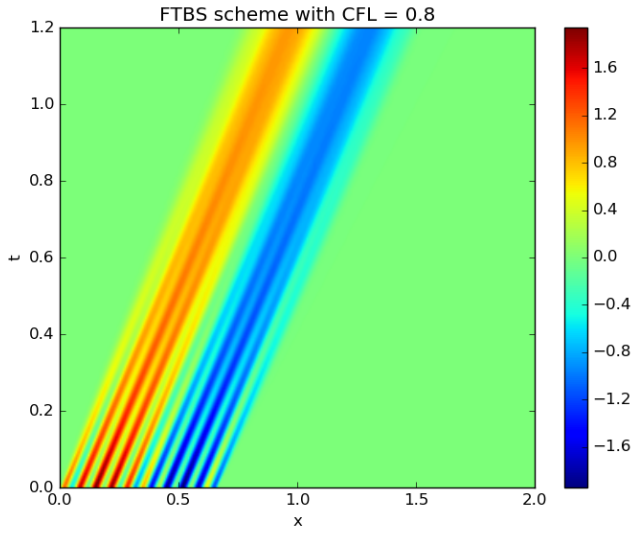

Figure 45: Color represents the magnitude of u at the given x and t

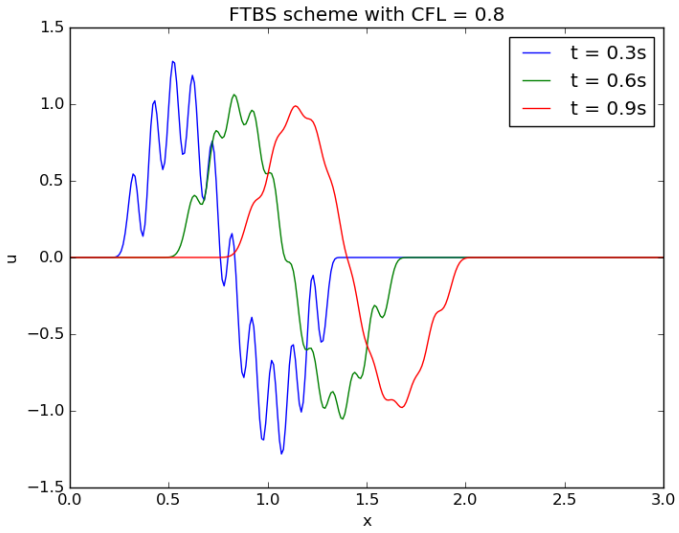

Figure 46: Plot of u v/s x for various t

We see that that the function starts smoothening and also dampens with time. It can also be noted that the high frequency components dampen faster

3.4.2 CFL = 1.0

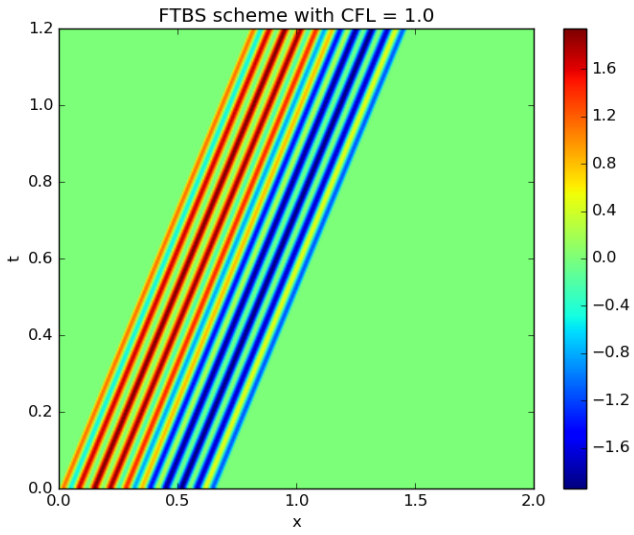

Figure 47: Color represents the magnitude of u at the given x and t

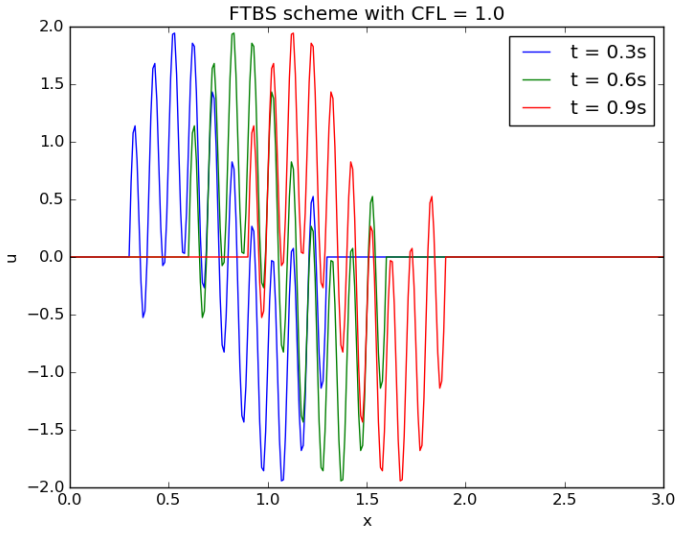

Figure 48: Plot of u v/s x for various t

The sine wave propogates without any damping.

3.4.3 CFL = 1.2

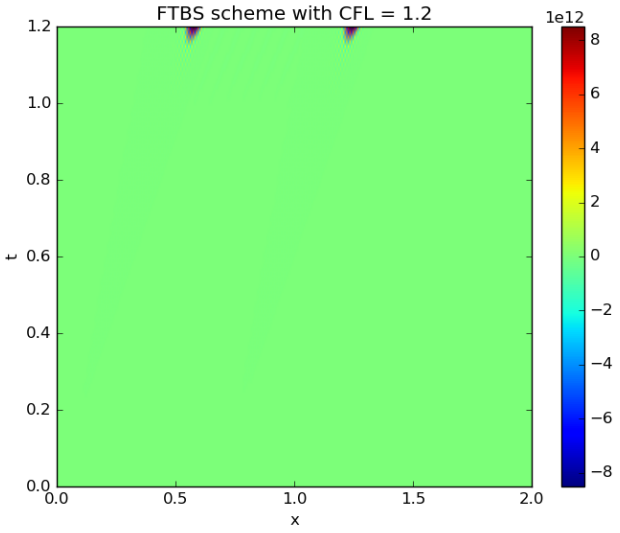

Figure 49: Color represents the magnitude of u at the given x and t

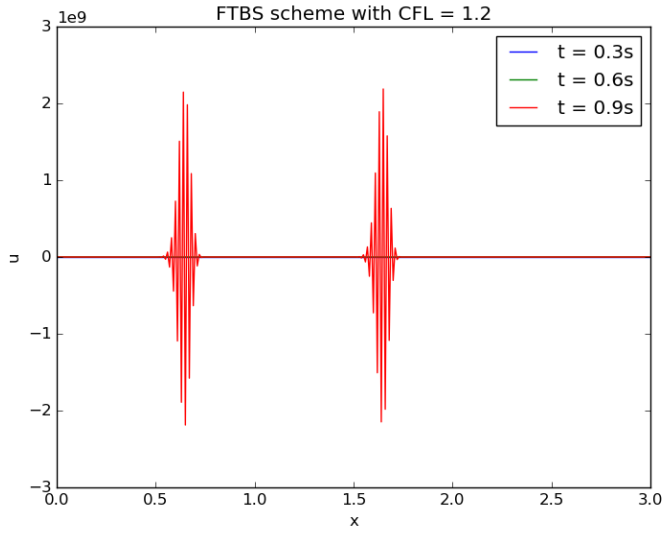

Figure 50: Plot of u v/s x for various t

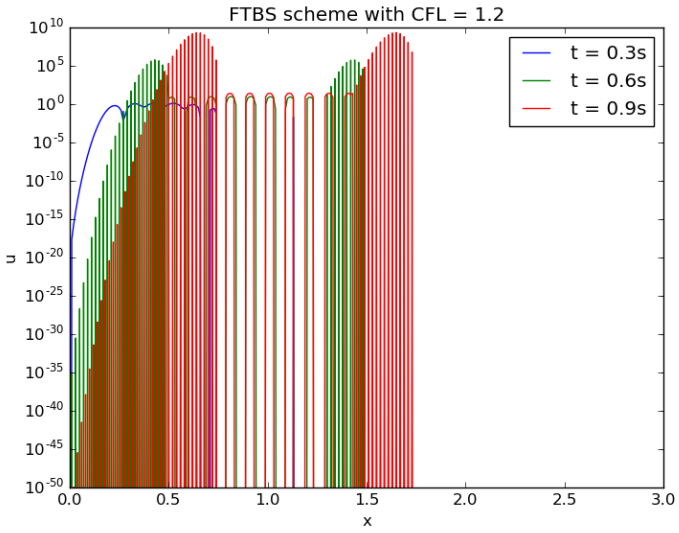

Figure 51: Plot of u v/s x for various t in log scale

We see that the solution diverges to very high values with time. i,e solution isn't stable

3.5 FTFS

3.5.1 CFL = 0.8

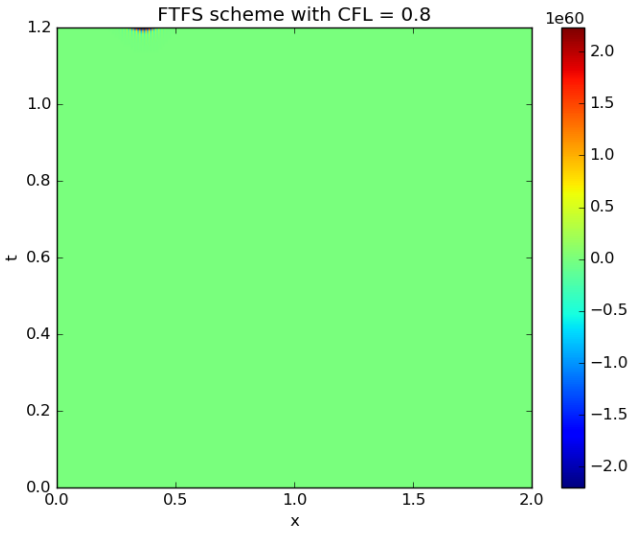

Figure 52: Color represents the magnitude of u at the given x and t

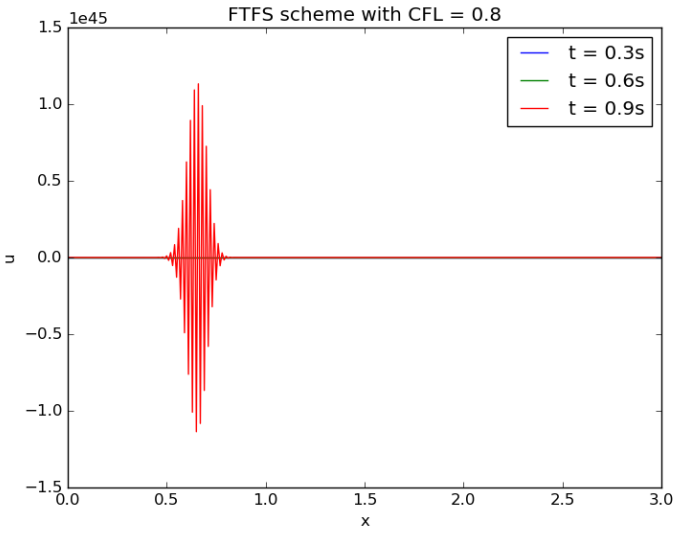

Figure 53: Plot of u v/s x for various t

3.5.2 CFL = 1.0

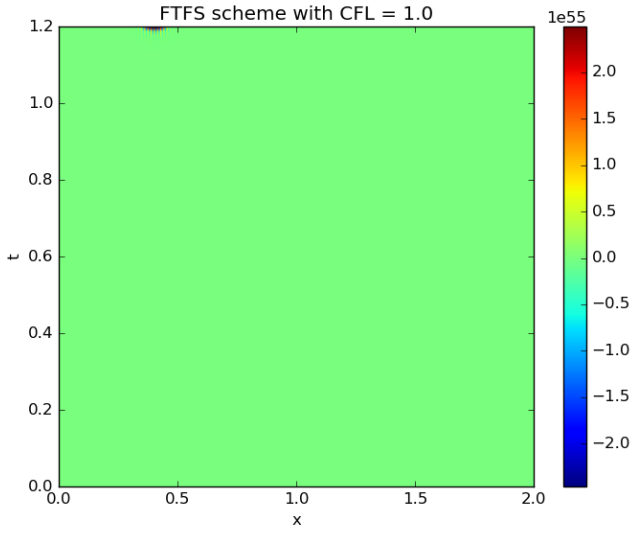

Figure 54: Color represents the magnitude of u at the given x and t

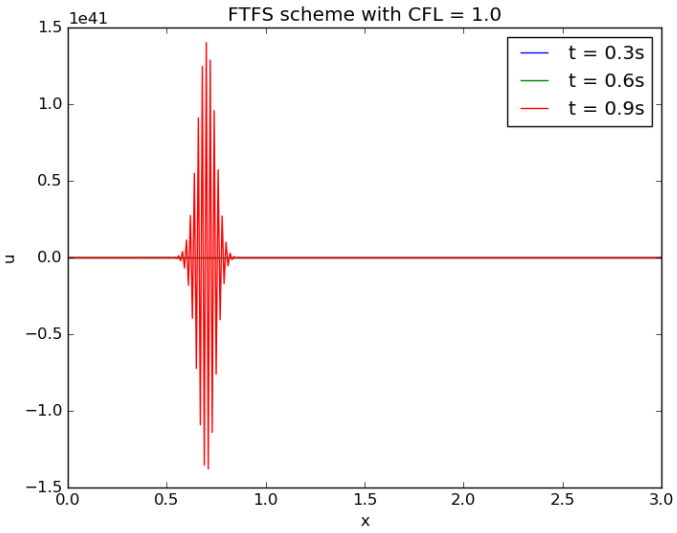

Figure 55: Plot of u v/s x for various t

3.5.3 CFL = 1.2

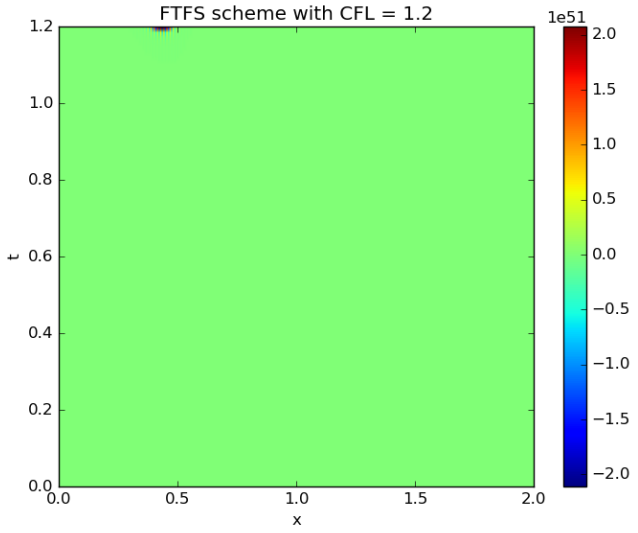

Figure 56: Color represents the magnitude of u at the given x and t

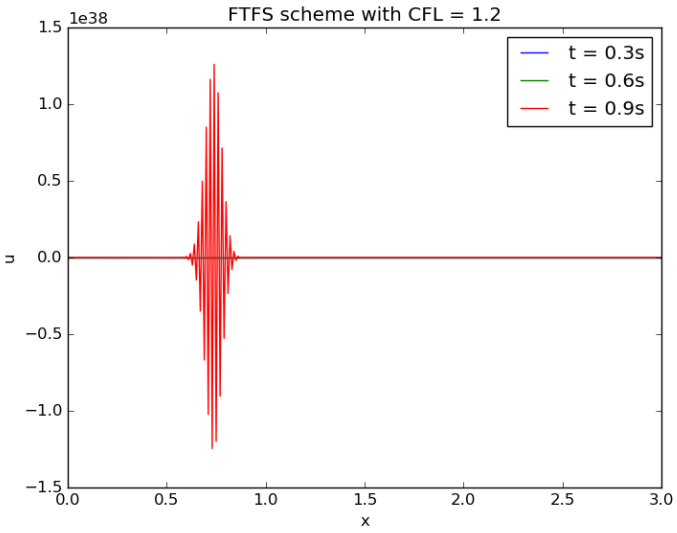

Figure 57: Plot of u v/s x for various t

We see that the solution diverges i.e the scheme is unstable

3.6 FTCS

3.6.1 CFL = 0.8

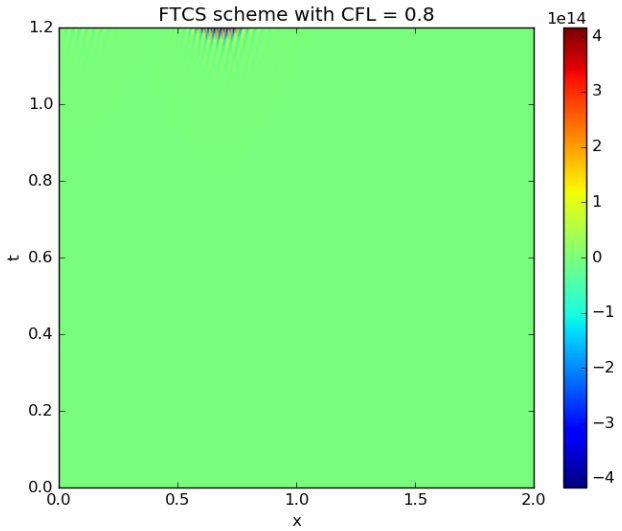

Figure 58: Color represents the magnitude of u at the given x and t

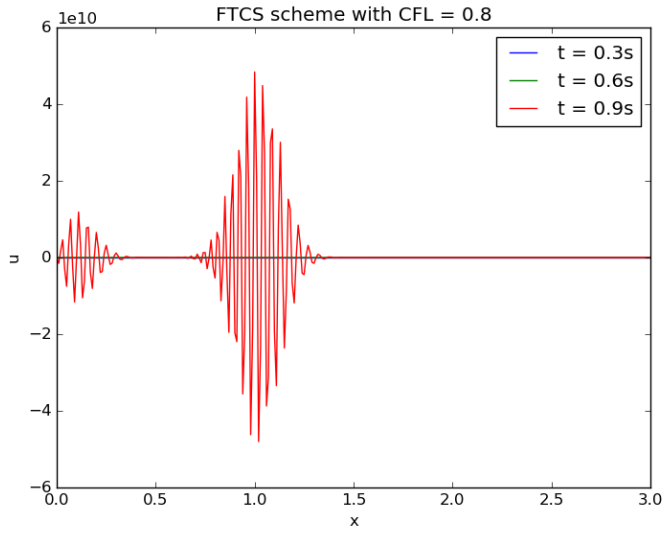

Figure 59: Plot of u v/s x for various t

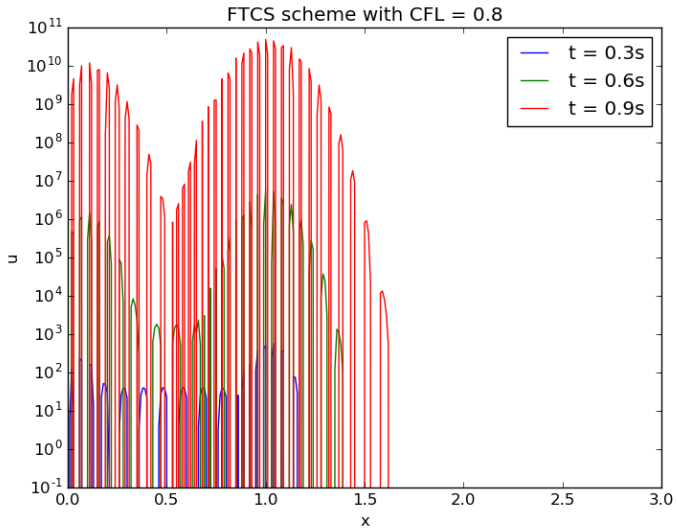

Figure 60: Plot of u v/s x for various t in log scale

3.6.2 CFL = 1.0

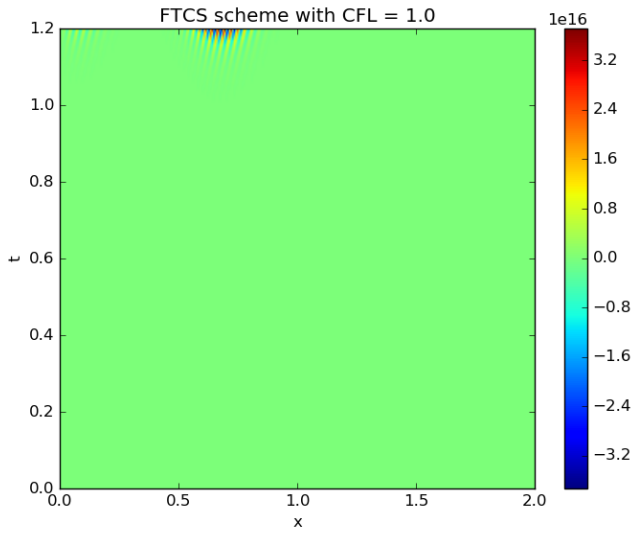

Figure 61: Color represents the magnitude of u at the given x and t

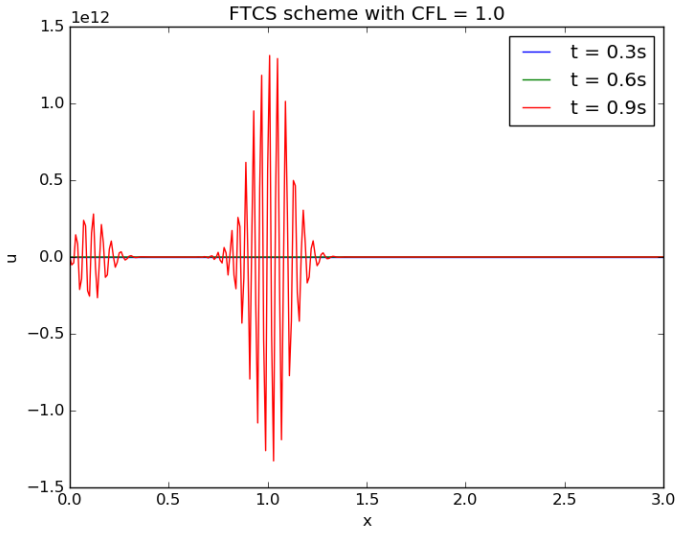

Figure 62: Plot of u v/s x for various t

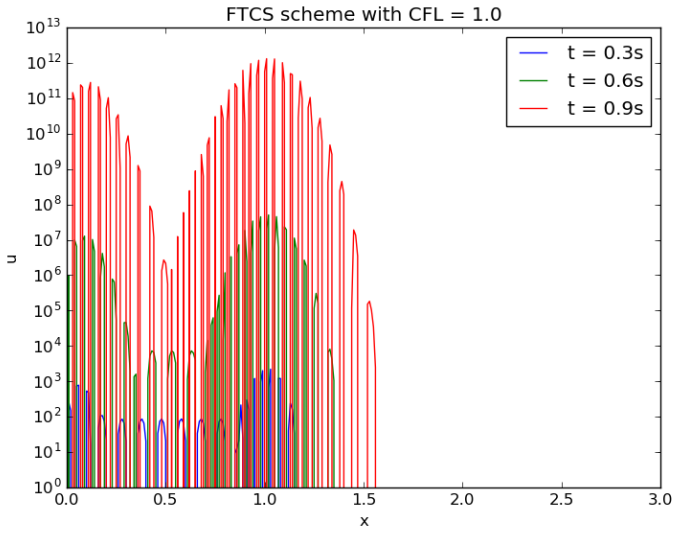

Figure 63: Plot of u v/s x for various t in log scale

3.6.3 CFL = 1.2

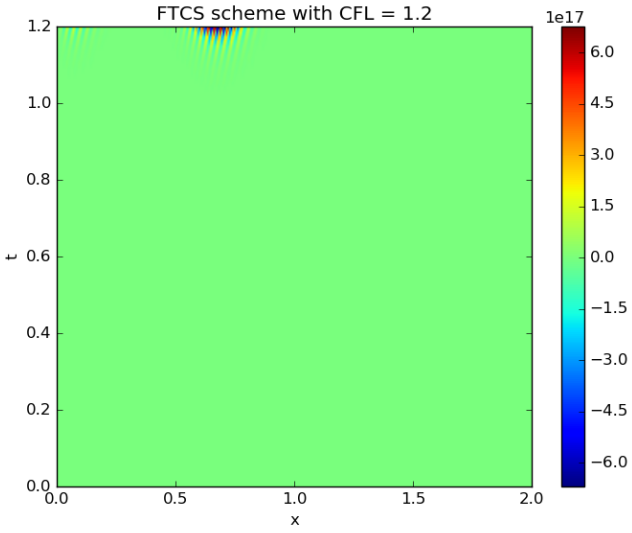

Figure 64: Color represents the magnitude of u at the given x and t

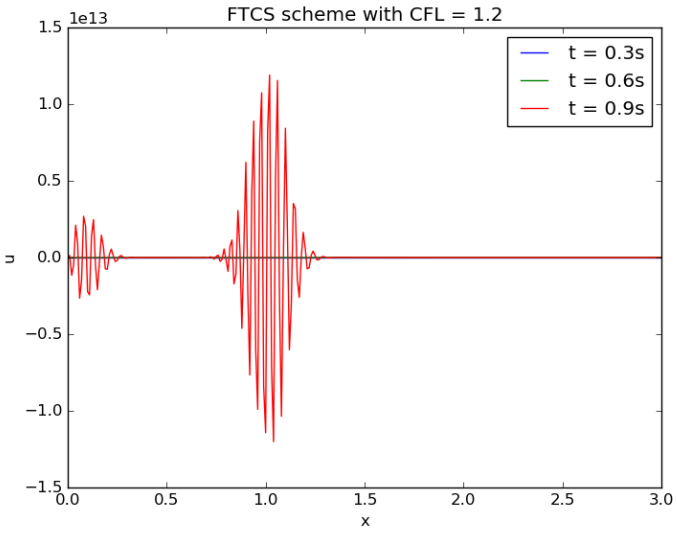

Figure 65: Plot of u v/s x for various t

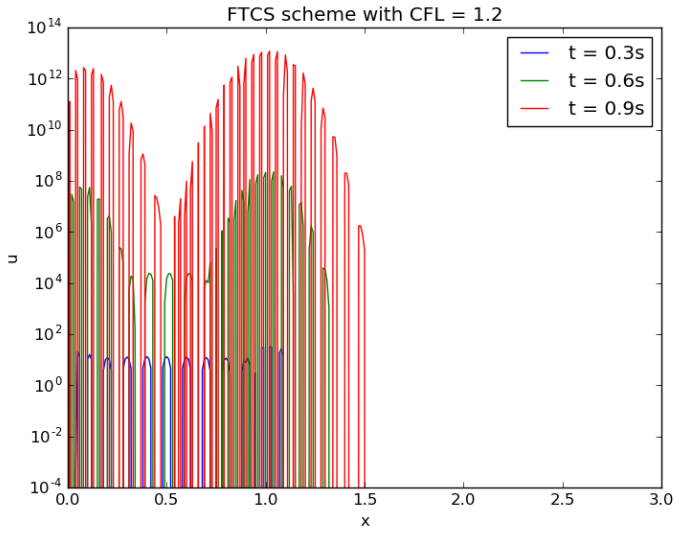

Figure 66: Plot of u v/s x for various t in log scale

We see that the solution doesn't converge.

4 Question 3

In this question we implement the different test cases implemented by Laney

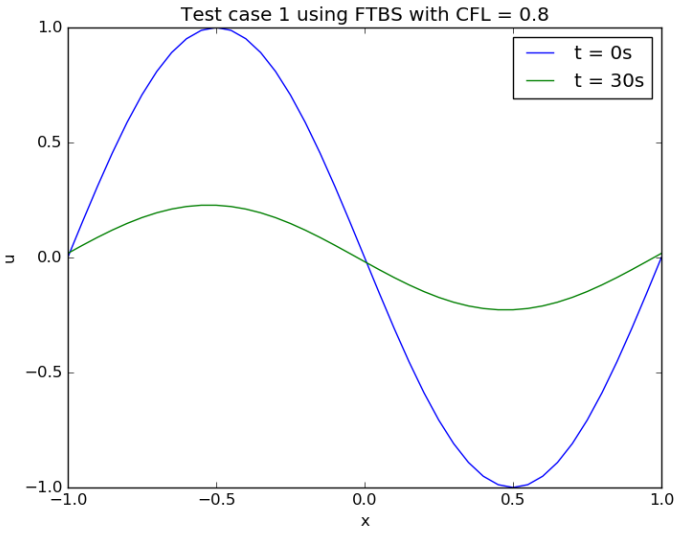

Figure 67: Laney test case 1 using FTBS

4.1 Test case 1

In this case we see that the wave has significantly damped from $t = 0$ to $t = 3 - s$.

The damping us neglugible even after $30s$

4.2 Test case 2

: In this case the wave starts dispersing as well as gets damped.

The dampening of waves is negligible but we cas see that different frequencies start travelling with different speeds

4.3 Test Case 3:

In this case the wave starts dispersing as well as gets damped.

Damping is negligible but the wave disperses.

5 Conclusion

This assignment gives us a better insight into the different schemes that can be used to solve Linear advection equation. We saw that only the FTBS and FTCS2 schemes are stable for $\sigma <= 1$. Also we saw that the soltution from the FTBS scheme dampens as well as disperses with time. The FTCS2 scheme eliminates the damping but dispersion still exists.

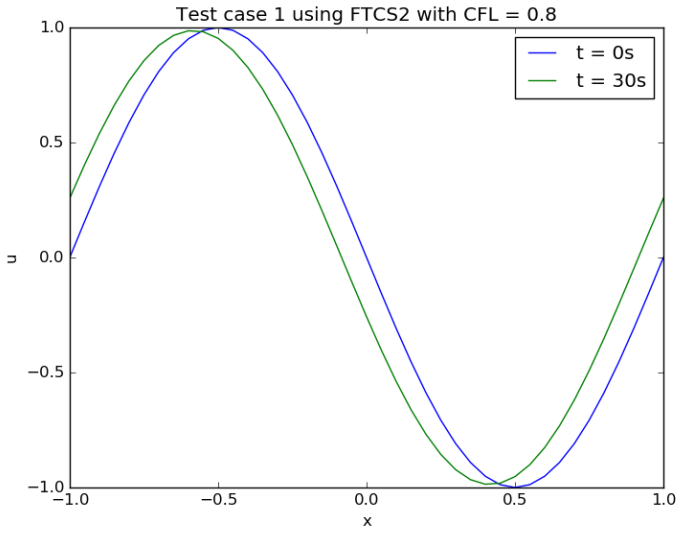

Figure 68: Laney test case 1 using FTCS2

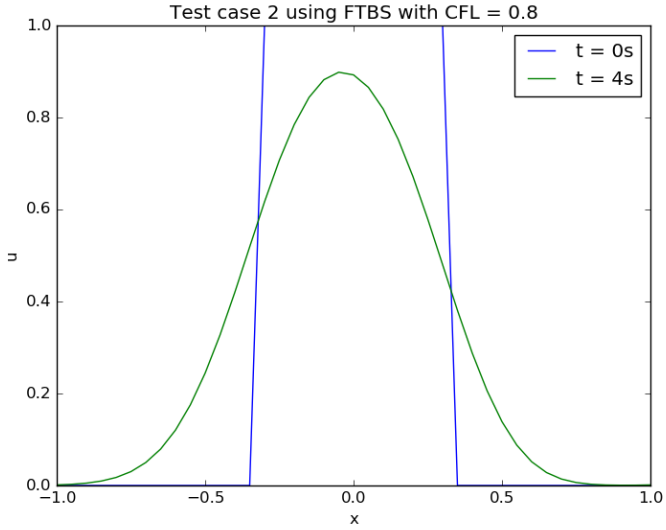

Figure 69: Laney test case 2 using FTBS

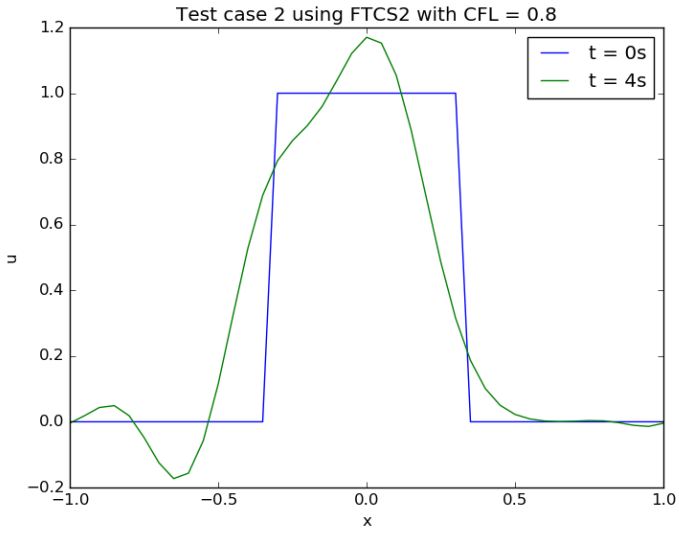

Figure 70: Laney test case 2 using FTCS2

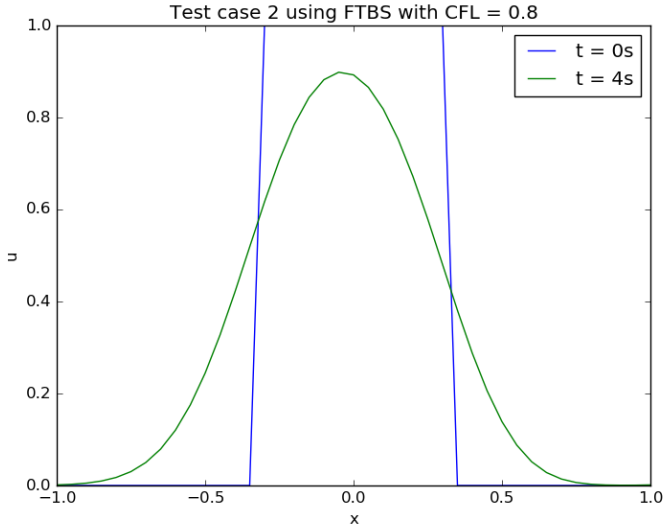

Figure 71: Laney test case 3 using FTBS

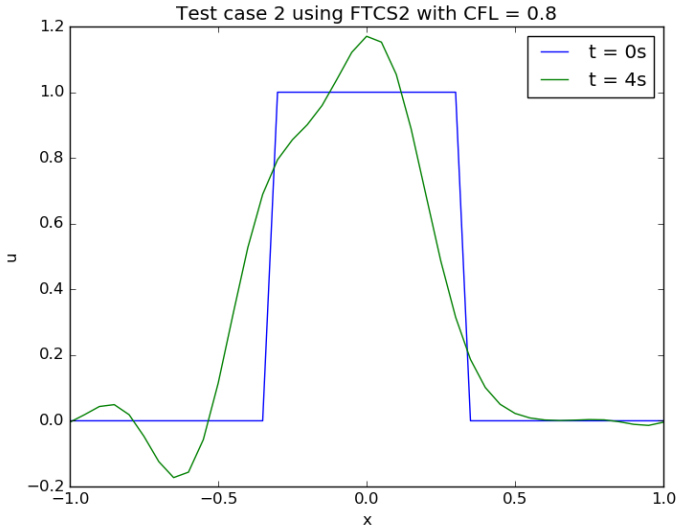

Figure 72: Laney test case 3 using FTCS2