Sequential Nondeterminism, Hoare Triples 1 & 2

CS 536: Science of Programming, Fall 2022

Due Wed Sep 28, 11:59 pm

upd soln p.1

A. Problems [60 points total]

Class 7: Sequential Nondeterminism

1. [13 = 3 + 3 + 7 points] Let DO be the nondeterministic loop

   $do\ x \neq 0 \rightarrow x := x\text{-}1;\ y := y\text{+}1 \ \square\ x \neq 0 \rightarrow x := x\text{-}1;\ y := y\text{+}2\ od$

   a. First, let's work on what what a typical loop iteration does over an arbitrary state $\sigma = \{x = \beta,\ y = \delta\}$. Assume $\beta \geq 2$ and calculate the two states we can be in after a single iteration of the loop. I.e., what are the τ where $\langle DO, \sigma\rangle \rightarrow^3 \langle DO, \tau\rangle$?

   b. Repeat part (a) but for two iterations to get three possible final states.

   c. Generalize parts (a) and (b) to κ iterations where $1 < \kappa \leq \beta$. I.e., what is Σ' such that $\langle DO, \sigma \rangle \rightarrow^{3*\kappa} \langle DO, \tau \rangle$ iff $\tau \in \Sigma'$?

Class 8: Hoare Triples, pt 1

The questions below have the form "If X, then Y _________ occur". To answer them, fill in the blank with "must", "can't", or "may or may not".

- ***Must occur*** means X implies Y. (E.g., if $x > 1$, then $x > 0$ must occur.)
- ***Can't occur*** means X implies $\neg Y$. (E.g., if $x > 1$, then $x < -3$ can't occur.)
- ***May or may not occur*** means that either $X \wedge Y$ or $X \wedge \neg Y$ can happen. (E.g., if $x > 1$, then $y = 0$ may or may not occur.)

You're not required to justify your answer, though you can if you want to (and you should be able to if asked in to in an exam).

2. [30 = 15 * 2 points] Below, unless specified, assume that $\sigma \neq \bot$ and S may or may not be deterministic.

   a. If $\sigma \models \{p\}\ S\ \{q\}$ and $\sigma \not\models p$, then $\bot \in M(S, \sigma)$ _________ occur.

   b. If $\sigma \models \{p\}\ S\ \{q\}$ and $\sigma \not\models p$, then $M(S, \sigma) - \{\bot\} \models q$ _________ occur.

   c. If $\sigma \models \{p\}\ S\ \{q\}$ and $\sigma \models p$, then $\bot \in M(S, \sigma)$ _________ occur.

   d. If $\sigma \models \{p\}\ S\ \{q\}$ and $\sigma \models p$, then $M(S, \sigma) - \{\bot\} \models q$ _________ occur.

   e. If $\models_{tot} \{p\}\ S\ \{q\}$ then $\models_{tot} \{p\}\ S\ \{T\}$ _________ occur.

   f. If $\models_{tot} \{p\}\ S\ \{T\}$ then $\models_{tot} \{p\}\ S\ \{q\}$ _________ occur.

g. If $\sigma \not\vDash \{p\}\ S\ \{q\}$ and S is deterministic, then $\sigma \vDash p$, $\bot \notin M(S, \sigma)$, and $M(S, \sigma) \vDash \neg q$ __________ all occur simultaneously.
h. If $\bot \notin M(S, \sigma)$, $M(S, \sigma) \not\vDash q$, and S is deterministic, then $M(S, \sigma) \vDash \neg q$ __________ occur.
i. If $\bot \notin M(S, \sigma)$, $M(S, \sigma) \not\vDash q$, and S is nondeterministic, then $M(S, \sigma) \vDash \neg q$ __________ occur.
j. If $M(S, \sigma) \not\vDash q$, $\tau \in M(S, \sigma)$, and S is nondeterministic, then $\tau \vDash q$ __________ occur.
k. If S is deterministic and $\sigma \vDash \{p\}\ S\ \{q\}$, then $\sigma \vDash \{p\}\ S\ \{\neg q\}$ __________ occur.
l. If $\sigma \not\vDash_{\text{tot}} \{p\}\ S\ \{q\}$ and S is deterministic, then $\sigma \vDash \{p\}\ S\ \{\neg q\}$ __________ occur.
m. If $\sigma \not\vDash_{\text{tot}} \{p\}\ S\ \{q\}$ and S is nondeterministic, then $\sigma \vDash \{p\}\ S\ \{\neg q\}$ __________ occur.
n. If $\sigma \not\vDash \{p\}\ S\ \{q\}$ and S is deterministic, then $\sigma \vDash_{\text{tot}} \{p\}\ S\ \{\neg q\}$ __________ occur.
o. If $\sigma \not\vDash \{p\}\ S\ \{q\}$ and S is non-deterministic, then $\sigma \vDash_{\text{tot}} \{p\}\ S\ \{\neg q\}$ __________ occur.

Class 9: Hoare Triples, pt 2

3. [2 points] Study the triple $\{???\}\ b := b+b\ \{b*c \le d-b\}$. Using backward assignment, what can we use for the precondition of the triple? (Hint: Be careful with parenthesization)

4. [3 points]
   a. Using backward assignment, what is u in $\{\ u\ \}\ \ x := m\ \{1 \le x*y \le n*m\}$?
   b. Using backward assignment, what is v in $\{\ v\ \}\ \ y := n\ \{\ u\ \}$?
   c. Joining parts (a) and (b), what is w in $\{\ w\ \}\ \ y := n\ ;\ x := m\ \{1 \le x*y \le n*m\}$?

5. [4 = 2*2 points] Let $p_0 \to p$, $p \to p_1$, $q_0 \to q$, and $q \to q_1$ all be valid. From $\{p\}\ S\ \{q\}$, there are four triples of the form $\{\ p_i\ \}\ S\ \{\ q_j\ \}$ that get by replacing p by p_0 or p_1 and q by q_0 or q_1.
   a. If $\sigma \vDash \{p\}\ S\ \{q\}$, which of the four triples $\sigma \vDash \{\ p_i\ \}\ S\ \{\ q_j\ \}$ is/are also satisfied by σ (under $\vDash$)? Briefly justify.
   b. If $\sigma \vDash_{tot} \{p\}\ S\ \{q\}$, which of the four triples $\sigma \vDash \{p_i\}\ S\ \{q_j\}$ is/are also satisfied by σ (under $\vDash_{\text{tot}}$)? Briefly justify.

6. [8 = 4*2 points] Say $\sigma \vDash \{p_1\}\ S\ \{q_1\}$ and $\sigma \vDash \{p_2\}\ S\ \{q_2\}$.⊥<
   a. Does $\sigma \vDash \{p_1 \wedge p_2\}\ S\ \{q_1 \wedge q_2\}$? Justify briefly.
   b. Does $\sigma \vDash \{p_1 \vee p_2\}\ S\ \{q_1 \vee q_2\}$? Justify briefly.
   c. Does $\sigma \vDash \{p_1 \wedge p_2\}\ S\ \{q_1 \vee q_2\}$? Justify briefly.
   d. Does $\sigma \vDash \{p_1 \vee p_2\}\ S\ \{q_1 \wedge q_2\}$? Justify briefly.

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Solutions

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1. (Nondeterministic loop)
   a. $\{x = \beta - 1, y = \delta+1\}$, $\{x = \beta - 1, y = \delta + 2\}$
   b. $\{x = \beta - 2, y = \delta'\}$ where δ' is $2, 3,$ or 4.
   2 times, we add 1 or 2 to δ, so we add a minimum of 2 and a maximum of 4 to δ.
   c. $\{x = \beta - \kappa, y = \delta''\}$ where $\kappa \le \delta'' \le 2\kappa$.
   κ times, we add 1 or 2 to δ, so we add a minimum of κ and a maximum of 2κ to δ.

2. (*wp* and *wlp* properties and relationships)
   Must happen: d, e, g, h, ~~i~~, l, n, ~~o~~. [2022-10-20] drop i, o
   Can't happen: ~~k~~. [2022-10-20] drop k
   May or may not happen: a, b, c, f, j, m. [2022-10-20] add i, k, o

3. (Backward assignment)
   $(b+b)*c \le d-(b+b)$

4. (Sequence of assignments)
   a. $u \equiv 1 \le m*y \le n*m$
   b. $v \equiv 1 \le m*n \le n*m$
   c. $w \equiv v \equiv 1 \le m*n \le n*m$

5. (Weakening and strengthening conditions)
   a. $\{p_0\}\ S\ \{q_1\}$ by pre. str. and post. weak.
   b. Same: $\{p_0\}\ S\ \{q_1\}$ by pre. str. and post. weak.

6. (Conjunctions and disjunctions of conditions)
   a. Yes. If $p_1 \wedge p_2$ holds then postcondition q_1 holds because of p_1 and q_2 holds because of p_2, so postcondition $q_1 \wedge q_2$ holds.
   b. Yes. Say $p_1 \vee p_2$ holds. If p_1 holds then q_1 (and therefore $q_1 \vee q_2$) holds; alternatively if p_2 holds then q_2 (and therefore $q_1 \vee q_2$) holds, and $q_1 \vee q_2$ either way.
   c. Yes. From part (a), we know $q_1 \wedge q_2$ holds, which implies that $q_1 \vee q_2$ holds.
   d. No. From part (b) we know that either q_1 or q_2 (or both) hold, but just knowing $q_1 \vee q_2$ doesn't imply that $q_1 \wedge q_2$ holds.

