

Syntactic Substitution, Forward Assignment, & sp

CS 536: Science of Programming, Fall 2022

Due Sat Oct 15, 11:59 pm

Problems [60 points total]

Class 12: Syntactic Substitutions [30 points]

For Problems 1 – 4, Let $p \equiv x * y < f(a) \vee \exists x. x \geq a * y \rightarrow \exists y. f(x * y) > a - y + z$ and calculate the substitutions below. Show some detail if you want partial credit for a wrong answer. Just do the syntactic calculations. Don't do any arithmetic or logical simplifications.

1. [4 points] $p[y+z/x]$
2. [6 points] $p[a-y/y]$
3. [9 points] $p[a*y/a]$
4. [11 points] $p[x \div y/a][y-z/x]$

Lecture 13: Forward Assignment; Strongest Postconditions [30 points]

5. [2 points] Give an example of an S such that $\models \{T\} S \{sp(T, S)\}$ but $\not\models_{tot} \{T\} S \{sp(T, S)\}$.
6. [3 points] Syntactically calculate $sp(x < y \wedge x + y \leq n, x := f(x+y); y := g(x*y))$. Don't simplify the result.

For Problems 7 – 10, calculate each sp or wp result syntactically. If simplification is requested, do the syntactic calculation first, then simplify, maintaining logical equivalence unless asked otherwise.

7. [3 points] Calculate and then logically simplify $sp(x = 2^k, x := x/2)$.
8. [3 points] Calculate (but don't simplify) $wp(x := x/2, x = 2^k)$.
9. [7 = 3+4 points]. Let $S \equiv \text{if even}(x) \text{ then } x := x+1 \text{ fi}$. (Let $\text{even}(x) \equiv x \% 2 = 0$ and $\text{odd}(x) \equiv x \% 2 \neq 0$.)
 - a. Calculate and then logically simplify $wp(S, \text{odd}(x))$.
 - b. Calculate and then logically simplify $sp(x = x_0, S)$. As part of simplification, drop x_0 . (The simplified result will be \Leftarrow but not \Leftrightarrow the unsimplified result.)
10. [12 = 7+5 points]. Let $S \equiv \text{if } x < b[\text{mid}] \text{ then } \text{right} := m \text{ else } \text{left} := m \text{ fi}$ and let $p' \equiv \text{left} = \text{left}_0 \wedge \text{right} = \text{right}_0$.
 - a. Calculate (but don't simplify $sp(p \wedge p', S)$ where $p \equiv \text{left} < \text{right} - 1 \wedge \text{mid} = (\text{left} + \text{right})/2 \wedge b[\text{left}] \leq x < b[\text{right}]$).
 - b. Calculate (but don't simplify $sp(p \wedge p', S)$ where $p \equiv -1 \leq \text{left} - 1 \leq \text{right} \wedge \text{mid} = (\text{left} + \text{right})/2 \wedge (x \in b[0 \dots n-1] \leftrightarrow x \in b[\text{left}.. \text{right}])$ (Notation: $x \in b[y..z] \equiv \text{member}(x, b, y, z)$)