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1.
a. m = x = 0; y = 1; while (m++ < n) { ++x; y *= x ; }
Ans:
x := 0; m := x; y := 1; while m < n do m := m + 1; x := x + 1; y := y * x od; m := m + 1

b. m = n; x = z = 1; while (—m < n) { x++; x += z ; }
Ans:
m = n; z = 1; x = z; m := m – 1; while m < n do x := x + 1; x = x + z; m := m – 1 od

c. m = n; v = x = 1; while (—m < n) { x++; v += x ; }
Ans:
m = n; x = 1; v = x; m := m – 1; while m < n do x := x + 1; v := v + x; m := m – 1 od

d. m = n; p = y = 1; while (—m < n) { ++p; p += y ; p++; p += z; }
Ans:
m := n; y := 1; p := y; m := m – 1; while m < n do p := p + 1; p := p + y; p := p + 1; p := p + z; m := m – 1 od

2.
Let S ≡ if x > 0 then x := x*z else if y > 0 then y := y*z f f
a. Evaluate ⟨S, {x = 3, y = 5, z = 9}⟩ to completion.
Ans:
⟨ S, {x = 3, y = 5, z = 9} ⟩
→⟨if x > 0 then x := x*z else if y > 0 then y := y*z fi fi, {x = 3, y = 5, z = 9}⟩
→⟨ x := x * z, {x = 3, y = 5, z = 9}⟩
→⟨ E, {x = 3, y = 5, z = 9} [x ↦ 27] ⟩ = ⟨ E, {x = 27, y = 5, z = 9}⟩

b. Evaluate ⟨S, {x = –2, y = 4, z = 3}⟩ to completion.
Ans:
⟨if x > 0 then x := x*z else if y > 0 then y := y*z fi fi, {x = –2, y = 4, z = 3} ⟩
→⟨ y := y * z, {x = –2, y = 4, z = 3}⟩
→⟨ E, {x = –2, y = 4, z = 3}[y ↦ 12] ⟩ = ⟨ E, {x = –2, y = 12, z = 3} ⟩

c. Evaluate ⟨S, {x = –5, y = –2, z = –2}⟩ to completion.
Ans:
⟨ S, {x = –5, y = –2, z = –2} ⟩
→⟨if x > 0 then x := x*z else if y > 0 then y := y*z fi fi, {x = –5, y = –2, z = –2}⟩
→⟨ skip, {x = –5, y = –2, z = –2} ⟩ —------- Since both x and y are negative.
→⟨ E, {x = –5, y = –2, z = –2} ⟩

3.

<W, {m=0, x=1, n=4}>
$\rightarrow^3$ <W, {m=0, x=1, n=4}[m ↦ 1][x ↦ 2] >
$\rightarrow^3$ <W, {m=1, x=2, n=4}[m ↦ 2][x ↦ 6] >
$\rightarrow^3$ <W, {m=2, x=6, n=4}[m ↦ 3][x ↦ 15] >
$\rightarrow^3$ <W, {m=3, x=15, n=4}[m ↦ 4][x ↦ 31] >
→ <*skip*, {m=4, x=31, n=4}>

4.

a. M(IF, {x = 3, y = 5, z = 9}) = {x=27, y=5, z=9}
b. M(IF, {x = –2, y = 4, z = 3}) = {x=-2, y=12, z=3}
c. M(IF, {x = –5, y = –2, z = –2}) = {x=-5, y=-2, z=-2}

5.
a.[3 points]
M(S, τ) = M(m := m+1; x := x+m*m, τ)
= M(x := x+m*m, τ[m↦ τ(m)+1])
= {τ[m↦ τ(m)+1][x ↦ β]}
where β = τ[m↦ τ(m)+1](x+m*m)= τ(x)+(τ(m)+1)*(τ(m)+1)

b.[3 points]
M(S, σk[m↦ α][x ↦ δ][n ↦ β]) = {σk+1}
where σk+1 = {m = α+1, x = (δ+(α+1)*(α+1)), n = β }
starting from m = 0, m ∈ {0,1,2,3…}
M(W, σ0[n ↦ β]) = ⊥d
→ m ≠ n for all m
→ β is a negative integer

c.[4 points]
From b solution:
M(S, σk[m↦ α][x ↦ δ][n ↦ β]) = {σk+1}
where σk+1 = {m = α+1, x = (δ+(α+1)*(α+1)), n = β }

starting from σ_0={m = 0, x = 0},
get M(W,σ_0[n↦β]) = {m = β, x = (0+1*1+2*2+3*3+……+β*β), n = β }
δ is the sum of squares of all the integers from 0 to β

6.[8 points]
M(S,σ)=M(x:=b[m+1] / sqrt(k),σ)= {σ[x↦Ω]}
where Ω= (σ(b)(σ(m+1))/sqrt(σ(k))) = (σ(b)(α+1)/sqrt(β))

M(S,σ)={⊥e}
→ Ω=⊥e
→
iff σ(b)(σ(m+1))=⊥e or σ(k)< 0 or sqrt(σ(k))) = 0
iff σ(b)(α+1)=⊥e or β< 0 or sqrt(β) = 0
iff (α+1 out of range for σ(b)] or β< 0 or β = 0
iff α < -1 or α ≥δ-1 or β≤ 0