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Q1

a. illegal: true (T) and false (y) cases evaluate to different types
b. legal: resulting expression would be a boolean (true/false)
c. illegal: since b3 is 2D data structure, we can't add 1D array type with an element

Q2

a. well-formed
b. not well-formed: w has reference to an identifier u
c. not well-formed: t has reference to identifiers r,s

Q3

a. $\sigma = \{ x = 2, b = \{ (0,7), (1,12), (2,3), (4,0) \} \}$
b. $\sigma = \{ x = 2, b[0] = 7, b[1] = 12, b[2] = 3, b[3] = 0 \}$

Q4 (6 = 3*2 points)
x = 100, y = 20, z = 5, b = (1,3,4)

Q5 (8 = 4*2 points)

a. well-formed, proper, terminate correctly
b. Well-formed, improper, misses the binding of b, k and b[k]
c. Well-formed, improper, misses the binding of b[k]
d. Well-formed, proper, runtime error on b[b[k]]

Q6 (9 points, d[3 points])

a. No difference. Because no binding of z in σ_0
b. Different. The first is well-formed, while the second is ill-formed.
c. $\sigma_1[\ b[0] \mapsto \sigma_1(b[2])\ ] = \sigma_1[b[0] \mapsto 4] = \{x= 5, y = 4, b = (4, 0, 4, 2)\}$.
d. $t[b[1] \mapsto \sigma_1(b[1]) + 8] = t[b[1] \mapsto 8] = \{x = 5, y = 4, b = (4, 8, 4, 2)\}$

Q7. (6 = 3 * 2 points)
(a). Does $\{x = 4, y = 7, b = (5, 4, 8)\} \vDash (\exists\ x.\ \exists\ m.\ b[m] < x < y)$? If not, why?
Ans:

Yes, 6 and 1 are witness values for x and m respectively.
Let $\sigma = \{x = 4, y = 7, b = (5, 4, 8)\}$
So, $\sigma[x \mapsto 6][m \mapsto 1] \vDash b[m] < x < y$

(b). Does $\{x = 1, b = (2, 8, 9)\} \vDash (\ \forall x.\ \forall k.\ 0 < k < 3 \rightarrow x < b[k]\ )$? If not, why?
Ans:

No, 12 for x and 1 for k are counterexample values.
Let σ = {x = 1, b = (2, 8, 9)}
So, $\sigma[x \mapsto 12][k \mapsto 1] \not\models 0 < k < 3 \rightarrow x < b[k]$

(c). Does {x = 0, b = (5, 3, 6)} $\models (\forall x.\ \forall k.\ 0 < k < 3 \wedge x < b[k])$? If not, why?
Ans:
No, similarly 12 for x and 1 for k are counterexample values.
Let σ = {x = 0, b = (5, 3, 6)}
So, $\sigma[x \mapsto 12][k \mapsto 1] \not\models 0 < k < 3 \wedge x < b[k]$

Q8. (9 = 3 * 3 points)
(a). $\not\models (\forall x \in V . (\exists y \in U . P(x, y)) \wedge (\forall z \in U . Q(x, z)))$
Ans:
iff for some σ state σ, for some α $\alpha \in V$, and for every β $\beta \in U$, we have $\sigma[x \mapsto \alpha][y \mapsto \beta] \not\models P(x, y)$
or
for some α $\alpha \in V$ and for some δ $\delta \in U$, we have $\sigma[x \mapsto \alpha][z \mapsto \delta] \not\models Q(x, z)$

(b). $\not\models \forall y \in V . ((\exists x \in W. P(x, y)) \rightarrow (\exists y \in U . Q(y, y)))$
Ans:
iff for some σ state σ, for some α $\alpha \in V$, if for some β $\beta \in W$, $\sigma[y \mapsto \alpha][x \mapsto \beta] \models q$ p(x, y), and for every δ $\delta \in U$, $\sigma[y \mapsto \alpha][[y \mapsto \delta] \not\models p$ q(y, y).

because the negation of $(\forall x . ((\exists y...) \rightarrow (\exists z...)))$ is $(\exists x . ((\exists y ...) \wedge \neg(\exists z ...)))$.

(c). $\sigma \not\models (\exists x \in W . (\forall y \in U . P(x, y)))$
Ans:
iff for this σ state σ, for every α $\alpha \in W$, and for some β $\beta \in U$, we have $\sigma[x \mapsto \alpha][y \mapsto \beta] \not\models P(x, y)$.

Q9. (6 points)

size(b1) <= size(b2) $\wedge$ 0$\leq$ x <size(b1) $\wedge$ 0$\leq$ y <size(b2) $\wedge$ ($\forall$ 0 $\leq$ i $\leq$ x .($\exists$ 0 $\leq$ j $\leq$ y . b1[i] =b2[j]))