Basics of Parallel Programs, v.2

CS 536: Science of Programming, Fall 2022

A. Why?

- Parallel programs are more flexible than sequential programs but their execution is more complicated.
- Parallel programs are harder to reason about because parts of a parallel program can interfere with other parts.
- Evaluation graphs can be used to show all possible execution paths for a parallel program.

B. Objectives

After this class, you should know

- The syntax and operational & denotational semantics of parallel programs.

C. Basic Definitions for Parallel Programs

- ***Syntax*** for parallel statements: $S := [S \,||\, S \,||\, \ldots \,||\, S]$. We say $[S_1 \,||\, S_2 \,||\, \ldots \,||\, S_n]$ is the ***parallel composition*** of the ***threads*** $S_1, S_2, \ldots, S_n$.
  - The threads must be sequential: You can't nest parallel programs. (But you can embed parallel programs within otherwise-sequential programs, such as in the body of a loop.)
- ***Example 1***: $[x := x+1 \,||\, x := x*2 \,||\, y := x^2]$ is a parallel program with three threads. Since it tries to nest parallel programs, $[x := x+1 \,||\, [x := x*2 \,||\, y := x^2]]$ is illegal.

Interleaving Execution of Parallel Programs

- We run sequential threads in parallel by ***interleaving*** their execution. I.e., we interleave the operational semantics steps for the individual threads.
- We execute one thread for some number of operational steps, then execute another thread, etc.
- Depending on the program and the sequence of interleaving, a program can have more than one final state (or cause an error sometimes but not other times).
- As an example, since evaluation of $[x := x+1 \,||\, x := x*2]$ is done by interleaving the operational semantics steps of the two threads, we can either evaluate $x := x+1$ and then $x := x*2$ or evaluate $x := x*2$ and then $x := x+1$.
- The difference between $[x := x+1 \,||\, x := x*2]$ and ***if*** $T \rightarrow x := x+1 \;\square\; T \rightarrow x := x*2$ ***fi*** is that the nondeterministic ***if-fi*** executes only one of the two assignments whereas the parallel composition executes both assignments but in an unpredictable order. The sequential nondeterministic ***if-fi***

that simulates the parallel assignments is ***if*** T → x := x+1; x := x*2 □ T → x := x*2; x := x+1 ***fi***. It nondeterministically chooses between the two possible traces of execution for the program.*

- Because of the nondeterminism, re-executions of a parallel program can use different orders. For example, two executions of ***while*** B ***do*** [x := x+1 || x := x*2] ***od*** can have the same sequence or different sequences of updates to x.

Difficult to Predict Parallel Program Behavior

- The main problem with parallel programs is that their properties can be very different from the behaviors of the individual threads.
- ***Example 2***:
  - ⊨ {x = 5} x := x+1 {x = 6} and ⊨ {x = 5} x := x*2 {x = 10}
  - But ⊨ {x = 5} [x := x+1 || x := x*2] {x = 11 ∨ x = 12}
- The problem with reasoning about parallel programs is that different threads can ***interfere*** with each other: They can change the state in ways that don't maintain the assumptions used by other threads.
- Full interference is tricky, so we're going to work our way up to it. First we'll look at simple, limited parallel programs that don't interact at all (much less interfere).
- But before that, we need to look at the semantics of parallel programs more closely.

D. Semantics of Parallel Programs

- To execute the sequential composition $S_1; \ldots; S_n$ for one step, we execute S_1 for one step.
- To execute the parallel composition $[S_1 \,||\, \ldots \,||\, S_n]$ for one step, we take one of the threads and evaluate it for one step.

Operational and Denotational Semantics of Parallel Programs

- ***Definition***: Given $[S_1 \,||\, \ldots \,||\, S_n]$, for each $k = 1, 2, \ldots, n$, if $\langle S_k, \sigma \rangle \rightarrow \langle T_k, \tau_k \rangle$, then

  $$\langle [S_1 \,||\, \ldots \,||\, S_n], \sigma \rangle \rightarrow \langle [S_1 \,||\, \ldots \,||\, S_{k-1} \,||\, T_k \,||\, S_{k+1} \,||\, \ldots \,||\, S_n], \tau_k \rangle$$

- We write E for sequential thread that has finished execution, so a parallel program that has finished execution is written $[E \,||\, \ldots \,||\, E \,||\, E]$. We'll treat E and $[E \,||\, \ldots \,||\, E \,||\, E]$ as being syntactically equal, i.e., $E \equiv [E \,||\, \ldots \,||\, E \,||\, E]$.

The → Notation*

- ***Notation***: The →* notation has the same meaning whether the configurations involved have parallel programs or not: →* means $\rightarrow^n$ for some $n \geq 0$, where $C_0 \rightarrow^n C_n$ means that there is actually a sequence of n+1 configurations, $C_0 \rightarrow C_1 \rightarrow \ldots \rightarrow C_{n-1} \rightarrow C_n$ where we've omitted writing the intermediate configurations.
- *Common Mistake:* Writing $\langle [E \,||\, E], \tau \rangle \rightarrow \langle E, \tau \rangle$ is a common mistake. Since $[E \,||\, E] \equiv E$, going from $\langle [E \,||\, E], \tau \rangle$ to $\langle E, \tau \rangle$ doesn't involve an execution step. But $\langle [E \,||\, E], \tau \rangle \rightarrow^0 \langle E, \tau \rangle$ is ok.

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* This trick doesn't scale up well to larger programs, but it helps with initially understanding parallel execution.

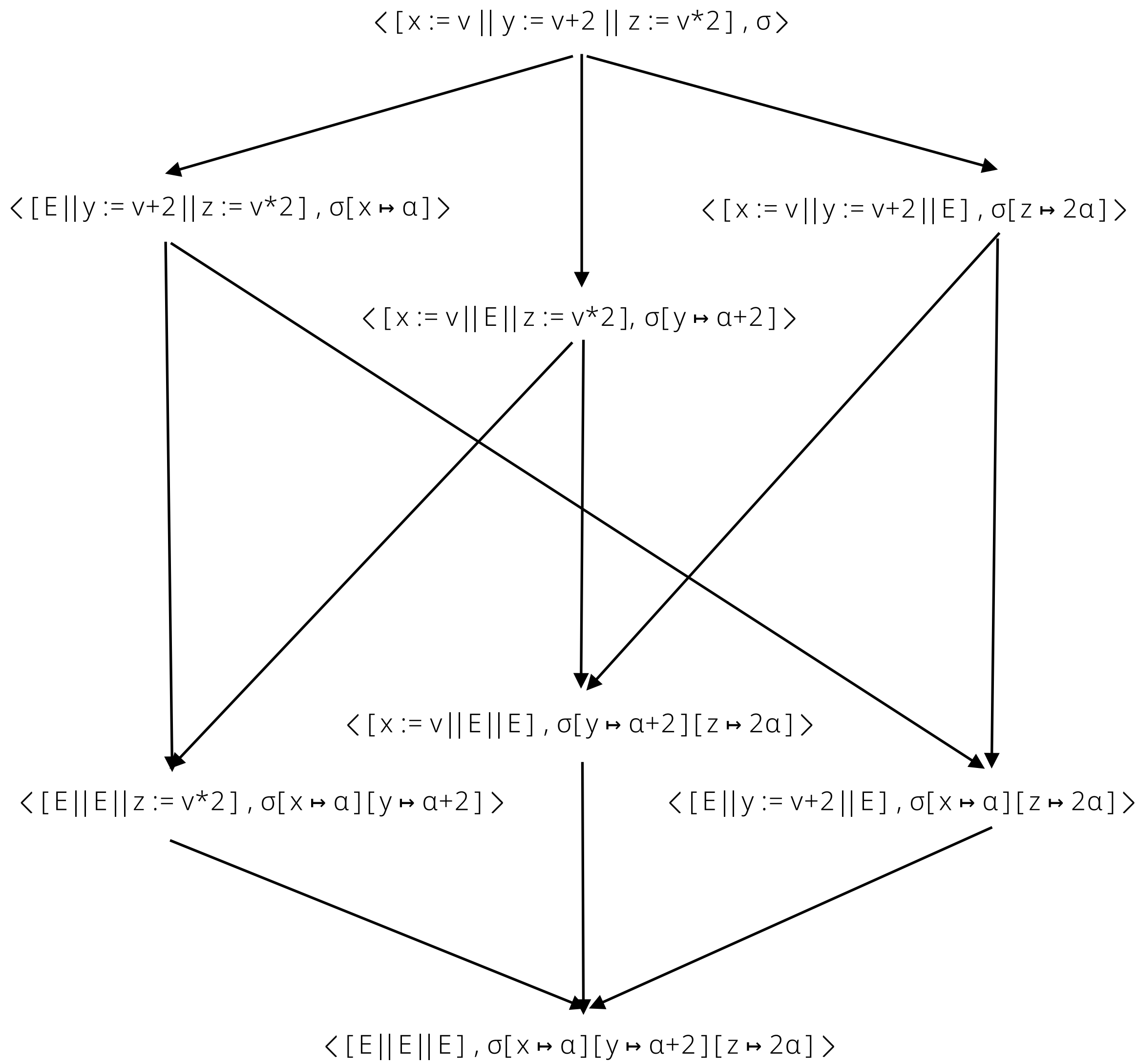
Evaluation Graph and Denotational Semantics

- Recall that the ***evaluation graph*** for $\langle S, \sigma \rangle$ is the directed graph of configurations and evaluation arrows leading from $\langle S, \sigma \rangle$.
- When drawing evaluation graphs, the configuration nodes need to be different.
  - (I.e., if the same configuration appears more than once, show multiple arrows into it — don't repeat the same node.)
- An evaluation graph shows all possible executions.
  - A program with n threads will have n out-arrows from its configuration.
- A path through the graph corresponds to one possible evaluation of the program.
- The ***denotational semantics*** of a program in a state is the set of all possible terminating states (plus possibly the pseudostates $\perp_d$ and $\perp_e$). I.e., the states found in the sinks (i.e., at the leaves) of an evaluation graph. (We'll modify this definition when we get to deadlocked programs.)
  - $M(S, \sigma) = \{\tau \in \sigma \mid \langle S, \sigma \rangle \rightarrow^* \langle E, \tau \rangle\}$
    $\cup \{\perp_d\}$ if S can diverge; i.e., if $\langle S, \sigma \rangle \rightarrow^* \langle \perp_d, \tau \rangle$ is possible
    $\cup \{\perp_e\}$ if S can produce a runtime error; i.e., $\langle S, \sigma \rangle \rightarrow^* \langle \perp_e, \tau \rangle$ is possible

- ***Example 3***: The evaluation graph below is for the same program as in Example 2, but starting with an arbitrary state σ where $\sigma(x) = \alpha$. The graph has two sinks for the two possible final states, so $M([x := x+1 \,||\, x := x*2], \sigma) = \{\sigma[x \mapsto 2\alpha+2], \sigma[x \mapsto 2\alpha+1]\}$.

$$
\begin{tikzcd}
& \langle [x := x+1 \,||\, x := x*2], \sigma[x \mapsto \alpha] \rangle \arrow[dl] \arrow[dr] & \\
\langle [E \,||\, x := x*2], \sigma[x \mapsto \alpha+1] \rangle \arrow[d] & & \langle [x := x+1 \,||\, E], \sigma[x \mapsto 2\alpha] \rangle \arrow[d] \\
\langle [E \,||\, E], \sigma[x \mapsto 2\alpha+2] \rangle & & \langle [E \,||\, E], \sigma[x \mapsto 2\alpha+1] \rangle
\end{tikzcd}
$$

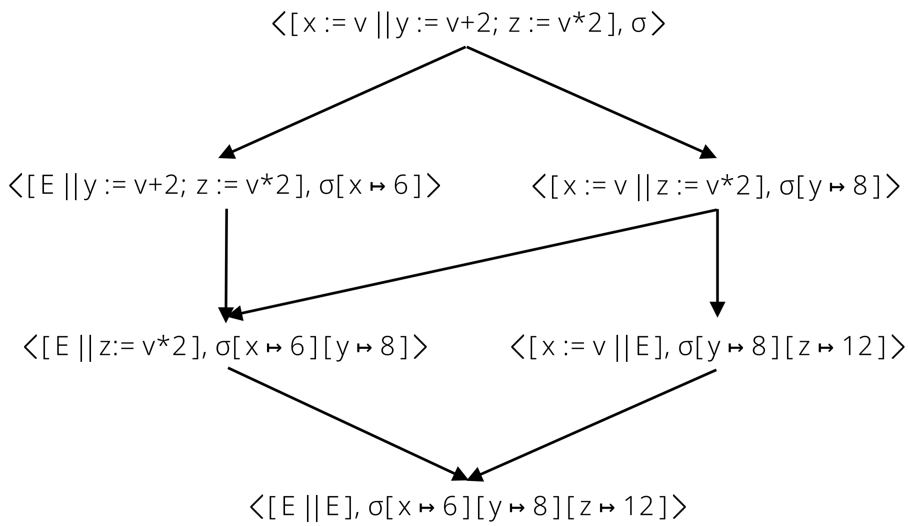
Example 3



Example 4

- ***Example 4***: For this example, the evaluation graph is for $\langle [x := v \,||\, y := v+2 \,||\, z := v*2], \sigma \rangle$, where $\sigma(v) = \alpha$. $M([x := v \,||\, y := v+2 \,||\, z := v*2], \sigma) = \{\sigma[x \mapsto \alpha][y \mapsto \alpha+2][z \mapsto 2\alpha]\}$. Note even though the program is nondeterministic, it produces the same result no matter what execution path it uses. (More generally, if S is parallel, then M(S, σ) can have more than 1 member, but the converse is not true: Having M(S, σ) of size 1 does not imply that S is nondeterministic.)

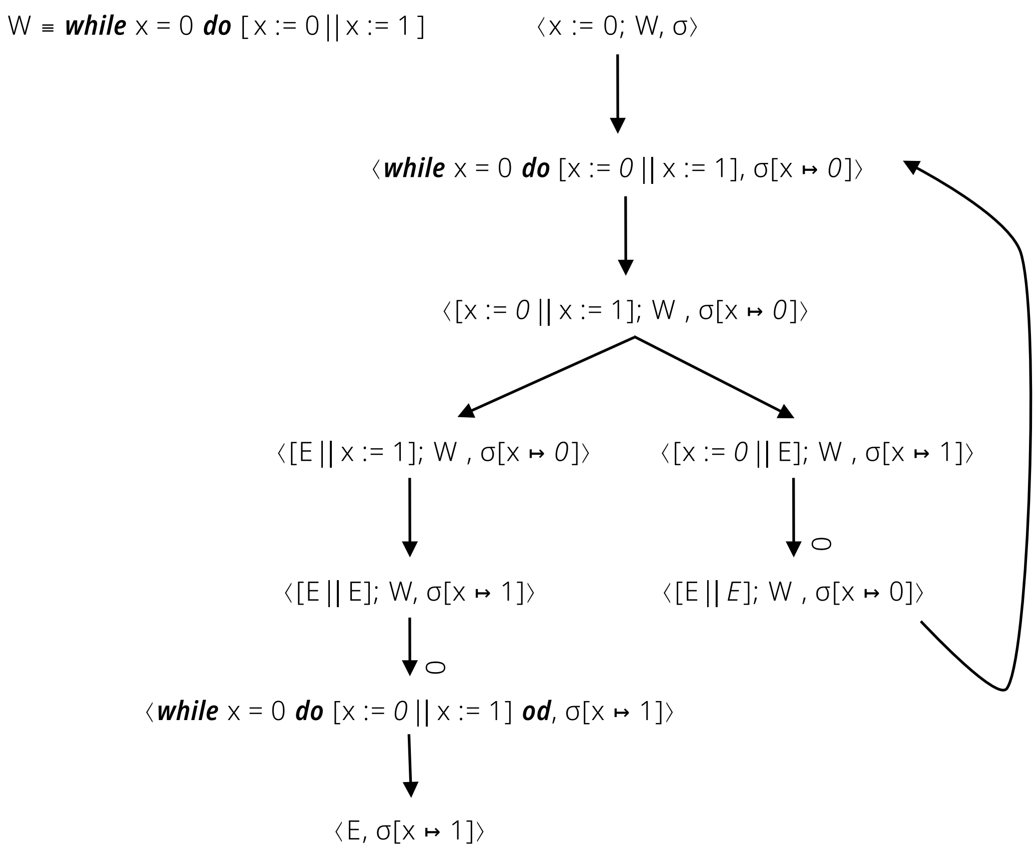
- ***Example 5***: If we take the program from Example 4 and combine the last two threads sequentially, then the evaluation graph for the resulting program is a subgraph of the Example 4 graph. Below, $\sigma(v) = 6$, and $M([\, x := v \,||\, y := v+2 \,||\, z := v*2 \,],\ \sigma) = \{\sigma[x \mapsto 6][y \mapsto 8][z \mapsto 12]\}$.



Example 5

- ***Example 6***: Let $W \equiv x := 0;$ ***while*** $x = 0$ ***do*** $[x := 0 \,||\, x := 1]$ ***od***. Then $M(W, \sigma) = \{\sigma[x \mapsto 1],\ \bot_d\}$. The problem here is possible divergence, but it only happens if we *always* choose thread 1 when we have to make the nondeterministic choice of $[x := 0 \,||\, x := 1]$. This is definitely unfair behavior, but it's allowed because of the unpredictability of our nondeterministic choices. In real life, we would want a fairness mechanism to ensure that all threads get to evaluate once in a while.
- If each thread is on a separate processor, then the nondeterministic choice corresponds to which processor is fastest, so the possible divergence of the program is a ***race condition***, where the correct behavior of a program depends on the relative speed of the processors involved. Here, divergence occurs if processor 1 is always faster than processor 2 (especially if processor 2 has died).
- Note that it's not necessarily a race condition to have a parallel program producing different results when run multiple times. As long as all results satisfy the specification, there's no race condition.

- ***Example 7***: The correctness triple $\{T\}\ [x := 0 \,||\, x := 1]\ \{x \geq 0\}$ does not have a race condition, but $\{T\}\ [x := 0 \,||\, x := 1]\ \{x > 0\}$ does.



Example 6

Note:

- The transitions ⟨[E || E]; W, σ[x ↦ …]⟩ $\rightarrow^0$ ⟨W, σ[x ↦ …]⟩ take 0 steps because [E || E]; W and W are the same program: [E || E]; W ≡ E; W ≡ W.

