

Solution to Homework 10 (Array Assignments)

1. (Calculate wp, simplification available.)

$$1a. \text{wp}(b[0] := 9, x > b[k]) \equiv (x > b[k])[9/b[0]] \equiv x > (b[k])[9/b[0]]$$

$$\equiv x > \text{if } k = 0 \text{ then } 9 \text{ else } b[k] \text{ fi. Possible simplification is if } k = 0 \text{ then } x > 9 \text{ else } x > b[k] \text{ fi}$$

$$1b. \text{wp}(b[k] := b[m], b[m] = z) \equiv (b[m] = z)[b[m]/b[k]] \equiv (b[m])[b[m]/b[k]] = z[b[m]/b[k]] \\ \equiv \text{if } m = k \text{ then } b[m] \text{ else } b[m] \text{ fi} = z, \text{ which simplifies to } b[m] = z.$$

$$1c. \text{wp}(b[k] := 1, b[k] = b[m]) \equiv (b[k])[1/b[k]] \equiv (b[m])[1/b[k]] \\ \equiv \text{if } k = k \text{ then } 1 \text{ else } b[k] \text{ fi} = \text{if } m = k \text{ then } 1 \text{ else } b[m] \text{ fi} \\ = (1 = \text{if } m = k \text{ then } 1 \text{ else } b[m] \text{ fi}) \\ = \text{if } m = k \text{ then } 1 = 1 \text{ else } 1 = b[m] \text{ fi} \\ = (m = k \vee b[m] = 1)$$

2. (Verify sequence of array assignments)

We want to calculate p_1 and p_2 in $\{p_2\} \ b[x] := b[m]; \{p_1\} \ b[y] := b[n] \ \{b[x] < b[y] \wedge x \neq y\}$.

$$p_1 \equiv \text{wp}(b[y] := b[n], b[x] < b[y]) \equiv (b[x] < b[y])[b[y]/b[n]] \\ \equiv (b[x])[b[n]/b[y]] < (b[y])[b[n]/b[y]] \\ \equiv \text{if } x = y \text{ then } b[n] \text{ else } b[x] \text{ fi} < \text{if } y = y \text{ then } b[n] \text{ else } b[y] \text{ fi} \\ \equiv \text{if } x = y \text{ then } b[n] \text{ else } b[x] \text{ fi} < b[n] \\ \equiv \text{if } x = y \text{ then } b[n] < b[n] \text{ else } b[x] < b[n] \text{ fi} \\ \equiv x \neq y \wedge b[x] < b[n]$$

$$p_2 \equiv \text{wp}(b[x] := b[m], p_1) \\ \equiv p_1[b[m]/b[x]] \\ \equiv (x \neq y \wedge b[x] < b[n])[b[m]/b[x]] \\ \Leftrightarrow x \neq y \wedge b[m] < (b[n])[b[m]/b[x]] \quad // \text{ compressed some steps here} \\ \Leftrightarrow x \neq y \wedge b[m] < \text{if } x = n \text{ then } b[m] \text{ else } b[n] \text{ fi} \\ \Leftrightarrow x \neq y \wedge \text{if } x = n \text{ then } b[m] < b[m] \text{ else } b[m] < b[n] \text{ fi} \\ \Leftrightarrow x \neq y \wedge x \neq n \wedge b[m] < b[n]$$

3. (Calculate $\text{wp}(b[x] := x, b[x] = b[b[y]])$)

$$a. \ e_1 \equiv (b[y])[x/b[x]] \equiv \text{if } x = y \text{ then } x \text{ else } b[y] \text{ fi}$$

$$b. \ e_2 \equiv e_1 = x \equiv x \Leftrightarrow \text{if } x = y \text{ then } x \text{ else } b[y] \text{ fi} = x \\ \Leftrightarrow \text{if } x = y \text{ then } x = x \text{ else } x = b[y] \text{ fi} \Leftrightarrow x = y \vee x = b[y]$$

$$c. \ e_3 \equiv b[e_1] = b[\text{if } x = y \text{ then } x \text{ else } b[y] \text{ fi}] \\ = \text{if } x = y \text{ then } b[x] \text{ else } b[b[y]] \text{ fi}$$

d. $e_4 \equiv (b[b[y]])(x/b[x])$
 $\equiv \text{if } x = e_1 \text{ then } x \text{ else } b[e_1] \text{ fi}$
 $= \text{if } e_2 \text{ then } x \text{ else } e_3 \text{ fi}$ // $e_2 \equiv (x = e_1), e_3 \equiv b[e_1]$
 $= \text{if } x = y \vee x = b[y] \text{ then } x \text{ else } e_3 \text{ fi}$ // Substitute for e_2
 $= \text{if } x = y \vee x = b[y] \text{ then } x \text{ else if } x = y \text{ then } b[x] \text{ else } b[b[y]] \text{ fi fi}$ // Substitute for e_3
 $= \text{if } x = y \vee x = b[y] \text{ then } x \text{ else } b[b[y]] \text{ fi}$ // Already checked $x=y$

e. $e_5 \equiv \text{wp}(b[x] := x, b[x] = b[b[y]])$
 $\Leftrightarrow x = e_4$
 $\Leftrightarrow x = \text{if } x = y \vee x = b[y] \text{ then } x \text{ else } b[b[y]] \text{ fi fi}$
 $\Leftrightarrow \text{if } x = y \vee x = b[y] \text{ then } x = x \text{ else } x = b[b[y]] \text{ fi fi}$
 $\Leftrightarrow x = y \vee x = b[y] \vee x = b[b[y]]$