Weakest Preconditions 1 & 2; Domain Predicates

CS 536: Science of Programming, Fall 2022

Due Sat Oct 8, 11:59 pm

Problems [60 points total]

Class 10: Weakest Preconditions part 1 [22 points]

1. [3 points] Let $IF_N \equiv$ ***if*** $B_1 \rightarrow S_1 \,\square\, B_2 \rightarrow S_2$ ***fi*** be a nondeterministic ***if-fi*** and let w_1 and w_2 be $wlp(S_1, q)$ and $wlp(S_2, q)$ respectively. Let p be $(B_1 \rightarrow w_1) \wedge (B_2 \rightarrow w_2)$ and let p' be $(B_1 \wedge w_1) \vee (B_2 \wedge w_2)$. Question: Why is $wlp(IF_N, q) \Leftrightarrow p$ but not p'?

2. [4 points] Which of the following (four) statements behave differently depending on whether S is deterministic or nondeterministic? Explain briefly.
   - $wp(S, p \vee q) \rightarrow$ and $\leftarrow wp(S, p) \vee wp(S, q)$
   - $wp(S, p \wedge q) \rightarrow$ and $\leftarrow wp(S, p) \wedge wp(S, q)$

3. [15 = 3*5 points] Let $w \Leftrightarrow wp(S, q)$ and let $b \rightarrow w$ and $w \rightarrow c$. Characterize each satisfiability / validity statement below as Always true, Always false, or Might be true (and might be false). Explain briefly.
   a. if S is deterministic, then $\vDash_{tot} \{b\}\ S\ \{q\}$.
   b. If S is nondeterministic, then there exists σ such that $\sigma \vDash \{\neg c\}\ S\ \{\neg q\}$.
   c. If S is nondeterministic, then there exist $\sigma \vDash \neg c$ and $\tau \in M(S, \sigma)$ such that $\tau \vDash q$.

Class 11: Weakest Preconditions part 2 [20 points]

Calculate the wlp for each of the following. You can omit intermediate calculations but they might be worth partial credit. Do only the syntactic calculations; don't simplify the result. E.g., $wlp(x := 2, x*x = 4) \equiv 2*2 = 4$, not T.

4. [10 points] $wlp(u := u*k;\ k := u,\ u > h(k))$.

5. [10 points] $wlp($***if*** $x < 0$ ***then*** $x := -x$ ***fi***$, x^2 \geq x)$. (Don't forget the implicit "***else skip***" clause.)

Class 11: Domain Predicates [18 points]

Calculate the *wp*'s below. Show your intermediate calculations. You can simplify your answer as you go and/or at the end or not at all (your preference). Suggestion: Define *S, q,* and *w* and separately calculate *D(S)*, $w \equiv wlp(S, q)$, and *D(w)*. Combine and give $wp(S, q) \Leftrightarrow D(S) \wedge w \wedge D(w)$.

6. [6 points] $wp(y := y/x, sqrt(y) < x)$.

7. [12 points] $wp(\textbf{if } y \geq 0 \textbf{ then } x := y / x \textbf{ else } x := -x / y \textbf{ fi}, r < x \leq y)$.