Array Assignments[1]

CS 536: Science of Programming, Fall 2021

Due Mon Nov 21, 11:59 pm

A. Why?

- Array assignments are more complicated than assignments to plain variables because they require information known only at runtime (the value of the index).

B. Objectives

After this homework, you should know how to

- Perform textual substitution to replace an array element.
- Calculate the *wp* of an array element assignment.

C. Problems [60 points total]

Class 21: Array Assignments

1. [9 = 3*3 points] Syntactically calculate the following; you may simplify if you want.
   a. wp(b[0] := 9, x > b[k])
   b. wp(b[k] := b[m], b[m] = z)
   c. wp(b[k] := 1, b[k] = b[m])

2. [6 points] Complete the full proof outline below for partial correctness by using wp to give definitions for p_1 and then p_2. Logically simplify as you go. (Hint: Try using $x \neq y$.)

   $\{p_2\}$ b[x] := b[m]; $\{p_1\}$ b[y] := b[n] $\{b[x] < b[y] \wedge x \neq y\}$

3. [35 points] The overall goal is to calculate wp(b[x] := x, b[x] = b[b[y]]). It's complicated, so we'll do it in parts. The hardest part is fully calculating (b[b[y]])[x/b[x]]. Unoptimized, (b[b[y]])[x/b[x]] ≡ ***if*** e_1 = x ***then*** x ***else*** b[e_1] ***fi***, where e_1 ≡ (b[y])[x/b[x]].
   a. [3 points] Expand e_1 ≡ (b[y])[x/b[x]] to get an ***if-fi*** expression.

   b. [9 points] Let e_2 ≡ (e_1 = x). Expand e_2 and logically simplify. (The simplest answer is a disjunction of two terms.)

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[1] This is the last homework for the semester!

c. [6 points] Let $e_3 \equiv b[e_1]$. Expand e_3 and arithmetically simplify it by changing it from a $b[\ldots]$ of an ***if-fi*** to an ***if-fi*** with $b[\ldots]$ inside.

d. [14 points] Let $e_4 \equiv$ (x = ***if*** $e_1 = x$ ***then*** x ***else*** $b[e_1]$ ***fi***). Substitute your e_2 for $(e_1 = x)$ and your e_3 for $b[e_1]$, and simplify the result: Push the x = ... part of the test into the arms of the ***if-fi*** and continue from there. The simplest result is a disjunct of three terms, but getting there is complicated. A hint: (***if*** T ***then*** B ***else*** ... ***fi*** $\Leftrightarrow B$) and (***if*** F ***then*** ... ***else*** B ***fi*** $\Leftrightarrow B$).

e. [3 points] Now calculate $wp(b[x] := x,\ b[x] = b[b[y]]) \equiv (b[x])[x/b[x]]) = (b[b[y]])[x/b[x]]$. (This should be easy.)

