

Weakest Preconditions

Part 2: Calculating wp , wlp ; Domain Predicates

CS 536: Science of Programming, Fall 2022

A. Why

- The weakest precondition and weakest liberal preconditions are the most general preconditions that a program needs in order to run correctly.

B. Objectives

At the end of this activity you should be able to

- Describe the relationship between $wp(S, q_1 \vee q_2)$, $wp(S, q_1)$, and $wp(S, q_2)$ and how it differs for deterministic and nondeterministic programs.
- Be able to calculate the wlp of a simple loop-free program.

C. Problems

- How are $wp(S, q_1 \vee q_2)$ and $wp(S, q_1)$ and $wp(S, q_2)$, related if S is deterministic? If S is nondeterministic?

For Problems 2 – 4, just syntactically calculate the wlp ; don't also logically simplify the result.

- Calculate the wlp in each of the following cases.
 - $wlp(k := k - s, n = 3 \wedge k = 4 \wedge s = -7)$.
 - $wlp(n := n * (n - k), n = 3 \wedge k = 4 \wedge s = -7)$.
 - $wlp(n := n * (n - k); k := k - s, n > k + s)$
- Let $Q(k, s) \equiv 0 \leq k \leq n \wedge s = \text{sum}(0, k)$ where $\text{sum}(u, v)$ is the sum of $u, u+1, \dots, v$ (when $u \leq v$) or 0 (when $u > v$).
 - Calculate $wp(k := k+1; s := s+k, Q(k, s))$.
 - Calculate $wp(s := s+k+1; k := k+1, Q(k, s))$.
 - Calculate $wp(s := s+k; k := k+1, Q(k, s))$. (This one isn't compatible with $s = \text{sum}(0, k)$.)

For Problems 4 – 6, don't forget the domain predicates. You can logically simplify as you go.

- Calculate the wp below.
 - $wp(\text{if } B \text{ then } x := x/2 \text{ fi}; y := x, x = 5 \wedge y = z)$.
 - $wp(\text{if } x \geq 0 \text{ then } x := x*2 \text{ else } x := y \text{ fi}; x := c*x, a \leq x < y)$
- Calculate p to be the wp in $\{p\} x := y/b[k] \{x > 0\}$.
- Calculate p_1 and p_2 to be the wp 's in $\{p_1\} y := \text{sqrt}(b[k]) \{z < y\}$ and $\{p_2\} k := x/k \{p_1\}$.

Solution to Practice 11 (Weakest Preconditions, pt. 2)

1. For deterministic S , $wp(S, q_1 \vee q_2) \Leftrightarrow wp(S, q_1) \vee wp(S, q_2)$.
For nondeterministic S , we have \Rightarrow but not \Leftarrow .
2. (Calculate wlp)
 - a. $wlp(k := k - s, n = 3 \wedge k = 4 \wedge s = -7) \equiv n = 3 \wedge k - s = 4 \wedge s = -7$
 - b. $wlp(n := n^*(n-k), n = 3 \wedge k = 4 \wedge s = -7) \equiv n^*(n-k) = 3 \wedge k = 4 \wedge s = -7$
 - c. $wlp(n := n^*(n-k); k := k-s, n > k+s)$
 $\equiv wlp(n := n^*(n-k), wlp(k := k-s, n > k+s))$
 $\equiv wlp(n := n^*(n-k), n > k-s+s)$
 $\equiv n^*(n-k) > k-s+s$
3. (wp involving sums) We have $Q(k, s) \equiv 0 \leq k \leq n \wedge s = \text{sum}(0, k)$.
 - a. $wp(k := k+1; s := s+k, Q(k, s))$
 $\equiv wp(k := k+1, wp(s := s+k, Q(k, s)))$
 $\equiv wp(k := k+1, Q(k, s+k))$
 $\equiv wp(k := k+1, 0 \leq k \leq n \wedge s+k = \text{sum}(0, k))$
 $\equiv 0 \leq k+1 \leq n \wedge s+k+1 = \text{sum}(0, k+1)$
 - b. $wp(s := s+k+1; k := k+1, Q(k, s))$
 $\equiv wp(s := s+k+1, wp(k := k+1, Q(k, s)))$
 $\equiv wp(s := s+k+1, Q(k+1, s))$
 $\equiv wp(s := s+k+1, 0 \leq k+1 \leq n \wedge s = \text{sum}(0, k+1))$
 $\equiv 0 \leq k+1 \leq n \wedge s+k+1 = \text{sum}(0, k+1)$
 - c. $wp(s := s+k; k := k+1, Q(k, s))$
 $\equiv wp(s := s+k, wp(k := k+1, Q(k, s)))$
 $\equiv wp(s := s+k, Q(k+1, s))$
 $\equiv wp(s := s+k, 0 \leq k+1 \leq n \wedge s = \text{sum}(0, k+1))$
 $\equiv 0 \leq k+1 \leq n \wedge s+k = \text{sum}(0, k+1)$. Note this isn't compatible with $s = \text{sum}(0, k)$.
4. (wp of *if-then*)
 - a. $wp(\text{if } B \text{ then } x := x/2 \text{ fi}; y := x, x = 5 \wedge y = z)$
 $\equiv wp(\text{if } B \text{ then } x := x/2 \text{ fi}, wp(y := x, x = 5 \wedge y = z))$
 $\equiv wp(\text{if } B \text{ then } x := x/2 \text{ fi}, x = 5 \wedge x = z)$
 $\equiv (B \rightarrow wp(x := x/2, x = 5 \wedge x = z)) \wedge (\neg B \rightarrow wp(\text{skip}, x = 5 \wedge x = z))$
 $\equiv (B \rightarrow x/2 = 5 \wedge x/2 = z) \wedge (\neg B \rightarrow x = 5 \wedge x = z)$

- b. $wp(\text{if } x \geq 0 \text{ then } x := x*2 \text{ else } x := y \text{ fi}; x := c*x, a \leq x < y).$
 $\equiv wp(S, wp(x := c*x, a \leq x < y))$ where S is the **if** statement
 $\equiv wp(S, a \leq c*x < y)$
 $\equiv wp(\text{if } x \geq 0 \text{ then } x := x*2 \text{ else } x := y \text{ fi}, a \leq c*x < y)$
 $\equiv (x \geq 0 \rightarrow wp(x := x*2, a \leq c*x < y)) \wedge (x < 0 \rightarrow wp(x := y, a \leq c*x < y))$
 $\equiv (x \geq 0 \rightarrow a \leq c*(x*2) < y) \wedge (x < 0 \rightarrow a \leq c*y < y)$

5. For $\{p\} x := y/b[k] \{x > 0\}$,
 let $p \Leftrightarrow wp(x := y/b[k], x > 0)$
 $\equiv wlp(x := y/b[k], x > 0) \wedge D(x := y/b[k])$
 $\equiv y/b[k] > 0 \wedge b[k] \neq 0 \wedge D(b[k])$
 $\equiv y/b[k] > 0 \wedge b[k] \neq 0 \wedge 0 \leq k < \text{size}(b)$
6. For $\{p_1\} y := \text{sqrt}(b[k]) \{z < y\}$
 let $p_1 \Leftrightarrow wp(y := \text{sqrt}(b[k]), z < y)$
 $\equiv wlp(y := \text{sqrt}(b[k]), z < y) \wedge D(y := \text{sqrt}(b[k]))$
 $\equiv z < \text{sqrt}(b[k]) \wedge b[k] \geq 0 \wedge D(b[k])$
 $\equiv z < \text{sqrt}(b[k]) \wedge b[k] \geq 0 \wedge 0 \leq D(b[k]) < \text{size}(b)$

For $\{p_2\} k := x/k; \{p_1\}$, let

- $p_2 \Leftrightarrow wp(k := x/k, p_1)$
 $\equiv wlp(k := x/k, p_1) \wedge D(k := x/k)$
 $\equiv p_1[x/k / k] \wedge k \neq 0$
 $\equiv (z < \text{sqrt}(b[k]) \wedge b[k] \geq 0 \wedge 0 \leq D(b[k]) < \text{size}(b)) [x/k / k] \wedge k \neq 0$
 $\equiv z < \text{sqrt}(b[x/k]) \wedge b[x/k] \geq 0 \wedge 0 \leq D(b[x/k]) < \text{size}(b) \wedge k \neq 0$