Types, Expressions, States, Quantified Predicates

CS 536: Science of Programming, Fall 2022

Due Tuesday (not Sat) Sep 13, 11:59 pm

2022-09-12 Group policy, Problems #6d, #9; 09-13 #8, #9

A. Formatting and Submitting Your Work

- Remember to use a word processor to write out your answers. Quantified variables range over $\mathbb{Z}$ unless otherwise specified.
- ~~*New Policy: For the group work, you can work in groups of 2 or 3 (preferably 3). You can work by yourself in a group of 1 but there's a 5% penalty (3 points out of 60).*~~ [no penalty for HW 2]
- [2022-09-13] To speed up grading, be sure to include your group number and a list of the group members' userids and A# numbers. Use *.pdfs, not *.doc files.

B. Problems [60 points total]

Class 3: Types, Expressions, and Arrays

1. (6 = 3 * 2 points) For each of the following, is the expression legal or illegal according to the syntax we're using. If illegal, why? If legal, what is the type of the resulting expression? Below, assume *b1*, and *b2* are one-dimensional and *b3* is two-dimensional.
   a. *(x < y ? T : y)* // Note < works on integers, not booleans
   b. *match(b1, b2, n)* // Are the first *n* elements of *b1* and *b2* equal (component-wise)?
      // I.e., *b1[0] = b2[0], b1[1] = b2[1], ...*
   c. *b3[0] + b3[2][3]*

2. (6 = 3 * 2 points) For each set below, is it a well-formed state? If not, why? *
   a. {*x* = (4), *y* = 5} // Is the value of *x* an integer or a short array?
   b. {*u* = (3, 4), *v* = 0, *w* = *u[1]* }
   c. {*r* = one, *s* = four, *t* = *r* + *s*}

3. (4 = 2 * 2 points) Let σ = {*x* = 2, *b* = β} where β = (7, 12, 3, 0).
   a. Rewrite σ giving the value of *b* as a set of ordered pairs.
   b. Rewrite σ giving the value of *b* as separate bindings for *b[0], b[1]*, etc.

4. (6 = 3 * 2 points) Let $\varphi \equiv x = y*z \wedge y = 4*z \wedge z = b[0] + b[2] \wedge 2 < b[1] < b[2] < 5$. Complete the definition of σ = {*x* = ____, *y* = ____, *z* = 5, *b* = _____ } so that $\sigma \vDash \varphi$. If some value is unconstrained, give it a greek letter name (δ, ζ, η, ... your choice).

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* For this assignment, a lot of semantic values are underlined for emphasis (but not necessarily all of them).

5. (8 = 4 * 2 points) Take the expression *0 * b[b[k]]*. For each state below, is it well-formed and proper for the expression? And if so, does the expression terminate correctly (and with what result)? If not, why?
   a. {*k* = 0, *b* = (3, 6, 1, 4), *c* = (2)}
   b. ∅
   c. {*k* = 0, *b* = 3}
   d. {*b* = (3), *k* = 0}

Class 4: State Updates, Satisfaction of Quantified Predicates

6. (9 points) Let σ_0 = {*x* = 2, *y* = 4, *b* = (-1, 4, 3, 9, 10)}.
   a. [2 points] Is there a difference between $\sigma_0[z \mapsto 1]$ and $\sigma_0 \cup \{(z, 1)\}$? Very briefly justify your answer. (A sentence should be fine.)
   b. [2 points] Repeat, on $\sigma_0[x \mapsto 4]$ and $\sigma_0 \cup \{(x, 4)\}$.
   c. [2 points] Let σ_1 = {*x* = β+3, *y* = 2 β, *b* = (β, 0, 2 β, β)} where β = 2. What is $\sigma_1[\, b[0] \mapsto \sigma_1(b[2])\,]$?
   d [3 points] Now let $\tau = \sigma_1[\, b[0] \mapsto \sigma_1(b[2])\,]$. What is $\tau[\, b[1] \mapsto \sigma_1(b[1]) + 8\,]$? [2022-09-12]

7. (6 = 3 * 2 points) For each of the following, say whether the state satisfies the quantified predicate (and if not, briefly why). Give a witness value (for satisfied existentials) or a counterexample (for unsatisfied universals).
   a. Does {*x* = 4, *y* = 7, *b* = (5, 4, 8)} ⊨ *(∃ x. ∃ m. b[m] < x < y)* ? If not, why?
   b. Does {*x* = 1, *b* = (2, 8, 9)} ⊨ *(∀x. ∀k. 0 < k < 3 → x < b[k])* ? If not, why?
   c. Does {*x* = 0, *b* = (5, 3, 6)} ⊨*(∀x. ∀k. 0 < k < 3 ∧ x < b[k])* ? If not, why?

8. (9 = 3 * 3 points) In English, explain briefly when each of the following holds.
   a. $\not\models (\forall\, x \in V\,.\, (\exists\, y \in U\,.\, P(x, y)) \wedge (\forall\, z \in U\,.\, Q(x, z)))$
   b. $\not\models \forall\, y \in V\,.\, ((\exists\, x \in W.\, P(x, y)) \rightarrow (\exists\, y \in U\,.\, Q(y, y)))$ [2022-09-13]
   c. $\sigma \not\models (\exists\, x \in W\,.\, (\forall\, y \in U\,.\, P(x, y)))$

9. (6 points) Write a definition for a predicate function *Subset(b1, b2, x, y)* ≡ ... such that *{b1[0], b1[1], ..., b1[x]}* ⊆ *{b2[0], b2[1], ..., b2[y]}*. For example, say we have a state where *b1* = (0, 3, 7, 2, 1, 4) and *b2*= (0, 2, 7, 3, 3, 5) [2022-09-13], then [See class 2 for discussion of predicate functions]
   - *Subset(b1, b2, 3, 5)* is true because {0, 3, 7, 2} ⊆ {0, 2, 7, 3, 3, 5}.
   - *Subset(b2, b1, 5, 3)* is false because {0, 2, 7, 3, 3, 5} ⊈ {0, 3, 7, 2}.
   - *Subset(b1, b2, 2, 2)* is false because {0, 3, 7} ⊈ {0, 2, 7}. [2022-09-13]
   - *Subset(b2, b1, 2, 3)* is true because {0, 2, 7} ⊆ {0, 3, 7, 2}. [2022-09-13]

If *x* or *y* are illegal as indexes for *b1* or *b2* respectively, have *P* return false. (You can add this as an explicit test or have it work implicitly -- either way is fine.) Feel free to write helper predicate functions if it makes your life easier.