Question 1-3 were graded by Lan Wei <lwei3@hawk.iit.edu>
Question 4-7 were graded by Suyog Bachhav <sbachhav@hawk.iit.edu>
Question 8-10 were graded by Nanda Kishore Reddy Velugoti <nvelugoti@hawk.iit.edu>

1).[8 = 4 * 2 points]
a. $p \rightarrow q$
b. $p \rightarrow q$
c. $q \rightarrow p$
d. $q \rightarrow p$

2). [4 = 2*2 points]

a). Yes. Syntactic Equality Implies Semantic Equality
b). No. 1 + 1 =2 while 1 + 1 ≢ 2

3).[6 = 3 * 2 points]
a). well formed, improper. Reason: It is missing a binding for z (even though the value of z is irrelevant).
b). well-formed, proper, runtime error. Reason: cannot execute sqrt(-4)
c). well-formed, proper, runtime error. Reason: denominator evaluates to 0.

4).

$p \wedge \neg(q \wedge r) \rightarrow q \wedge r \rightarrow \neg p$	
$\Leftrightarrow p \wedge \neg(q \wedge r) \rightarrow (\neg(q \wedge r) \vee \neg p)$	Defn →
$\Leftrightarrow \neg(p \wedge \neg(q \wedge r)) \vee (\neg(q \wedge r) \vee \neg p)$	Defn →
$\Leftrightarrow (\neg p \vee (q \wedge r)) \vee (\neg(q \wedge r) \vee \neg p)$	DeMorgan's law (on ¬(... ∧ ...)) and ¬¬
$\Leftrightarrow (q \wedge r) \vee \neg(q \wedge r) \vee \neg p \vee \neg p$	∨ associative and commutative
$\Leftrightarrow T \vee \neg p \vee \neg p$	Excluded Middle
$\Leftrightarrow T$	Domination

5)

$\neg(\forall\, x.\, (\exists\, y.\, x \leq y) \vee \exists\, z.\, x \geq z)$	
$\Leftrightarrow \exists\, x.\, \neg((\exists\, y.\, x \leq y) \vee \exists\, z.\, x \geq z)$	DeMorgan's Law (¬∀ ⇔ ∃ ¬)
$\Leftrightarrow \exists\, x.\, \neg(\exists\, y.\, x \leq y) \wedge \neg\exists\, z.\, x \geq z)$	DeMorgan's Law (¬ ∨ ⇔ ¬ ∧ ¬)
$\Leftrightarrow \exists\, x.\, (\forall\, y.\, x > y) \wedge \neg\exists\, z.\, x \geq z$	DeMorgan's Law (¬∃ ⇔ ∀ ¬) and ¬ of ≤
$\Leftrightarrow \exists\, x.\, (\forall\, y.\, x > y) \wedge \forall\, z.\, x < z$	DeMorgan's Law (¬∃ ⇔ ∀ ¬) and ¬ of ≥

6)
a. $((((p \wedge (\neg r)) \wedge s) \rightarrow (((\neg q) \vee r) \rightarrow (\neg p))) \leftrightarrow ((\neg s) \rightarrow t))$
b. $(\exists\; m\; .\; ((0 \leq m < n) \wedge (\forall\, j\; .\; ((0 \leq j < m) \rightarrow (b[0] \leq b[j] \leq b[m])))))$

7).

a. $\neg(p \vee q) \vee r \rightarrow \neg q \vee r \rightarrow p \vee \neg r \vee q \wedge s$
b. $\exists\ i\ .\ 0 \leq i < m \wedge (\forall\ j\ .\ m \leq j < n \rightarrow b[i] = b[j])$
c. $\forall\ x\ .\ (\exists\ y.\ p \rightarrow q) \rightarrow \forall\ z.\ q \vee r \wedge s$

8)
a. no
b. No
c. no
d. No

9)
a. Contingency

p	q	r	((p → (q → r)) ↔ ((p → q) → r))
F	F	F	F
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

b. Tautology

10)

GT(b, x, m, k) ≡ $\forall$ i.m ≤ i < m+k → x > b[i] is one solution. $\forall$ j.0 ≤ i < k → x > b[m+j] is another.