Quick Sort

I/p: A[1 . . . n]

O/p : Sorted order

Quick Sort

(1) Choose a pivot $x = A[p]$ $O(1)$

Partition (Loop invariant)

(2) Rearrange A[1 . . . n]

(a) A[1 . . . r-1] $\leq x$

(b) $A[r] = x$

(c) A[r+1 . . . n] $> x$

(3) QuickSort(A[1 . . . r-1])

QuickSort(A[r+1 . . . n])

$$T(n) = T(r) + T(n-r) + R(n)$$

$$= T(r) + T(n-r) + O(n)$$

If pivot is so good in all recursive steps

$$= \boxed{2T(n/2) + O(n)}$$

$$= O(n \log n)$$

$$T(n) \leq T(n/3) + T(2n/3) + \underline{O(n)}$$

Excessive

$$= \underline{O(n \log n)}$$

$$\leq \underline{2T(2n/3) + O(n)}$$

$A[1 - - - - n]$

↓

~~The~~ Get the element in the middle of the sorted order (Median)

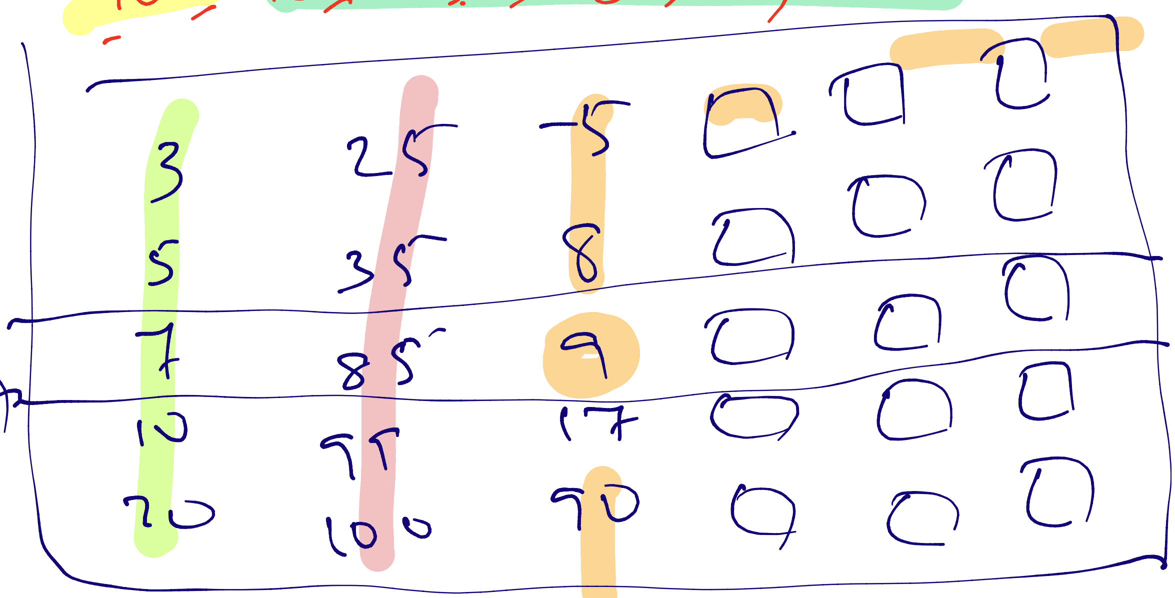
Objective: Find out median of n ~~ele~~ elements in $O(n)$ time

k^{th} smallest element:

Input: $A[1 - - n]$, k

Output: the k^{th} smallest element (the rank k element)

10, 20, 5, 3, 7, 99, 25, 35, 85, 100, 90, -5, 8, 9, 17



$n/5$ elements on the line

$M[1 \ldots n/5]$.

- recursively find the $n/10^{th}$ smallest element

10

of $M[1 \ldots n/5]$.

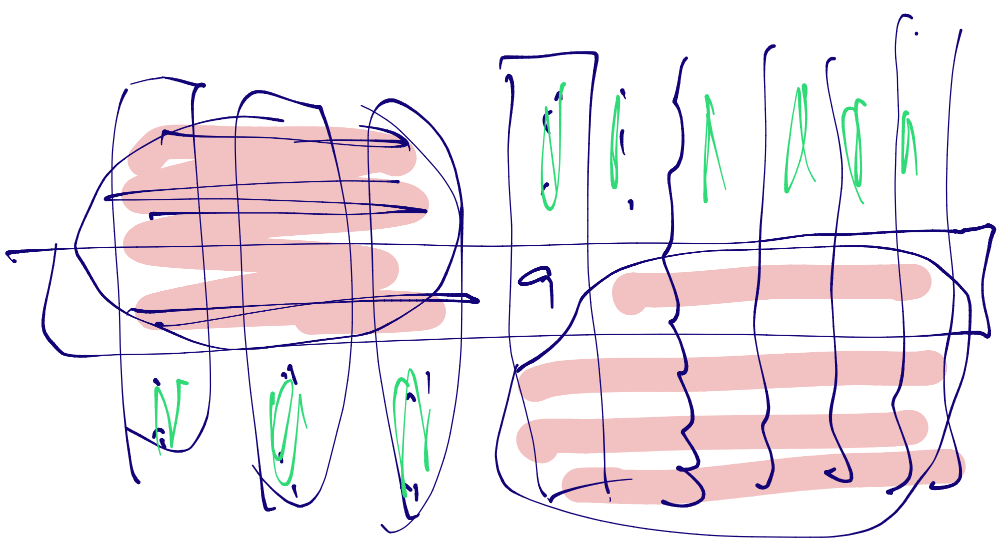
x

rearrange A

such that $A[1 \ldots r-1] \leq x$

$A[r] = x$

$A[r+1 \ldots n] > x$



elements $\leq$ ~~x~~ x
is at least $3n/10$

element $> x$ is at least $3n/10$

elements that are less than x is at most $7n/10$

element that are $> x$ is at most $7n/10$

SMALL(A[1...n], k)

Arrange them in blocks of 5

(2) Let x be the median of the middle row

$A[p] = \underline{\underline{x}}$.

- $\underline{\text{Partition}}(A[1...n], p)$

(1) $A[1...r] \leq x$ $\to \leq \frac{7n}{10}$

$O(n)$ ② $A[r] = x$

③ $A[r+1 \ldots n] > x$ — $7n/10$

If $r = k$ then output x

If $k < r$

— recursive $(A[1 \ldots r], k)$

Otherwise $k > r$

— recursive $(A[r+1 \ldots n], k-r)$

④ ~~Find the median~~

$M[1 \ldots n/5]$

$\text{SMALL}(M[1 \ldots n/5], n/2)$

(10)

$$T(n) = \boxed{T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + O(n)}$$

→ $O(n)$ ✓

→ $O(n\log n)$ ✓

$O(n^2)$

None of above.

$$\frac{n}{5} + \frac{7n}{10} = \frac{9n}{10}.$$

Def. Quick Sort

with $O(n \log n)$
worst-case time

Ner

$$T(n) = 2T(n/2) + O(n) = O(n \log n)$$

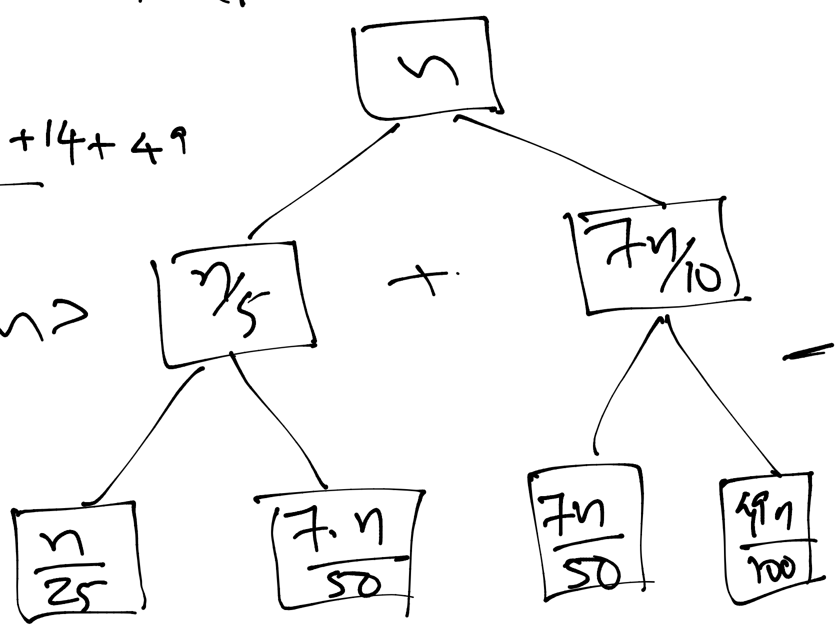
$$T(n) \le T(n/5) + T\left(\frac{7n}{10}\right) + cn$$

$$T(i) \le 25 \quad \text{for all } i \le 5$$

cn

$\frac{4+14+14+49}{100}$

$n >$



$$c\left(\frac{n}{5} + \frac{7n}{10}\right) = c\left(\frac{9}{10}n\right)$$

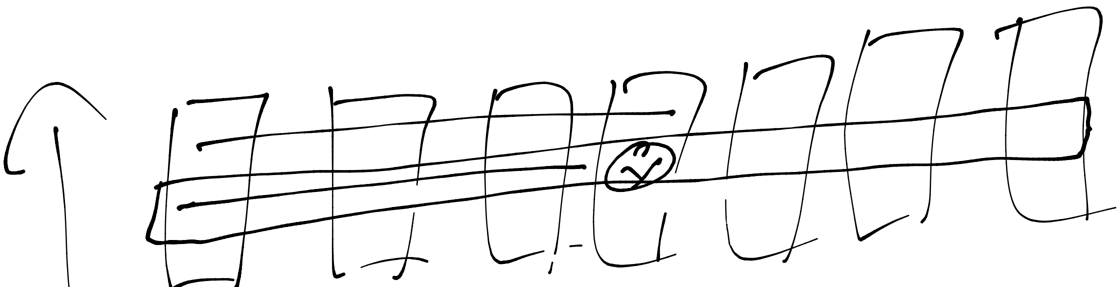
$$c \cdot \left(\frac{81}{100}n\right)$$

$$cn\left(1+\frac{9}{10}+\left(\frac{9}{10}\right)^2+\left(\frac{9}{10}\right)^3+\cdots\right)$$

$$cn\left(\frac{1}{1-9/10}\right)=\underline{10cn}$$

upper bound on the
element $\leq x$ in $= 2n/3$

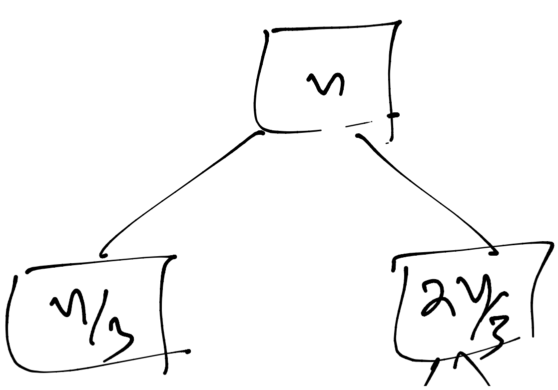
$$=\underline{O(n)}$$



$\geq 2\frac{n}{6}$ are $\leq x$

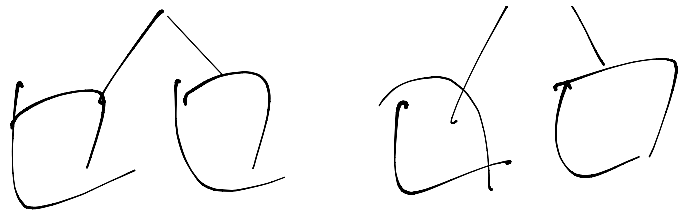
$\geq n/3$ elements are $\geq x$

$$T(n)=T\left(n/3\right)+T\left(2n/3\right)+cn$$



cn

cn



cn

$O(n \log n)$

cn · # levels in the recursion tree.

$\log_{3/2} n = O(\log n)$

- Divide into subproblem.
- Collect solutions ✓
- Combine them to a solution for our problem ✓

Backtracking

SUBSETSUM.

Ip: $X[1 \ldots \ldots n]$, T

Qn: $\exists$? a subset of X that ~~ad~~ sum to T.

$\{5, 7, 3, 2, 8, 9\}$, 14

Yes

$\cancel{16}$

25

4

No!

$X[1 \ldots n]$, $T \geq 0$

If $T > 0$ and $n = 0$ then No

If $T = 0$, Output Yes

$with \leftarrow SUBSETSUM(X[1 \ldots n-1], T - X[n])$

$without \leftarrow SUBSETSUM(X[1 \ldots n-1], T)$

Output $(with \vee without)$

$X[1 \ldots n]$

$$SUBSETSUM(i, T') = \begin{cases} 1 & \text{if } T' = 0 \\ SUBSETSUM(i-1, T' - X[i]) \vee SUBSETSUM(i-1, T') & \end{cases}$$

Output $SUBSETSUM(n, T)$

$T(0) = 1$

$T(n) = 2T(n-1) + 4$

$= O(2^n)$

Proof. by induction.

$T(n) \leq 2^n$

$T(0) = 2^0 = 1$ Base case

By I.H we have

$T(n-1) \leq 2^{n-1}$

$T(n) \leq 2T(n-1) + 4$

$\leq 2 \cdot 2^{n-1} + 4$

$$\leq 2^n + 4$$

$$\boxed{T(n) \leq c \cdot 2^n}$$

$$T(n) \leq 2 \cdot c \; 2^{n-1} + 4 \; =$$

$$\leq c \cdot 2^n + 4$$

$$T(n) \leq 2^n - 4$$

$$T(0) \leq 2^0 - 4 \leq -3$$

$$\boxed{T(n) \leq 5 \cdot 2^{n} - 4}$$

$$T(0) \leq 5 \cdot 1 - 4 = 1$$

$$T(n) \leq 2\left[5 \cdot 2^{n-1} - 4\right] + 4$$

$$\leq 5 \cdot 2^{n} - 4$$

Proof by induction

TEXT SEGMENTATION

IAMASTUDENT

I AM STUDENT

$s[i \ldots j \ldots n]$

$\text{ISWORD}(i, j) = s[i..j]$

$s\,\boxed{i \ldots n}$

$$\text{SPLITTABLE}(i) = \begin{cases} \bigvee_{j=i}^{n} \left(\underline{\text{ISWORD}(i, j)} \wedge \underline{\text{SPLITABLE}(j+1)} \right) \\ 1 & \text{if } i > n \end{cases}$$

# of ISWORD calls.

$$\boxed{\begin{aligned} T(n) &= \sum_{i=1}^{n} T(n-i) + \alpha n \\ T(0) &= 1 \end{aligned}}$$

$$T(n-1) = \sum_{i=1}^{n-1} T(n-1-i) + \alpha(n-1)$$

$$T(n) - T(n-1) = T(n-1) + \alpha$$

$$T(n) = 2T(n-1) + \alpha$$

$$= O(2^n)$$