CS 2443 - Quiz-1

MS17BTECH11013
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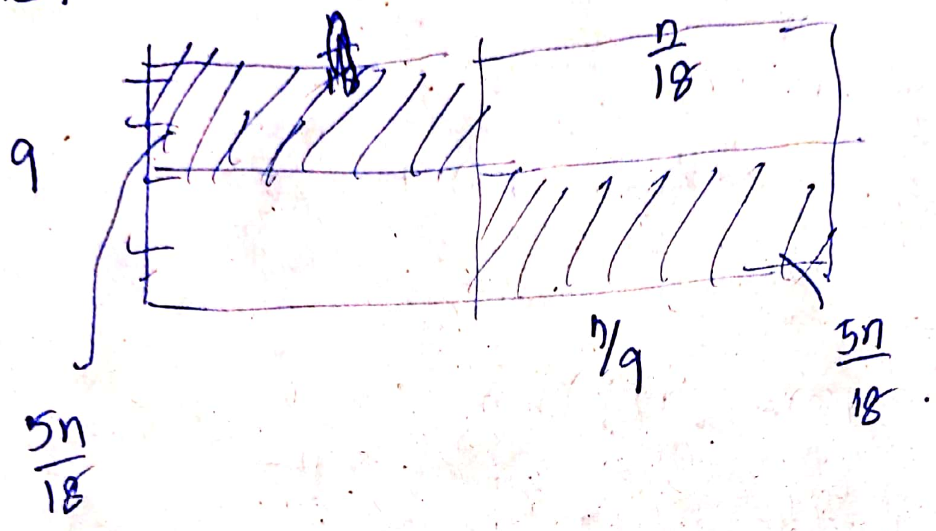
1)

if we ~~divid~~ make blocks of size 9 instead of 5 then



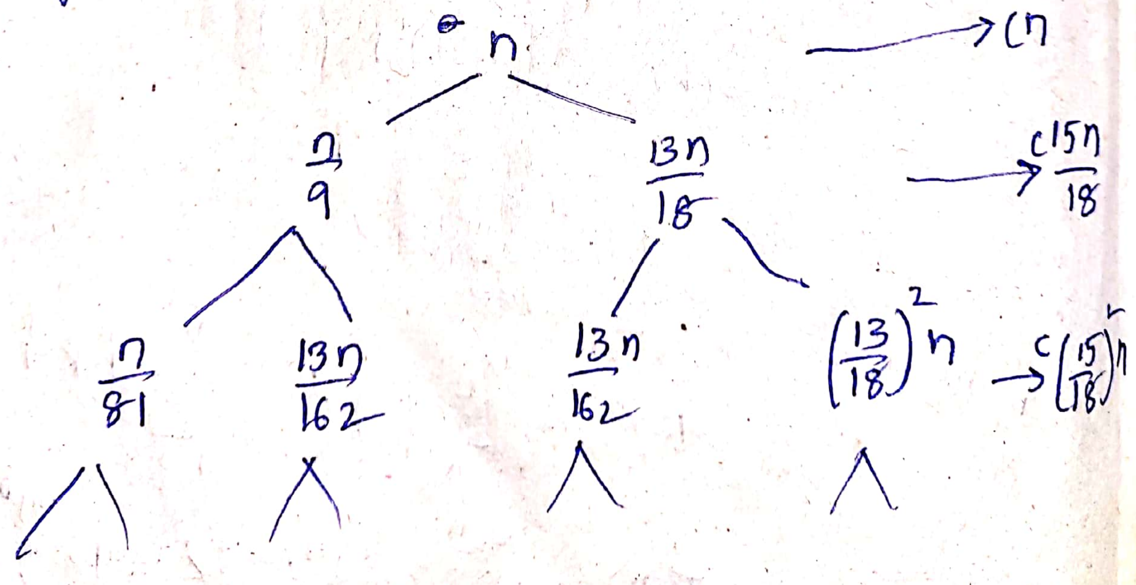
$n - \frac{5n}{18}$

$= \frac{13n}{18}$

The Recurrence Relation will be.

$$T(n) \leq T\left(\frac{n}{9}\right) + T\left(\frac{13n}{18}\right) + O(n)$$

solving by Recurrence Tree method



$$= cn\left[1+\frac{15}{18}+\left(\frac{15}{18}\right)^2+\ldots\ldots\right].$$

$\searrow$ ∞ GP

$a_0 = 1$

$r = \frac{15}{18}$.

$S_\infty = \frac{a}{1-r}$

$$= cn\left[\frac{1}{1-\frac{15}{18}}\right]$$

$$= cn\left[\frac{\frac{1}{1}}{\frac{3}{18}}\right]$$

$$= cn \cdot \frac{\cancel{18}^{6}}{\cancel{3}} = 6cn.$$

$$= O(n).$$

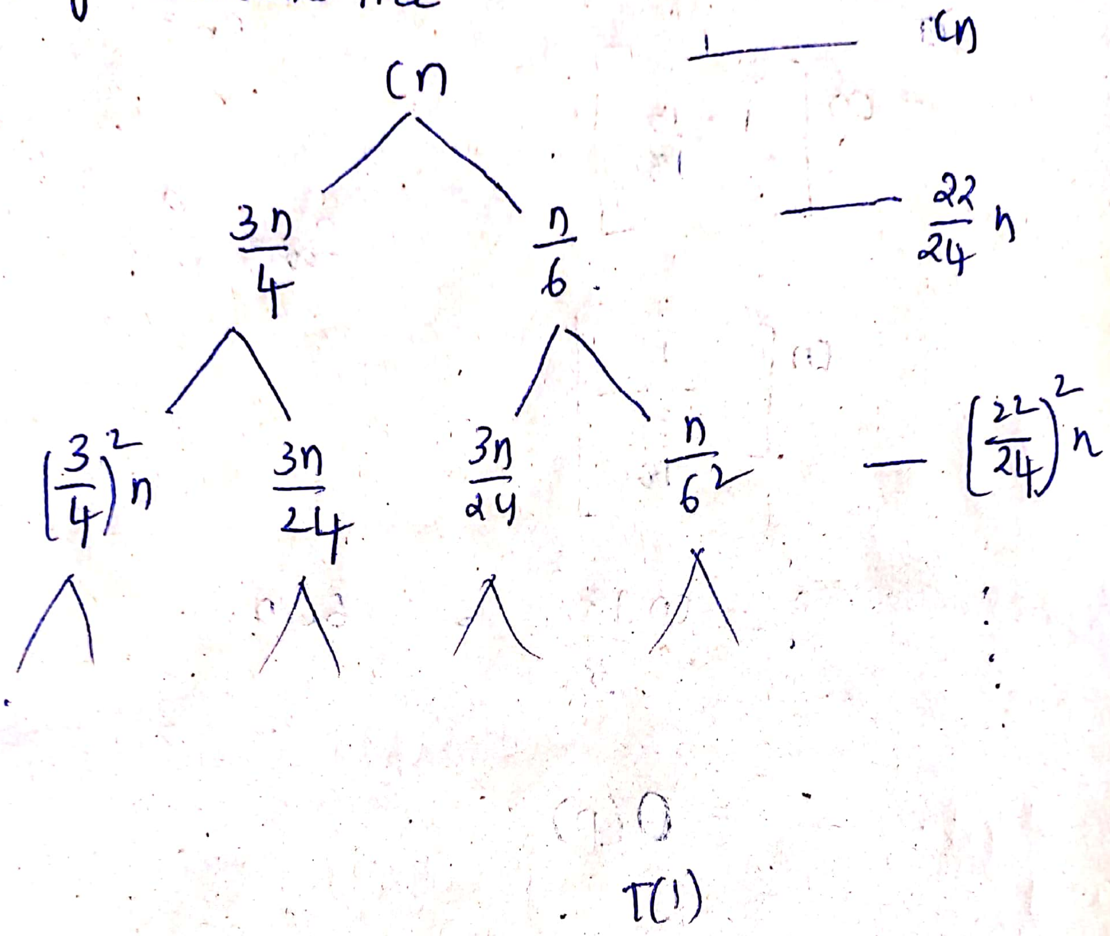
Q.

$T(n) =$

2.b

$$T(n) = 3T\left(\frac{n}{4}\right) + T\left(\frac{n}{6}\right) + n.$$

Using Recurrence Tree



$$= cn + \frac{23}{24}n + \left(\frac{22}{24}\right)^2 n + \dots$$

$$= cn\left[1 + \frac{22}{24} + \left(\frac{22}{24}\right)^2 + \dots\right]$$

$$= cn \cdot \left[\frac{1}{1 - \frac{22}{24}}\right]$$

$$= 12cn \qquad O(n)$$

3) $T(n) = 3T(n-1) + n^2$

$T(0) = 1$

To prove $T(n) = O(3^n)$

$T(n) = c.3^n$

Base case

$T(n) = O(3^n)$

$T(0) = c.3^0 = 1$

$= c$

Inductive hypothesis

$T(k) = O(3^k) \quad \forall k < n$

$T(k) = c.3^k \quad k = 1, 2, \ldots n-1$

$\forall k < n$

We know $T(n) = 3T(n-1) + n^2$

$= 3\left[c.3^{k-1}\right] + n^2$

$= 3.c.3^{n-1} + n^2$

$= c.3^n + n^2$

$T(n) = 3 \cdot 3^n + n^2$

$3^n + 2 \cdot 3^n + n^2$

3

$\leq c 3^n$

$T(n) \leq c 3^n + n^2$

$= c3^n - (-n^2)$ ≥ 0 no true

Now let $T(K) = O(3^n)$

$T(K) = c_1 3^n - c_2 n^2$ Tighter Bound.

W.K.T

$T(K) = 3T(n-1) + n^2$ $\forall K < n$

$K = 1, 2, \ldots n-1$

$= 3\left[c_1 3^{n-1} - c_2 (n-1)^2\right] + n^2$

$= c_1 3^n - 3c_2 (n-1)^2 + n^2$

$= c_1 3^n - \left[3c_2 (n-1)^2 - n^2\right]$

$$T(K) = c_1 3^n - \underbrace{[3c_2 (n-1)^2 - n^2]}_{>0}$$

$\vdots$

$< c \cdot 3^n$.

$3c_2 (n-1)^2 - n^2 > 0$

$3c_2 (n-1)^2 > n^2$

for a correct choice of c_2 we may prove

$3c_2 (n-1)^2 > n^2$.

$n = 1$

$\cancel{3c_2}\, 0 > 1$

$\cancel{3c}$

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