

CS 2443: Quiz 2

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- Total marks: 10.
- Read the question carefully and answer to the question only.
- Maintain academic honesty.

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1. Describe a recursive formula and prove its correctness for the following generalization of the SUBSET SUM problem. Given two arrays $X[1 \dots n]$ and $W[1 \dots n]$ of positive integers and an integer T , where each $W[i]$ denotes the weight of the corresponding element $X[i]$, compute the maximum weight subset of X whose elements sum to T . If no subset of X sums to T , then the output is $-\infty$. [3]
 2. Consider the algorithm $BFS(G, s)$ and finding shortest distance through BFS mentioned in the lecture, where G is a graph and s is a vertex in G . Prove that for any vertex v in the layer L_i , the distance from s to v is i . [2]
 3. Let G be a graph. Consider the following relation \sim . For any two vertices u and v , $u \sim v$ if and only if there are two edge-disjoint paths P_1 and P_2 from u to v (i.e., $E(P_1) \cap E(P_2) = \emptyset$). Is \sim an equivalence relation? Justify your answer. [2]
 4. Consider the digraph D mentioned in Figure 1. List all the strongly connected components in it. Consider the algorithm DFS-Loop mentioned in the lecture (see Figure 2). Suppose you run DFS-Loop on the graph D in Figure 1 (you may assume a labeling of vertices to $1, \dots, 5$). Then, list the finishing time of each vertex. [1+2]

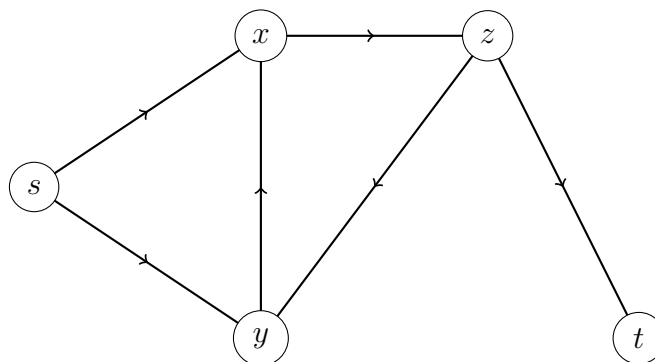


Figure 1: Digraph D

DFS-Loop (graph G)

Global variable $t = 0$

[# of nodes processed so far] For finishing times in 1st pass

Global variable $s = \text{NULL}$ For leaders in 2nd pass
[current source vertex]

Assume nodes labeled 1 to n

For $i = n$ down to 1

 if i not yet explored

$s := i$

 DFS(G, i)

DFS (graph G , node i)

-- mark i as explored For rest of DFS-Loop

-- set $\text{leader}(i) := \text{node } s$

-- for each arc (i, j) in G :

 -- if j not yet explored

 -- DFS(G, j)

-- $t++$

-- set $f(i) := t$

i 's finishing time

Figure 2: DFS-Loop