Quick Sort

I/p: A[1 - - - n]

O/p: Sorted order

Quick Sort

(1) Choose a pivot $x = A[p]$ — $O(1)$

(2) Partition (Loop invariant)

Rearrange $A[1 - - - n]$

(a) $A[1 - - - r-1] \leq x$

(b) $A[r] = x$

(c) $A[r+1 . . . n] > x$

(3) QuickSort(A[1 — r-1])

QuickSort(A[r+1 . . n])

$$T(n) = T(r) + T(n-r) + R(n)$$

$$= T(r) + T(n-r) + O(n)$$

If pivot is so good in all recursive steps

$$= \boxed{2T(n/2) + O(n)}$$

$$= O(n \log n)$$

$$T(n) \leq T(n/3) + T(2n/3) + \underline{O(n)}$$

Excessive

$$= \underline{O(n \log n)}$$

$$\leq \underline{2T(2n/3) + O(n)} =$$

$A[1 - - - - n]$

~~the~~ Get the element in the middle of the sorted order (Median)

Objective: Find out median of n ~~ben~~ elements in $O(n)$ time

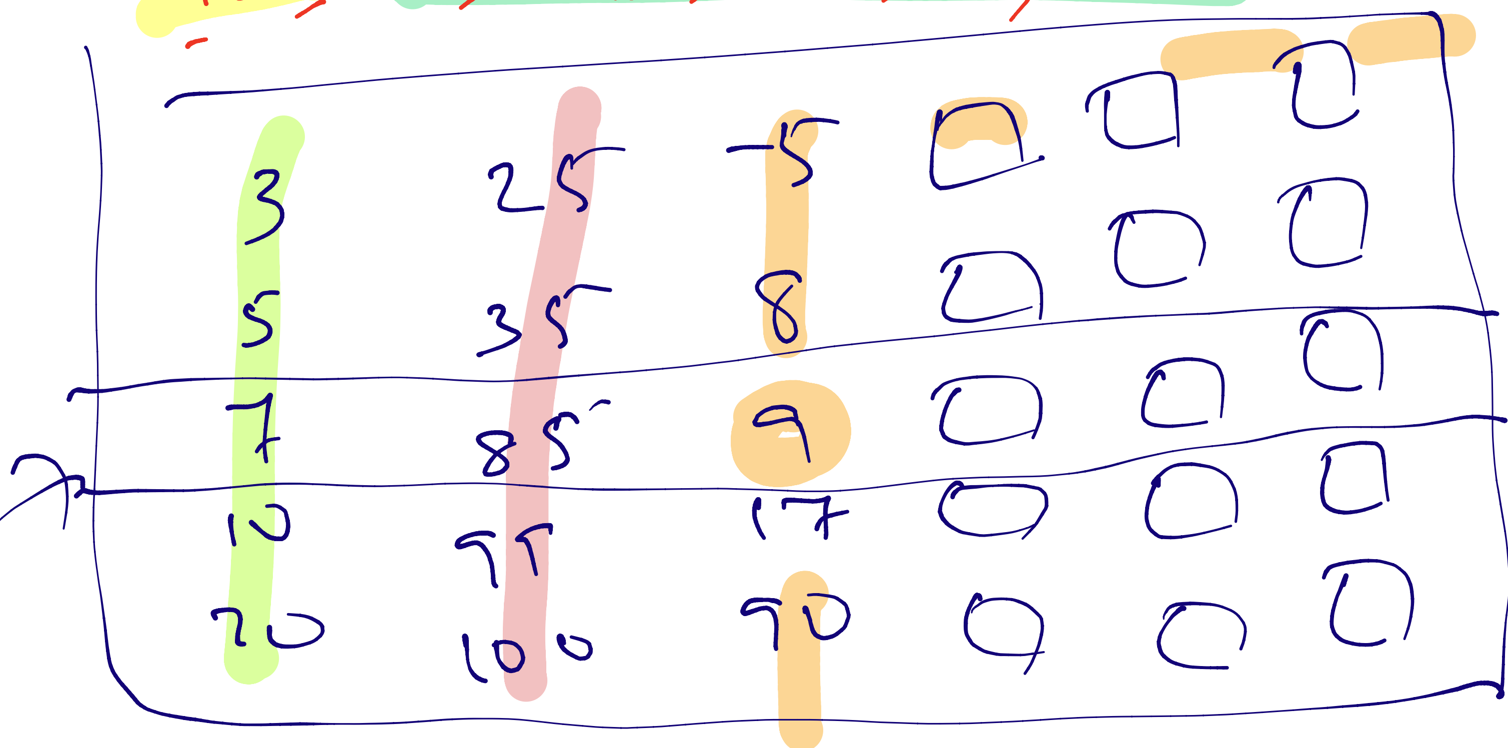
kth smallest element:

Input: $A[1 - - n]$, k

Output: the k^{th} smallest element (the rank k element)

10, 20, 5, 3, 7, 99, 25, 35, 85

100, 90, -5, 8, 9, 17



$n/5$ elements on line

$M[1 - - - n/5]$.

- recursively find the $n/10^{th}$ smallest element of $M[1 \ldots n/5]$.

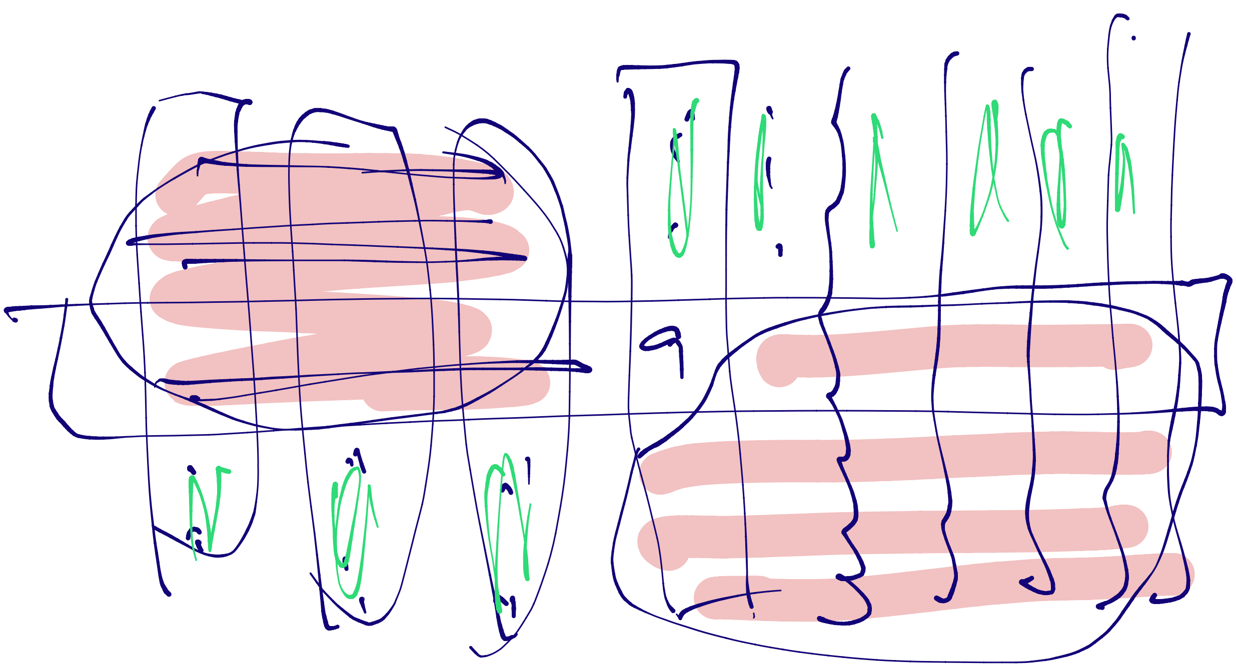
x

rearrange A

such that $A[1 \ldots r-1] \le x$

$A[r] = x$

$A[r+1 \ldots n] > x$



elements $\leq x$
is at least $3n/10$
element $> x$ is at least
$3n/10$

elements that are less than x is at most $7n/10$
element that are $> x$ is at most $7n/10$

SMALL($A[1 \ldots n]$, k)
Arrange them in blocks of 5
(I) Let x be the median of the middle row
$A[p] = x$.
- $\underline{\text{Partition}}(A[1 \ldots n], p)$
(1) $A[1 \ldots r] \leq x$
$\leq \frac{7n}{10}$

$O(n)$ (2) $A[r] = x$

(3) $A[r+1 \ldots n] > x$ — $7n/10$

If $r = k$ then output x

If $k < r$

— recursive $(A[1 \ldots r], k)$

Otherwise $k > r$

— recursive $(A[r+1 \ldots n], k - r)$

(4) ~~Find the median~~

$m[1 \ldots n/5]$

$\text{SMALL}(m[1 \ldots n/5], n/5)$

$\left(\frac{}{10}\right)$

$$T(n) = \boxed{T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + O(n)}$$

→ $O(n)$ ✓

→ $O(n \log n)$ ✓

$O(n^2)$

None of above.

$$\frac{n}{5} + \frac{7n}{10} = \frac{9n}{10}.$$

Def. Quick Sort

with $O(n \log n)$

worst-case time

Mer

$$T(n) = 2T(n/2) + O(n)$$
$$= O(n \log n)$$

$$T(n) \leq T(n/5) + T\left(\frac{7n}{10}\right) + cn$$

$T(i) \leq 25$ ✓ for all $i \leq 5$

cn?

$\boxed{n}$

$\frac{4+14+14+49}{100}$

$n >$

$\boxed{n/5}$ + $\boxed{7n/10}$ $\qquad c\left(\frac{n}{5} + \frac{7n}{10}\right)$

$= c\left(\frac{9}{10} n\right)$

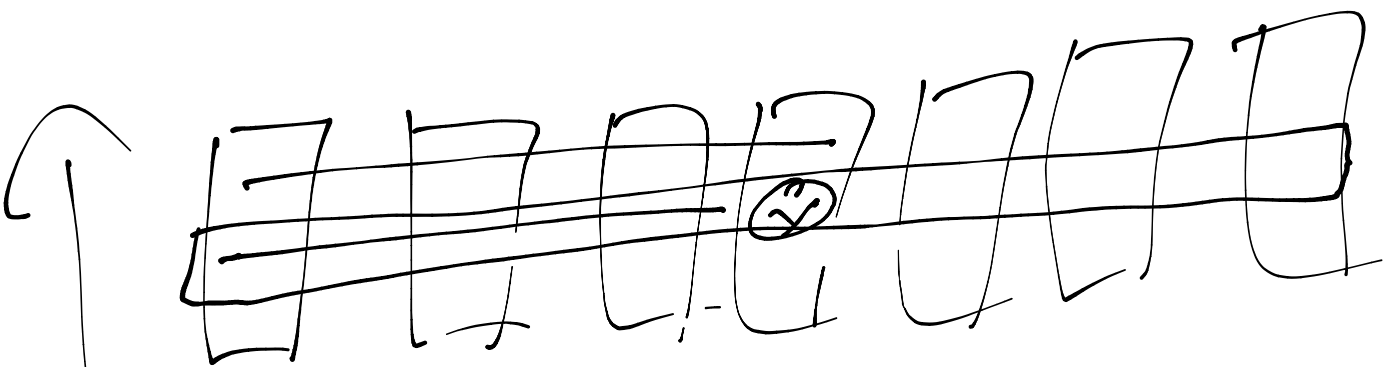
$\boxed{\frac{n}{25}}$ $\boxed{\frac{7 \cdot n}{50}}$ $\boxed{\frac{7n}{50}}$ $\boxed{\frac{49n}{100}}$ $\qquad c \cdot \left(\frac{81}{100} n\right)$

$$cn\left(1+\frac{9}{10}+\left(\frac{9}{10}\right)^2+\left(\frac{9}{10}\right)^3+\dots\right)$$

$$cn\left(\frac{1}{1-9/10}\right) = \underline{10cn}$$

upper bound on the # element $\le x$ is $= 2n/3$

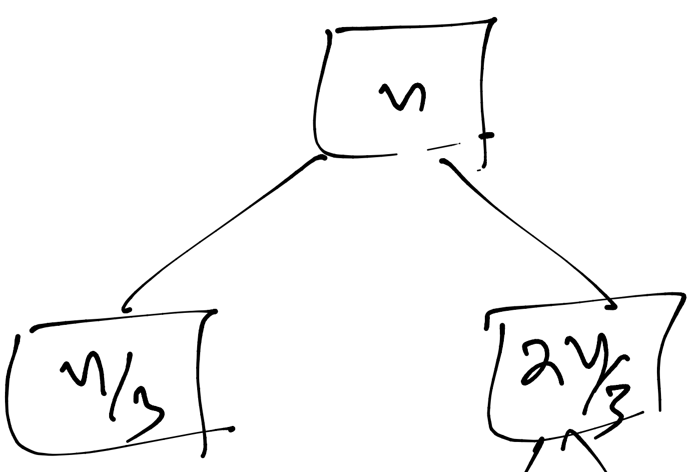
$$= \underline{O(n)}$$



$\geq 2\frac{n}{6}$ are $\le x$

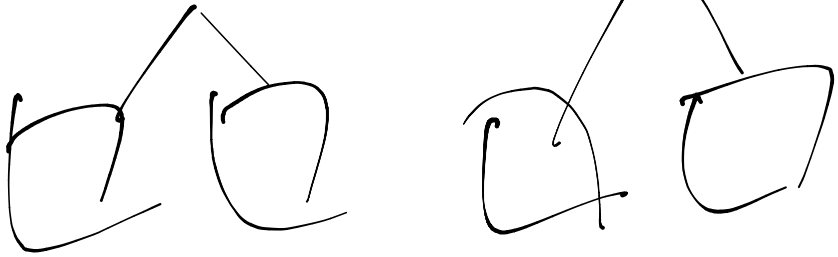
$\geq n/3$ elements are $\geq x$

$$\underline{T(n) = T\left(n/3\right) + T\left(2n/3\right) + cn}$$



cn

cn



cn

$O(n \log n)$

cn · # levels in the recursion tree

$\log_{3/2} n = O(\log n)$

- Divide into subproblem.
- Collect solutions ✓
- Combine them to a solution for own problem ✓

Backtracking

SUBSETSUM.

Ip: $X[1 \ldots n]$, T

Qn: $\exists$? a subset of X
that ~~ad~~ sum to T.

$\{5, 7, 3, 2, 8, 9\}$, 14

7

~~16~~

25

4

Yes

No!

$X[1 \ldots n]$, $T \geq 0$

If $T > 0$ and $n = 0$ then No

If $T = 0$, Output Yes

$with \leftarrow$ SUBSETSUM$(X[1 \ldots n-1], T - X[n])$

$without \leftarrow$ SUBSETSUM$(X[1 \ldots n-1], T)$

Output $(with \vee without)$

$X[1 \ldots n]$

$$\text{SUBSETSUM}(i, T') = \begin{cases} 1 & \text{if } T' = 0 \\ \text{SUBSETSUM}(i-1, T' - X[i]) \vee \text{SUBSETSUM}(i-1, T') \end{cases}$$

Output SUBSETSUM(n, T)

$T(0) = 1$

$T(n) = 2T(n-1) + 4$

$= O(2^n)$

Proof. by induction.

$T(n) \le 2^n$

$T(0) = 2^0 = 1$ Base case

By I.H we have

$T(n-1) \le 2^{n-1}$

$T(n) \le 2T(n-1) + 4$

$\le 2 \cdot 2^{n-1} + 4$

$$\leq 2^n + 4$$

$$\boxed{T(n) \leq c \cdot 2^n}$$

$$T(n) \leq 2 \cdot c\; 2^{n-1} + 4$$

$$\leq c \cdot 2^n + 4$$

$$T(n) \leq 2^n - 4$$

$$T(0) \leq 2^0 - 4 \leq -3$$

$$\boxed{T(n) \le 5 \cdot 2^n - 4}$$

$$T(0) \le 5 \cdot 1 - 4 = 1$$

$$T(n) \le 2\left[5 \cdot 2^{n-1} - 4\right] + 4$$

$$\le 5 \cdot 2^n - 4 \checkmark$$

Proof by induction

TEXT SEGMENTATION

IAMASTUDENT

I AM STUDENT

S[i . . . . . . n]

$ISWORD(i, j) = S[i..j]$

$S[\boxed{i . . . n}]$

$SPLITABLE(i) =$

$$\begin{cases} \bigvee_{j=i}^{n} \left(ISWORD(i,j) \wedge SPLITABLE(j+1) \right) \\ 1 \quad \text{if } i > n \end{cases}$$

# of ISWORD calls.

$$\boxed{\begin{aligned} T(n) &= \sum_{i=1}^{n} T(n-i) + \alpha n \\ T(0) &= 1 \end{aligned}}$$

$$T(n-1) = \sum_{i=1}^{n-1} T(n-1-i) + \alpha(n-1)$$

$$T(n) - T(n-1) = T(n-1) + \alpha$$

$$T(n) = 2T(n-1) + \alpha$$

$$= O(2^n)$$