

Slides for BST Q&A

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Plan

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Today we see how it implements the following

1. SEARCH
2. INSERT
3. SUCC (also PRED)
4. DELETE

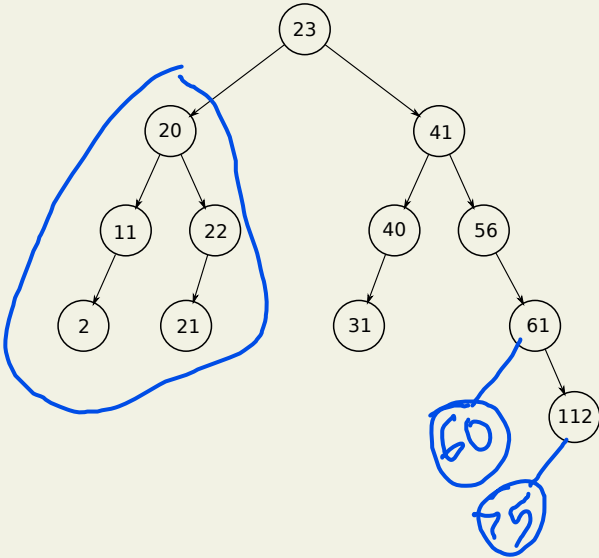
Binary Search Trees

Recall that a Binary Search Tree (BST) has the following crucial property:

For every node X in the BST, we have:

- ▶ Every node in the left subtree of X contains a value smaller than that of X .
- ▶ Every node in the right subtree of X contains a value larger than that of X .

Example BST



```

graph TD
    n0((0)) --> n2((2))
    n2 --> n3((3))
    n3 --> n0_1((0))
    n0_1 --> n0_2((0))
    n0_2 --> n0_3((0))
    n0_3 --> n7((7))
    n7 --> null((X))
  
```

INSERT procedure

The INSERT(*node*, *x*) procedure:

- ▶ If *node* = NULL, create new node with *x* and attach to parent.
- ▶ Else If $x < \text{value}(\textit{node})$,
 - ▶ INSERT(*node* → *left*, *x*)
- ▶ Else If $x > \text{value}(\textit{node})$ Then,
 - ▶ INSERT(*node* → *right*, *x*)

SEARCH procedure

- ▶ $\text{SEARCH}(node, x)$:
- ▶ If $node = \text{NULL}$, then return NULL
- ▶ Else If $x = \text{value}(node)$, then return $node$
- ▶ Else If $x < \text{value}(node)$, then
 - ▶ Return $\text{SEARCH}(node \rightarrow \text{left}, x)$
- ▶ Else
 - ▶ Return $\text{SEARCH}(node \rightarrow \text{right}, x)$

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- ▶ Find the node which stores *val*. Refer to this node as “*node*”.
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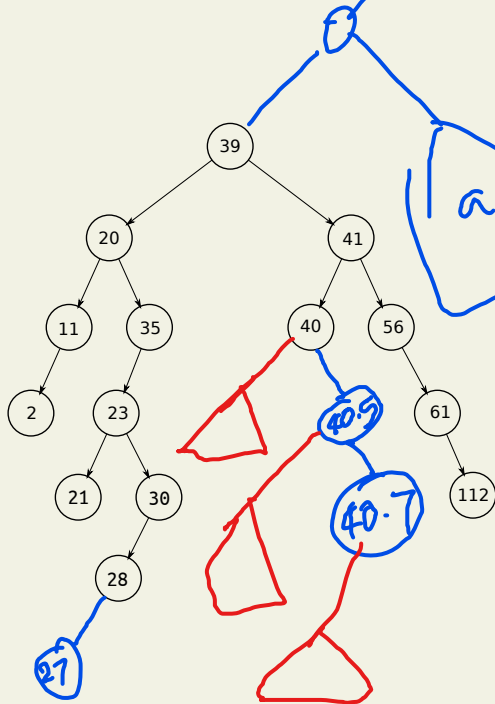
Case 1: *node* has a right child.

Find the smallest element of the right subtree.

Case 2: *node* does not have a right child.

Keep going to the parent till we reach an ancestor *x* that is a left child of its parent. The parent of *x* is the successor.

Example



DELETE

The DELETE(*VAL*) procedure is as follows:
Find the node that has value *val*.

DELETE

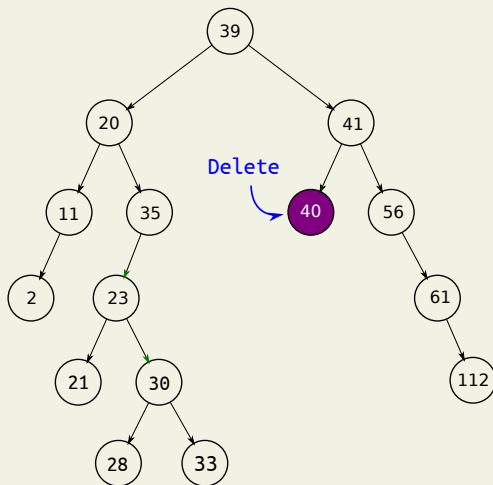
The DELETE(*val*) procedure is as follows:

Find the node that has value *val*.

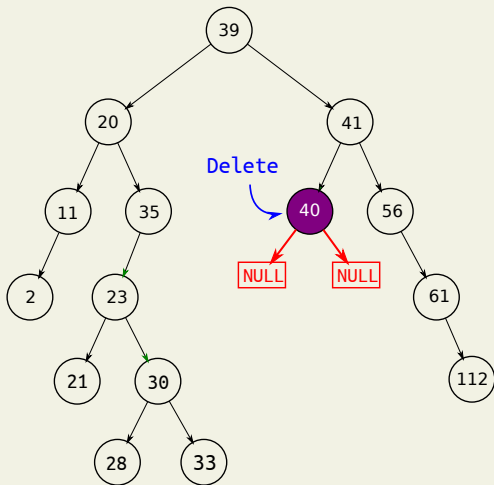
Three cases:

1. *node* has 0 children. (trivial)
2. *node* has 1 child. (splice)
3. *node* has 2 children:
 - ▶ Find successor node *X* with value *x*.
 - ▶ Splice *X* out of the tree.
 - ▶ Replace *val* with *x*.
 - ▶ Delete node *X*.

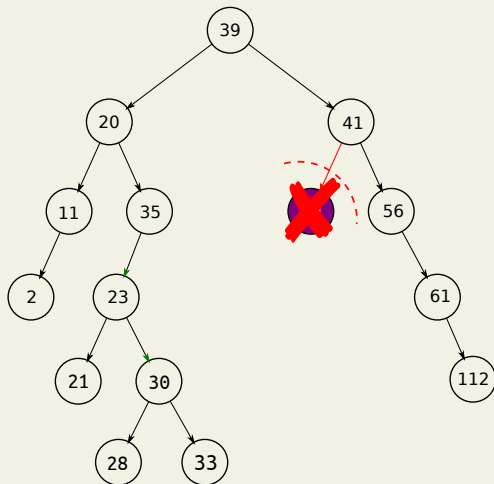
Example



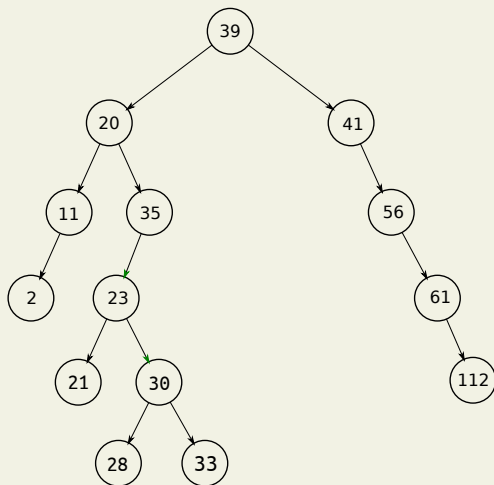
Example



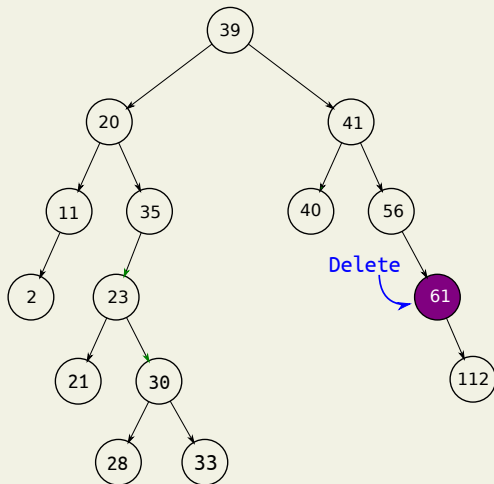
Example



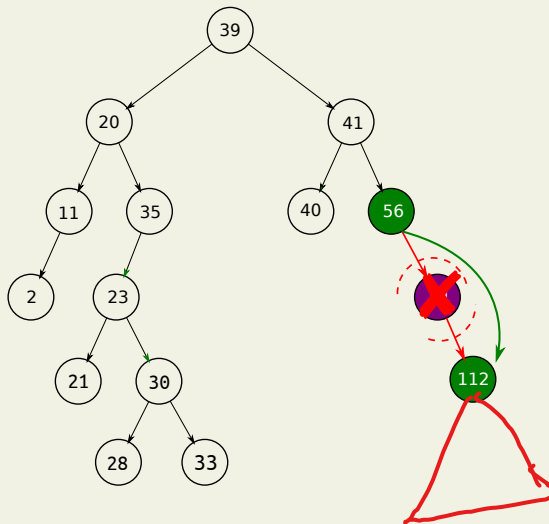
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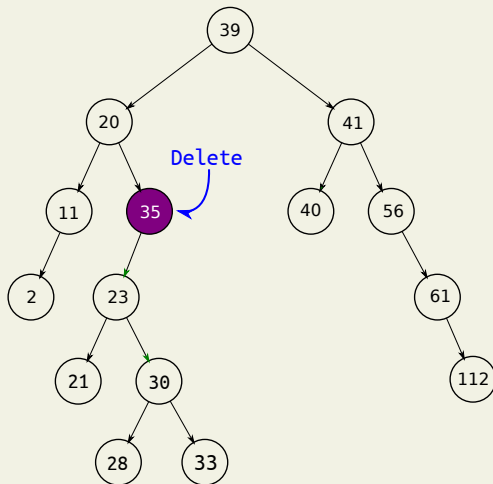
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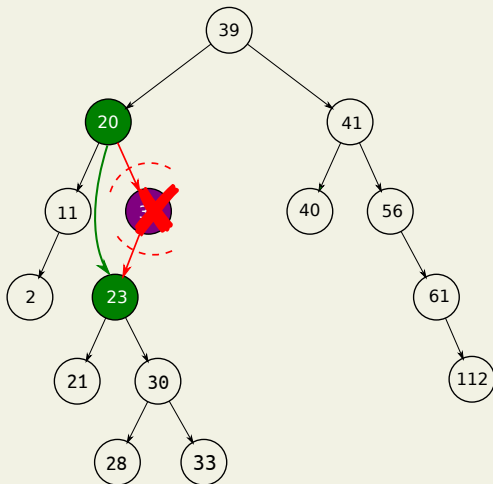
Example



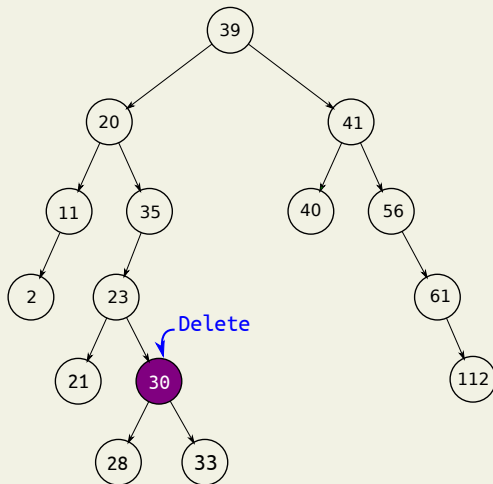
Example



Example

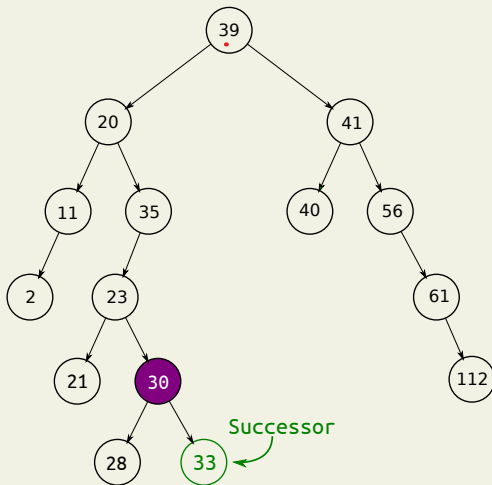


Example

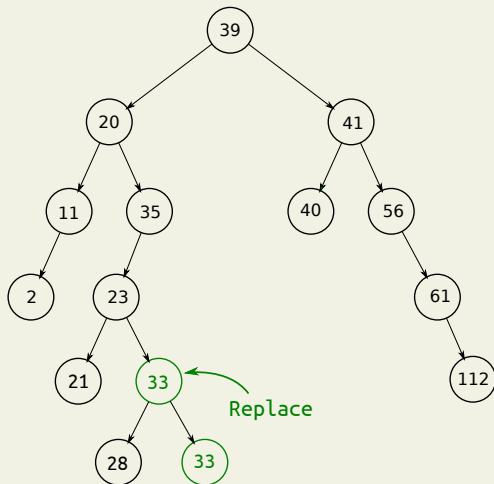


Example

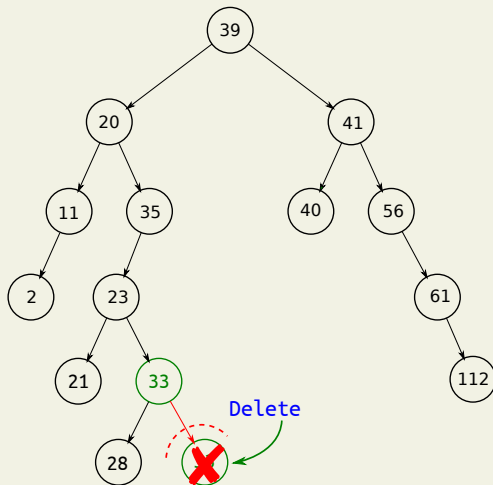
1, 2, 3, 4, 5, 18, 20, 26, ..



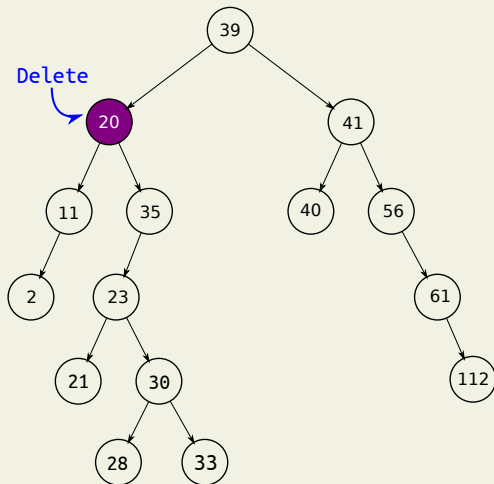
Example



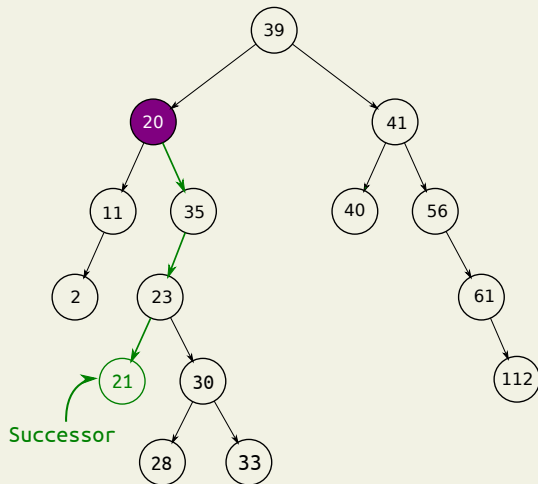
Example



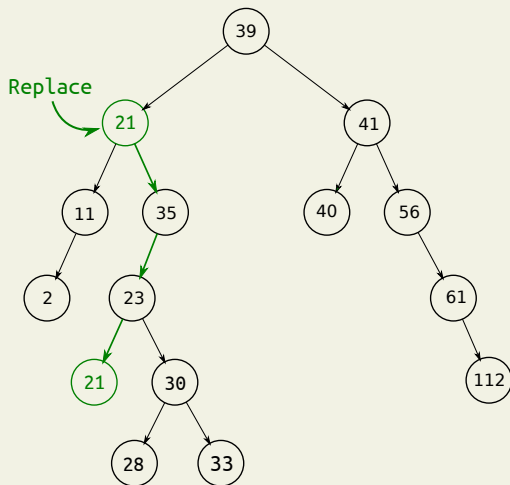
Example



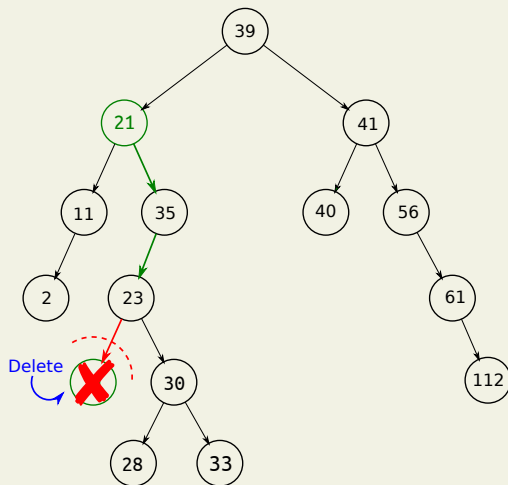
Example



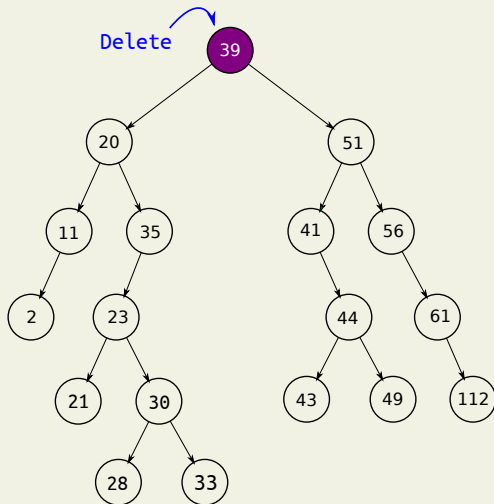
Example



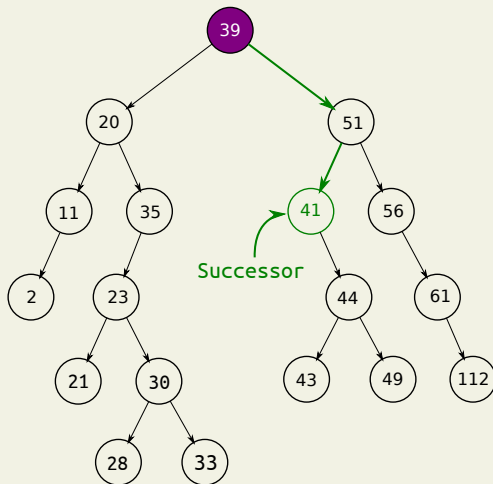
Example



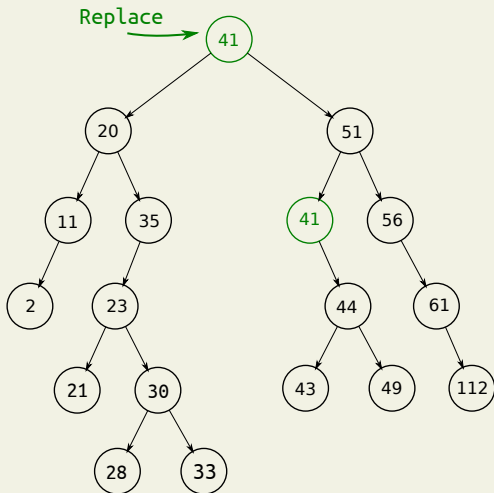
Example



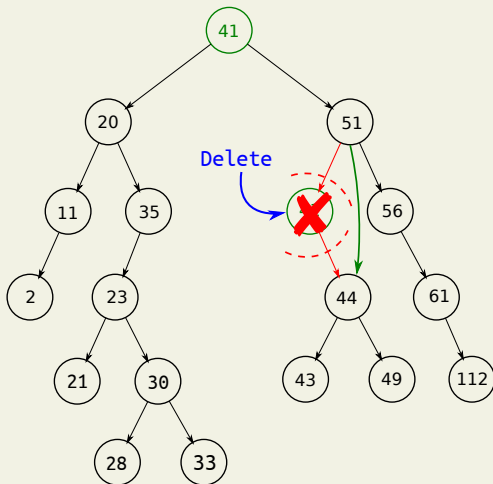
Example



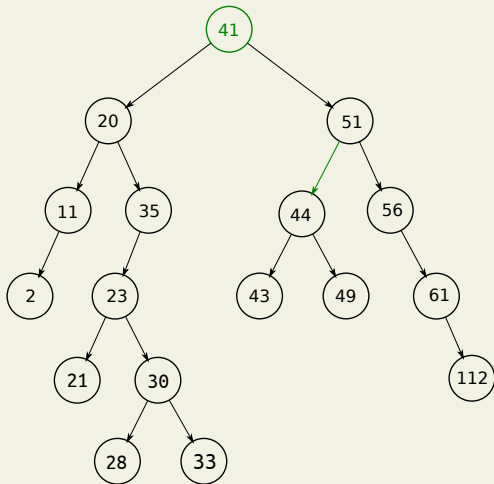
Example



Example



Example



Running Time

Worst case running times for a BST of height h :

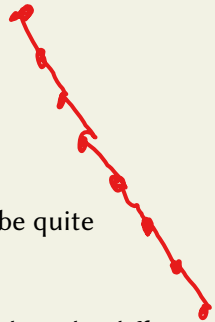
- ▶ INSERT – $O(h)$.
- ▶ SUCC – $O(h)$.
- ▶ SEARCH – $O(h)$.
- ▶ DELETE – $O(h)$.

The height of a BST depends on the input sequence and can be n after inserting n elements in the worst case.

Balancing a BST

The biggest drawback of BSTs are that they can be quite “unbalanced”.

One way to measure if a tree is balanced is to look at the difference between the longest path from root to leaf and the shortest path from root to leaf.



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We want to make sure this difference does not get too large.

Thank You!