Lecture 11

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Disjoint Set

Maintain a collection $\mathcal{F} = \{S_1, S_2, \ldots, S_k\}$ of disjoint sets.
One element from each set serves as a 'representative' for that set.

Disjoint Set supports the following procedures:

- $\textsc{MakeSet}(x)$ – Creates a singleton set with element x.
- $\textsc{Union}(x, y)$ – Performs union on sets containing x and y.
- $\textsc{FindSet}(x)$ – Find the set containing x.

MakeSet(x)

Creates a singleton set containing x.

We assume that x is not an element of any other set in $\mathcal{F}$.
We assign x as the representative for the set just created.

Union

Union(x, y)

Performs union on sets containing x and y.

Let $S, T \in \mathcal{F}$ such that $x \in S$ and $y \in T$.
Create a new set $U = S \cup T$.
Choose and assign a representative for U.
Remove S and T from $\mathcal{F}$.

FindSet

FindSet(x)
Find the set containing x.

Let $S \in \mathcal{F}$ such that $x \in S$. (Note: exactly one set contains x.)
Return a pointer to the representative element of S.

Implementation

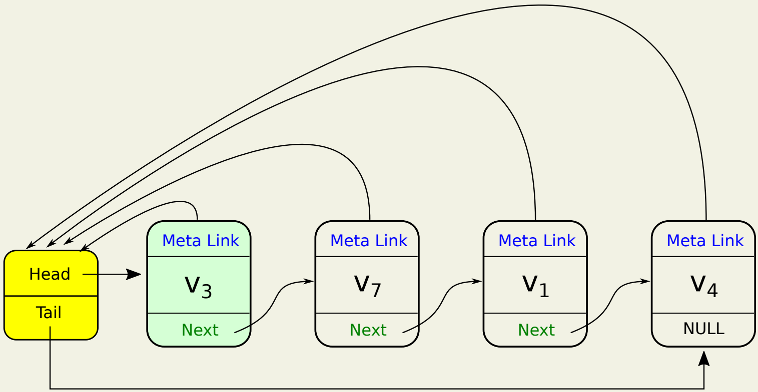
Disjoint Set using linked lists:

- For each set S, maintain:
  - a node with metadata
  - a linked list L_S with the objects in the set.
- The "Metadata Node" stores head and tail pointers to the linked list.
- Each node in the linked list consists of:
  - The value of the element.
  - A pointer to the next element.
  - A pointer to the Metadata Node.

The head of L_S is the representative of S.

Implementation

Linked list for set $\{v_1, v_3, v_4, v_7\}$.



Implementation

$\textsc{MakeSet}(x)$

Creates a singleton set with element x

- Create a new node for metadata
- Create a linked list containing just x.
- Node x is the head and tail of the list.
- Representative for this set is x itself.

Implementation

FindSet(x)

Find the set containing node x.

- Return a pointer to the representative.

Implementation

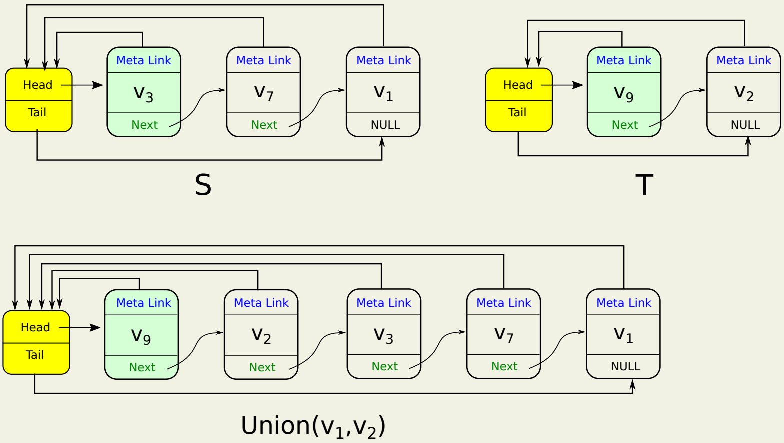
UNION(X,Y)

Union of sets containing x and y.

- Append linked list of set S containing x to set T containing y.
- Representative of new set is same as representative of T.
- Update meta pointers of nodes in S to the correct metadata node.
- Update tail pointer in metadata node of T.

Implementation

Union of sets $S = \{v_1, v_3, v_7\}$ and $T = \{v_2, v_9\}$.



Analysis

Running time under Linked List implementation:

- MakeSet(x) – $O(1)$
- FindSet(x) – $O(1)$
- Union(x, y) – ?

Analysis

Union(x, y) –

- $S \leftarrow$ FindSet(x) and $T \leftarrow$ FindSet(y) – $O(1)$ time.
- Appending linked list of S to tail end of T – $O(1)$ time.
- Updating the new metadata (tail) – $O(1)$ time.
- Updating the backward pointers of nodes in S takes $O(n)$ time.

We can show a case where after $O(n)$ operations, time taken would be $O(n^2)$.

Spanning Tree

A graph $T = (V, E')$ is a spanning tree of an undirected connected graph $G = (V, E)$ if:

- $E' \subseteq E$.
- T is a *tree*. i.e., there are no cycles in T.
- T is connected.

Informally: A spanning tree for G is a tree that can be found inside G which *spans* all vertices of G.

Recap: Minimum Spanning Tree Problem

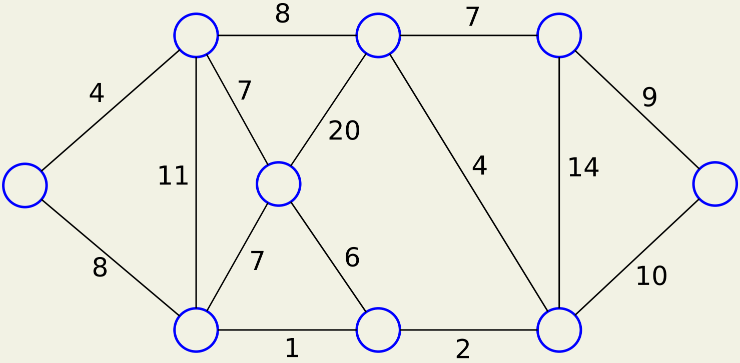
Input

- Undirected connected graph $G = (V, E)$
- Weight function $w : E \to \mathbb{Z}^+$

Goal

Compute a spanning tree for G with minimum total weight.

Kruskal’s algorthm example



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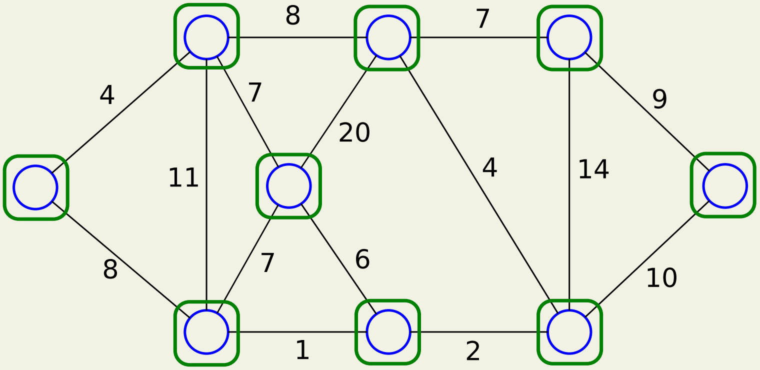

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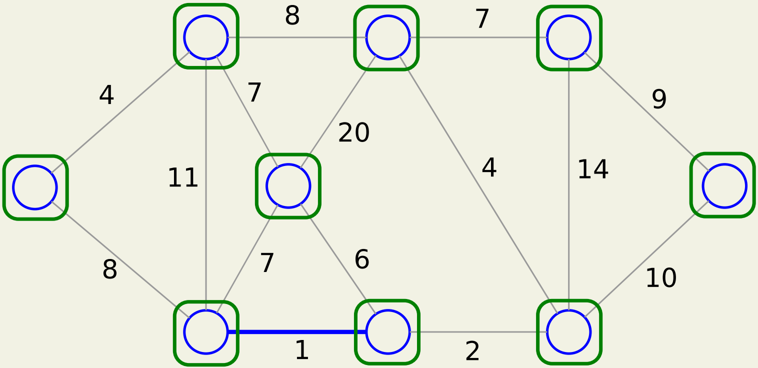

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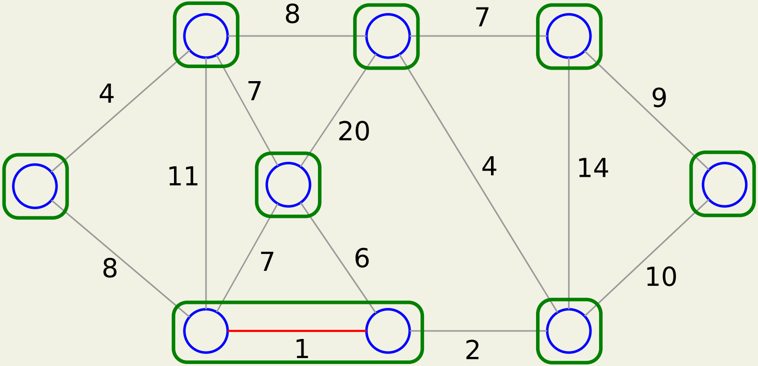

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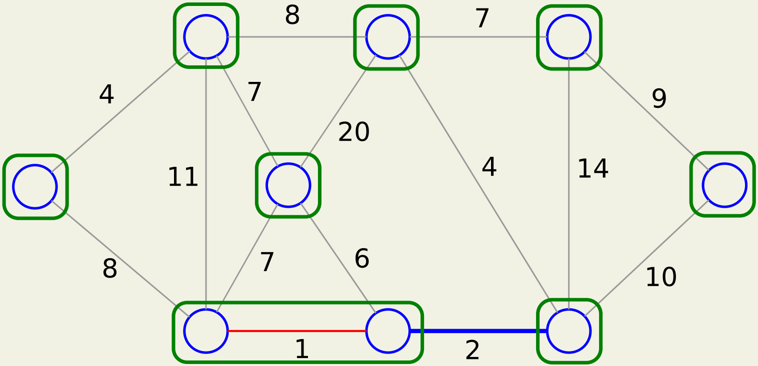

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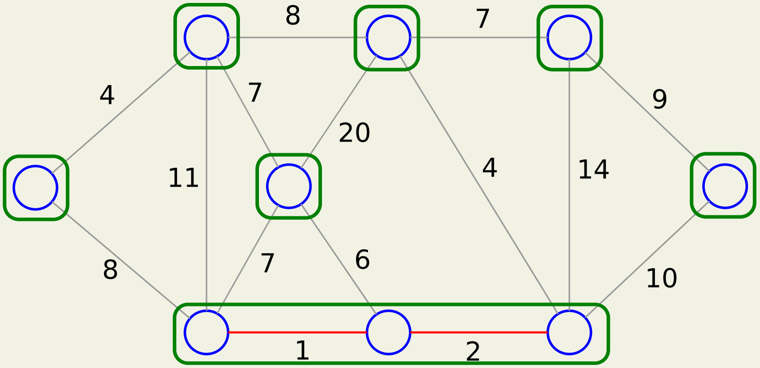

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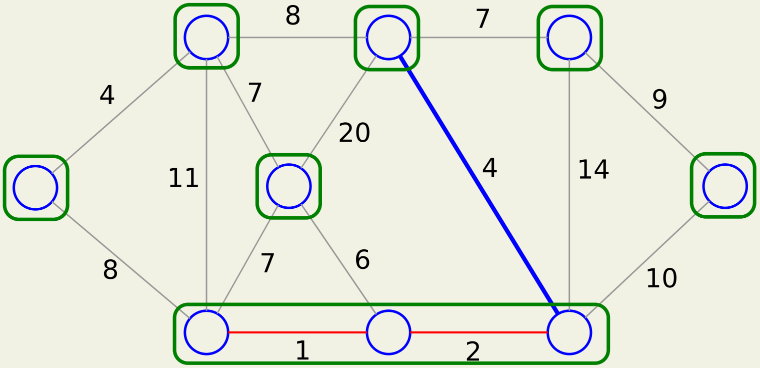

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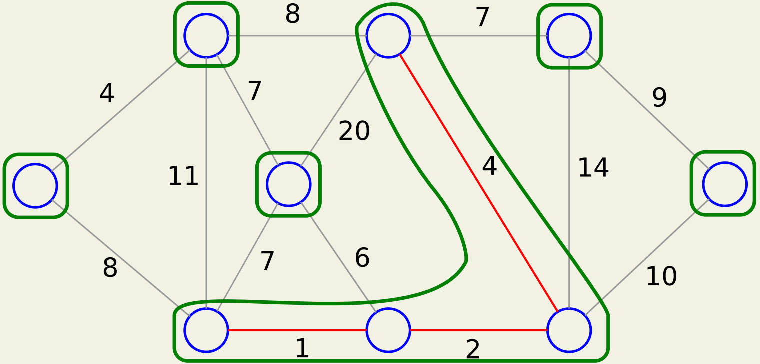

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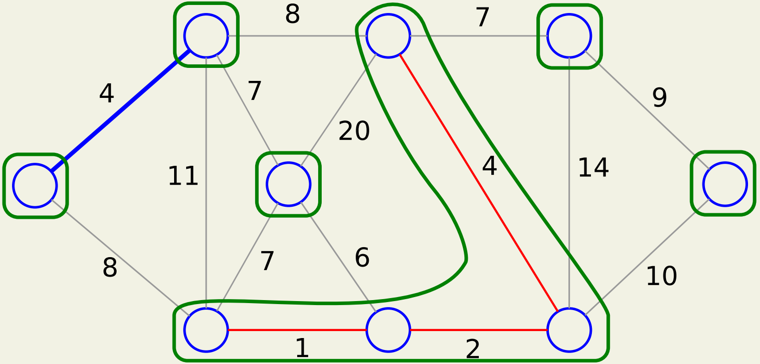

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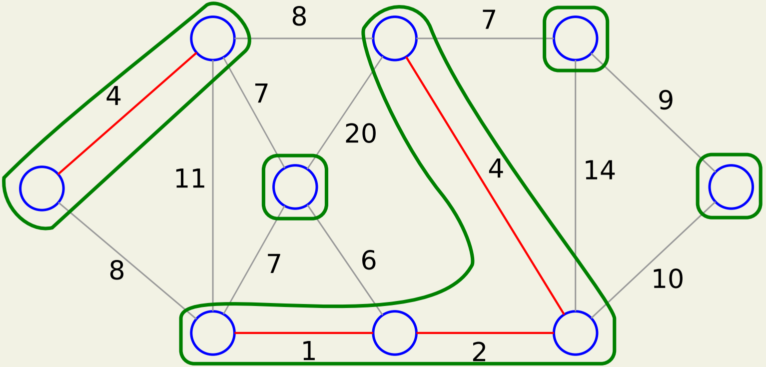

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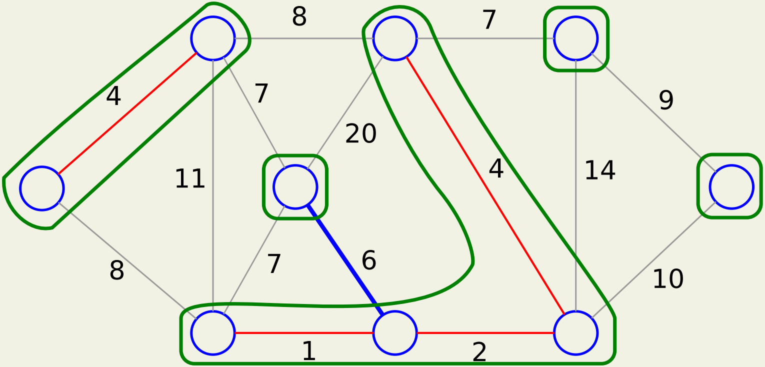

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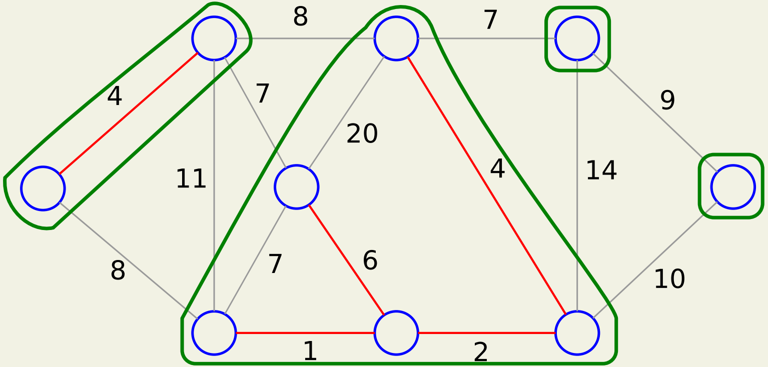

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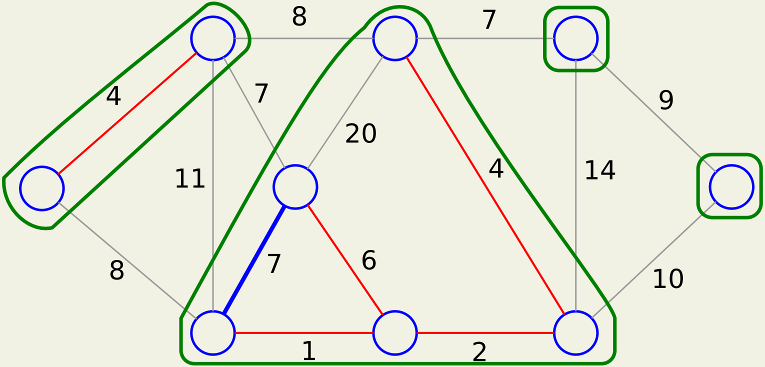

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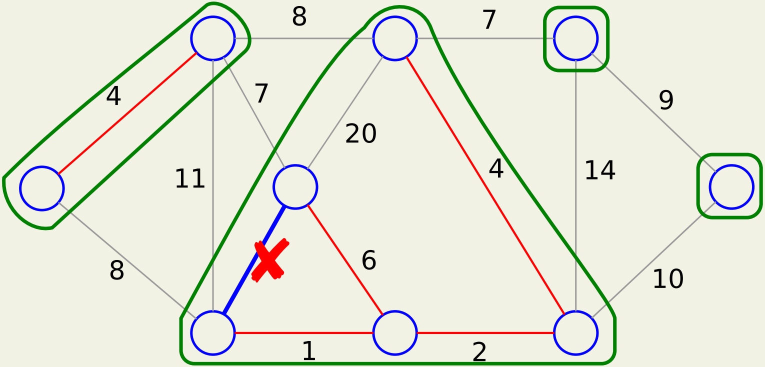

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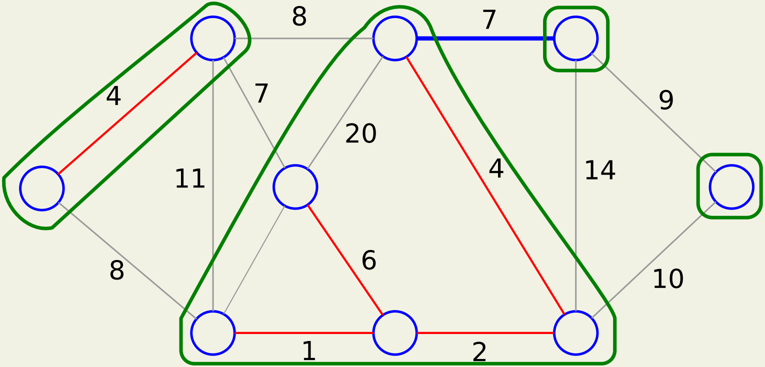

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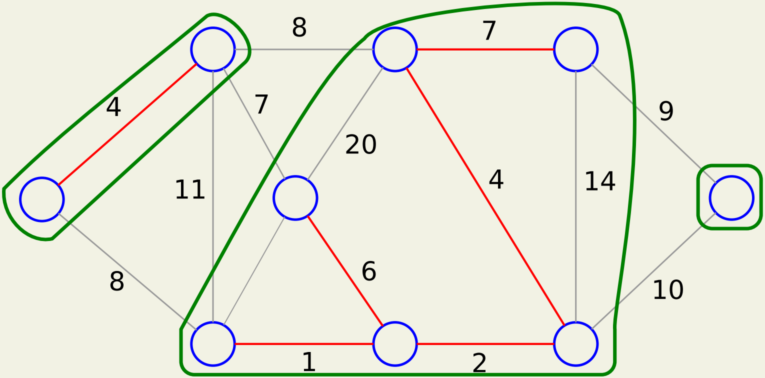

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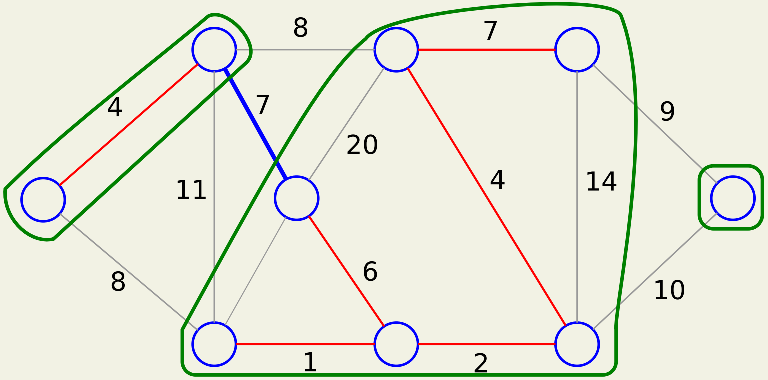

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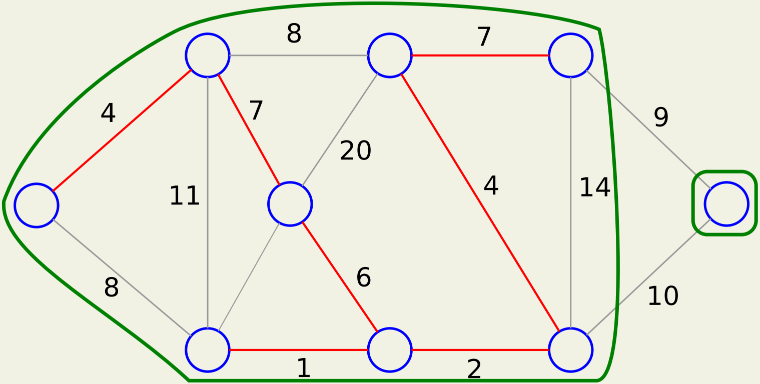

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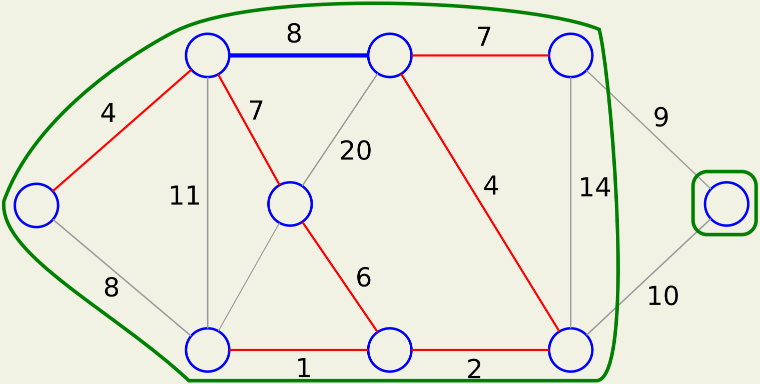

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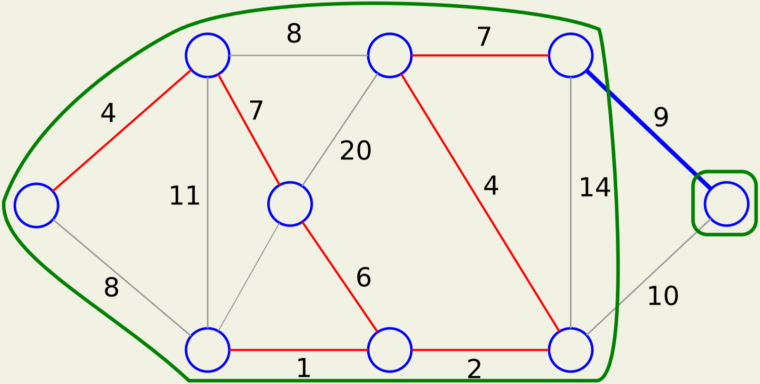

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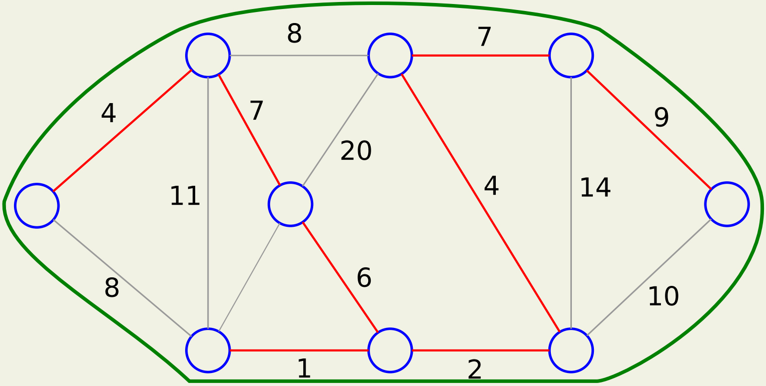

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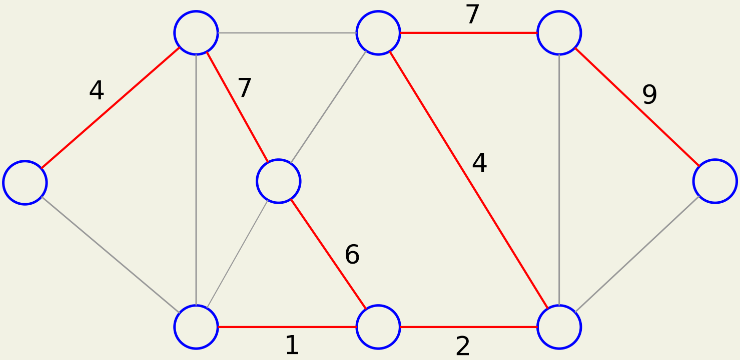

Pseudocode

Algorithm 1 Input: Undirected connected graph $G = (V, E)$

```
T ← ∅
for each v ∈ V do
    Create a node for v
    MakeSet(v)
end for
while T ≠ V do
    Choose smallest weight edge e = {u, v} ∈ E
    if FindSet(u) ≠ FindSet(v) then
        T ← T ∪ {e}
        Union(u, v)
    end if
end while
```