CS2443: Algorithms

MS17BTECH11013
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2)

Goal: Any vertex v in the layer $L_i$ the distance from s to v is i

in the BFS(G, s) ↳ for graph G source 's'.

Initialize

$$dist(v) = \begin{cases} 0 & \text{if } v = s \\ \infty & \text{if } v \neq s \end{cases}$$

After we run BFS(G, s) at termination $dist(v) = i \Leftrightarrow$ v is in the ith layer

Proof by Induction

Base case $i = 0$ 0th layer.

$dist(s) = 0$ [Trivial already given]

Inductive Hypothesis

Any vertex v in the layer $L_i$ the distance from s to v is i.

Scanned with CamScanner

Need to prove for $i+1$.

Since we know every layer-$i$, node $w$ is added to Quee by a layer $(i-1)$ node $v$ via the edge $(v, w)$

When considering edge $(v, w)$

If $w$ unexplored,

$$dist(w) = dist(v) + 1.$$

Assume $w$ is in the $(i+1)$ layer
$v$ is in the $i$ layer.

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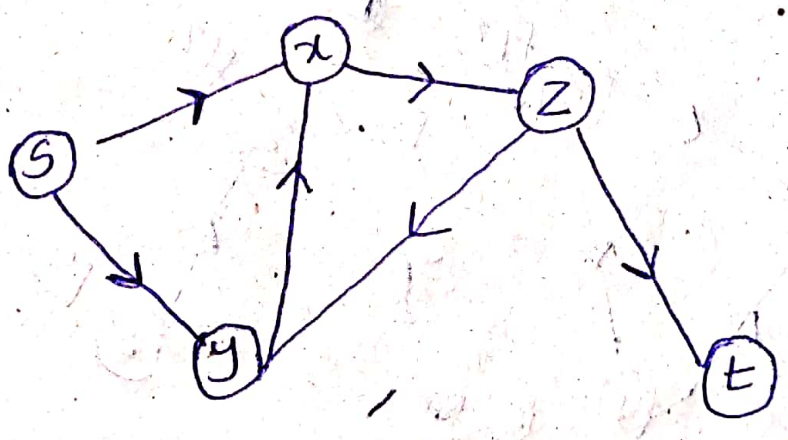
④

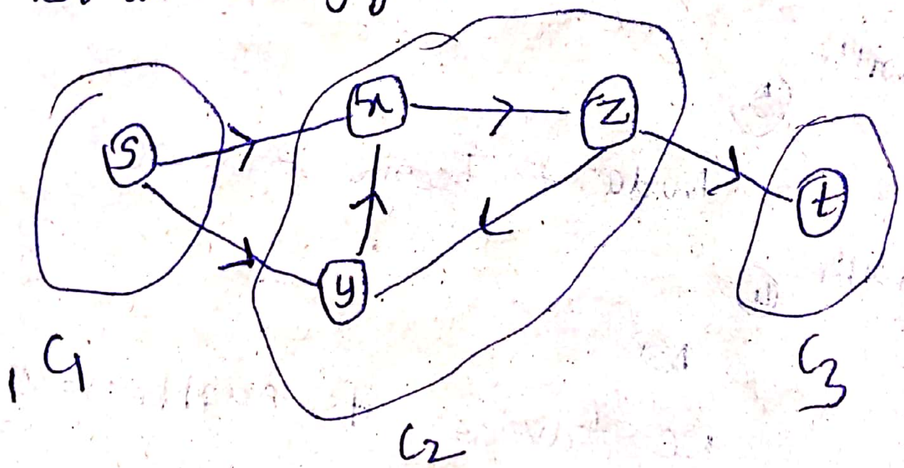



Digraph D

let assume an ordering.

$s < t < x < y < z$

**Qi** list all strongly connected components in G.

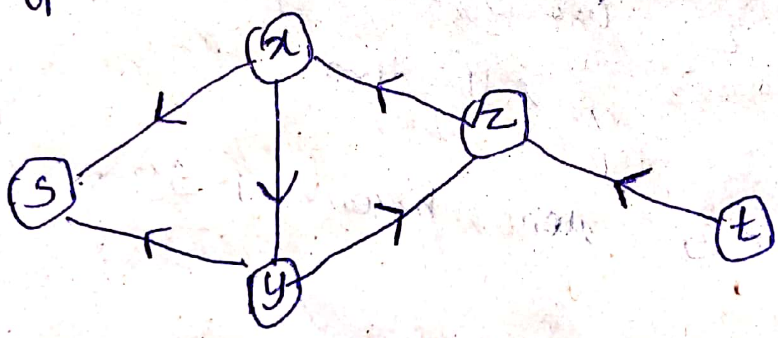



are → s

$x \to z \to y$

t

**Qii** $G^{rev}$ be

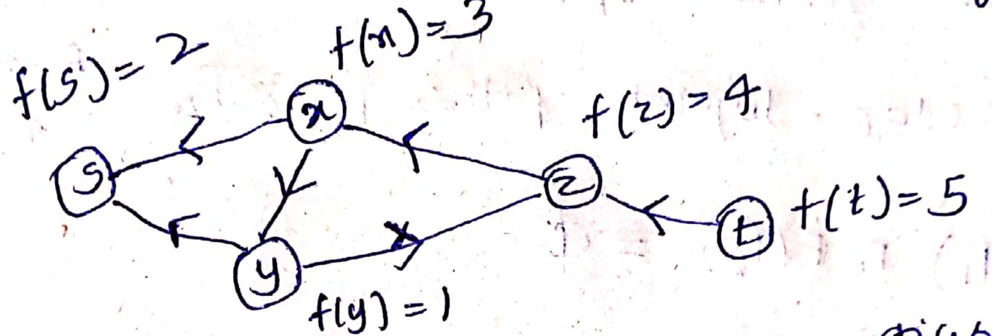


Start with z vertex.

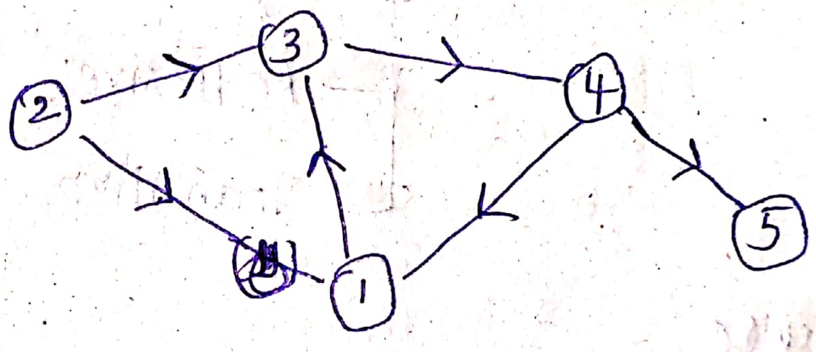
explore all its ~~xx~~ neighbours.

let $f(v)$ = "finishing time" of each $v \in V$

$V \rightarrow$ vertices of $G$.




calculate the finishing times of all vertices




start with 5

Node t becomes one scc's.

Finishing time of all vertices

$f(x) = 3$

$f(t) = 5$

$f(s) = 2$ $f(y) = 1$

$f(z) = 4$

③ Given

G be a graph

$\sim$ be a Relation

for any vertices u, v. $u \sim v$ iff there are two edge-disjoint paths $P_1$ & $P_2$ from u to v

$$E(P_1) \cap E(P_2) = \phi$$

To prove $\sim$ is equivalence relations

- Symmetric
- Reflexive
- Transitive

i) ~~Reflexive~~

i) Symmetric: ~~Given~~

If there is a two edge-disjoint path from u to v. We can trace back the ~~comes~~ path from v to reach u.

Hence there is a two edge disjoint path from v to u

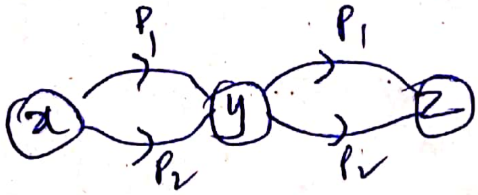
$u \sim v$

$v \sim u$

True only for undirected

False for directed

ii Transitive.




if there is a two edge path from $x$ to $y$ & also $y$ & $z$

$$x \sim y$$
$$y \sim z \longrightarrow x \sim z.$$

iii Reflexive

$$x \sim x.$$

Note ~~true as there is a '0' dist path from x to itself which is unique.~~

~~Here~~

by symmetric relation for a undirected graph it is true

$$\left.\begin{array}{l} x \sim y \\ y \sim x \end{array}\right\} \longrightarrow x \sim x.$$

For directed graph.

$x \sim y$ can't say about $y \sim x$.

Hence Not equivalence relation for directed graph.