# Asymptotic Notations.  $< \ \leq \ > \ \geq \ =$

Big-Oh, little-Oh, $\Omega$, $\omega$, $\Theta$

$$O(f(n)) = \{ g(n) : \exists c > 0 \ \exists n_0 > 0 \ \text{s.t.} \ \forall n \geq n_0 \ \ g(n) \leq c f(n) \}$$

$g(n) \geq c f(n)$

$$g(n) \in O(f(n))$$

$$g(n) = O(f(n))$$

## Example

$f_1(n) = 100 n^2 + 4n$

$g_1(n) = 10000 \, n \log n$

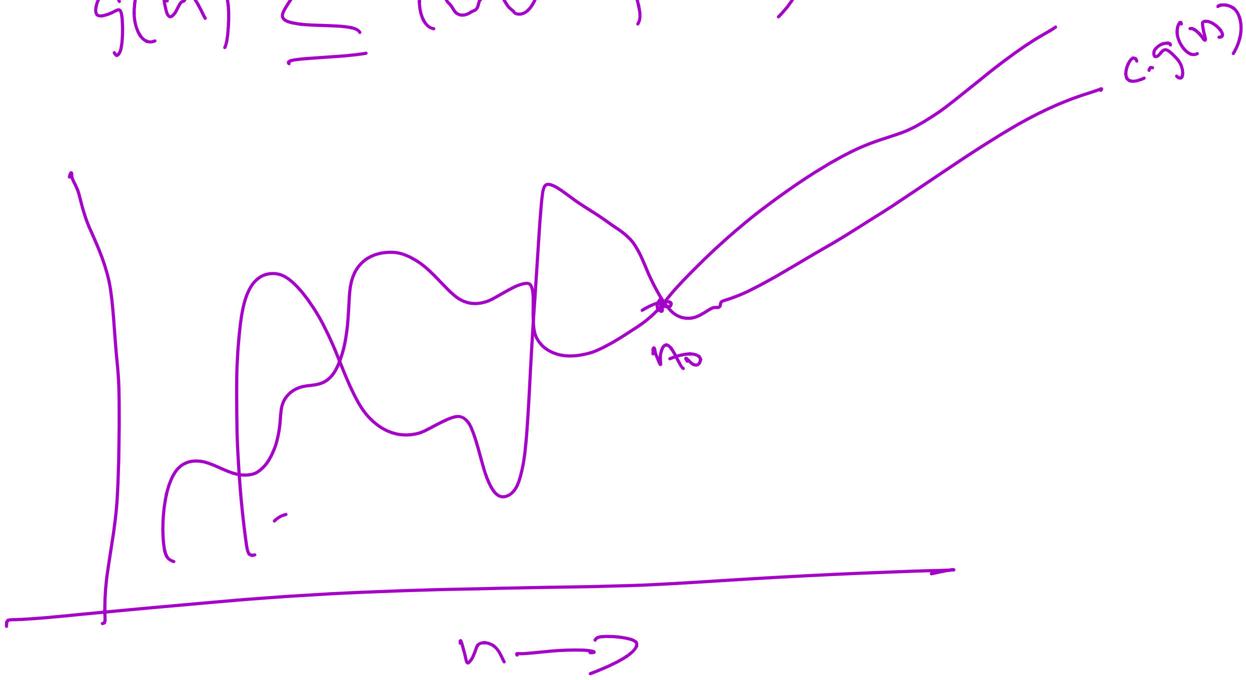
Is $g_1(n) = O(f_1(n))$ ? Yes.

$\exists c, n_0$ $\boxed{\text{s.t. } g_1(n) \leq c f(n) \ \forall n \geq n_0}$

$c = 1$ and $n_0 = 10000$

$c = 100$ , $n_0 = 1$

$g(n) \leq 100 \cdot f(n)$

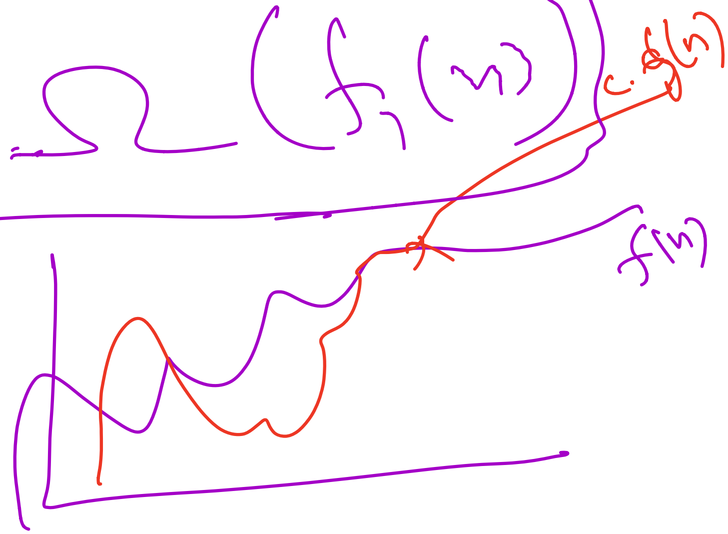



$c \cdot g(n)$

## $\underline{\underline{\Omega}}$

$$\Omega(f(n)) = \{ g(n) : \exists\ c > 0,\ n_0 > 0 \text{ s.t. } g(n) \geq c f(n)\ \forall\ n \geq n_0 \}$$

$$\boxed{\text{Is } g_1(n) = \Omega(f_1(n))}$$



$$10000\, n\log n \qquad 100n^2+4n$$

(A1) $\underline{g_1(n) \geq c \cdot f_1(n)}$ $\forall\ n \geq 10$

For $c = 1$ $\forall\ n \geq 100$, (A1) is not true

For $c = \frac{1}{1000}$ $\forall\ n \geq 100 \cdot \frac{1000}{\cancel{8}}$, (A1) is not true

<u>For any c</u>, $\forall\ n \geq 100 \cdot \frac{1}{c}$

<u>(A1) is not true</u>

<u>Example 2</u>

$$f_1(n) = \Omega(g_1(n)) \ ?$$

$$100n^2+4n \qquad 10000\, n\log n$$

$$f_1(n) \geq c \cdot g_1(n) \quad \forall\ n \geq n_0.$$

What is a value for $c$ and $n_0$

$c = \frac{1}{100}$ and $n_0 = 1$

## $\Theta$

$$\Theta(f(n)) = \{ g(n) : \exists\, c_1, c_2 > 0,\ \exists\, n_0 > 0 \text{ s.t. } c_2 f(n) \le g(n) \le c_1 f(n)\ \forall\, n \ge n_0 \}$$

$g(n) = O(f(n))$ and

$g(n) = \Omega(f(n))$.

Is $g_1(n) = \Theta(f_1(n))$ ?

No =

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$$\underline{\underline{n^2}} + 4n\log n + 100n \underline{\underline{+ 1000 \cdot 5n}} \le c n^2$$

$$\hookrightarrow O(n^2)$$

$$\hookrightarrow \Omega(n^2)$$

## Little-o

$$o(f(n)) = \{ g(n) : \forall c > 0, \exists n_0 > 0$$
$$\text{s.t. } g(n) < c f(n) \quad \forall n \geq n_0 \}.$$

$g(n) = 10 \cdot n \log n$ $\qquad f(n) = n^2$

Is $g(n) = o(f(n))$

Fix a constant $c$ $\qquad \forall n \geq n_0$

$g(n) \leq c \cdot f(n) \qquad \forall n \geq n_0$

$10 \cdot n \log n \leq c \cdot n^2$

$10 \log n \leq c \cdot n$

$\frac{10}{c} \leq \frac{n}{\log n} \qquad \exists n_0 .$

s.t. $\frac{n}{\log n} \geq \frac{10}{c} \quad \forall$

$n \geq n_0 .$

$f_1(n) = 100 n^2$

$g_1(n) = n^2$

① Is $g_1(n) = o(f_1(n))$ ? No

② Is $g_1(n) = O(f_1(n))$ ? Yes ✓

Need to come up with one value for $c$ and one value for $n_0$ s.t $g_1(n) \le c f_1(n)$ $\forall n \ge n_0$

① "For all $c$", there exists $n_0$ s.t $g_1(n) \le c f_1(n)$ $\forall n \ge n_0$

$\exists c$. $g_1(n) \not\le 10 \cdot f_1(n)$

Come up with a value for $c$, then $\exists$ infinitely many choices for $n$ such $g_1(n) \not\le c \cdot f_1(n)$

$\underline{g_1(n) = n^2}$  $\underline{f_1(n) = 100\, n^2}$

$g_1(n) \leq c \cdot 100 \cdot n^2$

$c = \frac{1}{200}$  $n^2 \leq \frac{n^2}{2}$  $\forall\, n > 0$

Analogous $\leq$

" $>$ "

$\underline{\underline{\omega}}$