Grand Assignment

MS3080 & MS5140

Newton-Raphson Method:

1. Determine the multiple real roots of the given equation by

a) Graphically
b) Using Newton-Raphson method to within the tolerance of 0.01%

$$f(x) = 0.5x^3 - 4x^2 + 5.5x - 1$$

2. Determine the roots of following simultaneous nonlinear equations using multiple Newton-Raphson method. Employ initial guesses of x = y = 1.2

$$y = -x^2 + x + 0.75$$
$$y + 5xy = x$$

3. Determine the roots of following simultaneous nonlinear equations using multiple Newton-Raphson method. Use a graphical approach to obtain initial guesses.

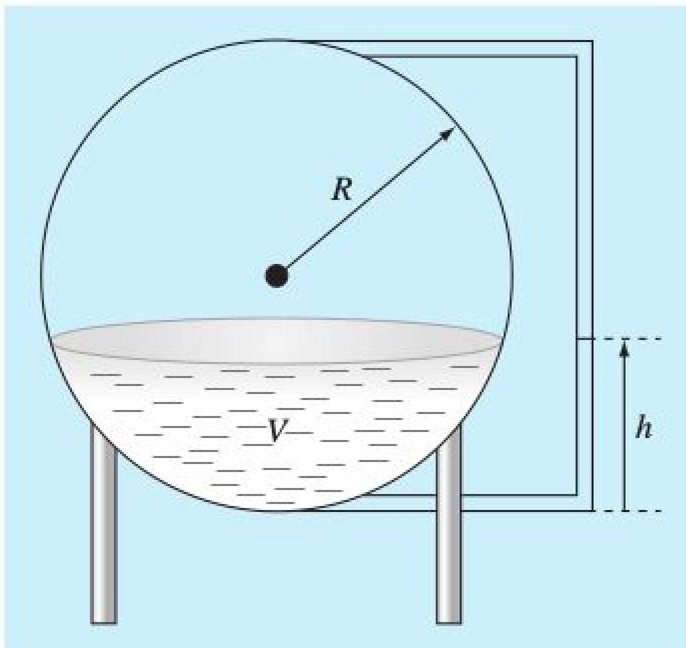
$$(x-4)^2 + (y-4)^2 = 5$$
$$x^2 + y^2 = 16$$

4. You are designing a spherical tank to hold water for a small village in a developing country. The volume of the liquid it can hold can be computed as

$$V = \pi h^2 \frac{[3R-h]}{3}$$

Where V = volume (mxmxm), h = depth of water in tank (m), and R = the tank radius (m).

If R = 3 m, what depth must the tank be filled so that it holds 30 mxmxm ? Use 3 iterations of the Newton-Raphson method to determine the answer. Determine the approximate relative error after each iteration. Note that an initial guess of R will always converge.



Curve Fitting & Interpolation

5. Construct the divided-difference table from these data and use it to interpolate for **f(x=0.4)**:

x	-0.2	0.3	0.7	-0.3	0.1
f(x)	1.23	2.34	-1.05	6.51	-0.06

a) Using the first 3 points
b) Using the last 3 points
c) Using the best set of 3 points. Which points should be used.

d) Using all points
e) Explain why the results are not all the same.

6. Given the data

x	1	2	3	5	7	8
f(x)	3	6	19	99	291	444

Calculate f(x=4) using Newton's Interpolation polynomials of order 1 through 4. Choose your base points to attain good accuracy. What do your results indicate regarding the order of the polynomial used to generate the data in the table?

7. Repeat Prob. 6 using Lagrange polynomials of order 1 through 3.

8. Use Newton's interpolating polynomial to determine y at x = 3.5 to the best possible accuracy. Compute the finite divided differences and order your points to attain optimal accuracy and convergence.

x	0	1	2.5	3	4.5	5	6
y	2	5.4375	7.3516	7.5625	8.4453	9.1875	12

9. Generate eight equally-spaced points from the function $f(t) = sin^2 t$ from t = 0 to 2π . Fit this data with

a) a seventh-order interpolating polynomial and
b) a cubic spline.

10. A useful application of Lagrange interpolation is called a *table look-up*. As the name implies, this involves “looking-up” an intermediate value from a table. To develop such an algorithm, the table of x and f (x) values are first stored in a pair of one-dimensional arrays. These values are then passed to a function along with the x value you wish to evaluate. The function then performs two tasks. First, it loops down through the table until it finds the interval within which the unknown lies. Then it applies a technique like Lagrange interpolation to determine the proper f (x) value. Develop such a function using a cubic Lagrange polynomial to perform the interpolation. For intermediate intervals, this is a nice choice because the unknown will be located in the interval in the middle of the four points necessary to generate the cubic. For the first and last intervals, use a quadratic Lagrange polynomial. Also have your code detect when the user requests a value outside the range of x’s. For such cases, the function should display an error message. Generate data for $f(x) = \ln(x)$ using x = 1, 2, ... , 10. And test for the following case a) 1.43, b) 5.78, c) 10.01

Least-Squares Regression

11. Given the data

0.90	1.42	1.30	1.55	1.63
1.32	1.35	1.47	1.95	1.66
1.96	1.47	1.92	1.35	1.05
1.85	1.74	1.65	1.78	1.71
2.29	1.82	2.06	2.14	1.27

Determine a) the mean b) the standard deviation c) the variance d) the coefficient of variation

12. It is known that the tensile strength of a plastic increases as a function of the time it is heat-treated. The following data are collected:

Time (min)	10	15	20	25	40	50	55	60	75
Tensile strength	5	20	18	40	33	54	70	60	78

Use least-squares regression to fit a straight line . Along with the slope and intercept, compute the standard error of the estimate and the correlation coefficient. Plot the data and the regression line. Use the fitted equation to determine the tensile strength at a time of 32 min

13. This is a problem on linearization of power equation. Fit the below data using logarithmic(base-10) transformation.

x	1	2	3	4	5
y	0.5	1.7	3.4	5.7	8.4

Power equation is expressed as $y = ax^b$. Use least-squares regression to find the coefficients of the power equation. Plot the given data(as points) and after determining the coefficients, plot the power equation that fits the data(line plot). And also plot the transformed data i.e log-log plot. Use the resulting power equation to predict at x=6

14. A material is tested for cyclic fatigue failure whereby a stress, in MPa, is applied to the material and the number of cycles needed to cause failure is measured. The results are in the table below. When a log-log plot of stress versus cycles is generated, the data trend shows a linear relationship. Use least-squares regression to determine a best-fit equation for this data.

N, cycles	1	10	100	1000	10000	100000	1000000
Stress, MPa	1100	1000	925	800	625	550	420

15. An object is suspended in a wind tunnel and the force measured for various levels of wind velocity. The results are tabulated below.

velocity (m/s)	10	20	30	40	50	60	70	80
Force (N)	25	70	380	550	610	1220	830	1450

Determine the coefficients that results in least-squares fit for a second-order polynomial.

Numerical Integration

16. Evaluate the following integral:

$$\int_0^{\frac{\pi}{2}} (8 + 4cosx)dx$$

a) Analytically
b) Trapezoidal rule
c) Composite trapezoidal rule with 2 and 4 segments
d) Simpson's ⅓ rule
e) Simpson's ⅜ rule
f) Composite Simpson's rule with 4 segments

For each of the numerical estimates (b) through (f), determine the percent relative error based on (a)

17. Use 2-point and 3-point Gauss-Legendre formula to evaluate the integral of

$$f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5$$

between the limits x = 0 to x = 0.8. Find the relative error w.r.t analytical solution for both formula.

18. Evaluate the following integral using:

$$\int_0^2 \frac{e^x \, sinx}{1+x^2} dx$$

a) 2-point Gauss-Legendre formula
b) 3-point Gauss-Legendre formula
c) 4-point Gauss-Legendre formula
d) 5-point Gauss-Legendre formula
e) 6-point Gauss-Legendre formula

Numerical Differentiation

19. Find the polynomial approximation to f(x) = $(1 - x)^{½}$ over [0,1] using Taylor series expansion about x=0. Find the number of terms required in the expansion to obtain results accurate to two decimal places.

20. Compute the first-order central difference approximations of $O(h^4)$for each of the following functions at the specified location and for the specified step size:

$$y = x^3 + 4x - 15 \ at \ x = 0, \ h = 0.25$$
$$y = x^2 cosx \ at \ x = 0.4, \ h = 0.1$$
$$y = \frac{sin(0.5\sqrt{x})}{x} \ at \ x = 1, \ h = 0.2$$

Compare your results with the analytical solutions.

21. Compute forward and backward difference approximations of O(h) and O(h^2), and central difference approximations of O(h^2) and O(h^4) for the first derivative of y = sin x at x = π/4 using a value of h = π/12. Estimate the true percent relative error for each approximation.

22. Use Richardson extrapolation to estimate the first derivative of y = cosx at x = π/4 using step sizes of h_1 = π/3 and h_2 = π/6. Employ centered differences of O(h^2) for the initial estimates.

23. Given the data:

x	1.5	1.9	2.1	2.4	2.6	3.1
f(x)	1.0628	1.3961	1.5432	1.7349	1.8423	2.0397

Compute first and second derivatives at x=1.75 using polynomial interpolation over 3 nearest neighbour points.

Ordinary Differential Equations

24. Solve the given initial value problem for y(x=1) by Taylor series method of order four with step length 0.2

$$y' = -2xy^2,\ y(0) = 1$$

Compare with analytical solution.

25. Using Taylor series method of order four solve the given initial value problem on [0,3] . Compare solutions for h = 1/2, 1/4 and 1/8.

$$y' = \frac{x-y}{2}\ with\ \ y(0) = 1$$

Plot the numerical solution with different h values and analytical solution on a single plot and compare.

x_i	y_i for h=1/2	y_i for h=1/4	y_i for h=1/8	y_i analytical
0.000	1.000000	1.000000	1.000000	1.000000
0.125	-	-		
0.250	-			
0.375	-	-		
0.500				
0.625	-	-		
0.750	-			
0.875	-	-		
1.000				

26. Repeat Problem 24 with Euler's method with h = ½, ¼, ⅛ on [0,3]. Plot the numerical solution with different h values and analytical solution on a single plot and compare.

27. Solve the following set of differential equations using Euler's method, assuming that at x = 0, $y_1 = 4$, and $y_2 = 6$. Integrate on [0,2] with a step size of 0.5.

$$\frac{dy_1}{dx} = -0.5y_1$$
$$\frac{dy_2}{dx} = 4 - 0.3y_2 - 0.1y_1$$

Plot the numerical solutions.

28. Use finite difference method to solve the boundary value problem given below.

$$y'' = y' + cosy\ with\ y(0) = 0, y(\pi) = 1$$

References:

1. Applied Numerical Analysis by Wheatley and Gerald

2. Numerical Methods for Engineers by Steven C. Chapra and Raymond P. Canale