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Date: February 3, 2026

Twenty-second International Olympiad, 1981

1981/1. P is a point inside a given triangle ABC. D, E, F are the feet of the perpendiculars from P to the lines BC, CA, AB respectively. Find all P for which

$$\frac{BC}{PD} + \frac{CA}{PE} + \frac{AB}{PF}$$

is least.

1981/2. Let $1 \leq r \leq n$ and consider all subsets of r elements of the set $\{1, 2, \ldots, n\}$. Each of these subsets has a smallest member. Let $F(n, r)$ denote the arithmetic mean of these smallest numbers; prove that

$$F(n, r) = \frac{n+1}{r+1}.$$

1981/3. Determine the maximum value of $m^3 + n^3$, where m and n are integers satisfying $m, n \in \{1, 2, \ldots, 1981\}$ and

$$(n^2 - mn - m^2)^2 = 1.$$

1981/4.

(a) For which values of $n > 2$ is there a set of n consecutive positive integers such that the largest number in the set is a divisor of the least common multiple of the remaining $n - 1$ numbers?

(b) For which values of $n > 2$ is there exactly one set having the stated property?

1981/5. Three congruent circles have a common point O and lie inside a given triangle. Each circle touches a pair of sides of the triangle. Prove that the incenter and the circumcenter of the triangle and the point O are collinear.

1981/6. The function $f(x, y)$ satisfies

$$\begin{aligned}
&(1)\ f(0, y) = y + 1,\\
&(2)\ f(x + 1, 0) = f(x, 1),\\
&(3)\ f(x + 1, y + 1) = f(x, f(x + 1, y)),
\end{aligned}$$

for all non-negative integers x, y. Determine $f(4, 1981)$.

Twenty-third International Olympiad, 1982

1982/1. The function $f(n)$ is defined for all positive integers n and takes on non-negative integer values. Also, for all m, n,

$$f(m+n) - f(m) - f(n) = 0 \text{ or } 1,$$

$f(2) = 0$, $f(3) > 0$, and $f(9999) = 3333$. Determine $f(1982)$.

1982/2. A non-isosceles triangle $A_1A_2A_3$ is given with sides a_1, a_2, a_3 (a_i opposite A_i). For all $i = 1, 2, 3$, M_i is the midpoint of side a_i, and T_i is the point where the incircle touches side a_i. Denote by S_i the reflection of T_i in the interior bisector of angle A_i. Prove that the lines M_1S_1, M_2S_2, M_3S_3 are concurrent.

1982/3. Consider the infinite sequences $\{x_n\}$ of positive real numbers with

$$x_0 = 1, \quad x_{i+1} \leq x_i \text{ for all } i \geq 0.$$

(a) Prove that for every such sequence there is an $n \geq 1$ such that

$$\frac{x_0^2}{x_1} + \frac{x_1^2}{x_2} + \cdots + \frac{x_{n-1}^2}{x_n} \geq 3.999.$$

(b) Find such a sequence for which

$$\frac{x_0^2}{x_1} + \frac{x_1^2}{x_2} + \cdots + \frac{x_{n-1}^2}{x_n} < 4.$$

1982/4. Prove that if n is a positive integer such that the equation

$$x^3 - 3xy^2 + y^3 = n$$

has a solution in integers (x, y), then it has at least three such solutions. Show that the equation has no solutions in integers when $n = 2891$.

1982/5. The diagonals AC and CE of the regular hexagon $ABCDEF$ are divided by the inner points M and N respectively, so that

$$\frac{AM}{AC} = \frac{CN}{CE} = r.$$

Determine r if B, M, N are collinear.

1982/6. Let S be a square with sides of length 100, and let L be a path within S which does not meet itself and which is composed of line segments $A_0A_1, A_1A_2, \ldots, A_{n-1}A_n$ with $A_0 \neq A_n$. Suppose that for every point P of the boundary of S there is a point of L at distance from P not greater than $1/2$. Prove that there are two points X and Y in L such that the distance between X and Y is not greater than 1, and the length of that part of L which lies between X and Y is not smaller than 198.