

MATHEMATICS

SECTION – A

Question numbers 1 to 6 carry 1 mark each.

1. If vectors \vec{a} and \vec{b} are such that $|\vec{a}| = \frac{1}{2}$, $|\vec{b}| = \frac{4}{\sqrt{3}}$, $|\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}}$, find $|\vec{a} \cdot \vec{b}|$.
2. If \vec{a} and \vec{b} are unit vectors, find the angle between them such that $\vec{a} - \sqrt{2}\vec{b}$ is also a unit vector.
3. Find the distance between the planes $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) - 4 = 0$ and $\vec{r} \cdot (6\hat{i} - 9\hat{j} + 18\hat{k}) + 30 = 0$.
4. If A is a square matrix such that $|A| = 5$, find $|AA^T|$.
5. If $A = \begin{bmatrix} 1 & 2 & 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 & 3 & -2 \end{bmatrix}$, find $|AB|$.
6. If $A = \begin{bmatrix} 0 & 3 & 2 & -5 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 4a & -8 & 5b \end{bmatrix}$, find the values of k and a .

SECTION – B

Question numbers 7 to 13 carry 4 marks each.

7. Solve for x : $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}(3x)$.
8. A typist charges Rs. 145 for typing 10 English and 3 Hindi pages, while charges Rs. 180 for typing 3 English and 10 Hindi pages. Using matrices, find the charges for typing one English page and one Hindi page separately. If the typist charged only Rs. 2 per page from a poor student for 5 Hindi pages, how much less was charged from the student?
9. If $f(x) = \begin{cases} \frac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \frac{\sqrt{1+bx} - 1}{x}, & x > 0 \end{cases}$ is continuous at $x = 0$, find the values of a and b .
10. If $x \cos(a+y) = \cos y$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. Hence show that $\sin a \frac{d^2y}{dx^2} + \sin 2(a+y) \frac{dy}{dx} = 0$.
11. Find the equation of tangents to the curve $y = x^3 + 2x - 4$, which are perpendicular to the line $x + 14y + 3 = 0$.
12. Evaluate: $\int_{-2}^2 \frac{x^2}{1+5^x} dx$.
13. Find: $\int (x+3)\sqrt{3-4x-x^2} dx$.