

# MATHEMATICS

## SECTION – A

Question numbers 1 to 6 carry 1 mark each.

1. If vectors  $\vec{a}$  and  $\vec{b}$  are such that  $|\vec{a}| = \frac{1}{2}$ ,  $|\vec{b}| = \frac{4}{\sqrt{3}}$ ,  $|\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}}$ , find  $|\vec{a} \cdot \vec{b}|$ .
2. If  $\vec{a}$  and  $\vec{b}$  are unit vectors, find the angle between them such that  $\vec{a} - \sqrt{2}\vec{b}$  is also a unit vector.
3. Find the distance between the planes  $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) - 4 = 0$  and  $\vec{r} \cdot (6\hat{i} - 9\hat{j} + 18\hat{k}) + 30 = 0$ .
4. If  $A$  is a square matrix such that  $|A| = 5$ , find  $|AA^T|$ .
5. If  $A = [1 \ 2; 3 \ -1]$  and  $B = [1 \ -4; 3 \ -2]$ , find  $|AB|$ .
6. If  $A = [0 \ 3; 2 \ -5]$  and  $kA = [0 \ 4a; -8 \ 5b]$ , find the values of  $k$  and  $a$ .

## SECTION – B

Question numbers 7 to 13 carry 4 marks each.

7. Solve for  $x$ :  $\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1}(3x)$ .
8. A typist charges Rs. 145 for typing 10 English and 3 Hindi pages, while charges Rs. 180 for typing 3 English and 10 Hindi pages. Using matrices, find the charges for typing one English page and one Hindi page separately. If the typist charged only Rs. 2 per page from a poor student for 5 Hindi pages, how much less was charged from the student?
9. If  $f(x) = \begin{cases} \frac{\sin(a+1)x + 2 \sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \frac{\sqrt{1+bx} - 1}{x}, & x > 0 \end{cases}$  is continuous at  $x = 0$ , find the values of  $a$  and  $b$ .
10. If  $x \cos(a+y) = \cos y$ , prove that  $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ . Hence show that  $\sin a \frac{d^2y}{dx^2} + \sin 2(a+y) \frac{dy}{dx} = 0$ .
11. Find the equation of tangents to the curve  $y = x^3 + 2x - 4$ , which are perpendicular to the line  $x + 14y + 3 = 0$ .
12. Evaluate:  $\int_{-2}^2 \frac{x^2}{1+5^x} dx$ .
13. Find:  $\int (x+3)\sqrt{3-4x-x^2} dx$ .