Assignment 14

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Download the latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment14

1 Problem

Let $\mathbf{T}$ be the linear operator on a $n-$ dimensional vector space $\mathbf{V}$ and suppose that $\mathbf{T}$ has an n distinct characteristic values.Prove that $\mathbf{T}$ is diagonalizable.

2 Results used

Diagonalizable	A linear operator $\mathbf{T}$ on a finite-dimensional vector space $\mathbf{V}$ is diagonalizable if and only if there exists an basis of $\mathbf{V}$, consisting of eigen vectors of $\mathbf{T}$
Theorem	If $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are eigenvectors of a linear operator $\mathbf{T}$ with distinct eigen values $\lambda_1, \lambda_2, \ldots, \lambda_k$,then $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly independent. Let $S_k = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$.Let $P(k) : S_k$ is linearly independent.S_1 is linearly independent. So,$P(1)$ holds.Assume $P(k)$ holds for $1 \leq k \leq n$.Therefore,S_k is linearly independent. $$\text{Let} \sum_{i=1}^{k+1} a_i\mathbf{v}_i = 0 \quad (2.0.1)$$ Applying $\mathbf{T}$ on both sides , we get $$\implies \mathbf{T}(\sum_{i=1}^{k+1} a_i\mathbf{v}_i) = 0 \quad (2.0.2)$$ $$\implies \sum_{i=1}^{k+1} a_i\mathbf{T}(\mathbf{v}_i) = 0 \quad (2.0.3)$$ $$\implies \sum_{i=1}^{k+1} a_i\lambda_i\mathbf{v}_i = 0 \quad (2.0.4)$$ $$\implies \sum_{i=1}^{k} a_i\lambda_i\mathbf{v}_i + a_{k+1}\lambda_{k+1}\mathbf{v}_{k+1} = 0 \quad (2.0.5)$$ Multiplying (2.0.1) by λ_{k+1},we get $$\lambda_{k+1}(\sum_{i=1}^{k+1} a_i\mathbf{v}_i) = 0 \quad (2.0.6)$$ $$\implies \sum_{i=1}^{k+1} a_i\lambda_{k+1}\mathbf{v}_i = 0 \quad (2.0.7)$$

	$$\implies \sum_{i=1}^{k} a_i\lambda_{k+1}\mathbf{v}_i + a_{k+1}\lambda_{k+1}\mathbf{v}_{k+1} = 0 \quad (2.0.8)$$ Subtracting (2.0.5) and (2.0.8),we get $$\sum_{i=1}^{k} a_i(\lambda_i - \lambda_{k+1})\mathbf{v}_i = 0 \quad (2.0.9)$$ $$\text{As } \lambda_i \text{ are distinct } \forall i \leq k, a_i = 0 \quad (2.0.10)$$ Substituting this in (2.0.1) $$\sum_{i=1}^{k+1} a_i\mathbf{v}_i = 0 \quad (2.0.11)$$ $$\implies a_{k+1}\mathbf{v}_{k+1} = 0 \quad (2.0.12)$$ As $\mathbf{v}_{k+1} \neq 0 \implies a_{k+1} = 0$ Since $\forall i \leq k+1, a_i = 0.S_{k+1}$ is linearly independent By principle of mathematic induction, S_n is linearly independent.

TABLE 1: Definitions and theorem used

3 Solution

Given	**T** has an n distinct characteristic values and dim(**V**) = n
T is diagonalizable	Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be distinct eigen values of **T** and let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ be the eigen vectors of **T**..From above results we can state that $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is linearly independent.And also given that dim(**V**) = n .So,this set forms a basis of **V**. $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a basis for **V** consisting of eigen vectors of **T**. So, **T** is diagonalizable.

TABLE 2: Solution