EE5609: Matrix Theory Assignment-11

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Abstract—**This document explains regarding the linear operator**

1 PROBLEM

Let $\mathbb{V}$ be the set of complex numbers regarded as vector space over the field of real numbers. We define a function $\mathbf{T}$ from $\mathbb{V}$ into the space of 2x2 matrices as follows. If z=x+iy with x and y real numbers, then

$$\mathbf{T}(z) = \begin{pmatrix} x + 7y & 5y \\ -10y & x - 7y \end{pmatrix} \quad (1.0.1)$$

Verify that

$$\mathbf{T}(z_1 z_2) = \mathbf{T}(z_1)\mathbf{T}(z_2) \quad (1.0.2)$$

2 SOLUTION

Given,

$$\mathbf{T}(x + iy) = \begin{pmatrix} x + 7y & 5y \\ -10y & x - 7y \end{pmatrix} \quad (2.0.1)$$

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\mathbf{T}(z_1 z_2) = \mathbf{T}\left((x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)\right) \quad (2.0.2)$$

Representing the result of (2.0.2) as

$$\mathbf{T}(z_1 z_2) = \begin{pmatrix} a_{11} & a_{12} \\ b_{11} & b_{12} \end{pmatrix} \quad (2.0.3)$$

where

$$a_{11} = (x_1 x_2 - y_1 y_2) + 7(x_1 y_2 + x_2 y_1)$$
$$a_{12} = 5(x_1 y_2 + x_2 y_1)$$
$$b_{11} = -10(x_1 y_2 + x_2 y_1)$$
$$b_{12} = (x_1 x_2 - y_1 y_2) - 7(x_1 y_2 + x_2 y_1)$$

Evaluating RHS of (1.0.2)

$$\mathbf{T}(z_1)\mathbf{T}(z_2) = \begin{pmatrix} x_1 + 7y_1 & 5y_1 \\ -10y_1 & x_2 - 7y_2 \end{pmatrix}\begin{pmatrix} x_2 + 7y_2 & 5y_2 \\ -10y_2 & x_2 - 7y_2 \end{pmatrix} \quad (2.0.4)$$

Representing the result of (2.0.4) as

$$\mathbf{T}(z_1)\mathbf{T}(z_2) = \begin{pmatrix} c_{11} & c_{12} \\ d_{11} & d_{12} \end{pmatrix} \quad (2.0.5)$$

where

$$c_{11} = (x_1 x_2 - y_1 y_2) + 7(x_1 y_2 + x_2 y_1)$$
$$c_{12} = 5(x_1 y_2 + x_2 y_1)$$
$$d_{11} = -10(x_1 y_2 + x_2 y_1)$$
$$d_{12} = (x_1 x_2 - y_1 y_2) - 7(x_1 y_2 + x_2 y_1)$$

From (2.0.3) and (2.0.5), we can say that

$$\mathbf{T}(z_1 z_2) = \mathbf{T}(z_1)\mathbf{T}(z_2) \quad (2.0.6)$$