EE5609: Matrix Theory
Assignment-12

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***Abstract*—This document deals regarding the linear functionals**

Download all latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment12

1 Problem

Let m and n be positive integers and field **F**.Let $f_1,\dots,f_m$ be linear functions in F^n.For α in F^n define

$$T\alpha = (f_1(\alpha),\dots,f_m(\alpha)) \tag{1.0.1}$$

show that T is a linear transformation from F^n into F^m.Then show that every linear transformation from F^n into F^m is of the above form ,for some $f_1,\dots,f_m$.

2 Solution

Let b,$\alpha \in F^n$ and a is a scalar

$$\begin{aligned} T(a\alpha+\mathbf{b}) &= (f_1(a\alpha+\mathbf{b}),\dots,f_m(a\alpha+\mathbf{b})) \\ &= (af_1(\alpha)+f_1(\mathbf{b}),\dots,af_m(\alpha)+f_m(\mathbf{b})) \\ &= a(f_1(\alpha),\dots,f_m(\alpha))+(f_1(\mathbf{b}),\dots,f_m(\mathbf{b})) \end{aligned} \tag{2.0.1}$$

The equation (2.0.1) can be written as

$$T(a\alpha+\mathbf{b}) = aT(\alpha)+T(\mathbf{b}) \tag{2.0.2}$$

So, T is a linear transformation.
Let the matrix A of order $m\times n$ represent any linear transformation $\mathbf{X} \mapsto A\mathbf{X}$ from F^n into F^m. For i=1,...,m ,let

$$f_i(x_1,\dots,x_n) = \sum_{j=1}^{n} A_{ij}x_j \tag{2.0.3}$$

The transformation into F^m, $A\mathbf{X}$ can be written as

$$(f_1(\mathbf{X}),\dots,f_m(\mathbf{X})) \tag{2.0.4}$$

This is of the form (1.0.1)