

Assignment 14

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Download the latex-tikz codes from

<https://github.com/pavanmanesh/EE5609/tree/master/Assignment14>

1 PROBLEM

Let \mathbf{T} be the linear operator on a n -dimensional vector space \mathbf{V} and suppose that \mathbf{T} has n distinct characteristic values. Prove that \mathbf{T} is diagonalizable.

2 RESULTS USED

Diagonalizable	A linear operator \mathbf{T} on a finite-dimensional vector space \mathbf{V} is diagonalizable if and only if there exists a basis of \mathbf{V} , consisting of eigen vectors of \mathbf{T}
Eigen vectors	If $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ are eigenvectors of a linear operator \mathbf{T} with distinct eigen values $\lambda_1, \lambda_2, \dots, \lambda_k$, then $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ are linearly independent

3 SOLUTION

Given	\mathbf{T} has n distinct characteristic values and $\dim(\mathbf{V}) = n$
\mathbf{T} is diagonalizable	Let $\lambda_1, \lambda_2, \dots, \lambda_n$ be distinct eigen values of \mathbf{T} and let $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ be the eigen vectors of \mathbf{T} . From above results we can state that $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ is linearly independent. And also given that $\dim(\mathbf{V}) = n$. So, this set forms a basis of \mathbf{V} . $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ is a basis for \mathbf{V} consisting of eigen vectors of \mathbf{T} . So, \mathbf{T} is diagonalizable.