Assignment 13

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Download the latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment13

1 Problem

Let $\mathbf{M} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z} \text{ and eigen values of} \mathbf{A} \in \mathbb{Q} \right\}$

1) $\mathbf{M}$ is empty
2) $\mathbf{M} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z} \right\}$
3) If $\mathbf{A} \in \mathbf{M}$ then the eigen values of $\mathbf{A} \in \mathbb{Z}$
4) If $\mathbf{A}$,$\mathbf{B} \in \mathbf{M}$ such that $\mathbf{AB}$=$\mathbf{I}$ then $\left|\mathbf{A}\right| \in \{+1,-1\}$

2 Solution

$\mathbf{M}$ is empty	Consider $\mathbf{A}$=$\mathbf{I}$=$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.The elements of $\mathbf{A} \in \mathbb{Z}$ and it's eigen values $1 \in \mathbb{Q}$. So, $\mathbf{M}$ is not empty.
$\mathbf{M} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z} \right\}$	Let $\mathbf{A}$=$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ where elements of $\mathbf{A} \in \mathbb{Z}$.The characteristic equation can be written as : $$\lambda^2 + 1 = 0 \implies \lambda = \pm i$$ We see that $\lambda \in \mathbb{C}$ which is contradicting the main definition of $\mathbf{M}$ .So,this is not correct.
Eigen values of $\mathbf{A} \in \mathbb{Z}$	Given ,$\mathbf{A} \in \mathbf{M}$.Let λ_1,λ_2 be the eigen values of $\mathbf{A}$.The characteristic polynomial can be written as: $$\lambda^2 - tr(\mathbf{A})\lambda + \det\mathbf{A} = 0 \text{ where } tr(\mathbf{A}) = \lambda_1 + \lambda_2, \det\mathbf{A} = \lambda_1\lambda_2$$ Given the eigen values λ_1,λ_2 $\in \mathbb{Q}$,For this to be possible the discriminant of above equation should $\in \mathbb{Z}$ $$\sqrt{(\lambda_1 + \lambda_2)^2 - 4\lambda_1\lambda_2} \in \mathbb{Z}$$ $$\implies \sqrt{(\lambda_1 - \lambda_2)^2} \in \mathbb{Z}$$ $$\implies \lambda_1 - \lambda_2 \in \mathbb{Z} \text{ This is possible when both } \lambda_1,\lambda_2 \in \mathbb{Z}.$$

If **AB**=**I** then $\lvert \mathbf{A} \rvert \in \{+1,-1\}$	As **A**,**B** $\in$ **M**, $\Longrightarrow$ $\lvert \mathbf{A} \rvert, \lvert \mathbf{B} \rvert \in \mathbb{Z}$ Given **AB**=**I** $\Longrightarrow$ $\lvert \mathbf{A} \rvert \lvert \mathbf{B} \rvert = 1$ This is possible only when $\lvert \mathbf{A} \rvert = \lvert \mathbf{B} \rvert = \pm 1$
Conclusion	options 3) and 4) are correct.