

EE5609: Matrix Theory

Assignment-11

M Pavan Manesh
EE20MTECH14017

Abstract—This document explains regarding the linear operator

Download all latex-tikz codes from

<https://github.com/pavanmanesh/EE5609/tree/master/Assignment11>

1 PROBLEM

Let \mathbb{V} be the set of complex numbers regarded as vector space over the field of real numbers. We define a function \mathbf{T} from \mathbb{V} into the space of 2×2 matrices as follows. If $z = x + iy$ with x and y real numbers, then

$$\mathbf{T}(z) = \begin{pmatrix} x + 7y & 5y \\ -10y & x - 7y \end{pmatrix} \quad (1.0.1)$$

Verify that

$$\mathbf{T}(z_1 z_2) = \mathbf{T}(z_1) \mathbf{T}(z_2) \quad (1.0.2)$$

2 THEORY

The product of two Kronecker products yields another Kronecker product:

$$(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = (\mathbf{AC}) \otimes (\mathbf{BD}) \quad (2.0.1)$$

3 SOLUTION

Given,

$$\mathbf{T}(z) = \begin{pmatrix} x + 7y & 5y \\ -10y & x - 7y \end{pmatrix} \quad (3.0.1)$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (3.0.2)$$

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} (1 \ 7)\mathbf{x} & (0 \ 5)\mathbf{x} \\ (0 \ -10)\mathbf{x} & (1 \ -7)\mathbf{x} \end{pmatrix} \quad (3.0.3)$$

$$= \begin{pmatrix} (1 \ 7)\mathbf{x} & (0 \ 5)\mathbf{x} \\ (0 \ -10)\mathbf{x} & (1 \ -7)\mathbf{x} \end{pmatrix} \quad (3.0.4)$$

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix} \quad (3.0.5)$$

$$\mathbf{B} = \begin{pmatrix} 0 & 5 \\ 1 & -7 \end{pmatrix} \quad (3.0.6)$$

$$\Rightarrow \mathbf{T}(\mathbf{x}) = (\mathbf{Ax} \ \mathbf{Bx}) \quad (3.0.7)$$

$$\mathbf{T}(\mathbf{x}) = (\mathbf{A} \ \mathbf{B}) \begin{pmatrix} \mathbf{x} & \mathbf{0} \\ \mathbf{0} & \mathbf{x} \end{pmatrix} \quad (3.0.8)$$

The diagonal block matrix can be expressed as the kronecker product of \mathbf{I} and \mathbf{x}

$$\mathbf{I} \otimes \mathbf{x} = \begin{pmatrix} \mathbf{x} & \mathbf{0} \\ \mathbf{0} & \mathbf{x} \end{pmatrix} \quad (3.0.9)$$

We can write (3.0.8) as

$$\mathbf{T}(\mathbf{x}) = (\mathbf{A} \ \mathbf{B})(\mathbf{I} \otimes \mathbf{x}) \quad (3.0.10)$$

Starting with RHS of (1.0.2)

$$\mathbf{T}(z_1) \mathbf{T}(z_2) = (\mathbf{A} \ \mathbf{B})(\mathbf{I} \otimes \mathbf{z}_1) (\mathbf{A} \ \mathbf{B})(\mathbf{I} \otimes \mathbf{z}_2) \quad (3.0.11)$$

using (2.0.1), We can say that

$$(\mathbf{I} \otimes \mathbf{z}_1)(\mathbf{I} \otimes \mathbf{z}_2) = \mathbf{I} \otimes (\mathbf{z}_1 \mathbf{z}_2) \quad (3.0.12)$$