Assignment 14

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Download the latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment14

1 Problem

Let $\mathbf{T}$ be the linear operator on a $n-$ dimensional vector space $\mathbf{V}$ and suppose that $\mathbf{T}$ has an n distinct characteristic values.Prove that $\mathbf{T}$ is diagonalizable.

2 Results used

Diagonalizable	A linear operator $\mathbf{T}$ on a finite-dimensional vector space $\mathbf{V}$ is diagonalizable if and only if there exists an basis of $\mathbf{V}$, consisting of eigen vectors of $\mathbf{T}$
Theorem	If $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are eigenvectors of a linear operator $\mathbf{T}$ with distinct eigen values $\lambda_1, \lambda_2, \ldots, \lambda_k$,then $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly independent. Let a_1,a_2,...,a_k be the scalars such that $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \cdots + a_k\mathbf{v}_k = 0 \quad \ldots(1)$ Applying $\mathbf{T}$ on both sides , we get $\mathbf{T}(a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \cdots + a_k\mathbf{v}_k)=0$ $\implies a_1\mathbf{T}(\mathbf{v}_1) + a_2\mathbf{T}(\mathbf{v}_2) + \cdots + a_k\mathbf{T}(\mathbf{v}_k)=0$ As $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are eigen vectors $\implies \mathbf{T}(\mathbf{v}_i)=\lambda_i\mathbf{v}_i$ for i=1,...,k $\implies a_1\lambda_1\mathbf{v}_1 + a_2\lambda_2\mathbf{v}_2 + \cdots + a_k\lambda_k\mathbf{v}_k = 0 \quad \ldots(2)$ Multiplying (1) by λ_k,we get $a_1\lambda_k\mathbf{v}_1 + a_2\lambda_k\mathbf{v}_2 + \cdots + a_k\lambda_k\mathbf{v}_k = 0 \quad \ldots(3)$ Subtracting (2) and (3),we get $a_1(\lambda_1 - \lambda_k)\mathbf{v}_1 + a_2(\lambda_2 - \lambda_k)\mathbf{v}_2 + \cdots + a_k(\lambda_{k-1} - \lambda_k)\mathbf{v}_{k-1} = 0$ As $\lambda_1, \lambda_2, \ldots, \lambda_k$ are distinct $\implies \lambda_i - \lambda_k \neq 0$ for i=1,2,..,k-1 So,We can say that $a_1 = a_2 = \cdots = a_{k-1} = 0$ Substituting this in (1), we get $a_k = 0$ As $a_1 = a_2 = \cdots = a_k = 0$,We can say that $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly independent

3 Solution

Given	**T** has an n distinct characteristic values and dim(**V**) = n
T is diagonalizable	Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be distinct eigen values of **T** and let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ be the eigen vectors of **T**..From above results we can state that $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is linearly independent.And also given that dim(**V**) = n .So,this set forms a basis of **V**. $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a basis for **V** consisting of eigen vectors of **T**. So, **T** is diagonalizable.