Assignment 14

M Pavan Manesh - EE20MTECH14017

Download the latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment14

1 Problem

Let $\mathbf{T}$ be the linear operator on a $n-$ dimensional vector space $\mathbf{V}$ and suppose that $\mathbf{T}$ has an n distinct characteristic values.Prove that $\mathbf{T}$ is diagonalizable.

2 Results used

Diagonalizable	A linear operator $\mathbf{T}$ on a finite-dimensional vector space $\mathbf{V}$ is diagonalizable if and only if there exists an basis of $\mathbf{V}$, consisting of eigen vectors of $\mathbf{T}$
Theorem	If $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are eigenvectors of a linear operator $\mathbf{T}$ with distinct eigen values $\lambda_1, \lambda_2, \ldots, \lambda_k$,then $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly independent. Let $\mathbf{v_1}$ and $\mathbf{v_2}$ be the eigen vectors corresponding to eigen values λ_1 and λ_2 $\implies \mathbf{T}(\mathbf{v}_1)=\lambda_1\mathbf{v}_1, \mathbf{T}(\mathbf{v}_2)=\lambda_2\mathbf{v}_2$ Let the linear combination of two eigen vectors be $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 = 0 \qquad \ldots(1)$ Applying $\mathbf{T}$ on both sides , we get $\mathbf{T}(a_1\mathbf{v}_1 + a_2\mathbf{v}_2)=0$ $\implies a_1\mathbf{T}(\mathbf{v}_1) + a_2\mathbf{T}(\mathbf{v}_2)=0$ $\implies a_1\lambda_1\mathbf{v}_1 + a_2\lambda_2\mathbf{v}_2 = 0 \qquad \ldots(2)$ Multiplying (1) by λ_1,we get $a_1\lambda_1\mathbf{v}_1 + a_2\lambda_1\mathbf{v}_2 = 0 \qquad \ldots(3)$ Subtracting (2) and (3),we get $a_2(\lambda_2 - \lambda_1)\mathbf{v}_2 = 0$ As λ_1, λ_2 are distinct $\implies \lambda_2 - \lambda_1 \neq 0$ So,We can say that $a_2 = 0$ Substituting this in (1), we get $a_1 = 0$ Therefore $\mathbf{v}_1, \mathbf{v}_2$ are linearly independent By induction ,for n distinct eigen values $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly independent.

3 Solution

Given	$\mathbf{T}$ has an n distinct characteristic values and dim($\mathbf{V}$) = n
$\mathbf{T}$ is diagonalizable	Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be distinct eigen values of $\mathbf{T}$ and let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ be the eigen vectors of $\mathbf{T}$..From above results we can state that $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is linearly independent.And also given that dim($\mathbf{V}$) = n .So,this set forms a basis of $\mathbf{V}$. $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a basis for $\mathbf{V}$ consisting of eigen vectors of $\mathbf{T}$. So, $\mathbf{T}$ is diagonalizable.