EE5609: Matrix Theory
Assignment-13

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Download all latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment13

1 Problem

Let $\mathbf{M} = \{\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z}$ and eigen values of $\mathbf{A} \in \mathbb{Q}\}$

1) $\mathbf{M}$ is empty
2) $\mathbf{M} = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z}\}$
3) If $\mathbf{A} \in \mathbf{M}$ then the eigen values of $\mathbf{A} \in \mathbb{Z}$
4) If $\mathbf{A}$,$\mathbf{B} \in \mathbf{M}$ such that $\mathbf{AB}$=$\mathbf{I}$ then $\left|\mathbf{A}\right| \in \{+1,-1\}$

2 Solution

2.1 Option 1

Consider $\mathbf{A}$=$\mathbf{I}$=$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
The elements of $\mathbf{A} \in \mathbb{Z}$ and it's eigen values $1 \in \mathbb{Q}$. So, $\mathbf{M}$ is not empty.

2.2 Option 2

Let $\mathbf{A}$=$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ where elements of $\mathbf{A} \in \mathbb{Z}$
The characteristic equation can be written as

$$\lambda^2 + 1 = 0 \implies \lambda = \pm i \tag{2.2.1}$$

We see that $\lambda \in \mathbb{C}$ which is contradicting to the definition of $\mathbf{M}$ .So,this option is not correct.

2.3 Option 3

Let λ_1,λ_2 be the eigen values of $\mathbf{A}$.The characteristic polynomial can be written as:

$$\lambda^2 - tr\left(\mathbf{A}\right) + \det \mathbf{A} = 0 \tag{2.3.1}$$

where

$$tr\left(\mathbf{A}\right) = \lambda_1 + \lambda_2, \det \mathbf{A} = \lambda_1\lambda_2$$

Given the eigen values λ_1,$\lambda\_2 \in \mathbb{Q}$,For this to be possible the discriminant of equation (2.3.1) should $\in \mathbb{Z}$

$$\sqrt{(\lambda_1 + \lambda_2)^2 - 4\lambda_1\lambda_2} \in \mathbb{Z}$$
$$\sqrt{(\lambda_1 - \lambda_2)^2} \in \mathbb{Z}$$
$$\lambda_1 - \lambda_2 \in \mathbb{Z}$$

This is possible when both λ_1,$\lambda_2 \in \mathbb{Z}$.

2.4 Option 4

As $\mathbf{A}$,$\mathbf{B} \in \mathbf{M} \implies \left|\mathbf{A}\right|, \left|\mathbf{B}\right| \in \mathbb{Z}$
Given $\mathbf{AB}$=$\mathbf{I} \implies \left|\mathbf{A}\right|\left|\mathbf{B}\right| = 1$
This is possible only when $\left|\mathbf{A}\right|$=$\left|\mathbf{B}\right|$=$\pm$ 1

3 Conclusion

The correct options would be 3 and 4.