EE5609: Matrix Theory Assignment-9

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***Abstract*—This document explains how to find the basis for the given vector space**

Download all latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment9

1 Problem

Let $\mathbb{V}$ be a vector space which is spanned by the rows of matrix

$$\mathbf{A} = \begin{pmatrix} 3 & 21 & 0 & 9 & 0 \\ 1 & 7 & -1 & -2 & -1 \\ 2 & 14 & 0 & 6 & 1 \\ 6 & 42 & -1 & 13 & 0 \end{pmatrix} \tag{1.0.1}$$

a. Find a basis for $\mathbb{V}$

b. Tell which vectors $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$ are elements of $\mathbb{V}$

c. If $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$ is in $\mathbb{V}$,what are its coordinates in the basis chosen in part(a)?

2 Solution

Row reducing (1.0.1)

$$\begin{pmatrix} 3 & 21 & 0 & 9 & 0 \\ 1 & 7 & -1 & -2 & -1 \\ 2 & 14 & 0 & 6 & 1 \\ 6 & 42 & -1 & 13 & 0 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix} 1 & 7 & 0 & 3 & 0 \\ 1 & 7 & -1 & -2 & -1 \\ 2 & 14 & 0 & 6 & 1 \\ 6 & 42 & -1 & 13 & 0 \end{pmatrix}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - R_1]{R_3 \leftarrow R_3 - 2R_1} \begin{pmatrix} 1 & 7 & 0 & 3 & 0 \\ 0 & 0 & -1 & -5 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -5 & 0 \end{pmatrix}$$

$$\xleftrightarrow{R_4 \leftarrow R_4 - R_2} \begin{pmatrix} 1 & 7 & 0 & 3 & 0 \\ 0 & 0 & -1 & -5 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow -R_2} \begin{pmatrix} 1 & 7 & 0 & 3 & 0 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - R_3]{R_4 \leftarrow R_4 - R_3} \begin{pmatrix} 1 & 7 & 0 & 3 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.1}$$

a. For the basis of $\mathbb{V}$, we can take the non zero rows of (2.0.1)

$$\rho_1 = \begin{pmatrix} 1 \\ 7 \\ 0 \\ 3 \\ 0 \end{pmatrix} \rho_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 5 \\ 0 \end{pmatrix} \rho_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.2}$$

b. Vectors which are elements of $\mathbb{V}$ are of the form:

$$c_1\rho_1 + c_2\rho_2 + c_3\rho_3 = \begin{pmatrix} c_1 \\ 7c_1 \\ c_2 \\ 3c_1 + 5c_2 \\ c_3 \end{pmatrix} \tag{2.0.3}$$

where c_1,c_2,c_3 are scalars.

c. Expressing (2.0.3) in matrix form,if $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$ is in $\mathbb{V}$,it must be of the form

$$\begin{pmatrix} 1 & 0 & 0 \\ 7 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \tag{2.0.4}$$

The augmented matrix form

$$\begin{pmatrix} 1 & 0 & 0 & x_1 \\ 7 & 0 & 0 & x_2 \\ 0 & 1 & 0 & x_3 \\ 3 & 5 & 0 & x_4 \\ 0 & 0 & 1 & x_5 \end{pmatrix} \tag{2.0.5}$$

Converting the above matrix into row reduced echelon form

$$\begin{aligned} &\begin{pmatrix} 1 & 0 & 0 & x_1 \\ 7 & 0 & 0 & x_2 \\ 0 & 1 & 0 & x_3 \\ 3 & 5 & 0 & x_4 \\ 0 & 0 & 1 & x_5 \end{pmatrix} \xleftrightarrow[R_2 \leftarrow R_2 - 7R_1]{R_4 \leftarrow R_4 - 3R_1} \begin{pmatrix} 1 & 0 & 0 & x_1 \\ 0 & 0 & 0 & x_2 - 7x_1 \\ 0 & 1 & 0 & x_3 \\ 0 & 5 & 0 & x_4 - 3x_1 \\ 0 & 0 & 1 & x_5 \end{pmatrix} \\ &\xleftrightarrow{R_2 \leftarrow R_3} \begin{pmatrix} 1 & 0 & 0 & x_1 \\ 0 & 1 & 0 & x_3 \\ 0 & 0 & 0 & x_2 - 7x_1 \\ 0 & 5 & 0 & x_4 - 3x_1 \\ 0 & 0 & 1 & x_5 \end{pmatrix} \\ &\xleftrightarrow{R_4 \leftarrow R_4 - 5R_2} \begin{pmatrix} 1 & 0 & 0 & x_1 \\ 0 & 1 & 0 & x_3 \\ 0 & 0 & 0 & x_2 - 7x_1 \\ 0 & 0 & 0 & x_4 - 3x_1 - 5x_3 \\ 0 & 0 & 1 & x_5 \end{pmatrix} \\ &\xleftrightarrow{R_5 \leftarrow R_3} \begin{pmatrix} 1 & 0 & 0 & x_1 \\ 0 & 1 & 0 & x_3 \\ 0 & 0 & 1 & x_5 \\ 0 & 0 & 0 & x_4 - 3x_1 - 5x_3 \\ 0 & 0 & 0 & x_2 - 7x_1 \end{pmatrix} \end{aligned} \tag{2.0.6}$$

From (2.0.6),the coordinates of $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$ in the basis is

$$\begin{pmatrix} x_1 \\ x_3 \\ x_5 \end{pmatrix} \tag{2.0.7}$$