EE5609: Matrix Theory
Assignment-10

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***Abstract*—This document explains regarding the linear operator**

Download all latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment11

1 Problem

Let $\mathbb{V}$ be the set of complex numbers regarded as vector space over the field of real numbers.We define a function $\mathbf{T}$ from $\mathbb{V}$ into the space of 2x2 matrices as follows.If z=x+iy with x and y real numbers,then

$$\mathbf{T}(z) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} \tag{1.0.1}$$

Verify that

$$\mathbf{T}(z_1 z_2) = \mathbf{T}(z_1)\mathbf{T}(z_2) \tag{1.0.2}$$

2 Solution

Given,

$$\mathbf{T}(x+iy) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} \tag{2.0.1}$$

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\mathbf{T}(z_1 z_2) = \mathbf{T}\left((x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)\right) \tag{2.0.2}$$

Representing (2.0.2) as

$$\mathbf{T}(z_1 z_2) = \begin{pmatrix} a_{11} & a_{12} \\ b_{11} & b_{12} \end{pmatrix} \tag{2.0.3}$$

where

$$\begin{aligned} a_{11} &= (x_1x_2 - y_1y_2) + 7(x_1y_2 + x_2y_1) \\ a_{12} &= 5(x_1y_2 + x_2y_1) \\ b_{11} &= -10(x_1y_2 + x_2y_1) \\ b_{12} &= (x_1x_2 - y_1y_2) - 7(x_1y_2 + x_2y_1) \end{aligned}$$

Evaluating RHS of (1.0.2)

$$\mathbf{T}(z_1)\mathbf{T}(z_2) = \begin{pmatrix} x_1+7y_1 & 5y_1 \\ -10y_1 & x_2-7y_2 \end{pmatrix}\begin{pmatrix} x_2+7y_2 & 5y_2 \\ -10y_2 & x_2-7y_2 \end{pmatrix} \tag{2.0.4}$$

Representing (2.0.4) as

$$\mathbf{T}(z_1)\mathbf{T}(z_2) = \begin{pmatrix} c_{11} & c_{12} \\ d_{11} & d_{12} \end{pmatrix} \tag{2.0.5}$$

where

$$\begin{aligned} c_{11} &= (x_1x_2 - y_1y_2) + 7(x_1y_2 + x_2y_1) \\ c_{12} &= 5(x_1y_2 + x_2y_1) \\ d_{11} &= -10(x_1y_2 + x_2y_1) \\ d_{12} &= (x_1x_2 - y_1y_2) - 7(x_1y_2 + x_2y_1) \end{aligned}$$

From (2.0.3) and (2.0.5),we can say that

$$\mathbf{T}(z_1 z_2) = \mathbf{T}(z_1)\mathbf{T}(z_2) \tag{2.0.6}$$