Assignment 18

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Download the latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment18

1 Problem

Let $\mathbf{A}$ be a (6×6) matrix over $\mathbb{R}$ with characteristic polynomial $=(x-3)^2\,(x-2)^4$ and the minimum polynomial $=(x-3)\,(x-2)^2$.Then jordan canonical form of $\mathbf{A}$ can be

1) $\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$
2) $\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$
3) $\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$
4) $\begin{pmatrix} 3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

2 Definitions

Jordan canonical form	If **A** is a matrix of order n×n ,then the Jordan canonical form of **A** is a matrix of order n×n expressed as $$\mathbf{J} = \begin{pmatrix} \mathbf{J_1} & & \\ & \ddots & \\ & & \mathbf{J_k} \end{pmatrix} \quad (2.0.1)$$ where $\mathbf{J_1}, ..., \mathbf{J_k}$ are the Jordan blocks.
Algebraic multiplicity A_M	Algebraic multiplicity of characteristic value λ in the characteristic polynomial determines the size of Jordan block for that eigen value $$A_M = \text{Size of Jordan block for that } \lambda \quad (2.0.2)$$
Geometric multiplicity G_M	Geometric multiplicity determines the number of Jordan sub-blocks in a Jordan block for λ
Minimal Polynomial	The multiplicity of λ in the minimal polynomial determines the size of the largest sub-block.

TABLE 1: Definition and Properties used

3 Solution

Characteristic polynomial	$$p(x) = (x-3)^2(x-2)^4 \quad (3.0.1)$$
Algebraic Multiplicity	$$\text{For } \lambda = 3, A_M = 2 \quad (3.0.2)$$ $$\text{For } \lambda = 2, A_M = 4 \quad (3.0.3)$$
Minimal polynomial	$$m(x) = (x-3)(x-2)^2 \quad (3.0.4)$$
Finding Jordan blocks for λ_1=3	For λ_1=3,We can write from table1 that $$\text{The highest order of Jordan block} = 1$$ $$\text{Size of Jordan block} = A_M = 2$$

	The Jordan blocks for λ_1=3 $$\mathbf{J_1} = \begin{pmatrix}3\end{pmatrix}, \mathbf{J_2} = \begin{pmatrix}3\end{pmatrix} \tag{3.0.5}$$
Finding Jordan blocks for λ_1=2	For λ_1=2,We can write from table1 that $$\text{The highest order of Jordan block} = 2$$ $$\text{Size of Jordan block} = A_M = 4$$ The Jordan blocks for λ_1=3 $$\mathbf{J_3} = \begin{pmatrix}2 & 1\\0 & 2\end{pmatrix}, \mathbf{J_4} = \begin{pmatrix}2 & 1\\0 & 2\end{pmatrix} \tag{3.0.6}$$ or $$\mathbf{J_3} = \begin{pmatrix}2 & 1\\0 & 2\end{pmatrix}, \mathbf{J_4} = \begin{pmatrix}2\end{pmatrix}, \mathbf{J_5} = \begin{pmatrix}2\end{pmatrix} \tag{3.0.7}$$
Jordan canonical form	Jordan canonical form of **A** is $$\mathbf{J} = \begin{pmatrix}\mathbf{J_1} & & & \\ & \mathbf{J_2} & & \\ & & \mathbf{J_3} & \\ & & & \mathbf{J_4}\end{pmatrix} \text{or} \begin{pmatrix}\mathbf{J_1} & & & & \\ & \mathbf{J_2} & & & \\ & & \mathbf{J_3} & & \\ & & & \mathbf{J_4} & \\ & & & & \mathbf{J_5}\end{pmatrix} \tag{3.0.8}$$ $$\begin{pmatrix}3&0&0&0&0&0\\0&3&0&0&0&0\\0&0&2&1&0&0\\0&0&0&2&0&0\\0&0&0&0&2&1\\0&0&0&0&0&2\end{pmatrix} \text{or} \begin{pmatrix}3&0&0&0&0&0\\0&3&0&0&0&0\\0&0&2&1&0&0\\0&0&0&2&0&0\\0&0&0&0&2&0\\0&0&0&0&0&2\end{pmatrix} \tag{3.0.9}$$
Conclusion	From above,we can say that options 2) and 3) are correct.

TABLE 2: Finding Jordan canonical form