EE5609: Matrix Theory Assignment-10

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***Abstract*—This document explains how to find the basis for the given vector space**

Download all latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment10

1 Problem

For the linear operator T

$$T\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}3x_1\\x_1-x_2\\2x_1+x_2+x_3\end{pmatrix} \tag{1.0.1}$$

Prove that

$$(T^2-I)(T-3I)=0 \tag{1.0.2}$$

2 Solution

Expressing (1.0.1) in matrix form

$$T=\begin{pmatrix}3&0&0\\1&-1&0\\2&1&1\end{pmatrix} \tag{2.0.1}$$

Using (2.0.1),We can write that

$$T^2=\begin{pmatrix}3&0&0\\1&-1&0\\2&1&1\end{pmatrix}\begin{pmatrix}3&0&0\\1&-1&0\\2&1&1\end{pmatrix}$$

$$\implies T^2=\begin{pmatrix}9&0&0\\2&1&0\\9&0&1\end{pmatrix} \tag{2.0.2}$$

$$T^2-I=\begin{pmatrix}8&0&0\\2&0&0\\9&0&0\end{pmatrix} \tag{2.0.3}$$

$$T-3I=\begin{pmatrix}0&0&0\\1&-4&0\\2&1&-2\end{pmatrix} \tag{2.0.4}$$

From (2.0.3) and (2.0.4),

$$(T^2-I)(T-3I)=\begin{pmatrix}8&0&0\\2&0&0\\9&0&0\end{pmatrix}\begin{pmatrix}0&0&0\\1&-4&0\\2&1&-2\end{pmatrix}$$

$$\implies (T^2-I)(T-3I)=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix} \tag{2.0.5}$$