Assignment 16

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Download the latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment16

1 Problem

True or false? If the triangular matrix $\mathbf{A}$ is similar to a diagonal matrix, then $\mathbf{A}$ is already diagonal.

2 Definitions

Characteristic Polynomial	For an $n \times n$ matrix $\mathbf{A}$, characteristic polynomial is defined by, $$p(x) = \left|x\mathbf{I} - \mathbf{A}\right|$$
Minimal Polynomial	Minimal polynomial $m(x)$ is the smallest factor of characteristic polynomial $p(x)$ such that, $$m(\mathbf{A}) = 0$$
Theorem	Let $\mathbf{V}$ be a finite-dimensional vector space over the field $\mathbf{F}$ and let $\mathbf{T}$ be a linear operator on $\mathbf{V}$.Then $\mathbf{T}$ is diagonalizable if and only if the minimal polynomial for $\mathbf{T}$ has the form $$p = (x - c_1)\dots(x - c_k) \quad (2.0.1)$$ where $c_1, c_2, ..., c_k$ are distinct elements of F.
Diagonalizable	$\mathbf{A}$ is called diagonalizable if it is similar to diagnol matrix $\mathbf{B}$ i.e., if $\exists$ an invertible matrix $\mathbf{P}$ such that $$\mathbf{B} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P} \quad (2.0.2)$$

TABLE 1: Definitions and theorem used

3 Solution

Given	The triangular matrix $\mathbf{A}$ is similar to a diagonal matrix.
Example	Let $$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} \tag{3.0.1}$$ We can see that $\mathbf{A}$ is triangular but not diagonal.
Characteristic polynomial	$$\left\lvert x\mathbf{I} - \mathbf{A} \right\rvert = \begin{vmatrix} x-1 & 2 \\ 0 & x-3 \end{vmatrix} \tag{3.0.2}$$ $$= (x-1)(x-3) \tag{3.0.3}$$
Minimal polynomial	As the eigen values are distinct,minimal polynomial $$m(x) = (x-1)(x-3) \tag{3.0.4}$$
Diagonalizable	From theorem (2.0.1),We can say that $\mathbf{A}$ diagonalizable i.e., it is similar to a diagnol matrix.
Finding matrix similar to $\mathbf{A}$	The eigen values are $$\lambda_1 = 1, \lambda_2 = 3 \tag{3.0.5}$$ The eigen vectors are $$(\mathbf{A} - \lambda_i \mathbf{I})\mathbf{x_i} = 0 \tag{3.0.6}$$ $$\lambda_1 = 1 \implies \begin{pmatrix} 0 & 2 \\ 0 & 2 \end{pmatrix}\mathbf{x_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \implies \mathbf{x_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3.0.7}$$ $$\lambda_2 = 3 \implies \begin{pmatrix} -2 & 2 \\ 0 & 0 \end{pmatrix}\mathbf{x_2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \implies \mathbf{x_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{3.0.8}$$ The invertible matrix $$\mathbf{P} = \begin{pmatrix} \mathbf{x_1} & \mathbf{x_2} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \tag{3.0.9}$$ The diagnol matrix similar to $\mathbf{A}$

	$$\mathbf{B} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{pmatrix}1 & -1\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\0 & 3\end{pmatrix}\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix} \quad (3.0.10)$$ $$\mathbf{B} = \begin{pmatrix}1 & 0\\0 & 3\end{pmatrix} \quad (3.0.11)$$
Conclusion	From above,we can say that **A** need not be diagonal to satisfy given conditions.So, given statement is false.

TABLE 2: Finding minimal polynomial and simila