EE5609: Matrix Theory Assignment-11

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***Abstract*—This document explains regarding the linear operator**

Download all latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment11

1 Problem

Let $\mathbb{V}$ be the set of complex numbers regarded as vector space over the field of real numbers.We define a function $\mathbf{T}$ from $\mathbb{V}$ into the space of 2x2 matrices as follows.If z=x+iy with x and y real numbers,then

$$\mathbf{T}(z) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} \tag{1.0.1}$$

Verify that

$$\mathbf{T}(z_1 z_2) = \mathbf{T}(z_1)\mathbf{T}(z_2) \tag{1.0.2}$$

2 Theory

The product of two Kronecker products yields another Kronecker product:

$$(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = (\mathbf{AC}) \otimes (\mathbf{BD}) \tag{2.0.1}$$

3 Solution

Given,

$$\mathbf{T}(\mathbf{z}) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} \tag{3.0.1}$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{3.0.2}$$

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} \begin{pmatrix} 1 & 7 \end{pmatrix}\mathbf{x} & \begin{pmatrix} 0 & 5 \end{pmatrix}\mathbf{x} \\ \begin{pmatrix} 0 & -10 \end{pmatrix}\mathbf{x} & \begin{pmatrix} 1 & -7 \end{pmatrix}\mathbf{x} \end{pmatrix} \tag{3.0.3}$$

$$= \begin{pmatrix} \begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix}\mathbf{x} & \begin{pmatrix} 0 & 5 \\ 1 & -7 \end{pmatrix}\mathbf{x} \end{pmatrix} \tag{3.0.4}$$

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix} \tag{3.0.5}$$

$$\mathbf{B} = \begin{pmatrix} 0 & 5 \\ 1 & -7 \end{pmatrix} \tag{3.0.6}$$

$$\implies \mathbf{T}(\mathbf{x}) = \begin{pmatrix} \mathbf{Ax} & \mathbf{Bx} \end{pmatrix} \tag{3.0.7}$$

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \mathbf{x} & \mathbf{0} \\ \mathbf{0} & \mathbf{x} \end{pmatrix} \tag{3.0.8}$$

The diagonal block matrix can be expressed as the kronecker product of $\mathbf{I}$ and $\mathbf{x}$

$$\mathbf{I} \otimes \mathbf{x} = \begin{pmatrix} \mathbf{x} & \mathbf{0} \\ \mathbf{0} & \mathbf{x} \end{pmatrix} \tag{3.0.9}$$

We can write (3.0.8) as

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} (\mathbf{I} \otimes \mathbf{x}) \tag{3.0.10}$$

Starting with RHS of (3.0.11)

$$\mathbf{T}(z_1)\mathbf{T}(z_2) = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} (\mathbf{I} \otimes \mathbf{z_1}) \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} (\mathbf{I} \otimes \mathbf{z_2}) \tag{3.0.11}$$

If

$$(\mathbf{I} \otimes \mathbf{z_1}) \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} (\mathbf{I} \otimes \mathbf{z_2}) = (\mathbf{I} \otimes \mathbf{z_1 z_2}) \tag{3.0.12}$$

then,we can write (3.0.11) as

$$\mathbf{T}(z_1)\mathbf{T}(z_2) = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} (\mathbf{I} \otimes \mathbf{z_1 z_2}) = \mathbf{T}(z_1 z_2) \tag{3.0.13}$$