EE5609: Matrix Theory Assignment-10

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Abstract—**This document explains regarding the use of Cayley-Hamilton theorem**

Download all latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment10

1 Problem

For the linear operator $\mathbf{T}$

$$\mathbf{T}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3x_1 \\ x_1 - x_2 \\ 2x_1 + x_2 + x_3 \end{pmatrix} \tag{1.0.1}$$

Prove that

$$(\mathbf{T}^2 - I)(\mathbf{T} - 3I) = 0 \tag{1.0.2}$$

2 Solution

Expressing (1.0.1) in matrix form

$$\mathbf{T} = \begin{pmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \tag{2.0.1}$$

The characteristic equation of $\mathbf{T}$ is given as follows,

$$\left|\mathbf{T} - \lambda\mathbf{I}\right| = \begin{vmatrix} 3-\lambda & 0 & 0 \\ 1 & -1-\lambda & 0 \\ 2 & 1 & 1-\lambda \end{vmatrix} = 0 \tag{2.0.2}$$

$$\implies -\lambda^3 + 3\lambda^2 + \lambda - 3 = 0 \tag{2.0.3}$$

By the Cayley-Hamilton theorem,We can write (2.0.3) as

$$-\mathbf{T}^3 + 3\mathbf{T}^2 + \mathbf{T} - 3I = 0 \tag{2.0.4}$$

$$\implies \mathbf{T}^3 - 3\mathbf{T}^2 - \mathbf{T} + 3I = 0 \tag{2.0.5}$$

Rearranging (2.0.5)

$$\mathbf{T}^2(\mathbf{T} - 3I) - (\mathbf{T} - 3I) = 0$$

$$(\mathbf{T}^2 - I)(\mathbf{T} - 3I) = 0 \tag{2.0.6}$$