ECE-415 (Computer Vision I)
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Homework-2

Given,

pixel coordinates

$$
\begin{bmatrix}
37 & 37 & 37 & 37 & 37 & 37 & 37 \\
37 & 37 & 37 & 37 & 37 & 37 & 37 \\
37 & 37 & 37 & 37 & 37 & 37 & 37 \\
37 & 37 & 125 & 125 & 37 & 37 & 37 \\
37 & 37 & 37 & 125 & 37 & 37 & 37 \\
37 & 37 & 37 & 37 & 37 & 37 & 37 \\
37 & 37 & 37 & 37 & 37 & 37 & 37
\end{bmatrix}
$$

a).

Cartesian coordinates of pixels in the objects

$$
x \downarrow \quad y \rightarrow
$$

$$
\begin{bmatrix}
(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) & (1,7) \\
(2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) & (2,7) \\
(3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) & (3,7) \\
(4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) & (4,7) \\
(5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) & (5,7) \\
(6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) & (6,7) \\
(7,1) & (7,2) & (7,3) & (7,4) & (7,5) & (7,6) & (7,7)
\end{bmatrix}
$$

$\therefore$ Cartesian coordinates of pixels in the object are

$(4,3), (4,4), (5,4)$.

b). Spatial coordinates of pixel in the object in homogeneous system; $\tilde{w} = 2$.

Wkt $\tilde{m} = \tilde{w}\bar{m}$

scalar — augmented vector.

$$= \tilde{w}\begin{bmatrix} x \\ y \\ \tilde{w} \end{bmatrix} = \tilde{w}\begin{bmatrix} 4 & 4 & 5 \\ 3 & 4 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= 2\begin{bmatrix} 4 & 4 & 5 \\ 3 & 4 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\tilde{m} = \begin{bmatrix} 8 & 8 & 10 \\ 6 & 8 & 8 \\ 2 & 2 & 2 \end{bmatrix}$$

c). Spatial coordinates of pixel in the object in homogeneous system as augmented vector. ($\tilde{w} = 1$)

$$\tilde{m} = \tilde{w}\bar{m}$$

$$= 1.\begin{bmatrix} 4 & 4 & 5 \\ 3 & 4 & 4 \\ 1 & 1 & 1 \end{bmatrix}.$$

$\because \bar{m} = [x, y, 1]$

d). Given,

transformation matrix.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

which is of the form.

$$\begin{bmatrix} I_{2\times2} & t_{2\times1} \\ 0^T & 1 \end{bmatrix}.$$

$$\therefore \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

$\Rightarrow$ Inhomogeneous coordinates $p' = [I \ t]\bar{p}$ ↳ augmented vector.

$$\therefore \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 5 \\ 3 & 4 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+2 & 4+0+2 & 5+0+2 \\ 0+3+1 & 0+4+1 & 0+4+1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 6 & 6 & 7 \\ 4 & 5 & 5 \end{bmatrix}$$

e). The matrix corresponds to "<u>Translation</u>".

f). Rotation in cartesian is given by $p' = R.p$.

$\because t = 0$.

$\therefore$ In Homogeneous coordinate system.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$\because t = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ as there is no translation.

$\therefore$ Transformation matrix will be,

$$\begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

g). Spatial coordinates in Cartesian system. after rotating by 45° →

wkt $p' = R.p$ } in Cartesian

where $R = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix}$

$= \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix}$

$p = \begin{bmatrix} x \\ y \end{bmatrix}$.

$\therefore p' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & +0.707 \end{bmatrix} \begin{bmatrix} 4 & 4 & 5 \\ 3 & 4 & 4 \end{bmatrix}$

$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0.707 & 0 & 0.707 \\ 4.949 & 5.66 & 6.36 \end{bmatrix}$ } Cartesian system.

h). Affine + Rotation matrix.

Given, transformation matrix $\begin{bmatrix} 0.5 & -2 & -1 \\ 0.5 & 1 & 1.5 \\ 0 & 0 & 1 \end{bmatrix}$

assuming rotating angle as 45°.

Rotation matrix $= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore$ Combined transformation

$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = A.R.\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 & -2 & -1 \\ 0.5 & 1 & 1.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\left.\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1.06 & -1.767 & -1 \\ 1.06 & 0.35 & 1.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \right\} \text{Homogenous system.}$$

∴ Transformation to be performed in single step is given by the matrix.

$$\begin{bmatrix} -1.06 & -1.767 & -1 \\ 1.06 & 0.35 & 1.5 \\ 0 & 0 & 1 \end{bmatrix}.$$

i). Cartesian coordinates of pixel (4,3) after applying transformation is:

$$p' = A.R.p.$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1.06 & -1.767 & -1 \\ 1.06 & 0.35 & 1.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -10.5425 \\ 6.8025 \\ 1 \end{bmatrix}$$ as $\tilde{w} = 1$ the augmented vector.

The Cartesian coordinates are $\begin{bmatrix} -10.5425 \\ 6.8025 \end{bmatrix}$. //