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Homework - 4

Problem 1)

a). 5x5 moving average filter

Kernel \rightarrow

$$\frac{1}{25} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \because M=5 \quad \therefore \frac{1}{M^2} = \frac{1}{25}$$

The neighborhood matrix at location (4,4) \rightarrow

222 129

73

1	19	204	85	214
64	132	195	154	73
184	234	82	22	216
232	153	152	175	109
65	107	191	101	10

1	19	204	85	214
64	132	195	154	73
184	234	82	22	216
232	153	152	175	109
65	107	191	101	10

\therefore applying the moving average filter (5x5) to the neighborhood matrix:

$$= \frac{1}{25} [1 \times 1 + 19 \times 1 + 204 \times 1 + \dots + 191 \times 1 + 101 \times 1 + 10 \times 1]$$

$$= \frac{1}{25} \times 3174 = 126.96 \approx \boxed{127}$$

\therefore Intensity of pixel at location 4x4 is 127

1. 5x5 Gaussian filter

$$g(x,y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}} \rightarrow 0.$$

considering $\sigma=1$

$$\textcircled{1} \rightarrow g(x,y) = \frac{1}{\sqrt{\pi}} e^{-\left(\frac{x^2+y^2}{2}\right)}$$

$$\therefore \text{Gaussian filter} = \frac{1}{\substack{\text{sum of all} \\ \text{elements in} \\ \text{Gaussian} \\ \text{kernel}}} \times \begin{bmatrix} e^{-\frac{(-2)^2 + (-2)^2}{2}} & e^{-\frac{(-2)^2 + (0)^2}{2}} & & & \\ e^{-\frac{(-1)^2 + (-2)^2}{2}} & \cdot & \cdot & & \\ \vdots & \vdots & \ddots & & \\ & & & \ddots & \\ & & & & \end{bmatrix}$$

$$= \frac{1}{\substack{\text{sum of} \\ \text{all kernel} \\ \text{elements}}} \times \begin{bmatrix} e^{-4} & e^{-2.5} & e^{-2} & e^{-2.5} & e^{-4} \\ e^{-2.5} & e^{-1} & e^{-0.5} & e^{-1} & e^{-2.5} \\ e^{-2} & e^{-0.5} & e^{0.5} & e^{-0.5} & e^{-2} \\ e^{-2.5} & e^{-1} & e^{-0.5} & e^{-1} & e^{-2.5} \\ e^{-4} & e^{-2.5} & e^{-2} & e^{-2.5} & e^{-4} \end{bmatrix}$$

$$= \frac{1}{6.1689} \times \begin{bmatrix} 0.0183 & 0.0821 & 0.1353 & 0.0821 & 0.0183 \\ 0.0821 & 0.3678 & 0.6065 & 0.3678 & 0.0821 \\ 0.1353 & 0.6065 & 1 & 0.6065 & 0.0353 \\ 0.0821 & 0.3678 & 0.6065 & 0.3678 & 0.0821 \\ 0.0183 & 0.0821 & 0.1353 & 0.0821 & 0.0183 \end{bmatrix}$$

$$= \begin{bmatrix} 0.003 & 0.0133 & 0.0219 & 0.0133 & 0.003 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.003 & 0.0133 & 0.0219 & 0.0133 & 0.003 \end{bmatrix}$$

c). The neighborhood matrix at location (3,6) is :

217	62	210	57	118
204	85	214	113	182
195	154	73	228	214
82	22	216	40	48
152	175	109	203	155

$$\rightarrow \begin{bmatrix} 217 & 62 & 210 & 57 & 118 \\ 204 & 85 & 214 & 113 & 182 \\ 195 & 154 & 73 & 228 & 214 \\ 82 & 22 & 216 & 40 & 48 \\ 152 & 175 & 109 & 203 & 155 \end{bmatrix}$$

∴ applying the 5×5 Gaussian filter to the neighborhood matrix,
we get:

$$\begin{bmatrix} 217 & 62 & 210 & 57 & 118 \\ 204 & 85 & 214 & 113 & 182 \\ 195 & 154 & 73 & 228 & 214 \\ 82 & 22 & 216 & 40 & 48 \\ 152 & 175 & 109 & 203 & 155 \end{bmatrix} \xrightarrow{\text{element wise multiplication}} \begin{bmatrix} 0.003 & 0.0133 & 0.0219 & 0.0133 & 0.003 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.003 & 0.0133 & 0.0219 & 0.0133 & 0.003 \end{bmatrix}$$

$$= 217 \times 0.003 + 62 \times 0.0133 + 210 \times 0.0219 + \dots$$

$$+ \dots + 109 \times 0.0219 + 203 \times 0.0133 + 155 \times 0.003$$

$$= 138.89$$

$$\underline{139}$$

∴ Intensity of pixel at location (3,6) is 139.

d). The neighborhood matrix at location (2,2) is:

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 222 & 129 & 159 & 217 \\ 0 & 43 & \boxed{1} & 19 & 204 \\ 0 & 43 & 64 & 132 & 195 \\ 0 & 123 & 184 & 234 & 82 \end{matrix}$$

with zero padding the neighborhood matrix is

$$\rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 222 & 129 & 159 & 217 \\ 0 & 43 & 1 & 19 & 204 \\ 0 & 43 & 64 & 132 & 195 \\ 0 & 123 & 184 & 234 & 82 \end{bmatrix}$$

∴ applying 5x5 Gaussian filter to the neighborhood matrix,
we get:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 222 & 129 & 159 & 217 \\ 0 & 43 & 1 & 19 & 204 \\ 0 & 43 & 64 & 132 & 195 \\ 0 & 123 & 184 & 234 & 82 \end{bmatrix}$$

element
wise
multiplication

$$\begin{bmatrix} 0.003 & 0.0133 & 0.0219 & 0.0123 & 0.003 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.003 & 0.0133 & 0.0219 & 0.0123 & 0.003 \end{bmatrix}$$

$$= 0 \times 0.003 + 0 \times 0.0133 + 0 \times 0.0219 + \dots$$

$$+ \dots + 184 \times 0.0219 + 234 \times 0.0133 + 82 \times 0.003$$

$$= 82 \times 0.074$$

$$\approx 60.$$

∴ Intensity of pixel at location (2,2) is 82.

Problem 2)

a). Plot the image.



b). After applying the 5x5 moving average filter to the input image. The output image is as follows:



It can be seen that the output image is **smoothed** compared to the input image. The smoothing can be confirmed by seeing the Label on the bottle, the title of the book and the letters in the frame.

c). After applying the 5x5 Gaussian filter to the input image. The output image is as follows:



As compared to the input image the Gaussian filter image is slightly smoothed/blurred.

In comparison to the output image of the Moving average filter the Gaussian filter is slightly less smoothed/blurred. In moving average filter we can hardly read the letters "INTRODUCTION TO FOURIOER OPTICS" as observed in the Gaussian or the Input image.