

## assignment-5

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07/11/2021

```
setwd("C:/Users/pavankumar pendela/Desktop/MSBA/Quantitative management  
Dr.Wu/assignment 5")
```

### installing packages

```
library(Benchmarking)

## Loading required package: lpSolveAPI
## Loading required package: ucminf
## Loading required package: quadprog

library(tidyverse)

## -- Attaching packages ----- tidyverse
1.3.1 --

## v ggplot2 3.3.5      v purrr  0.3.4
## v tibble  3.1.5      v dplyr  1.0.7
## v tidyr   1.1.4      v stringr 1.4.0
## v readr   2.0.2      v forcats 0.5.1

## -- Conflicts -----
tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

### COMPUTE THE FORMULATION

```
# creating the vectors with our values
input <- matrix(c(150,400,320,520,350, 320, 200, 700, 1200, 2000,
1200, 700),ncol = 2)
output <-
matrix(c(14000,14000,42000,28000,19000,14000,3500,21000,10500,42000,25000,
15000),ncol = 2)
# Assigning column names
colnames(output) <- c("staff_daily_hours","supplies_daily")
colnames(input) <- c("daily_reimbursed_patient",
"daily_privately_paid_patient")
# values of input & output
input
```

```
##      daily_reimbursed_patient daily_privately_paid_patient
## [1,]                150                200
## [2,]                400                700
## [3,]                320               1200
## [4,]                520               2000
## [5,]                350               1200
## [6,]                320                700
```

output

```
##      staff_daily_hours supplies_daily
## [1,]            14000            3500
## [2,]            14000            21000
## [3,]            42000            10500
## [4,]            28000            42000
## [5,]            19000            25000
## [6,]            14000            15000
```

As you can see, we're obtaining the same results as the performance data table from Hope Valley Health Care Association's six nursing facilities.

Now, we use "DEA" tool that can help organizations to identify and allocate their resources to enhance their efficiency and have better practices.

## DEA analysis using FDH

```
analysis_fdh<- dea(input,output,RTS = "fdh")

eff_fdh <- as.data.frame(analysis_fdh$eff)

colnames(eff_fdh) <- c("efficiency_fdh")

peer_fdh <- peers(analysis_fdh)

colnames(peer_fdh) <- c("peer1_fdh")

lambda_fdh <- lambda(analysis_fdh)

colnames(lambda_fdh) <- c("L1_fdh", "L2_fdh", "L3_fdh", "L4_fdh", "L5_fdh",
"L6_fdh")

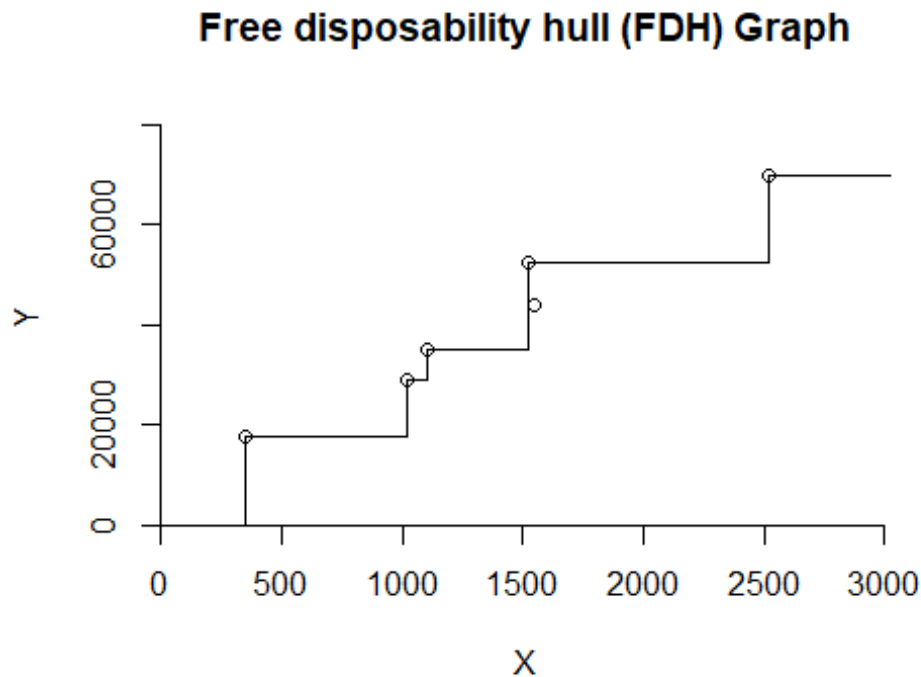
peer_lamb_eff_fdh <- cbind(peer_fdh, lambda_fdh, eff_fdh)

peer_lamb_eff_fdh

##      peer1_fdh L1_fdh L2_fdh L3_fdh L4_fdh L5_fdh L6_fdh efficiency_fdh
## 1           1     1     0     0     0     0     0           1
## 2           2     0     1     0     0     0     0           1
## 3           3     0     0     1     0     0     0           1
## 4           4     0     0     0     1     0     0           1
```

```
## 5      5      0      0      0      0      1      0      1
## 6      6      0      0      0      0      0      1      1

# Plot the results
dea.plot(input,output,RTS="fdh", main="Free disposability hull (FDH) Graph")
```



## DEA analysis using CRS

```
analysis_crs <- dea(input,output,RTS = "crs")

eff_crs <- as.data.frame(analysis_crs$eff)

colnames(eff_crs) <- c("efficiency_crs")

peer_crs <- peers(analysis_crs)

colnames(peer_crs) <- c("peer1_crs", "peer2_crs", "peer3_crs")

lambda_crs <- lambda(analysis_crs)

colnames(lambda_crs) <- c("L1_crs", "L2_crs", "L3_crs", "L4_crs")

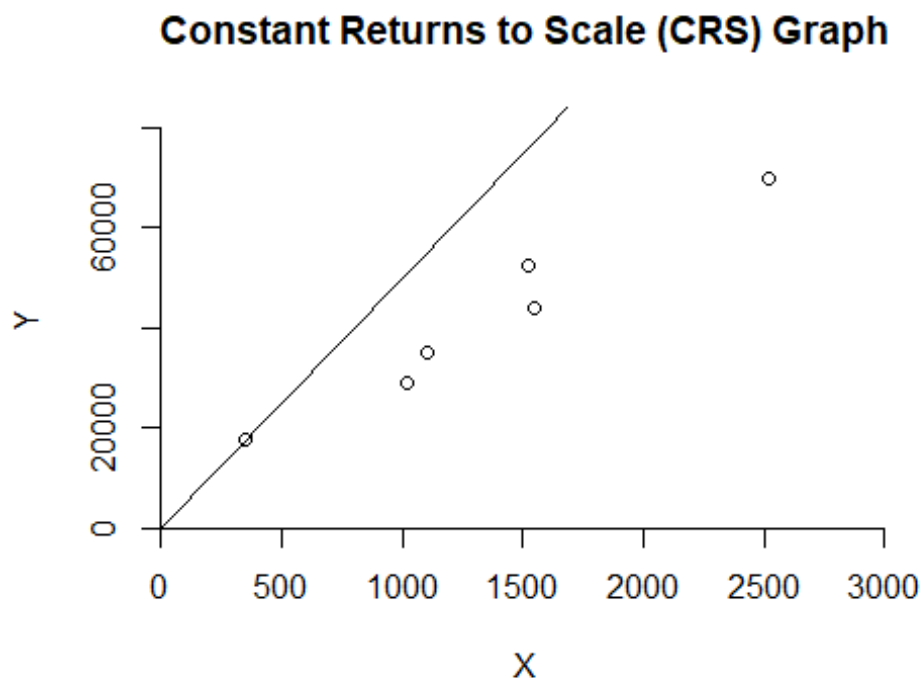
peer_lamb_eff_crs <- cbind(peer_crs, lambda_crs, eff_crs)

peer_lamb_eff_crs
```

```
## peer1_crs peer2_crs peer3_crs L1_crs L2_crs L3_crs L4_crs
## 1 1 NA NA 1.0000000 0.0000000 0 0.0000000
## 2 2 NA NA 0.0000000 1.0000000 0 0.0000000
## 3 3 NA NA 0.0000000 0.0000000 1 0.0000000
## 4 4 NA NA 0.0000000 0.0000000 0 1.0000000
## 5 1 2 4 0.2000000 0.08048142 0 0.5383307
## 6 1 2 4 0.3428571 0.39499264 0 0.1310751
## efficiency_crs
## 1 1.0000000
## 2 1.0000000
## 3 1.0000000
## 4 1.0000000
## 5 0.9774987
## 6 0.8674521
```

*# Plot the results*

```
dea.plot(input,output,RTS="crs", main="Constant Returns to Scale (CRS)
Graph")
```



## DEA Analysis using VRS

```
analysis_vrs <- dea(input,output,RTS = "vrs")
eff_vrs <- as.data.frame(analysis_vrs$eff)
colnames(eff_vrs) <- c("efficiency_vrs")
```

```

peer_vrs <- peers(analysis_vrs)

colnames(peer_vrs) <- c("peer1_vrs", "peer2_vrs", "peer3_vrs")

lambda_vrs <- lambda(analysis_vrs)

colnames(lambda_vrs) <- c("L1_vrs", "L2_vrs", "L3_vrs", "L4_vrs", "L5_vrs")

peer_lamb_eff_vrs <- cbind(peer_vrs, lambda_vrs, eff_vrs)

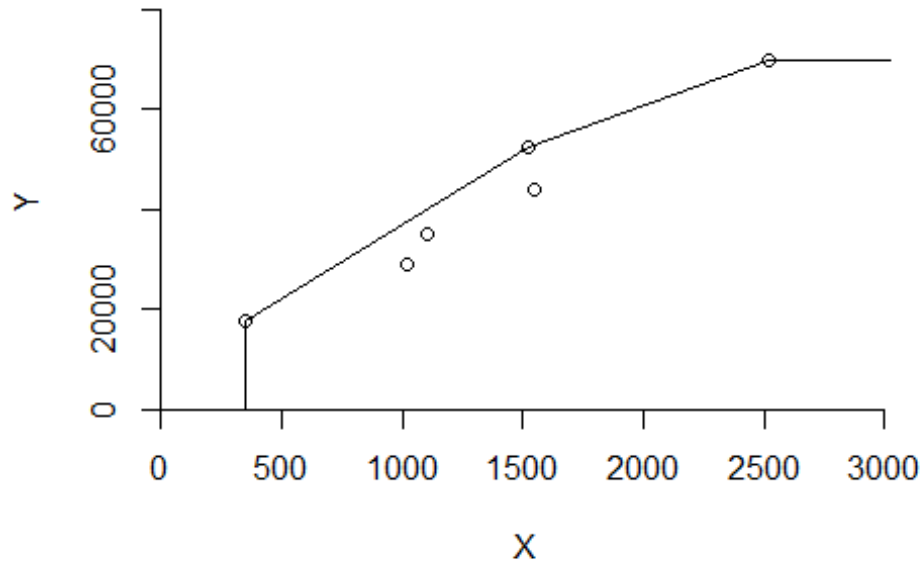
peer_lamb_eff_vrs

##   peer1_vrs peer2_vrs peer3_vrs    L1_vrs    L2_vrs L3_vrs L4_vrs
L5_vrs
## 1          1         NA         NA 1.0000000 0.0000000      0      0
0.0000000
## 2          2         NA         NA 0.0000000 1.0000000      0      0
0.0000000
## 3          3         NA         NA 0.0000000 0.0000000      1      0
0.0000000
## 4          4         NA         NA 0.0000000 0.0000000      0      1
0.0000000
## 5          5         NA         NA 0.0000000 0.0000000      0      0
1.0000000
## 6          1          2          5 0.4014399 0.3422606      0      0
0.2562995
##   efficiency_vrs
## 1          1.0000000
## 2          1.0000000
## 3          1.0000000
## 4          1.0000000
## 5          1.0000000
## 6          0.8963283

# Plot the results
dea.plot(input,output,RTS="vrs", main="Variable Returns to Scale (VRS)
Graph")

```

## Variable Returns to Scale (VRS) Graph



# DEA Analysis

using IRS

```
analysis_irs <- dea(input,output,RTS = "irs")
eff_irs <- as.data.frame(analysis_irs$eff)
colnames(eff_irs) <- c("efficiency_irs")
peer_irs <- peers(analysis_irs)
colnames(peer_irs) <- c("peer1_irs", "peer2_irs", "peer3_irs")
lambda_irs <- lambda(analysis_irs)
colnames(lambda_irs) <- c("L1_irs", "L2_irs", "L3_irs", "L4_irs", "L5_irs")
peer_lamb_eff_irs <- cbind(peer_irs, lambda_irs, eff_irs)
```

peer\_lamb\_eff\_irs

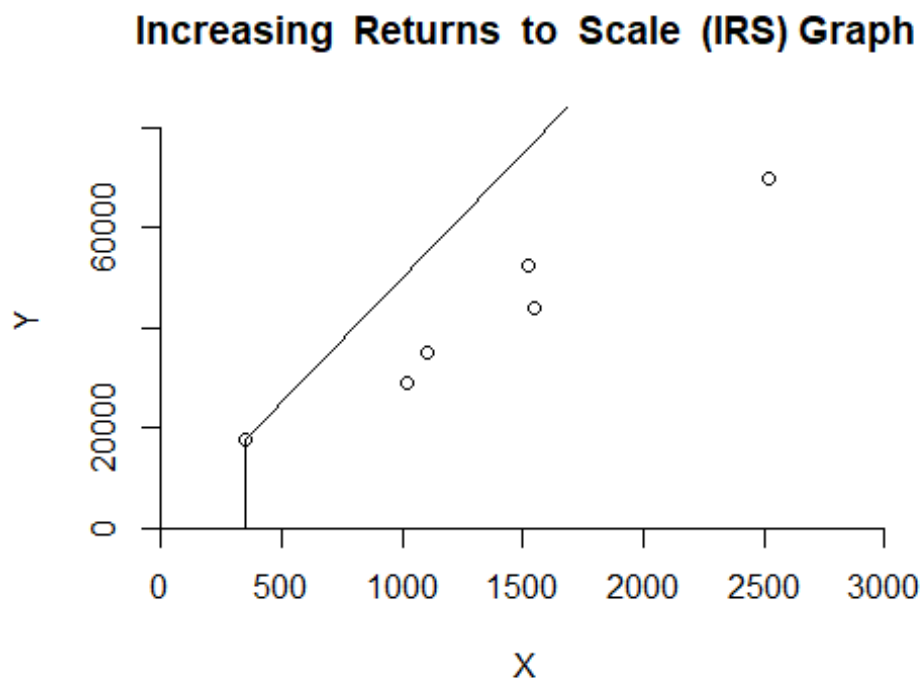
	peer1_irs	peer2_irs	peer3_irs	L1_irs	L2_irs	L3_irs	L4_irs	L5_irs
## 1	1	NA	NA	1.0000000	0.0000000	0	0	0.0000000
## 2	2	NA	NA	0.0000000	1.0000000	0	0	0.0000000
## 3	3	NA	NA	0.0000000	0.0000000	1	0	

```

0.0000000
## 4      4      NA      NA 0.0000000 0.0000000      0      1
0.0000000
## 5      5      NA      NA 0.0000000 0.0000000      0      0
1.0000000
## 6      1      2      5 0.4014399 0.3422606      0      0
0.2562995
## efficiency_irs
## 1      1.0000000
## 2      1.0000000
## 3      1.0000000
## 4      1.0000000
## 5      1.0000000
## 6      0.8963283

# Plot the results
dea.plot(input,output,RTS="irs", main="Increasing Returns to Scale (IRS)
Graph")

```



# DEA Analysis

using DRS

```

analysis_drs <- dea(input,output,RTS = "drs")
eff_drs <- as.data.frame(analysis_drs$eff)
colnames(eff_drs) <- c("efficiency_drs")

```

```

peer_drs <- peers(analysis_drs)

colnames(peer_drs) <- c("peer1_drs", "peer2_drs", "peer3_drs")

lambda_drs <- lambda(analysis_drs)

colnames(lambda_drs) <- c("L1_drs", "L2_drs", "L3_drs", "L4_drs")

peer_lamb_eff_drs <- cbind(peer_drs, lambda_drs, eff_drs)

peer_lamb_eff_drs

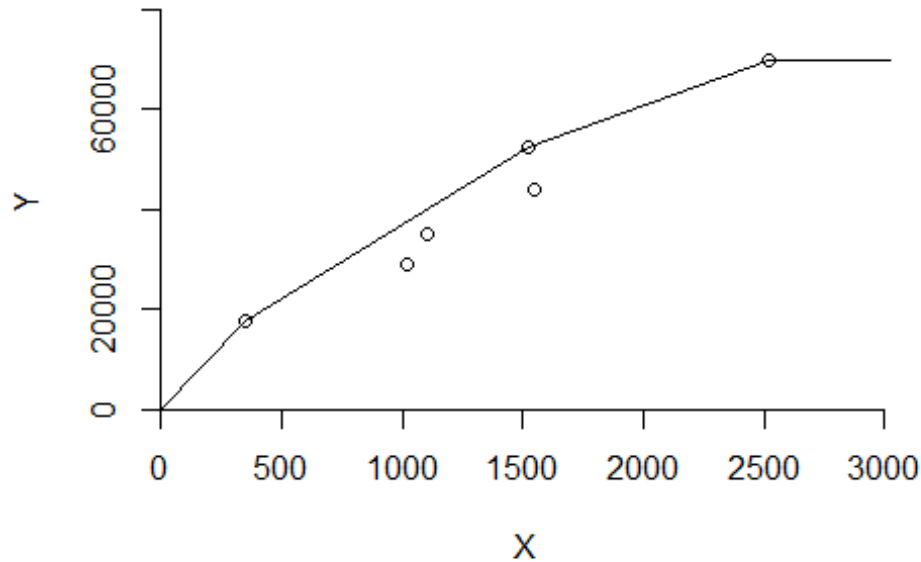
##   peer1_drs peer2_drs peer3_drs    L1_drs    L2_drs L3_drs    L4_drs
## 1         1         NA         NA 1.0000000 0.0000000    0 0.0000000
## 2         2         NA         NA 0.0000000 1.0000000    0 0.0000000
## 3         3         NA         NA 0.0000000 0.0000000    1 0.0000000
## 4         4         NA         NA 0.0000000 0.0000000    0 1.0000000
## 5         1         2         4 0.2000000 0.08048142    0 0.5383307
## 6         1         2         4 0.3428571 0.39499264    0 0.1310751
##   efficiency_drs
## 1      1.0000000
## 2      1.0000000
## 3      1.0000000
## 4      1.0000000
## 5      0.9774987
## 6      0.8674521

# Plot the results
dea.plot(input,output,RTS="drs", main="Decreasing Returns to Scale (DRS)
Graph")

```



## Decreasing Returns to Scale (DRS) Graph



# DEA Analysis

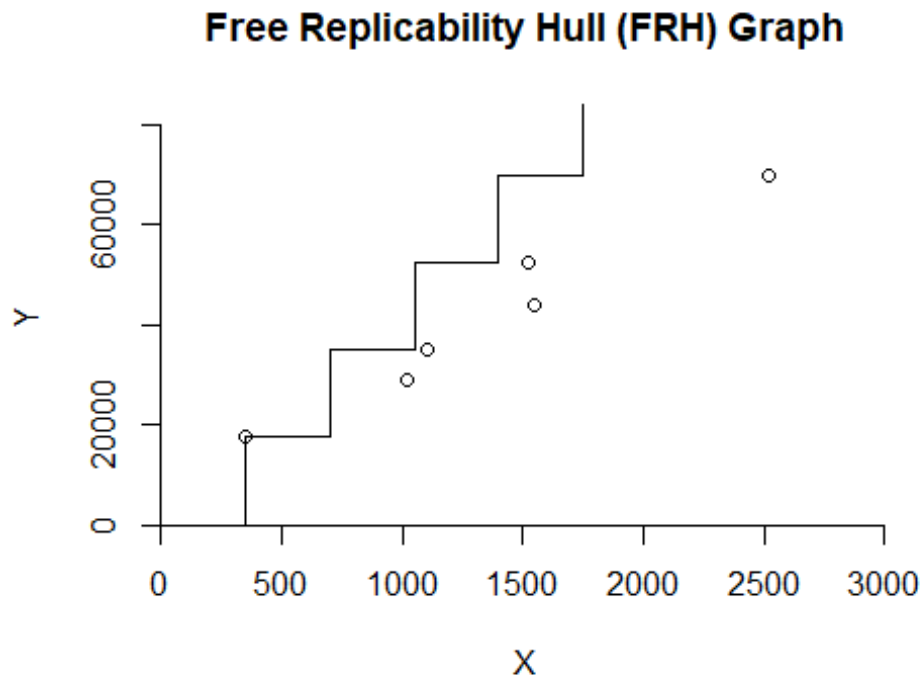
using FRH

```
analysis_frh <- dea(input,output,RTS = "add")
eff_frh <- as.data.frame(analysis_frh$eff)
colnames(eff_frh) <- c("efficiency_frh")
peer_frh <- peers(analysis_frh)
colnames(peer_frh) <- c("peer1_frh")
lambda_frh <- lambda(analysis_frh)
colnames(lambda_frh) <- c("L1_frh", "L2_frh", "L3_frh", "L4_frh", "L5_frh",
"L6_frh")
peer_lamb_eff_frh <- cbind(peer_frh, lambda_frh, eff_frh)
peer_lamb_eff_frh
```

##	peer1_frh	L1_frh	L2_frh	L3_frh	L4_frh	L5_frh	L6_frh	efficiency_frh
## 1	1	1	0	0	0	0	0	1
## 2	2	0	1	0	0	0	0	1
## 3	3	0	0	1	0	0	0	1
## 4	4	0	0	0	1	0	0	1

```
## 5      5      0      0      0      0      1      0      1
## 6      6      0      0      0      0      0      1      1

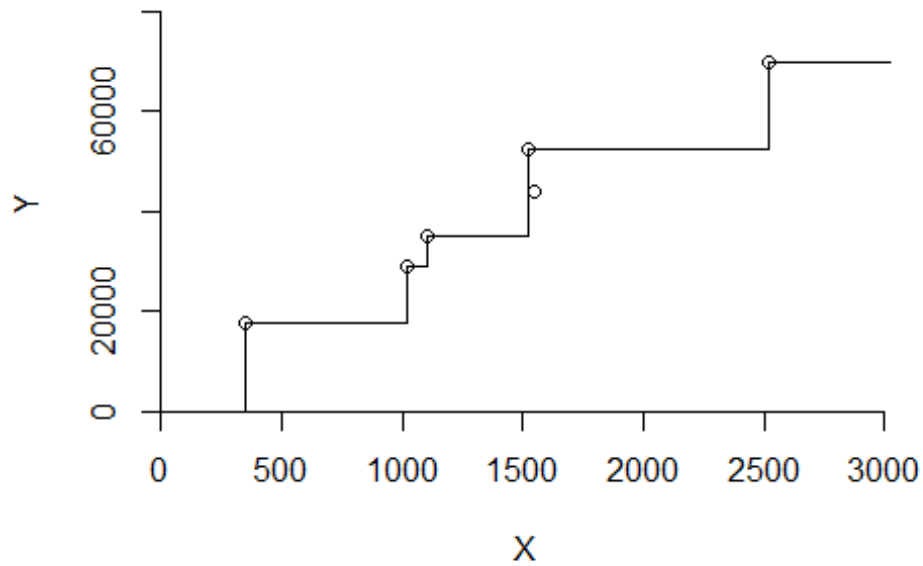
# Plot the results
dea.plot(input,output,RTS="add", main="Free Replicability Hull (FRH) Graph")
```



# Compare  
between different assumptions

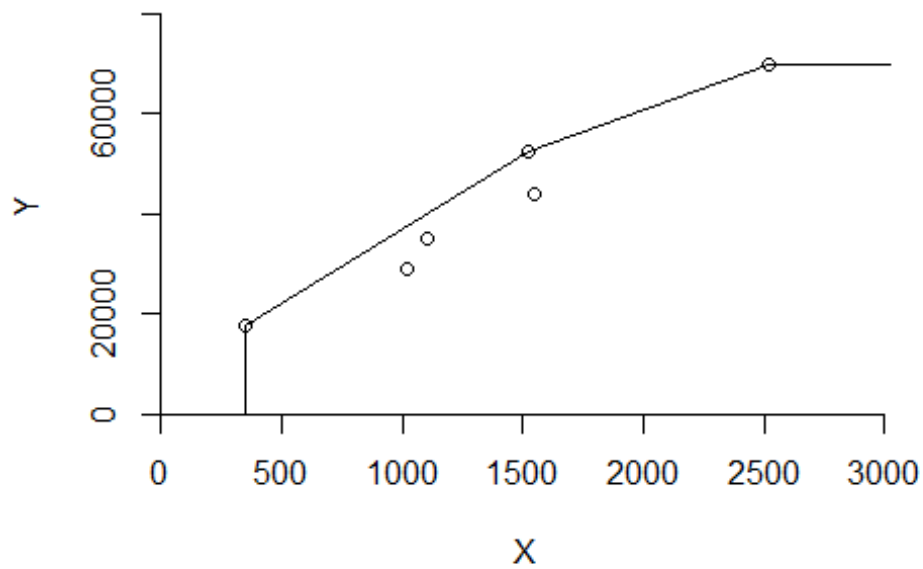
```
dea.plot(input,output,RTS="fdh", main="Free disposability hull (FDH) Graph")
```

**Free disposability hull (FDH) Graph**

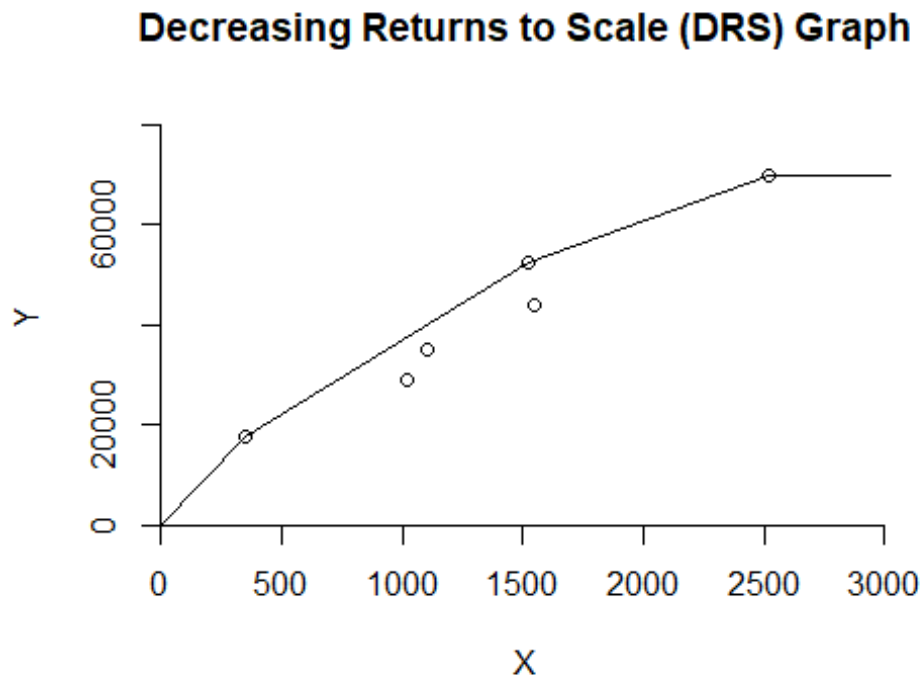


```
dea.plot(input,output,RTS="vrs", main="Variable Returns to Scale (VRS)  
Graph")
```

**Variable Returns to Scale (VRS) Graph**

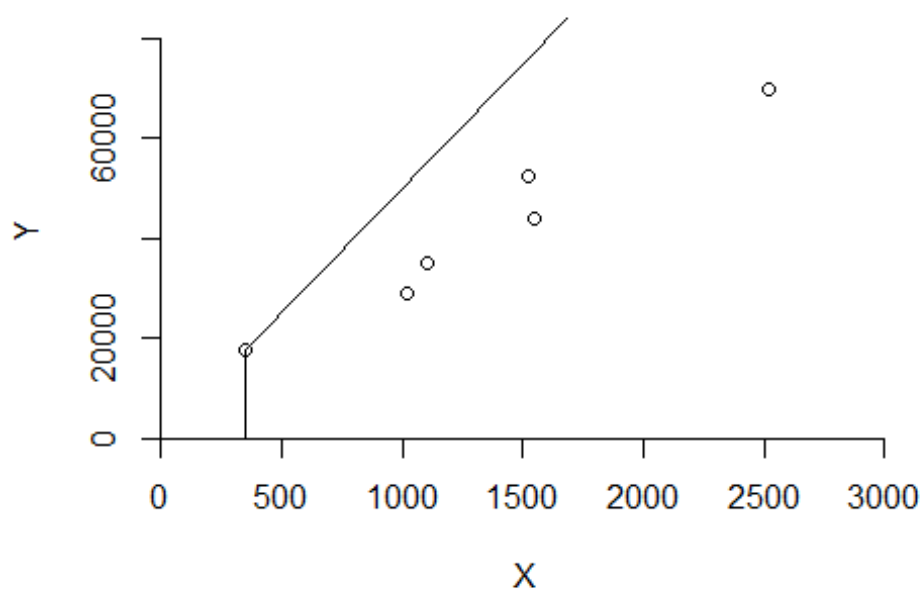


```
dea.plot(input,output,RTS="drs", main="Decreasing Returns to Scale (DRS) Graph")
```



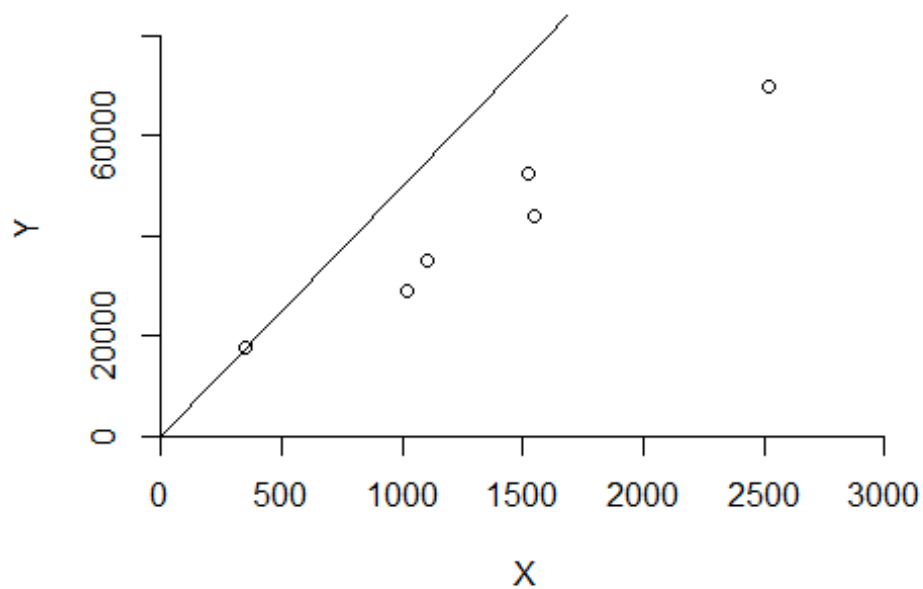
```
dea.plot(input,output,RTS="irs", main="Increasing Returns to Scale (IRS) Graph")
```

### Increasing Returns to Scale (IRS) Graph

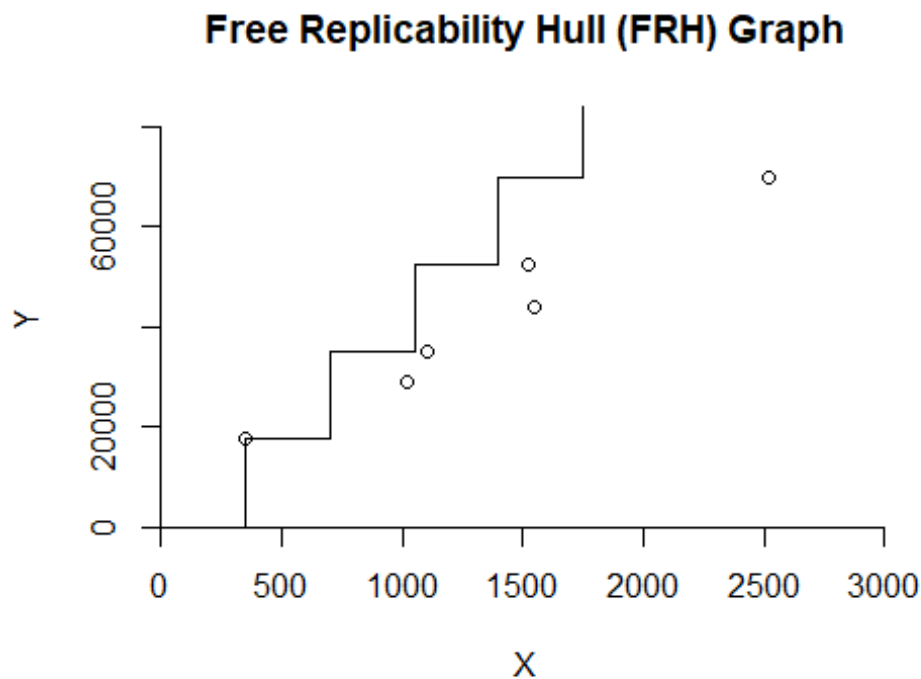


```
dea.plot(input,output,RTS="crs", main="Constant Returns to Scale (CRS) Graph")
```

### Constant Returns to Scale (CRS) Graph



```
dea.plot(input,output,RTS="add", main="Free Replicability Hull (FRH) Graph")
```



These charts allow us to compare the results of each DEA model.

“All DEA models share the premise of estimating the technology using a minimum extrapolation technique,” according to what we studied in this session (DEA Slides).

FDH is the smallest technology set, as can be shown. It seeks to create fewer outputs (number of patient days reimbursed by third-party sources and number of patient days reimbursed privately) with more inputs (number of patient days reimbursed by third-party sources and number of patient days reimbursed privately) (staffing labor and the cost of supplies). FDH is the most popular model among businesses, yet it has several flaws owing to its assumptions. As we can see, all of the efficiencies in this model are 1, however it is not as efficient as we thought when compared to other models since we identify areas/units to improve.

VRS is larger than FDH because it “fills-out” the spaces that FDH reduced. Here we can see that unit 6 can improve its efficiency.

As the graphs show, DRS and IRS are bigger than VRS. For smaller input values, DRS seeks to enlarge the set, whereas the IRS tries to raise the technology. Units 5 and 6 might increase their efficiency, according to DRS, and facility 6 could improve as well, according to IRS.

CRS is the largest technology set, allowing us to assess whether there are any conceivable scaling up or down combinations. Units 5 and 6 require improvement based on the efficiency numbers.

The purpose of FRH, which is larger than FDH but less than CRS, is to replace deterministic data with random variables.

Q2. The Research and Development Division of the Emax Corporation has developed three new products. A decision now needs to be made on which mix of these products should be produced. Management wants primary consideration given to three factors: total profit, stability in the workforce, and achieving an increase in the company's earnings

next year from the \$75 million achieved this year. In particular, using the units given in the following table, they want to

Maximize  $Z = P - 6C - 3D$ , where

$P$  = total (discounted) profit over the life of the new products,

$C$  = change (in either direction) in the current level of employment,

$D$  = decrease (if any) in next year's earnings from the current year's level.

The amount of any increase in earnings does not enter into  $Z$ , because management is concerned primarily with just achieving some increase to keep the stockholders happy. (It has mixed feelings about a large increase that then would be difficult to surpass in subsequent years.)

The impact of each of the new products (per unit rate of production) on each of these factors is shown in the following table:

Factor	Unit Contribution			Goal	Units
	Product:				
	1	2	3		
Total profit	20	15	25	Maximize	Millions of dollars
Employment level	6	4	5	= 50	Hundreds of employees
Earnings next year	8	7	5	≥ 75	Millions of dollars

Objective function:

max:  $20X_1 + 15X_2 + 25X_3 - 6Y_{1P} - 6Y_{1M} - 3Y_{2M}$

S.T : Employment Level

$6x_1 + 4x_2 + 5x_3 - (Y_{1P} - Y_{1M}) = 50$



Earnings Next Year

$$8x_1 + 7x_2 + 5x_3 - (Y_2P - Y_2M) = 75$$

Nonnegativity constraint

$$X_1, X_2, X_3 \geq 0$$

$$Y_1P, Y_1M, Y_2P, Y_2M \geq 0$$

```
library(lpSolveAPI)
setwd("C:/Users/pavankumar pendela/Desktop/MSBA/Quantitative management
Dr.Wu/assignment 5")
# Load the data
emax <- read.lp("emax.lp")
emax

## Model name:
##      X1      X2      X3      Y1P      Y1M      Y2M      Y2P
## Maximize    20     15     25     -6     -6     -3      0
## R1          6      4      5     -1      1      0      0 = 50
## R2          8      7      5      0      0      1     -1 = 75
## Kind        Std      Std      Std      Std      Std      Std      Std
## Type        Real     Real     Real     Real     Real     Real     Real
## Upper       Inf      Inf      Inf      Inf      Inf      Inf      Inf
## Lower        0        0        0        0        0        0        0

solve(emax)

## [1] 0
```

As we can see, the solver is returning 0, indicating that it is finding a solution.

```
get.objective(emax)

## [1] 225
```

We are maximizing profit while reducing other company goals such as manpower and profits in this scenario. The penalty for failing to meet the goals on the objective function is 225.

```
get.variables(emax)

## [1] 0 0 15 25 0 0 0
```

This order is from how the variables were written in the objective function. In our case, the results are as follows:  $X_1 = 0, X_2 = 0, X_3 = 15, Y_1P = 25, Y_1M = 0, Y_2M = 0, Y_2P = 0$  which means that the earnings ( $y_2$ ) expectations are fully satisfied. Regarding the workforce, the goal projected is exceeded by 25 and based on the total profit of product 3, it has a negative result on its profit by 15.