# assignment-5

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```
setwd("C:/Users/pavankumar pendela/Desktop/MSBA/Quantiative management
Dr.Wu/assignment 5")
```

## installing packages

```
library(Benchmarking)

## Loading required package: lpSolveAPI

## Loading required package: ucminf

## Loading required package: quadprog

library(tidyverse)

## -- Attaching packages --------------------------------------- tidyverse
1.3.1 --

## v ggplot2 3.3.5     v purrr   0.3.4
## v tibble  3.1.5     v dplyr   1.0.7
## v tidyr   1.1.4     v stringr 1.4.0
## v readr   2.0.2     v forcats 0.5.1

## -- Conflicts ------------------------------------------
tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

COMPUTE THE FORMULATION

```
# creating the vectors with our values
input  <-  matrix(c(150,400,320,520,350,  320,  200,  700,  1200,  2000,
1200,  700),ncol  =  2)
output <-
matrix(c(14000,14000,42000,28000,19000,14000,3500,21000,10500,42000,25000,
15000),ncol = 2)
# Assigning column names
colnames(output) <- c("staff_daily_hours","supplies_daily")
colnames(input) <- c("daily_reimbursed_patient",
"daily_privately_paid_patient")
# values of input & output
input
```

```
##      daily_reimbursed_patient daily_privately_paid_patient
## [1,]                      150                          200
## [2,]                      400                          700
## [3,]                      320                         1200
## [4,]                      520                         2000
## [5,]                      350                         1200
## [6,]                      320                          700
```

```
output
```

```
##      staff_daily_hours supplies_daily
## [1,]             14000           3500
## [2,]             14000          21000
## [3,]             42000          10500
## [4,]             28000          42000
## [5,]             19000          25000
## [6,]             14000          15000
```

As you can see, we're obtaining the same results as the performance data table from Hope Valley Health Care Association's six nursing facilities.

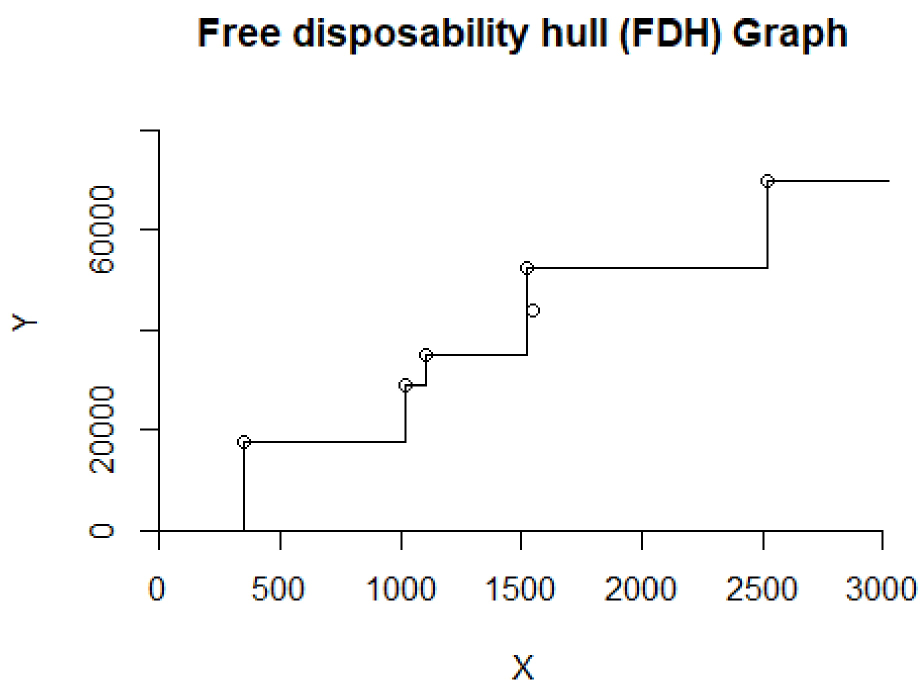
Now, we use "DEA" tool that can help organizations to identify and allocate their resources to enhance their efficiency and have better practices.

## DEA analysis using FDH

```
analysis_fdh<-  dea(input,output,RTS  =  "fdh")

eff_fdh <- as.data.frame(analysis_fdh$eff)

colnames(eff_fdh)  <-  c("efficiency_fdh")

peer_fdh  <-  peers(analysis_fdh)

colnames(peer_fdh) <- c("peer1_fdh")

lambda_fdh <- lambda(analysis_fdh)

colnames(lambda_fdh) <- c("L1_fdh", "L2_fdh", "L3_fdh", "L4_fdh", "L5_fdh",
"L6_fdh")

peer_lamb_eff_fdh  <-  cbind(peer_fdh,  lambda_fdh,  eff_fdh)

peer_lamb_eff_fdh
```

```
##   peer1_fdh L1_fdh L2_fdh L3_fdh L4_fdh L5_fdh L6_fdh efficiency_fdh
## 1         1      1      0      0      0      0      0              1
## 2         2      0      1      0      0      0      0              1
## 3         3      0      0      1      0      0      0              1
## 4         4      0      0      0      1      0      0              1
```

```
## 5          5     0     0     0     0     1     0           1
## 6          6     0     0     0     0     0     1           1

# Plot the results
dea.plot(input,output,RTS="fdh", main="Free disposability hull (FDH) Graph")
```

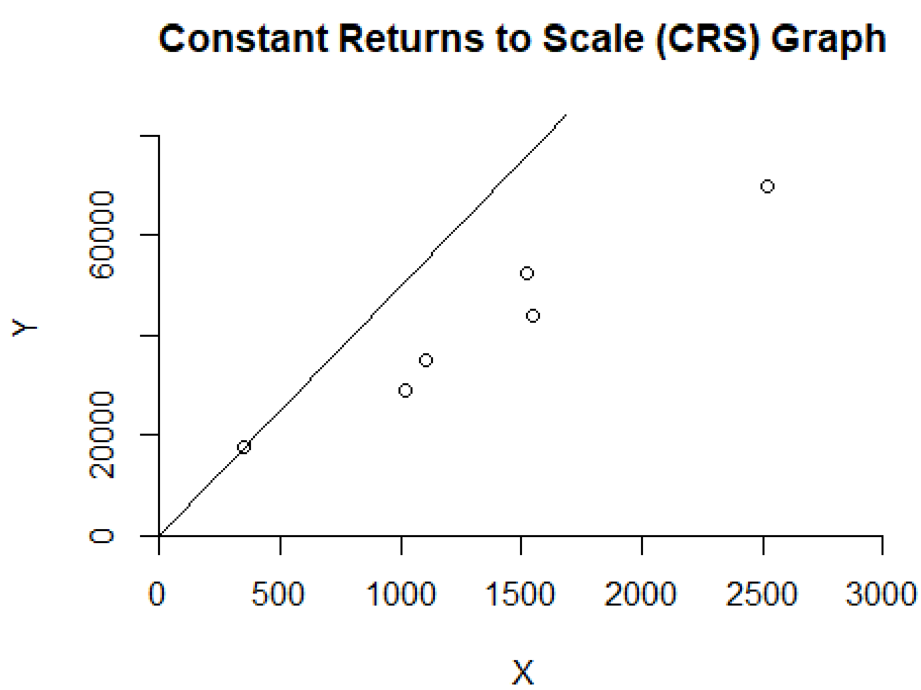



## DEA analysis using CRS

```
analysis_crs <- dea(input,output,RTS = "crs")

eff_crs <- as.data.frame(analysis_crs$eff)

colnames(eff_crs) <- c("efficiency_crs")

peer_crs <- peers(analysis_crs)

colnames(peer_crs) <- c("peer1_crs", "peer2_crs", "peer3_crs")

lambda_crs <- lambda(analysis_crs)

colnames(lambda_crs) <- c("L1_crs", "L2_crs", "L3_crs", "L4_crs")

peer_lamb_eff_crs <- cbind(peer_crs, lambda_crs, eff_crs)

peer_lamb_eff_crs
```

```
##   peer1_crs peer2_crs peer3_crs    L1_crs     L2_crs L3_crs    L4_crs
## 1         1        NA        NA 1.0000000 0.00000000      0 0.0000000
## 2         2        NA        NA 0.0000000 1.00000000      0 0.0000000
## 3         3        NA        NA 0.0000000 0.00000000      1 0.0000000
## 4         4        NA        NA 0.0000000 0.00000000      0 1.0000000
## 5         1         2         4 0.2000000 0.08048142      0 0.5383307
## 6         1         2         4 0.3428571 0.39499264      0 0.1310751
##   efficiency_crs
## 1      1.0000000
## 2      1.0000000
## 3      1.0000000
## 4      1.0000000
## 5      0.9774987
## 6      0.8674521
```

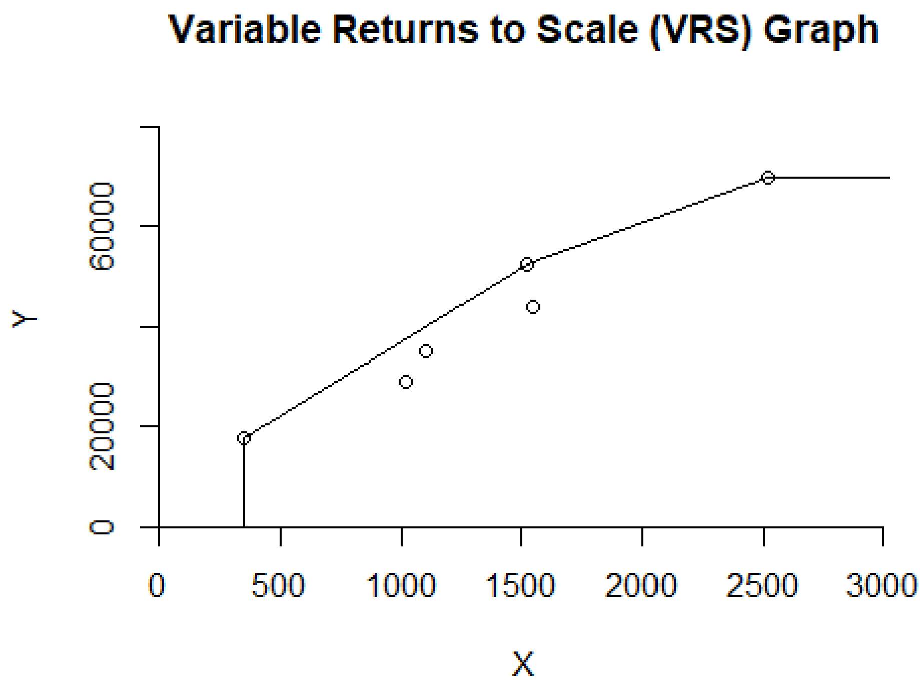
```
# Plot the results
dea.plot(input,output,RTS="crs", main="Constant Returns to Scale (CRS)
Graph")
```




## DEA Analysis using VRS

```
analysis_vrs  <-  dea(input,output,RTS  =  "vrs")

eff_vrs <- as.data.frame(analysis_vrs$eff)

colnames(eff_vrs)  <-  c("efficiency_vrs")
```

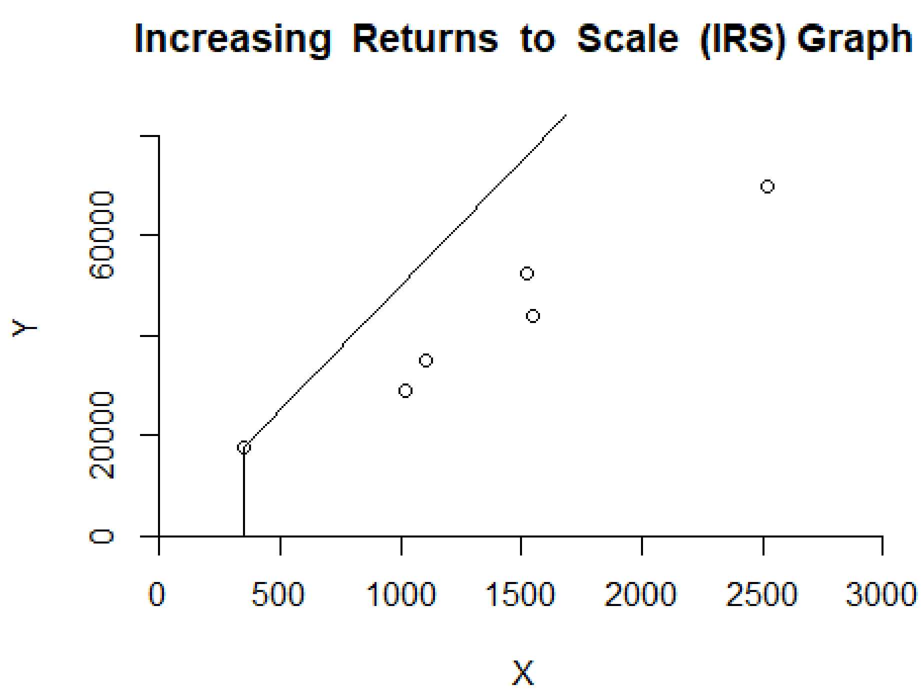
```
peer_vrs <- peers(analysis_vrs)

colnames(peer_vrs) <- c("peer1_vrs", "peer2_vrs", "peer3_vrs")

lambda_vrs <- lambda(analysis_vrs)

colnames(lambda_vrs) <- c("L1_vrs", "L2_vrs", "L3_vrs", "L4_vrs", "L5_vrs")

peer_lamb_eff_vrs <- cbind(peer_vrs, lambda_vrs, eff_vrs)

peer_lamb_eff_vrs

##   peer1_vrs peer2_vrs peer3_vrs    L1_vrs    L2_vrs L3_vrs L4_vrs
L5_vrs
## 1         1        NA        NA 1.0000000 0.0000000      0      0
0.0000000
## 2         2        NA        NA 0.0000000 1.0000000      0      0
0.0000000
## 3         3        NA        NA 0.0000000 0.0000000      1      0
0.0000000
## 4         4        NA        NA 0.0000000 0.0000000      0      1
0.0000000
## 5         5        NA        NA 0.0000000 0.0000000      0      0
1.0000000
## 6         1         2         5 0.4014399 0.3422606      0      0
0.2562995
##   efficiency_vrs
## 1      1.0000000
## 2      1.0000000
## 3      1.0000000
## 4      1.0000000
## 5      1.0000000
## 6      0.8963283

# Plot the results
dea.plot(input,output,RTS="vrs", main="Variable Returns to Scale (VRS)
Graph")
```




# DEA Analysis using IRS

```
analysis_irs  <-  dea(input,output,RTS  =  "irs")

eff_irs <- as.data.frame(analysis_irs$eff)

colnames(eff_irs)  <-  c("efficiency_irs")

peer_irs <- peers(analysis_irs)

colnames(peer_irs)  <-  c("peer1_irs",  "peer2_irs",  "peer3_irs")

lambda_irs  <-  lambda(analysis_irs)

colnames(lambda_irs) <- c("L1_irs", "L2_irs", "L3_irs", "L4_irs", "L5_irs")

peer_lamb_eff_irs <- cbind(peer_irs, lambda_irs, eff_irs)

peer_lamb_eff_irs

##   peer1_irs peer2_irs peer3_irs    L1_irs    L2_irs L3_irs L4_irs
L5_irs
## 1         1        NA        NA 1.0000000 0.0000000      0      0
0.0000000
## 2         2        NA        NA 0.0000000 1.0000000      0      0
0.0000000
## 3         3        NA        NA 0.0000000 0.0000000      1      0
```

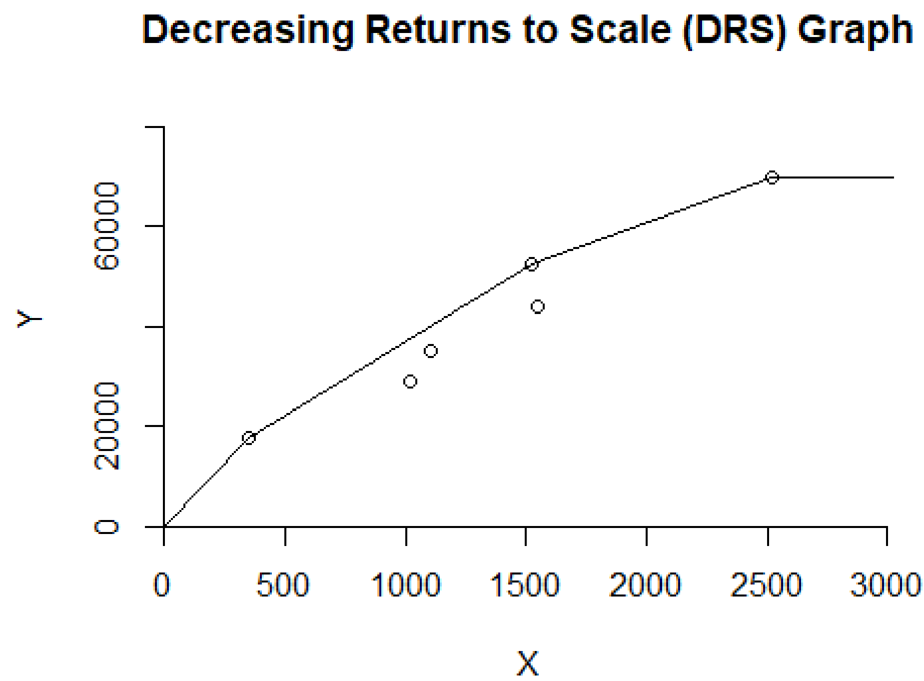
```
0.0000000
## 4         4         NA         NA 0.0000000 0.0000000      0      1
0.0000000
## 5         5         NA         NA 0.0000000 0.0000000      0      0
1.0000000
## 6         1          2          5 0.4014399 0.3422606      0      0
0.2562995
##   efficiency_irs
## 1      1.0000000
## 2      1.0000000
## 3      1.0000000
## 4      1.0000000
## 5      1.0000000
## 6      0.8963283

# Plot the results
dea.plot(input,output,RTS="irs",  main="Increasing  Returns  to  Scale  (IRS)
Graph")
```




# DEA Analysis using DRS

```
analysis_drs  <-  dea(input,output,RTS  =  "drs")

eff_drs <- as.data.frame(analysis_drs$eff)

colnames(eff_drs)  <-  c("efficiency_drs")
```

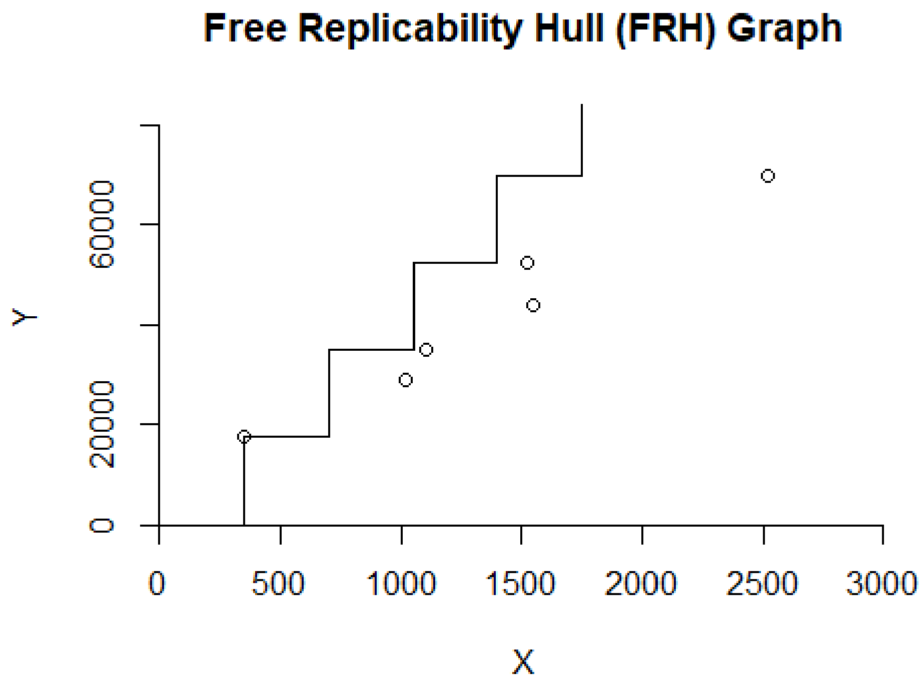
```r
peer_drs <- peers(analysis_drs)

colnames(peer_drs) <- c("peer1_drs", "peer2_drs", "peer3_drs")

lambda_drs <- lambda(analysis_drs)

colnames(lambda_drs) <- c("L1_drs", "L2_drs", "L3_drs", "L4_drs")

peer_lamb_eff_drs <- cbind(peer_drs, lambda_drs, eff_drs)

peer_lamb_eff_drs

##   peer1_drs peer2_drs peer3_drs    L1_drs     L2_drs L3_drs    L4_drs
## 1         1        NA        NA 1.0000000 0.00000000      0 0.0000000
## 2         2        NA        NA 0.0000000 1.00000000      0 0.0000000
## 3         3        NA        NA 0.0000000 0.00000000      1 0.0000000
## 4         4        NA        NA 0.0000000 0.00000000      0 1.0000000
## 5         1         2         4 0.2000000 0.08048142      0 0.5383307
## 6         1         2         4 0.3428571 0.39499264      0 0.1310751
##   efficiency_drs
## 1      1.0000000
## 2      1.0000000
## 3      1.0000000
## 4      1.0000000
## 5      0.9774987
## 6      0.8674521

# Plot the results
dea.plot(input,output,RTS="drs", main="Decreasing Returns to Scale (DRS)
Graph")
```




# DEA Analysis using FRH

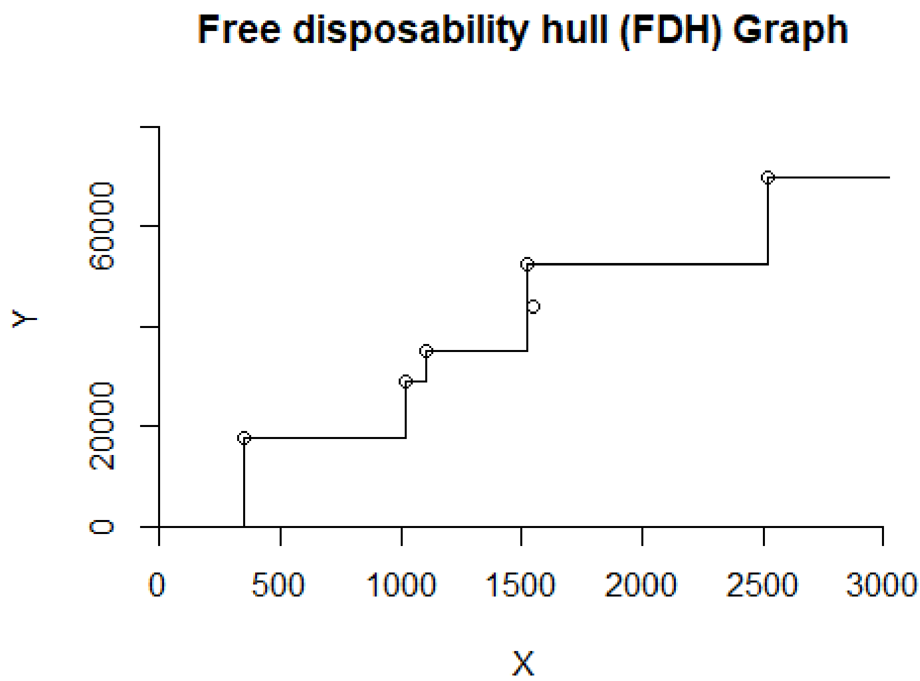
```
analysis_frh  <-  dea(input,output,RTS  =  "add")

eff_frh <- as.data.frame(analysis_frh$eff)

colnames(eff_frh) <- c("efficiency_frh")

peer_frh <- peers(analysis_frh)

colnames(peer_frh) <- c("peer1_frh")

lambda_frh <- lambda(analysis_frh)

colnames(lambda_frh) <- c("L1_frh", "L2_frh", "L3_frh", "L4_frh", "L5_frh",
"L6_frh")

peer_lamb_eff_frh <- cbind(peer_frh, lambda_frh, eff_frh)

peer_lamb_eff_frh

##   peer1_frh L1_frh L2_frh L3_frh L4_frh L5_frh L6_frh efficiency_frh
## 1         1      1      0      0      0      0      0              1
## 2         2      0      1      0      0      0      0              1
## 3         3      0      0      1      0      0      0              1
## 4         4      0      0      0      1      0      0              1
```

```
## 5          5      0      0      0      0      1      0              1
## 6          6      0      0      0      0      0      1              1

# Plot the results
dea.plot(input,output,RTS="add", main="Free Replicability Hull (FRH) Graph")
```

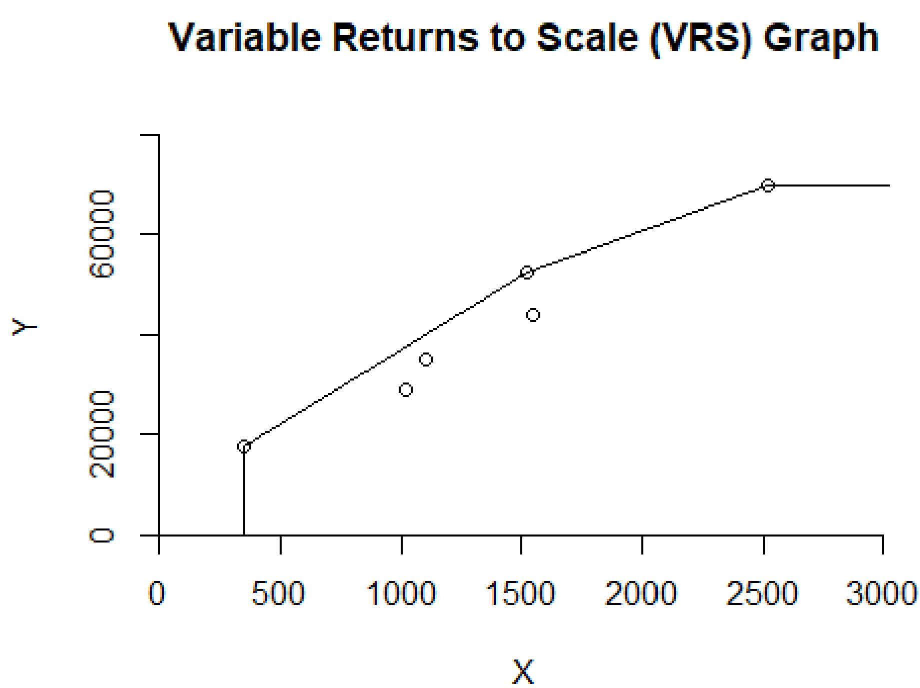



# Compare between different assumptions

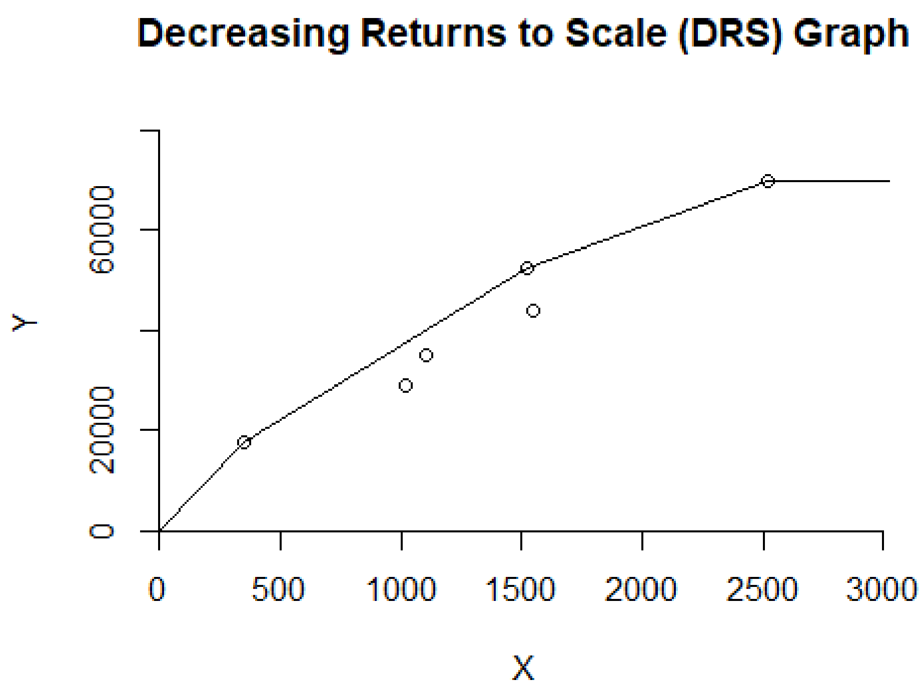
```
dea.plot(input,output,RTS="fdh", main="Free disposability hull (FDH) Graph")
```



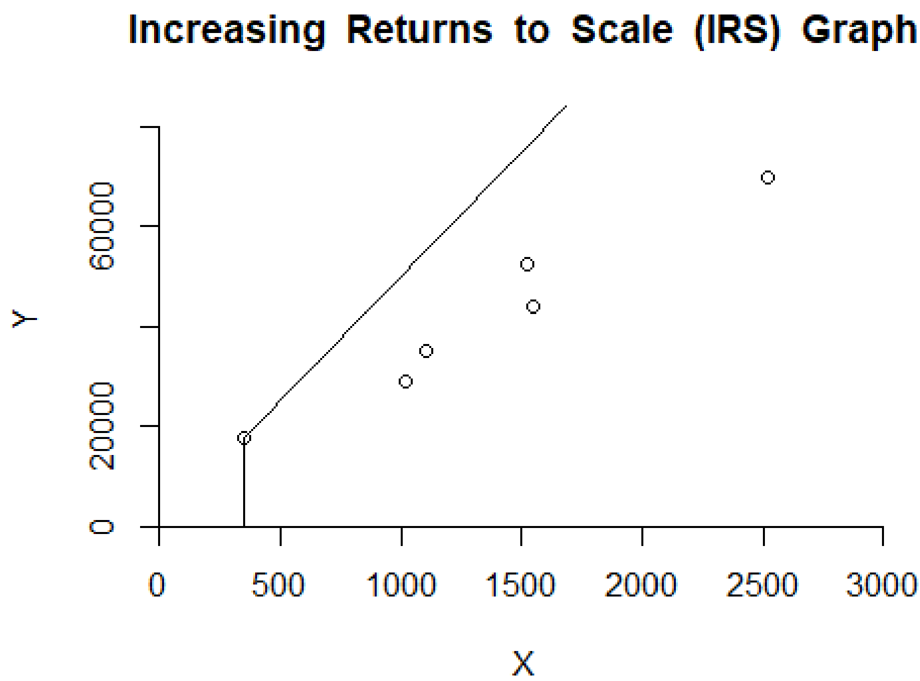

```
dea.plot(input,output,RTS="vrs", main="Variable Returns to Scale (VRS)
Graph")
```



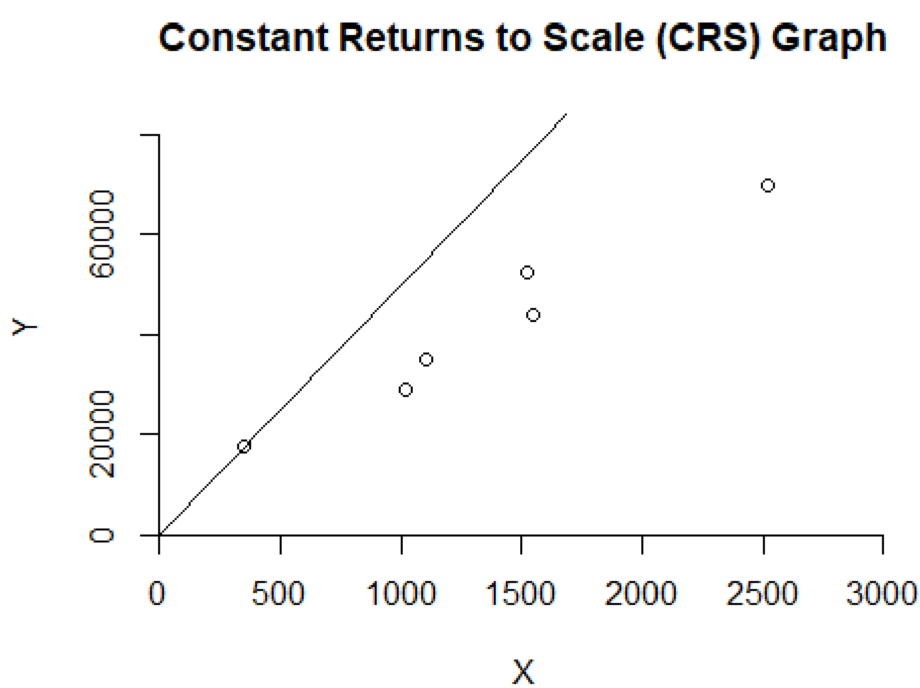
```
dea.plot(input,output,RTS="drs", main="Decreasing Returns to Scale (DRS)
Graph")
```



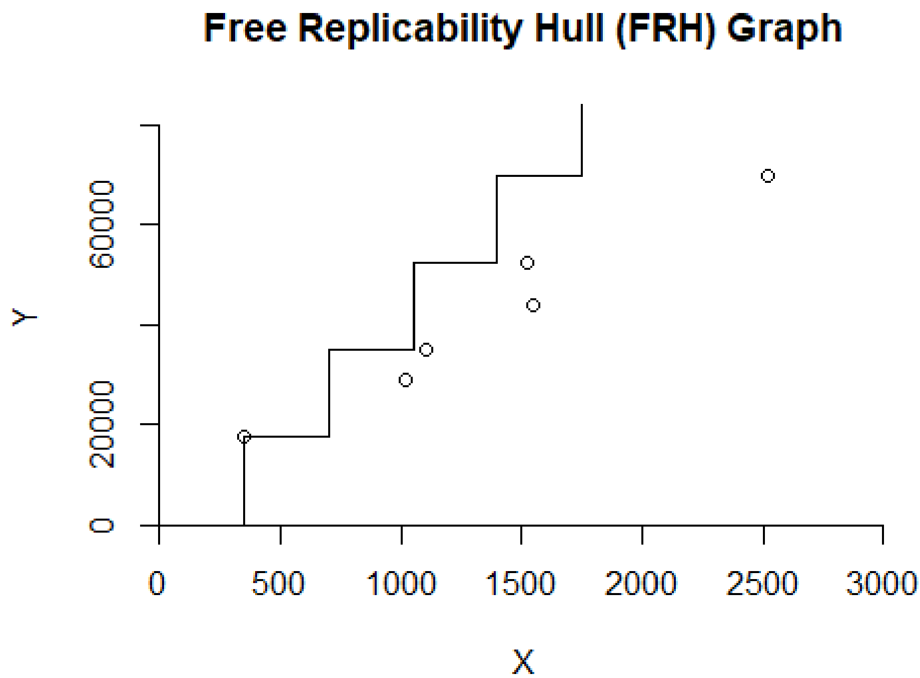

```
dea.plot(input,output,RTS="irs",  main="Increasing  Returns  to  Scale  (IRS)
Graph")
```




```
dea.plot(input,output,RTS="crs", main="Constant Returns to Scale (CRS)
Graph")
```



```
dea.plot(input,output,RTS="add", main="Free Replicability Hull (FRH) Graph")
```




These charts allow us to compare the results of each DEA model. “All DEA models share the premise of estimating the technology using a minimum extrapolation technique,” according to what we studied in this session (DEA Slides).

FDH is the smallest technology set, as can be shown. It seeks to create fewer outputs (number of patient days reimbursed by third-party sources and number of patient days reimbursed privately) with more inputs (number of patient days reimbursed by third-party sources and number of patient days reimbursed privately) (staffing labor and the cost of supplies). FDH is the most popular model among businesses, yet it has several flaws owing to its assumptions. As we can see, all of the efficiencies in this model are 1, however it is not as efficient as we thought when compared to other models since we identify areas/units to improve.

VRS is larger than FDH because it “fills-out” the spaces that FDH reduced. Here we can see that unit 6 can improve its efficiency.

As the graphs show, DRS and IRS are bigger than VRS. For smaller input values, DRS seeks to enlarge the set, whereas the IRS tries to raise the technology. Units 5 and 6 might increase their efficiency, according to DRS, and facility 6 could improve as well, according to IRS.

CRS is the largest technology set, allowing us to assess whether there are any conceivable scaling up or down combinations. Units 5 and 6 require improvement based on the efficiency numbers.

The purpose of FRH, which is larger than FDH but less than CRS, is to replace deterministic data with random variables.

## Q 2 - Research and Development Division of Emax Corporation

Q2.The Research and Development Division of the Emax Corporation has developed three new products. A decision now needs to be made on which mix of these products should be produced. Management wants primary consideration given to three factors: total profit, stability in the workforce, and achieving an increase in the company's earnings

next year from the $75 million achieved this year. In particular, using the units given in the following table, they want to

Maximize Z = P - 6C - 3D, where

P = total (discounted) profit over the life of the new products,

C = change (in either direction) in the current level of employment,

D = decrease (if any) in next year's earnings from the current year's level.

The amount of any increase in earnings does not enter into Z, because management is concerned primarily with just achieving some increase to keep the stockholders happy. (It has mixed feelings about a large increase that then would be difficult to surpass in subsequent years.)

The impact of each of the new products (per unit rate of production) on each of these factors is shown in the following table:

| | Unit Contribution | | | | |
|---|---|---|---|---|---|
| | Product: | | | | |
| **Factor** | **1** | **2** | **3** | **Goal** | **Units** |
| Total profit | 20 | 15 | 25 | Maximize | Millions of dollars |
| Employment level | 6 | 4 | 5 | = 50 | Hundreds of employees |
| Earnings next year | 8 | 7 | 5 | ≥ 75 | Millions of dollars |

Objective function:

max: 20 X1 + 15 X2 + 25 X3 - 6 Y1P - 6 Y1M - 3 Y2M S.T : Employment Level

6x1 + 4x2 + 5x3 - (Y1P - Y1M) = 50

Earnings Next Year

8x1 + 7x2 + 5x3 - (Y2P - Y2M) = 75

Nonnegativity contraint

X1, X2, X3 >= 0 Y1P, Y1M, Y2P, Y2M >= 0

```
library(lpSolveAPI)
setwd("C:/Users/pavankumar pendela/Desktop/MSBA/Quantiative management
Dr.Wu/assignment 5")
# Load the data
emax <-  read.lp("emax.lp")
emax

## Model name:
##               X1    X2    X3   Y1P   Y1M   Y2M   Y2P
## Maximize      20    15    25    -6    -6    -3     0
## R1             6     4     5    -1     1     0     0  =  50
## R2             8     7     5     0     0     1    -1  =  75
## Kind         Std   Std   Std   Std   Std   Std   Std
## Type        Real  Real  Real  Real  Real  Real  Real
## Upper        Inf   Inf   Inf   Inf   Inf   Inf   Inf
## Lower          0     0     0     0     0     0     0

solve(emax)

## [1] 0
```

As we can see, the solver is returning 0, indicating that it is finding a solution.

```
get.objective(emax)

## [1] 225
```

We are maximizing profit while reducing other company goals such as manpower and profits in this scenario. The penalty for failing to meet the goals on the objective function is 225.

```
get.variables(emax)

## [1]  0  0 15 25  0  0  0

get.constraints(emax)

## [1] 50 75
```

### We can see that 25000 people were hired. In order to bring Y1P as 0 . We added so,e variable

```
setwd("C:/Users/pavankumar pendela/Desktop/MSBA/Quantiative management
Dr.Wu/assignment 5")
```

## load data

```
S_emax <-  read.lp("second emax.lp")
S_emax

## Model name:
##                X1     X2     X3    Y1P    Y1M    Y2M    Y2P
## Maximize       20     15     25  -2500     -6     -3      0
## R1              6      4      5     -1      1      0      0  =  50
## R2              8      7      5      0      0      1     -1  =  75
## Kind          Std    Std    Std    Std    Std    Std    Std
## Type         Real   Real   Real   Real   Real   Real   Real
## Upper         Inf    Inf    Inf    Inf    Inf    Inf    Inf
## Lower           0      0      0      0      0      0      0
```

### solve lp

```
solve(S_emax)

## [1] 0
```

### Getting Objective Function

```
get.objective(S_emax)

## [1] 208.3333

get.variables(S_emax)

## [1] 0.000000 8.333333 3.333333 0.000000 0.000000 0.000000 0.000000

get.constraints(S_emax)

## [1] 50 75
```

The variables in the objective function were written in this sequence. In our situation, the results are as follows: X1 = 0,X2 = 8.33,X3 = 3.33,Y1P = 0,Y1M = 0,Y2M = 0,Y2P = 0 and it clearly satisfies all the expectations.