Forward and Futures Contracts

Outline

- Forward contracts
    - No-arbitrage argument for the fair forward price
    - Assets producing an income
    - Assets with storage costs
- Futures contracts
    - Stock indices and index futures
    - Commodity futures
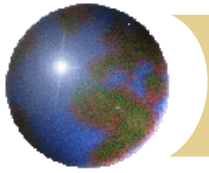   - Interest rates futures: T-bill futures, Eurodollar futures, SOFR futures

Forward contract

- Long forward contract: obligation to pay $F in cash for an asset at time T in the future
- Short forward contract: obligation to deliver an asset at some time in the future in return for $F.
- Asset:
  - Physical: e.g. commodity (oil, gold, etc.)
 - Financial: Bonds, currency, stock or stock index

Forward – spot relationship

- What is the relation between spot price of the asset S and the forward price F(T)?

- F(T) is *not* the asset price at time T! But can be locked in by a risk-free arbitrage strategy.
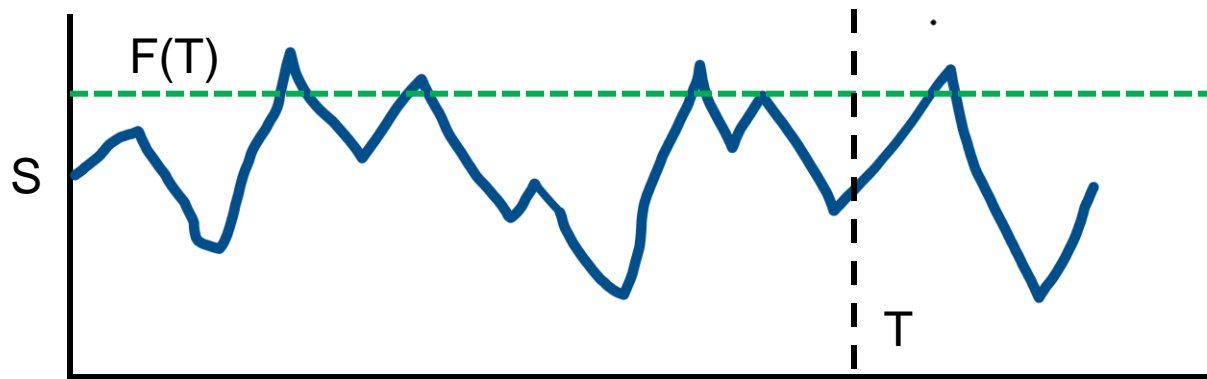
An Arbitrage Opportunity?

- Suppose that:
  - The spot price of a non-dividend-paying stock is $40
  - The 3-month forward price is $43
  - The 3-month US$ interest rate is 5% per annum
- Is there an arbitrage opportunity?
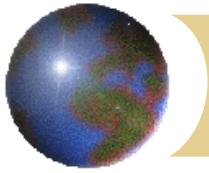
Another Arbitrage Opportunity?

- Suppose that:
  - The spot price of nondividend-paying stock is $40
  - The 3-month forward price is US$39
  - The 1-year US$ interest rate is 5% per annum (continuously compounded)
- Is there an arbitrage opportunity?
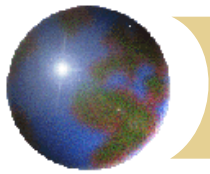
Notation for Valuing Futures and Forward Contracts

S_0: Spot price today

F_0: Futures or forward price today

T: Time until delivery date

r: Risk-free interest rate for maturity T
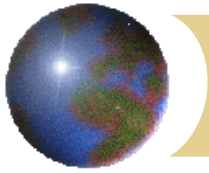
The Forward Price

If the spot price of an investment asset is S_0 and the futures price for a contract deliverable in T years is F_0, then

$$F_0 = S_0 e^{rT}$$

where r is the T-year risk-free rate of interest.

In our examples, S_0 =40, T=0.25, and r=0.05 so that
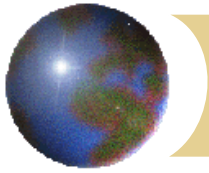
$$F_0 = 40 e^{0.05 \times 0.25} = 40.50$$

When an Investment Asset Provides a Known Income *(page 113, equation 5.2)*

$$F_0 = (S_0 - I)e^{rT}$$

where I is the present value of the income during life of forward contract

When an Investment Asset Provides a Known Yield *(Page 115, equation 5.3)*

$$F_0 = S_0\, e^{(r-q)T}$$

where q is the average yield during the life of the contract

(expressed with continuous compounding)

Valuing a Forward Contract

- The *forward price* of an asset is different from the *value of a forward contract*

- A forward contract is worth zero when it is first negotiated

- Later it may have a positive or negative value

- This is the "value of the forward contract"

Valuing a Forward Contract

(*pages 115-117)*

- Denote K the delivery price and F_0 the forward price for a contract that would be negotiated today

- By considering the difference between a contract with delivery price K and a contract with delivery price F_0 we can deduce that:

  - the value of a long forward contract is
    $$(F_0 - K)e^{-rT}$$

 - the value of a short forward contract is
    $$(K - F_0)e^{-rT}$$

Futures Contracts

- Futures contracts are similar to forward contracts.
- Both are obligations by the long/short side to purchase/delivery an asset at delivery date
- The asset is standardized (quantity and quality) and the contract can be sold to another market participant
- The mechanics of the futures contract involves also a margin and daily mark-to-market process.
- Futures contracts are traded on exchanges.

Forwards vs futures contracts

Forward contract

asset

Payment in the amount predetermined F(T)

Futures contract

asset

Last payment at price S(T)
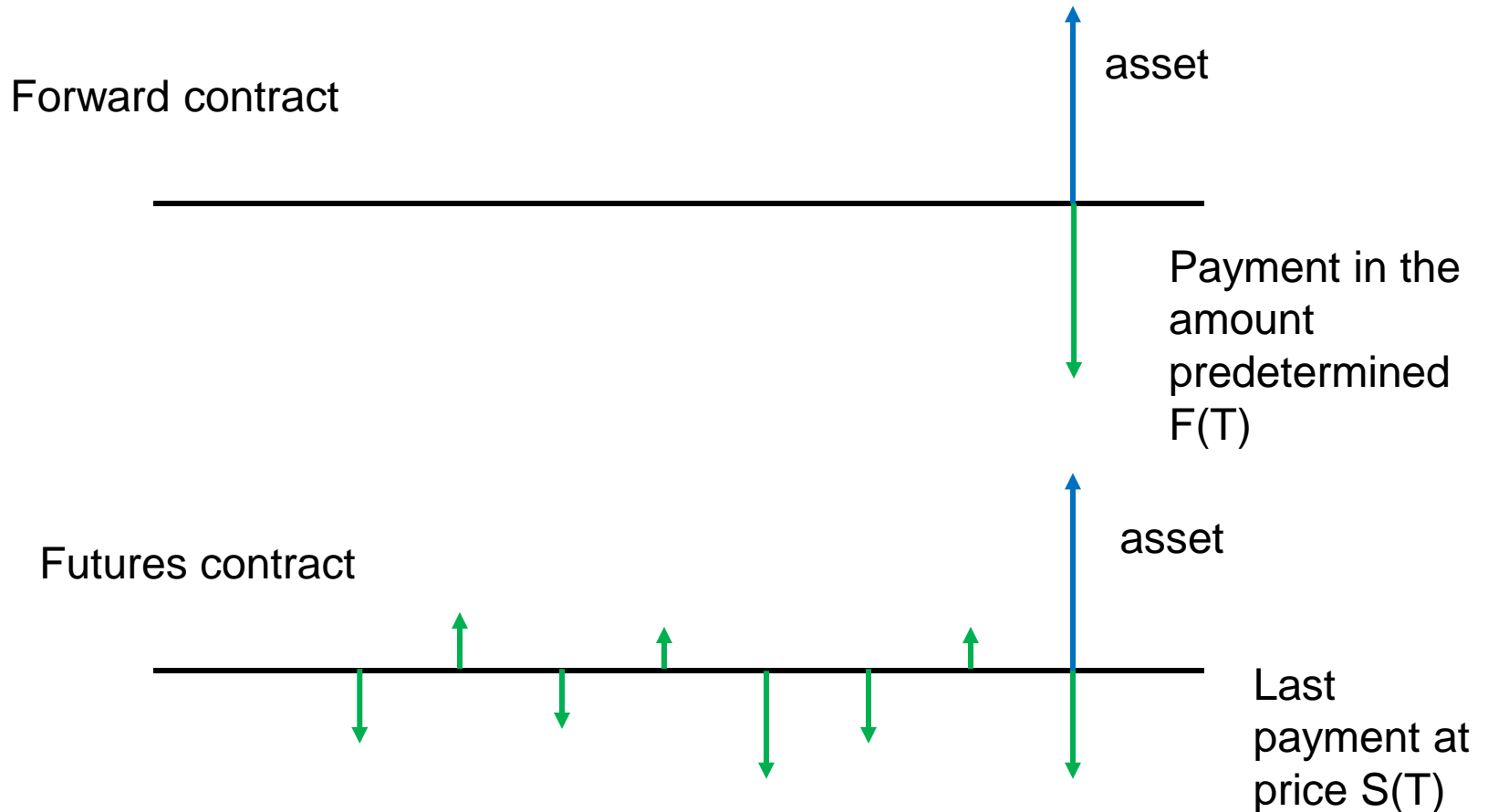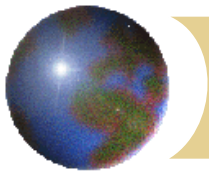
Mechanics of a futures contract

The asset buyer enters into a corn futures contract for delivery at T for $100/bushel. Both sides are required to post $20 margin (which will be returned at the end)
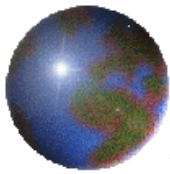
	Farmer (short side)		Asset buyer (long side)
Time	Margin	Settlement price	Margin
0	$20	$100	$20
1	$30	$90	$10
2	$25	$95	$15

Delivers corn. Receives $95 (+ $5 margin profit) = $100

Pays $95 (+ $5 margin loss) = $100. Receive corn.

Futures contracts traded

- Equity index futures: SP500, Nasdaq-100, …
- Interest rates futures: T-notes, Eurodollar futures, 30day Fed Funds futures, SOFR futures
- FX futures
- Commodity futures:
  - Energy (crude oil, power, natural gas)
  - Agricultural (corn, wheat, …)
  - Metals

https://www.cmegroup.com

CRUDE OIL FUTURES - QUOTES

| MONTH	OPTIONS	CHART	LAST	CHANGE	PRIOR SETTLE
NOV 2024 CLX4	OPT		69.50	-2.06 (-2.88%)	71.56
DEC 2024 CLZ4	OPT		68.92	-1.92 (-2.71%)	70.84
JAN 2025 CLF5	OPT		68.56	-1.77 (-2.52%)	70.33
FEB 2025 CLG5	OPT		68.30	-1.66 (-2.37%)	69.96
MAR 2025 CLH5	OPT		68.09	-1.59 (-2.28%)	69.68

17

The futures curve

- What determines the shape of the futures curve?

- Two types of curves:
    - Backwardation (down-sloping). Observed in commodities when there is a shortage.
    - Contango (up-sloping).

- Another factor is whether the asset is meant for consumption or is an investment asset

Futures curve (WTI – Feb.23)
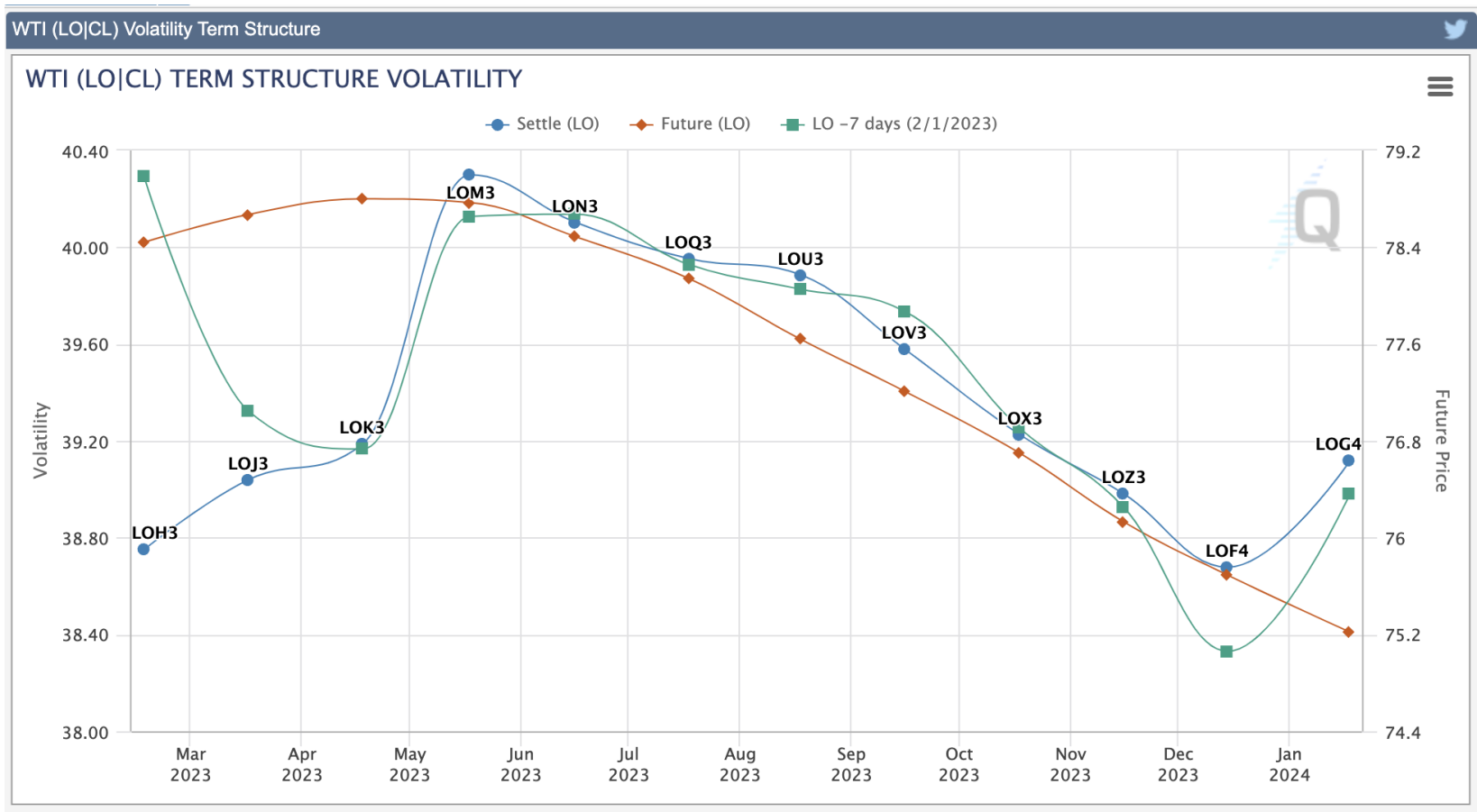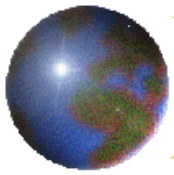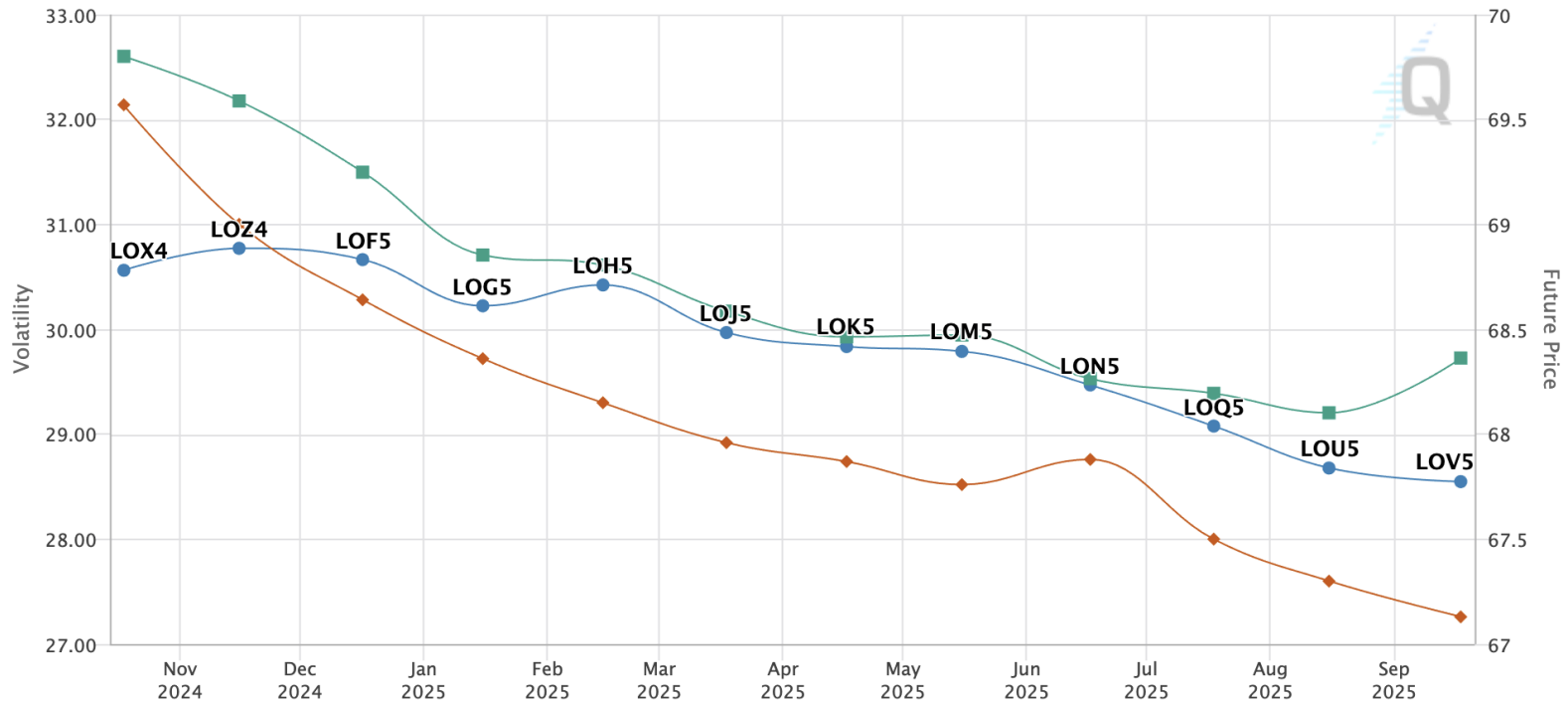
Futures curve (WTI – Sep.24)

Relation of futures and spot price

- One can show that the futures prices are equal to the forward prices, provided that interest rates are deterministic (do not have any randomness)
- Thus we can use the no-arbitrage arguments for forward contracts to price also futures contracts.
- The exception are futures on interest rates. When interest rates are uncertain, they are slightly different:
  - A strong positive correlation between interest rates and the asset price implies the futures price is slightly higher than the forward price
  - A strong negative correlation implies the reverse

Consumption vs Investment Assets

- <u>Investment assets</u> are assets held by significant numbers of people purely for investment purposes
  - E.g. stocks, bonds, gold
- <u>Consumption assets</u> are assets held primarily for consumption
  - E.g. copper, oil, corn

Stock Indices

- Stock index: tracks changes in the value of a basket of stocks.

- Examples:

  - Dow Jones Industrial Average (DJI). 30 blue-chip US stocks

  - Standard and Poor's 500. Portfolio of 500 stocks: 400 industrials, 40 utilities, 20 transportation and 40 financials

  - Nasdaq-100. 100 stocks on Nasdaq.

Stock Index Futures

- Futures are traded on exchanges which deliver an amount linked to the price of the index at maturity (Cash settled)

- SP500 Index Futures ($250x, $50 x index)
- Nasdaq-100 Futures ($100x, $20 x index)
- Russell 2000 Index Futures

https://www.cmegroup.com/trading/equity-index/

Stock Index *(Page 118-120)*

- Stocks pay dividends. The stock index can be viewed as an investment asset paying a dividend yield
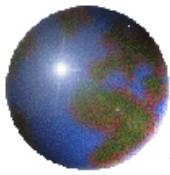
- The futures price and spot price relationship is therefore

$$F_0 = S_0 \, e^{(r-q)T}$$

where q is the average dividend yield on the portfolio represented by the index during life of contract
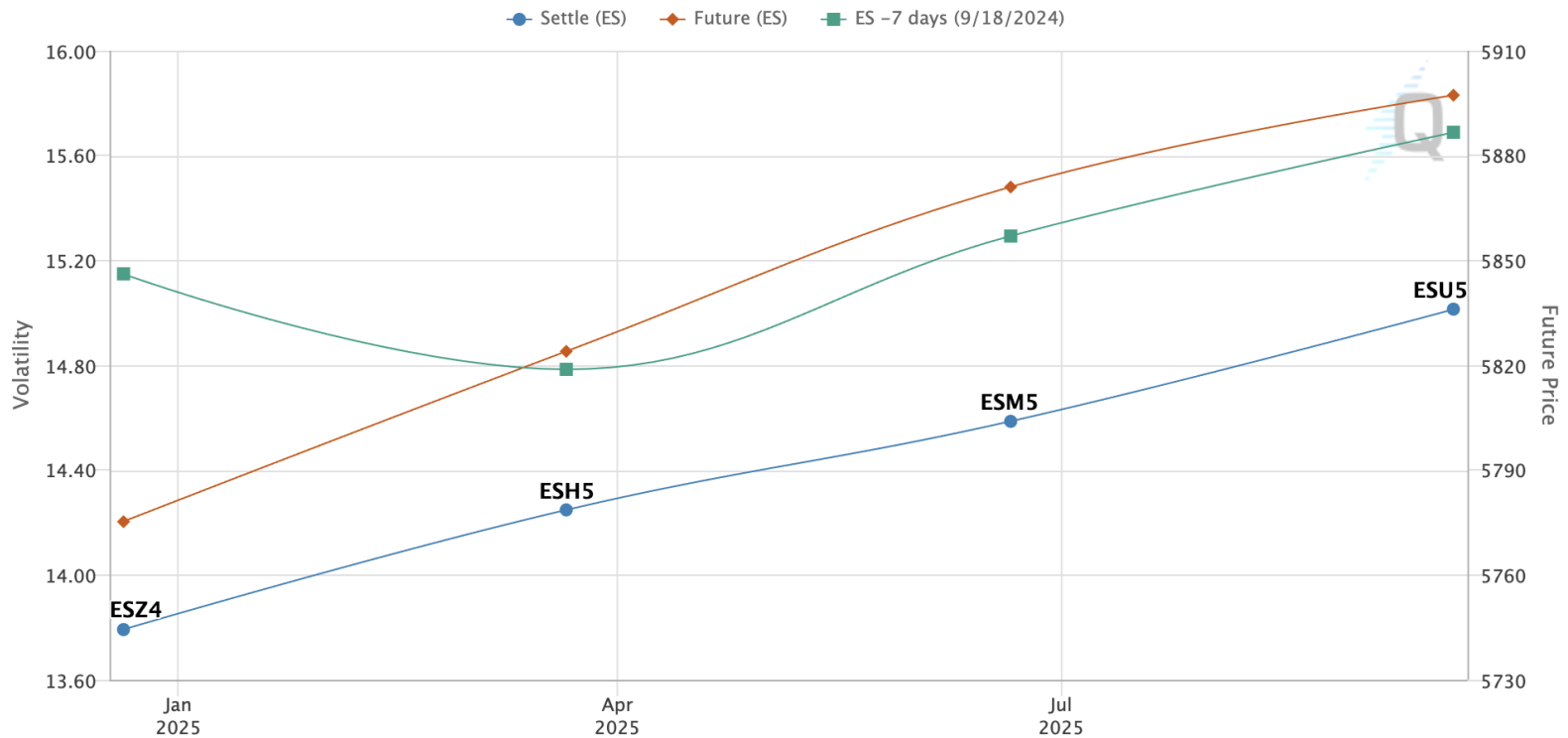
Stock Index *(continued)*

⬥ For the formula to be true it is important that the index represent an investment asset

⬥ In other words, changes in the index must correspond to changes in the value of a tradable portfolio

⬥ The Nikkei index viewed as a dollar number does not represent an investment asset (See Business Snapshot 5.3, page 119)

The futures curve for SPX futures

Futures and Forwards on Currencies
(Page 120-123)

- ◈ A foreign currency is analogous to a security providing a yield
- ◈ The yield is the foreign risk-free interest rate
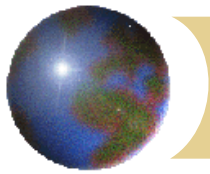
Forwards on foreign currencies

- Denote S_0 the price in USD of one unit of foreign currency, and F_0 the forward price of the same quantity for delivery at time T
- Note that S_0 is just the spot FX rate
- Denoting r_f the risk-free interest rate in the foreign currency we have

$$F_0 = S_0 e^{(r - r_f)T}$$

Consumption Assets: Storage is Negative Income

$$F_0 \leq S_0\, e^{(r+u)T}$$

where u is the storage cost per unit time as a percent of the asset value.

Alternatively,

$$F_0 \leq (S_0 + U)e^{rT}$$

where U is the present value of the storage costs.

Interest rates futures

In the US, the main interest rates futures are:

- Treasury bond futures
- Fed Fund Futures
- SOFR Futures

Treasury Bond Futures

Pages 138-143

A Treasury Bond futures contract delivers at maturity:

- Treasury Bond (physical settlement)
- The value of the bond (cash settlement)

| GRADE AND QUALITY | U.S. Treasury bonds that have remaining term to maturity of at least 15 years and less than 25 years from the first day of the futures delivery month.* The delivery invoice amount equals the futures settlement price times a conversion factor, plus accrued interest. The conversion factor is the price of the delivered bond ($1 par value) to yield 6 percent.

*For more information, please refer to T-Bond Basket Gap Decision. |

Treasury Bond Futures - Quotes

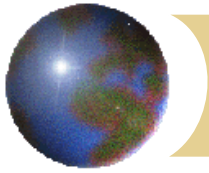## U.S. TREASURY BOND FUTURES - QUOTES

AUTO-REFRESH IS OFF   *Last Updated 08 Feb 2023 04:37:27 PM CT.   Market data is delayed by at least 10 minutes.*

| MONTH	OPTIONS	CHART	LAST	CHANGE	PRIOR SETTLE
MAR 2023 ZBH3	OPT	..ıl	128'28	+0'15 (+0.37%)	128'13
JUN 2023 ZBM3	OPT	..ıl	130'01	+0'05 (+0.12%)	129'28
SEP 2023 ZBU3	OPT	..ıl	130'25	UNCH (UNCH)	130'25

Eurodollar Futures *(Page 143-148)*

- Eurodollar futures are (were) futures on 3-month LIBOR
- One contract is on the rate earned on $1 million
- A change of one basis point or 0.01 in a Eurodollar futures quote corresponds to a contract price change of $25
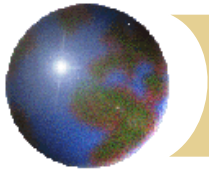
Fed Fund Futures

- The 30 Day Fed Fund Futures are cash settled and pay an amount proportional to 100 - (arithmetic average of the daily effective Fed Funds Rates during the contract month)

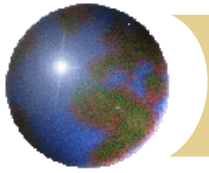- Provide a gauge of market expectations about the Fed's action at future FOMC meetings
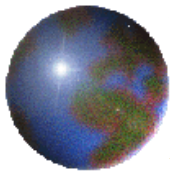
Fed Funds Futures
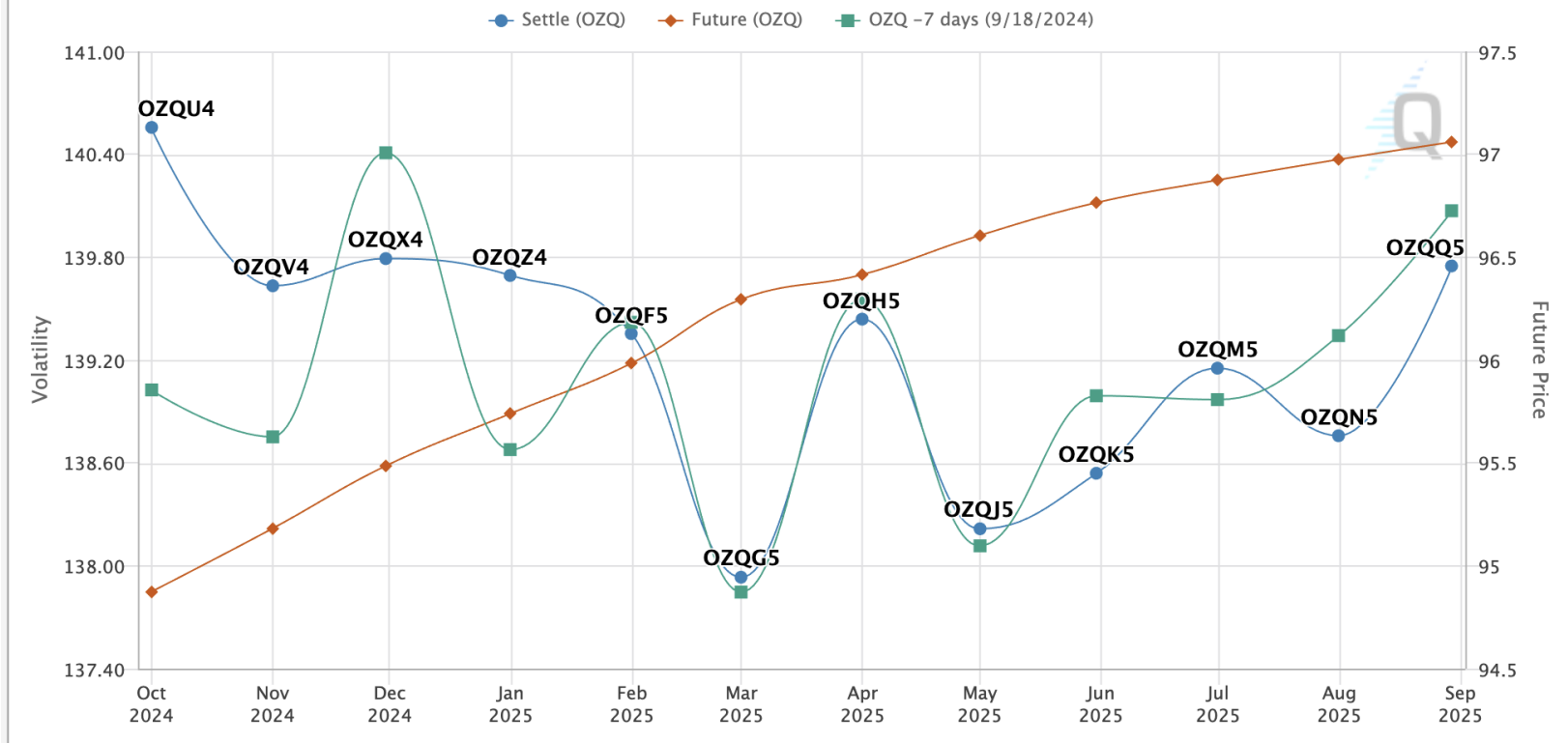
SOFR Futures

- The one-month SOFR futures is designed to be as similar as possible to the one-month Fed Fund futures contract and is based on an arithmetic average of overnight rates

- The three-month SOFR futures is designed to be as similar as possible to the three-month Eurodollar futures and is based on the result of compounding overnight rates
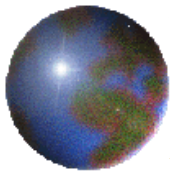
ONE-MONTH SOFR FUTURES - QUOTES

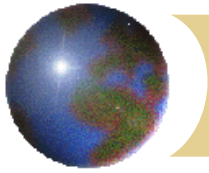AUTO-REFRESH IS OFF   *Last Updated 25 Sep 2024 12:53:29 PM CT.   Market data is delayed by at least 10 minutes.*

MONTH	OPTIONS	CHART	LAST	CHANGE	PRIOR SETTLE
SEP 2024 SR1U4	OPT	📊	94.8575	−0.0025 (UNCH)	94.86
OCT 2024 SR1V4	OPT	📊	95.165	+0.01 (+0.01%)	95.155
NOV 2024 SR1X4	OPT	📊	95.455	+0.005 (+0.01%)	95.45
DEC 2024 SR1Z4	OPT	📊	95.69	+0.005 (+0.01%)	95.685

Using SOFR for Hedging

- A company has agreed to pay three month SOFR plus 200 basis points on $100 million for three months starting on December 16, 2021

- The December SOFR futures price is 99.990

- What rate can the company lock in?

Uses of Eurodollar/SOFR futures

- The Eurodollar futures are popular hedging instruments for interest rates risk

- Their quoted prices are also used to construct the Libor curve (the curve of forward Libor rates), along with:

  - Deposit rates (Libor rates) – short maturities
  - Eurodollar futures – middle maturities
  - Swap rates – long maturities

- They will be replaced by SOFR futures for these purposes